

**Implications of LHCb measurement**  
**23-25 October 2024, CERN**

# **Workshop summary** **'Challenges in** **semileptonic B decays'**



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# Challenges in Semileptonic B decays

- **Mainz 20-24 Apr 2015**
  - P. Gambino, A. Kronfeld, M. Rotondo, C. Schwanda, S. Turczyk
  - 33 Participants
- **Mainz 9-13 Apr 2018**
  - P. Gambino, A. Kronfeld, M. Rotondo, C. Schwanda
  - 35 Participants
  - Challenges in Semileptonic B Decays [arXiv:2006.07297](https://arxiv.org/abs/2006.07297)
- **Barolo 19-23 Apr 2022**
  - P. Gambino, A. Kronfeld, M. Rotondo, C. Schwanda, M.Jung, T. Mannel
  - 39 Participants
- **Vienna 23-27 Sept 2024**
  - F.Bernlochner, A.Kronfeld, M.Rotondo, C.Schwanda, M.Jung, T.Mannel
  - 48 Participants

# Challenges in Semileptonic B decays

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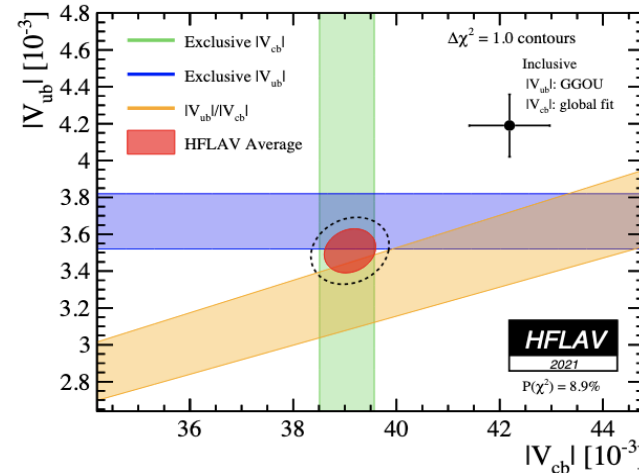
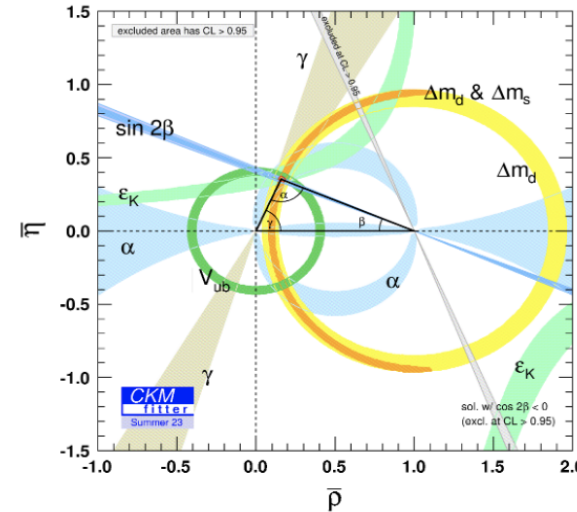
Gather together **experimentalists**, **lattice** and **continuum** theorists: develop a medium-term strategy of analyses and calculations aimed at solving the puzzling **discrepancies between inclusive and exclusive determinations**, for both  $|V_{ub}|$  and  $|V_{cb}|$

Understand the discrepancy between SM prediction and measurements of “**semitauonic**” decays

**prospects** of semileptonic decays at LHCb, on the implications of the first Belle-II data

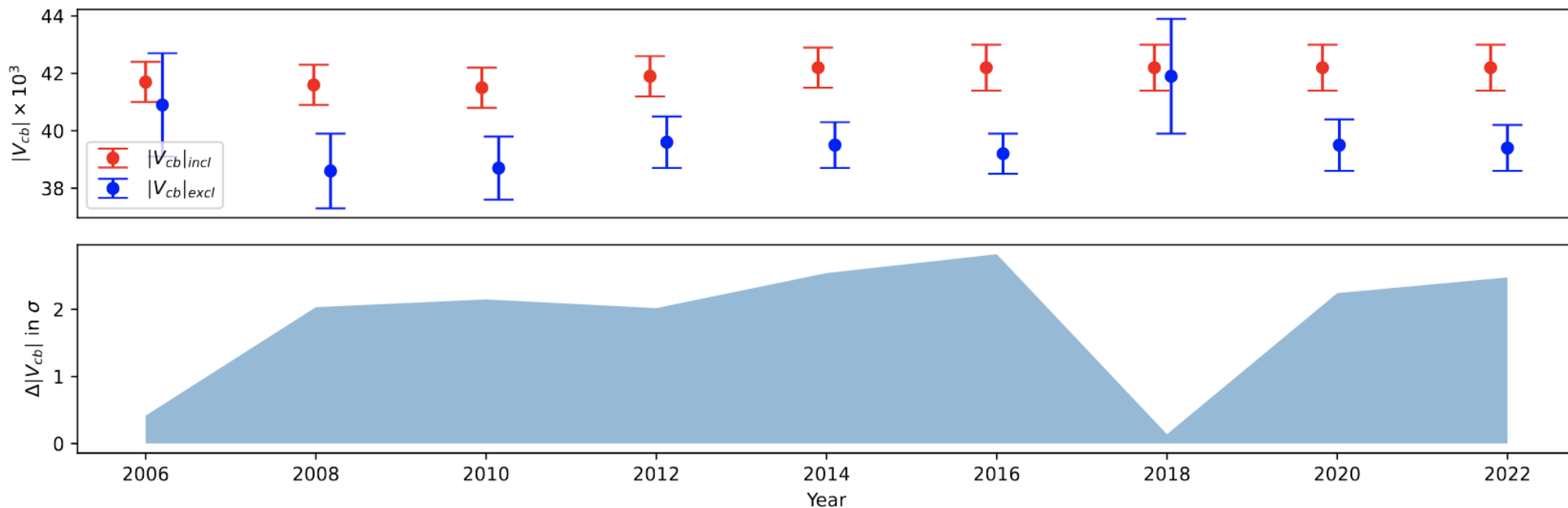
## Why is $V_{cb}$ so important?

- Fundamental parameter of the Standard Model.
- Important role in Unitarity Triangle.
- Important for prediction of  $\epsilon_K \propto x |V_{cb}|^4 + \dots$
- Important for predictions of FCNCs.
- Ratio  $|V_{ub}/V_{cb}|$  directly constrains one side of the Unitarity Triangle.



# Exclusive $b \rightarrow c$

A. Vaquero

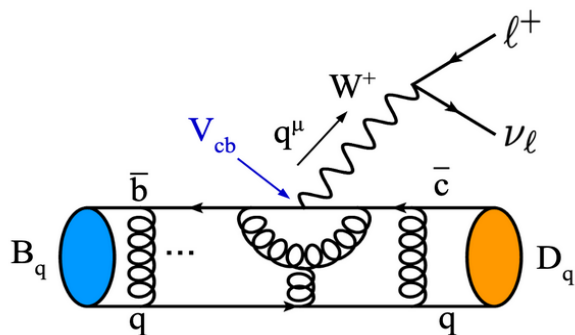


- Current values (PDG 2024):

$$|V_{cb}|_{excl} \times 10^{-3} = 39.8(6) \longrightarrow \text{Dominated by } B \rightarrow D^*$$

$$|V_{cb}|_{incl} \times 10^{-3} = 42.2(5)$$

# Exclusive $b \rightarrow c: B \rightarrow D^* \ell \bar{\nu}$



$$w = \frac{M_B^2 + M_{D^*}^2 - q^2}{2M_B M_{D^*}}$$

$$q_\mu = (p_B - p_{D^*})_\mu$$

$$\underbrace{\frac{d\Gamma}{dw} \left( B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \bar{\nu}_\ell \right)}_{\text{Experiment}} = \underbrace{K_{B_{(s)} \rightarrow D_{(s)}^{(*)}}(w, m_\ell)}_{\text{Known factors}} \underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} \times |V_{cb}|^2$$

$$\langle D^*(\varepsilon, p') | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle = i g \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\nu^* p_\alpha p'_\beta,$$

$$\langle D^*(\varepsilon, p') | \bar{c} \gamma^\mu \gamma^5 b | \bar{B}(p) \rangle = f \varepsilon^{*\mu} + (\varepsilon^* \cdot p) [a_+ (p + p')^\mu + a_- (p - p')^\mu],$$

CLN used only for cross checks

## BGL parameterization

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^N a_n z^n,$$

$$f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^N b_n z^n,$$

$$\mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^N c_n z^n,$$

Combination of f and a.

Conformal variable z:

$$z = \frac{\sqrt{w+1} - \sqrt{2}a}{\sqrt{w+1} + \sqrt{2}a}$$

QCD encoded in coefficients:

$$\{a_n, b_n, c_n\}$$

$$c_0 = \text{constants} \times b_0$$

$$\sum_{j=0}^{\infty} a_j^2 \lesssim 1$$

$$\sum_{j=0}^{\infty} (b_j^2 + c_j^2) \lesssim 1.$$

$$\sum_{j=0}^{\infty} d_j^2 \lesssim 1$$

# Exclusive $b \rightarrow c$ : LQCD $B \rightarrow D^* l \nu$

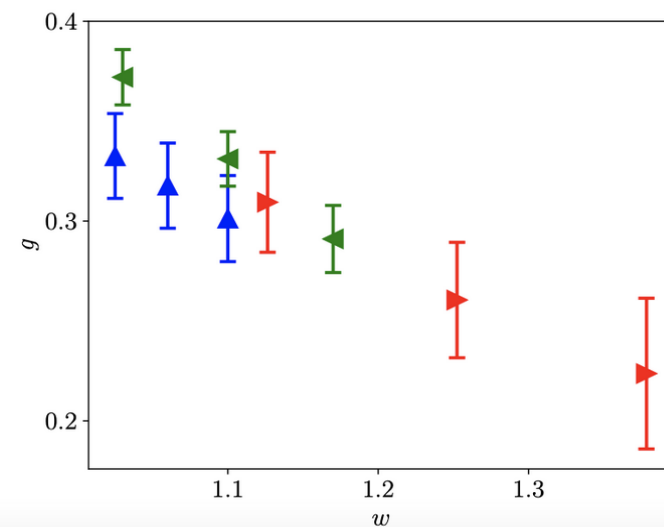
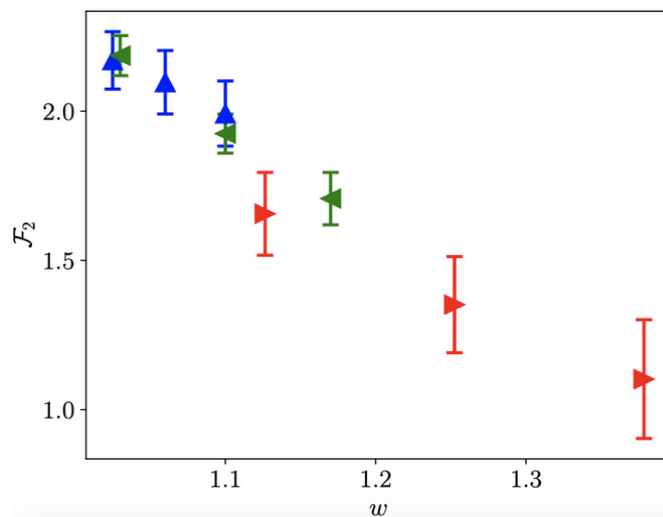
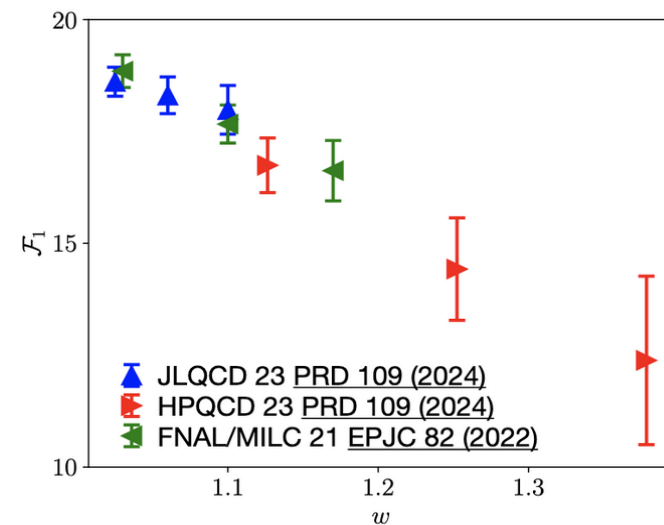
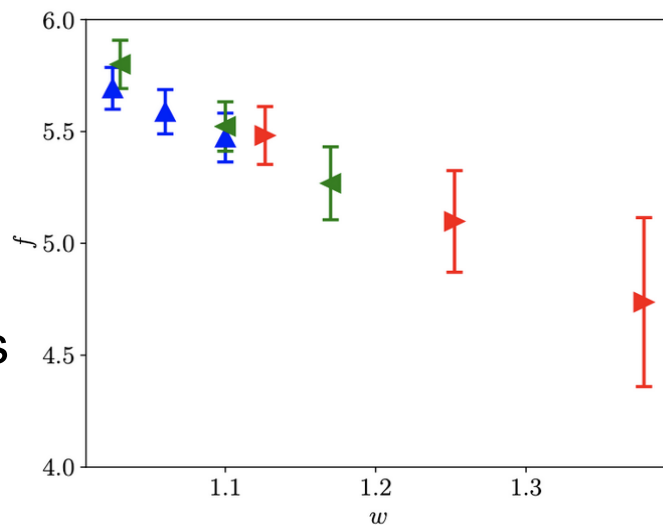
## Lattice Data

- first time that lattice data covers kinematical range
- three different and independent collaborations

[JLQCD 23 PRD 109 \(2024\)](#)

[HPQCD 23 PRD 109 \(2024\)](#)

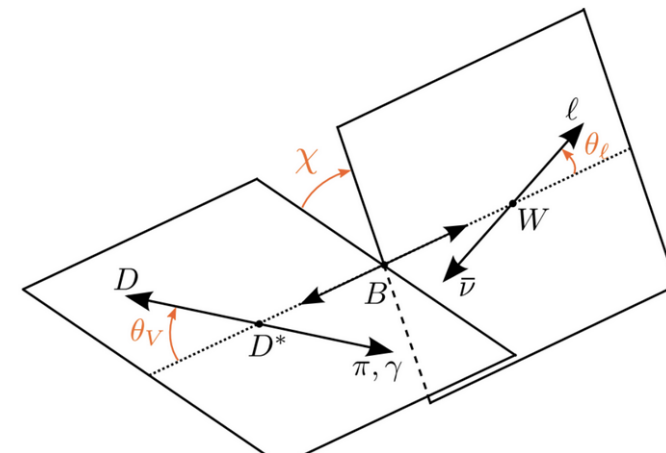
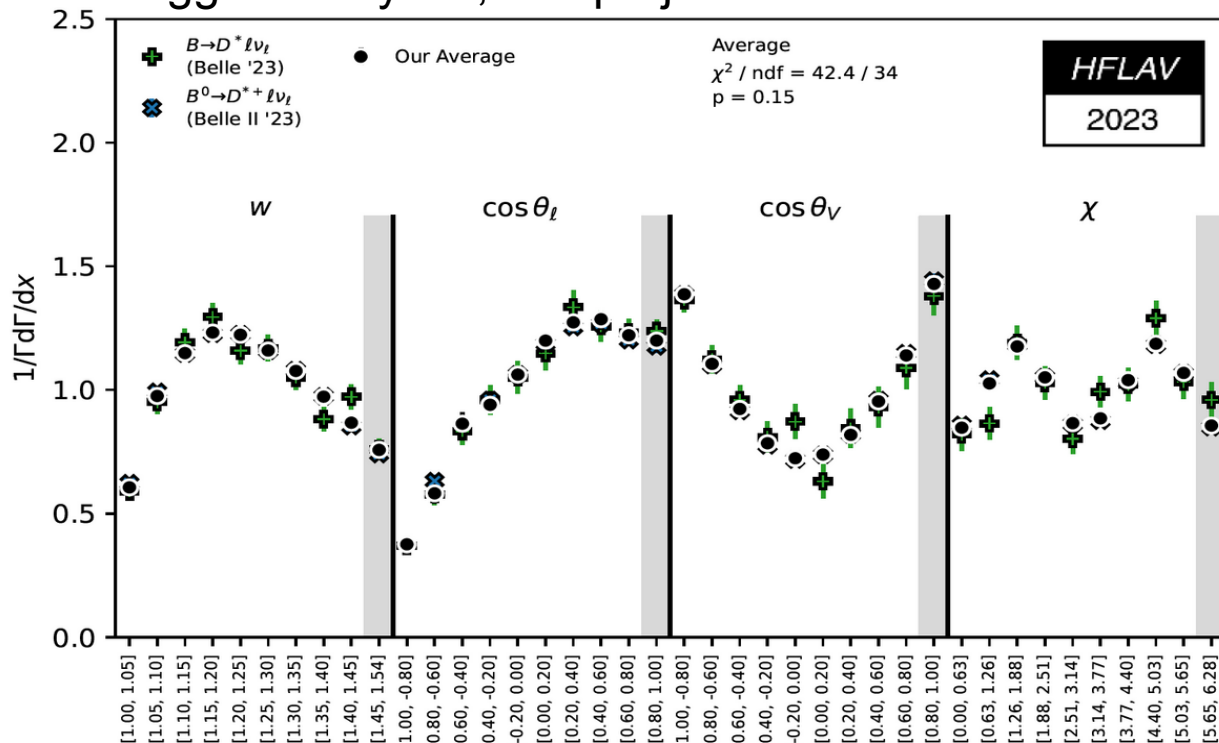
[FNAL/MILC EPJC 82 \(2022\)](#)



# Exclusive $b \rightarrow c$ : experimental data $B \rightarrow D^* \ell \nu$

M. Primm

## Tagged analyses, 1-D projections



$$\begin{aligned}
 \frac{d\Gamma(B \rightarrow D^* \ell \nu_\ell)}{dw d \cos \theta_\ell d \cos \theta_V d \chi} &= \frac{6 m_B m_{D^*}^2}{8 (4\pi)^4} \sqrt{w^2 - 1} (1 - 2wr + r^2) G_F^2 \eta_{EW}^2 |V_{cb}|^2 \\
 &\times \left( (1 - \cos \theta_\ell)^2 \sin^2 \theta_V H_+^2 + (1 + \cos \theta_\ell)^2 \sin^2 \theta_V H_-^2 \right. \\
 &\quad + 4 \sin^2 \theta_\ell \cos^2 \theta_V H_0^2 - 2 \sin^2 \theta_\ell \sin^2 \theta_V \cos 2\chi H_+ H_- \\
 &\quad - 4 \sin \theta_\ell (1 - \cos \theta_\ell) \sin \theta_V \cos \theta_V \cos \chi H_+ H_0 \\
 &\quad \left. + 4 \sin \theta_\ell (1 + \cos \theta_\ell) \sin \theta_V \cos \theta_V \cos \chi H_- H_0 \right),
 \end{aligned}$$

### B Tagged analysis:

Belle Phys.Rev.D 108 (2023) 1, 012002

Belle II Phys.Rev.D 108 (2023) 9, 9

### B untagged analyses:

Belle Phys. Rev. D 100 (2019) 052007

Belle II Phys.Rev.D 108 (2023) 092013

**Informations available in HepData**



## How to best analyse this new quality of data as part of a precision test of the SM?

- Where should we truncate the series?
- How shall we include the unitarity constraints?
- Which statistical method is “best” to fit data?

Truncation order dilemma:

The choice of where to truncate the BGL expansion can impact the extracted value:

- Truncate too soon: Model dependence in extracted result for  $|V_{cb}|$ ?
- Truncate too late: Unnecessarily increase variance on  $|V_{cb}|$ ?

# Exclusive $b \rightarrow c: B \rightarrow D(^*)lv$

Introduction

Convergence of the  $z$ -expansion

Impact of different statistical methods

Angular observables: Experiments and Theory

S. Schacht

## “Standard”

(frequentist)

- $\chi^2$  fit with hard unitarity constraints as side condition.
- Increase BGL order until  $\chi^2$  stable. [Gambino Jung Schacht 1905.08209]

## Feldman Cousins

(frequentist)

- Well-defined CL including hard unitarity constraints, using toy Monte Carlo data. [Gambino Jung Schacht 24xx.soon]

## Nested Hypothesis Test to determine truncation order.

(frequentist)

- Go to order  $N + 1$  if  $\Delta\chi^2 = \chi_N^2 - \chi_{N+1}^2 \geq 1$ . Check unitarity a posteriori. [Bernlochner Ligeti Robinson 1902.09553]
- Alternative: Akaike Information Criterion. [Persson Bernlochner Ligeti Prim Robinson 2024]

## Bayesian inference

- BGL unitarity constraints as prior [Flynn Jüttner Tsang 2303.11285, Bordone Jüttner 2406.10074]
- Dispersive Matrix approach [Di Carlo, Martinelli, Naviglio, Sanfilippo, Simula, Vittorio 2105.02497]

Next slide  
F. Bernlochner

Next next slide  
A. Jüttner

# Exclusive $b \rightarrow c: B \rightarrow D(*)|v$

## Different approaches on the market

F. Bernlochner

	Evaluation Metric	Selection Rule	Search Algorithm
<b>Bernlocher et al. (2019)</b>	$\chi^2$	Choose nested model if $\Delta\chi^2 > 1$	Forward stepwise selection
<b>Gambino, Jung, Schacht</b>	$\chi^2 + \text{unitarity penalty}$	Higher complexity until stable	Forward selection
<b>Current paper</b>	AIC	Lowest metric (w/ and w/o UT)	Exhaustive search

Akaike Information Criterion  
 $\text{AIC} = 2 \cdot k - 2 \log(L)$

Toys studies to show effectiveness of AIC in choosing BGL order: unbiased  $|V_{cb}|$  with correct coverage properties

Beyond single Model selection  
 Global AIC

An approach that weighs multiple models based on their AIC scores, rather than selecting a single best model:

$$w_i = \exp(-\frac{1}{2}\Delta_i) / \sum_j \exp(-\frac{1}{2}\Delta_j) \quad \Delta_i = \text{AIC}_i - \text{AIC}_{\min} \quad |V_{cb}| = \sum_i w_i |V_{cb}|_i$$

## Bayesian form-factor fit

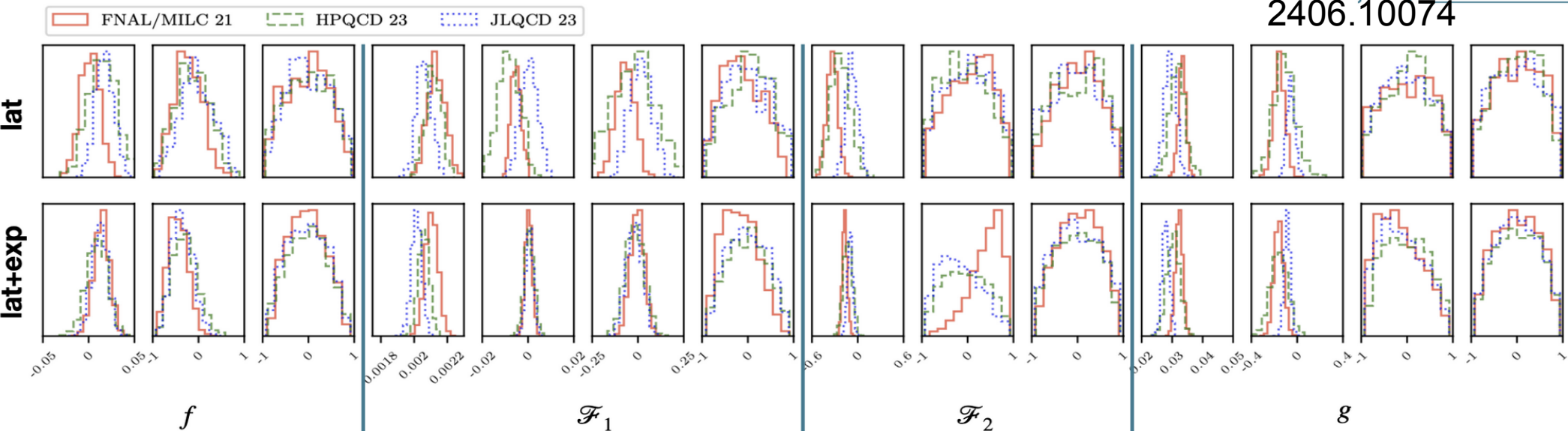
Flynn, AJ, Tsang, *JHEP* 12 (2023) 175

Compute BGL parameters as expectation values  $\langle g(\mathbf{a}) \rangle = \mathcal{N} \int d\mathbf{a} g(\mathbf{a}) \pi(\mathbf{a} | \mathbf{f}, C_f) \pi_a$

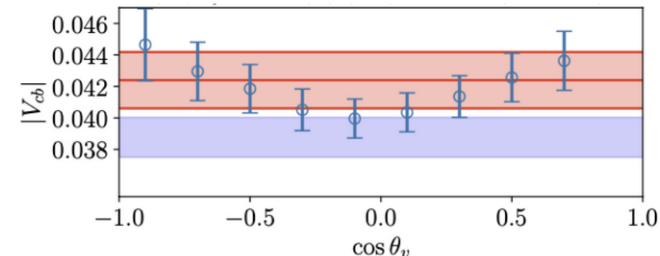
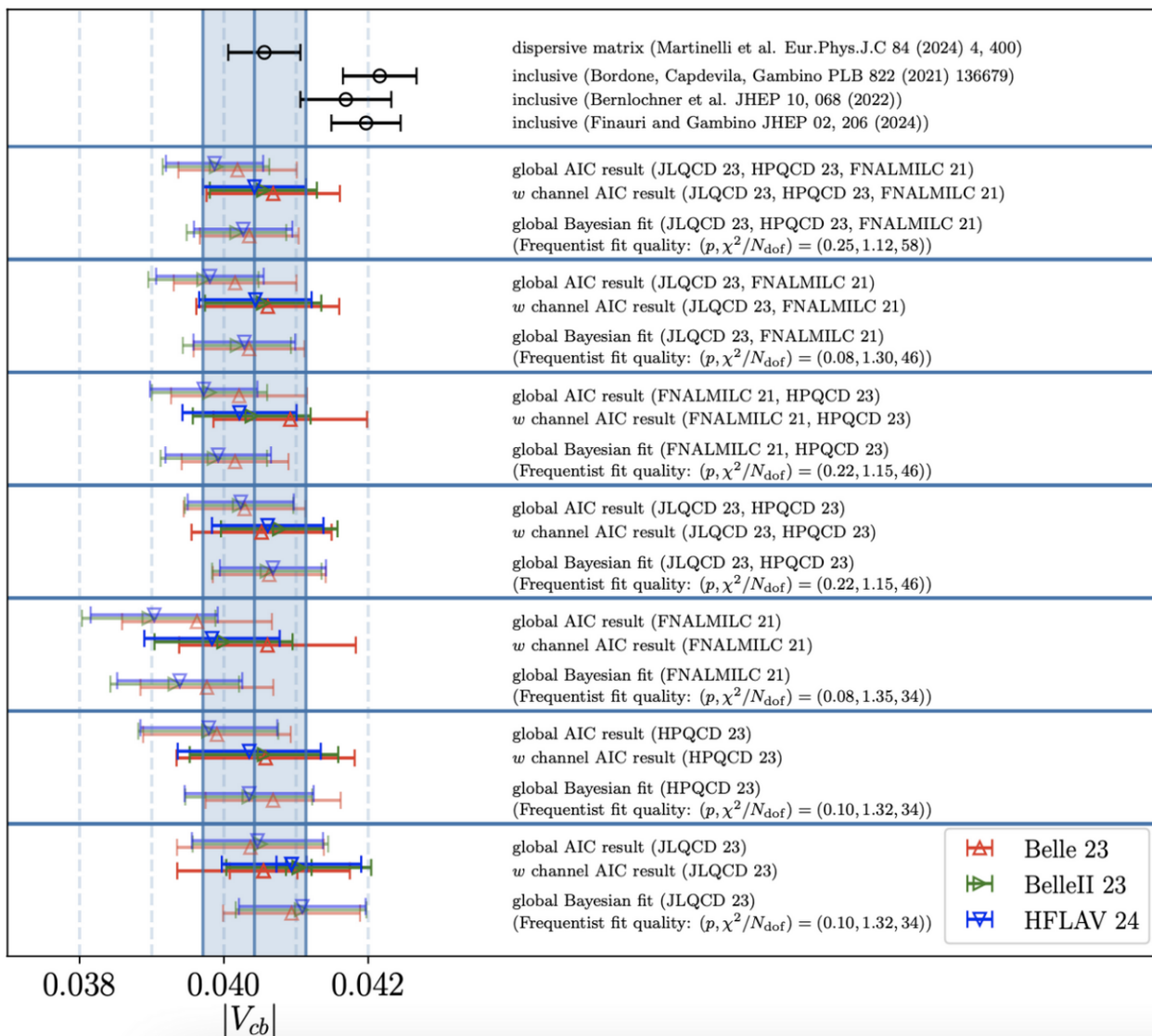
where *probability for parameters given model and data (assume input Gaussian)*

$$\pi(\mathbf{a} | \mathbf{f}, C_f) \propto \exp\left(-\frac{1}{2} \chi^2(\mathbf{a}, \mathbf{f})\right) \quad \text{where} \quad \chi^2(\mathbf{a}, \mathbf{f}) = [\mathbf{f} - \mathbf{Z}\mathbf{a}]^T C_f^{-1} [\mathbf{f} - \mathbf{Z}\mathbf{a}]$$

Bordone, Juttner,  
2406.10074



# Exclusive $b \rightarrow c: B \rightarrow D^{(*)}l\nu$



- blue:** • Frequentist fit  $(p, \chi^2/N_{\text{dof}}, N_{\text{dof}}) = (0.00, 2.82, 8)$
- d'Agostini Bias? [d'Agostini, Nucl.Instrum.Meth.A 346 (1994)]

- red:** • Akaike-Information-Criterion analysis [H. Akaike IEEE TAC (19.6,1974)]  
 average over all possible fits with at least two data points  
 and then weighted average:

$$w_{\{\alpha,i\}} = \mathcal{N}^{-1} \exp\left(-\frac{1}{2}(\chi^2_{\{\alpha,i\}} - 2N_{\text{dof},\{\alpha,i\}})\right) \quad \mathcal{N} = \sum_{\text{set} \in \{\alpha,i\}} w_{\text{set}}$$

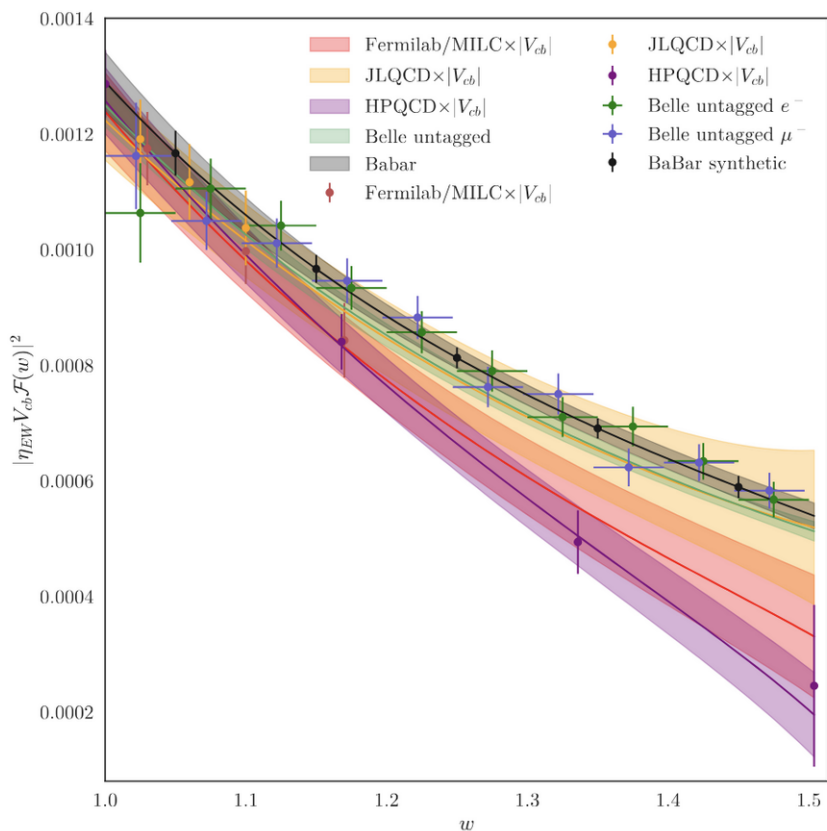
$$|V_{cb}| = \langle |V_{cb}| \rangle \equiv \sum_{\text{set} \in \{\alpha,i\}} w_{\text{set}} |V_{cb}|_{\text{set}}$$

- result more *sensible* and bias apparently reduced

- We find no noteworthy tensions between the results from both strategies
- This analysis confirms a slight tension with inclusive analyses

# Exclusive $b \rightarrow c$ : combined fit Data + LQCD

A. Vaquero

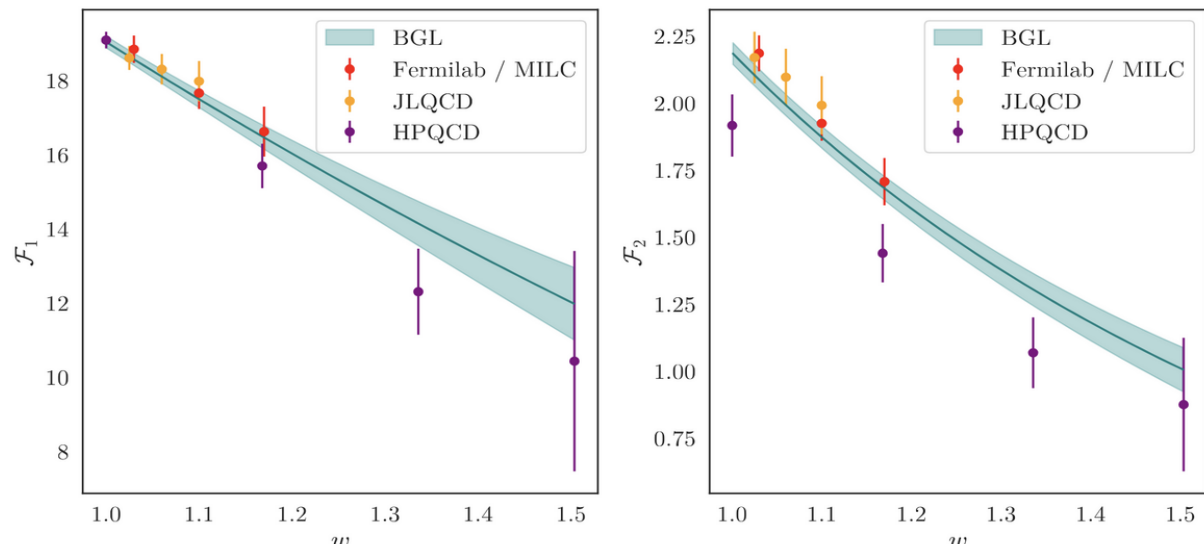


$$|V_{cb}|^{\text{FM}} = 38.40(78) \times 10^{-3}$$

$$|V_{cb}|^{\text{JLQCD}} = 39.19(90) \times 10^{-3}$$

$$|V_{cb}|^{\text{HPQCD}} = 39.31(74) \times 10^{-3}$$

## Combined fit of only Lattice results



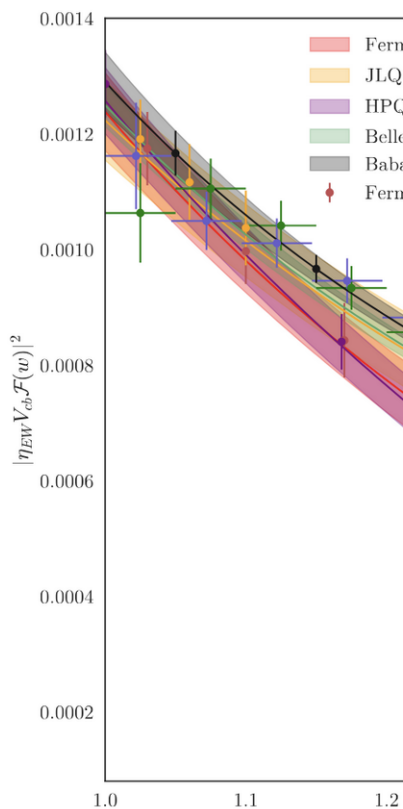
● BGL fit 2222

	w Constraint		w/o Constraint	
	$p$	$R_2(1)$	$p$	$R_2(1)$
MILC	0.51	1.20(12)	0.43	1.27(13)
JLQCD	0.52	0.98(19)	0.25	0.97(19)
HPQCD	0.77	1.39(16)	0.65	1.39(16)
MILC+JLQCD	0.40	1.118(97)	0.36	1.16(11)
MILC+HPQCD	0.44	1.262(93)	0.37	1.262(93)
JLQCD+HPQCD	0.73	1.18(12)	0.67	1.18(12)
All	0.56	1.193(83)	0.50	1.193(83)

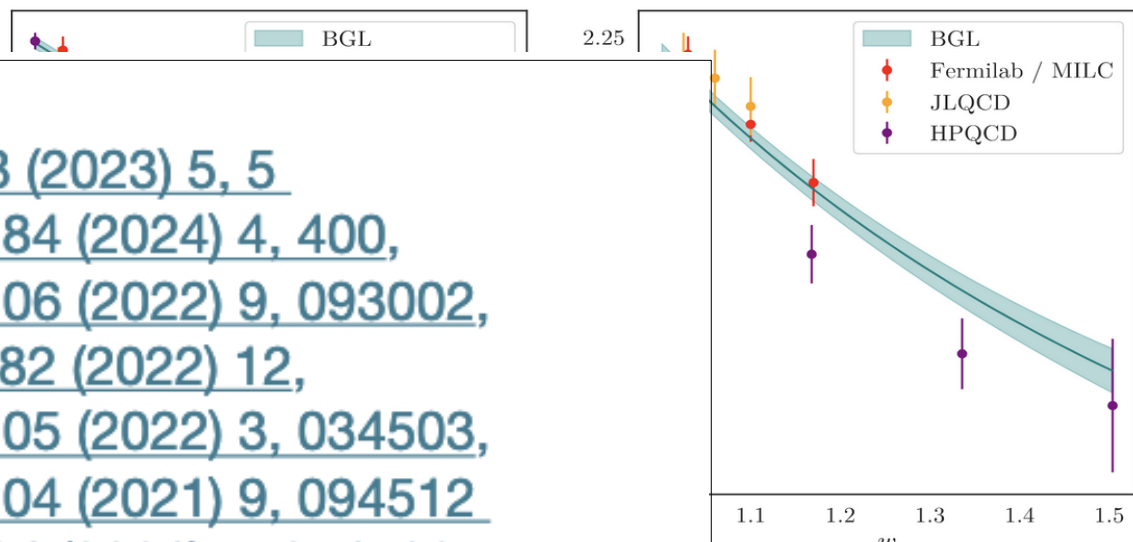
●  $p$ -value of Belle untagged + BaBar BGL fit 223 is  $\approx 0.04$

# Exclusive $b \rightarrow c$ : combined fit Data + LQCD

A. Vaquero



## Combined fit of only Lattice results



### Other related work:

Fedele et al. [PRD 108 \(2023\) 5, 5](#)

Martinelli et al. [EPJC 84 \(2024\) 4, 400](#),

[PRD 106 \(2022\) 9, 093002](#),

[EPJC 82 \(2022\) 12](#),

[PRD 105 \(2022\) 3, 034503](#),

[PRD 104 \(2021\) 9, 094512](#)

Di Carlo et al. [PRD 104 \(2021\) 5, 054502](#)

Gambino [PLB 795 \(2019\) 386-390](#)

Bigi [PLB 769 \(2017\) 441-445](#), [JHEP 11 \(2017\) 061](#)

Bernlochner et al. [PRD 100 \(2019\) 1, 013005](#)

$$|V_{cb}|^{\text{FM}} = 38.40(78) \times 10^{-3}$$

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	$p$	$R_2(1)$
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MILC+HPQCD	0.44	1.262(93)
JLQCD+HPQCD	0.73	1.18(12)
All	0.56	1.193(83)

w/o Constraint

	$p$	$R_2(1)$
)	0.43	1.27(13)
)	0.25	0.97(19)
)	0.65	1.39(16)

•  $p$ -value of Belle untagged + BaBar BGL fit 223 is  $\approx 0.04$

# Exclusive $b \rightarrow c$ : $B \rightarrow D\nu$ , $B \rightarrow D^*\nu$

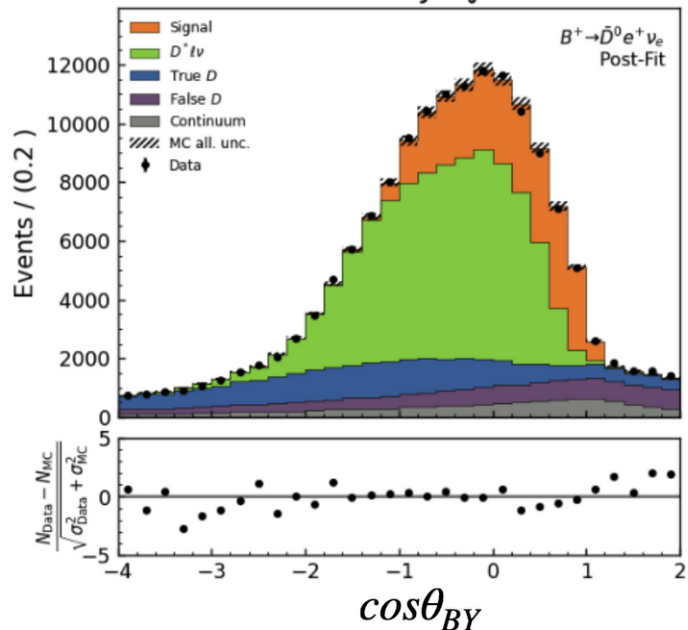
$\bar{B} \rightarrow D^{(*)}\ell^{-}\bar{\nu}_\ell$  with hadronic tagging at *BABAR*

[PRL123,091801(2019) + PRD110,032018(2024)]

Different approach:  
unbinned analyses

B. Dey

**Belle II Preliminary**  $\int \mathcal{L} dt = 189.2 \text{ fb}^{-1}$



$B^0 \rightarrow D^-(K\pi\pi) I^+ \nu$  and  $B^+ \rightarrow D^0(K\pi) I^+ \nu$  : untagged

Exploit isospin symmetry: reduced systematics  
Extract signal yields from fit to  $\cos\theta_{BY}$  (10 bins in  $w$ )

## Update to 362/fb ongoing

	$ V_{cb,BGL} $ [%]	$ V_{cb,BCL} $ [%]
Stat. Error	0.7	0.6
MC Stat. Error	0.4	0.3
$N_{bb}$	0.8	0.8
$f_{00}/f_{+-}$	< 0.1	< 0.1
$\mathcal{B}(D \rightarrow K\pi(\pi))$	0.4	0.4
Selection	0.2	0.2
$\mathcal{B}(B \rightarrow X_c \ell \nu)$	0.2	0.1
LeptonID	0.1	0.1
KaonID	0.4	0.4
Tracking efficiency	0.5	0.5
$B \rightarrow D\ell\nu_\ell$ form factor	0.8	0.4
$B \rightarrow D^*\ell\nu_\ell$ form factor	0.1	0.1
$\cos\theta_{BY}$ background modelling	0.2	0.2
$w$ background modelling	0.5	0.4
$\tau_{B^{0/\pm}}$	0.1	0.1
Total systematic	1.5	1.4
Theory <a href="#">PRD 79, 013008</a> , <a href="#">PRD 93, 119906</a>	1.3	1.2
Total	2.1	1.9

Can we analyse  $B \rightarrow D^*\ell\nu$  and  $B \rightarrow D\ell\nu$  together?

[inspired by a BaBar analysis from 2008 ([PRD 79, 012002](#))]

**Ongoing analysis:** no soft pion and independent from  $f_{00}$



# Exclusive $b \rightarrow c$ : $B \rightarrow D^* l \nu$ @ LHCb

## Exploiting angular observables: $B^0 \rightarrow D^* \mu \nu$

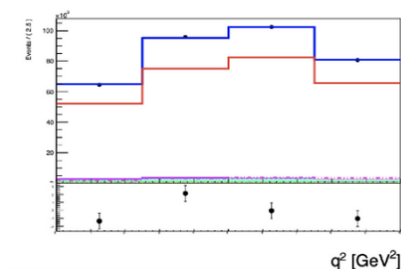
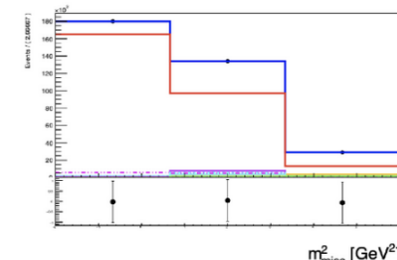
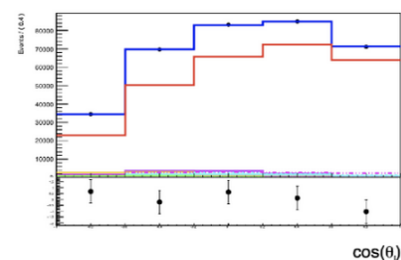
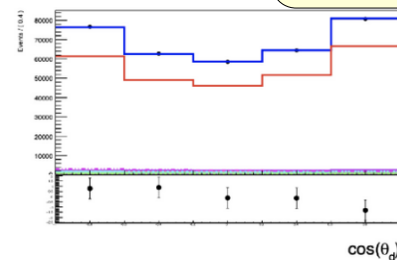
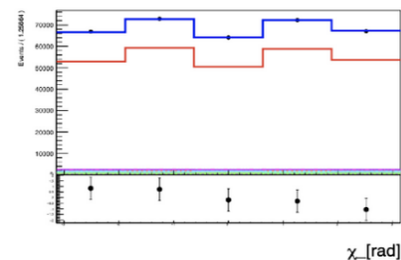
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L. Grillo

Enough statistics  
for a 5D fit !

- ▶ Extract directly Wilson Coefficients and FF parameters from fit to data
- ▶ Shape analysis - no attempt to measure  $|V_{cb}|$
- ▶ SM fits: CLN ([Nuclear Physics B 530 \(1998\) 153-181](#)), BGL ([Phys.Rev. D56 \(1997\) 6895-6911](#)) and BLPR parametrisation for hadronic FF
- ▶ NP fits: BLPR parametrisation (F. Bernlochner et. al. [Phys. Rev. D 95, 115008 \(2017\)](#))
- ▶ High statistics  $B^0 \rightarrow D^* \mu \nu$  sample(s), could fit for hadronic FF parameters and NP WC at the same time, if correlations allow
- ▶ First sensitivity estimates [B. Mitreska CERN-THESIS-2022-105](#)

Pseudo-experiments



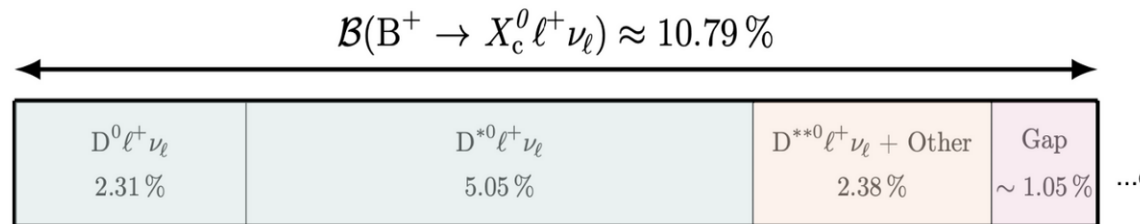
Different advantages and challenges wrt measurements performed at the b-factories

# Exclusive $b \rightarrow c: B \rightarrow D^{**}$

## Some blocks are still missing...

$$B \rightarrow D^{**} \ell \nu$$

A leading systematic for many analyses (not just semileptonic):



- $D_0^*$  and friends are not what we think they are. We need more spectra (not BFs) to disentangle different resonances.
- The  $D^*$  tail should be correctly handled in MC!



Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \rightarrow D^* \ell^+ \nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \rightarrow D_1 \ell^+ \nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \rightarrow D_2^* \ell^+ \nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \rightarrow D_1' \ell^+ \nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \rightarrow D \pi \pi \ell^+ \nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \rightarrow D^* \pi \pi \ell^+ \nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \rightarrow D \eta \ell^+ \nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \rightarrow D^* \eta \ell^+ \nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \rightarrow X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

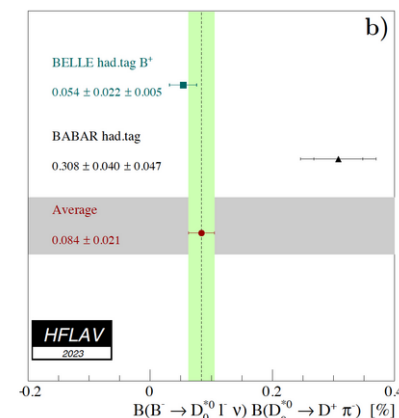
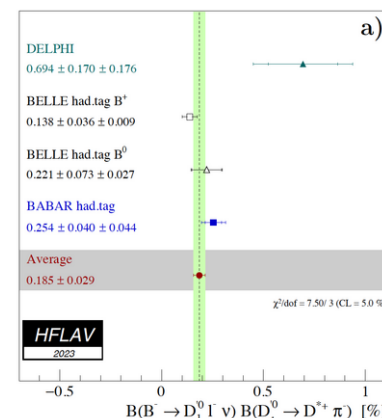
Fairly well known.

Broad states based on 3 measurements. (BaBar, Belle, DELPHI)

Some hints from BaBar & recent Belle result.

Fill the gap with current "best guess".

## Broad resonances



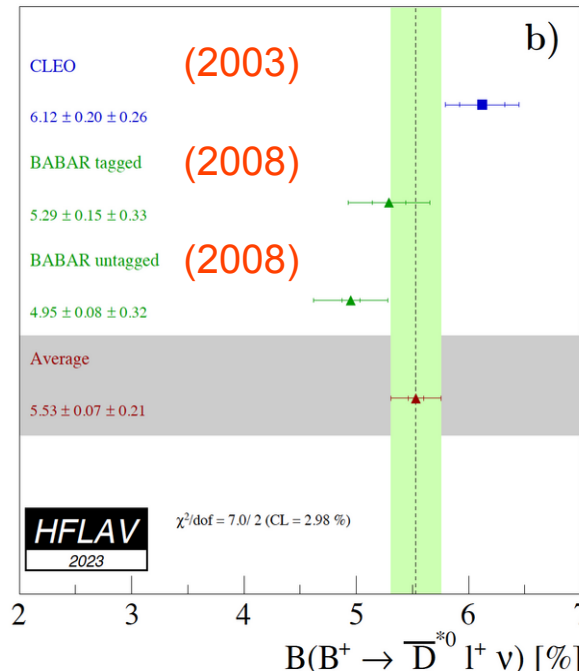
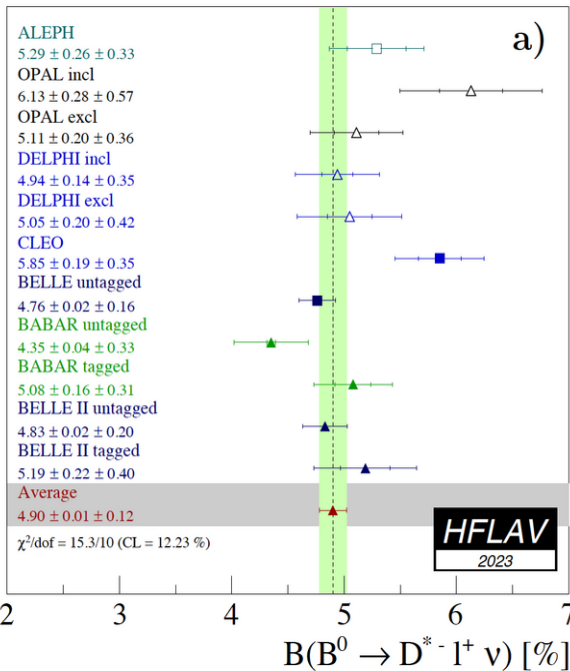
# Exclusive $b \rightarrow c$ : discussion points

- Are there alternative approaches to HPQCD to cover full  $w$  range?
- Not only LQCD, LCSR + HQE approaches (N. Gubernari)
- Paramount to provide data in a form that can be reused: provide more informations? Final tuples, code...

- Crucial to study  $B \rightarrow D^{**}$  semileptonic decays
  - New measurements required: BFs +  $q^2$  shapes
- Combine with analyses of fully hadronic  $b \rightarrow c$  decays

$B^0 \rightarrow D^{*+} l \nu$

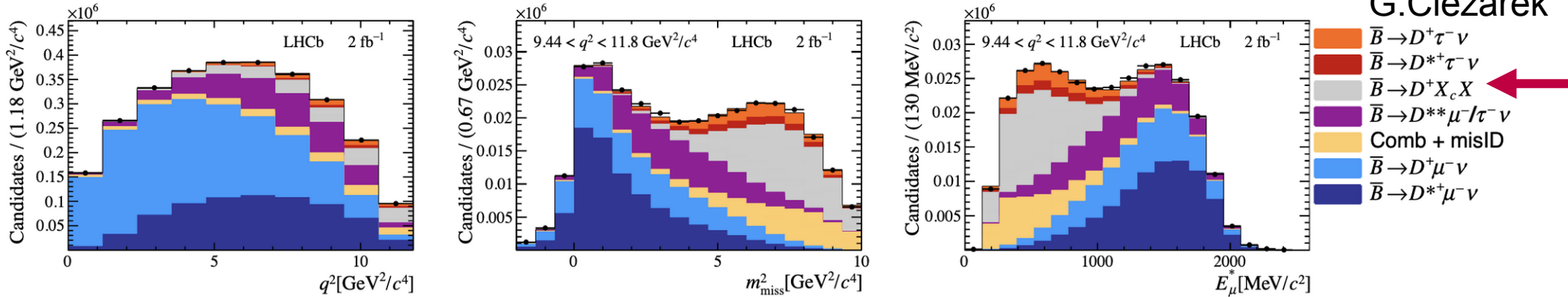
$B^+ \rightarrow D^{*0} l \nu$



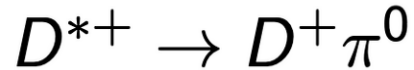
- Re-measure BFs
- How should experiments (and LQCD groups) deal with old measurements (predictions) ?

# Exclusive $b \rightarrow c$ : $R(D^+)$ - $R(D^{*+})$

G.Ciezarek



- Very nice additional sample reconstructed with neutral!



- Important caveat: measurements assume SM shape+uncertainties for  $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_{\tau}$ 
  - Fine for a SM null test
  - If there is non lefthanded vector new physics, measurements of  $\mathcal{R}(D^{(*)})$  no longer valid
- Much more to come!

# Exclusive $b \rightarrow c: R(D^+)-R(D^{*+})$

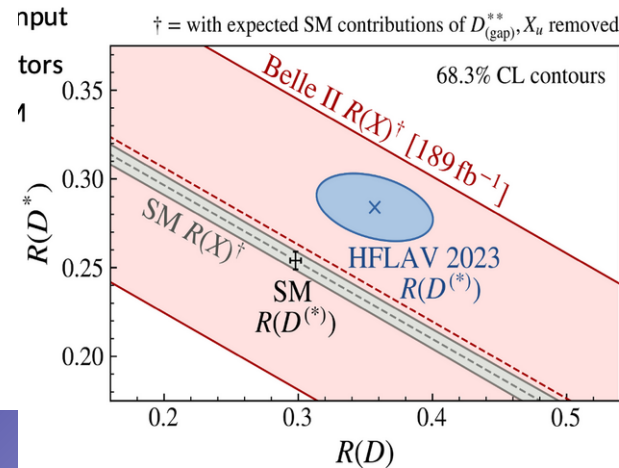
- **Background from  $b \rightarrow c \bar{c} q$  decays (double-charm)**
  - Need additional measurements: absolute BF of specific modes (also inclusive BF)
    - For the LHCb ongoing analysis in B2OC: please, extract also branching fractions!
- $R(D)-R(D^*)$  with LHCb hadronic tau: need external inputs:  $B \rightarrow D^*3\pi$

## Common systematics

- $B \rightarrow D^{**} / FF$ : different treatment between different meas./exp.
- Assumptions are implied on correlations in combining measurements

## Other observables

- Angular observables, tau polarization: try to go differential as much as possible
- Inclusive  $R(X_c)$



## SysVar

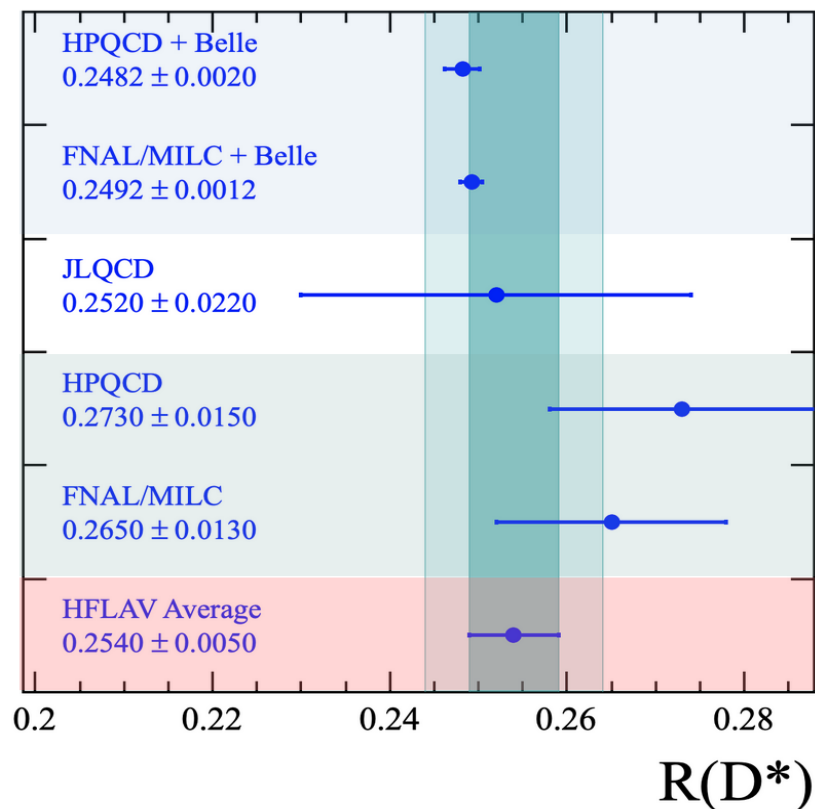
A New Tool for Enhancing Consistency in the Treatment of Systematic Uncert.

- properly combine Belle II measurements

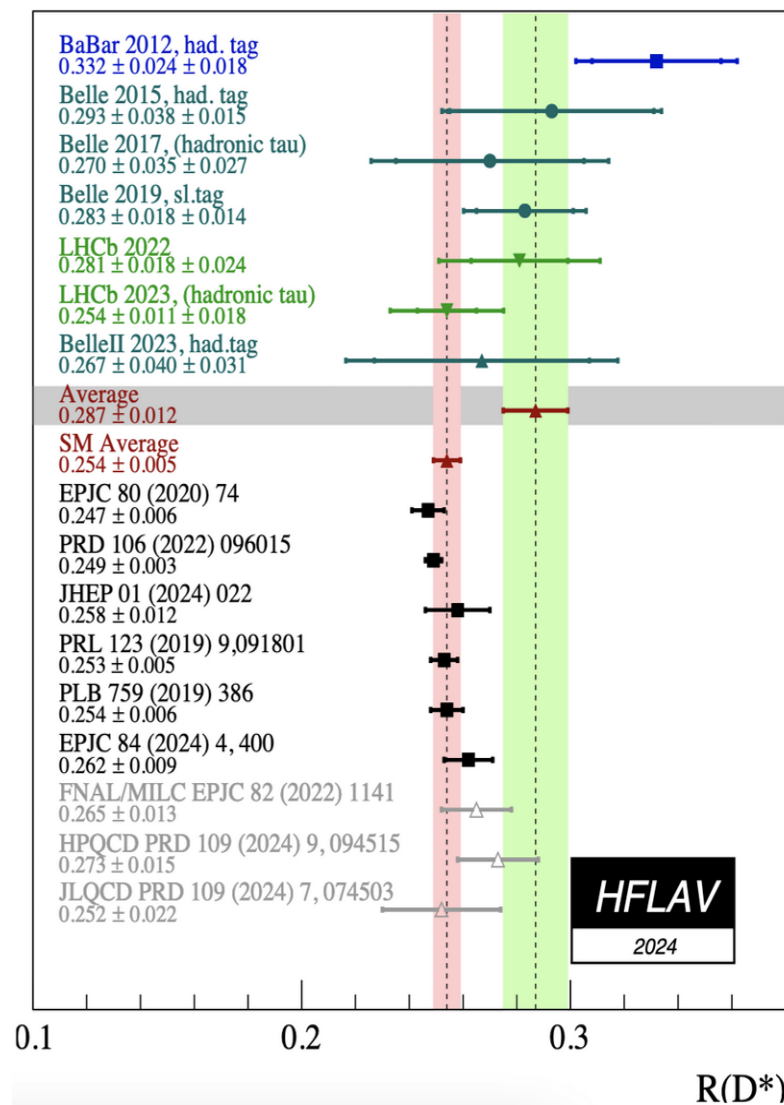
I. Tsakidis

# Exclusive $b \rightarrow c$ : SM predictions of $R(D)$

Most of the SM predictions use fit of theory inputs (mostly LQCD) and experimental data of  $B \rightarrow D/D^* \ell \nu$  with light leptons

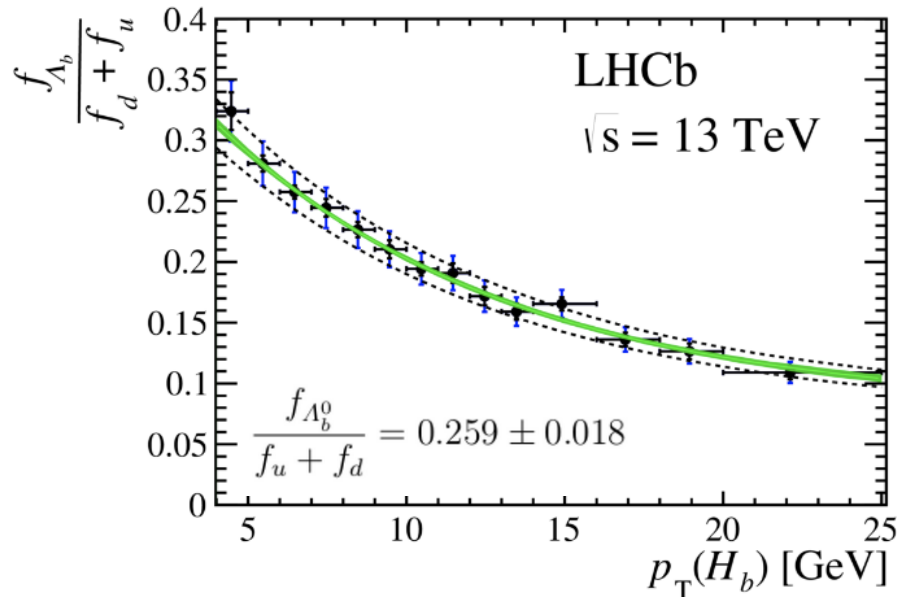


Impact of including  $B \rightarrow D^* \ell \nu$  data from Belle  
[PRD 100, 052007 \(2019\)](#)



# Exclusive $b \rightarrow c$ : b-baryons

A. Lupato



- Broad SL physics program at LHCb: measurement of CKM matrix elements, angular analysis, Wilson coefficients, LFU tests...
- Successful Run1 and Run2: 3+6 fb<sup>-1</sup>, still many analysis ongoing

b-baryons should be investigated more

- Additional LQCD would be desirable!
- But HQET is very predictive for  $\Lambda_b$  ( $ud$  pair in spin-0 configuration)
- Additional baryons?
  - $\Omega_b(bss) \rightarrow \Omega_c(css) \mu \nu$
  - ...

## FLAG2021

Process	Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment
$\Lambda_b \rightarrow \Lambda_c^*(2625) \ell^- \bar{\nu}_\ell$	Meinel 21	[147]	2+1	A	○	○	■	○	✓
$\Lambda_b \rightarrow \Lambda_c^*(2595) \ell^- \bar{\nu}_\ell$	Meinel 21	[147]	2+1	A	○	○	■	○	✓
$\Lambda_b \rightarrow \Lambda^*(1520) \ell^+ \ell^-$	Meinel 20	[157]	2+1	A	○	○	■	○	✓
$\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$	Detmold 16	[152]	2+1	A	○	○	■	○	✓
$\Lambda_b \rightarrow p \ell^- \bar{\nu}_\ell$	Detmold 15	[145]	2+1	A	○	○	■	○	✓
$\Lambda_b \rightarrow \Lambda_c \ell^- \bar{\nu}_\ell$	Detmold 15, Datta 17	[145, 146]	2+1	A	○	○	■	○	✓

# Inclusive $b \rightarrow c$

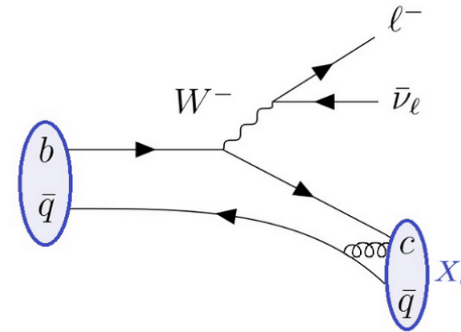
The inclusive semi-leptonic decay rate in the SM

$$\Gamma(\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell) = |V_{cb}|^2 G_F^2 \frac{m_b^5}{16\pi^3} f(m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \dots)$$

is computed through a power expansion in  $\Lambda_{\text{QCD}}/m_b \sim 0.1$

## Heavy Quark Expansion (HQE)

$$f(m_b, m_c, \dots) = f^{\text{LP}} + f^{\text{NLP}, \pi} \frac{\mu_\pi^2}{m_b^2} + f^{\text{NLP}, G} \frac{\mu_G^2}{m_b^2} + f^{\text{NNLP}, D} \frac{\rho_D^3}{m_b^3} + f^{\text{NNLP}, LS} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^4}{m_b^4}\right)$$



G.Finauri

## Theory State of the Art in $\bar{B} \rightarrow X_c \ell \bar{\nu}$

	$dE_\ell$	$dm_X^2$	$dq^2$	$\Gamma$
1	$\alpha_s^2$ [Melnikov 2008] [Pak, Czarnecki 2008]	$\alpha_s^2$	$\alpha_s^2$ [Fael, Herren 2024]	$\alpha_s^3$ [Fael, Schönwald, Steinhauser 2020]
$1/m_b^2$	$\alpha_s$ [Alberti, Ewerth, Gambino, Nandi 2012, 2013]	$\alpha_s$	$\alpha_s$	$\alpha_s$
$1/m_b^3$	1 [Gremm, Kapustin 1997]	1	$\alpha_s$ [Mannel, Moreno Pivovarov 2021]	$\alpha_s$ [Mannel, Pivovarov 2019]
$1/m_b^{4,5}$ $1/(m_b^3 m_c^2)$	1 [Mannel, Turczyk, Uraltsev 2010]	1	1 [Mannel, Milutin, Vos 2023]	1 [Mannel, Turczyk, Uraltsev 2010]

$$\langle O^n \rangle_{\text{cut}} = \frac{\int_{\text{cut}} dO O^n \frac{d\Gamma}{dO}}{\int_{\text{cut}} dO \frac{d\Gamma}{dO}}$$

## Experimental inputs from:

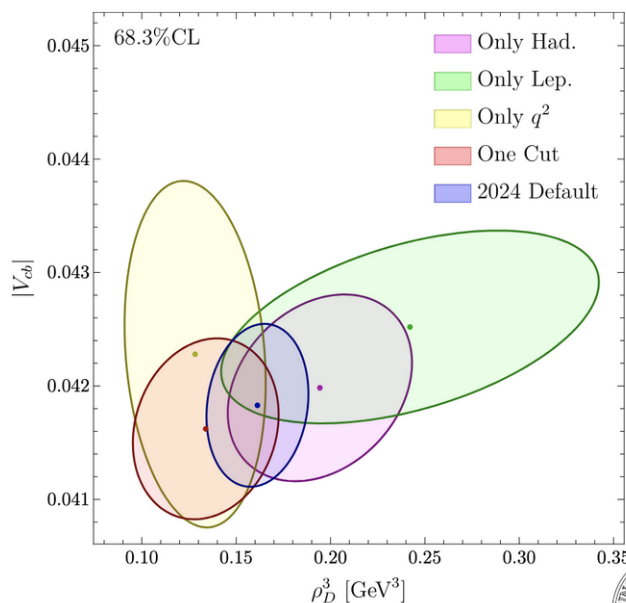
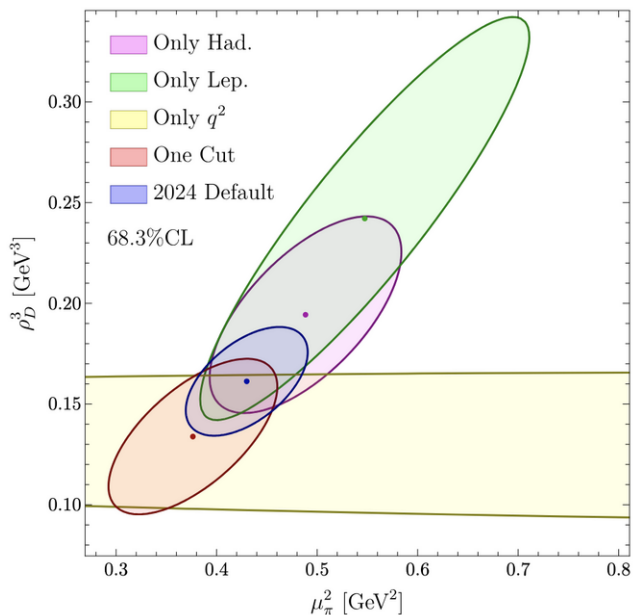
- Delphi, CDF, BaBar, Belle
  - $E_{\text{lep}}, M_X, \Delta\text{BF}$
- Belle, Belle II
  - $q^2$



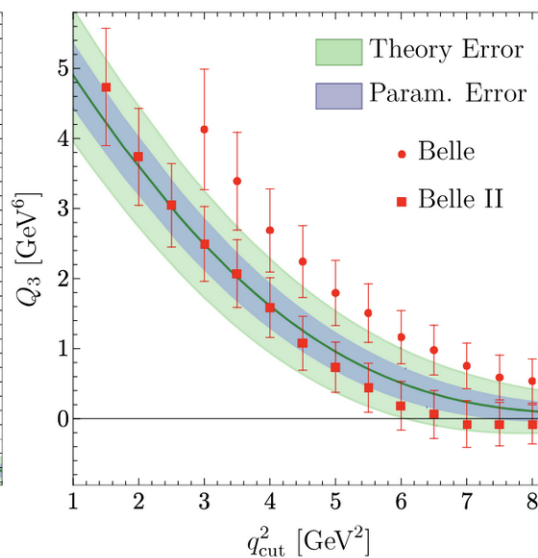
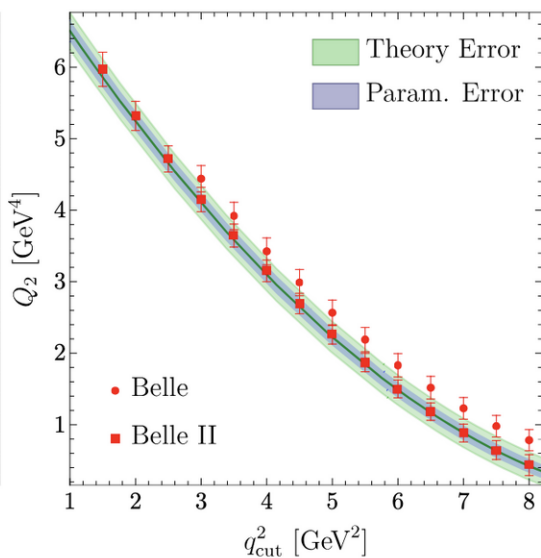
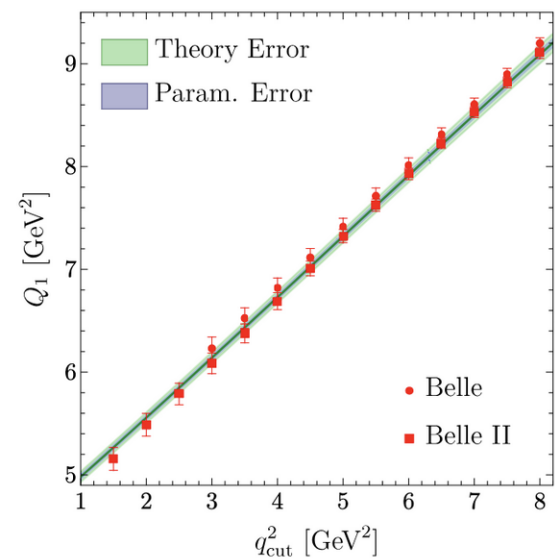
# Inclusive $b \rightarrow c$

G.Finauri

$\sigma = 1.1\%$



$m_b^{\text{kin}}$	$\bar{m}_c(2 \text{ GeV})$	$\mu_\pi^2$	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	$\rho_{LS}^3$	$\text{BR}_{cl\nu}$	$10^3  V_{cb} $
4.573	1.090	0.454	0.288	0.176	-0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48
1	0.380	-0.219	0.557	-0.013	-0.172	-0.063	-0.428
	1	0.005	-0.235	-0.051	0.083	0.030	0.071
		1	-0.083	0.537	0.241	0.140	0.335
			1	-0.247	0.010	0.007	-0.253
				1	-0.023	0.023	0.140
					1	-0.011	0.060
						1	0.696
							1



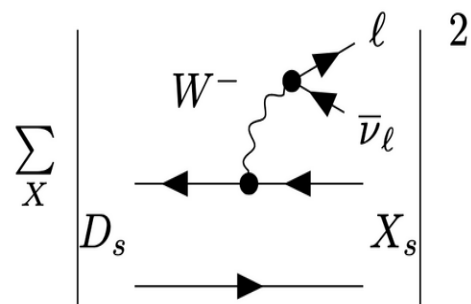
Systematic shift between Belle and Belle II data  $\sim 2\sigma$

Inclusion of NLO and NNLO terms have big impact on HQE parameters

## Inclusive semileptonic decays from Lattice QCD

Lattice QCD might be able to provide input of non-perturbative parameters

On-going analysis of inclusive semileptonic decay rate  $D_s \rightarrow X_s \ell \bar{\nu}_\ell$



$$\frac{d\Gamma}{dq^2 dq_0^2 dE_l} = \frac{G_F^2 |V_{cs}|^2}{8\pi^2} L_{\mu\nu} W^{\mu\nu}$$

$L_{\mu\nu}$ : Leptonic tensor (analytically known)

$W^{\mu\nu}$ : Hadronic tensor (nonperturbative QCD)

### Challenges on the lattice

- Require external states
  - Long time separations
- Large number of states
  - Identify all of them?
- Extraction of  $W_{\mu\nu}$  from correlator  
ill-posed problem (**inverse problem**)

### Future:

- Consider various observables:  $q^2$  moments,  $E_l, \dots$ ?
- Extend to  $B_s \rightarrow \text{LHCb}$  prospects

# Inclusive $c \rightarrow s/d$

A. Gilman

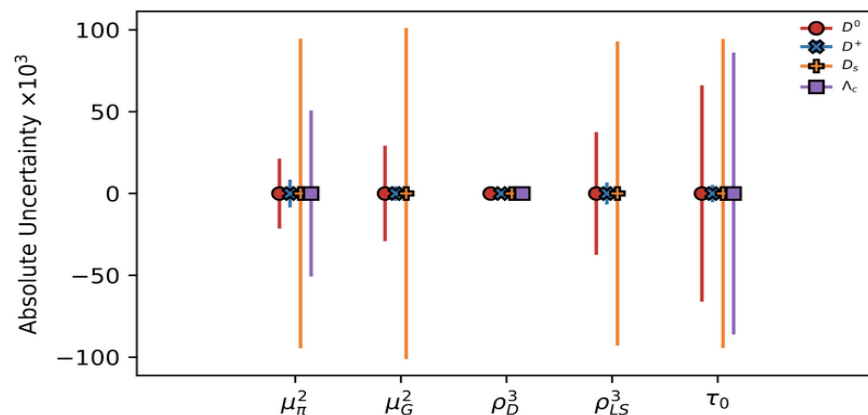
Prospects for inclusive semileptonic charm decays  
based on arXiv:2408.10063

The (not as) Heavy Quark Expansion



- ▶ Heavy-Quark Expansion (HQE) expands in  $\frac{\Lambda_{QCD}}{m_q}$
- ▶ Since  $m_c < m_b$ , converges more slowly for charm than but more sensitive to hadronic matrix elements

Estimated sensitivity to HQE parameters  
Based on fast simulation of currently available data



Very strong prospects for first determinations of charm HQE parameters with current data and first HQE analysis of inclusive heavy baryon decays.

BESIII datasets

	$D^0$	$D^+$	$D_s^+$	$\Lambda_c^+$
$E_{cm}$ GeV	3.773	3.773	4.130–4.230	4.600–4.699
Int. Lumi. [ $\text{fb}^{-1}$ ]	21	21	7.1	4.5
Estimated DT Yields	200000	700000	30000	4300

Strong prospects for competitive inclusive determinations of  $|V_{cs}|$ , maybe also for  $|V_{cd}|$

# Inclusive $b \rightarrow u$



Full data: 711fb<sup>-1</sup>



Run1 data: 364fb<sup>-1</sup>, ongoing

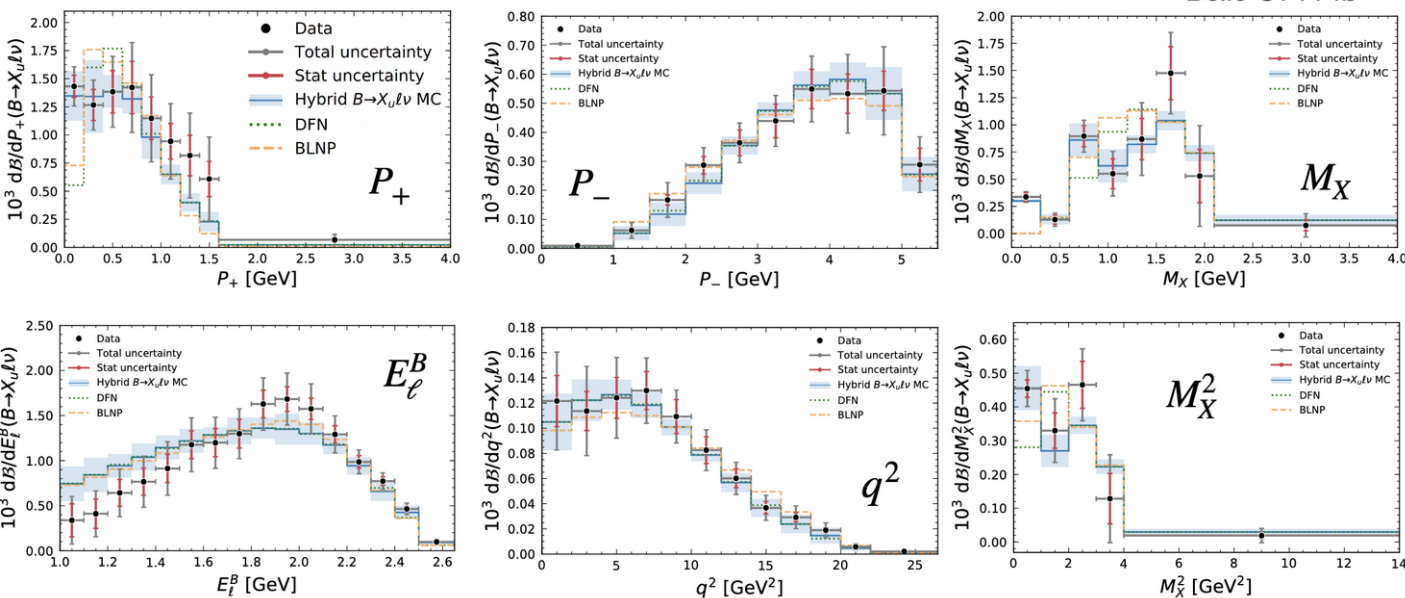
L.Cao

- $\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)$  and  $|V_{ub}|$  [PRD.104..012008 (2021)]
- $d\mathcal{B}/dx, x = M_X, M_X^2, q^2, P^\pm, E_\ell^B$  [PRL.127.261801 (2021)]
- Simultaneous determination of excl. & incl.  $|V_{ub}|$  [PRL.131.211801 (2023)]



- $\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)$  and  $|V_{ub}|$
- $d\mathcal{B}/dx, x = x_{\text{Belle}} + \cos\theta_\ell$
- Weak annihilation in  $B \rightarrow X_u \ell \nu$

Belle @711 fb<sup>-1</sup>



Unfolded/eff. corrected  
1-d projections

Improvements on  
inclusive signal  
modeling are urgently  
needed!

# Inclusive $b \rightarrow u$ : theory developments

## Expected updates for BLNP and GGOU

GGOU, Gambino2007

B. Capdevila

⇒ We have developed a robust numerical approach to extract the  $b \rightarrow uW^*$  form factors at NLO and NNLO, overcoming the challenges posed by missing analytic structures

BLNP, Bosch2004 and Lange2005

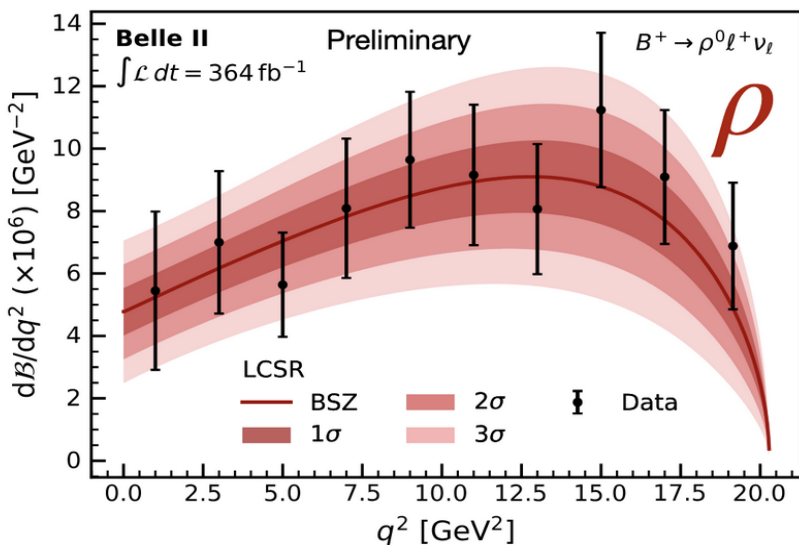
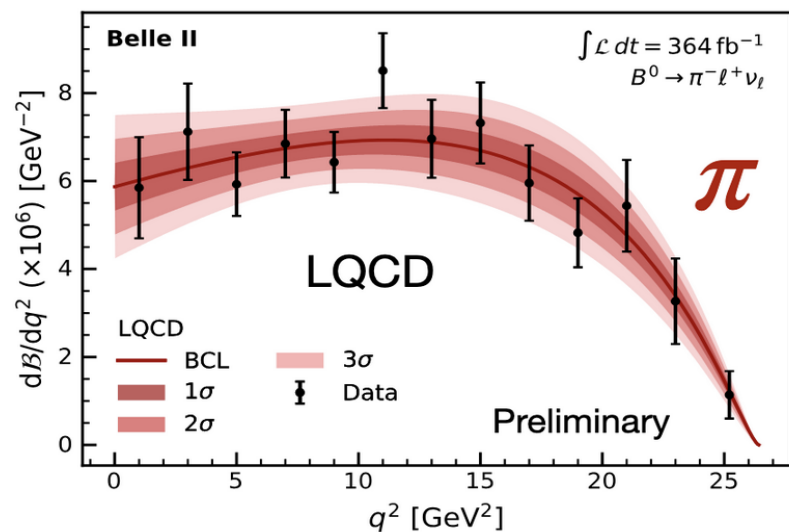
B. O. Lange

### BLNP mandatory maintenance

- reduced perturbative uncertainty at NNLO.
- reduced SF uncertainty by including  $\rho_D^3$
- updating power corrections

# Exclusive $b \rightarrow u$ @ Belle II

F. Bernlochner



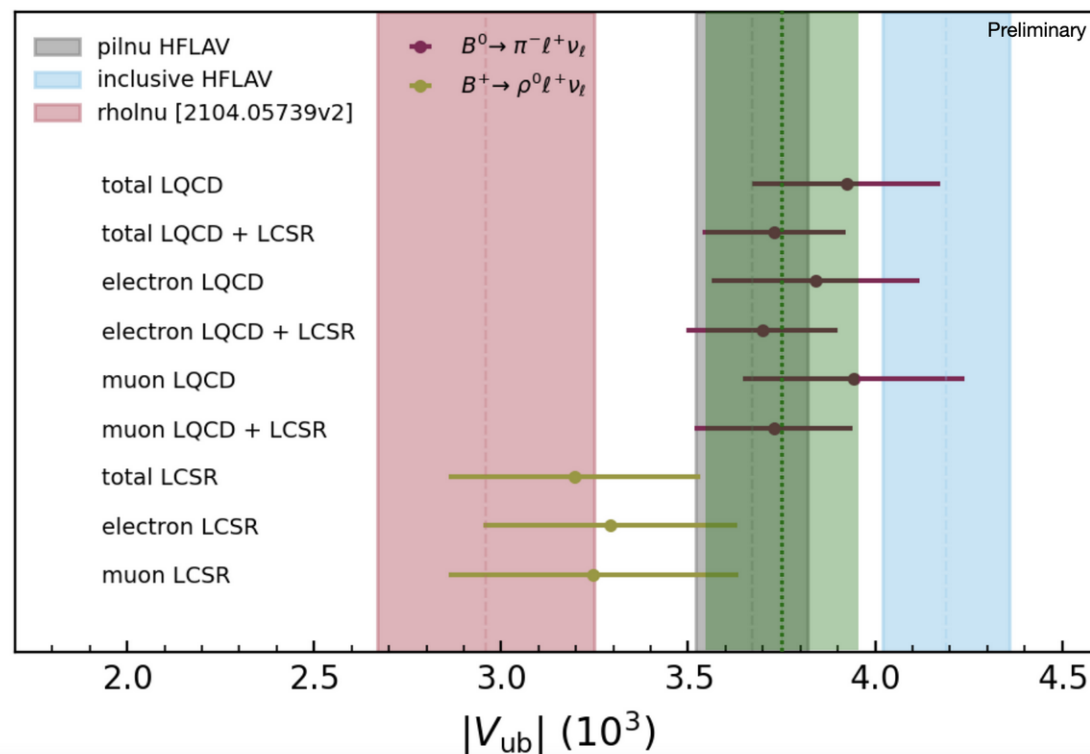
Determine  $|V_{ub}|$  by minimising  $\chi^2$ :

$$\chi^2 = \sum_{i,j}^N (\Delta B_i - \Delta \Gamma_i \tau) C_{ij}^{-1} (\Delta B_j - \Delta \Gamma_j \tau) + \chi_{\text{Theory}}^2$$

- Experimental observation
- Experimental covariance
- Theoretical prediction

	$B^0 \rightarrow \pi^+ l^- \bar{\nu}_l$	$B^- \rightarrow \rho^0 l^- \bar{\nu}_l$
Form factor param.	Bourrely-Caprini-Lellouch (BCL) <a href="#">Phys. Rev. D 82, 099902</a>	Bharucha-Straub-Zwicky (BSZ) <a href="#">J. High Energ. Phys. 2016, 98</a>
Theory input	LQCD <a href="#">Eur. Phys. J. C 82 (2022) 869</a> LQCD + LCSR <a href="#">J. High Energ. Phys. 2021, 36</a>	LCSR <a href="#">J. High Energ. Phys. 2016, 98</a>

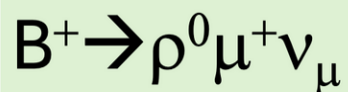
Updated HFLAV 2024 (LQCD only, including latest JLQCD)





Chosen normalisation mode is  $B^- \rightarrow J/\psi(\mu^+\mu^-)K^-$

- Very clean and large sample
  - small additional statistical uncertainty
- Same number of tracks and similar topology to signal when one muon is neglected.
  - reduced systematic uncertainty on efficiencies ratio

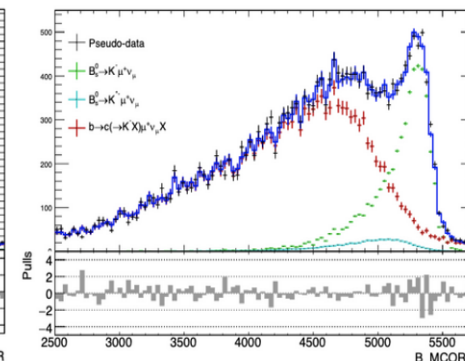
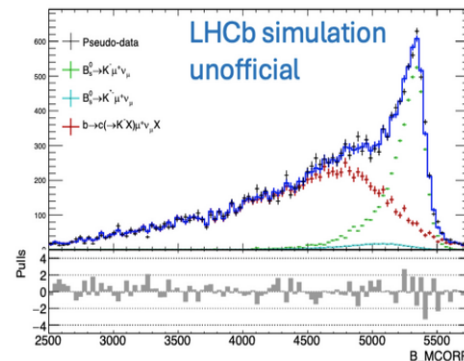


- Goal: measure the differential branching fraction in bins of  $q^2$  and extract the FF
- Normalization mode chosen as  $B^+ \rightarrow \bar{D}^0 \mu^+ \nu_\mu$ ,  $\bar{D}^0 \rightarrow \pi^+ \pi^-$

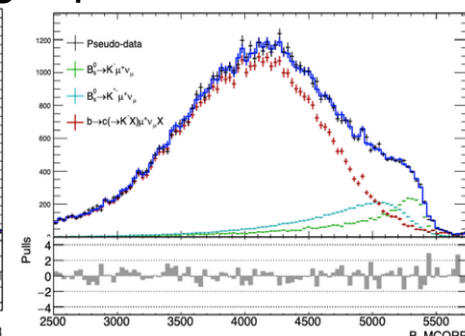
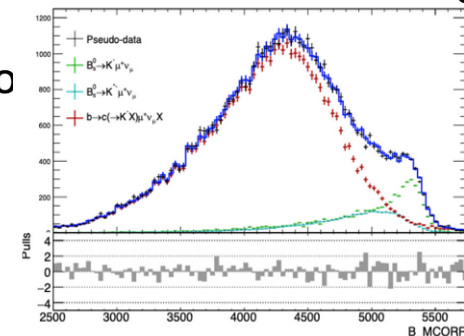
$$B(B^+ \rightarrow \bar{D}^0(\pi^+ \pi^-) \mu^+ \nu_\mu) = (3.34 \pm 0.14) 10^{-5} \quad \text{with a 4.2\% relative uncertainty}$$

- Same set of final state tracks to reduce systematic uncertainty in efficiency ratio

Low  $q^2$



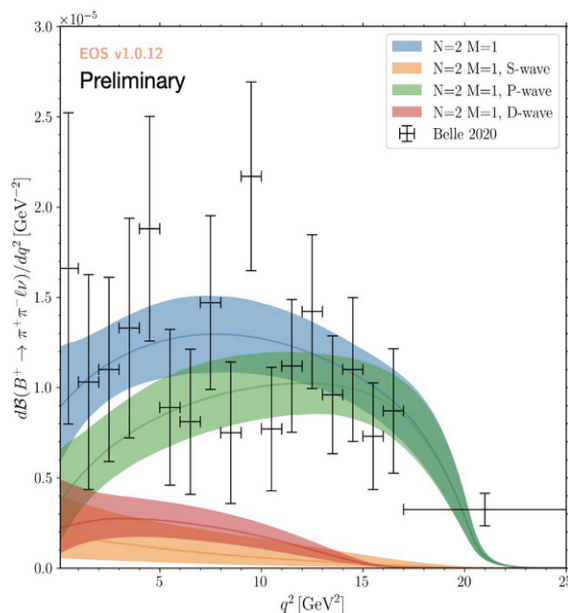
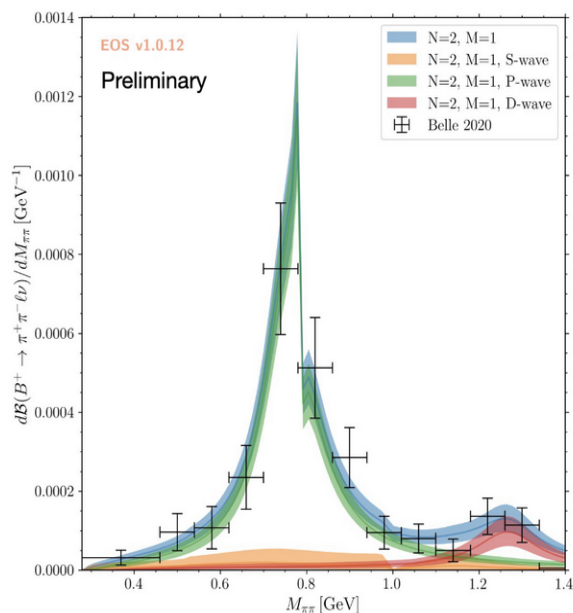
High  $q^2$



# Exclusive $B \rightarrow 2\pi \ell \nu$

R. von Tonder  
(paper in preparation)

A recent Belle measurement of  $B^+ \rightarrow \pi^+ \pi^- \ell^+ \nu$  decays provides 2-D correlated spectra of  $q^2$  &  $M_{\pi^+ \pi^-}$ .  
We fit to the 2-D spectra using the [EOS package](#).



## Discussed points:

- $B \rightarrow \rho, \omega$  form factors: rely on LCSR, but computed for narrow states
- Start from general  $B \rightarrow \pi^+ \pi^-$ : require additional data: Belle II + LHCb?
- $B^+ \rightarrow \pi^+ \pi^0 \ell^+ \nu$ ? simpler than  $\pi^+ \pi^-$  (absence of  $f_0$  states)



# Exclusive $b \rightarrow u$

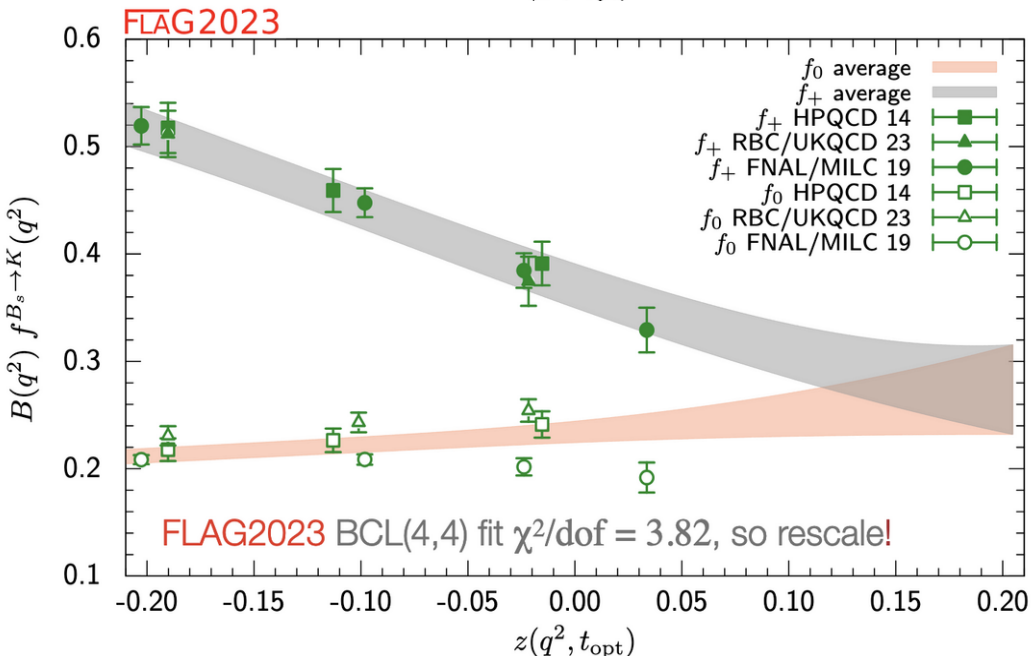
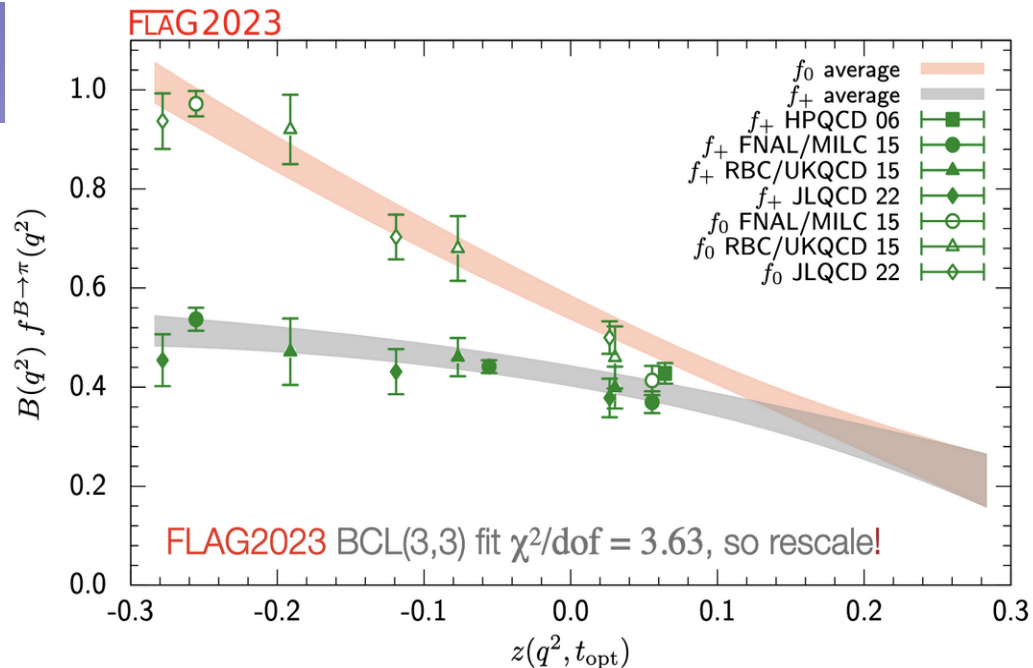
## $B \rightarrow \pi l \nu$ in FLAG2023

A.Kronfeld

- **HPQCD 2006** [[arXiv:hep-lat/0601021](https://arxiv.org/abs/hep-lat/0601021)]: NRQCD  $b$  quark on 2+1 asqtad sea (MILC); six ensembles at two lattice spacings; smallest  $m_l = m_s/8$ .
- **RBC/UKQCD 2015** [[arXiv:1501.05373](https://arxiv.org/abs/1501.05373)]: RHQ  $b$  quark on 2+1 DWF sea; five ensembles at two lattice spacings; smallest  $m_l = m_s/8$ .
- **Fermilab-MILC 2015** [[arXiv:1503.07839](https://arxiv.org/abs/1503.07839)]: Fermilab  $b$  quark on 2+1 asqtad sea (MILC); twelve ensembles at four lattice spacings; smallest  $m_l = m_s/20$ . Supersedes [arXiv:0811.3640](https://arxiv.org/abs/0811.3640).
- **JLQCD 2022** [[arXiv:2203.04938](https://arxiv.org/abs/2203.04938)]: DWF  $h$  quark,  $m_h \leq 2.44m_c$ , on 2+1 DWF sea; eleven ensembles at three lattice spacings; smallest  $m_l = 7m_s/80$ .

## $B_s \rightarrow K l \nu$ in FLAG2023

- **HPQCD 2014** [[arXiv:1406.2279](https://arxiv.org/abs/1406.2279)]: NRQCD  $b$  quark on 2+1 asqtad sea (MILC); five ensembles at two lattice spacings; smallest  $m_{l,sea} = m_{s,sea}/10$ ,  $m_{l,val} = m_{s,val}/7$ .
- **Fermilab-MILC 2019** [[arXiv:1901.02561](https://arxiv.org/abs/1901.02561)]: Fermilab  $b$  quark on 2+1 asqtad sea (MILC); six ensembles at three lattice spacings; smallest  $m_l = m_s/20$ .
- **RBC/UKQCD 2023** [[arXiv:2303.11280](https://arxiv.org/abs/2303.11280)]: RHQ  $b$  quark on 2+1 DWF sea; six ensembles at three lattice spacings; smallest  $m_l = m_s/10$ . Supersedes RBC/UKQCD 2015 [[arXiv:1501.05373](https://arxiv.org/abs/1501.05373)], says FLAG.



# Exclusive $b \rightarrow u$

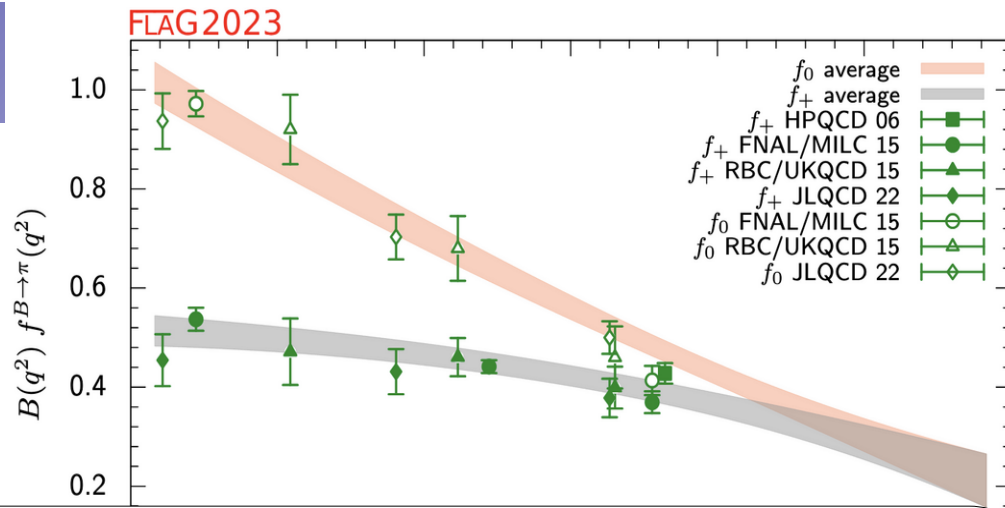
## $B \rightarrow \pi l \nu$ in FLAG2023

A.Kronfeld

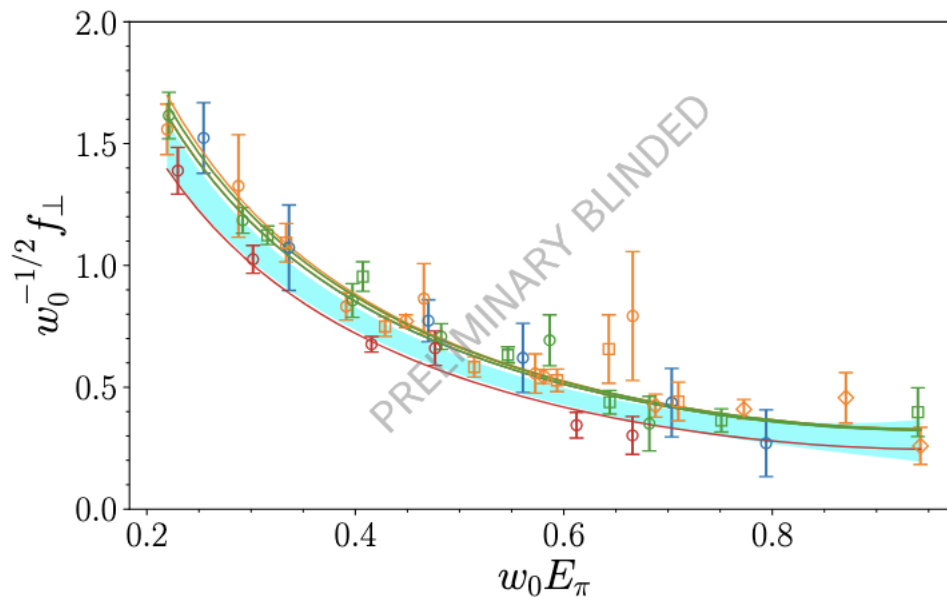
- **HPQCD 2006** [[arXiv:hep-lat/0601021](https://arxiv.org/abs/hep-lat/0601021)]: NRQCD  $b$  quark on  $2+1$  asqtad sea (MILC); **six** ensembles at **two** lattice spacings; smallest  $m_l = m_s/8$ .
- **RBC/UKQCD 2015** [[arXiv:1501.05373](https://arxiv.org/abs/1501.05373)]: RHQ  $b$  quark on  $2+1$  DWF sea; **five** ensembles at **two** lattice spacings; smallest  $m_l = m_s/8$ .
- **Fermilab-MILC 2015** [[arXiv:1503.07839](https://arxiv.org/abs/1503.07839)]: Fermilab  $b$  quark on  $2+1$  sea (MILC); **twelve** ensembles at **four** lattice spacings; Supersedes [arXiv:0811.3640](https://arxiv.org/abs/0811.3640).
- **JLQCD 2022** [[arXiv:2203.04938](https://arxiv.org/abs/2203.04938)]: DWF  $h$  quark,  $m_l = m_s/8$  sea; **eleven** ensembles at **three** lattice spacings; sr

## $B_s \rightarrow K l \nu$ in FLAG2023

- **HPQCD 2014** [[arXiv:1406.2279](https://arxiv.org/abs/1406.2279)]: NRQCD  $b$  quark on  $2+1$  sea (MILC); **five** ensembles at **two** lattice spacings; sr  $m_{l,\text{val}} = m_{s,\text{val}}/7$ .
- **Fermilab-MILC 2019** [[arXiv:1901.02561](https://arxiv.org/abs/1901.02561)]: Fermilab  $b$  quark on  $2+1$  sea (MILC); **six** ensembles at **three** lattice spacings; sr  $m_{l,\text{val}} = m_{s,\text{val}}/7$ .
- **RBC/UKQCD 2023** [[arXiv:2303.11280](https://arxiv.org/abs/2303.11280)]: RHQ  $b$  quark on  $2+1$  sea (RBC/UKQCD 2015 [[arXiv:1501.05373](https://arxiv.org/abs/1501.05373)]), says FLAG2023



Fermilab-MILC on  $2+1+1$ -Flavor Ensembles: update ongoing





Fruitful discussions in a nice venue

More than I summarized here!

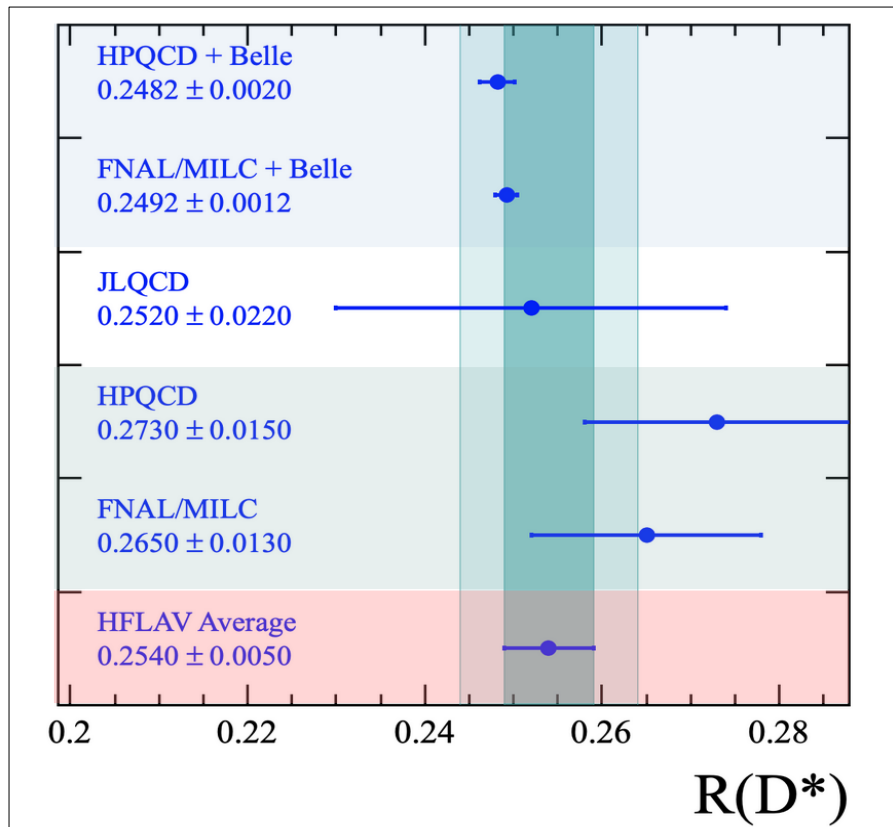
(I apologize for not mentioning the many interesting contributions)

# Thanks!

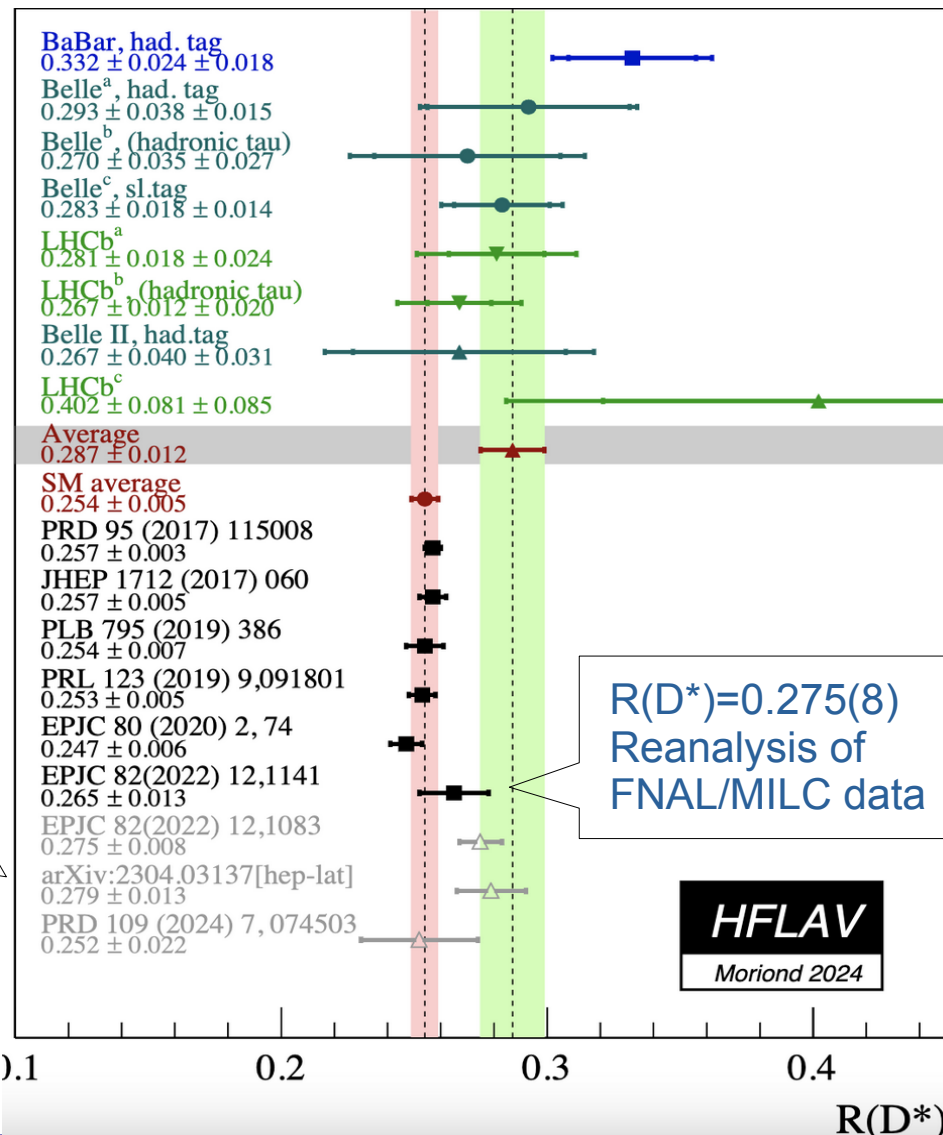
# BACKUP

# Status of $R(D^*)$ predictions

Most of the SM predictions use fit of theory inputs (mostly LQCD) and experimental data of  $B \rightarrow D/D^* \ell \nu$  with light leptons



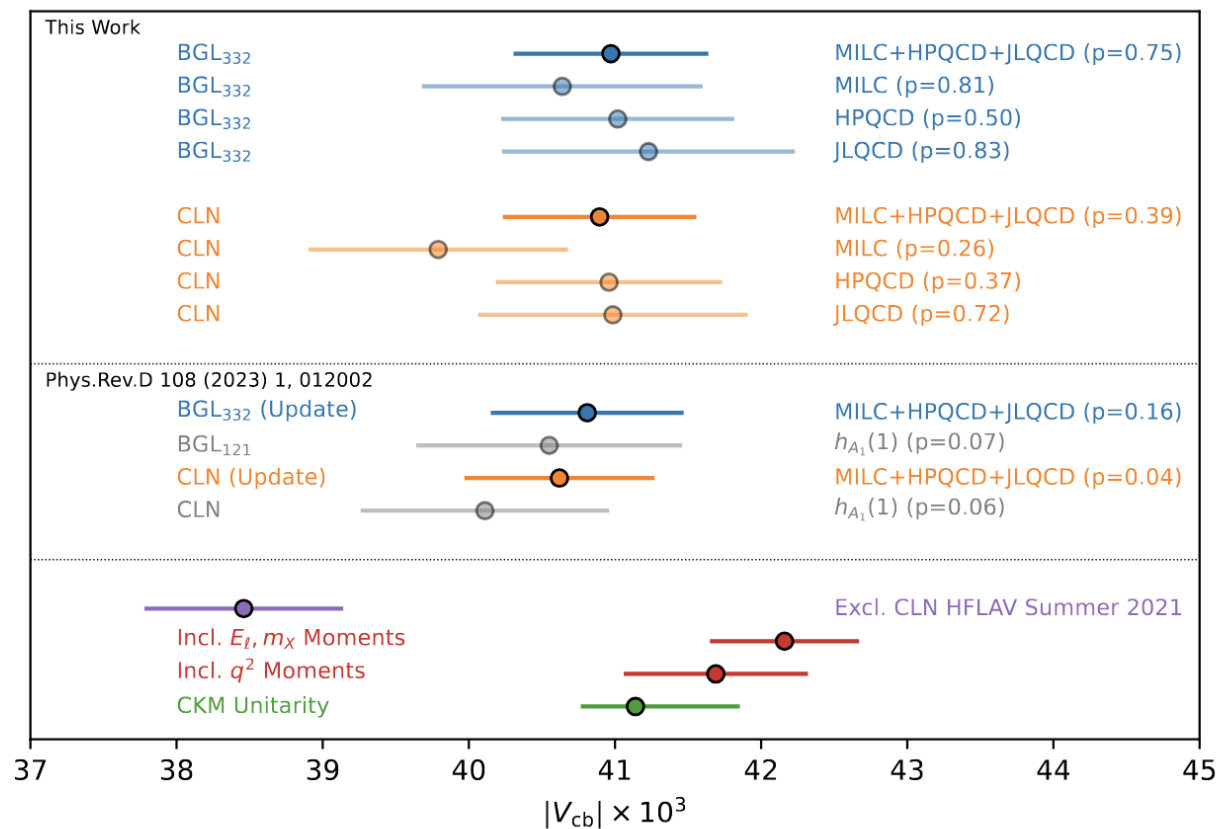
Impact of including  $B \rightarrow D^* \ell \nu$  data from Belle  
[PRD 100, 052007 \(2019\)](#)



# Exclusive $b \rightarrow c$

At  $\sim 1\%$  precision things become complicated

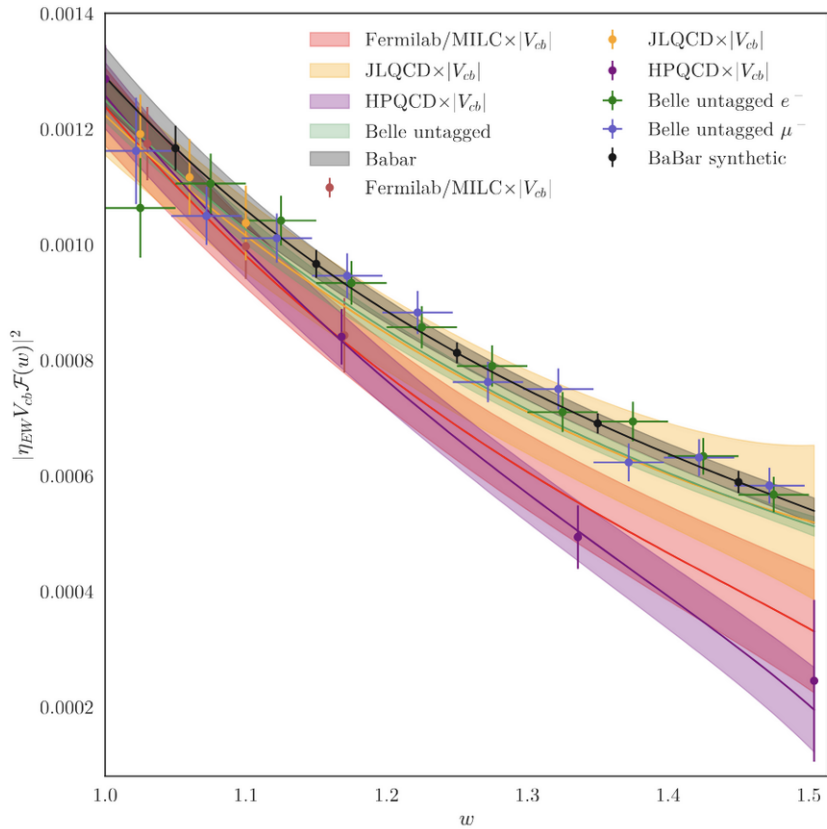
S. Schacht  
M. Primm



[overview from Belle 2310.20286]

# Exclusive $b \rightarrow c$ : combined fit Data + LQCD

A. Vaquero

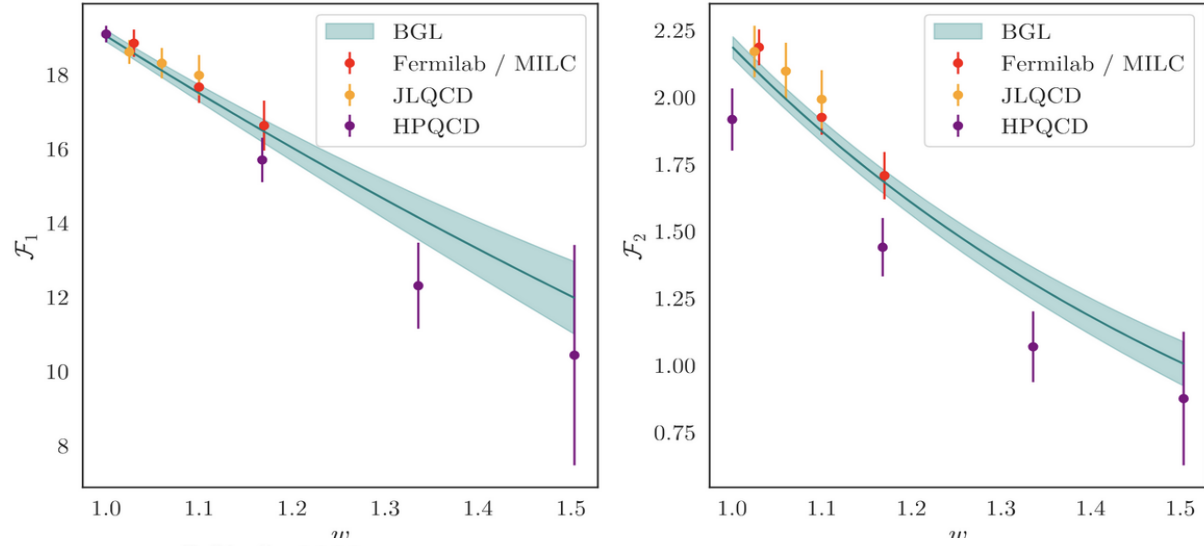


$$|V_{cb}|^{\text{FM}} = 38.40(78) \times 10^{-3}$$

$$|V_{cb}|^{\text{JLQCD}} = 39.19(90) \times 10^{-3}$$

$$|V_{cb}|^{\text{HPQCD}} = 39.31(74) \times 10^{-3}$$

## Combined fit of only Lattice results



● BGL fit 2222

	w Constraint		w/o Constraint	
	$p$	$R_2(1)$	$p$	$R_2(1)$
MILC	0.51	1.20(12)	0.43	1.27(13)
JLQCD	0.52	0.98(19)	0.25	0.97(19)
HPQCD	0.77	1.39(16)	0.65	1.39(16)
MILC+JLQCD	0.40	1.118(97)	0.36	1.16(11)
MILC+HPQCD	0.44	1.262(93)	0.37	1.262(93)
JLQCD+HPQCD	0.73	1.18(12)	0.67	1.18(12)
All	0.56	1.193(83)	0.50	1.193(83)

●  $p$ -value of Belle untagged + BaBar BGL fit 223 is  $\approx 0.04$

# Exclusive $b \rightarrow c$ : not only LQCD

## Heavy Quark Expansion (HQE)

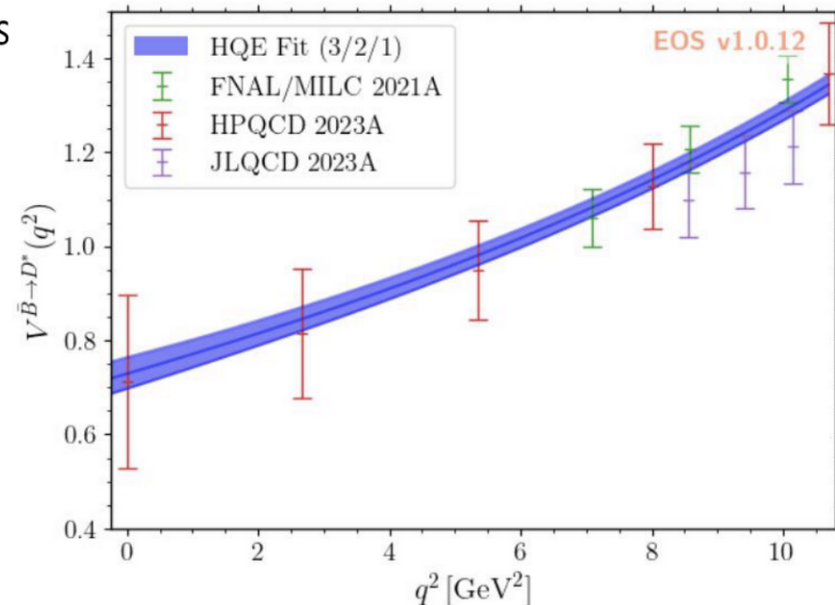
N. Gubernari

use HQE: perform a power series expansion in  $\Lambda_{\text{QCD}}/m_c$  and  $\Lambda_{\text{QCD}}/m_b$  of FFs

$$\mathcal{F}_{\text{HQE}}^i(q^2) = \xi(q^2) \left( c_0^i + c_1^i \frac{\alpha_s}{\pi} \right) + c_2^i \frac{1}{m_b} L_k(q^2) + c_3^i \frac{1}{m_c} L_k(q^2) + c_4^i \frac{1}{m_c^2} l_k(q^2)$$

all the  $B^{(*)} \rightarrow D^{(*)}$  FFs can be expressed in terms of 10 Isgur-Wise functions  
(1 leading, 3 subleading, 6 subsubleading)

HQE analyses are important for  
the interpretation of FFs  
calculations and measurements





# Exclusive $b \rightarrow c$ : BaBar

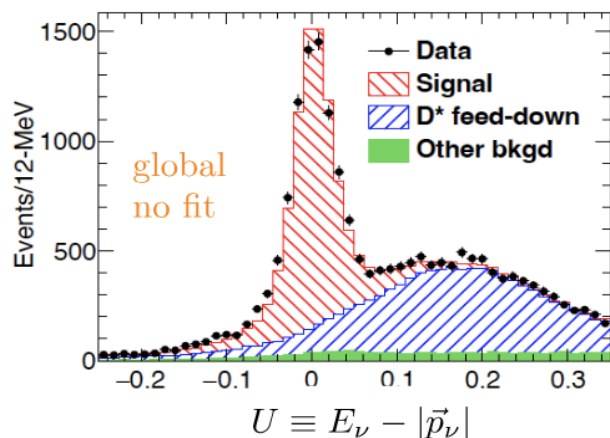
B. Dey

## $\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell$ with hadronic tagging at BABAR

[PRL123,091801(2019) + PRD110,032018(2024)]

Novel approach:  
unbinned analysis

### $B \rightarrow D \ell^- \bar{\nu}_\ell$ : ANALYSIS SETUP



- Full *BABAR* dataset w/ hadronic tagging. **2d problem** in  $\{q^2, \cos \theta_\ell\}$ .

$$\frac{d\Gamma}{dq^2 d \cos \theta_\ell} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{32\pi^3} k^3 |f_+(q^2)|^2 \sin^2 \theta_\ell$$

- Rate factorizes but efficiency and background **correlated**.

$\mathcal{B}$ measurement (had-tagged)	$ V_{cb}  \times 10^3$	
	$N = 2$	$N = 3$
BaBar-10, $B^0$	$40.02 \pm 1.76$	$39.60 \pm 1.76$
BaBar-10, $B^+$	$38.67 \pm 1.41$	$38.25 \pm 1.42$
Belle-16, $B^0$	$41.66 \pm 1.21$	$41.22 \pm 1.24$
Belle-16, $B^+$	$41.27 \pm 1.22$	$40.82 \pm 1.25$

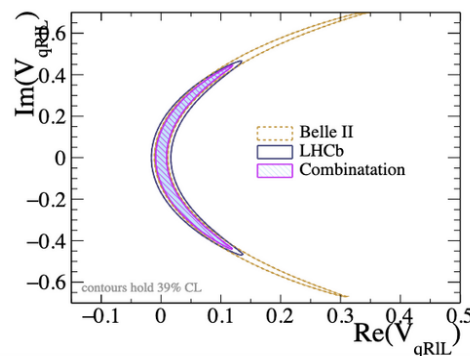
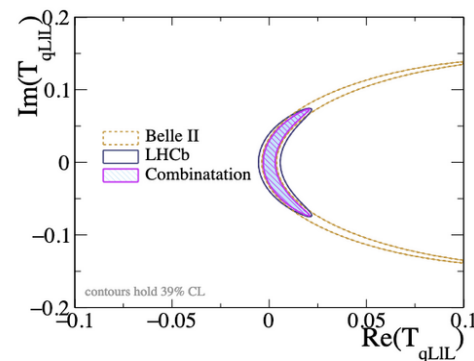
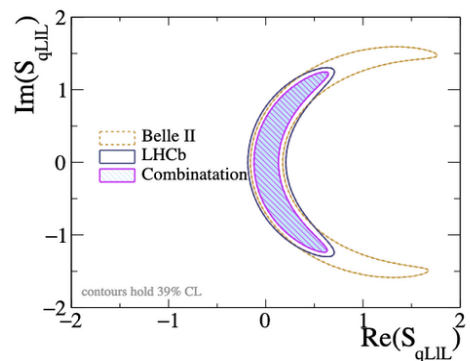
Simultaneous fit between  
 $B \rightarrow D$  and  $B \rightarrow D^*$  ongoing

Release synthetic data

# Exclusive $b \rightarrow c$

B. Mitreska

- New Physics with  $b \rightarrow c$  ?
- Global Wilson coefficient fit with LHCb and Belle II
  - **LHCb input:** Toy MC with  $B^0 \rightarrow D^* \tau \nu$  (with  $\tau \rightarrow \mu \nu \nu$ )
  - **Belle II input:** Toy MC with  $B^0 \rightarrow D^* \tau \nu$  (with  $\tau \rightarrow \pi / \rho \nu$ )



- The combination (dashed pink contour) demonstrates improvement in sensitivity
- No backgrounds or systematic uncertainties considered in the plots

## Recent review of $D^{**}$ feed-down recipes in

Florian U. Bernlochner, Manuel Franco Sevilla, Dean J. Robinson, and Guy Wormser, Rev. Mod. Phys. 94, 015003

- $R(D^{**})$  assumptions for feed-down subtraction :

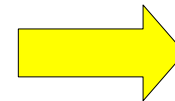
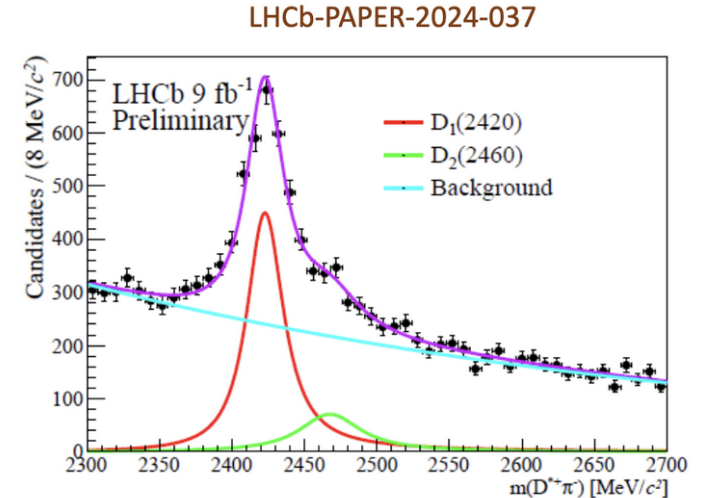
- **BABAR 0.18**
- **BELLE  $\sim 0.15$**
- **LHCb  $-\mu$ onic : 0.12**
- **LHCb-hadronic  $-\text{Run1}$  : Sum is 11% of  $D^* \tau \nu$  yield**

### $B^- \rightarrow D^{**0} \tau^- \bar{\nu}_\tau$ physics results

- Using the normalisation channel yield , one finds:

$$\frac{\mathcal{B}(B^- \rightarrow (D_1(2420)^0 + D_2^*(2460)^0) \tau^- \bar{\nu}_\tau)}{\mathcal{B}(B^- \rightarrow (D_1(2420)^0 + D_2^*(2460)^0) D_s^{(*)-})} = 0.19 \pm 0.05,$$

LHCb-PAPER-2024-037  
*Preliminary*



External input:  
LHCb-PAPER-2024-001

$R(D_1 + D_2^*) =$   
0.13 0.03 0.01 0.02

Kolya package, Fael, Milutin, KKV [2409.15007]

Open source Python package:

<https://gitlab.com/vcb-inclusive/kolya>

- Add total rate to extract  $|V_{cb}|$
- Use  $\mathcal{B} = (10.48 \pm 0.13)\%$  Bernlochner, Fael, KKV

Order	$ V_{cb}  \cdot 10^{-3}$	Moments Unc.	Rate Unc.
$O_3$	41.64	0.27	0.34
$O_3$ no NNLO	41.95	0.32	0.35
$O_4$	42.03	0.34	0.35
$O_4$ no NNLO	42.24	0.24	0.35
$O_5$	42.00	0.31	0.36
$O_5$ no NNLO	42.06	0.35	0.34