Inclusive $B_s^0 \to X_c \ell \nu$ measurements at LHCb

Exclusively for you

Michel De Cian, October 23, 2024

Based on M. De Cian, N. Feliks, M. Rotondo, K. Vos, JHEP 06 (2024) 158

The CKM triangle

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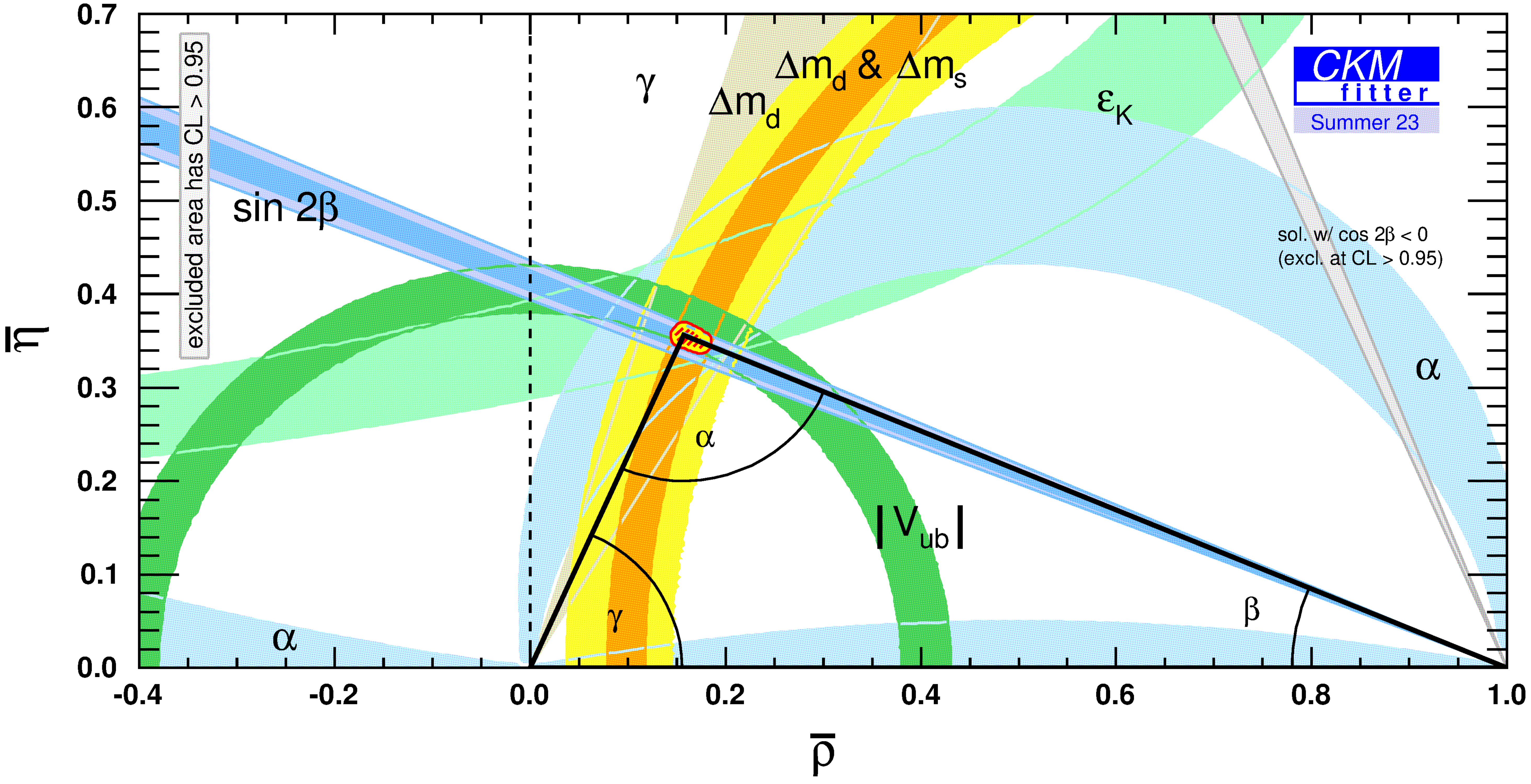
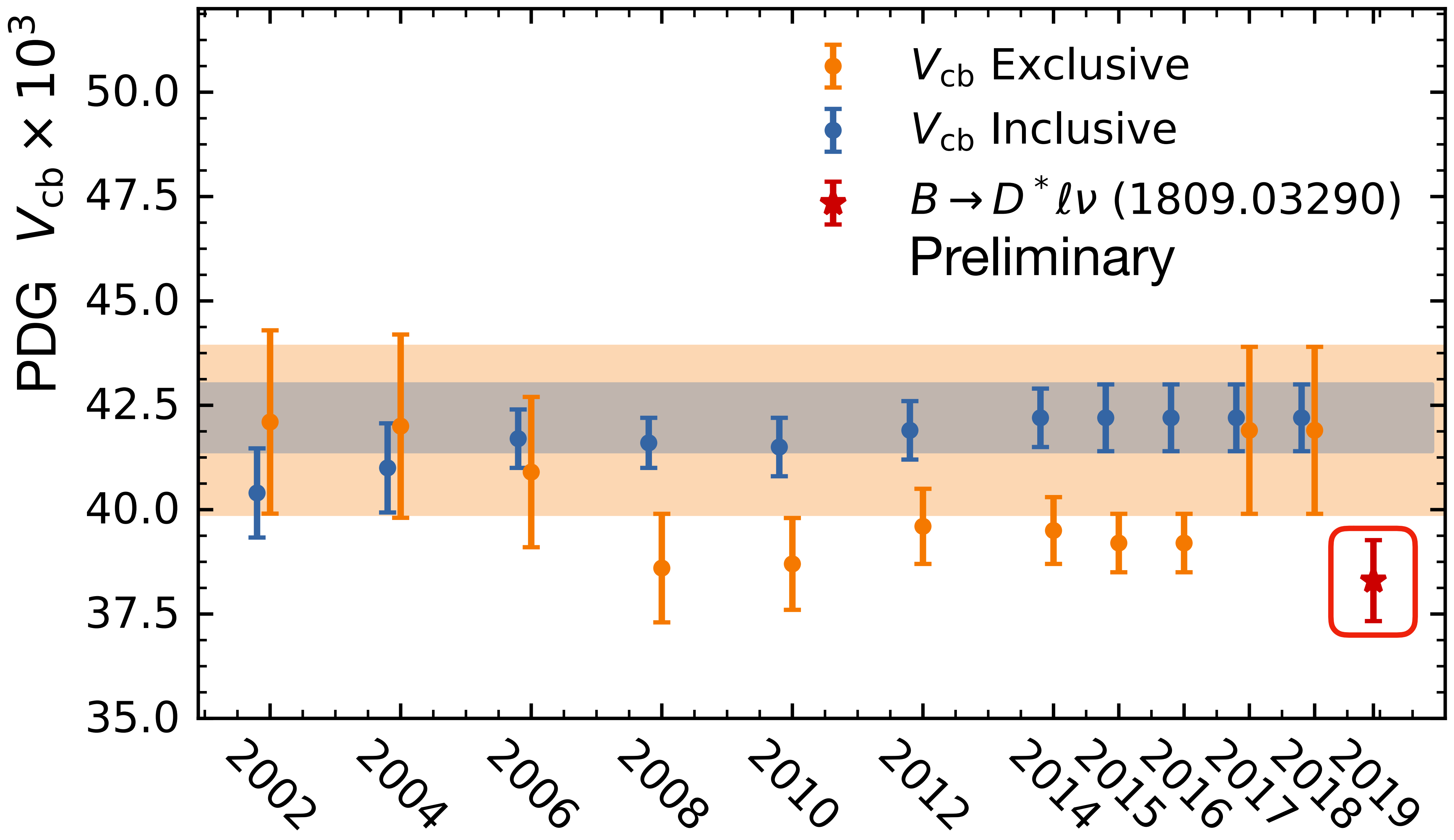


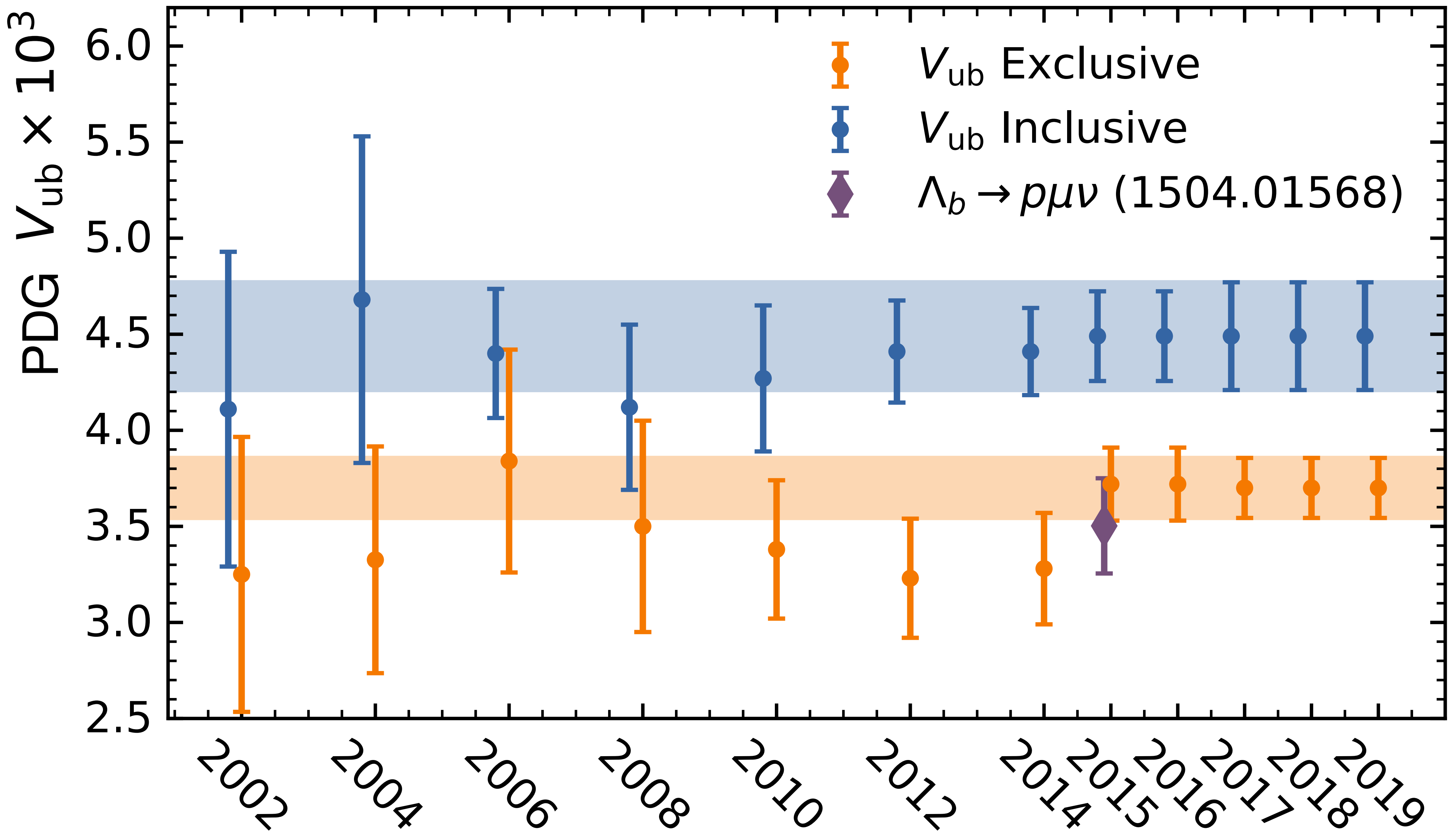
- V_{cb} and V_{ub} determine height of CKM triangle
- Consistent discrepancy in different measurement methods for V_{ub} and V_{cb} in the last 15+ years

Inclusive vs Exclusive

A long-standing puzzle

- Inclusive decays consider all $B \to X_{c/u}\,\mu\nu$ decays
- Exclusive decays consider one specific $B \to X_{c/u}\mu\nu$ decay, e.g. $B \to D^{*}\mu\nu$
- Discrepancy is not just an „aesthetic problem“, it limits the precision of e.g. the prediction on $\varepsilon_K \sim V_{cb}^4$



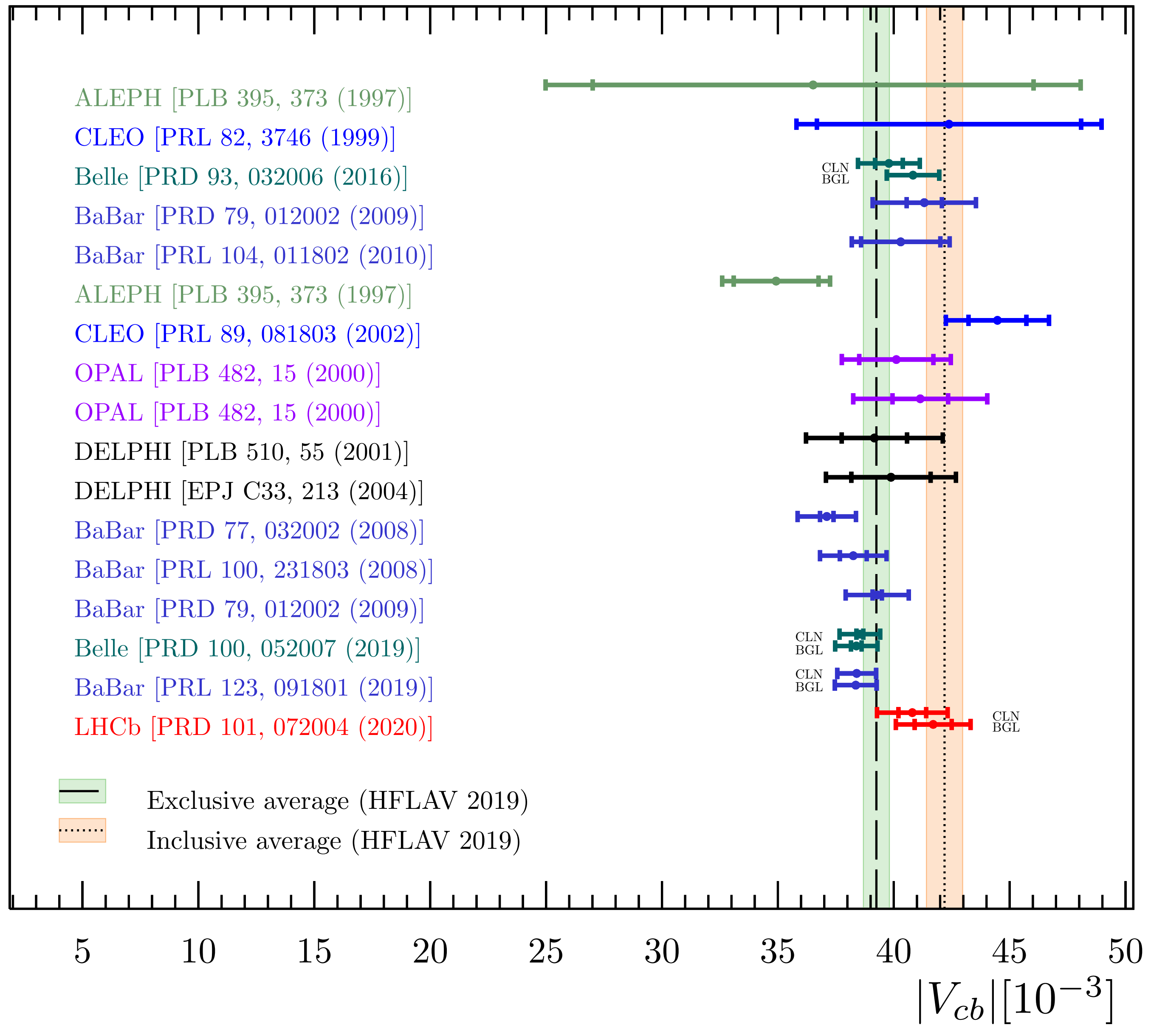


Inclusive vs Exclusive

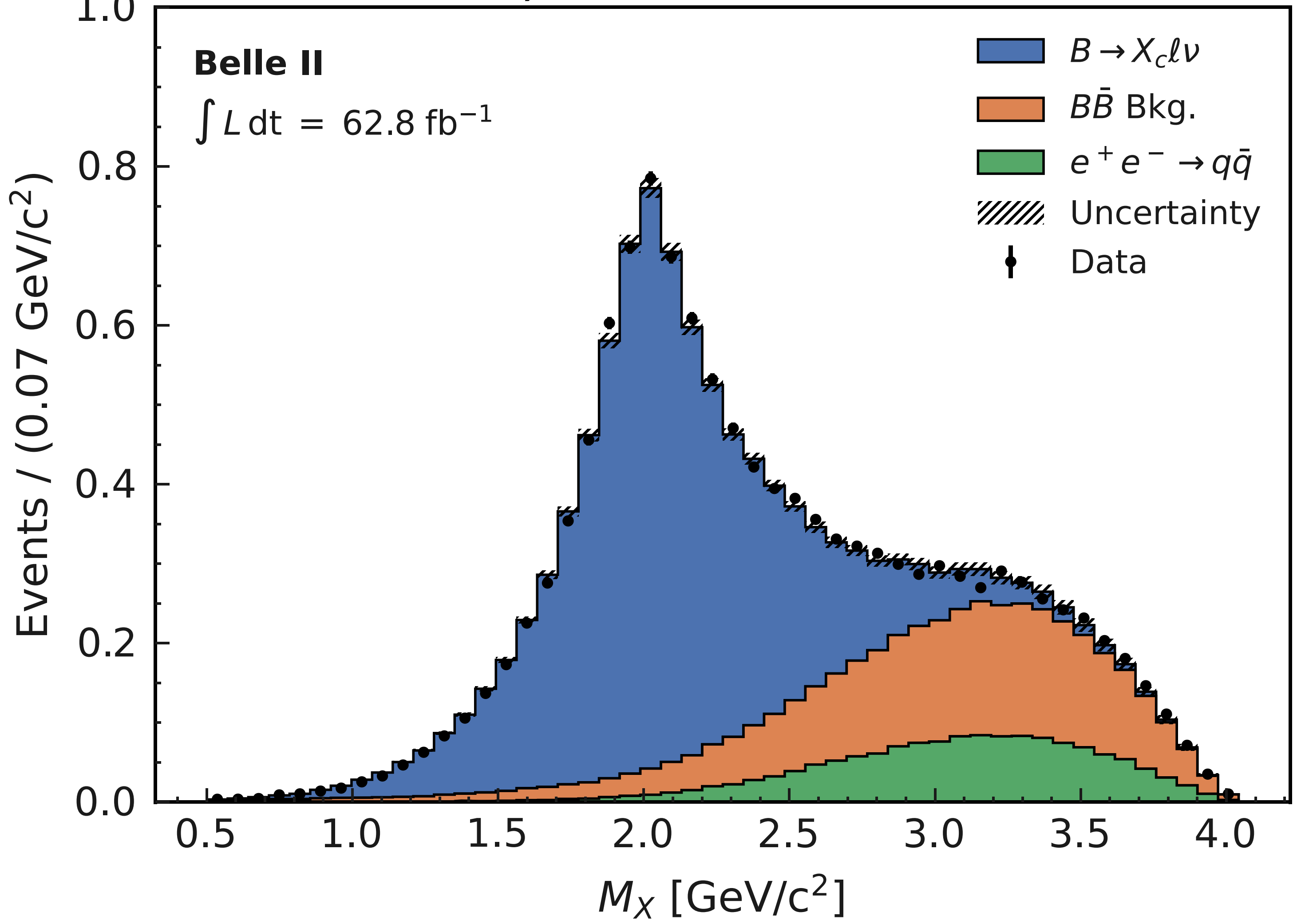
A long-standing puzzle

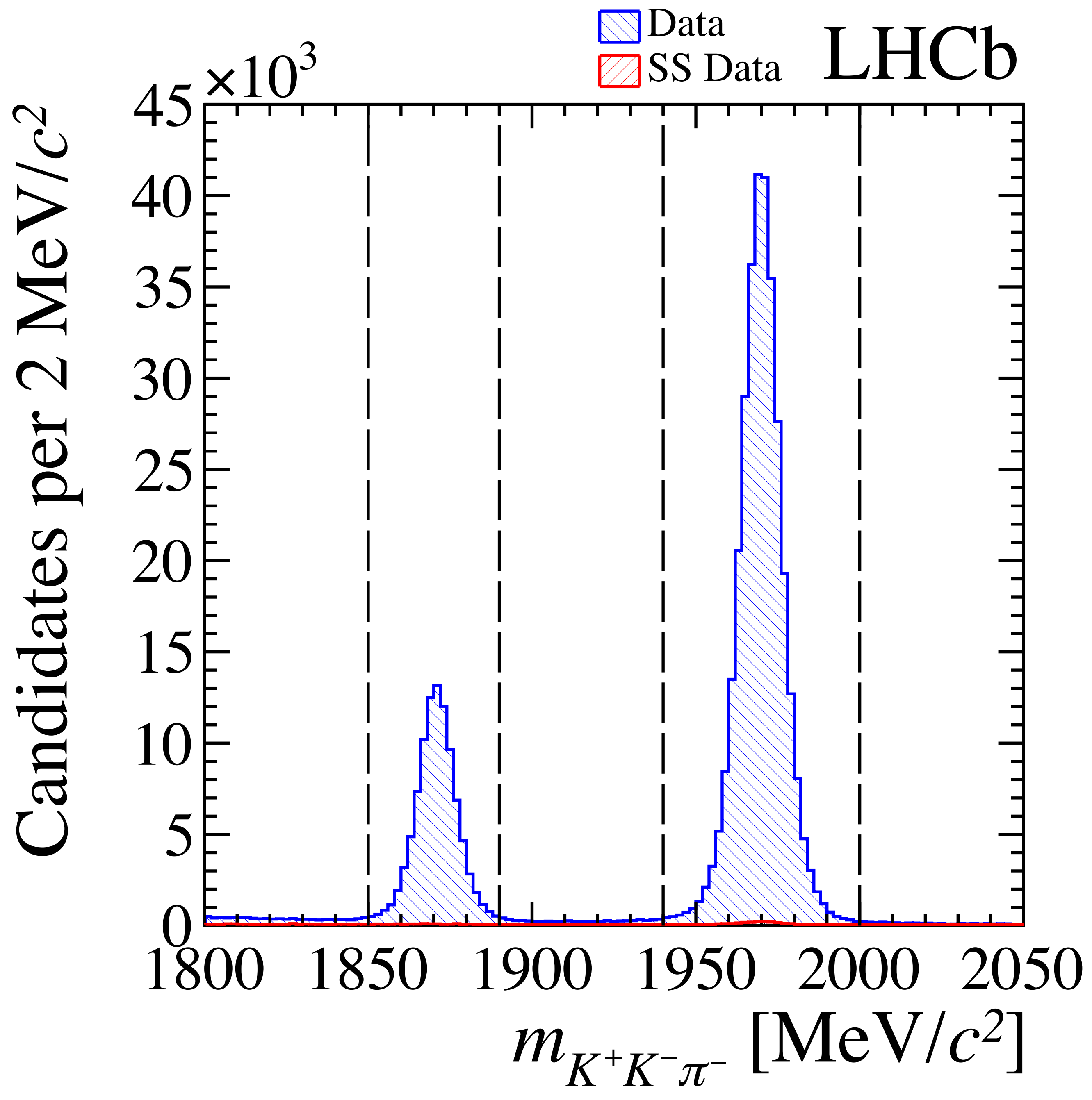
- Exclusive measurements performed by B-factories and LHCb.
- Inclusive measurements only by B-factories.
- Lack of unique final state very hard for detectors at hadron colliders.

Phys.Rev.D 101 (2020) 7, 072004



Phys.Rev.D 107 (2023) 7, 072002

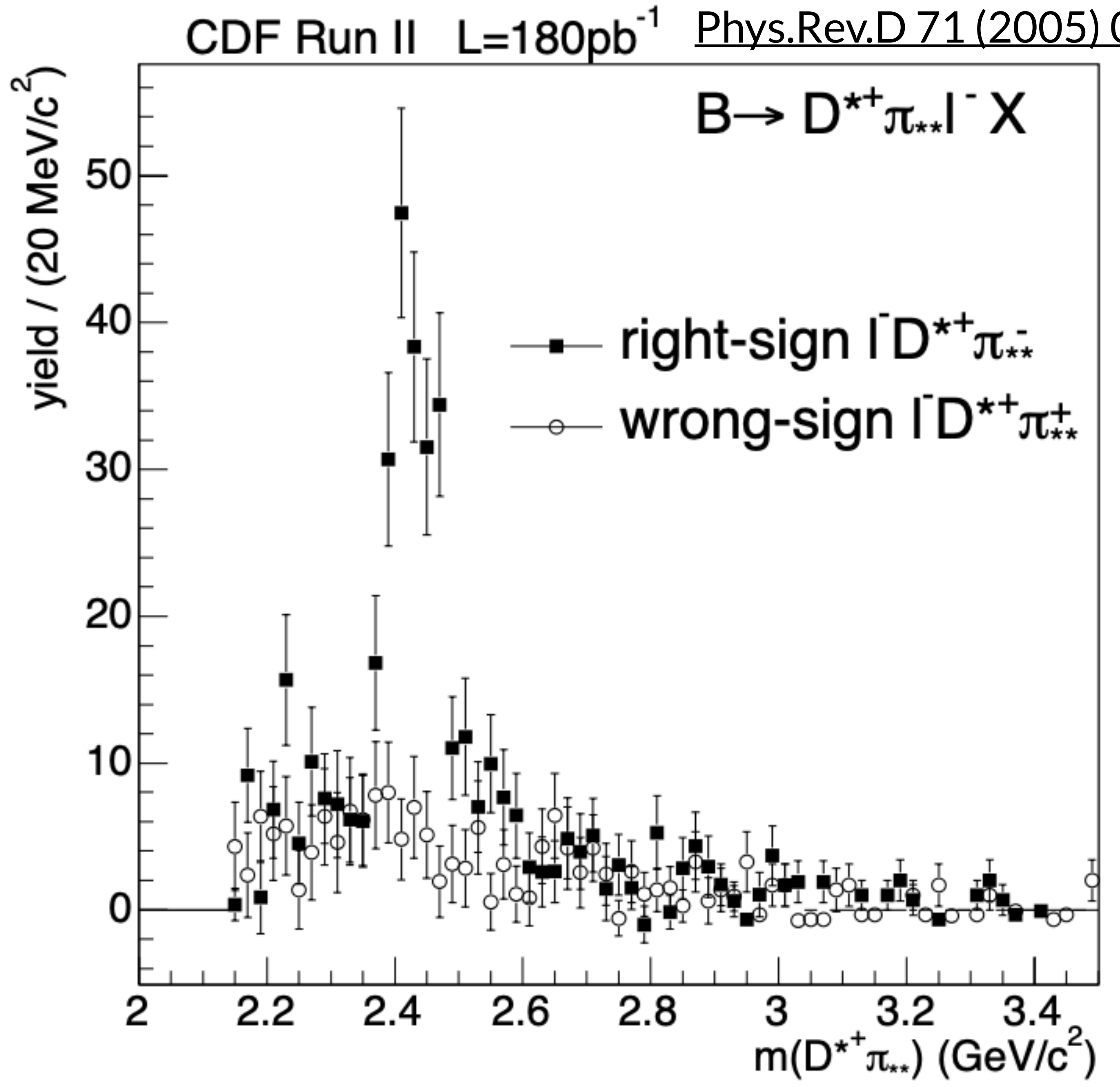




Sum-of-exclusives

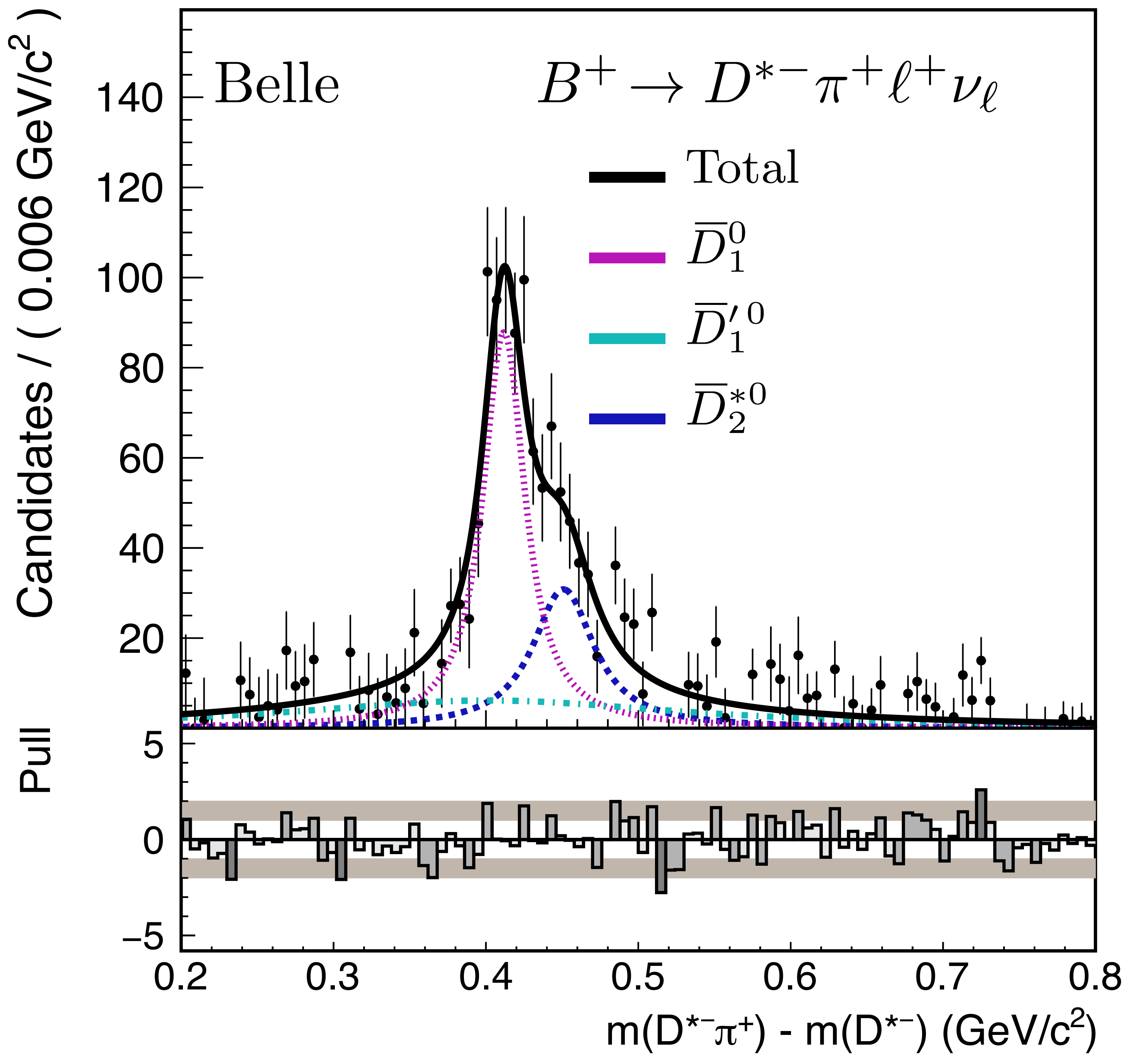
„The whole is more than the sum of its parts“

- Instead of a „true“ inclusive measurement, sum all final states.
- Pioneering measurement by CDF (from 2005!), however by now outdated by knowledge about the D^{**} spectrum
- $D^{(*(*))}$ spectrum complicated by interference effects.



Phys.Rev.D 71 (2005) 051103

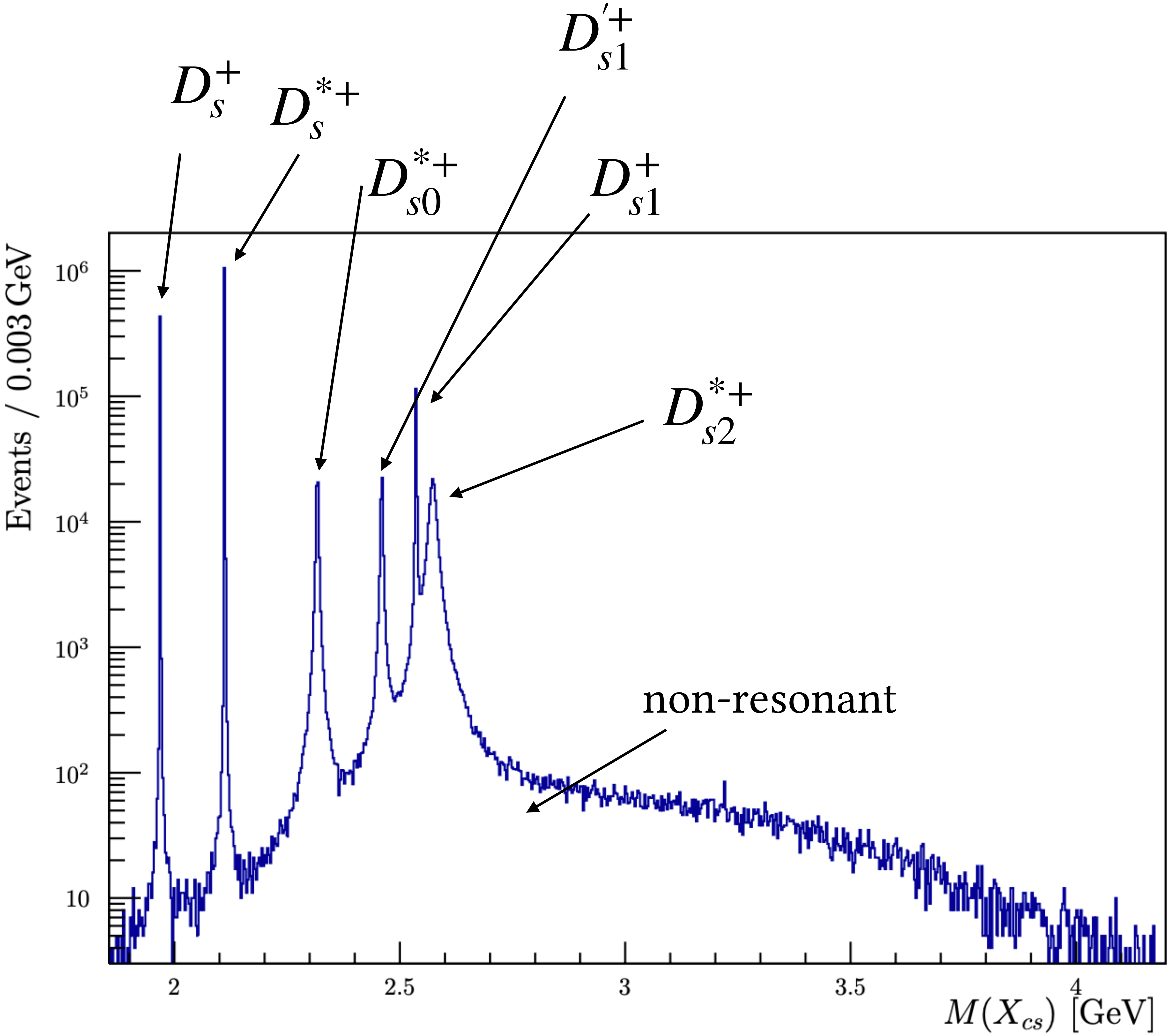
Phys.Rev.D 107 (2023) 9, 092003



Plenty of B_s^0 mesons

Provided to you by the LHC

- LHCb reconstructs many B_s^0 and Λ_b^0 hadrons.
- $D_s^{(*(*))}$ has mostly well-separated resonances - no interference effects to consider.
- $B_s^0 \to D_s^{(*(*))}\ell\nu$ abundant at LHCb.
- But how do we actually determine V_{cb} ?



Heavy Quark Expansion

And its parameters

- Decay rate of $B_s^0 \to D_s^{(*(*))}\ell\nu$ given by V_{cb}, and expansion in $1/m_b^n$ with perturbatively calculable parts and non-perturbative parameters.
- Corrections only enter at $1/m_b^2$
- Need to determine μ_π, μ_G, ρ_D and ρ_{LS} from data.

$$
\begin{aligned}
\Gamma = &\boxed{|V_{cb}|^2} \frac{G_F^2 m_b^5(\mu)}{192\pi^3}\eta_{ew}\times \\
&\boxed{\left[z_0^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_0^{(1)} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 z_0^{(2)}(r) + \ldots\right.} \\
&+ \frac{\mu_\pi^2}{m_b^2}\left(z_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_2^{(1)}(r) + \ldots\right) \\
&+ \frac{\mu_G^2}{m_b^2}\left(y_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_2^{(1)}(r) + \ldots\right) \\
&+ \frac{\rho_D^3}{m_b^3}\left(z_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_3^{(1)}(r) + \ldots\right) \\
&+ \frac{\rho_{LS}^3}{m_b^3}\left(y_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_3^{(1)}(r) + \ldots\right) + \ldots\Big]
\end{aligned}
$$

Moments

Now wait a moment

- Can link μ_π, μ_G, ρ_D and ρ_{LS} to statistical moments of the E_ℓ^*, q^2 or m_{X_c} spectra.
- E_ℓ^* and q^2 not directly accessible at LHCb, but sum-of-exclusives m_{X_c} is.
- So all we need to know is the $m_{X_c} = m_{D_s^{(*(*))}}$ spectrum, and we can extract the non-perturbative parameters of the HQE

$$M_n' = \langle (m_H^2 - \langle m_H^2 \rangle)^n \rangle = \int (m_H^2 - M_1)^n \frac{1}{\Gamma_{SL}} \frac{d\Gamma_{SL}}{dm_H^2} dm_H^2 .$$

$$M_1 = 4.85 + 0.30\alpha_s + 0.46\frac{\mu_G^2}{\mathrm{GeV}^2} - 0.68\frac{\mu_\pi^2}{\mathrm{GeV}^2} + 0.99\frac{\rho_D^3}{\mathrm{GeV}^3} - 0.12\frac{\rho_{LS}^3}{\mathrm{GeV}^3} ,$$

$$M_2' = 0.28 + 1.47\alpha_s - 0.30\frac{\mu_G^2}{\mathrm{GeV}^2} + 4.77\frac{\mu_\pi^2}{\mathrm{GeV}^2} - 6.0\frac{\rho_D^3}{\mathrm{GeV}^3} + 0.28\frac{\rho_{LS}^3}{\mathrm{GeV}^3} ,$$

$$M_3' = -0.058 + 3.3\alpha_s + 0.04\frac{\mu_G^2}{\mathrm{GeV}^2} + 3.6\frac{\mu_\pi^2}{\mathrm{GeV}^2} + 23.96\frac{\rho_D^3}{\mathrm{GeV}^3} + 0.96\frac{\rho_{LS}^3}{\mathrm{GeV}^3}$$

Current knowledge

Of semileptonic B_s^0 decays

- 1 ground state,
  1 excited state,
  4 higher excited states,
  „non-resonant“ contribution
- Each $D_s^{(*(*))}$ meson has different BRs into different final states - need to know at least one precisely.

B_s^0 Decay	$\mathcal{B}[\%]$ (Conf. A)	$\mathcal{B}[\%]$ (Conf. B)
$\overline{B}_s^0 \to X_{cs}\ell\bar{\nu}_\ell$	10.05±0.31	10.05±0.31
$\overline{B}_s^0 \to D_s^+\ell^-\bar{\nu}_\ell$ [38]	2.44±0.23	2.44± 0.10
$\overline{B}_s^0 \to D_s^{*+}\ell^-\bar{\nu}_\ell$ [38]	5.3±0.5	5.30 ±0.22
$\overline{B}_s^0 \to D_{s0}^{*+}\ell^-\bar{\nu}_\ell$ (see text)	0.3±0.3	0.30±0.03
$\overline{B}_s^0 \to D_{s1}^{\prime +}\ell^-\bar{\nu}_\ell$ (see text)	0.3±0.3	0.30±0.03
$\overline{B}_s^0 \to D_{s1}^+\ell^-\bar{\nu}_\ell$	0.98±0.20	0.98±0.05
$\overline{B}_s^0 \to D_{s2}^{*+}\ell^-\bar{\nu}_\ell$	0.58±0.20	0.58±0.04
$\overline{B}_s^0 \to D^{(*)}K\ell^-\bar{\nu}_\ell$ (see text)	0.15±0.15	0.150±0.015

First two states

The basics

- $\mathscr{B}(B_s^0 \to D_s^+ \mu^- \nu)$
  - Known with about 10% relative precision.
  - Potential for further reduction.

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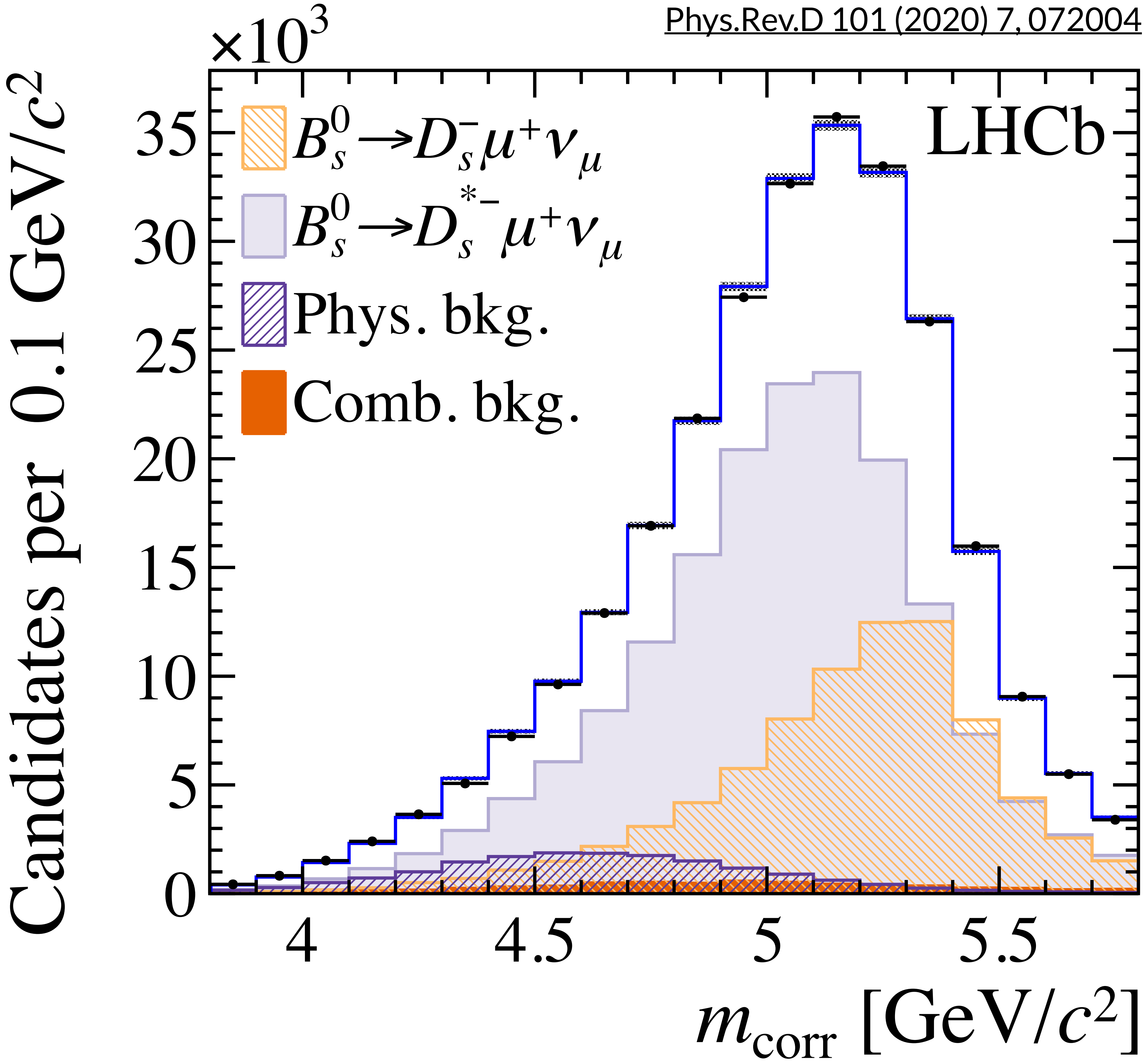


First excited states

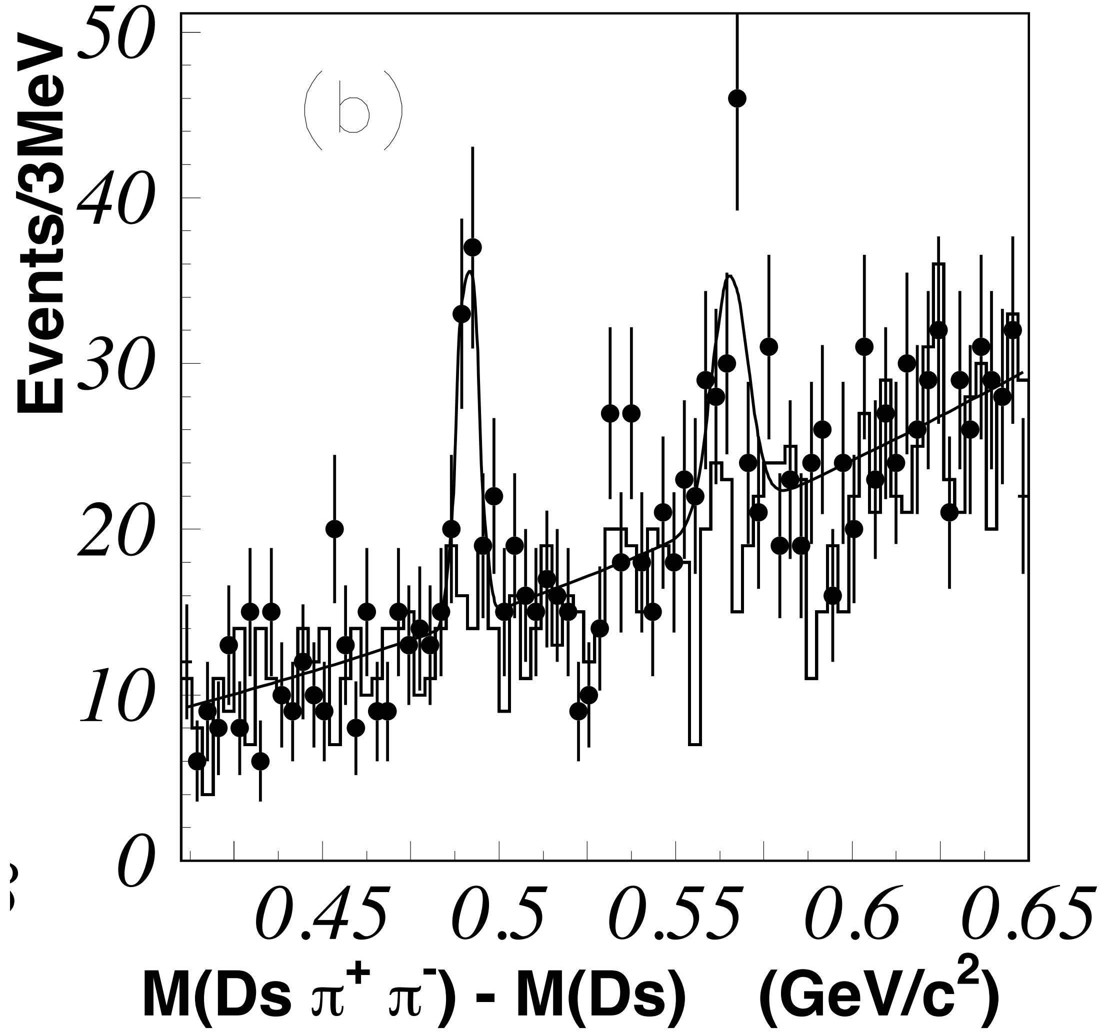
Below threshold

- The first two higher resonances are below the DK threshold, so exclusively decay to D_s^+ mesons.

- $\mathscr{B}(B_s^0 \to D_{s0}^{*+}\mu^-\nu)$
  - No measurement has been published. We assume $\mathscr{B} = (0.3 \pm 0.3)\,\%$
  - $\mathscr{B}(D_{s0}^{*+} \to D_s^+\pi^0)$ known with about 20% relative uncertainty.
  - Soft π^0 makes the reconstruction inefficient, but clearly doable.

- $\mathscr{B}(B_s^0 \to D_{s1}^{'+}\mu^-\nu)$
  - No measurement has been published. We assume $\mathscr{B} = (0.3 \pm 0.3)\,\%$
  - $\mathscr{B}(D_{s1}^{'+} \to D_s^{*+}\pi^0)$ known with about 20% relative uncertainty
  - $D_{s1}^{'+} \to D_s^+\pi^+\pi^-$ also seen and experimentally easier, but a bit larger uncertainty

PRL 92, 012002



Second excited states

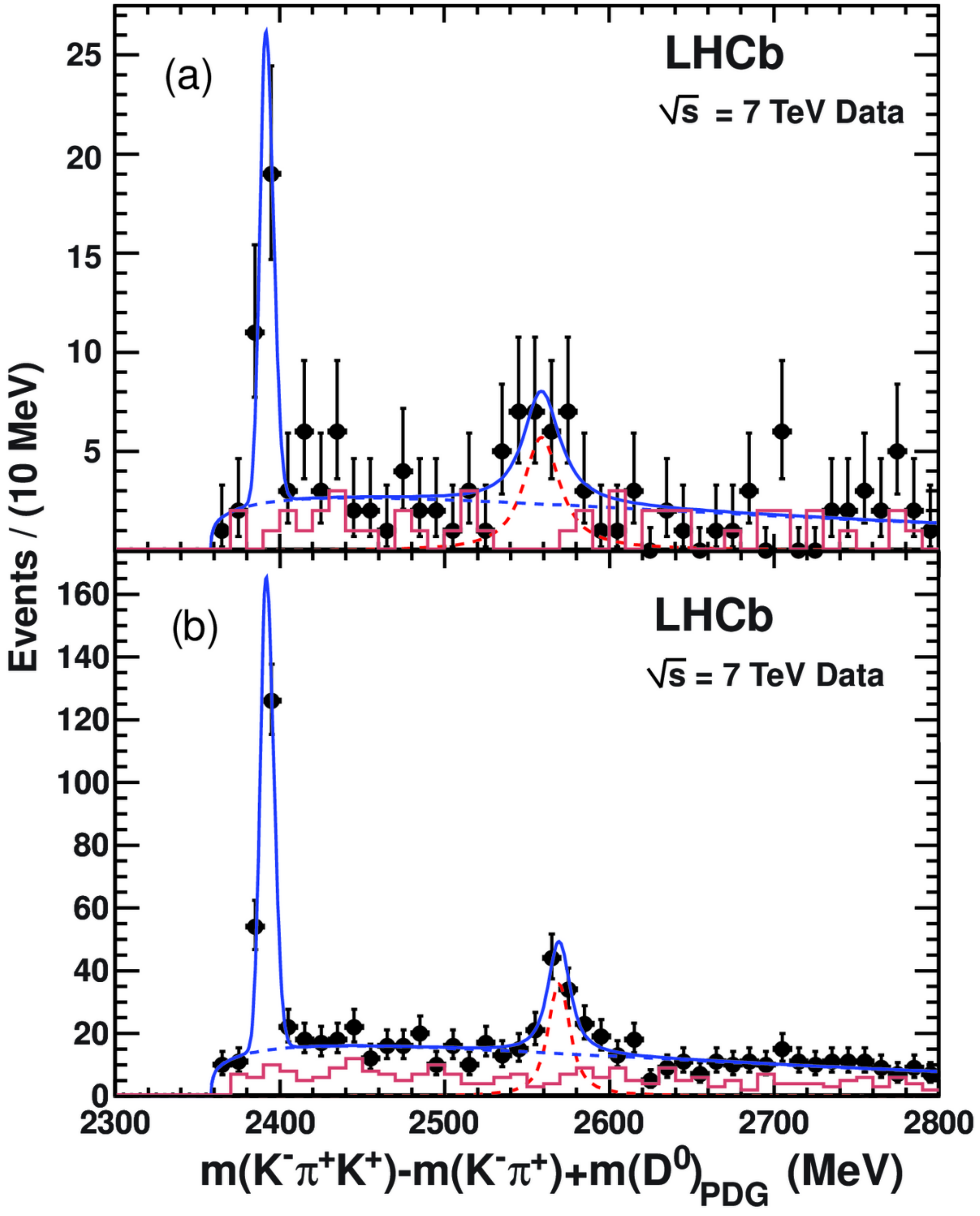
Above threshold

- The second two higher resonances are above the DK threshold, and decay to DK mesons.

- $\mathscr{B}(B_s^0 \to D_{s1}^+ \mu^- \nu)$
  - Measured by DØ and LHCb with about 20% relative uncertainty. Easy to improve.
  - $\mathscr{B}(D_{s1}^+ \to D^{0*} K^+)$ known with about 15% relative uncertainty, thanks to recent BESIII result, arXiv:2407.07651 (not yet used in the following)
  - Experimentally easy, reconstruct D^{*0} as D^0.

- $\mathscr{B}(B_s^0 \to D_{s2}^{*+} \mu^- \nu)$
  - Measured by LHCb with about 35% relative uncertainty. Easy to improve.
  - $\mathscr{B}(D_{s2}^{*+} \to D^0 K^+)$ with about 15% relative uncertainty, thanks to recent BESIII result, arXiv:2407.07651 (not yet used in the following)
  - Experimentally easy

Phys.Lett.B 698 (2011) 14-20



Even higher states

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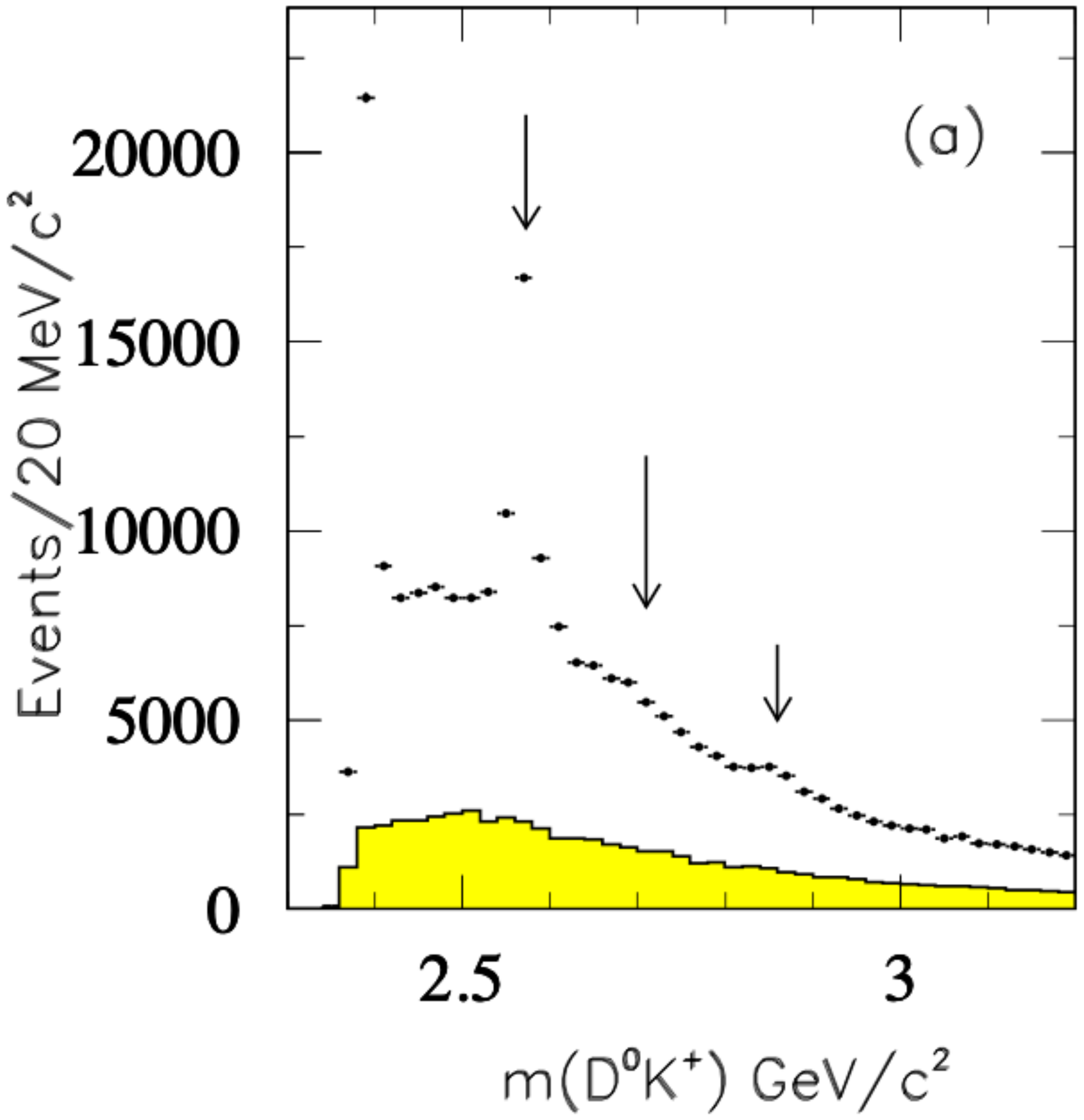
- Resonances with higher mass than the D_{s2}^{*+} have been observed.

- Measuring $\mathscr{B}(B_s^0 \to D_{sJ}^{*+}\mu^-\nu)$ with $D_{sJ}^{*+} \to D^0K^+$ is experimentally straightforward, but $\mathscr{B}(D_{sJ}^{*+} \to D^0K^+)$ cannot be measured at LHCb.

- Might be possible at Belle II (?)

Phys.Rev.D 80 (2009) 092003

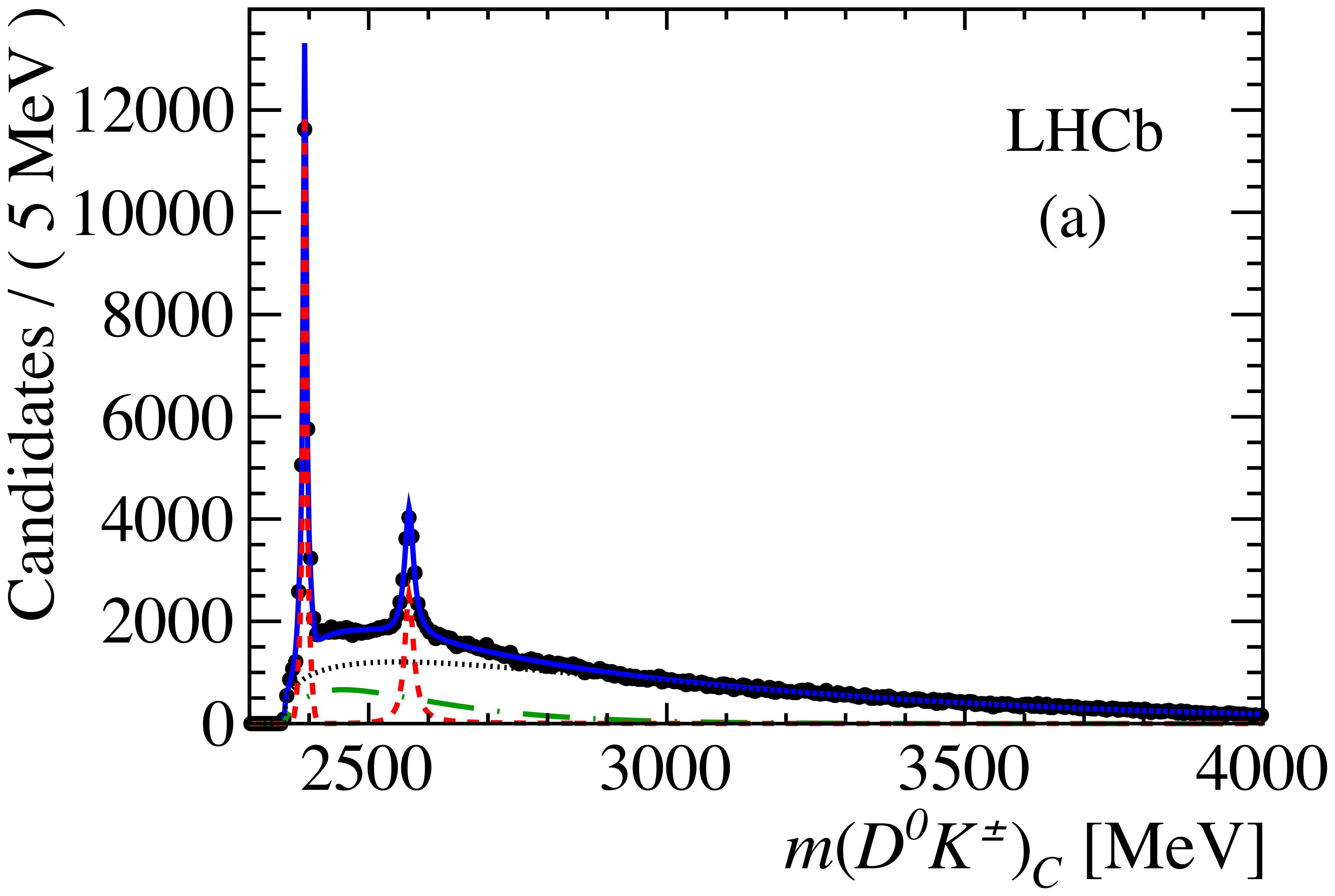


„Non-resonant“ decays

And their modelling

- $B_s^0 \to D^0 K^+ \mu\nu$ has been observed at LHCb, but no branching fraction was published.
- For this study we extract the shape from a „modified Goity-Roberts model“ (used for $B \to D\pi\ell\nu$), accounting for the $K - \pi$ difference.
- A new approach is under development, following arxiv:2311.00864 for $B \to D\pi\ell\nu$
(E. Gustafson, F. Herren, R. S. Van de Water, R. van Tonder, M. L. Wagman)

Phys.Rev.D 100 (2019) 3, 031102



Total Branching fraction

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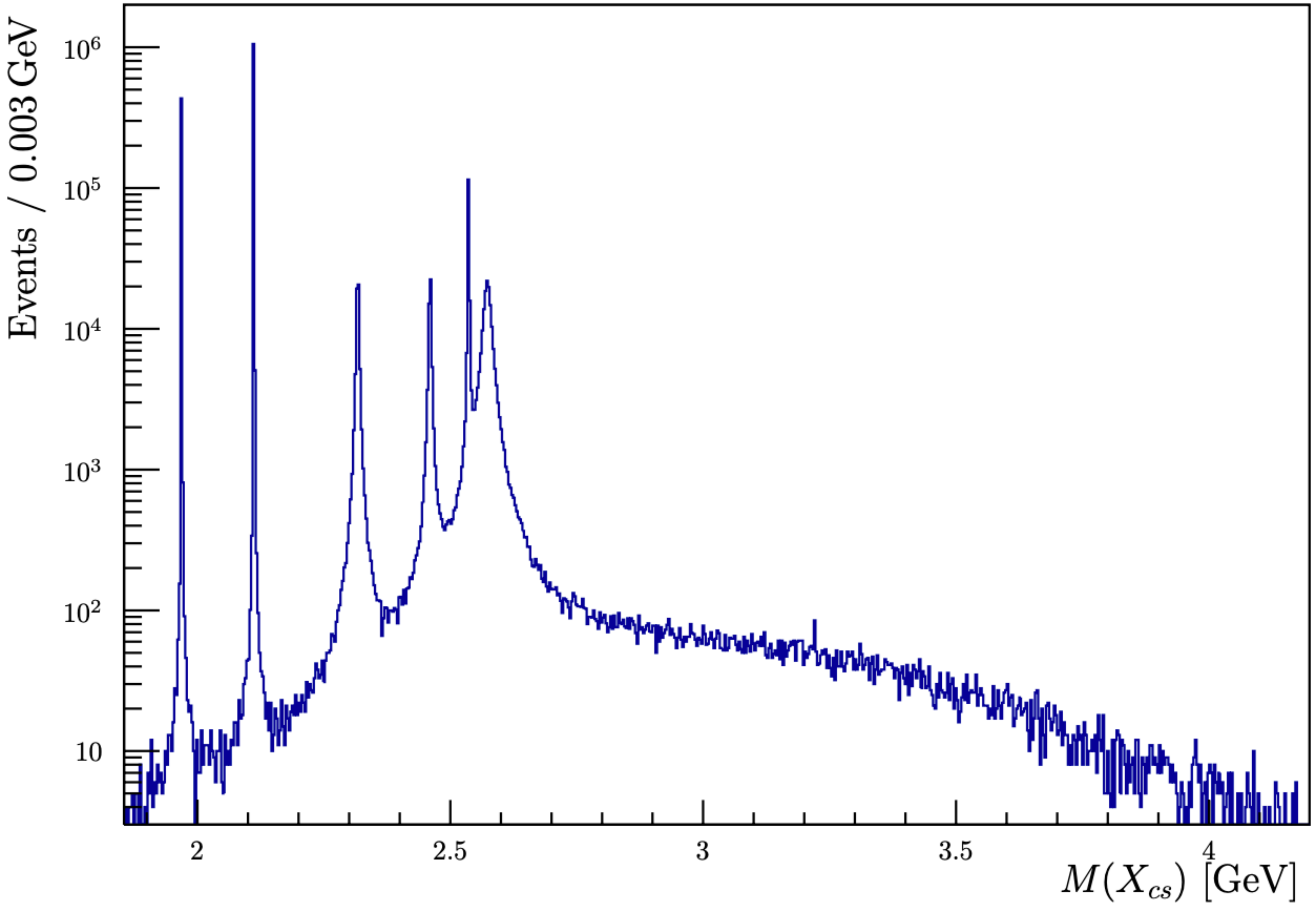
- Summing up all exclusive branching fractions, including an estimate of the non-resonant contribution from Phys.Rev.D 100 (2019) 3, 031102, we got more than the prediction for the semileptonic branching fraction
- Given the uncertainty on the non-resonant component, we constrained it to:

- $\Gamma_{SL}(B_s^0)/\Gamma_{SL}(B^0) = 1 - (0.018 \pm 0.008)$
- $\mathscr{B}(B_s^0 \to X_c \mu \nu) = (10.05 \pm 0.31)\,\%$
- $\mathscr{B}(B_s^0 \to D^{(*)0} K^+ \ell \nu) = \mathscr{B}(B_s^0 \to X_c \mu \nu) - \sum_{res} \mathscr{B}_{res}$

The spectrum

Of semileptonic B_s^0 decays



SM „predictions“

And where they are coming from

- μ_G can be obtained from $B_s^{*0} - B_s^0$ hyperfine splitting.
- μ_π can be obtained from the B_s^0/B^0 and D_s^0/D^0 mass differences
- ρ_D can be linked to the decay constant
- For ρ_{LS} we take the value from B^0 and increase the uncertainty due to $SU(3)_F$ breaking effects

- $(m^2_{B_s^{*0}} - m^2_{B_s^0}) = \frac{4}{3}\mu_G^2(B_s^0) + \mathcal{O}(1/m_b)$
  $\mu_G^2(B_s^0) = (0.35 \pm 0.07)\ \mathrm{GeV}^2$
- $\mu_\pi^2(B_s^0) = (0.58 \pm 0.10)\ \mathrm{GeV}^2$
- $\rho_D^3(B_s^0) \simeq (0.26 \pm 0.03)\ \mathrm{GeV}^3$
- $\rho_{LS}^3(B_s) \simeq -(0.13 \pm 0.10)\ \mathrm{GeV}^3$

Fit to spectrum

And value of moments

- Conf. A uses the currently known experimental precision
- Conf. B a future with improved precisions
- L = 0 and L = 1 only considers spin 0 and spin 1 resonances
- Using these values, and constraining μ_G and ρ_{LS} we can obtain „measurements“ for all HQE parameters

Moments	Conf. A	M_2'	M_3'	Conf. B	M_2'	M_3'	$L=0$ and $L=1$
M_1 [GeV^2]	4.82± 0.08	0.74	0.55	4.78±0.02	0.72	0.45	4.79±0.02
M_2' [GeV^4]	1.36 ± 0.29		0.96	1.22±0.05		0.90	0.82±0.09
M_3' [GeV^6]	4.7 ± 1.8			3.86±0.28			1.07±0.11

Fit to spectrum

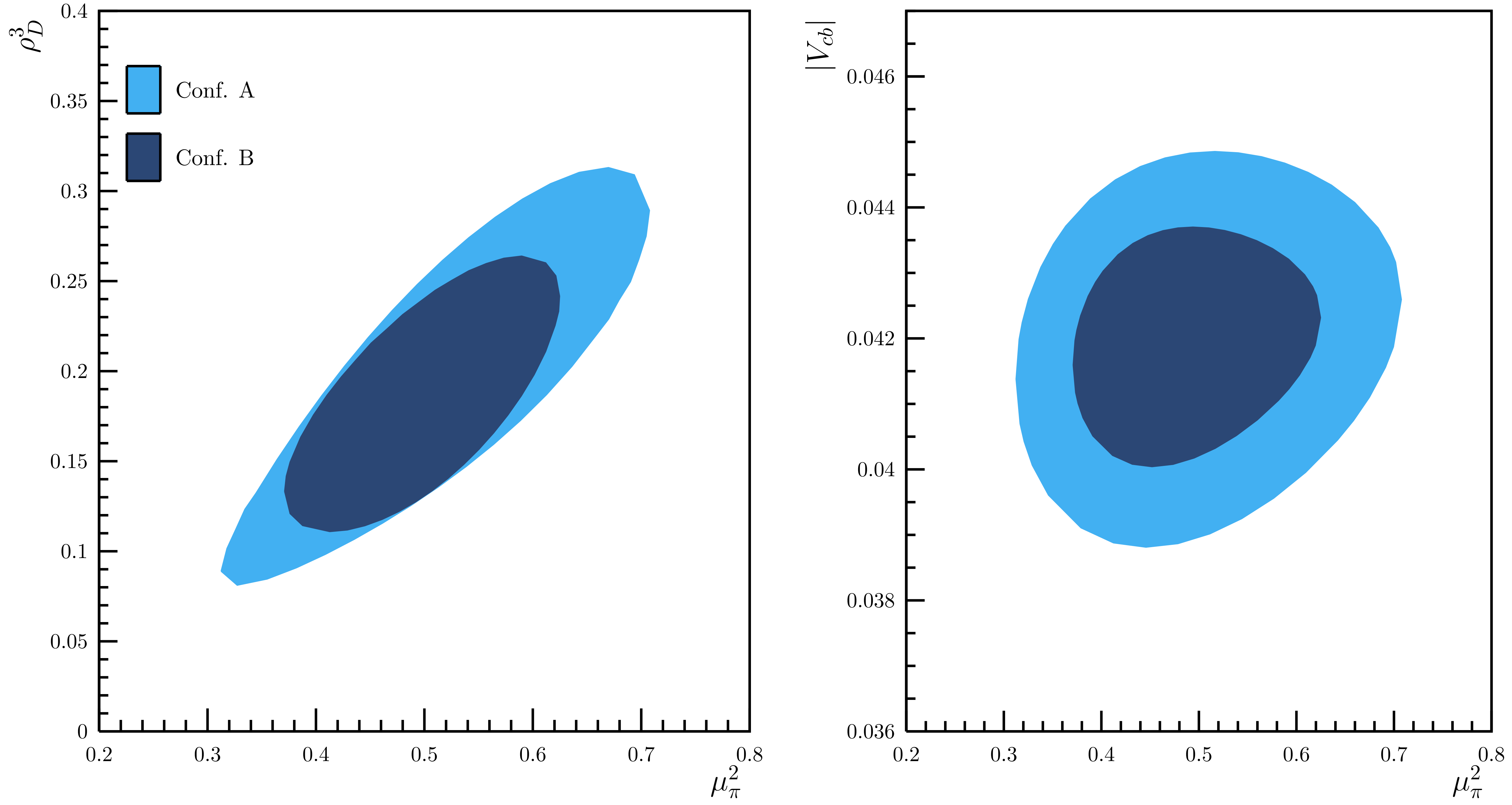
And value of HQE parameters

- We obtain:
- $\mu_\pi^2 = (0.46 \pm 0.12)\ \mathrm{GeV}^2$ vs $(0.58 \pm 0.10)\ \mathrm{GeV}^2$ (predicted)
  and therefore $\dfrac{\mu_\pi^2(B_s^0)}{\mu_\pi^2(B^0)} \sim 0.96$
- $\rho_D^3 = (0.16 \pm 0.06)\ \mathrm{GeV}^3$ vs $(0.26 \pm 0.03)\ \mathrm{GeV}^3$ (predicted)
  and therefore $\dfrac{\rho_D^3(B_s^0)}{\rho_D^3(B^0)} \sim 0.86$
- The constrained values of μ_G and ρ_{LS} are very close to their input values.

V_{cb} and correlation

Between HQE parameters

- Using the experimentally measured $\mathscr{B}(B_s^0 \to X_c \ell \nu) = (9.6 \pm 0.8)\,\%$
- We calculate $V_{cb} = (41.8 \pm 2.0) \cdot 10^{-3}$
- Largely driven by branching fraction number.
- Strong correlation between ρ_D^3 and μ_π^2



Towards precision

Many interesting things to tackle

- $\mathscr{B}(B_s^0 \to D_s^+ \mu^- \nu)$ and $\mathscr{B}(B_s^0 \to D_s^{*+} \mu^- \nu)$ are the dominating contributions. Need a precise measurement of the branching fractions (mostly experimental task)
- (Improved) measurements of $\mathscr{B}(B_s^0 \to D_s^{**+} \mu^- \nu)$, and measurements / predictions of $\mathscr{B}(D_{s0}^+ \to D_s^+ X)$ and $\mathscr{B}(D_{s1}^{\prime +} \to D_s^+ Y)$ (theory & experiment)
- Improved theoretical & experimental treatment of $B_s^0 \to D^{(*)0} K \mu^- \nu$ decay

Conclusion

- Presence of mostly narrow resonances in $B_s^0 \to X_c \ell \nu$ allows for a sum-of-exclusives approach to an inclusive measurement.
- Performed a proof-of-concept study, using literature values as input to the spectrum and the SM „predictions“.
- Most input measurements can be theoretically and/or experimentally improved.
- With these improvements precise values for the HQE parameters (and V_{cb}?) can be obtained.

_BACKUP

Decay channels

D_{s0}^{*+}		$D_{s1}^{\prime +}$		D_{s1}^{+}		D_{s2}^{*+}	
$2317.8 \pm 0.5\,\text{MeV}$		$2459.5 \pm 0.6\,\text{MeV}$		$2535.11 \pm 0.06\,\text{MeV}$		$2569.1 \pm 0.8\,\text{MeV}$	
$< 3.8\,\text{MeV}$		$< 3.5\,\text{MeV}$		$0.92 \pm 0.05\,\text{MeV}$		$16.9 \pm 0.7\,\text{MeV}$	
$D_s^+\pi^0$	$100^{+0}_{-20}\%$	$D_s^{*+}\pi^0$	$48 \pm 11\%$	$D^{*+}K_S^0$	$85 \pm 12\%$	D^0K^+	seen
$D_s^+\gamma$	$< 5\%$	$D_s^+\gamma$	$18 \pm 4\%$	$D^{*0}K^+$	100%	$D^+K_S^0$	seen
$D_s^{*+}\gamma$	$< 6\%$	$D_s^+\pi^+\pi^-$	$4.3 \pm 1.3\%$	$D^+\pi^-K^+$	$2.8 \pm 0.5\%$	$D^{*+}K_S^0$	seen
$D_s^+\gamma\gamma$	$< 18\%$	$D_s^{*+}\gamma$	$< 8\%$	$D_s^+\pi^+\pi^-$	seen		
		$D_{s0}^{*+}\gamma$	$3.7^{+5.0}_{-2.4}\%$	D^+K^0	$< 34\%$		
				D^0K^+	$< 12\%$		

Numbers before update by BESIII