Recent progress on non-local contributions in $B \to K^{(*)}\bar{\ell}\ell$

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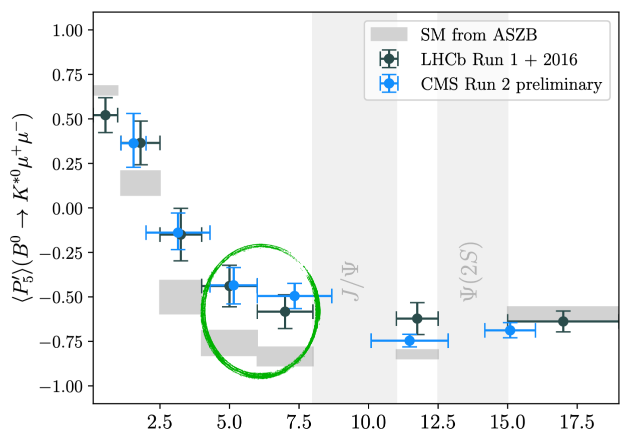
Implications of LHCb measurements and future prospects, 23-25 October 2024

Based on S. Mutke, MH, B. Kubis *JHEP* 07 (2024) 276 and G. Isidori, Z. Polonsky, AT (2405.17551)

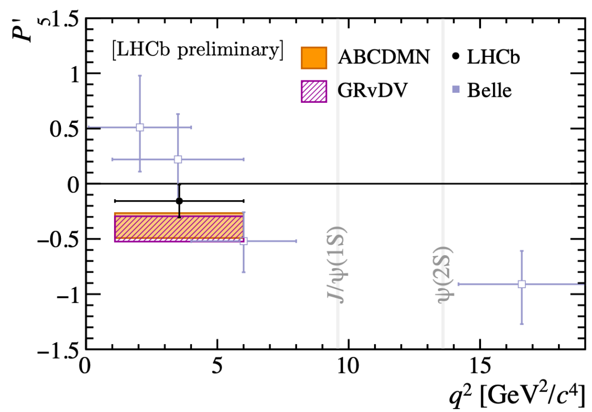
$b \to s\bar{\ell}\ell$ decays

- Flavor-changing neutral transitions are prime candidates in the search for BSM physics.
- Long-standing tension with the SM in the exclusive $b \to s\bar{\ell}\ell$ in rates and angular distributions, especially in the low-q^2 region (q^2 is the invariant mass of the lepton pair).

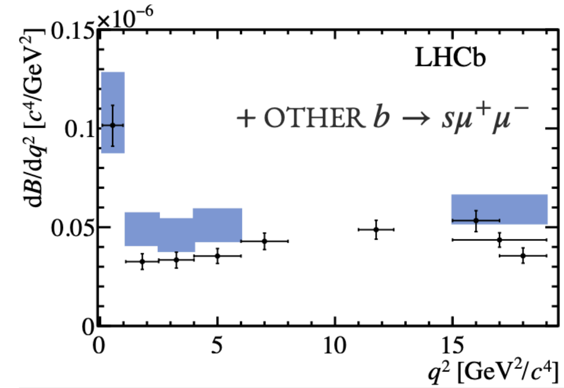


CMS-PAS-BPH-21-002 PRL. 125 (2020) 011802

[Plot by M. Andersson]



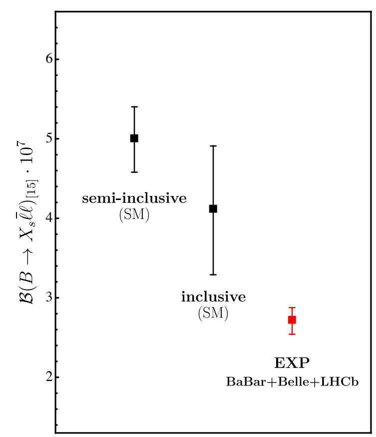
[LHCb-PAPER-2024-022, angular analysis of $B \to K^*\bar{e}e$]



[LHCb, JHEP 11 (2016) 047]

- Tension also at the inclusive level at high q^2.
- The inclusive rate has a different sensitivity to non-perturbative effects associated with charm-rescattering and is insensitive to local form factors uncertainties).

See talk by J. Jenkins



[G.Isidori, Z. Polonsky, AT, 2305.03076]

Theoretical Challenges in $B \to K^{(*)}\bar{\ell}\ell$

- $b \to s\bar{\ell}\ell$ decays are described by:

$$\mathscr{L} = \mathscr{L}_{\text{QCD+QED}}^{[N_f=5]} + \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i \mathcal{O}_i$$

$$\mathcal{O}_1 = (\bar{s}_L^\alpha \gamma_\mu c_L^\beta)(\bar{c}_L^\beta \gamma^\mu b_L^\alpha) \qquad \mathcal{O}_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

- To leading order in QED:

$$\mathscr{A}(B \to M\ell^+\ell^-) = \frac{G_F \alpha V_{ts}^* V_{tb}}{\sqrt{2}\pi} \Big[(C_9\, \ell\gamma^\mu\ell + C_{10}\, \ell\gamma^\mu\gamma_5\ell) \langle M | \bar{s}\gamma_\mu P_L b | \bar{B} \rangle - \frac{1}{q^2} \ell\gamma^\mu\ell \, (2im_b C_7 \langle M | \bar{s}\sigma_{\mu\nu} q^\nu P_R b | B \rangle + \mathscr{H}_\mu) \Big]$$

Local form factors

Non-local form factors

Lattice QCD + Light-Cone Sum Rules

matrix elements of the four-quark operators (only $\mathcal{O}_1, \mathcal{O}_2$ give a significant contribution)

Non-local contributions in $B \to K^{(*)}\bar{\ell}\ell$

- The non-local form factors contain the matrix elements of the **four-quark operators** $\mathcal{O}_{1-6}$.

- Note that to all orders in α_s, and to first order in $\alpha_{\rm em}$, **these matrix elements have the same structure as the matrix elements of $\mathcal{O}_7$ and $\mathcal{O}_9$:**

$$\mathcal{M}(B \to H_\lambda \ell^+\ell^-)|_{C_{1-6}} = -i\frac{32\pi^2\mathcal{N}}{q^2}\bar{\ell}\gamma^\mu\ell \int d^4x\, e^{iqx}\langle H_\lambda | T\{j^{\rm em}_\mu(x), \sum_{i=1,6} C_i\mathcal{O}_i(0)\} | B\rangle$$
$$= \left(\Delta_9^\lambda(q^2) + \frac{m_B^2}{q^2}\Delta_7^\lambda\right)\langle H_\lambda\, \ell^+\ell^- | \mathcal{O}_9 | B\rangle$$

- The (regular for $q^2 \to 0$) contributions of the non-local matrix elements of the four-quark operators can be effectively taken into account by the **shift**

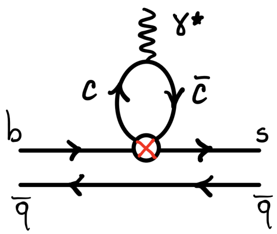
$$C_9 \to C_9^\lambda(q^2) = C_9^{\rm SM} + \Delta_9^\lambda(q^2) + C_9^{\rm SD}$$

LD + NP ?

- Therefore, even though the tension with the data could be well described by a shift in C_9 of $\mathcal{O}(25\%)$ with respect to the SM value, this shift could come from an inaccurate description of the non-local matrix elements.

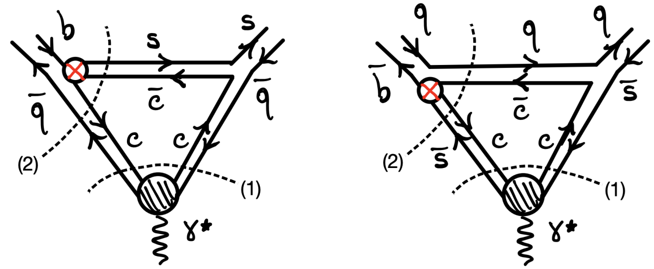
Non-local contributions in $B \to K^{(*)}\bar{\ell}\ell$

- The correlator in $\int d^4x\; e^{iqx}\langle H_\lambda \,|\, T\{j_\mu^{\mathrm{em}}(x), \sum_{i=1,6} C_i \mathcal{O}_i(0)\}\,|\, B\rangle$ receives two kinds of contributions:



[N. Gubernari et al, 2206.03797]

(a)



(b)

Pictures from [Ciuchini, Fedele et al, 2212.10516]

[Ladisa, Santorelli, 2208.00080]

- For the second kind of contributions, applying dispersive methods is tricky because the analytic structure is quite involved; in particular, an additional singularity in the case of an **anomalous threshold** could move into the q^2 integration domain, requiring a non trivial deformation of the path.
- The effect of these contributions is indistinguishable from a short-distance effect, since they show a **reduced q^2- or λ- dependence.**

A primer on form factor singularities

- **Kinematic invariants**
  - **Meson masses** $(q+k)^2 = M_B^2$, $k^2 = (M_P^2, M_V^2)$

    $\hookrightarrow$ only defined on-shell, otherwise model dependence from choice of interpolating field
  - **Photon virtuality** q^2

    $\hookrightarrow$ can define analytic continuation for q^2 arbitrary in the complex plane
- **Singularities in q^2**
  - **Poles**: (infinitely) narrow states $\Rightarrow q^2 = M_{J/\psi}^2, M_{\psi(2S)}^2$
  - **Normal thresholds**: branch points of $\gamma^* \to \{\pi^+\pi^-, D\bar{D}, \ldots\}$ cuts

    $\hookrightarrow q^2 = \{4M_\pi^2, 4M_D^2, \ldots\}$
  - **Anomalous thresholds**: anomalous branch points

    $\hookrightarrow$ can arise depending on left-hand-cut structure of $B \to (P, V)\gamma^*$ amplitude
- Conformal parameterizations Gubernari, Reboud, van Dyk, Virto

$$z(q^2) = \frac{\sqrt{s_{\text{thr}} - q^2} - \sqrt{s_{\text{thr}} - s_0}}{\sqrt{s_{\text{thr}} - q^2} + \sqrt{s_{\text{thr}} - s_0}} \qquad s_{\text{thr}} = 4M_D^2$$

do not account for anomalous contributions

$\hookrightarrow$ how big is the impact? MH, Kubis, Mutke 2024

Anomalous thresholds: when do they matter?

Dispersion relation for scalar loop function $C_0(s) \equiv C_0(p_1^2, s, p_3^2, m_1^2, m_2^2, m_3^2)$

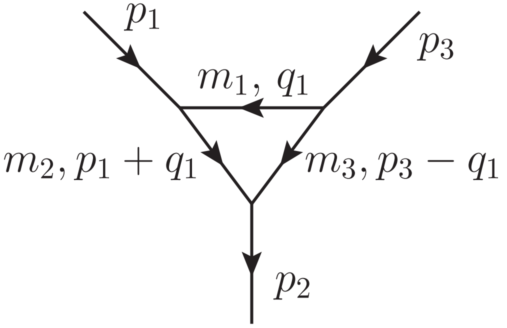
$$C_0(s) = \frac{1}{2\pi i} \int_{(m_2+m_3)^2}^{\infty} ds' \, \frac{\text{disc}\, C_0(s')}{s' - s}$$

- Discontinuity takes the form (modulo analytic continuations)

$$\text{disc}\, C_0(s) = \frac{2\pi i \theta(s - (m_2 + m_3)^2)}{\sqrt{\lambda(s, p_1^2, p_3^2)}} \log \frac{a - b}{a + b}$$

$$a = s^2 - s(p_1^2 + p_3^2 + m_2^2 + m_3^2 - 2m_1^2) + (p_1^2 - p_3^2)(m_2^2 - m_3^2)$$

$$b = \sqrt{\lambda(s, m_2^2, m_3^2)\lambda(s, p_1^2, p_3^2)}$$



$\hookrightarrow$ anomalous thresholds $s_\pm$ correspond to the zeros of the logarithm

- For sufficiently small p_1^2, p_3^2, $s_\pm$ lie on the second sheet, but $p_1^2 = M_B^2$ large
- Anomalous branch point moves onto first sheet for

$$m_3 p_1^2 + m_2 p_3^2 - (m_2 + m_3)(m_1^2 + m_2 m_3) > 0$$

$\hookrightarrow$ anomalous term in dispersion relation

Anomalous thresholds: when do they matter?

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- Discontinuity takes the form (modulo analytic continuations)

$$\text{disc}\, C_0(s) = \frac{2\pi i \theta(s-(m_2+m_3)^2)}{\sqrt{\lambda(s,p_1^2,p_3^2)}}\log\frac{a-b}{a+b}$$

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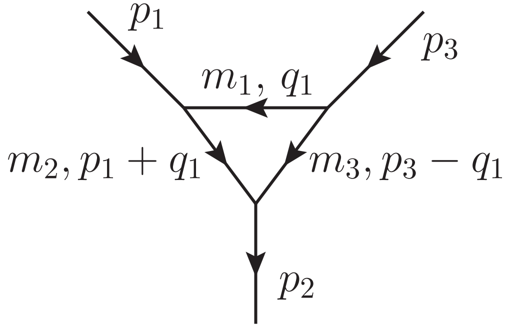

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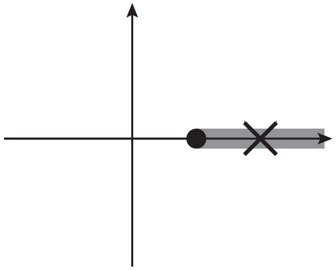
Anomalous thresholds: deformation of the integration contour

- Anomalous branch point on first sheet (can be either s_+ or s_-) requires **deformation of the integration contour**

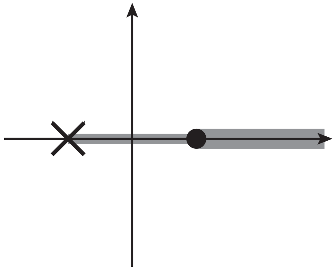
$$s_x = x\left(m_2 + m_3\right)^2 + (1-x)s_\pm$$

- Three cases:

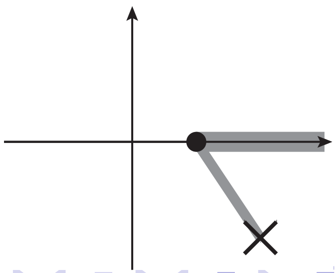
1. $s_\pm$ on normal cut
   ↪ analytic continuation of normal discontinuity



2. $s_\pm$ on negative real axis
   ↪ integration deformed along real axis



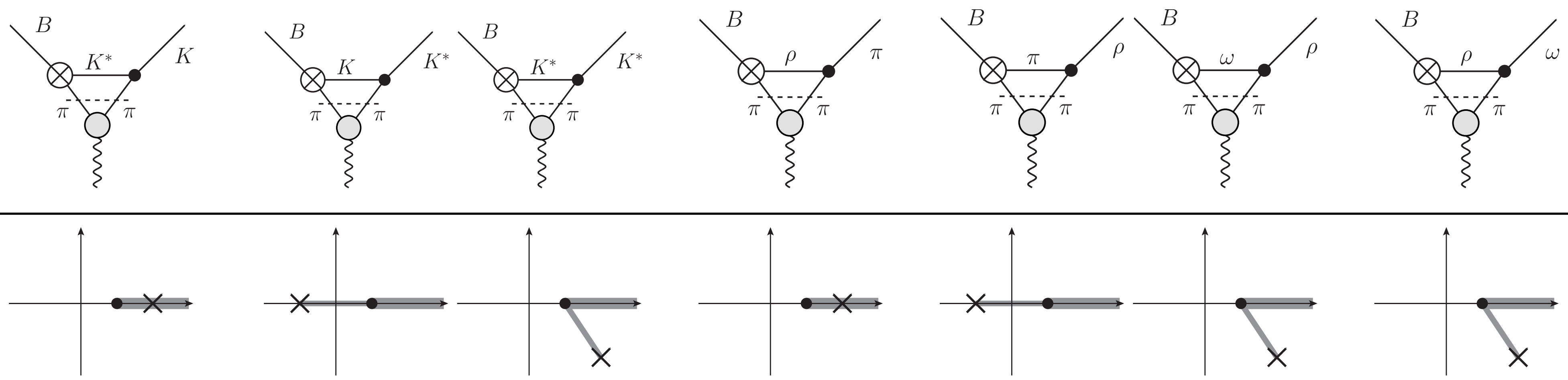
3. $s_\pm$ in complex plane
   ↪ integration deformed into complex plane



Anomalous thresholds for $B \to (P, V)\gamma^*$: strategy

- Start with ***u*-quark loop** and **$\pi\pi$ intermediate states**:
  - Pion form factor well known
  - Most branching fractions $B \to (P, V)\pi\pi$ known
  - Phenomenological knowledge of $B \to (PP, PV, VV)$
  - Sizable energy gap to next state $\pi\omega$
    $\hookrightarrow$ cf. various $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$ for hadronization of charm loop
- **CKM scaling**
  - $b \to s$: u-quark loop, $\mathcal{O}(\lambda^4)$, CKM suppressed compared to charm, $\mathcal{O}(\lambda^2)$
  - $b \to d$: both scale as $\mathcal{O}(\lambda^3)$
- Strategy/goals of this exercise:
  - Consider left-hand cuts that lead to anomalous thresholds
  - Fix parameters from $B \to (P, V)\pi\pi$ and $B \to (PP, PV, VV)$ input
  - Quantify the size of anomalous contributions to $B \to (P, V)\gamma^*$ form factors

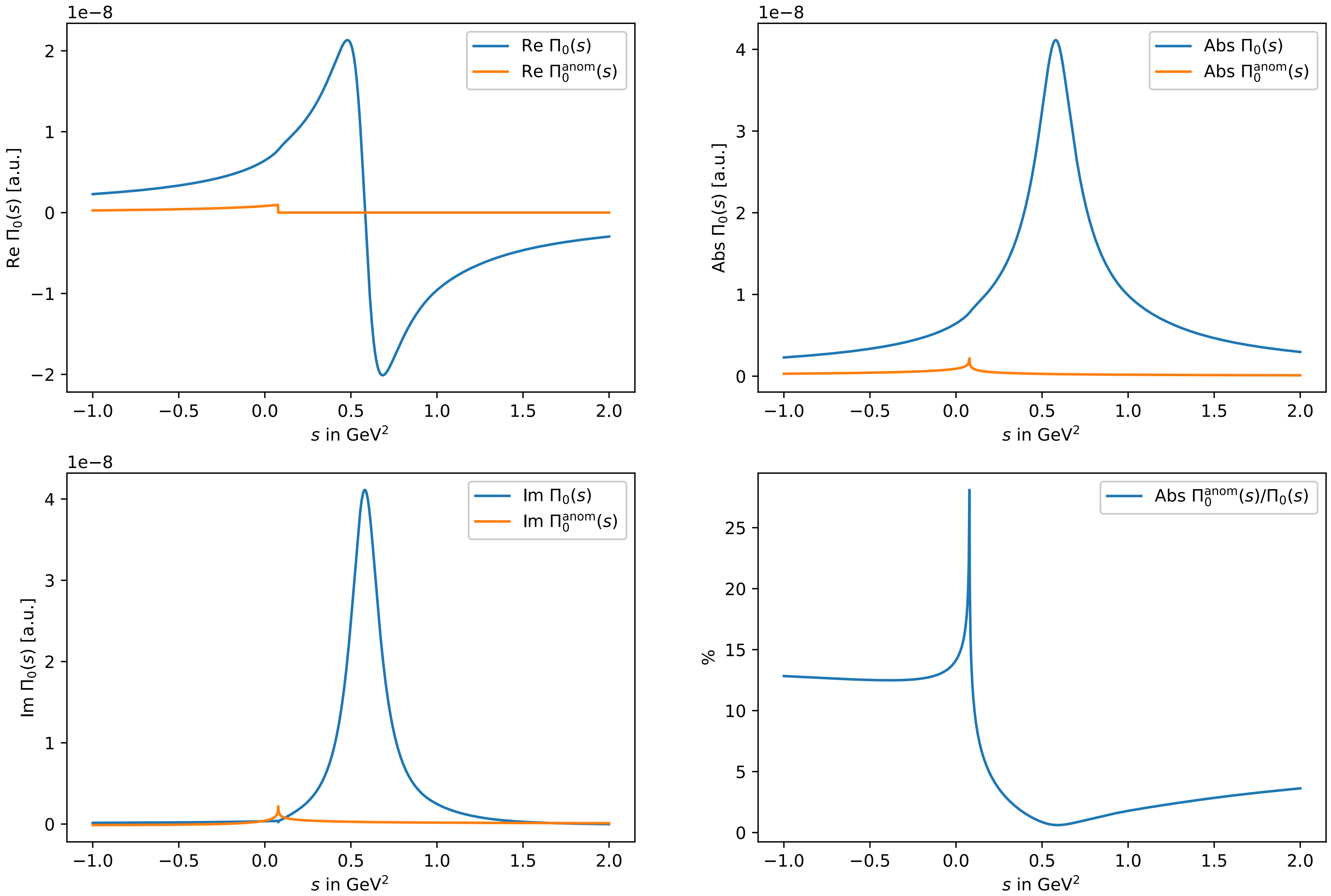
Anomalous thresholds for $B \to (P, V)\gamma^*$: list of processes



$B \to K\gamma^*$	$B \to K^*\gamma^*$	$B \to \pi\gamma^*$	$B \to \rho\gamma^*$	$B \to \omega\gamma^*$
$s_+ = 18.6\,\text{GeV}^2$	-57.8 $0.5 - 4.2i$	26.4	-859.3 $0.7 - 4.8i$	$0.2 - 4.9i$
$\text{Br}[B \to K^*\pi]$	$\text{Br}[B \to K^{(*)}\pi]$	$\text{Br}[B \to \rho\pi]$	$\text{Br}[B \to \pi\pi, \pi\omega]$	$\text{Br}[B \to \rho\pi]$
$\text{Br}[B \to K\pi\pi]$	$\text{Br}[B \to K^*\pi\pi]$	$\text{Br}[B \to 3\pi]$	$\text{Br}[B \to \rho\pi\pi]$	$\text{Br}[B \to \omega\pi\pi]$

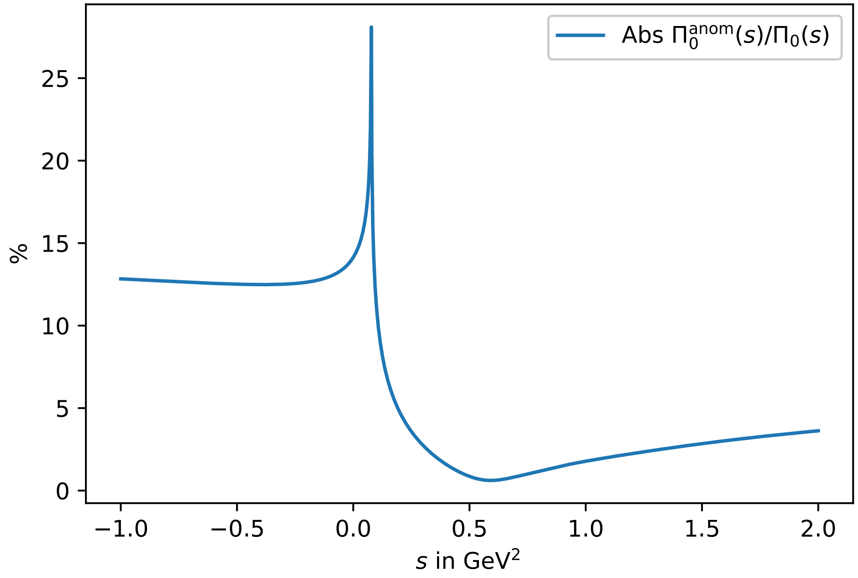
- $s_{\text{thr}} = 4M_\pi^2 = 0.08\,\text{GeV}^2$
- The branching fractions in the last line assume $\pi\pi$ in a P-wave.
- Consider K^*, ρ, ω narrow for now (could integrate over spectral functions).
- To disentangle helicity amplitudes, not only branching ratios, but polarization fractions are required.

Example: anomalous contribution to the longitudinal $B^+ \to K^{*+}\gamma^*$ FF



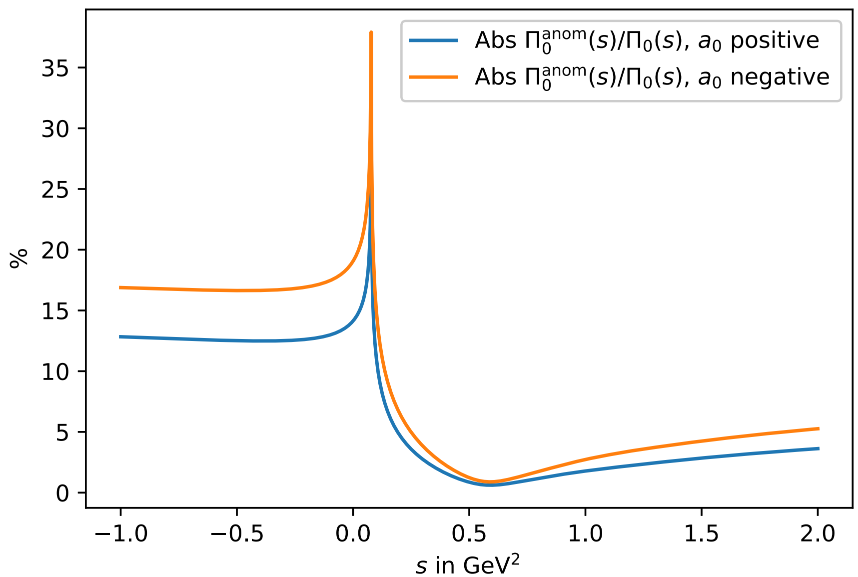
Example: anomalous contribution to the longitudinal $B^+ \to K^{*+}\gamma^*$ FF

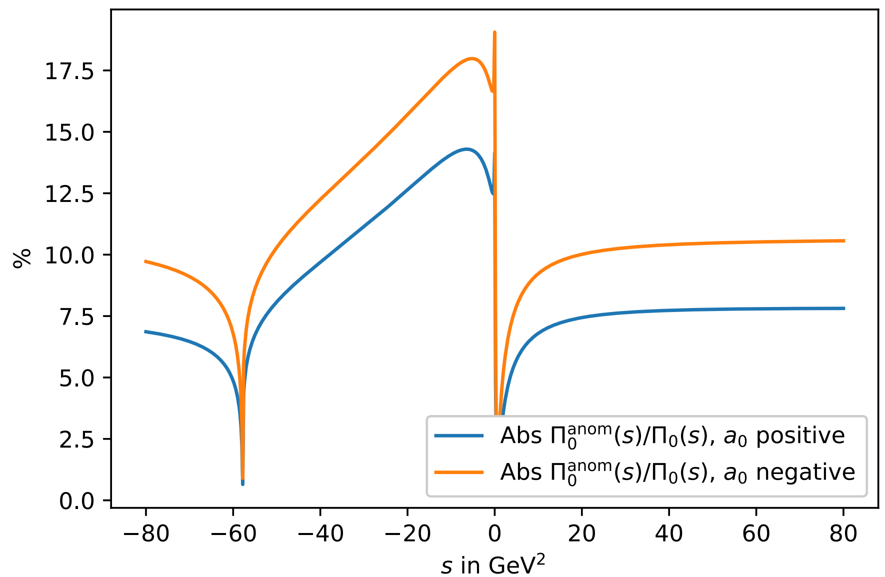
- Anomalous fraction $|\Pi_0^{\text{anom}}(s)/\Pi_0(s)| \sim 10$ % in this case
- Suppression around ρ-peak and singularity at threshold



Example: anomalous contribution to the longitudinal $B^+ \to K^{*+}\gamma^*$ FF

- Anomalous fraction $|\Pi_0^{\text{anom}}(s)/\Pi_0(s)| \sim 10\ \%$ in this case
- Suppression around ρ-peak and singularity at threshold
- No qualitative difference due to sign ambiguity of subtraction constant $a_{P,\lambda}$
- Even more relevant for space-like s





- Results qualitatively similar in other cases, percentage (slightly) lower

Key results and future plans

- Investigated **anomalous contributions to $B \to (P, V)\gamma^*$** for u-quark loop

  $\hookrightarrow$ estimated their relevance on basis of experimental data

- Key finding: **anomalous contributions can be as large as 10% off resonance**

  $\hookrightarrow$ relevant for matching in the space-like region?

- Consequences for ***u*-quark loop**:
  - Phenomenological estimates of non-local contributions

    $\hookrightarrow$ combination of dispersion relations, data input, QCD factorization, LCSRs, ...
  - Extension to higher intermediate states

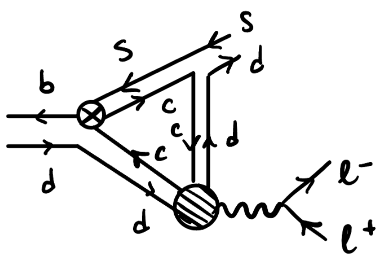
    $\hookrightarrow$ $4\pi \simeq \pi\omega$ could be feasible

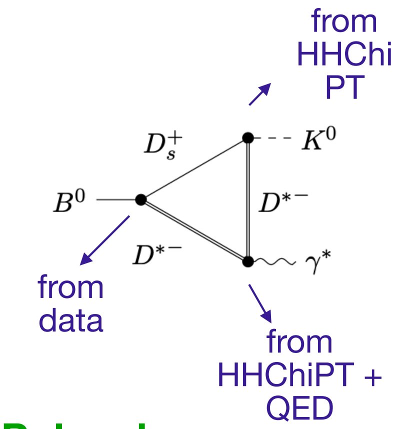
- Generalization to ***c*-quark loop**:
  - Expect impact of anomalous thresholds to be qualitatively similar

    $\hookrightarrow$ **always case with s_+ in lower complex plane**
  - More intermediate states in close proximity: $D\bar{D}$, $D^*\bar{D}$, $D^*\bar{D}^*$, ...
  - Phenomenology of form factors and amplitudes less well understood

Charm rescattering in $B^0 \to K^0 \bar{\ell}\ell$

- We give an estimate of long-distance effects associated with the rescattering of a charmed and a charmed-strange mesons.





[G.Isidori, Z. Polonsky, AT, 2405.17551]

- We look at the simplest rescattering contribution from the leading two-body intermediate state $D_s D^*$ and $D_s^* D$.

- We estimate this diagram using an effective description in terms of hadronic degrees of freedom, using **data** on $B \to DD^*$ and **Heavy Hadron Chiral Perturbation Theory** for the $DD_s^*(D_s D^*)K$ vertex.

- We obtain an accurate description in the low recoil (or **high** q^2) limit; we extrapolate to the whole kinematical region introducing appropriate form factors.

Model

- **Dynamics of $D_{(s)}^{(*)}$ mesons close to their mass shell, determined by:**
  - Lorentz invariance
  - Gauge invariance under QED
  - $SU(3)$ light-flavor symmetry
  - Heavy-quark spin symmetry

$$\begin{aligned}\mathcal{L}_{D,\text{free}} = &-\frac{1}{2}\left(\Phi^{\mu\nu}_{D^*}\right)^\dagger \Phi_{D^*\,\mu\nu} - \frac{1}{2}\left(\Phi^{\mu\nu}_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu\nu} \\ &+ \left(D_\mu \Phi_D\right)^\dagger D^\mu \Phi_D + \left(D_\mu \Phi_{D_s}\right)^\dagger D^\mu \Phi_{D_s} \\ &+ m_D^2 \left[\left(\Phi^\mu_{D^*}\right)^\dagger \Phi_{D^*\,\mu} + \left(\Phi^\mu_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu}\right] \\ &- m_D^2 \left[\Phi_D^\dagger \Phi_D + \Phi_{D_s}^\dagger \Phi_{D_s}\right] + \text{h.c.}\,.\end{aligned}$$

- **Weak $B \to DD^*$ transition described by (using heavy-quark spin symmetry + data)**

$$\mathcal{L}_{BD} = g_{DD^*}\left(\Phi^{\mu\dagger}_{D_s^*}\,\Phi_D \partial_\mu \Phi_B + \Phi^\dagger_{D_s}\,\Phi^\mu_{D^*}\partial_\mu \Phi_B\right) + \text{h.c.}$$

$$g_{DD^*} = \sqrt{2} G_F\,|V_{tb}^* V_{ts}| m_B m_D\,\bar{g} \qquad \bar{g} \approx 0.04$$

- **From HHChiPT** (valid close to endpoint $q^2 \approx m_B^2$):

$$\mathcal{L}_{DK} = \frac{2 i g_\pi m_D}{f_K}\left(\Phi^{\mu\dagger}_{D^*}\Phi_{D_s}\partial_\mu \Phi_K^\dagger - \Phi_D^\dagger \Phi^\mu_{D_s^*}\partial_\mu \Phi_K^\dagger\right) + \text{h.c.}$$

Form Factors

In order to obtain a reliable estimate **over the entire kinematical range**, we need to take into account the fact that the hadrons are not well described by fundamental fields far from their mass shell:

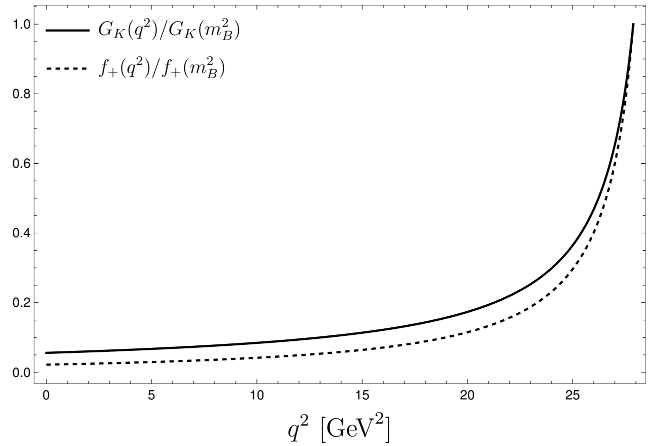
- Correction for **QED vertex** **(using Vector Meson Dominance):**

$$e \to eF_V(q^2)\,, \qquad F_V(q^2) = \frac{m_{J/\psi}^2}{m_{J/\psi}^2 - q^2}$$

- Correction for DD^*K **vertex**:

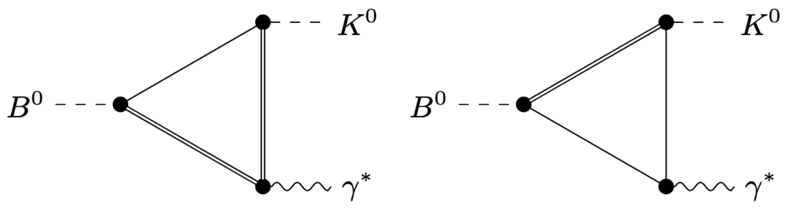
$$\frac{1}{f_K} \to \frac{1}{f_K} G_K(q^2)\,,$$
$$G_K(q^2) = \frac{1}{1 + E_K(q^2)/f_K} = \frac{2m_B f_K}{2m_B f_K + m_B^2 - q^2}$$

Useful consistency check: G_K has a similar scaling to the vector form factor $f_+(q^2)$ for $B_0 \to K_0$



Results

- We compute the **one-loop diagrams** appearing in the model presented.
- In the $SU(3)$-symmetry limit, the diagrams obtained by swapping $D_s^{(*)} \leftrightarrow D^{(*)}$ are symmetric.



$$\mathcal{M}_{\rm LD} = -\frac{e g_{DD^*} g_\pi F_V(q^2) G_K(q^2)}{8\pi^2 f_K m_D}(p_B \cdot j_{\rm em}) \times \left[(2 + L_\mu) - \delta L(q^2, m_B^2, m_D^2)\right],$$

$$L_\mu = \log(\mu^2/m_D^2)$$

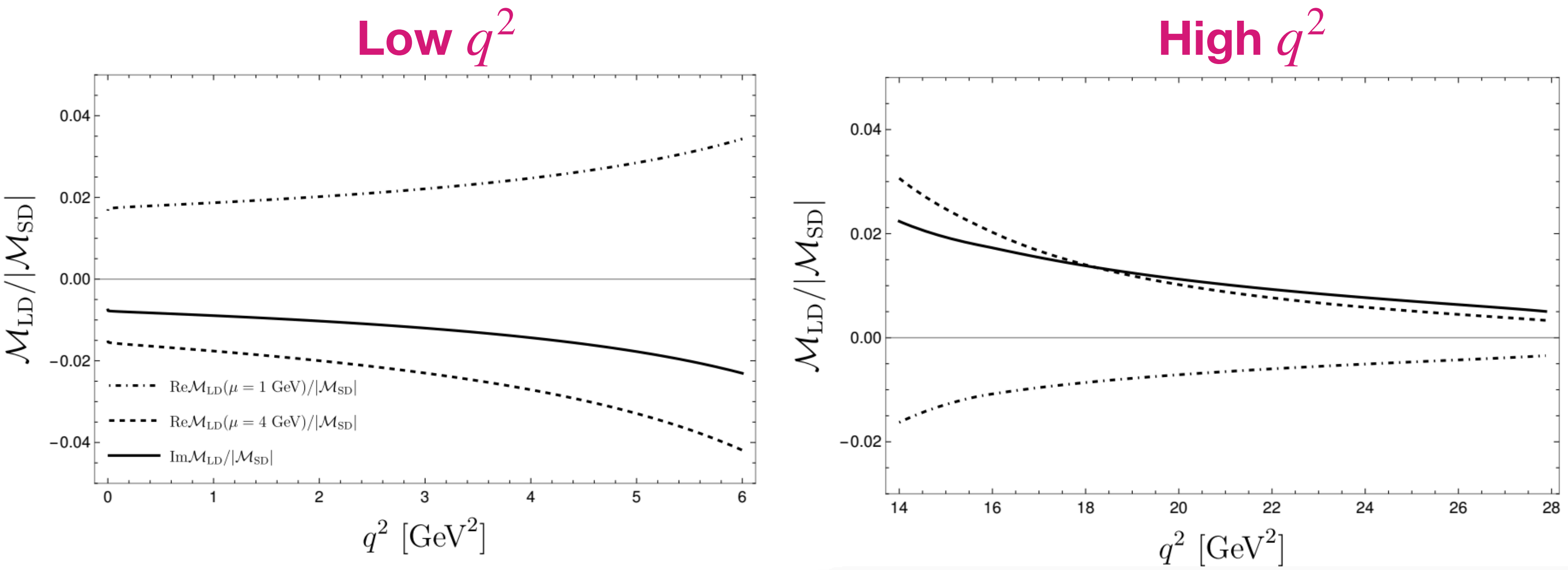
$$\delta L(q^2, m_B^2, m_D^2) = \frac{L(m_B^2, m_D^2) - L(q^2, m_D^2)}{q^2 - m_B^2},$$

$$L(x,y) = \log\left(\frac{2y - x + \sqrt{x(x-4y)}}{2y}\right) \times \left[\sqrt{x(x-4y)} + y\log\left(\frac{2y - x + \sqrt{x(x-4y)}}{2y}\right)\right]$$

- Compare it to the **short-distance matrix element**: $\mathcal{M}_{\rm SD} = \frac{4G_F}{\sqrt{2}}\frac{e}{16\pi^2} V_{tb}^* V_{ts}(p_B \cdot j_{\rm em}) f_+(q^2)(2C_9)$

Results

- Ratios of long-distance vs short-distance matrix elements:



- The **absorptive part** of the matrix element is independent of the renormalization scheme used, and and it is given by the discontinuity of the amplitude corresponding to the kinematical regions where the internal mesons go on-shell.

- We find these contributions are relatively **flat** in q^2 and therefore can mimic short-distance effects.

- However, these LD contributions are **not large**: the one from the D^*D_s and D_s^*D intermediate states does not exceed a few percent relative to the SD amplitude.

Quantify effect on C_9

- We can encode the effect of the $\mathscr{M}_{LD}$ via a q^2-dependent **shift in C_9.**

$$\delta C^{\rm LD}_{9,DD^*}(q^2,\mu) = \bar{g}\,\Delta(q^2)\Big[2 + L_\mu - \delta L(q^2, m_B^2, m_D^2)\Big] \qquad \Delta(q^2) = -\frac{g_\pi m_B F_V(q^2) G_K(q^2)}{2 f_K f_+(q^2)}$$

- Averaging over the low- and high-q^2 regions, we find:

$$\delta\bar{C}^{\rm LD,low}_{9,DD^*}(\mu) = -0.003 - 0.059\,i - 0.156\log\left(\frac{\mu}{m_D}\right)$$
$$\delta\bar{C}^{\rm LD,high}_{9,DD^*}(\mu) = 0.009 + 0.053\,i + 0.063\log\left(\frac{\mu}{m_D}\right).$$

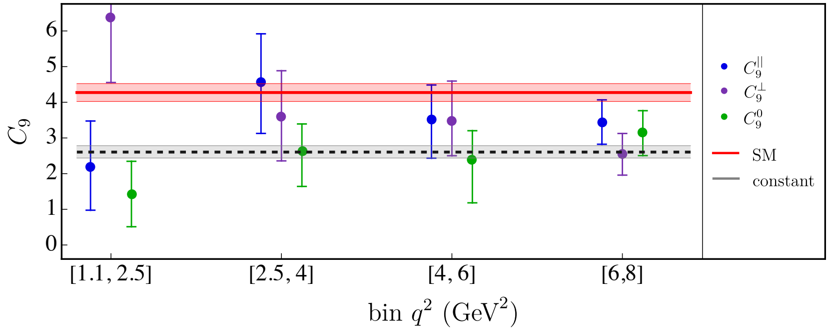
- Varying the renormalization scale μ in the range $[1,4]$ GeV:

$$|\delta\bar{C}^{\rm LD}_{9,DD^*}| \leq 0.11 \longrightarrow \boxed{\frac{\delta C_9}{C_9^{SM}} \approx 2.5\,\%}$$

Sign of δC_9

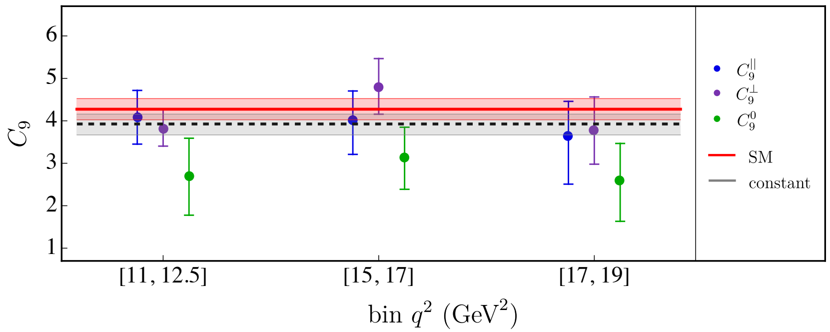
- The sign of δC_9 is **opposite** in the two cases (regardless of the phase of g_{DD^*}): comparing the extraction of C_9 at low- and high-q^2 provides a useful data-driven check for such long-distance contributions.

- From a fit of C_9 from the branching ratio and angular observables in $B \to K^* \bar{\mu}\mu$, we get, assuming that C_9 could depend on the bin and on the polarization:



$$C_9 = 2.6^{+0.2}_{-0.2}$$

Using resonance parameters found by LHCb recently (2405.17347)



$$C_9 = 3.9^{+0.2}_{-0.3}$$

$$C_9^{\perp} = 4.0^{+0.4}_{-0.4}$$

$$C_9^{||} = 4.0^{+0.5}_{-0.5}$$

$$C_9^{0} = 2.9^{+0.6}_{-0.6}$$

→ **Importance of extracting the value of C_9 at different values of q^2**

Additional intermediate states

- So far we focused on the D^*D_s or D_s^*D intermediate states, but in principle there are **other states** with $\bar{c}c\bar{s}d$ valence structure.
- Consider all intermediate states the allow parity-conserving strong interactions with the kaon:

B^0 Decay	$\mathcal{B}(B^0 \to X) \times 10^3$
D^*D_s	8.0 ± 1.1
DD_s^*	7.4 ± 1.6
$D^*D_s^*$	17.7 ± 1.4
$DD_{s0}(2317)$	1.06 ± 1.6
$D^*D_{s1}(2457)$	9.3 ± 2.2
$D^*D_{s1}(2536)$	0.50 ± 0.14
$DD_{s2}(2573)$	$(3.4 \pm 1.8) \times 10^{-2}$
$D^*D_{s2}(2573)$	< 0.2
$DD_{s1}(2700)$	0.71 ± 0.12

- Conservative **multiplicity factor** accounting for all possible intermediate states:

$$\mathcal{N} = \frac{\sum_X \mathcal{M}(B^0 \to X)}{\mathcal{M}(B^0 \to D^*D_s) + \mathcal{M}(B^0 \to DD_s^*)} \approx \frac{1}{2} \sum_X \sqrt{\frac{\mathcal{B}(B^0 \to X)}{\mathcal{B}(B^0 \to DD_s^*)}} \approx 3$$

$$\longrightarrow \quad |\delta C_9^{\text{LD}}| \leq \mathcal{N} |\delta \bar{C}_{9,DD^*}^{\text{LD}}| \leq 0.33 \quad \longrightarrow \quad \boxed{\frac{\delta C_9}{C_9^{SM}} \approx 8 - 10\,\%}$$

Conclusions

- We have presented an estimate of $B^0 \to K^0 \bar{\ell}\ell$ long-distance contributions induced by the rescattering of a charmed and a charmed-strange meson;
- At **high-q^2** the estimate is based on controlled approximations from VMD and HHChiPT;
- At **low-q^2** the extrapolation via form factors is only meant to provide a conservative upper bound;
- We find these contributions are relatively **flat in q^2** and therefore can mimic short-distance effects, but they don't exceed a few percent relative to the SD contribution.
- We neglected some effects ($SU(3)$-breaking effects, higher-mass charmonium resonances, baryonic modes, higher-multipole photon couplings), but we were conservative adding all possible intermediate states coherently -> $10\,\%$ **shift in C_9**

Comparison (current status)

	MH, Kubis, Mutke 2024	Isidori, Polonsky, AT 2024
Framework	dispersion relations	loop integrals
	discontinuities $\simeq \left[B \to (P,V)X\bar{X}\right] \times \left[X\bar{X} \to \gamma^*\right]^*$ agree	
Key input	**couplings from *B*, *D*, …decays, form factors**	
Key output	size of anomalous contributions by comparing normal and anomalous terms in dispersion relation for $B \to (P,V)\gamma^*$	size of $D\bar{D}$ rescattering via $D_s^*\bar{D}$, $D^*\bar{D}_s$ loop diagrams (including form factors) and coherent sum over other channels
Applied to	*u*-quark loop	charm loop
$\{P,V\}$	$\{\pi, K, \rho, K^*\}$	$\{K\}$

Conclusions

- Several **indications** that **uncertainty** from **charm-loop effects** is **not dramatic**
  - Anomalous thresholds lie in lower complex plane, small effect for *u*-quark loop
  - Loop estimate of $D\bar{D}$ rescattering diagrams for $B \to K\ell^+\ell^-$ gives $\Delta C_9/C_9 \lesssim 10\%$.
- Possible theory improvements
  - Extensions/refinements within each method see earlier slides
  - Combination of and deeper comparison among methods
- So, one should (and can) do better, but **need data input**
  - *D*-meson form factors
  - $B \to K^{(*)}D\bar{D}$ branching fractions and Dalitz plots
  - Differential information important to disentangle phases and relative importance of decay mechanisms
- **q^2 dependence critical** to test different scenarios and differentiate hadronic effects from BSM

  $\hookrightarrow$ please report results in as many separate q^2 bins as possible!

Thank you for your attention!

Backup Slides

Anomalous thresholds: where do they come from?

- **Landau equations**: singularities of general loop integral

$$\int \prod_{\ell=1}^{L} \frac{d^4 q_\ell}{(2\pi)^4} \prod_{i=1}^{n} \frac{i}{k_i^2 - m_i^2 + i\epsilon} \quad \text{singular when} \quad \begin{cases} \lambda_i (k_i^2 - m_i^2) = 0 & \text{for all } i = 1, \ldots, n \\ \sum_{i=1}^{n} \lambda_i k_i \cdot \frac{\partial k_i}{\partial q_\ell} = 0 & \ell = 1, \ldots, L \end{cases}$$

$\hookrightarrow$ "leading singularity" $\Leftrightarrow$ all $\lambda_i \neq 0$

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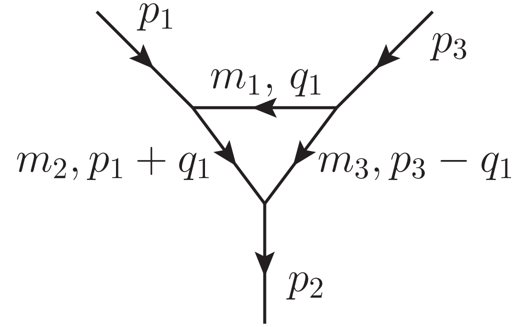
$\hookrightarrow$ "leading singularity" $\Leftrightarrow$ all $\lambda_i \neq 0$

- **Triangle diagram**: $L = 1$, $n = 3$, Landau equations become

$$\lambda_i(k_i^2 - m_i^2) = 0 \qquad \sum_{i=1}^{3} \lambda_i k_i = 0$$



  - **Normal thresholds**: e.g., $\lambda_3 = 0 \Rightarrow p_1^2 = (m_1 \pm m_2)^2$

    [zeros of $\lambda(p_1^2, m_1^2, m_2^2)$, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$]

  - **Anomalous threshold**: all $\lambda_i \neq 0$

$$\hookrightarrow p_2^2 = s_\pm \equiv p_1^2 \frac{m_1^2 + m_3^2}{2m_1^2} + p_3^2 \frac{m_1^2 + m_2^2}{2m_1^2} - \frac{p_1^2 p_3^2}{2m_1^2} - \frac{(m_1^2 - m_2^2)(m_1^2 - m_3^2)}{2m_1^2} \pm \frac{1}{2m_1^2} \sqrt{\lambda(p_1^2, m_1^2, m_2^2)\lambda(p_3^2, m_1^2, m_3^2)}$$

Helicity amplitudes for $B \to (P, V)\gamma^*$ and $B \to (P, V)\pi\pi$

- Decompose **non-local form factors** $\mathcal{H}_\mu^{B\to(P,V)}(q^2)$ into covariant structures with scalar coefficients $\Pi_{P,\lambda}(q^2)$ free of kinematic singularities/zeros:

$$\mathcal{H}_\mu^{B\to P}(q^2) = \tilde{\mathcal{S}}_\mu \Pi_P(q^2)$$
$$\mathcal{H}_\mu^{B\to V}(q^2) = \eta^{*\alpha} \left[\tilde{\mathcal{S}}^{\perp}_{\alpha\mu}\Pi_\perp(q^2) + \tilde{\mathcal{S}}^{\parallel}_{\alpha\mu}\Pi_\parallel(q^2) + \tilde{\mathcal{S}}^{0}_{\alpha\mu}\Pi_0(q^2)\right]$$

  - For V: polarization vector η^α, helicity components $\lambda \in \{\perp, \parallel, 0\}$
  - Longitudinal component $\Pi_0(q^2)$

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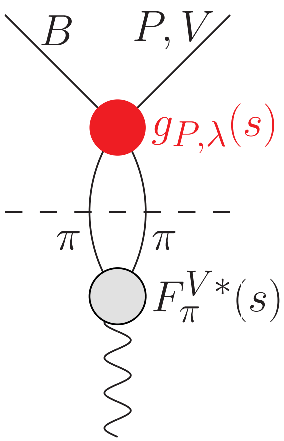
  - For V: polarization vector η^α, helicity components $\lambda \in \{\perp, \parallel, 0\}$
  - Longitudinal component $\Pi_0(q^2)$
- Also decompose $B \to (P, V)\pi\pi$ amplitude into helicity amplitudes:

$$\mathcal{M}^{B\to P\pi\pi}(s,t,u) = \mathcal{F}_P(s,t,u)$$
$$\mathcal{M}^{B\to V\pi\pi}(s,t,u) = \eta^{*\alpha}\left[\tilde{T}^{\perp}_{\alpha}\mathcal{F}_\perp(s,t,u) + \tilde{T}^{\parallel}_{\alpha}\mathcal{F}_\parallel(s,t,u) + \tilde{T}^{0}_{\alpha}\mathcal{F}_0(s,t,u)\right]$$

- Polarization fractions $f_\lambda = |\mathcal{F}_\lambda|^2/\sum_{\lambda'}|\mathcal{F}_{\lambda'}|^2$
  - Know longitudinal fraction $f_L \equiv f_0$ from experiment
  - Set $f_\perp = f_\parallel = (1 - f_L)/2$, motivated by QCD factorization Beneke, Rohrer, Yang 2007

Left-hand cuts in $B \to (P, V)\pi\pi$



- Unitarity relations for $\pi\pi$ intermediate states FF ($s = q^2$)

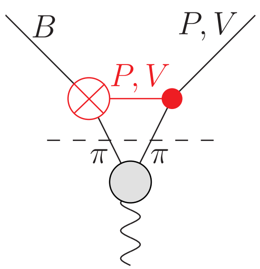
$$\text{disc}\,\Pi_{P,\lambda}(s) = 2i\,\nu_{P,\lambda}(s)\,g_{P,\lambda}(s)\,F_\pi^{V*}(s)$$

  - Kinematic factors $\nu_{P,0}(s) \equiv \sigma_\pi^3(s)$, $\nu_{\perp,\parallel}(s) \equiv s\,\sigma_\pi^3(s)$, where $\sigma_\pi(s) = \sqrt{1 - 4M_\pi^2/s}$
  - $g_P(s)$, $g_\lambda(s)$ denote P-waves of $\mathcal{F}_P(s,t,u)$, $\mathcal{F}_\lambda(s,t,u)$

- Consider left-hand cuts from t, u-channel Born exchange
  - P-wave projection leads to triangle topology:

$$g_{P,\lambda}(s) \sim \text{disc}\,C_0(s)$$

  - Anomalous thresholds in FF dispersion relations:



$$\Pi_{P,\lambda}(s) = \underbrace{\frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\nu_{P,\lambda}(s')\,g_{P,\lambda}(s')\,F_\pi^{V*}(s')}{s'-s}}_{\equiv \Pi_{P,\lambda}^{\text{norm}}(s)} + \underbrace{\frac{1}{\pi}\int_0^1 dx\,\frac{\partial s_x}{\partial x}\,\frac{\nu_{P,\lambda}(s_x)\,\text{disc}\,g_{P,\lambda}(s_x)\,F_\pi^{V*}(s_x)}{s_x - s}}_{\equiv \Pi_{P,\lambda}^{\text{anom}}(s)}$$

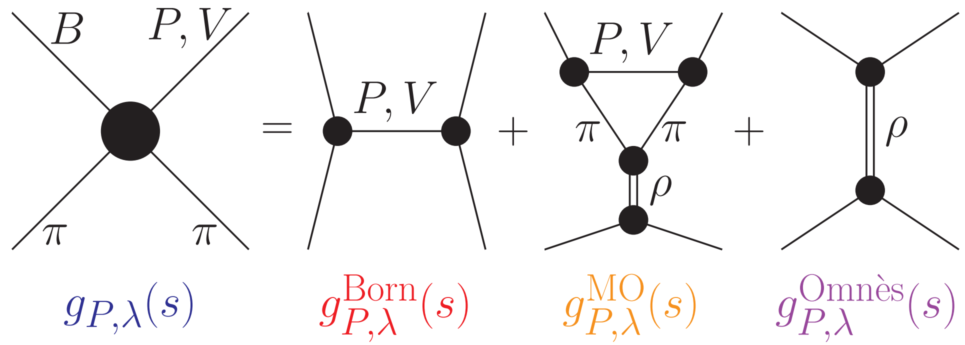
Unitarization of $B \to (P, V)\pi\pi$

- Simple Born amplitude violates unitarity (Watson's theorem):

$$\text{disc}\, g_{P,\lambda}(s) = 2i\, g_{P,\lambda}(s) \sin\delta(s)\, e^{-i\delta(s)} = 2i\, g_{P,\lambda}(s)\, \sigma_\pi(s)\, t^*(s)$$

$\hookrightarrow$ $\pi\pi$ elastic scattering phase shift $\delta(s)$ ($I = 1, L = 1$), $t(s) = \sin\delta(s)\, e^{i\delta(s)}/\sigma_\pi(s)$

- Unitarize via Muskhelishvili–Omnès representation:



$$g^{\text{MO}}_{P,\lambda}(s) = \Omega(s)\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{g^{\text{Born}}_{P,\lambda}(s')\sin\delta(s')}{|\Omega(s')|(s'-s)} + \frac{s}{\pi}\int_0^1 \frac{dx}{s_x}\frac{\partial s_x}{\partial x}\frac{\text{disc}\, g^{\text{Born}}_{P,\lambda}(s_x)\,\sigma_\pi(s_x)\, t(s_x)}{\Omega(s_x)(s_x-s)}\right]$$

$$g^{\text{Omnès}}_{P,\lambda}(s) = a_{P,\lambda}\,\Omega(s)$$

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s}\right] = |\Omega(s)|\, e^{i\delta(s)}$$

- One subtraction constant $a_{P,\lambda}$ needed due to $g_{P,\lambda}(s) \sim \mathcal{O}(s^{-1})$

Fixing the subtraction constant

- Use experimental values of branching fractions $\text{Br}[B \to \rho(P, V)]$ and polarization fractions $f_L[B \to \rho V]$ to fix the subtraction constants $a_{P,\lambda}$
  - P-wave contributions to differential decay rate

$$d\Gamma_{P,\lambda}[B \to (P, V)\pi\pi] = \phi_{P,\lambda}(s)|g_{P,\lambda}(s)|^2 ds$$

  ↪ Phase space factors $\phi_{P,\lambda}(s)$

  ↪ Remember $g_{P,\lambda}(s) = g^{\text{Born}}_{P,\lambda}(s) + g^{\text{MO}}_{P,\lambda}(s) + a_{P,\lambda}\Omega(s)$

  - Demand that ρ-band of width $\Delta = 2M_\rho\Gamma_\rho$ be saturated:

$$\Gamma[B \to \rho P] \stackrel{!}{=} \mathcal{N}_P^{-1} \int_{M_\rho^2-\Delta}^{M_\rho^2+\Delta} ds \frac{d\Gamma_P[B \to P\pi\pi]}{ds}$$

$$f_\lambda\, \Gamma[B \to \rho V] \stackrel{!}{=} \mathcal{N}_\lambda^{-1} \int_{M_\rho^2-\Delta}^{M_\rho^2+\Delta} ds \frac{d\Gamma_\lambda[B \to V\pi\pi]}{ds}$$

$$\text{with } \mathcal{N}_{P,\lambda} = \left(\int_{M_\rho^2-\Delta}^{M_\rho^2+\Delta} ds\, \phi_{P,\lambda}(s)|\Omega(s)|^2\right) \Big/ \left(\int_{4M_\pi^2}^{(M_B-M_{P/V})^2} ds\, \phi_{P,\lambda}(s)|\Omega(s)|^2\right)$$

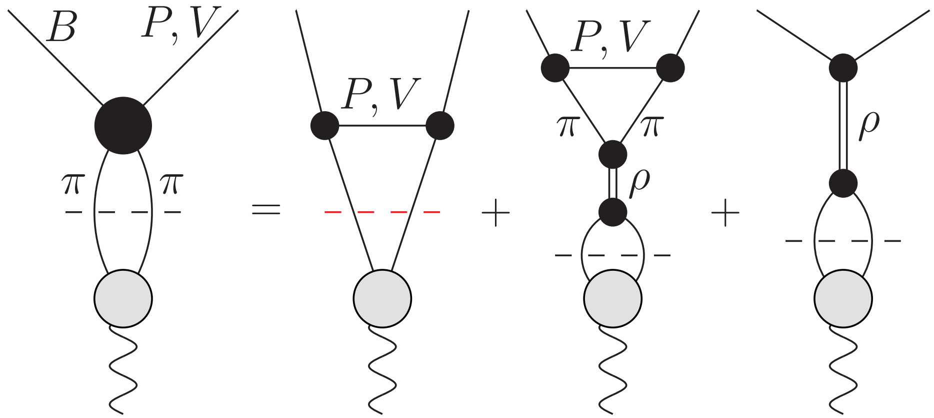
  ↪ Normalization factor $\mathcal{N}_{P,\lambda}$ as we only integrated over the ρ-band

  - Solve this for $a_{P,\lambda}$ assuming $a_{P,\lambda}$ is real (sign ambiguity!)

FF dispersion relations

- Plugging the unitarized P-waves into the FF unitarity relation:

$$\mathrm{disc}\,\Pi_{P,\lambda}(s) = 2i\,\nu_{P,\lambda}(s)\,g_{P,\lambda}(s)\,F_\pi^{V*}(s)$$

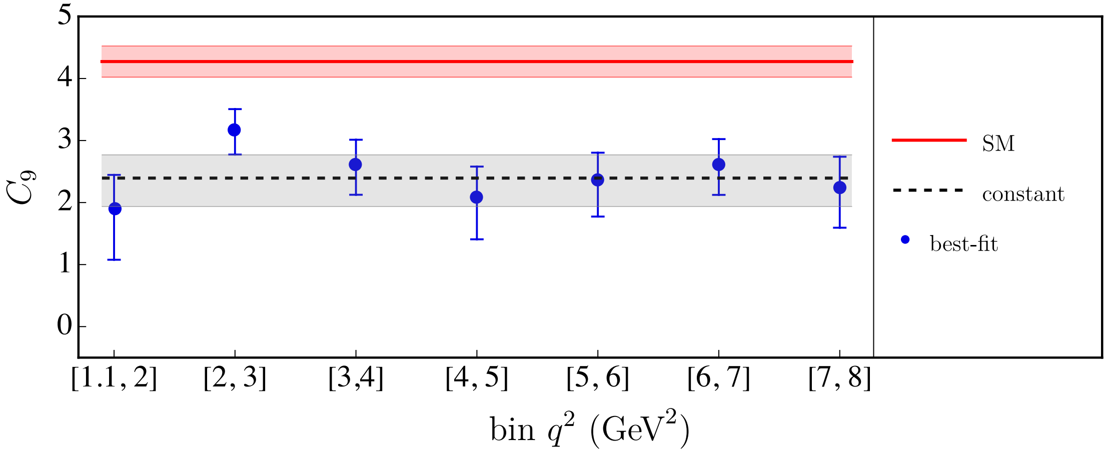


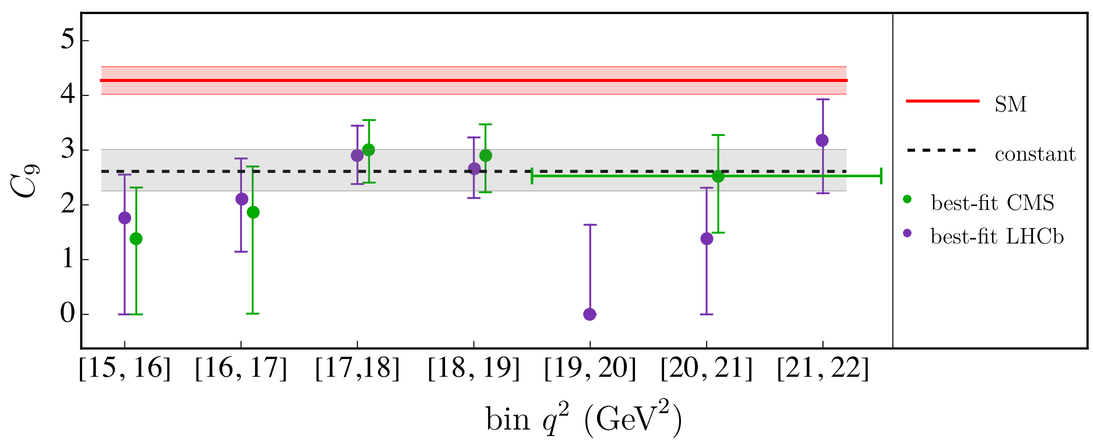
$\hookrightarrow$ Only gives an anomalous contribution $\sim \mathrm{disc}\,g_{P,\lambda}(s) = \mathrm{disc}\,g^{\mathrm{Born}}_{P,\lambda}(s)$

- Approximate pion form factor simply as $F_\pi^V(s) = \Omega(s)$
- Obtain unsubtracted FF dispersion relation

$$\Pi_{P,\lambda}(s) = \underbrace{\frac{1}{\pi}\int_{4M_\pi^2}^\infty ds'\,\frac{\nu_{P,\lambda}(s')\,g_{P,\lambda}(s')\,\Omega^*(s')}{s'-s}}_{\equiv\Pi^{\mathrm{norm}}_{P,\lambda}(s)} + \underbrace{\frac{1}{\pi}\int_0^1 dx\,\frac{\partial s_x}{\partial x}\,\frac{\nu_{P,\lambda}(s_x)\,\mathrm{disc}\,g^{\mathrm{Born}}_{P,\lambda}(s_x)\,\Omega^*(s_x)}{s_x-s}}_{\equiv\Pi^{\mathrm{anom}}_{P,\lambda}(s)}$$

Fit $B \to K\bar{\ell}\ell$





[M. Bordone, G.Isidori, S. Mächler, AT, 2401.18007]

Fit $B \to K\bar{\ell}\ell$

$$C_9 \to C_9^\lambda(q^2) + Y_{c\bar{c}}^\lambda(q^2) + Y_{q\bar{q}}^{[0]}(q^2) + Y_{b\bar{b}}^{[0]}(q^2)$$

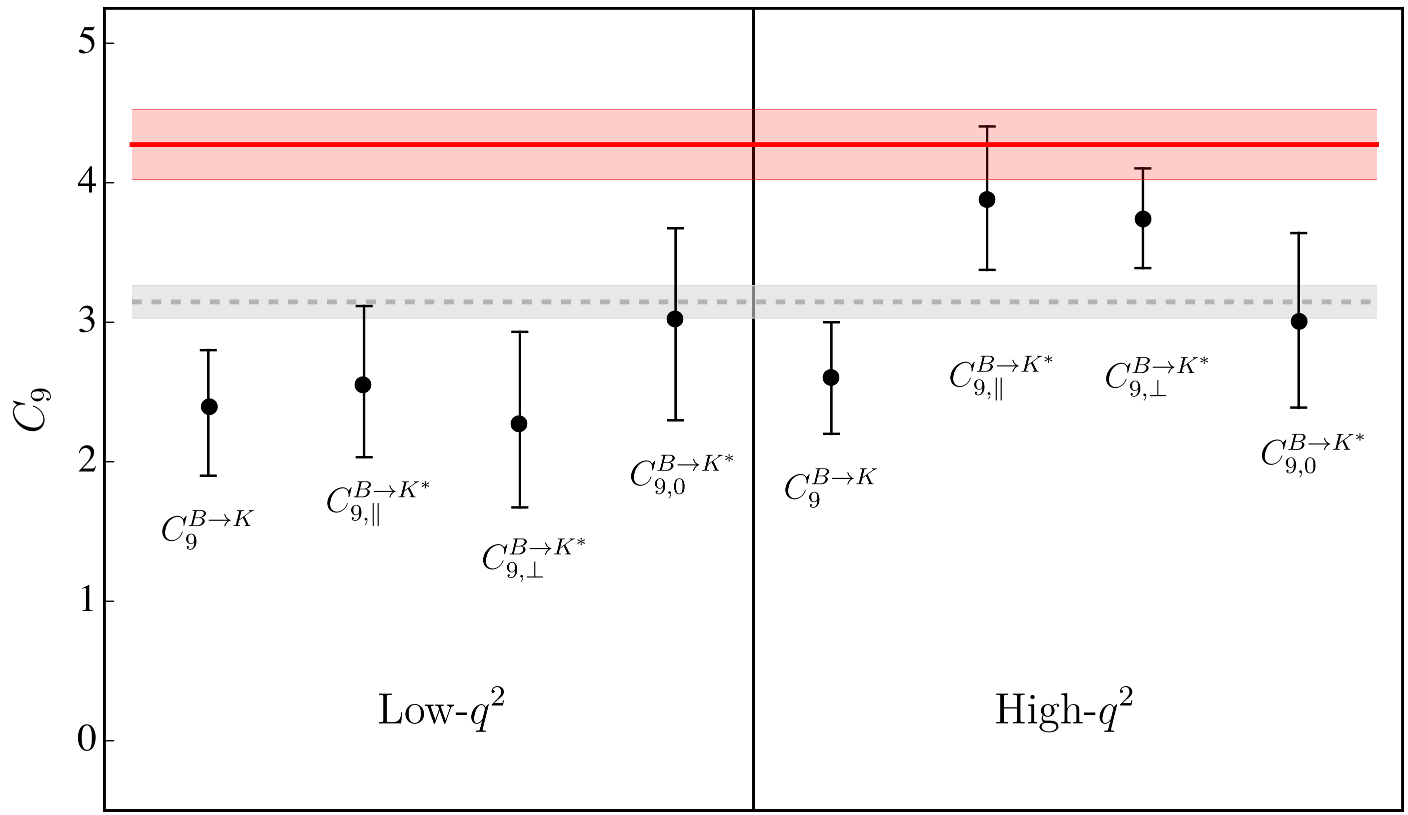
$$Y_{c\bar{c}}^\lambda(q^2) = Y_{c\bar{c}}^\lambda(q_0^2) + \frac{16\pi^2}{\mathcal{F}_\lambda(q^2)} \Delta\mathcal{H}_{c\bar{c}}^\lambda(q^2)$$

$$\Delta\mathcal{H}_{c\bar{c}}^{\lambda,1P}(q^2) = \sum_V \eta_V^\lambda e^{i\delta_V^\lambda} \frac{(q^2 - q_0^2)}{(m_V^2 - q_0^2)} A_V^{\text{res}}(q^2)\bigg|_{q_0^2=0} = \sum_V \eta_V^\lambda e^{i\delta_V^\lambda} \frac{q^2}{m_V^2} A_V^{\text{res}}(q^2)$$

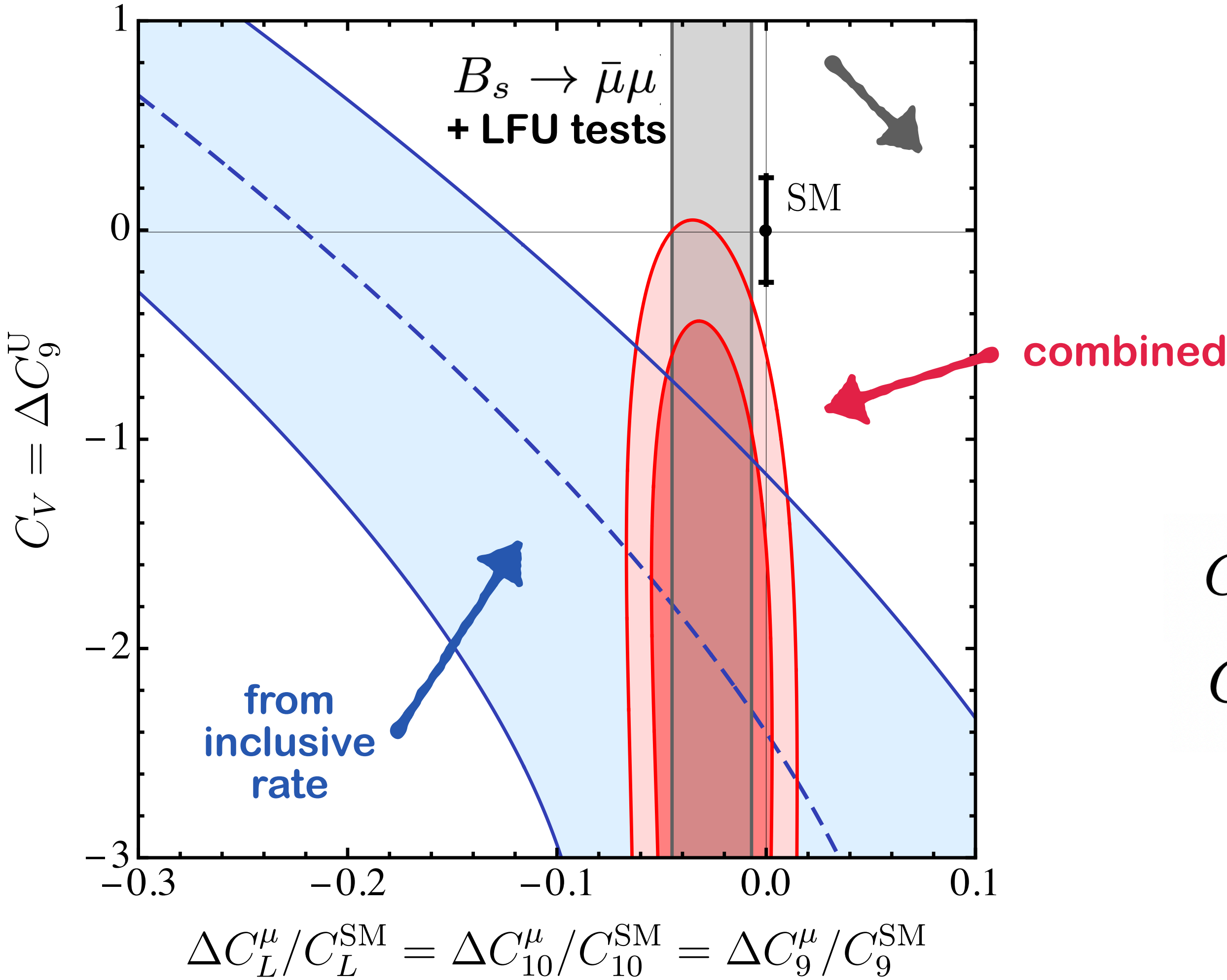
$$A_V^{\text{res}}(q^2) = \frac{m_V \Gamma_V}{m_V^2 - q^2 - i m_V \Gamma_V}$$

$$Y_{c\bar{c}}^\lambda(q^2) = -\frac{4}{9}\left[\frac{4}{3}C_1(\mu) + C_2(\mu)\right]\left[1 + \ln\left(\frac{m^2}{\mu^2}\right)\right] + \frac{16\pi^2}{\mathcal{F}_\lambda(q^2)} \sum_V \eta_V^\lambda e^{i\delta_V^\lambda} \frac{q^2}{m_V^2} A_V^{\text{res}}(q^2)$$

Fit $B \to K, K^* \bar{\ell}\ell$



Inclusive rate



$$C_V = C_9 + C_{10}$$

$$C_L = -2C_{10}$$

Inclusive rate

SM prediction for the **inclusive rate:**

[Z. Ligeti and F. J. Tackmann, 0707.1694]

$$R_{\text{incl}}^{(\ell)}(q_0^2) = \frac{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}} = \frac{|V_{tb}V_{ts}^*|^2}{|V_{ub}|^2}\left[\mathcal{R}_L + \Delta\mathcal{R}_{[q_0^2]}\right]$$

$$\mathcal{R}_L = \frac{\alpha_e^2 C_L^2}{16\pi^2}$$

$q_0^2 = 15\ \text{GeV}^2$

from Belle, arXiv:2107.13855

$$\Delta\mathcal{R}_{[15]} = \frac{\alpha_e^2}{8\pi^2}\Big[C_V^2 + C_V C_L + 0.485 C_L + 0.97 C_V + 0.93 + \Delta_{\text{n.p.}} + C_7(1.91 + 2.05 C_L + 4.27 C_7 + 4.1 C_V)\Big]$$

Change of basis: $\{\mathcal{O}_9, \mathcal{O}_{10}\} \to \{\mathcal{O}_V, \mathcal{O}_L\}$

$$\mathcal{O}_V = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_L = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \ell_L)$$

$$C_L = -2C_{10}$$

$$C_V = C_9 + C_{10}$$

Significant cancellation of non-perturbative uncertainties since **the hadronic structure is very similar** ($b \to q_{light}$, left-handed current)

Inclusive rate

- Agreement in the SM between the inclusive rate and the sum over the leading exclusive modes $B \to K\bar{\ell}\ell, B \to K^*\bar{\ell}\ell, B \to K\pi\bar{\ell}\ell$ (via HHChPT)

$$\sum_i \mathscr{B}(B \to X_s^i \bar{\ell}\ell)^{SM}_{[15]} = (5.07 \pm 0.42) \times 10^{-7}$$

$$\mathscr{B}(B \to X_s \bar{\ell}\ell)^{SM}_{[15]} = (4.10 \pm 0.81) \times 10^{-7}$$

- This compatibility opens up the possibility of comparing the inclusive SM prediction and a sum-over-exclusive experimental result (from LHCb):

$$B \to K\bar{\ell}\ell = (0.85 \pm 0.05) \times 10^{-7}$$

$$B \to K^*\bar{\ell}\ell = (1.58 \pm 0.35) \times 10^{-7}$$

$$B \to K\pi\bar{\ell}\ell = (0.05 \pm 0.09) \times 10^{-7}$$

$$B \to K\pi\pi\bar{\ell}\ell = (0.06 \pm 0.05) \times 10^{-7}$$

$$B \to K\pi\pi\pi\bar{\ell}\ell = (0.00 \pm 0.04) \times 10^{-7}$$

$$\rightarrow \boxed{\mathscr{B}(B \to X_s \bar{\ell}\ell)^{exp}_{[15]} = (2.65 \pm 0.17) \times 10^{-7}}$$

- **Confirmation of sizable suppression** on the $b \to s\bar{\mu}\mu$ rates at low q^2 compared to SM predictions
- Independent verification **not sensitive to uncertainties on the form factors**
- Sizable uncertainty but mainly **experimental** on $B \to X_u \ell \bar{\nu}$