Progress in lattice QCD in studying CPV and mixing in charm

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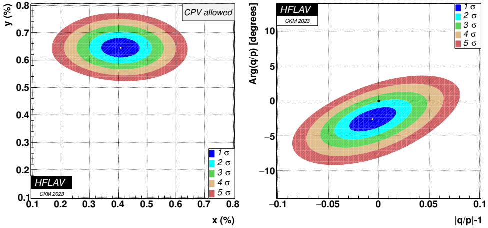
Implications of LHCb measurements, CERN

23 October 2024

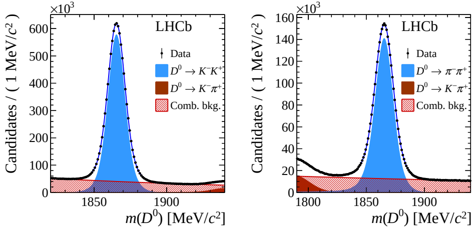
$D^0 - \bar{D}^0$ mixing



[HFLAV, PRD 23]

- no-mixing point $x = y = 0$ excluded at $> 11.5\sigma$
- no evidence for indirect CP violation $|q/p| \neq 1$ or $\phi = \arg(q/p) \neq 0$

hadronic D decays

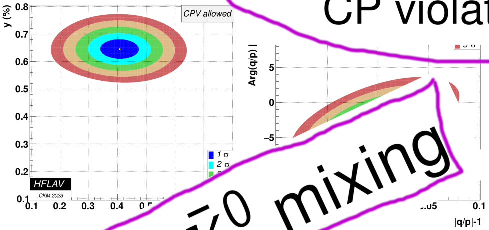


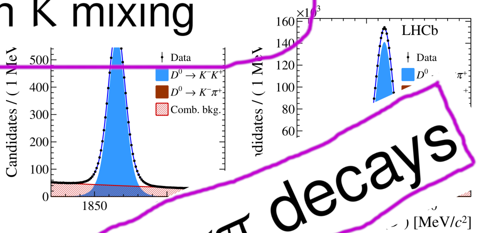
[LHCb, PRL 19]

- $\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-15.4 \pm 2.9) \times 10^{-4}$

First, what lattice can do today: CP violation in K mixing

$D^0 - \bar{D}^0$ ... decays





$K^0 - \bar{K}^0$ mixing

$K \to \pi\pi$ decays

- ... $x = y = 0$ excluded at ...
- no evidence for indirect CP violation $|q/p| \neq 1$ or $\phi = \arg(q/p) \neq 0$
- ... $= A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-15.4 \pm 2.9) \times 10^{-4}$

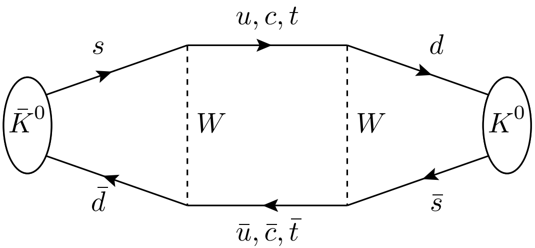
Neutral Meson Mixing

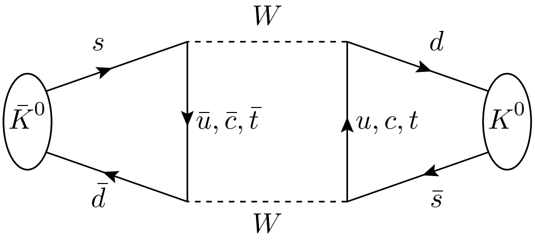
Warm-up exercise, CP violation in K mixing

$$|K_L\rangle \approx \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$
$$|K_S\rangle \approx \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

with a long-lived $|K_L\rangle$ and a short-lived $|K_S\rangle$.
Indirect CP violation parameter ϵ_K can be parameterized by mass and widths splittings

$$\Delta M_K = M_{K_L} - M_{K_S}, \qquad \Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$$
$$\phi_\epsilon = \frac{\Delta M_K}{\Delta\Gamma_K/2}$$
$$\epsilon_K = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left(\frac{-\mathrm{Im}M_{\bar{0}0}}{\Delta M_K} + \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0}\right)$$





K MESON MIXING

$$\epsilon_K = e^{i\phi_\epsilon} \sin(\phi_\epsilon) \left(\frac{-\mathrm{Im} M_{12}}{\Delta M_K} + \frac{\mathrm{Re} A_0}{\mathrm{Im} A_0} \right)$$

A_0 is the $K \to (\pi\pi)_{I=0}$ decay amplitude
M_{12} splits into

$$\begin{aligned} M_{12} = \langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle &= \langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{SD} + \langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{LD} \\ &= \langle K^0 | \mathcal{H}_W^{\Delta S=2} | \bar{K}^0 \rangle + \sum_n \frac{\langle K^0 | \mathcal{H}_W^{\Delta S=1} | n \rangle \langle n | \mathcal{H}_W^{\Delta S=1} | \bar{K}^0 \rangle}{M_K - E_n} \end{aligned}$$

On the lattice, we can compute both:

- $\langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{SD}$ [Kaon mixing beyond the standard model with physical masses; FE et al., PRD 24]
- $\langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{LD}$ [Long-distance contribution to ϵ_K from lattice QCD; Bai et al., PRD 24]

CHALLENGES IN COMPUTING ϵ_K

extracting the $K - \bar{K}$ mixing amplitude from finite-volume correlators [Christ et al., PRD 13]

- closest Euclidean correlation function: integrated 4pt correlator $\int dt_1 dt_2 \langle 0|T[\bar{K}^0(t_f)H_W(t_2)H_W(t_1)\bar{K}^0(t_i)]|0\rangle$
- on-shell intermediate states $|n\rangle\langle n|$ between H_W complicate calculation:

growing exponentials

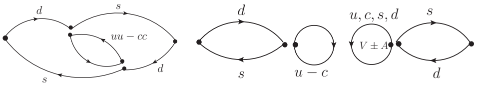
- FV states E_n with mass $M_n < M_K$ lead to unphysical growing exponentials
- these must be removed explicity and then added back in later

finite-volume effects

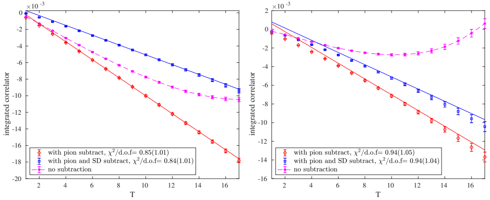
- consequently, FV estimator has poles at removed energies
- power-like volume effects are understood and described by $K \to \pi\pi$ and $\pi\pi \to \pi\pi$ scattering amplitudes

$\Rightarrow$ Precise knowledge of **excited-state spectrum** needed to extract long-distance amplitude from Euclidean finite-volume correlators

- RBC/UKQCD Domain-Wall Fermion ensembles
- one coarse lattice spacing $a^{-1} = 1.78$ GeV
- 2 pion masses 339 MeV and 592 MeV
- non-perturbative renormalization
- result: $\epsilon_K^{LD} = 0.195(77)e^{i\phi_\epsilon} \times 10^{-3}$
- comparison: $\epsilon_K^{SD} = 1.360(154)e^{i\phi_\epsilon} \times 10^{-3}$
- smaller than experimental value: $|\epsilon_K| = 2.228(11) \times 10^{-3}$
- discrepancy not understood, but $|V_{cb}|$ contributes to ϵ_K determination, present uncertainty in incl. vs excl.



a selection of topologies to be computed



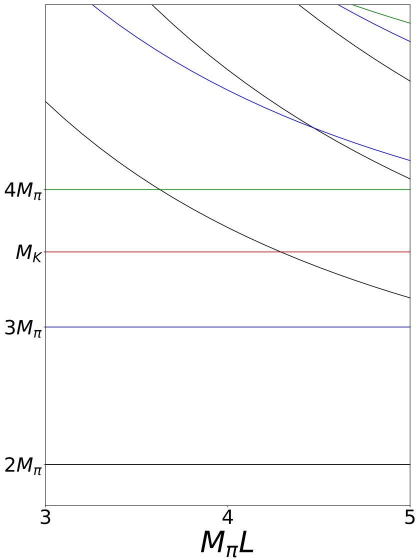
integrated 4pt-correlator, with subtractions

Calculation at physical pion mass underway, **progress report at this year's lattice conference** [Yikai Huo, Lattice 24]

$K - \bar{K}$ MIXING

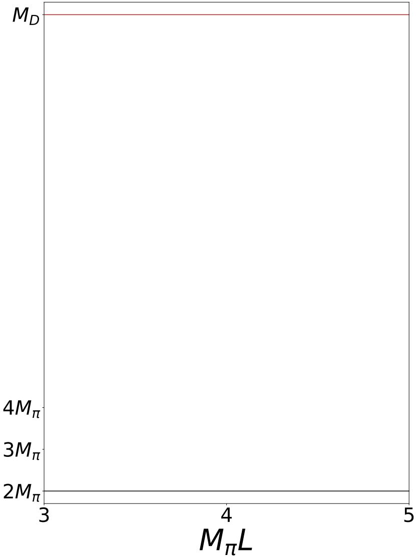
- Kaon decay spectrum on lattices $M_\pi L \sim 4$
- removal of $2-3$ states $\rightarrow$ conceptually clear
- 3-pion state kinematically suppressed, not removed in RBC/UKQCD work
- formalism for explicit removal known

[Jackura, Briceño, Hansen; PoS Lattice22]

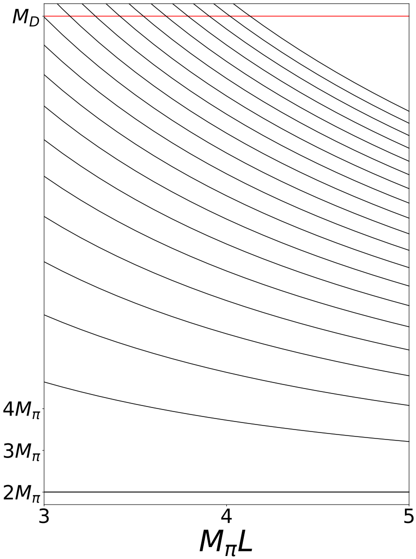


$D - \bar{D}$ MIXING

- D-meson decay spectrum on lattices $M_\pi L \sim 4$



- D-meson decay spectrum on lattices $\mathrm{M}_\pi \mathrm{L} \sim 4$
- 17 interacting states below M_D at $\mathrm{M}_\pi \mathrm{L} \sim 4$



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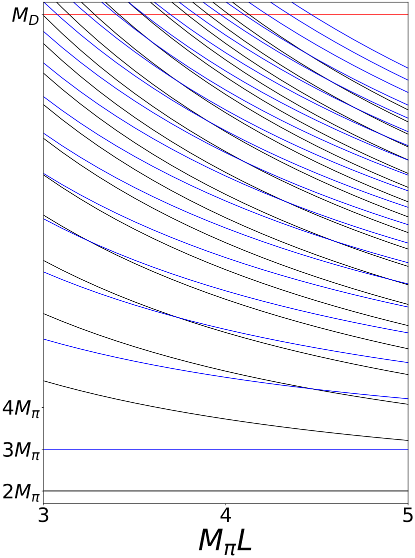
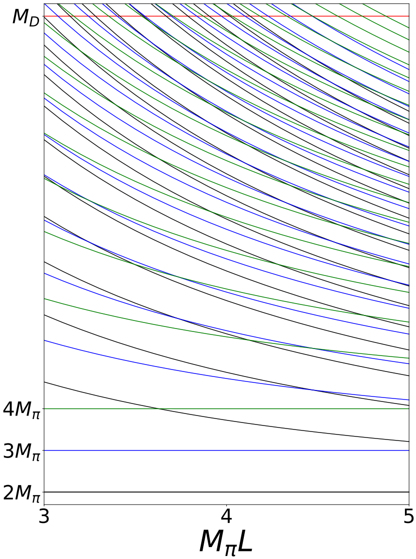


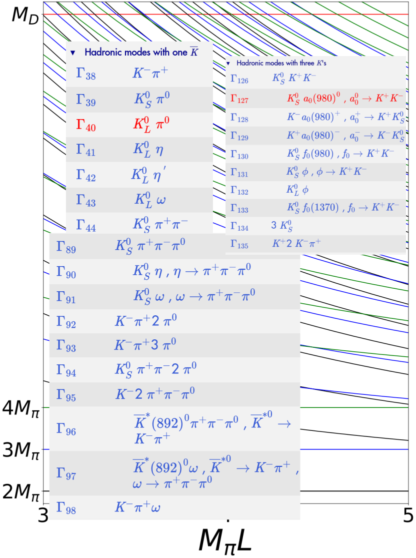
$D - \bar{D}$ MIXING

- D-meson decay spectrum on lattices $M_\pi L \sim 4$
- 17 interacting states below M_D at $M_\pi L \sim 4$
- 3π states $\rightarrow$ conceptually clear
- 4π states $\rightarrow$ no formalism yet



$D - \bar{D}$ MIXING

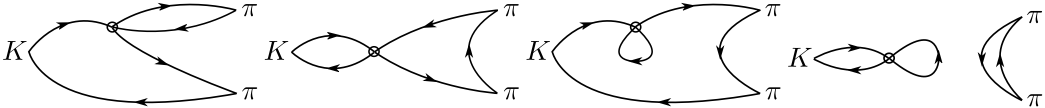
- D-meson decay spectrum on lattices $M_\pi L \sim 4$
- 17 interacting states below M_D at $M_\pi L \sim 4$
- 3π states → conceptually clear
- 4π states → no formalism yet
- $K\pi$, $K3\pi$, $3K$, 6π, . . . → 🤯



$\mathrm{D}-\bar{\mathrm{D}}$ MIXING

- no hope to extract full D-meson decay spectrum from lattice QCD with current formalisms and techniques
- exciting recent developments in **spectral-function methods** in lattice QCD
  - $\mathrm{O}(3)$ non-linear σ model [Bulava et al.; JHEP 22]
  - R-Ratio [Alexandrou et al.; PRL 23], can help constrain $(g-2)_{\mu}^{\mathrm{HVP}}$
  - inclusive decays [Hansen et al.; PRD 17] [Gambino, Hashimoto; PRL 21] [Barone, Lattice@CERN 24] [De Santis, Lattice24] [Groß, Lattice24] [Kellermann, Lattice24]
- **similar methods would apply here**, but formalisms do not exist as of yet
- a whole week was dedicated to these problems at our Lattice Theory Institute this year [Lattice@CERN 24, week 1]

Hadronic Decays



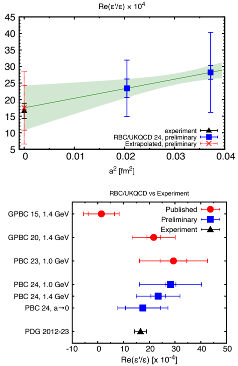
- One project where DWF at M_π^{phys} led to success: K $\to \pi\pi$
- Domain-Wall Fermions lead to **clean 4-quark-operator renormalization**
- long-standing puzzle $\text{Re}(A_0)/\text{Re}(A_2) = 22.45$ - factor 10 larger than perturbatively
- cancellation at M_π^{phys} between Wick contractions of $\text{Re}(A_2)$, **very sensitive to** m_l

$\Rightarrow$ $\text{Re}(A_0)/\text{Re}(A_2) = 19.9(2.3)(4.4)$

- Progress on direct Kaon CPV parameter ϵ'
  - second lattice spacing added to existing approach (G-parity boundary)

    [Kelly; Lattice24]
  - Periodic boundary conditions, excited state from variational techniques

    [Tomii; Lattice24]



Towards hadronic D decays on the lattice

- first exploratory calculation: $D \to K\pi$ at $SU(3)_F$ symmetric point

  [F Joswig, FE, MT Hansen, N Lachini, A Portelli; PoS Lattice22] [MT Hansen, Lattice23]

- Need to compute the $D \to K\pi$ amplitude:

$$A = C^{LL} Z^{\overline{MS}} \langle n, L | \mathcal{H}_W | D, L \rangle$$

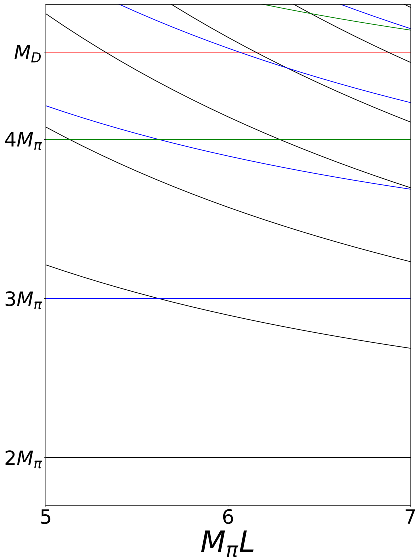
which includes the challenges:

- Lellouch-Lüscher factor C^{LL}
- Renormalization $Z^{\overline{MS}}$
- Finite-volume state $\langle n, L |$
- 3-point matrix element $\langle n, L | \mathcal{H}_W | D, L \rangle$

FINITE-VOLUME SPECTRUM AT $SU(3)_F$ SYMMETRIC POINT

	$T \times L^3$	a[fm]	$M_\pi L$
a12m400	96×24^3	0.12	5.988(28)
a094m400	96×32^3	0.094	6.201(19)
a064m400	96×48^3	0.064	6.383(14)

- ensembles by the OpenLat initiative [OpenLat]
- D-meson decay spectrum at $SU(3)_F$ symmetric point, $M_\pi = 410$ MeV
- even around $M_\pi L \sim 6$ spectrum seems relatively manageable
- $M_K = M_\pi \Rightarrow$ no further states present
- $4M_\pi \sim M_D \Rightarrow$ Lellouch-Lüscher factor C^{LL} is known [Hansen, Sharpe; PRD 12]
    - simple form for 2π spectrum factor
    - including heavily suppressed 3π contributions is conceptually possible



FURTHER CHALLENGES

Renormalization:

- working on non-perturbative renormalization [Martinelli et al., 1995]
- fix renormalization conditions via tree-level matrix elements

$$Z_\Gamma \langle p|O_\Gamma|p\rangle\Big|_{p^2=-\mu^2} = \langle p|O_\Gamma|p\rangle\Big|_{tree}$$

- four distinct flavours in $D \to K\pi$ avoid power-divergent mixing

$$Q = (\bar{d}u)_{V-A}(\bar{c}s)_{V-A}$$

finite-volume state:

Distillation technique allows large basis of two-hadron operators

$$K(\mathbf{p}_1)\pi(\mathbf{p}_1') = |K\pi\rangle_1$$

$$\ldots$$

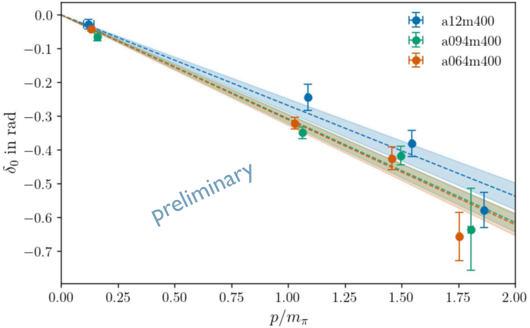
$$K(\mathbf{p}_m)\pi(\mathbf{p}_m') = |K\pi\rangle_m$$

$\Rightarrow$ Generalized Eigenvalue Problem technique allows spectrum extraction

$$|n, L\rangle = v_1^n|K\pi\rangle_1 + \cdots + v_m^n|K\pi\rangle_m$$

Status of the calculation

- S-wave phase shift δ_0 consistent over all ensembles
- $SU(3)_F$ symmetric point renders otherwise inachievable computation possible
- code & strategy for most complex contractions have been finalized

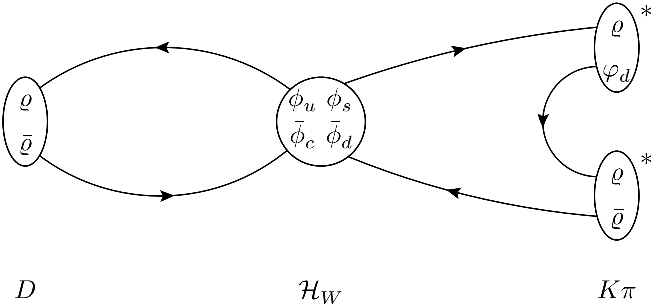


rest-frame phase shift δ_0 on the 3 ensembles

3-point matrix element:

- Incorporating the local 4-quark operator for $\mathcal{H}_W$ into the distillation framework is challenging but straightforward
- The four distinct flavours in $\mathrm{Q} = (\bar{\mathrm{d}}\mathrm{u})_{\mathrm{V-A}}(\bar{\mathrm{c}}s)_{\mathrm{V-A}}$ significantly simplifies the challenge
- The code for computing $\mathrm{D} \to \mathrm{K}\pi$ matrix elements has been developed already

$\Rightarrow$ Reduced number of Wick-contractions



Conclusions

$D - \bar{D}$ mixing

- formalism and computations exist for $K - \bar{K}$ mixing
- full excited-state spectrum inachievable for D case
- spectral reconstruction methods would be a more feasible approach

$\Rightarrow$ fast progress is made, but no formalism exists as of yet

hadronic D decays

- formalism and computations exist for $K \to \pi\pi$
- full excited-state spectrum also inachievable
- exploratory calculation studies $D \to K\pi$ decays at $SU(3)_F$ symmetric point
- symmetries and high pion masses render computation achievable

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