Theory of Exclusive $c \to u\ell^+\ell^-$ Transitions

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Why $c \to u\ell^+\ell^-$?

- $c \to u\ell^+\ell^-$ are the Flavour Changing Neutral Current (FCNC) decays in the up-sector : Rare decays
- FCNCs are loop level processes : Provides excellent opportunities for BSM searches.
- However, in the up sector FCNCs are more challenging:
    1. Stronger GIM cancellations
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Major differences in the FCNCs in the up and down quark sectors

FCNC in B-decays	FCNC in D-decays
Short distance dominated	Long distance dominated
Weak annihilation contribution is negligible	Weak annihilation is the main contribution
Loop contribution is the major source of long-distance uncertainties	Loop contribution is suppressed due to GIM cancellation
Highly suppressed in SM and provides an excellent opportunity for BSM searches	BSM search is not straightforward because of pollution due to long distance effects.
Cleaner signal at experiments	Experimentally challenging due to resonances

[See talk by L. Madhan & S. Celani (A. Tinari & M. Hoferichter) for Exp. (Theory) status of $b \to s\ell\ell$]

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This talk

[H. Gisbert, M. Golz, D. Mitzel, 2011.09478], [G. Hiller et. al., 2202.02331, 2410.00115], [S. Fajfer, et. al., 2312.07501]

$D \to \pi\ell^+\ell^-$: Simplest decay mode to study $c \to u\ell^+\ell^-$

- Dominated by weak singly Cabibbo suppressed (SCS) $D \to \pi$ transition combined with an electromagnetic emission of the lepton pair.

- A simple mechanism: $D \to \pi\ell^+\ell^- \approx D \to \pi V(\to \ell^+\ell^-)$ (with $V = \rho, \omega, \phi, \ldots$).

V	$BR(D^+ \to \pi^+ V)$	$BR(V \to \mu^+\mu^-)$	$BR(D^+ \to \pi^+ V)_{V\to\mu^+\mu^-}$
$\rho^0(770)$	$(8.3 \pm 1.4) \times 10^{-4}$	$(4.55 \pm 0.28) \times 10^{-5}$	$(3.78 \pm 0.68) \times 10^{-8}$
$\omega(782)$	$(2.8 \pm 0.6) \times 10^{-4}$	$(7.4 \pm 1.8) \times 10^{-5}$	$(2.1 \pm 0.7) \times 10^{-8}$
$\phi(1020)$	$(5.7 \pm 0.14) \times 10^{-3}$	$(2.85 \pm 0.19) \times 10^{-4}$	$(1.62 \pm 0.12) \times 10^{-6}$

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[PDG]

- SM predictions below and above resonances :

$$BR(D^+ \to \pi^+\mu^+\mu^-)_{q^2\in[0.250^2,0.525^2]} = (8.1^{5.9}_{-6.1}) \times 10^{-9}$$

$$BR(D^+ \to \pi^+\mu^+\mu^-)_{q^2>1.25^2} = (2.7^{+4.0}_{-2.6}) \times 10^{-9}$$

[A. Bharucha, D. Boito, C. Méaux (2011.12856)]

Effective Operators

- The effective Hamiltonian for $D \to \pi \ell^+ \ell^-$ (SCS)

$$\mathscr{H}_{eff}^{\Delta S=0} = \frac{4G_F}{\sqrt{2}} \sum_{\mathscr{D}=d,s} \lambda_{\mathscr{D}} \left[C_1(\mu) O_1^{\mathscr{D}} + C_2(\mu) O_2^{\mathscr{D}}\right] - \lambda_b \sum_{i=3}^{10} C_i(\mu) O_i$$

$\lambda_{\mathscr{D}}$: $V_{u\mathscr{D}} V_{c\mathscr{D}}^* \approx \lambda$

λ_b: $V_{ub} V_{cb}^* \approx \lambda^5$ → **suppressing factor**

$O_1^{\mathscr{D}} = (\bar{u}_L \gamma_\mu \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu c_L)$

$O_2^{\mathscr{D}} = (\bar{u}_L \gamma_\mu t^a \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu t^a c_L)$

$\sum_{i=3}^{10} C_i(\mu) O_i$: $\ll C_{1,2} @ \mathscr{O}(m_c)$

WCs @ $\mu = 1.3$ GeV

	C_1	C_2	C_3	C_4	C_5	C_6
LL	-1.035	1.094	-0.004	-0.061	0.000	0.001
NLL	-0.712	1.038	-0.006	-0.093	0.000	0.001
NNLL	-0.633	1.034	-0.008	-0.093	0.000	0.001
	C_7^{eff}	C_8^{eff}	C_9	C_{10}	C_9^{NNLL}	C_{10}^{NNLL}
LL	0.078	-0.055	-0.098	0	-0.488	0
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[S. de Boer, B. Müller, D. Siegel, (1606.05521)]

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- Hamiltonian in the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):

$$\mathscr{H}^{(\Delta_s=0,\lambda_b=0)}_{eff} = \frac{4G_F}{\sqrt{2}} \lambda_d \left[C_1(O_1^d - O_1^s) + C_2(O_2^d - O_2^s)\right]$$

- The largest effect beyond GIM limit $\sim \lambda_b C_9$ ($C_9 = -0.488$) : The short distance contribution.

Most commonly adopted approach

- Treat the resonances as a correction to C_9 .

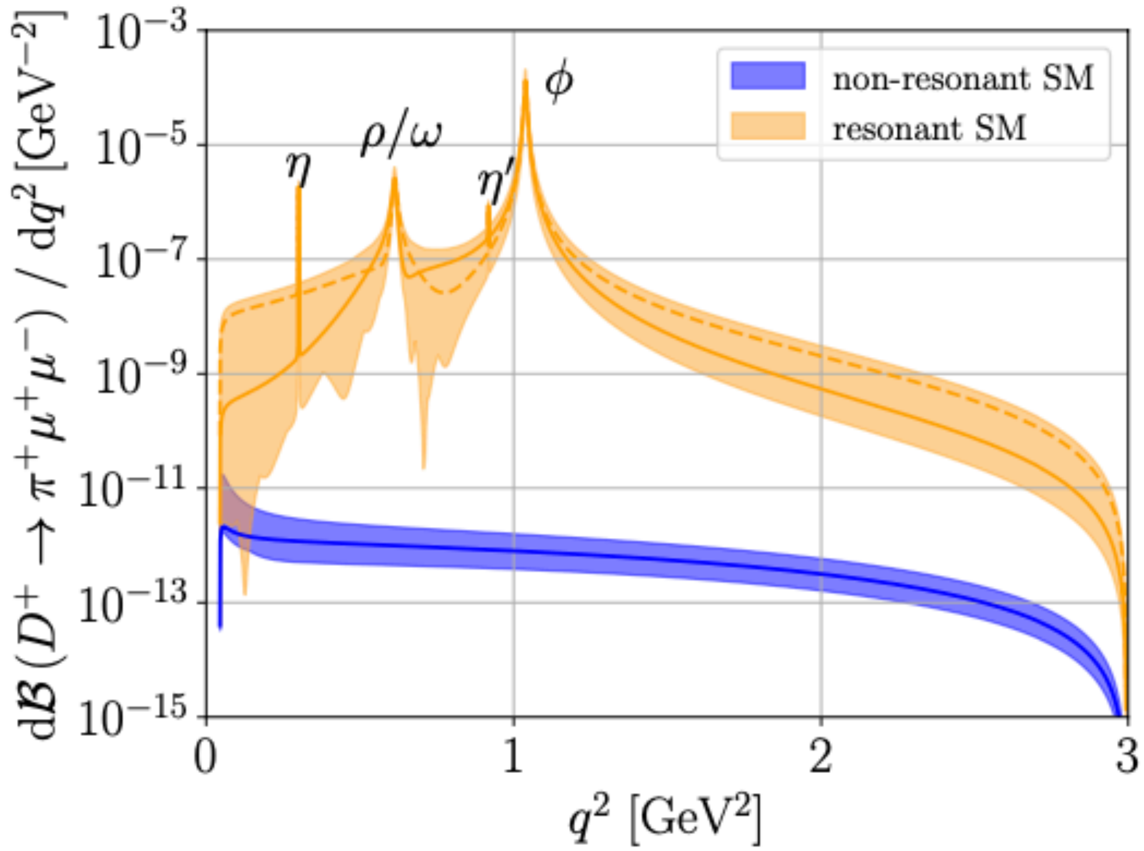
[G. Hiller et al. 1510.00311, 1909.11108, 2410.00115]
[S. Fajfer, N. Kośnik, 1510.00965]

- Model the resonances $\rho, \omega, \phi, \eta, \eta'$, using Breit Wigner parametrization.

$$C_9^R(q^2) = a_\rho e^{\mathrm{i}\,\delta_\rho}\left(\frac{1}{q^2 - m_\rho^2 + \mathrm{i}\,m_\rho\Gamma_\rho} - \frac{1}{3}\frac{1}{q^2 - m_\omega^2 + \mathrm{i}\,m_\omega\Gamma_\omega}\right) + \frac{a_\phi e^{\mathrm{i}\,\delta_\phi}}{q^2 - m_\phi^2 + \mathrm{i}\,m_\phi\Gamma_\phi},$$

$$C_P^R(q^2) = \frac{a_\eta e^{\mathrm{i}\,\delta_\eta}}{q^2 - m_\eta^2 + \mathrm{i}\,m_\phi\Gamma_\eta} + \frac{a_{\eta'}}{q^2 - m_{\eta'}^2 + \mathrm{i}\,m_{\eta'}\Gamma_{\eta'}},$$

- The major source of uncertainties : unknown strong phases



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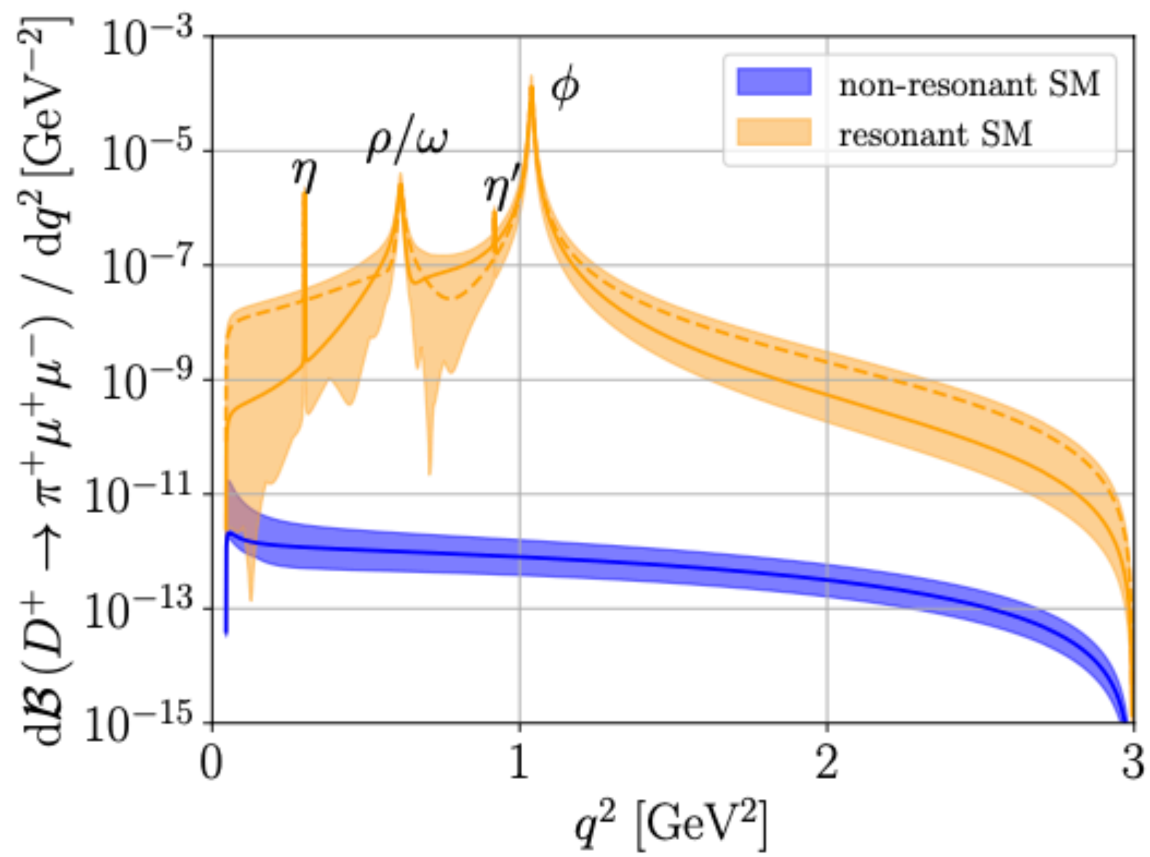
- Ways to look for new physics:
  - Put kinematical cuts ($q^2 > m_\phi^2$) : Branching fraction less polluted by resonances $\Longrightarrow$ More sensitivity to new physics
  - Null Tests: Look for observables like CP asymmetries, lepton flavour universality ratios, based on approximate symmetries of SM.



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A similar approach at Experiments

- Most recent upper bound on $(D^+ \to \pi^+\mu^+\mu^-)$: vetoing the resonance region (using integrated luminosity of 1.6 fb^{-1}).

[LHCb, (JHEP06 (2021) 044)]

- Upper bounds from PDG:

Decay mode	Cabibbo hierarchy	BR, exp. upper limit
$D^+ \to \pi^+\ell^+\ell^-$	SCS	$1.1 \times 10^{-6}(\ell = e)$ $6.7 \times 10^{-8}(\ell = \mu)$
$D^+ \to K^+\ell^+\ell^-$	DCS	$8.5 \times 10^{-7}(\ell = e)$ $5.4 \times 10^{-8}(\ell = \mu)$
$D^0 \to \bar{K}^0\ell^+\ell^-$	CF	$2.4 \times 10^{-5}(\ell = e)$ $2.6 \times 10^{-4}(\ell = \mu)$
$D^0 \to \pi^0\ell^+\ell^-$	SCS	$4 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta\ell^+\ell^-$	SCS	$3 \times 10^{-6}(\ell = e)$ $5.3 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta'\ell^+\ell^-$	SCS	-
$D^0 \to K^0\ell^+\ell^-$	DCS	-
$D_s^+ \to \pi^+\ell^+\ell^-$	CF	$5.5 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-7}(\ell = \mu)$
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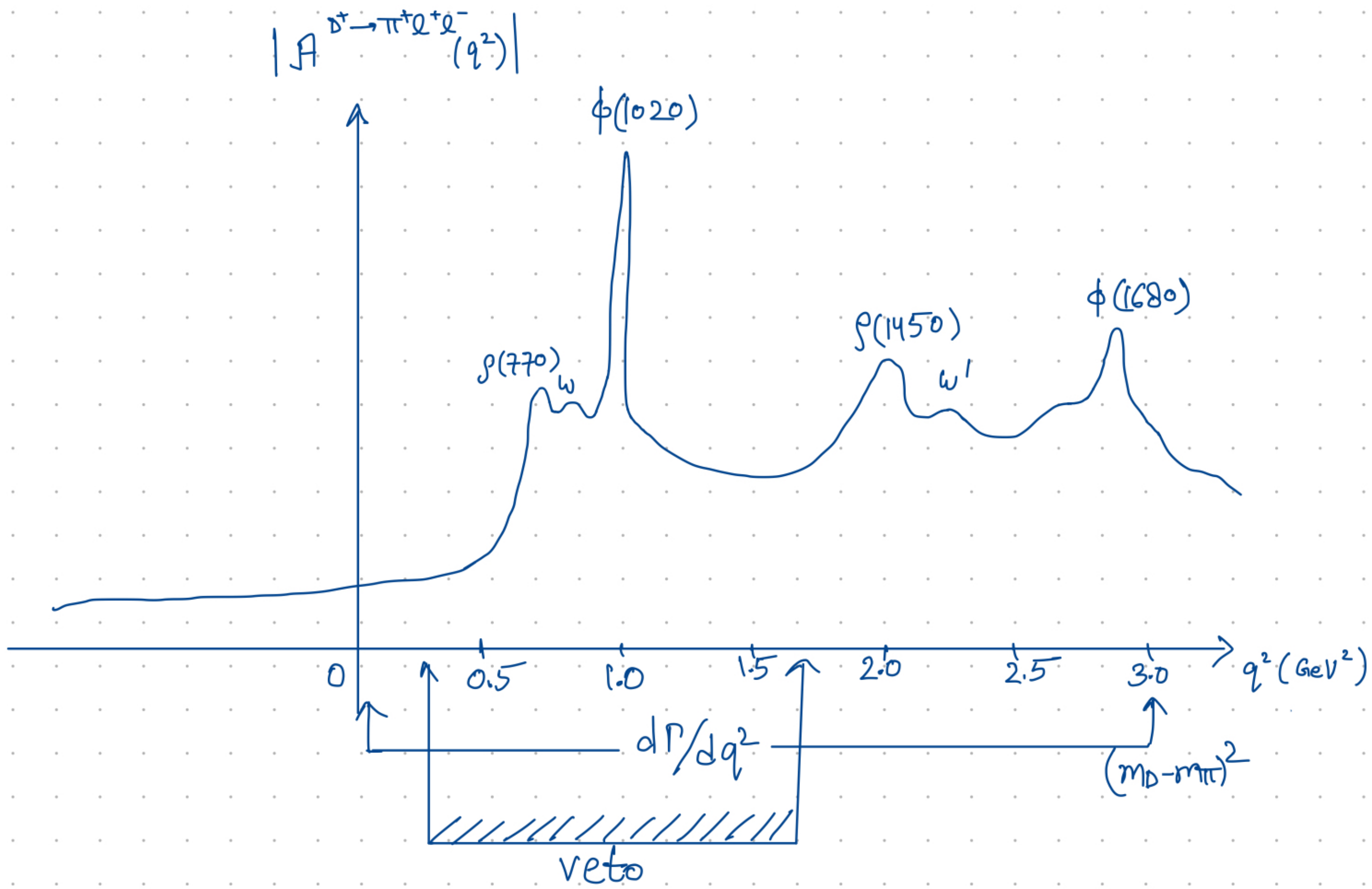
[PDG]

Can we really isolate resonances?



- The full amplitude represented via hadronic dispersion relation :

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$

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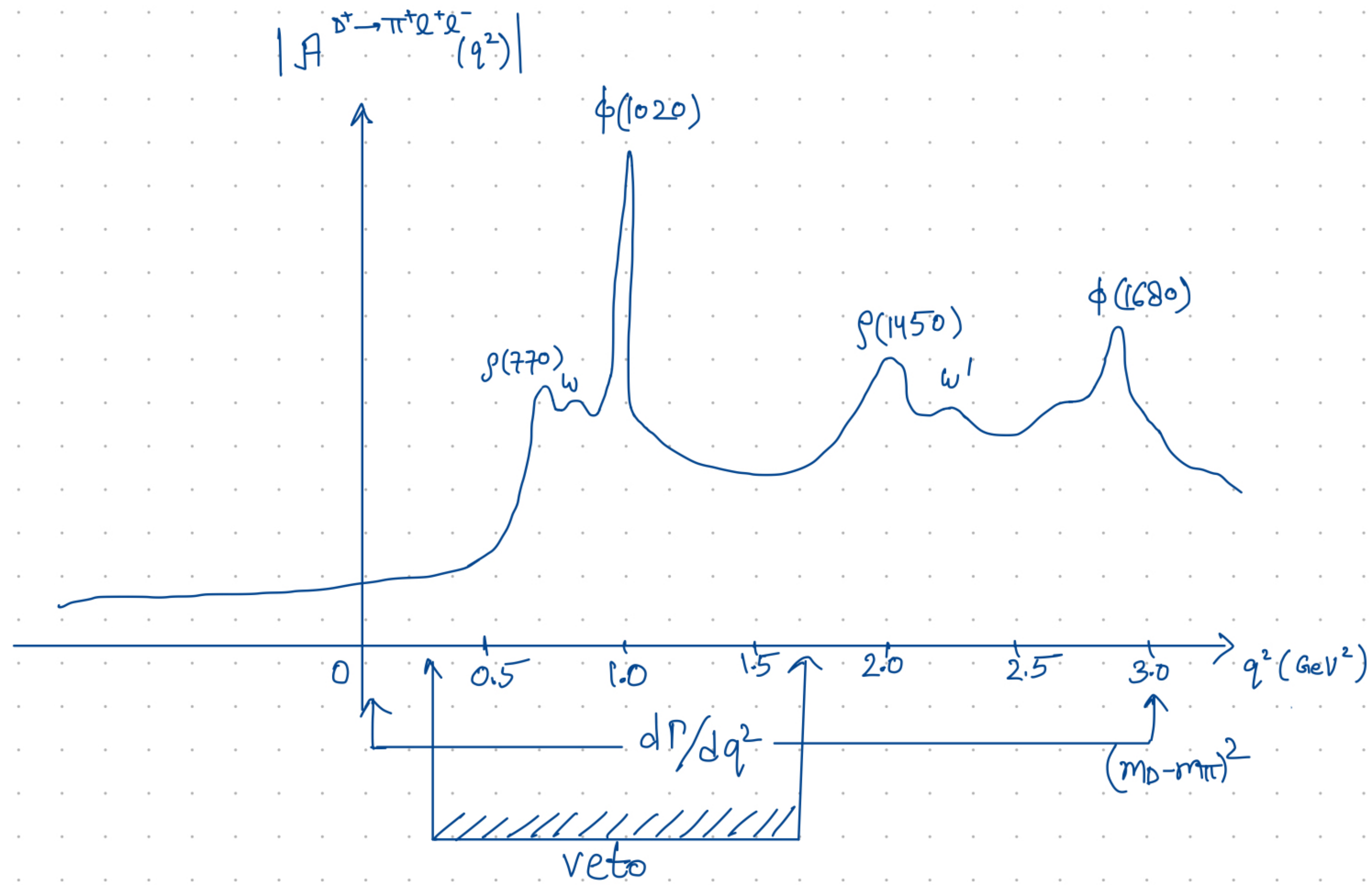

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Decay constant — Amplitude for $D \to \pi V$ — Continuum and higher resonances

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- Dispersion relation tells us: vetoing a certain q^2- region does not remove resonances from the amplitude.
- The radial excitations of ρ, ω, ϕ and the "tail" at $s > (m_D - m_\pi)^2$ are indispensable.

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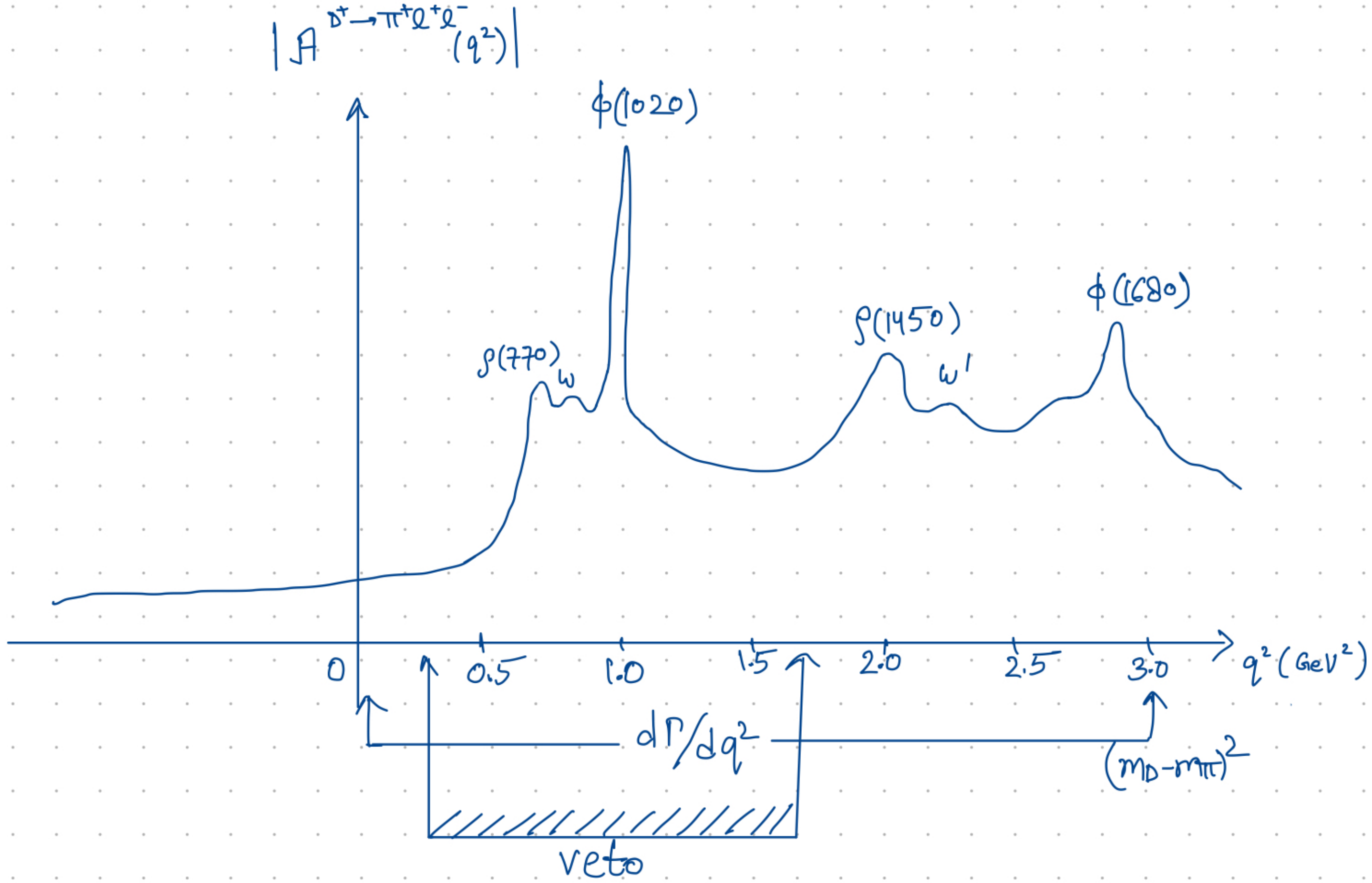

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Is a QCD based analysis possible?

Amplitude and Hadronic Matrix Element

- In the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):,

$$\mathscr{A}(D^+ \to \pi^+ \ell^+ \ell^-) = \left(\frac{16\pi\alpha_{em} G_F}{\sqrt{2}} \right) \lambda_d \frac{\bar{u}_\ell \gamma^\mu \nu_\ell}{q^2} \mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q)$$

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The hadronic part (hadronic matrix element)

$$\mathcal{A}_\mu^{D^+\to\pi^+\gamma^*}(p,q) = i\int d^4x e^{iq.x}\langle \pi^+(p)\,|\,T\left\{j_\mu^{em}(x), \mathcal{H}_{eff}^{(\Delta_s=0,\lambda_b=0)}\right\}|\,D^+(p+q)\rangle$$

$$= \left[(p\,.\,q)q_\mu - q^2 p_\mu\right] \mathcal{A}_{\mathcal{D}}^{D^+\to\pi^+\gamma^*}(q^2)$$

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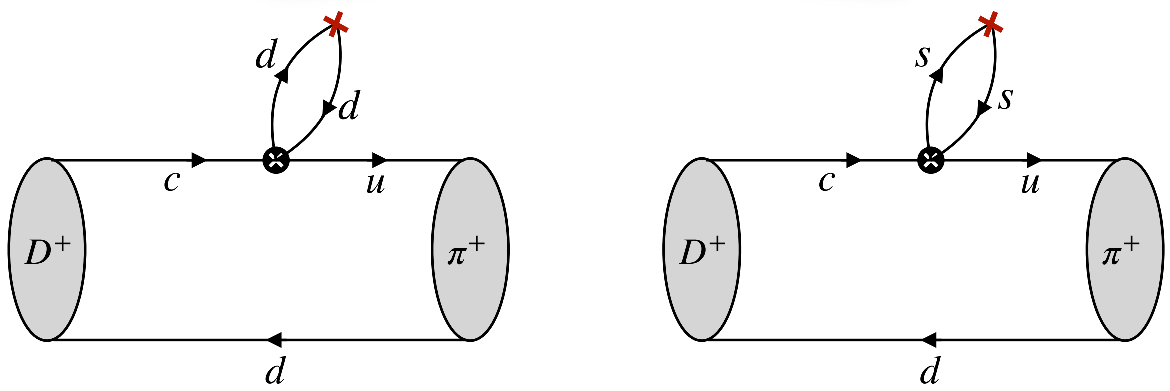
The non-local form factor : **dominated by long distance effects in the physical region of q^2.**

$(4m_\ell^2 < q^2 < (m_D - m_\pi)^2)$

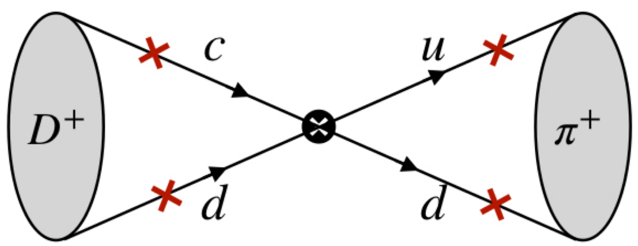
The object of our interest

Quark Topologies for $\mathscr{A}_{\mu}^{D^+ \to \pi^+ \gamma^*}(p, q)$

Loop Topology
(Only possible in SCS decays)



Annihilation Topology



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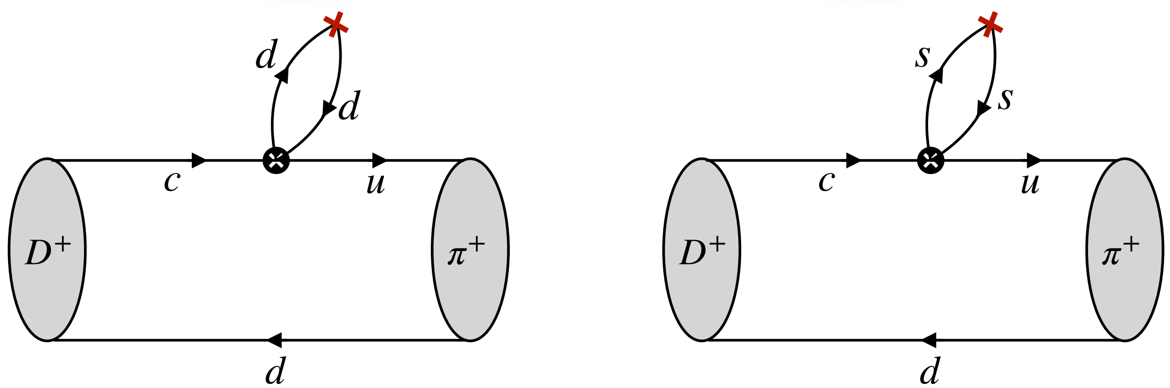

In the $SU(3)_f$ (along with $\lambda_b \approx 0$),
a complete GIM cancellation

L-topology has non-zero contribution only due to $m_s \neq m_d$.

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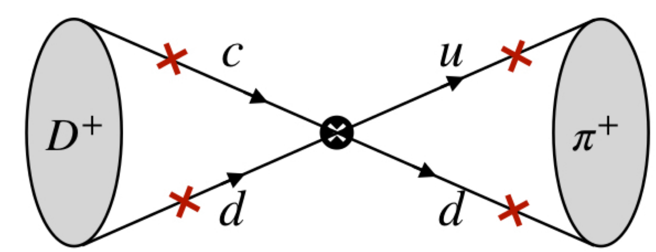

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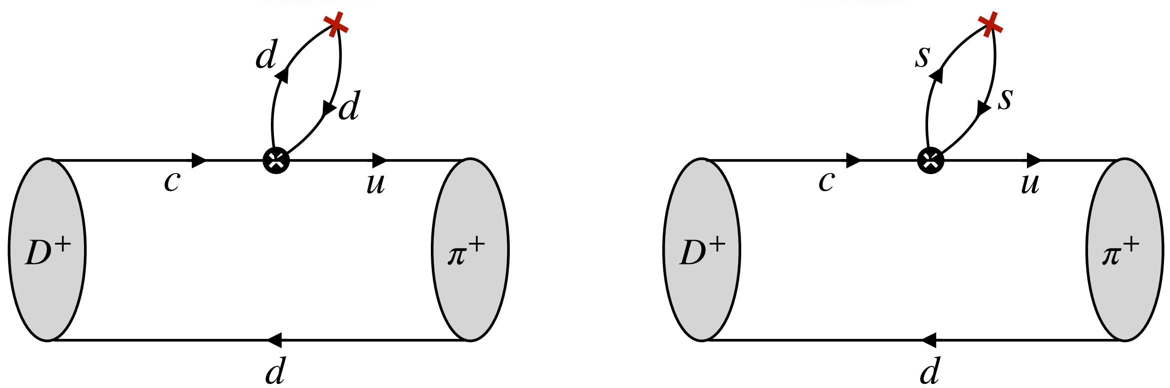

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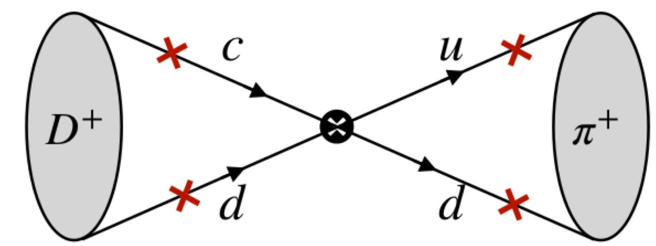

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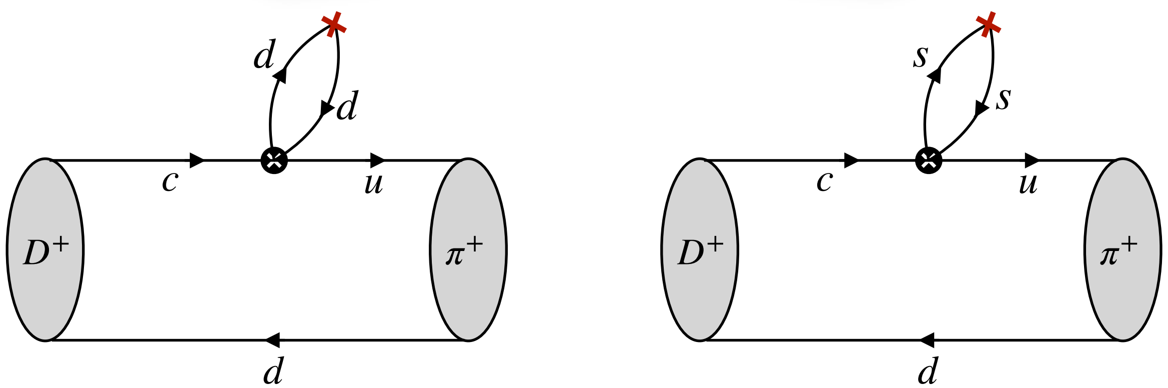

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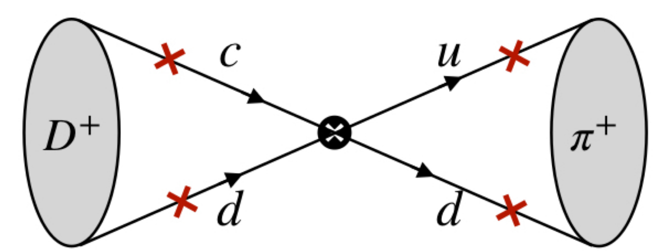

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- At NLO, there will be multiple diagrams with the exchange of virtual gluons : Out of the scope of the present study.

What do we already know from QCD?

- Available estimates are are based on QCD factorization.
- The method was originally suggested for $B \to K^* \ell^+ \ell^-$ decays. [M. Beneke, T. Feldmann, D. Seidel (hep-ph/0106067)]
- The result for $b \to s\ell\ell$ were then first used for $D \to \rho\ell\ell$: The resonances were modelled using Shifman's infinite resonance model. [T. Feldmann, B. Müller, D. Seidel (1705.05891)]

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$$BR(D^+ \to \pi^+\mu^+\mu^-)_{q^2 > 1.25^2} = (2.7^{+4.0}_{-2.6}) \times 10^{-9}$$

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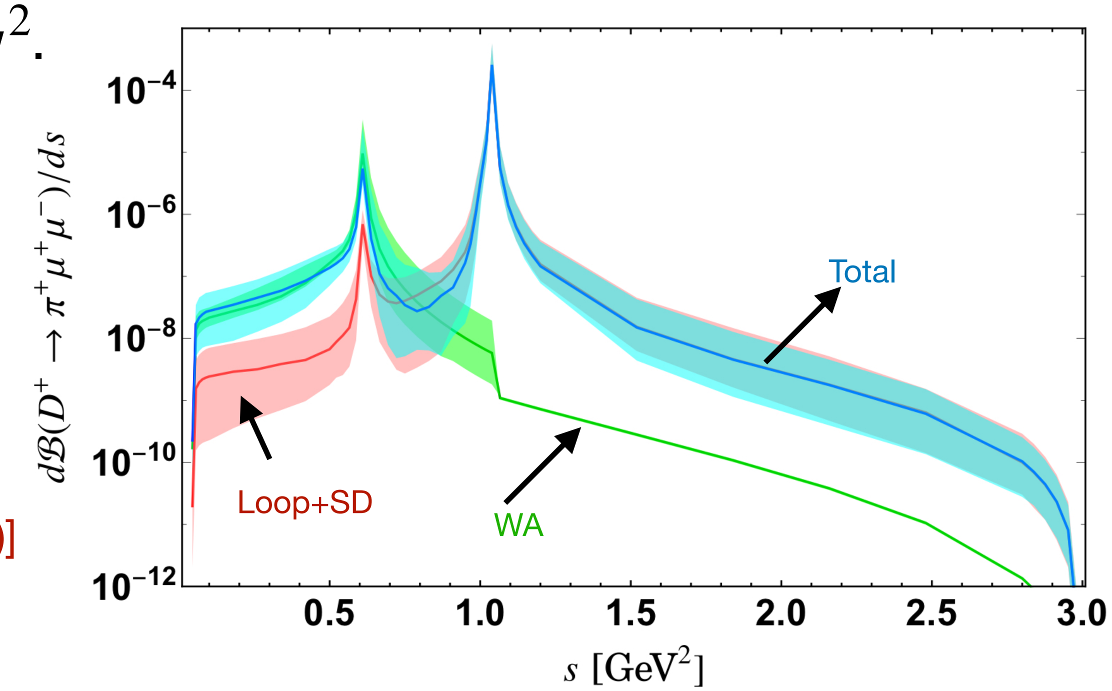

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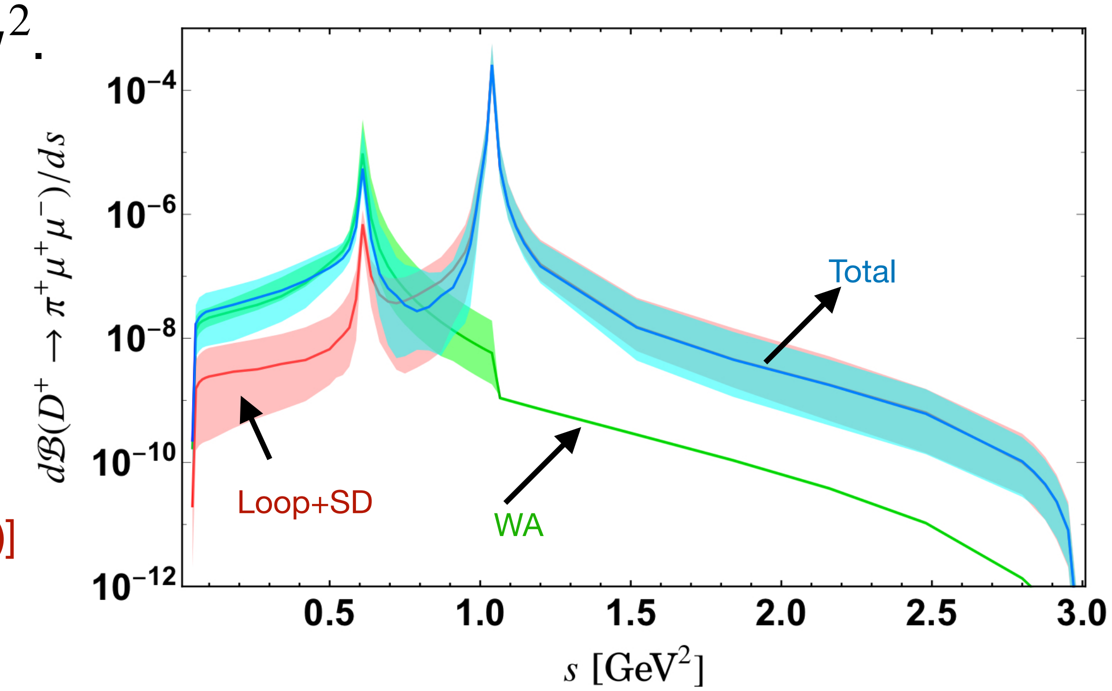

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    - Includes only one of the four annihilation diagrams (emission from the initial d-quark) :
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    - $\frac{1}{m_c}$ corrections eg. from the use of D-meson distribution amplitudes:
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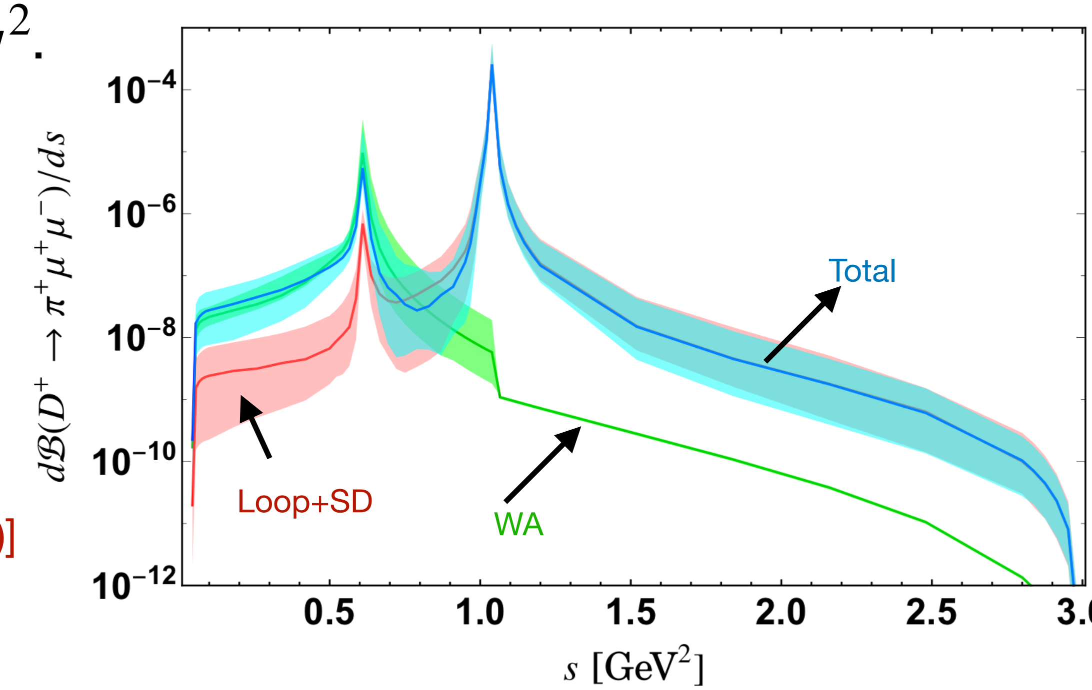

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As the Experimental bounds are now approaching theory predictions, it is important to look for alternative QCD based methods.

The use of U-spin

[In preparation, AB, Alexander Khodjamirian and Thomas Mannel]

- Combining : GIM limit, $\lambda_b = 0, \lambda_d = -\lambda_s$ with $SU(3)_{fl}$ limit, $m_s = m_{u,d}$ **(Only annihilation topology)**
- The Hamiltonians of CF, SCS, and DSC modes form a U-triplet:

$$O_1^{(U=1)} \equiv \begin{pmatrix} (\bar{u}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu c_L) \\ \frac{1}{\sqrt{2}} \left[(\bar{u}_L \gamma_\mu d_L)(\bar{d}_L \gamma^\mu c_L) - (\bar{u}_L \gamma_\mu s_L)(\bar{s}_L \gamma^\mu c_L) \right] \\ (\bar{u}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu c_L) \end{pmatrix} = \begin{pmatrix} |1, +1\rangle \\ -|1,0\rangle \\ |1, -1\rangle \end{pmatrix}$$

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U-spin relations

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = -\mathcal{A}^{(D_s^+\to K^+\gamma^*)}(q^2) = \mathcal{A}^{(D_s^+\to\pi^+\gamma^*)}(q^2) = \mathcal{A}^{(D^+\to K^+\gamma^*)}(q^2)$$

$$\mathcal{A}^{(D^0\to\bar{K}^0\gamma^*)}(q^2) = \mathcal{A}^{(D^0\to K^0\gamma^*)}(q^2) = -\frac{1}{2}\mathcal{A}^{(D^0\to\pi^0\gamma^*)}(q^2) + \frac{\sqrt{3}}{2}\mathcal{A}^{(D^0\to\eta^0\gamma^*)}(q^2)$$

$$\mathcal{A}^{(D^0\to\eta_8\gamma^*)}(q^2) = -\sqrt{3}\mathcal{A}^{(D^0\to\pi^0\gamma^*)}(q^2)$$

$$\mathcal{A}^{(D^0\to\eta'\gamma^*)}(q^2) = 0$$

D^0, η': U-spin singlets.

- Other $D_{(s)} \to P\ell^+\ell^-$ channels ($P = \pi, K, \eta$)), Cabibbo favoured(CF) and doubly Cabibbo suppressed(DCS) are also interesting : can help to disentangle the annihilation topology.

$D \to \pi\ell^+\ell^-$ using LCSR supported Dispersion relation

[In preparation, AB, Alexander Khodjamirian and Thomas Mannel]

- **Benefits:**
  - An independent alternative to QCDf.
  - Finite m_c .

Our methodology: LCSR-supported dispersion relation

Ways to compute $\mathscr{A}_{\mathscr{D}}^{D^+\to\pi^+\gamma^*}(q^2)$

Dispersion relation
(Valid for all q^2 values)

Light Cone Sum Rules
(Only valid in space-like q^2)

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$$|A_{DV\pi}| = \left(\frac{8\pi BR(D^+ \to V\pi^+)}{\tau(B) G_F^2 |V_{cs}|^2 |V_{ud}|^2 m_{D^+}^3 \lambda_{D^+V\pi^+}^{3/2}}\right)^{1/2}$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Normalized to the valence quark content of V

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Challenges:

- Unknown strong phases
- Unknown spectral densities: too complicated to be parameterised.

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using **z-parametrization** (valid only for $q^2 < s_0^h$)

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)} = \sum_{k=0}^{K} a_k [z(q^2)]^k$$

with,

$$z(q^2) = \frac{\sqrt{s_0^h - q^2} - \sqrt{s_0^h}}{\sqrt{s_0^h - q^2} + \sqrt{s_0^h}}$$

a_k = Complex coefficients

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Our methodology: LCSR-supported dispersion relation

Dispersion relation
(Valid for all q^2 values)

$=$ at $q^2 < 0$

Light Cone Sum Rules
(Only valid in space-like q^2)

- Fit the unknown parameters using the data from LCSR computation.
- Make predictions for $q^2 > 0$ using the fitted parameters in the dispersion relation.

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Summary of the Main idea :

Step-1: Compute $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ using Light Cone Sum Rules (valid only for $q^2 < 0$)

Step-2: Write the hadronic dispersion relation in terms of unknown phases and z-parameters (valid for all values of q^2).

Step-3: Match the LCSR results with the dispersion relation at $q^2 < 0$ and estimate the unknown parameters.

Step-4: Estimate $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ in the physical region using dispersion relation.

(Resembling partly the analysis of nonlocal effects in $B \to K^{(*)}\ell^+\ell^-$)

[A. Khodjamirian, T. Mannel, A. Pivovarov, Y. Wang, 1211.0234]

[A. Khodjamirian, A. V. Rusov, 1703.04765] , N. Gubernari, M. Rebound, D. van Dyk, J. Virto, 2011.09813

A brief overview of LCSR method

- The correlation function:

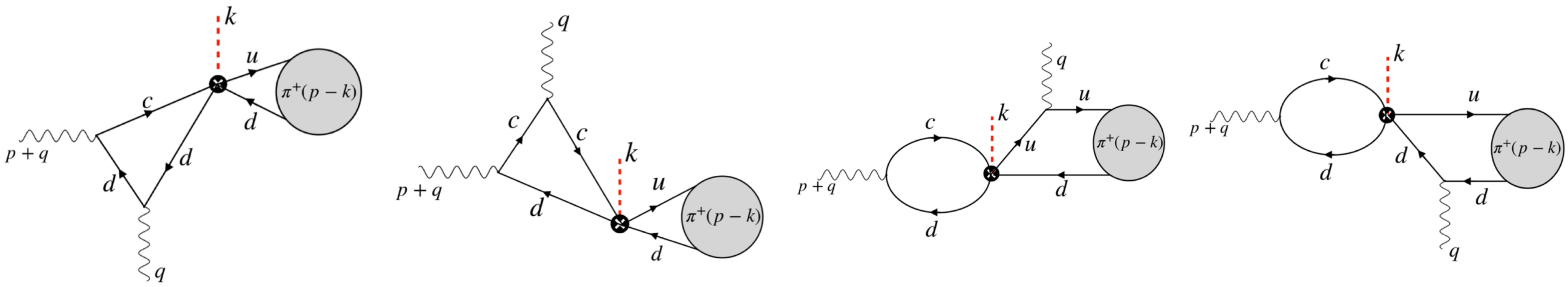
$$F_\mu(p,q,k) = -\int d^4x\, e^{iq\cdot x}\int d^4y\, e^{-i(p+q)\cdot y}\langle \pi^+(p-k)\,|\,T\{J^{em}_\mu(x)\mathcal{H}^{(\Delta_s=0,\lambda_b=0)}_{eff}(0)J^D_5(y)\}\,|0\rangle$$

$$\sum_{q=u,d,c} Q_q\bar{q}(x)\gamma_\mu q(x)$$

$$im_c\bar{c}(y)\gamma_5 d(y)$$

TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a convolution of perturbative hard scattering kernel and pion DAs)



(a) (b) (c) (d)

Weak Annihilation Diagrams (LO) in terms of pion DAs

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TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a convolution of perturbative hard scattering kernel and pion DAs)

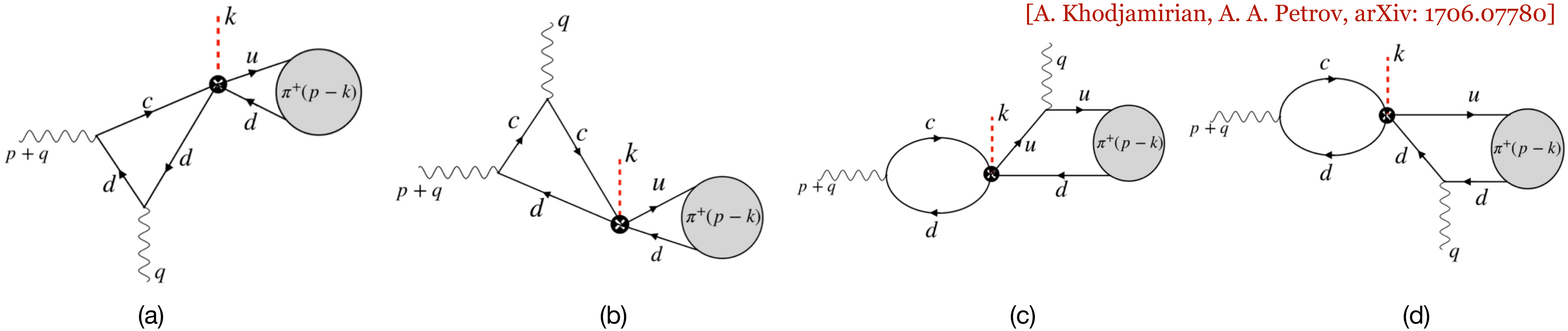
- The artificial momentum k is introduced at the four vertex to avoid parasitic contributions in the dispersion relation.

(Used before in LCSR analysis of $B \to 2\pi$ and $D \to 2\pi, K\bar{K}$)

[A. Khodjamirian, arXiv: hep-ph/0012271]

[A. Khodjamirian, M. Melcher, B. Melic, arXiv: hep-ph/0304179, hep-ph/0509049]

[A. Khodjamirian, A. A. Petrov, arXiv: 1706.07780]



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TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a convolution of perturbative hard scattering kernel and pion DAs)

Only O_1^d contributes. The O_2^d contribution vanishes after Fierz transformation.

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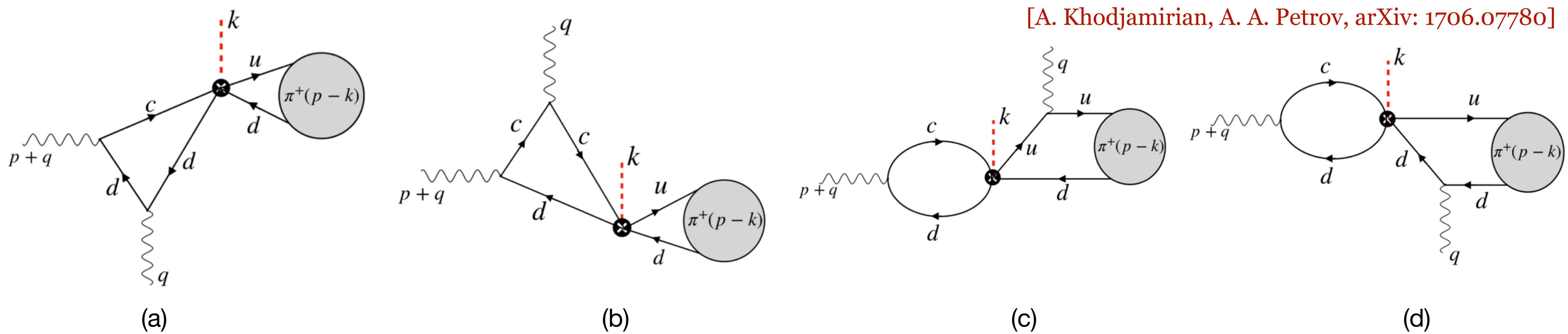

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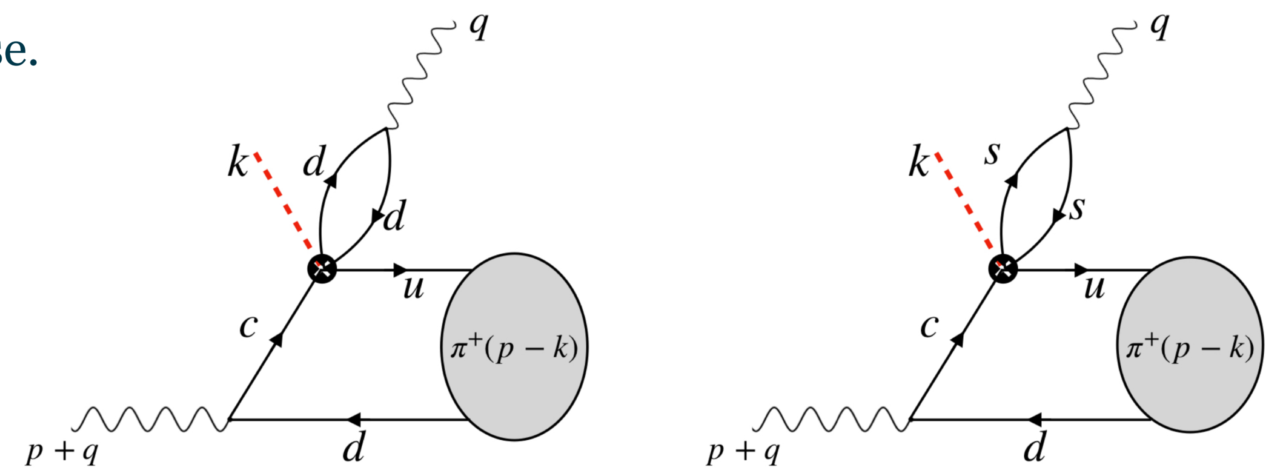
TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a convolution of perturbative hard scattering kernel and pion DAs)

- The correlation function factorises into loop function and a simpler $D \to \pi$ matrix element.
- Both Wilson Coefficients (C_1 and C_2) contribute in this case.
- The contribution is small due to GIM suppression.



Loop Diagrams (LO) in terms of pion DAs

A brief overview of LCSR method

TOOLS TO DERIVE LCSR

Light cone OPE
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Dispersion Relation in D-meson channel
(Enables to relate the calculated correlation function to the sum over $D \to \pi\gamma^*$ hadronic matrix elements.)

$$\frac{1}{\pi}\int_{m_c^2}^{\infty} ds \frac{ImF^{(OPE)}(s, q^2, P^2 = m_D^2)}{s-(p+q)^2} = \frac{m_D^2 f_D A^{(D^+\to\pi^+\gamma^*)}(q^2)}{m_D^2-(p+q)^2} + \int_{s_{h_d}}^{\infty} ds \frac{\rho_{h_D}(s, q^2, P^2 = m_D^2)}{s-(p+q)^2}$$

A brief overview of LCSR method

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Borel Transformation
(To suppress the effect of continuum and higher resonances to reduce the uncertainty due to duality approximation)

LCSR Results

- The final sum rule read as (for $q^2 < 0$):

$$m_D^2 f_D \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) e^{-m_D^2/M^2} = \frac{1}{\pi}\int_{m_c^2}^{s_0^D} ds e^{-s/M^2} \mathrm{Im}(F^{(OPE)}(s, q^2, m_D^2))$$

- M^2 (Borel parameter) and s_0^D (effective threshold) are the sum rule parameters taken to be:

$$M^2 = (4.5 \pm 1.0)\ \text{GeV}^2$$

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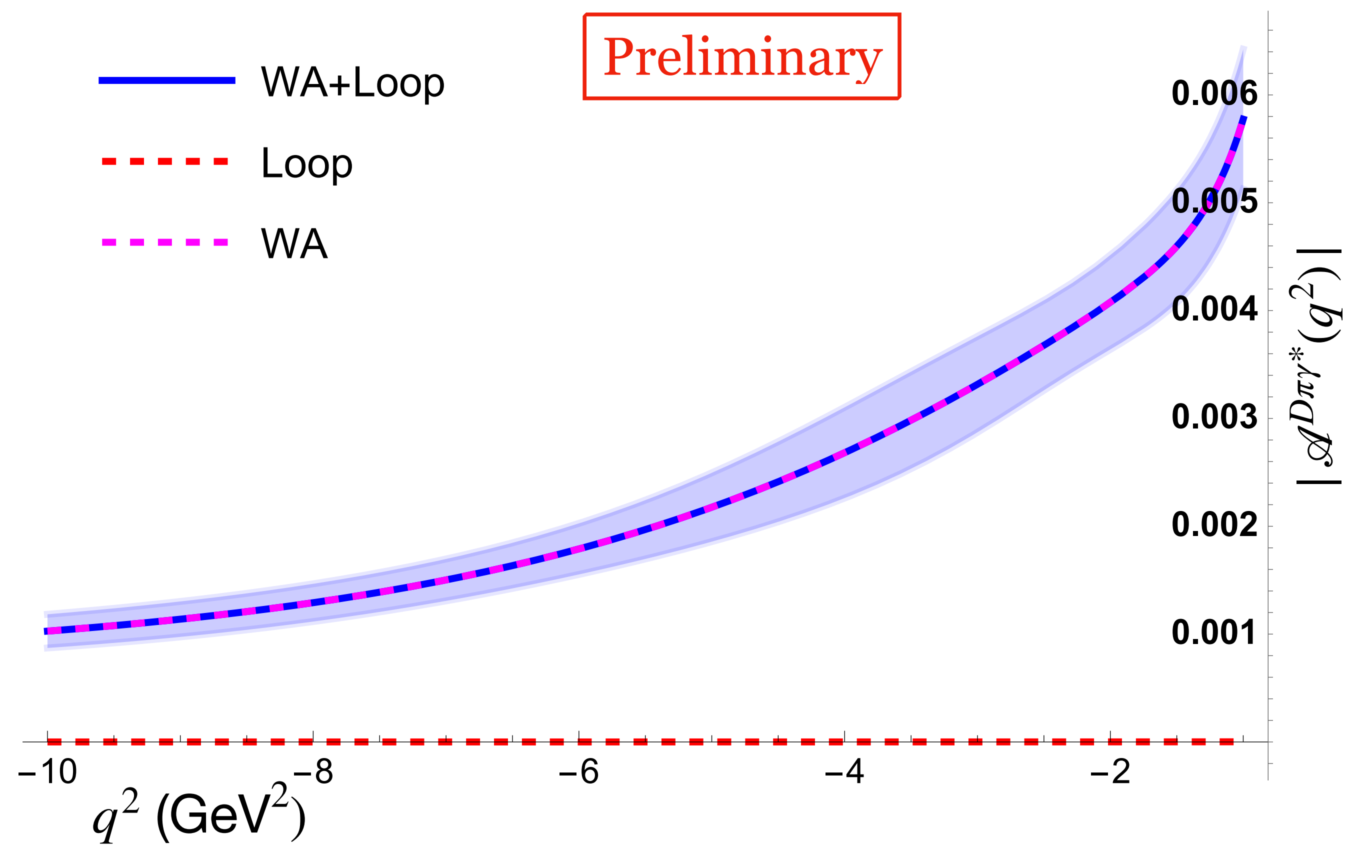

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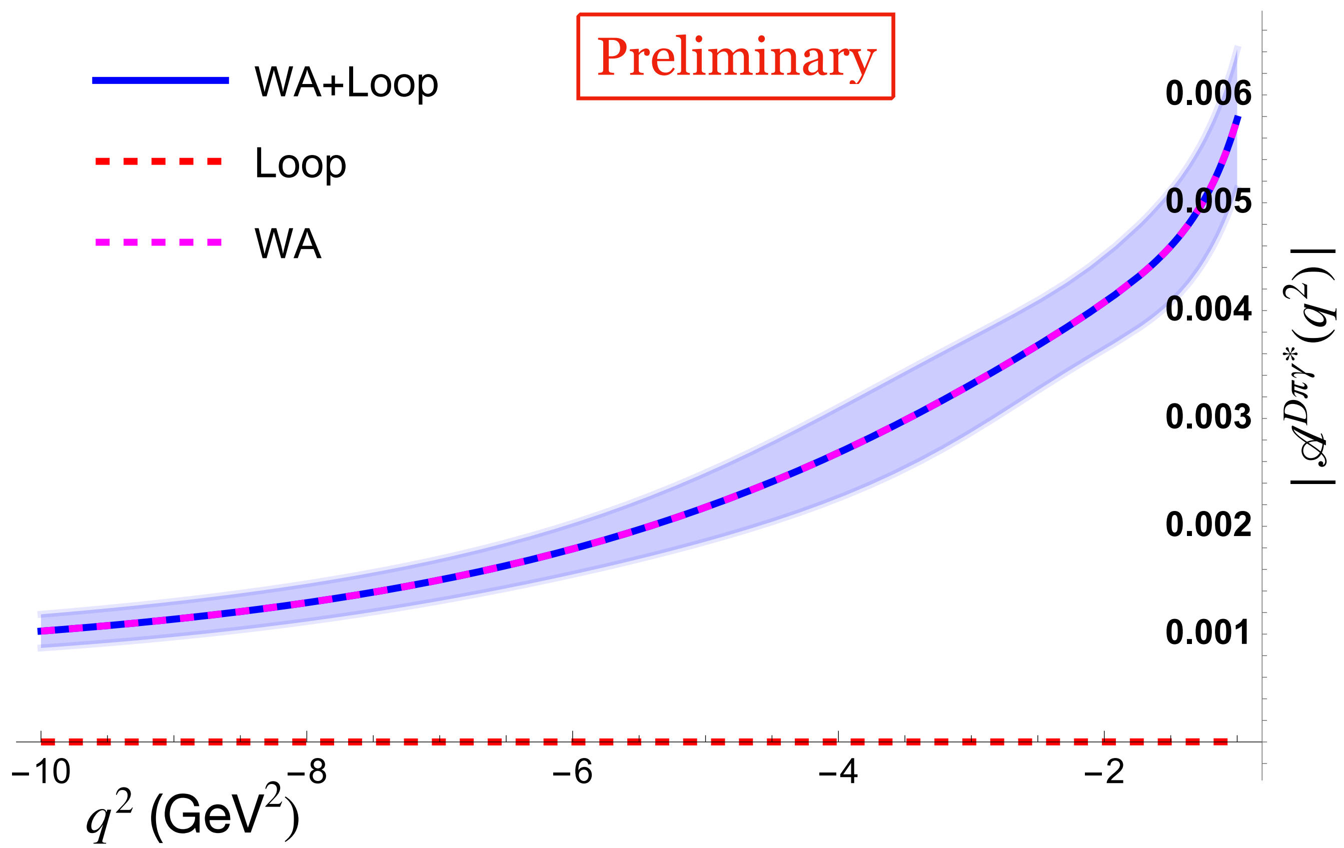

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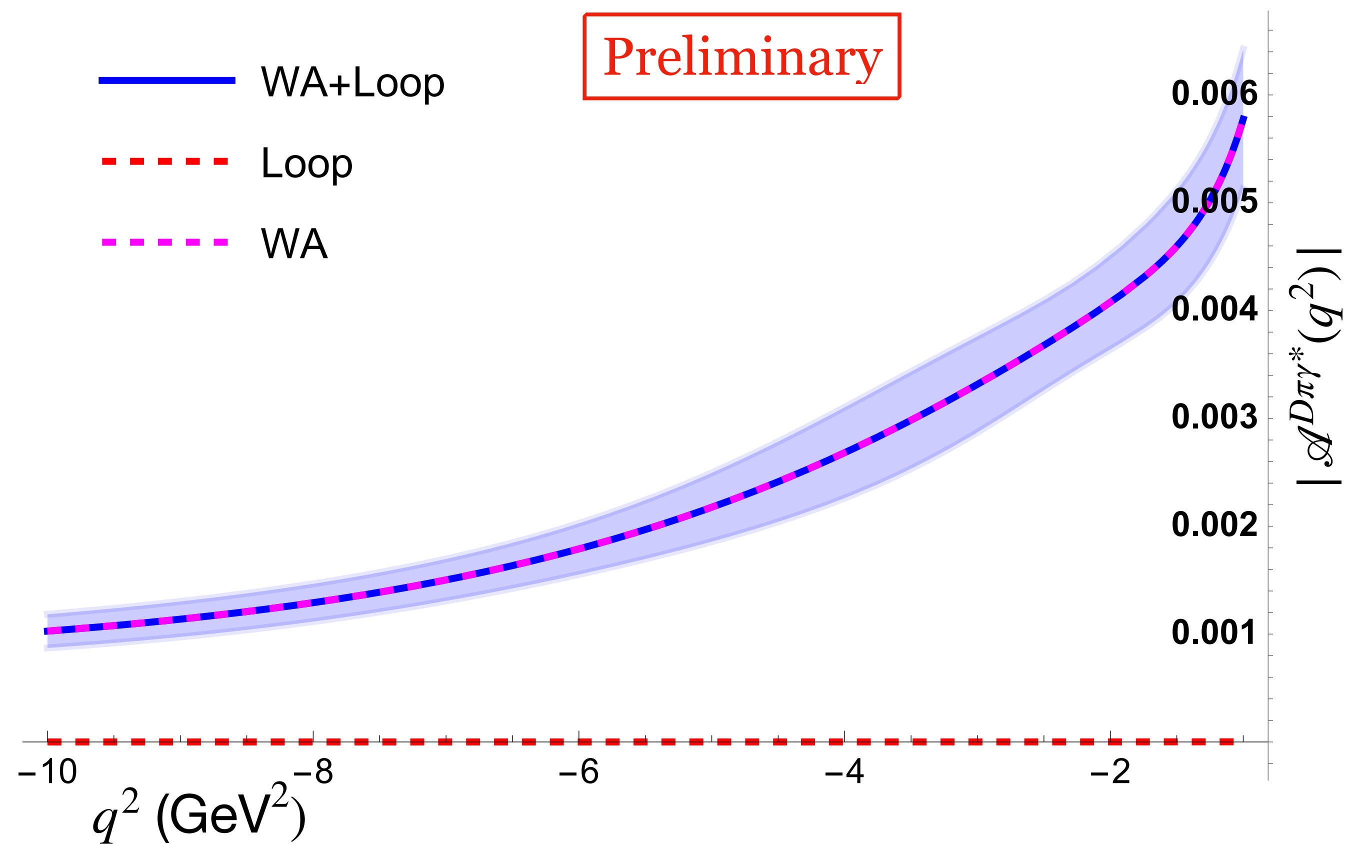

- The contribution to the decay amplitude from O_9 varies from $\sim 1.5 \times 10^{-6}$ to $\sim 7.5 \times 10^{-6}$ at $0 < q^2 < (m_D - m_\pi)^2$: at least three order of magnitudes smaller than the WA+loop amplitude

Preliminary Results

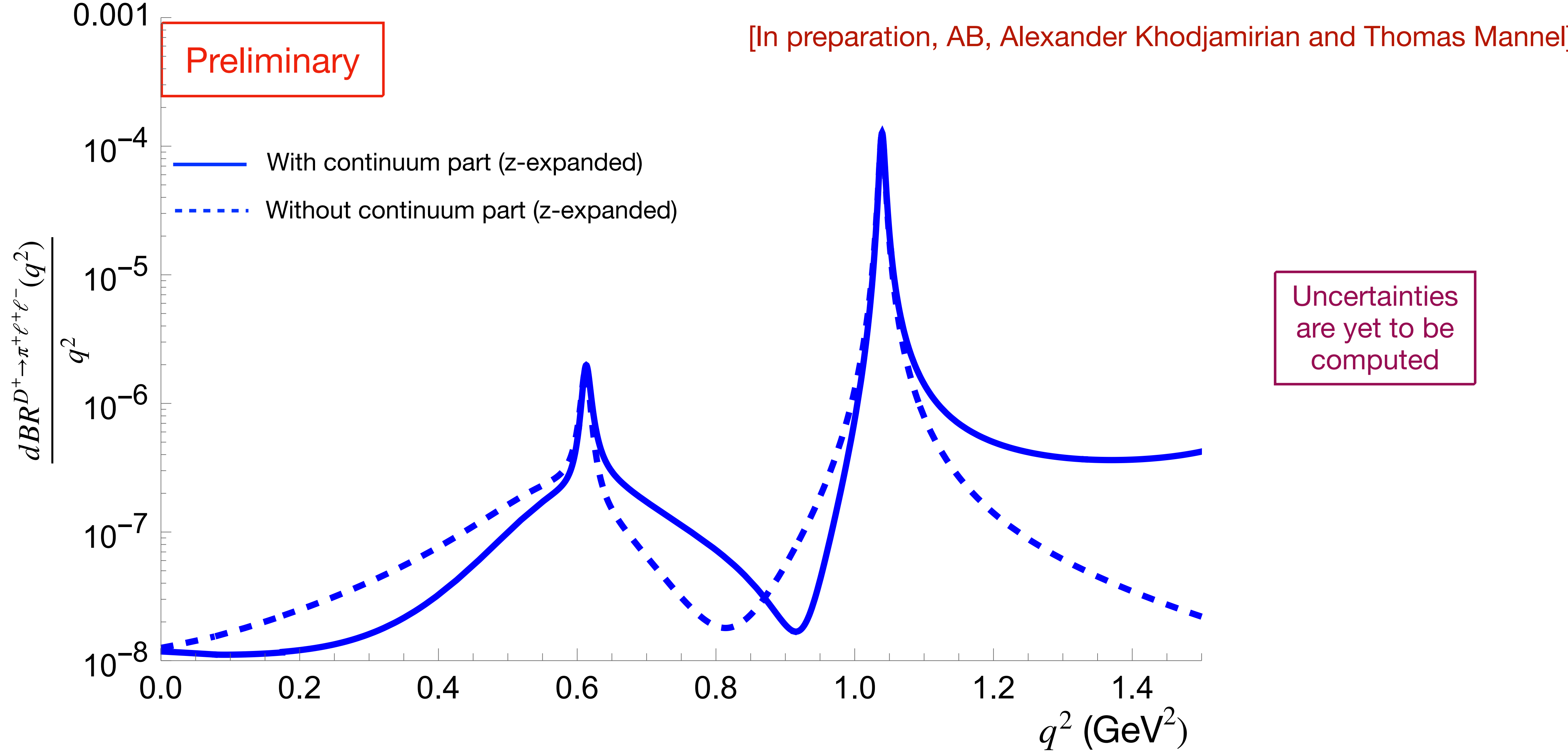

Figure: The results for the differential branching fraction using the dispersion relation with the fitted parameters

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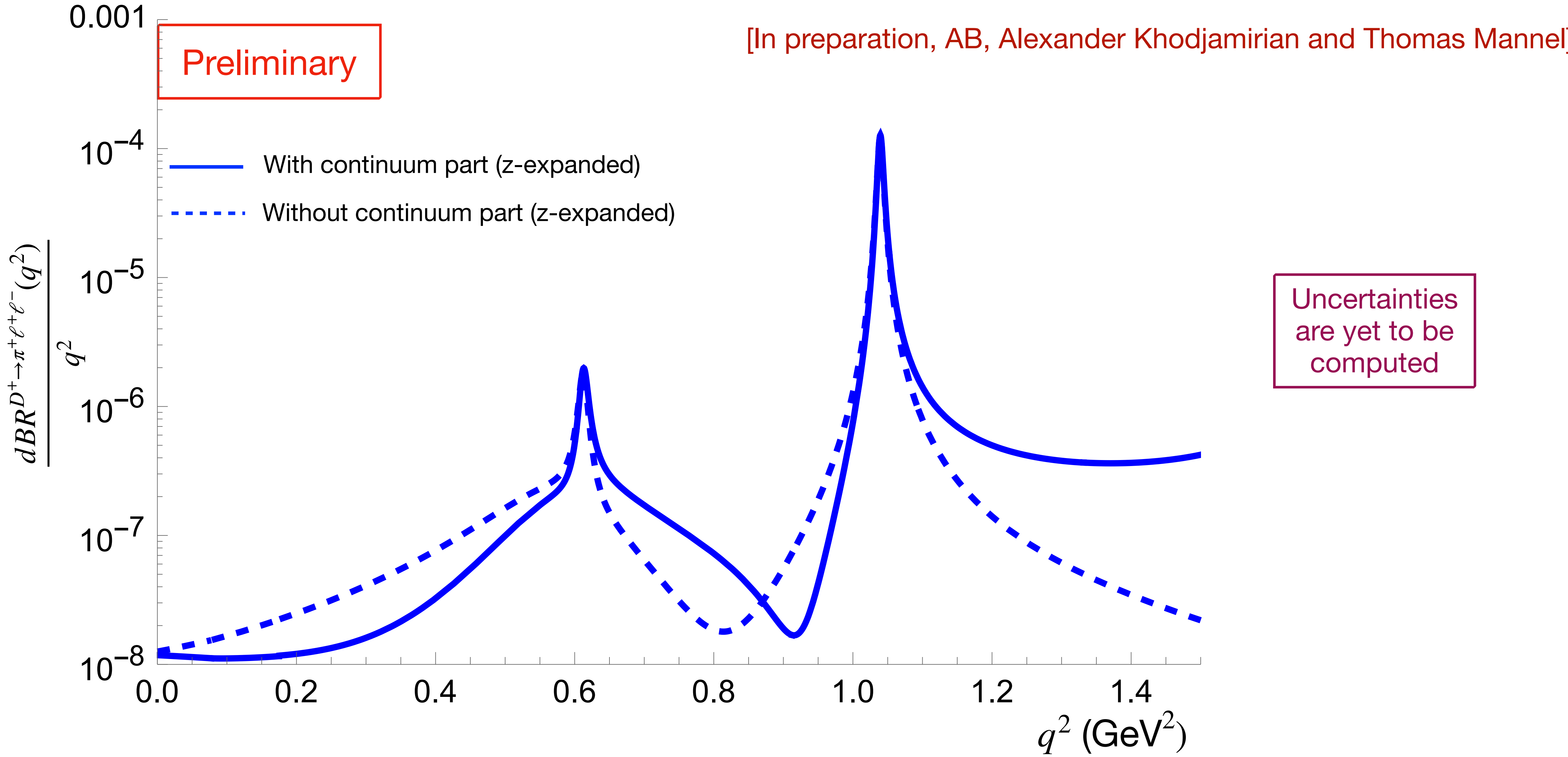

Uncertainties are yet to be computed

Figure: The results for the differential branching fraction using the dispersion relation with the fitted parameters

- Our method works only for $q^2 < m_{\rho'}^2$ due to the restrictions of z-expansion.
- The low q^2 region is generated by the "tail" of the resonances, the intermediate and high q^2 region is influenced by excited states.
- The low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$), integrated branching fraction $\sim 4.0 \times 10^{-9}$ ($\sim \frac{1}{2}$ times the QCDf predictions) [Preliminary].

[A. Bharucha, D. Boito, C. Méaux, JHEP 04 (2021) 158]

Summary and Outlook

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- The amplitude is mainly dominated by the weak annihilation topology generated by $O_{1,2}$, "Short distance" contributions e.g. due to O_9 are tiny.

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Message for LHCb analysis

There is no way to isolate long distance effects in $D_{(s)} \to P\ell^+\ell^-$ decays by simply vetoing resonances, one needs measurements of the differential decay rates in the whole q^2 region.

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