Inclusive $\bar{B} \to X_s \ell^+ \ell^-$ Theory

Implications of LHCb measurements and future prospects
CERN, Oct 24 2024

Huber, Hurth, JJ, Lunghi, Qin, Vos [2404.03517v1] (v2 soon)

TP1 Theoretical Particle Physics

Jack Jenkins

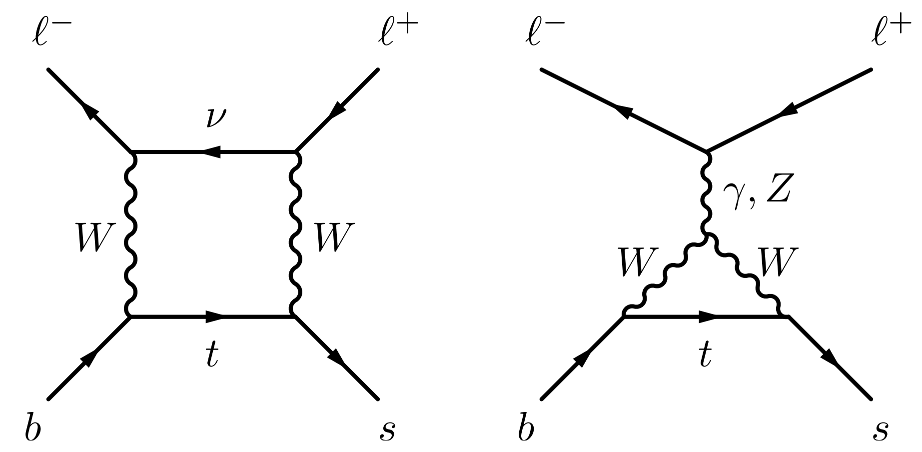
Universität Siegen

Overview

Branching fractions for $b \to s\ell\ell$ are small (QED and $1/16\pi^2$) $\sim 10^{-7} - 10^{-6}$

.. but not too small, large m_t and $V_{ts} \sim V_{cb}$
$\to$ no GIM or CKM suppression

Promising landscape to look for BSM as interference with SM



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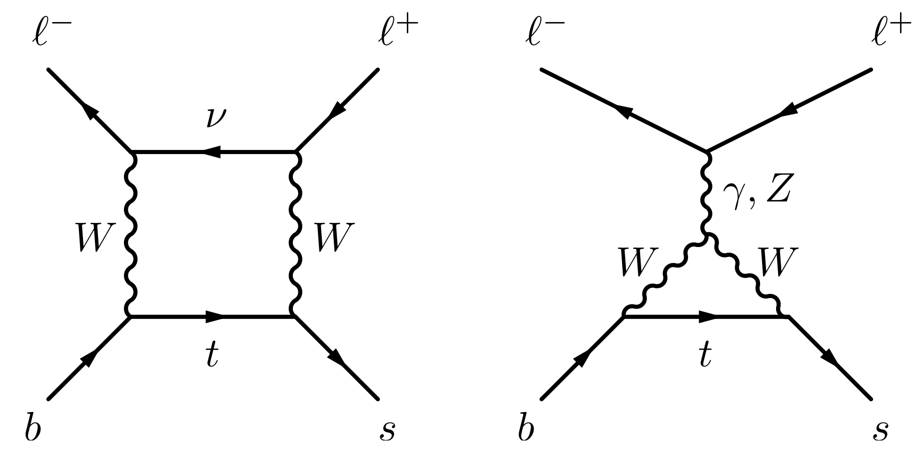

LHCb has established record in exclusive decays:

$\bar{B}_s \to \mu^+\mu^-$
$\bar{B} \to K^{(*)}\mu^+\mu^-, \bar{B}_s \to \phi\mu^+\mu^-, \Lambda_b \to \Lambda_s\mu^+\mu^-$

Can LHCb measure the (semi)-inclusive modes?

$$\bar{B} \to \sum_i X_s^i \mu^+\mu^- \qquad X_b \to K^+\mu^+\mu^- X$$

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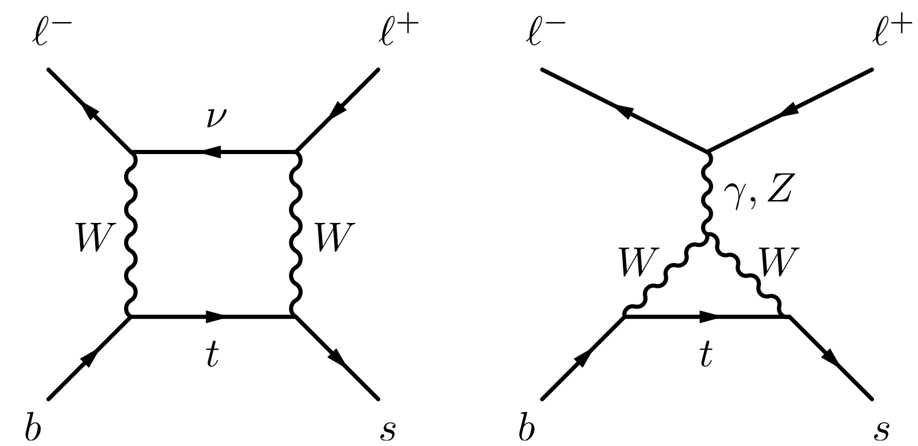

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Complementarity of inclusive and exclusive $b \to s\ell\ell$

Inclusive:
HQE, SCET I, quark-hadron duality

Exclusive:
Local and nonlocal $H_b \to H_s$ form factors

B tagging: prospects

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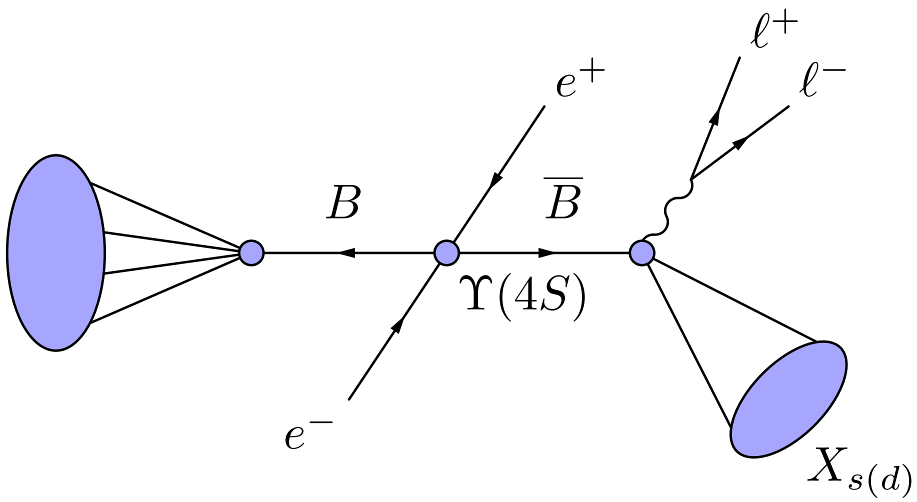
B factories



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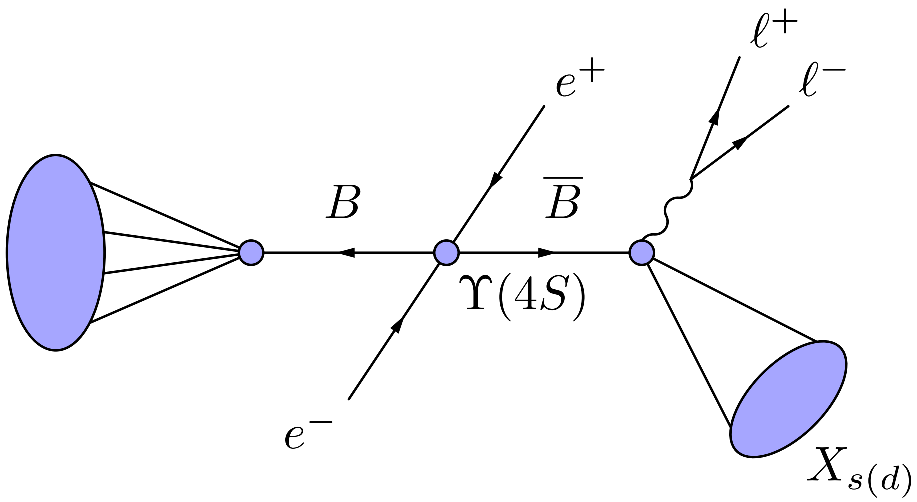

Reconstruct $\bar{B}$ momentum from tagging recoil B
(low efficiency, gain in systematics)

Belle & Babar used sum over exclusive modes
(also $\pi^0 \to \gamma\gamma$)

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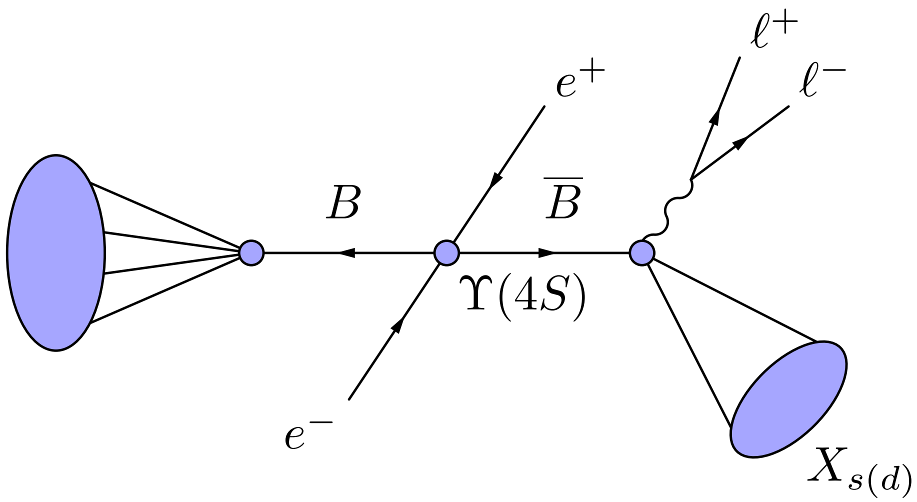

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Belle [0503044]	$\mathscr{B}$	152M
Belle [1402.7134]	$\mathscr{A}_{FB}$	772M
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BaBar [1312.5364]	$\mathscr{B}, \mathscr{A}_{CP}$	471M

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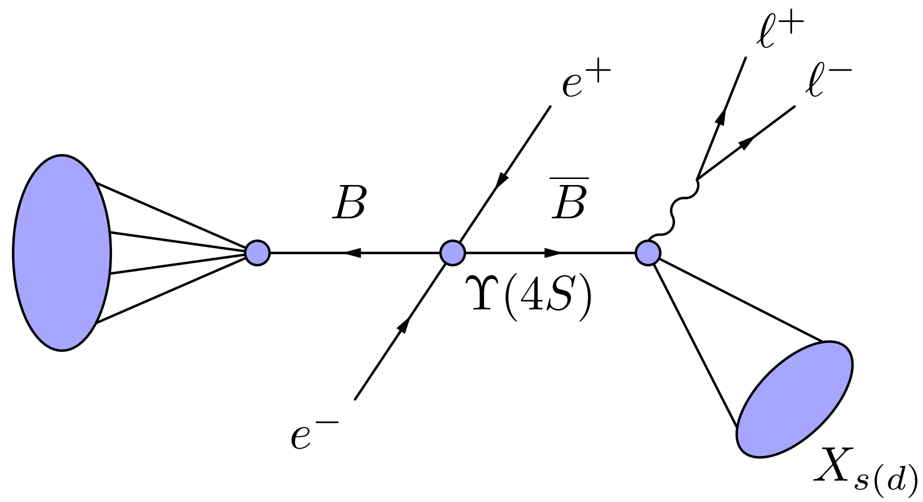
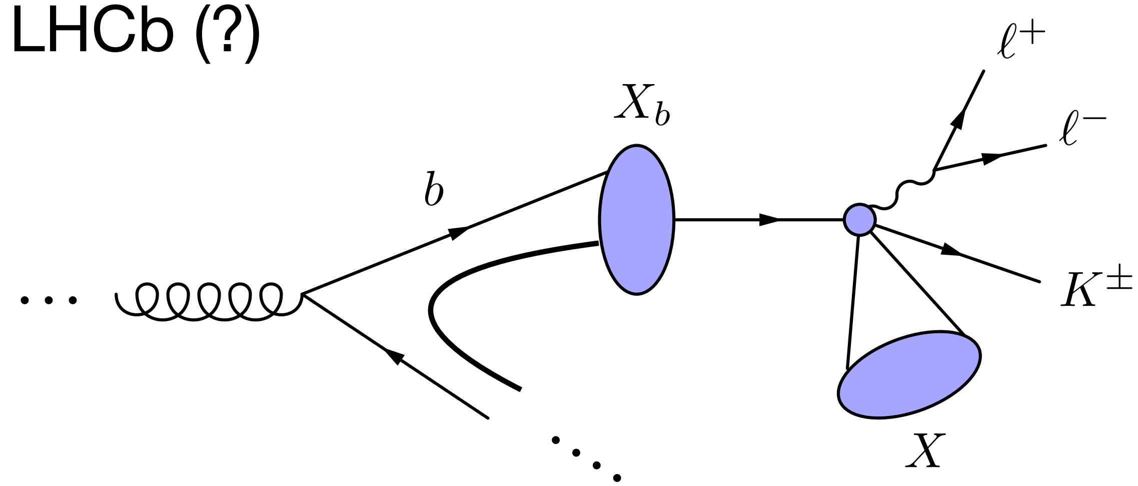


LHCb (?)



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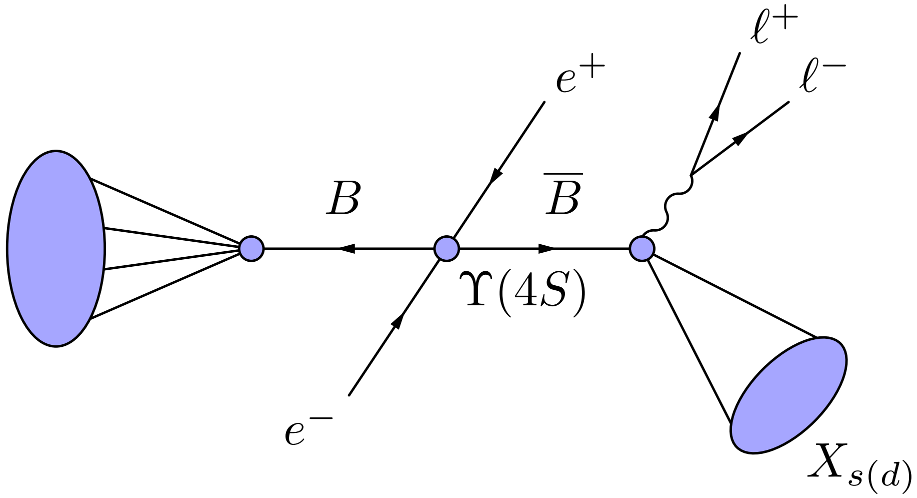

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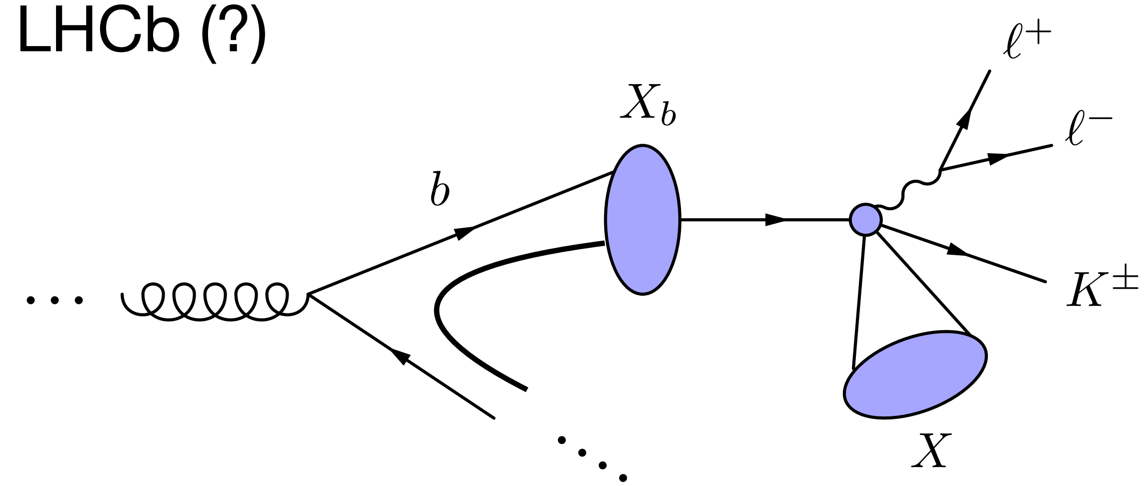

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Sum over exclusive modes and isospin reweighting $B^{0,+} \to K^+(+n\pi^\pm)\mu^+\mu^-$ (avoid π^0s)

Koppenburg [CERN-THESIS-2002-010]

B tagging: prospects

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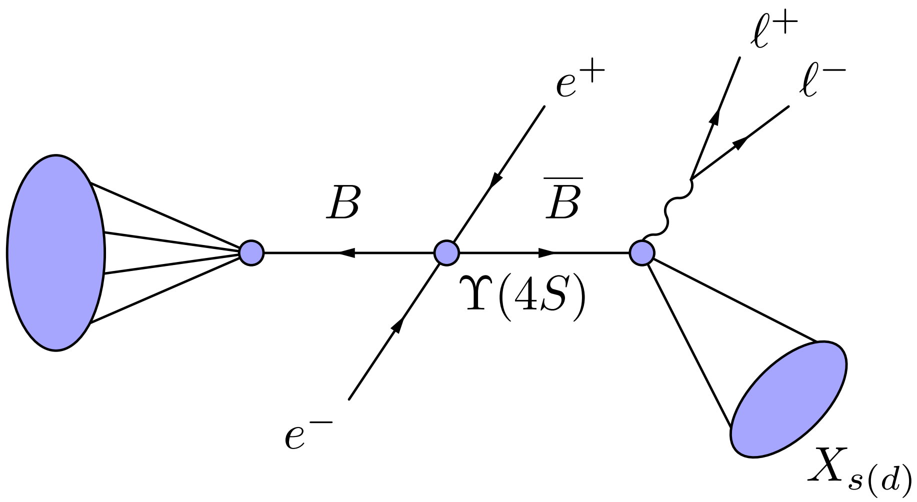

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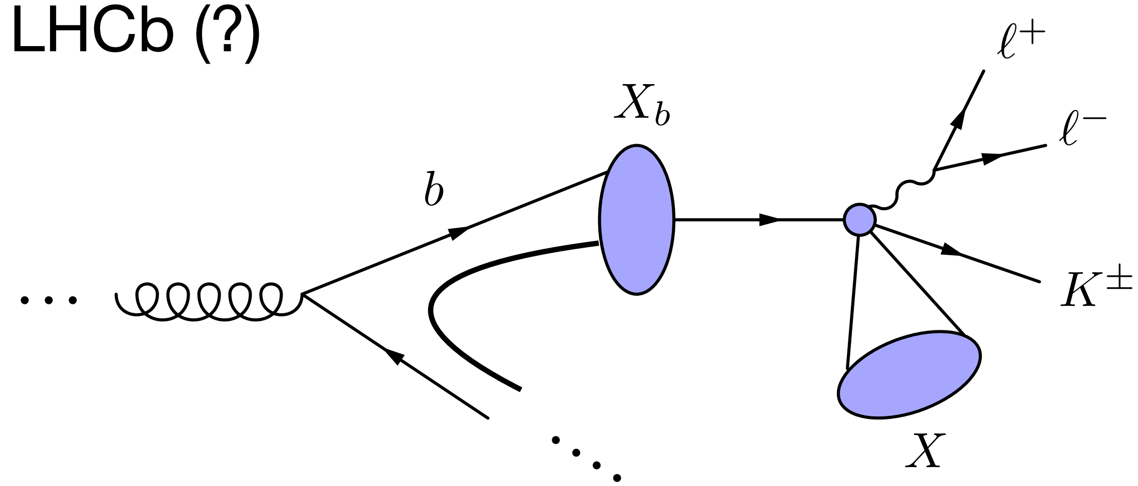

Sum over exclusive modes and isospin reweighting
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Koppenburg [CERN-THESIS-2002-010]

Isospin extrapolation from semi-inclusive
$X_b \to K^+\mu^+\mu^-X$ (vertex 3 charged particles)

Amhis, Owen [2106.15943]

Separately measure and subtract $\bar{B}_s$ and Λ_b^0 contaminations to X_b using an additional K or p

$\bar{B}_s$: $\quad X_b \to K^+K^-\mu^+\mu^-X$

Λ_b^0: $\quad X_b \to pK^-\mu^+\mu^-X$

LHCb sum-over-exclusive ($q^2 > 15$)

Dominant K, K^* LHCb [1403.8044] [1606.04731]

$\mathscr{B}(B^0 \to K^0\mu^+\mu^-) = (0.67 \pm 0.12) \times 10^{-7}$

$\mathscr{B}(B^+ \to K^+\mu^+\mu^-) = (0.85 \pm 0.05) \times 10^{-7}$

$\mathscr{B}(B^0 \to K^{*0}\mu^+\mu^-) = (1.74 \pm 0.14) \times 10^{-7}$

$\mathscr{B}(B^+ \to K^{*+}\mu^+\mu^-) = (1.58 \pm 0.32) \times 10^{-7}$

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Isospin avg.

$\mathscr{B}(B \to K^*\mu^+\mu^-) = (1.72 \pm 0.13) \times 10^{-7}$

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Total $K + K^*$: $\mathscr{B}(B \to K^{(*)}\mu^+\mu^-) = (2.54 \pm 0.14) \times 10^{-7}$

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$$\mathcal{B}(B \to (K\pi)_S\mu^+\mu^-) = \frac{3}{2}\mathcal{B}(B^0 \to (K^+\pi^-)\mu^+\mu^-) = (0.05 \pm 0.09) \times 10^{-7}$$

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Grand total:

$$\mathcal{B}(B \to X_s\mu\mu) = (2.65 \pm 0.17) \times 10^{-7}$$

Structure of RGEs

The amplitude is proportional to the running QED $\alpha_e(\mu_0) \to \alpha_e(\mu_b)$

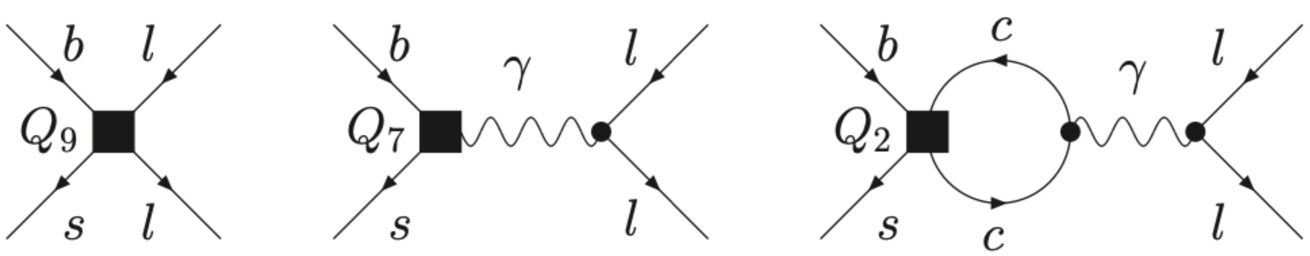
$$\eta_s = \frac{\alpha_s(\mu_0)}{\alpha_s(\mu_b)} = 1 + \beta_s^{(00)} \frac{\alpha_s(\mu_0)}{4\pi} \ln \frac{\mu_b^2}{\mu_0^2} = O(1)$$

$$\frac{\alpha_e(\mu_0)}{\alpha_e(\mu_b)} = 1 - \beta_e^{(00)} \frac{\alpha_e(\mu_0)}{4\pi} \ln \frac{\mu_b^2}{\mu_0^2}$$

$f(\eta_s)/\alpha_s(\mu_0)$

SM → LEFT matching at 2 loops (NLO QCD)
Bobeth, Misiak, Urban [9910220]

All relevant three-loop ADMs in QEDxQCD
Gorbahn, Haisch [0411071]
Huber, Lunghi, Misiak, Wyler [0512066]



$$\mu \frac{dC_i}{d\mu} = \gamma_{ij}^T(\alpha_s, \alpha_e) C_j(\mu)$$

$$C_i(\mu_b) = V_{ij} \left[\eta_s^{a_j} \delta_{jk} + \ldots \right] V_{kl}^{-1} C_l(\mu_0)$$

$C_9(\mu_b) \sim \alpha_e/\alpha_s$ is superleading (N^{-1}LO because α_s^{-1})
(Depending on the counting of N's $1/\alpha_s$ is LO and α_e is NLO)

Perturbation theory stable at $\mu_b \sim 5$ GeV after many terms in expansion in α_s and α_e/α_s which are generated by the solutions to the RGEs

Leading power amplitude

$C_{9,10} \sim 2, -4$ and $C_{1,2} \sim -1{,}1$ justifies going to higher orders in $Q_{9,10} - Q_{9,10}$ interference (large m_t)

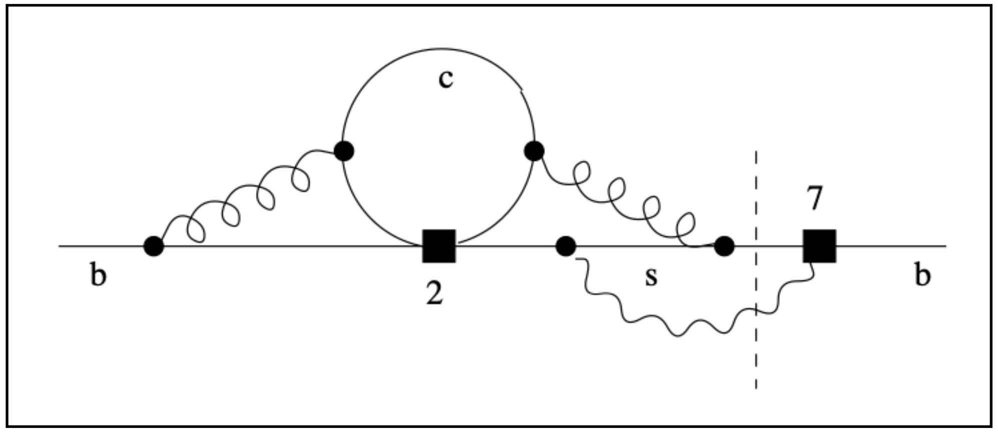
$Q_{9,10} - Q_{9,10}$ at NNLO

Progress at N3LO for $\bar{B} \to X_u \ell \nu$ Fael, Usovitsch [2310.03685]

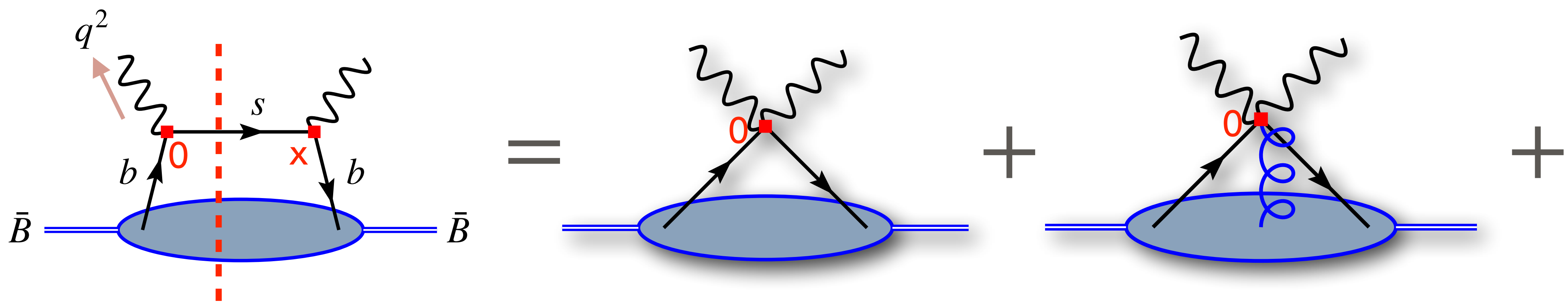
$Q_{9,10} - Q_{1,2}$ interference at NLO (two loops, q^2/m_b^2 and m_c^2/m_b^2)

Asatryan, Asatrian, Greub, Walker [0204341]
de Boer [1707.00988]

Three loop $Q_{1,2} - Q_7$ interference for $\bar{B} \to X_s \gamma$ ($q^2 = 0$) on the horizon
Greub, Asatrian, Asatryan, Born, Eicher [2407.17270]
Czaja, Czakon, Huber, Misiak, Niggetiedt, Rehman, Schönwald, Steinhauser [2309.14707]
Fael, Lange, Schönwald, Steinhauser [2309.14706]



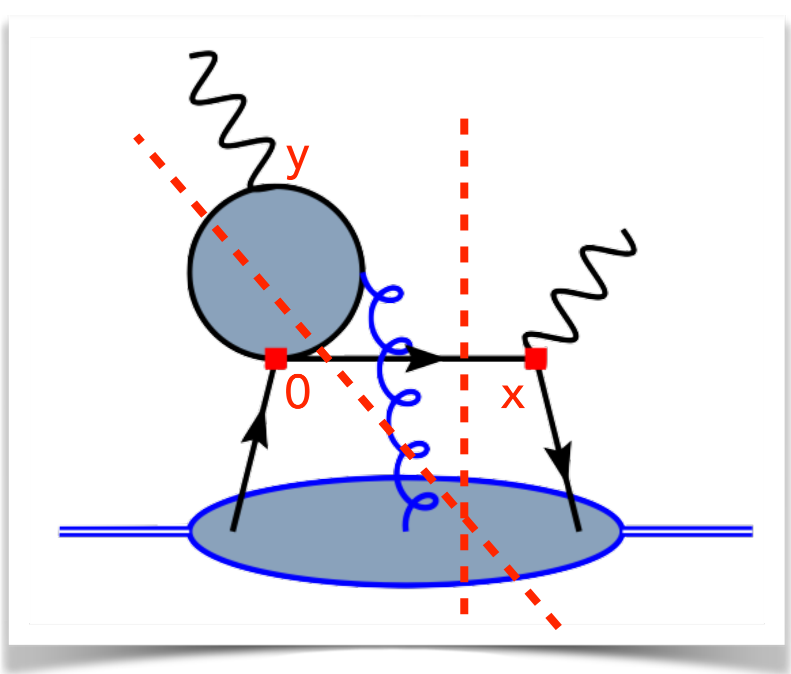
HQE, Krüger-Seghal



$$\langle \bar{B} | \bar{b}\gamma^\mu b | \bar{B} \rangle \sim p_B^\mu$$

$$\langle \bar{B} | \bar{h}_v (iD)^2 h_v | \bar{B} \rangle$$

$$\langle \bar{B} | \bar{h}_v (-i\sigma^{\mu\nu}) G^a_{\mu\nu} T^a h_v | \bar{B} \rangle$$



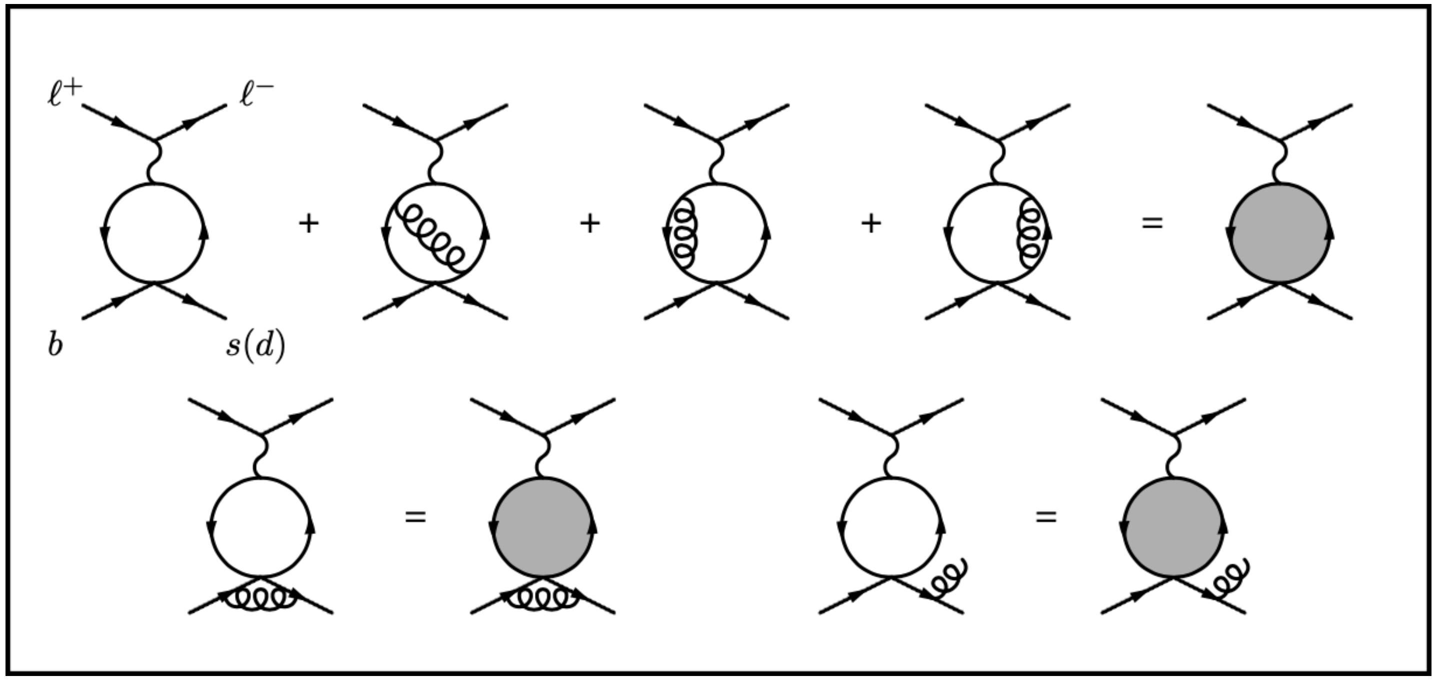
Nonfactorizable charm loop contributions can be re-expanded in local operators at high q^2 ($\alpha_s \Lambda^2 / m_c^2$)

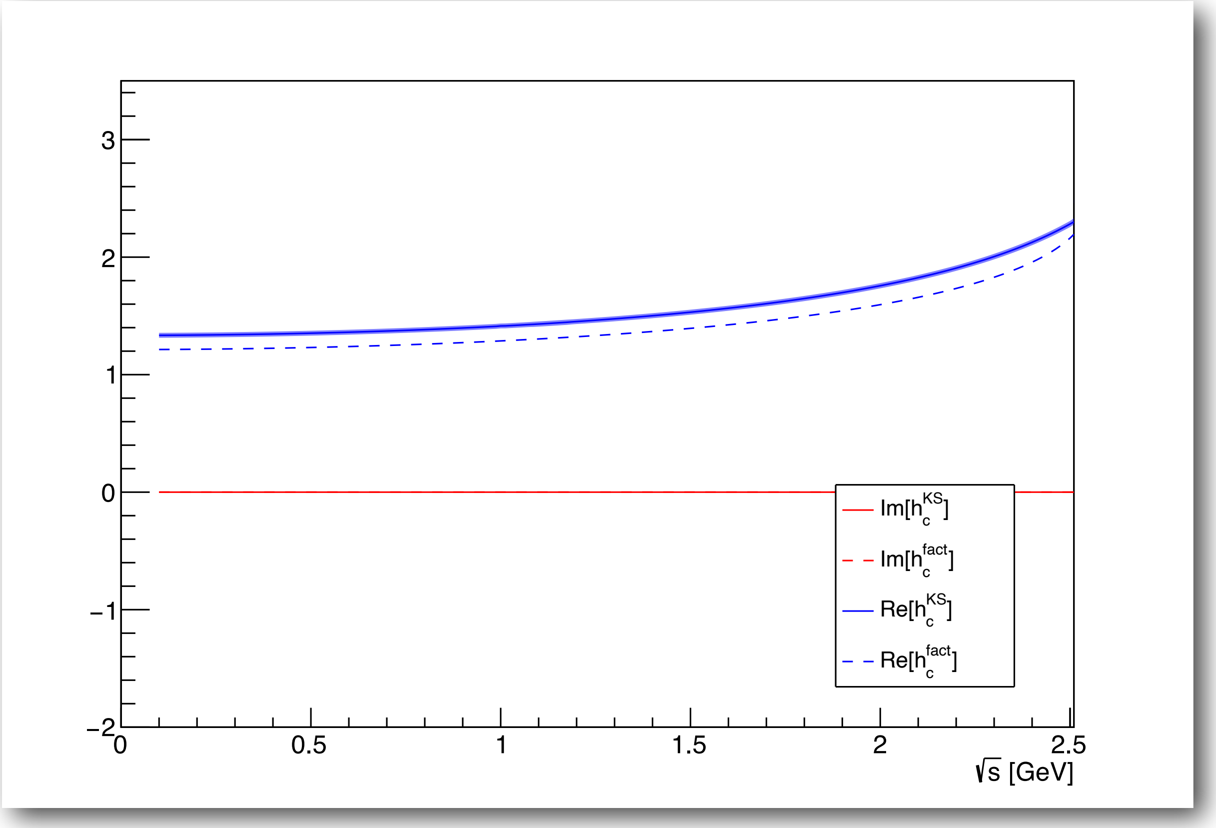
Buchalla, Isidori, Rey [9705253]

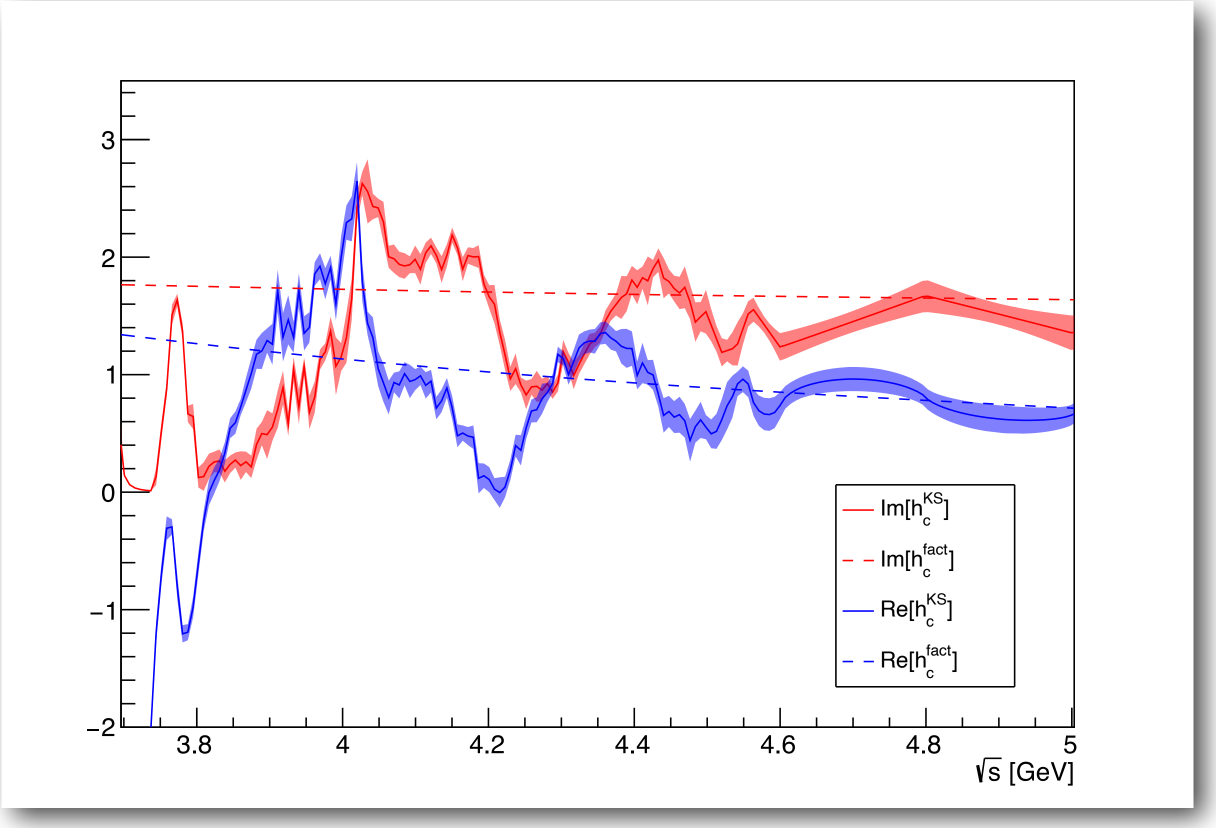
The *factorizable* charm loop appears at every term in the power expansion and can be treated nonperturbatively using data on $e^+e^- \to$ hadrons and dispersion relations

Krüger, Sehgal [9603237] [9608361]

HQE, Krüger-Seghal







Normalisation

Two strategies for normalising to semileptonic BRs

$$\int_{q_0^2}^{M_B^2} dq^2 \frac{d\mathscr{B}(\bar{B} \to X_s \ell\ell)}{dq^2} \Big/ \mathscr{B}(B \to X_c \ell \nu)$$

$|V_{cb}|^2$ and m_b^5 prefactors cancel ✓

Cons: Introduces spurious charm mass dependence and $(b \to c\ell\nu)_{exp}$ (ok)

$$\mathscr{R}(q_0^2) = \int_{q_0^2}^{M_B^2} dq^2 \frac{d\mathscr{B}(\bar{B} \to X_s \ell\ell)}{dq^2} \Big/ \int_{q_0^2}^{M_B^2} dq^2 \frac{d\mathscr{B}(\bar{B} \to X_u \ell \nu)}{dq^2}$$

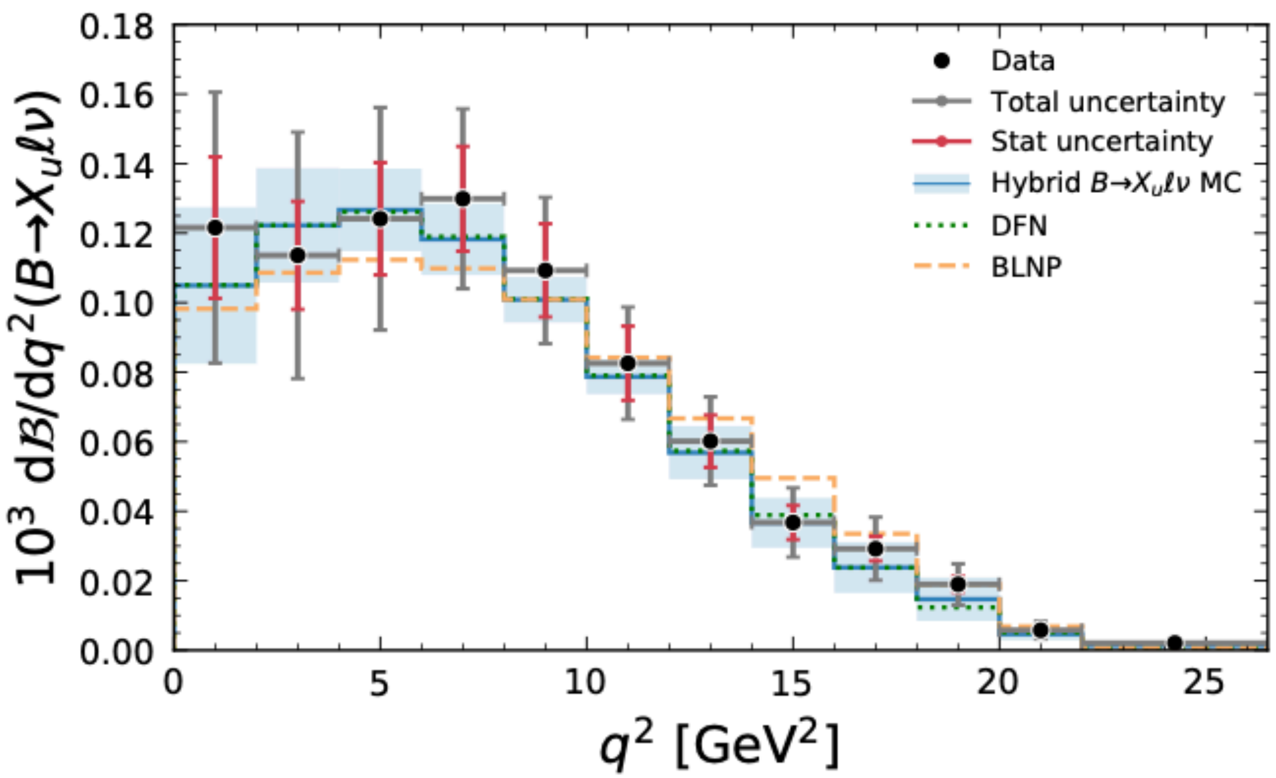
Non-perturbative and perturbative $Q_{9,10}$ matrix elements are suppressed at the *differential* level ✓

Resilient to the M_X cut at low-q^2 needed to suppress huge cascading backgrounds $B \to D(\to K\ell\nu X)\ell\nu X$ and $B \to \psi(\to X\ell\ell)X$ ✓

Cons: $|V_{ub}/V_{cb}|^2$ prefactor, $(b \to u\ell\nu)_{exp}$

reducible (?)

Belle [2107.13855]



$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>14.4]_{exp} = (1.76 \pm 0.32) \times 10^{-4}$$

$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>15]_{exp} = (1.52 \pm 0.28) \times 10^{-4}$$

Results

q^2 range [GeV^2]	$[1,6]$	$[1,3.5]$	$[3.5,6]$
$\mathcal{B}$ [10^{-7}]	16.87 ± 1.25	9.17 ± 0.61	7.70 ± 0.65
$\mathcal{H}_T$ [10^{-7}]	3.14 ± 0.25	1.49 ± 0.09	1.65 ± 0.17
$\mathcal{H}_L$ [10^{-7}]	13.65 ± 1.00	7.63 ± 0.54	6.02 ± 0.49
$\mathcal{H}_A$ [10^{-7}]	-0.27 ± 0.21	-1.08 ± 0.08	0.81 ± 0.16

q^2 range [GeV^2]	> 14.4	> 15
$\mathcal{B}$ [10^{-7}]	3.04 ± 0.69	2.59 ± 0.68
$\mathcal{R}(q_0^2)$ [10^{-4}]	26.02 ± 1.76	27.00 ± 1.94

For LHCb
(after PHOTOS)

Table 1: Phenomenological results without logarithmically enhanced electromagnetic effects. The slight changes compared to [9] are due to the change in the input parameters.

$\bar{B} \to X_s \ell^+ \ell^-$ ($\ell = e, \mu$ average)

q^2 range [GeV^2]	$[1,6]$	$[1,3.5]$	$[3.5,6]$
$\mathcal{B}$ [10^{-7}]	17.41 ± 1.31	9.58 ± 0.65	7.83 ± 0.67
$\mathcal{H}_T$ [10^{-7}]	4.77 ± 0.40	2.50 ± 0.18	2.27 ± 0.22
$\mathcal{H}_L$ [10^{-7}]	12.65 ± 0.92	7.085 ± 0.48	5.56 ± 0.45
$\mathcal{H}_A$ [10^{-7}]	-0.10 ± 0.21	-0.989 ± 0.080	0.89 ± 0.16

q^2 range [GeV^2]	> 14.4
$\mathcal{B}$ [10^{-7}]	2.66 ± 0.70
$\mathcal{R}(q_0^2)$ [10^{-4}]	24.12 ± 2.01

For Belle II
(before/without PHOTOS)

Table 2: Phenomenological results including log-enhanced QED corrections to the $\bar{B} \to X_s \ell^+ \ell^-$ process. All quantities are obtained by averaging $\ell = e, \mu$. The denominator of the ratio $\mathcal{R}(q_0^2)$ (i.e. the $\bar{B} \to X_u \ell \bar{\nu}$ rate for $q^2 > q_0^2$), on the other hand, does not include effects which correspond to log-enhanced QED corrections on the theory side. See text for further details.

Results

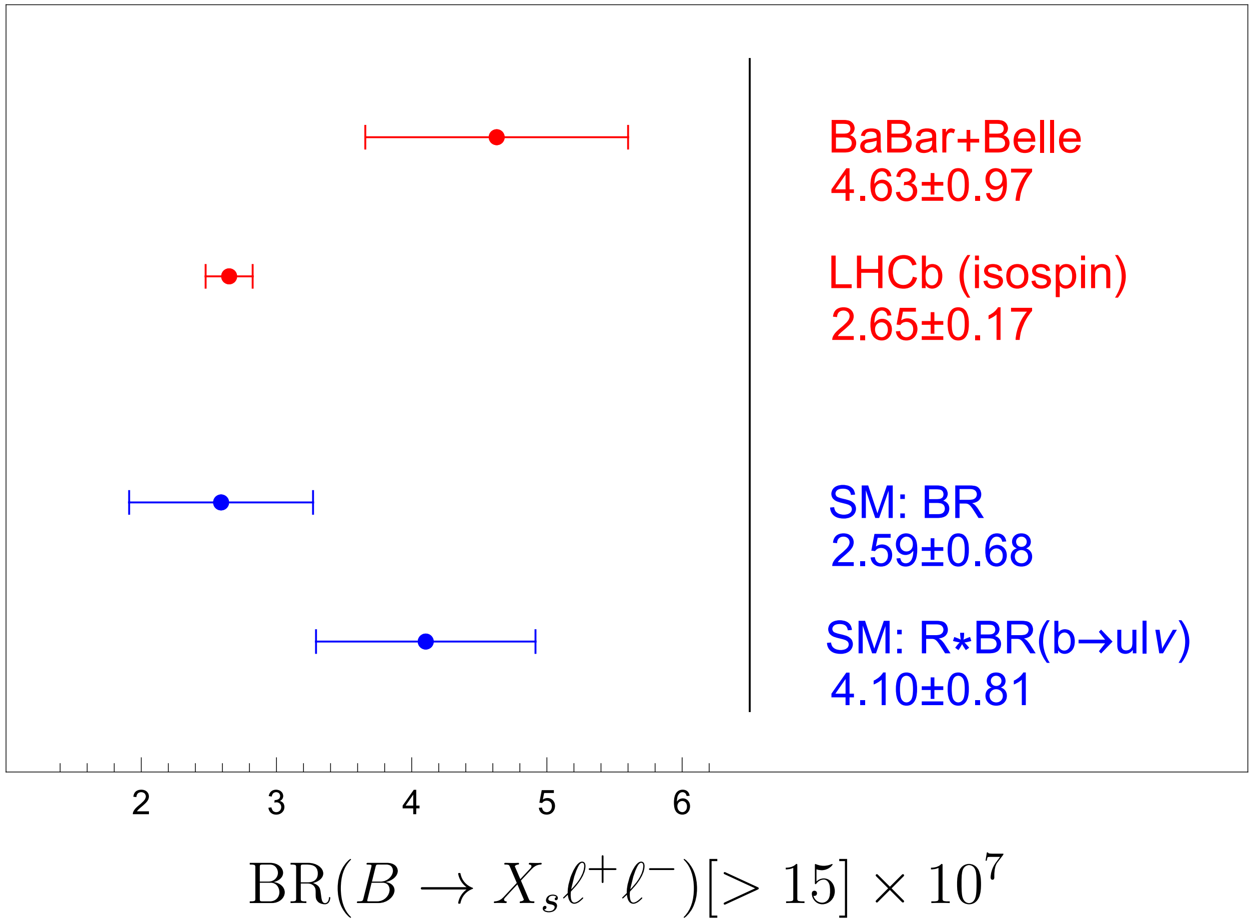
Using $\mathcal{R}_{SM} \times \mathcal{B}(B \to X_u \ell \nu)_{exp}$:

$\mathcal{B}(B \to X_s \mu\mu)[>15] = (4.5 \pm 1.0) \times 10^{-7}$ Isidori, Polonsky, Tinari [2305.03076]

$\mathcal{B}(B \to X_s \mu\mu)[>15] = (4.10 \pm 0.81) \times 10^{-7}$ Huber, Hurth, JJ, Lunghi, Qin, Vos [2404.03517]

Implementation of $Q_{1,2} - Q_{7,9}$ at NLO
Krüger-Sehgal for cuts through charm

Both effects decrease the BR → towards LHCb



Using exclusive FFs

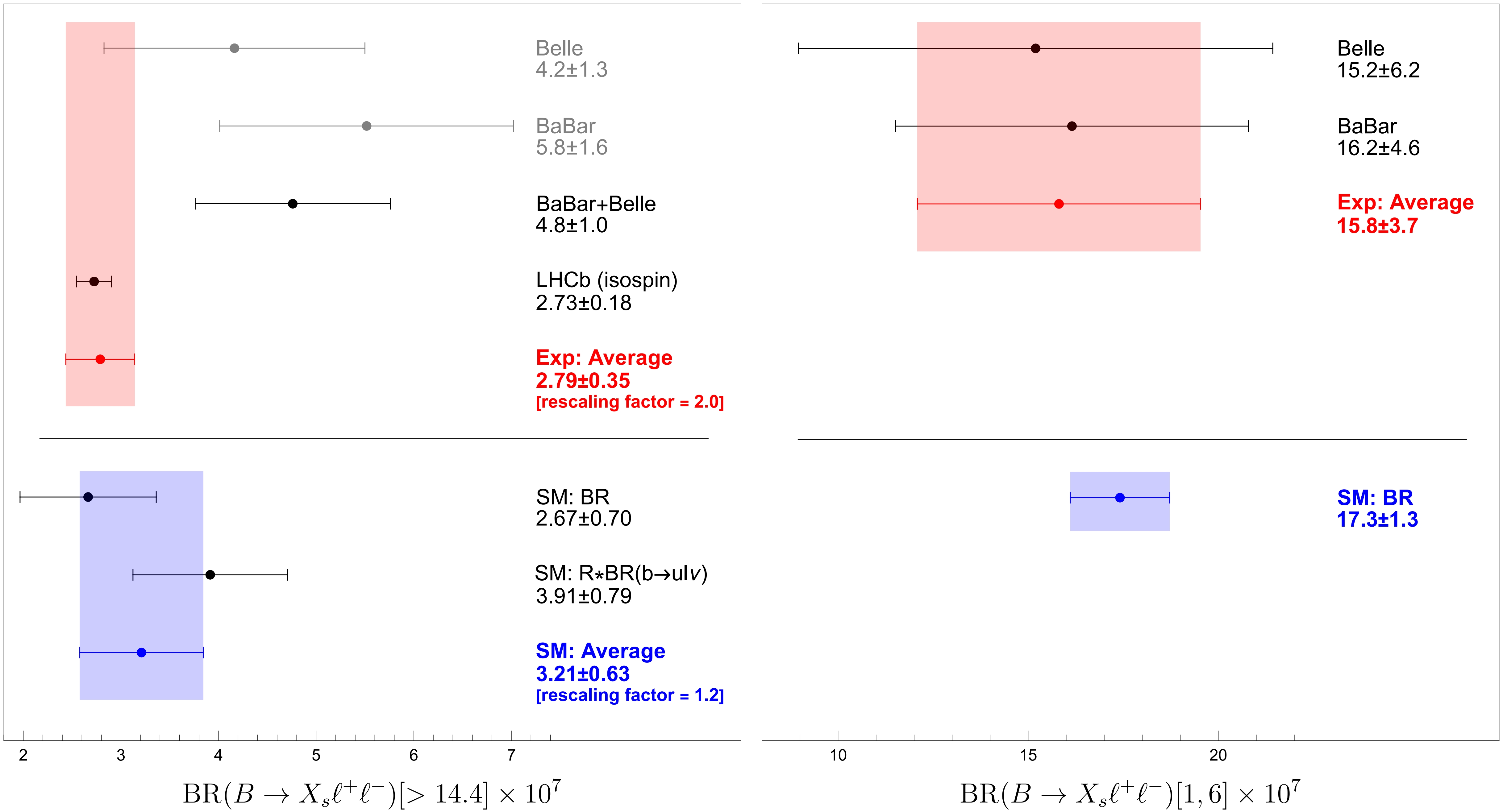
$$\sum_i \mathcal{B}(\bar{B} \to X_s^i \mu\mu)[>15] = (5.07 \pm 0.42) \times 10^{-7}$$

Isidori, Polonsky, Tinari [2305.03076]

$$\left(\frac{\mathcal{B}[>14.4]_{with\,QED}}{\mathcal{B}[>14.2]_{with\,QED}}\right)_{SM} = 0.96$$

$$\left(\frac{\mathcal{B}[>15]_{no\,QED}}{\mathcal{B}[>14.4]_{with\,QED}}\right)_{SM} = 0.97$$

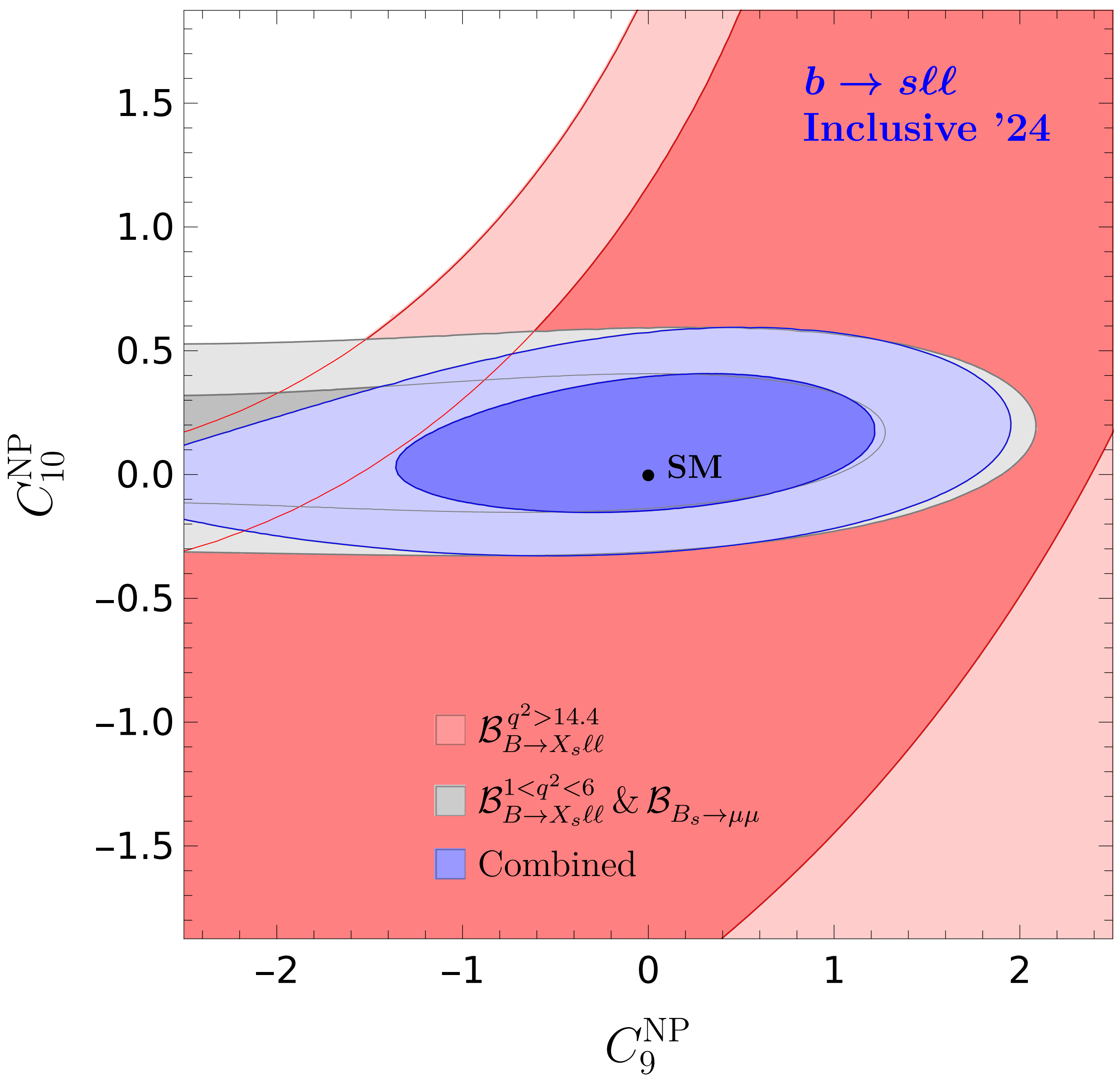
Results



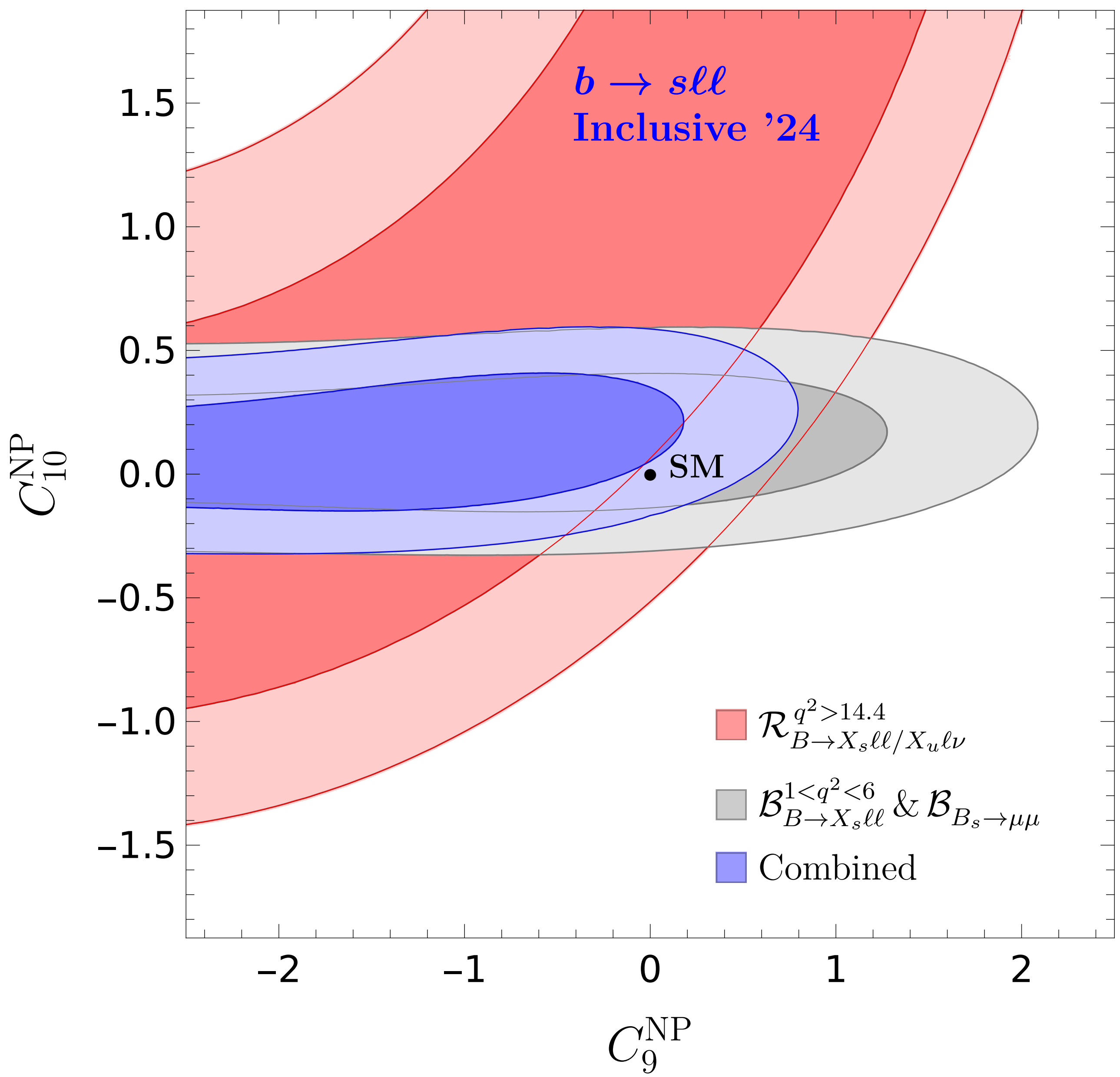
Belle
4.2±1.3
BaBar
5.8±1.6
BaBar+Belle
4.8±1.0
LHCb (isospin)
2.73±0.18
Exp: Average
2.79±0.35
[rescaling factor = 2.0]
SM: BR
2.67±0.70
SM: R∗BR(b→ulν)
3.91±0.79
SM: Average
3.21±0.63
[rescaling factor = 1.2]
2
3
4
5
6
7
$\mathrm{BR}(B \to X_s \ell^+ \ell^-)[> 14.4] \times 10^7$
Belle
15.2±6.2
BaBar
16.2±4.6
Exp: Average
15.8±3.7
SM: BR
17.3±1.3
10
15
20
$\mathrm{BR}(B \to X_s \ell^+ \ell^-)[1,6] \times 10^7$

New physics constraints (SM coefficients)

$B \to X_c \ell \nu$ normalisation



$B \to X_u \ell \nu$ normalisation



Final Thoughts / LHCb wishlist

- Branching ratio at high-q^2 (LHCb, Belle, BaBar) is a bit scattered (SF=2).
- Updates on fully-charged $\bar{B} \to (K, K\pi, K\pi\pi)\mu^+\mu^-$ branching ratios from LHCb needed to confirm $B \to K^{(*)}\mu\mu$ saturation at high-q^2 and clarify situation with B factories
- A dedicated analysis to combine the modes would be ideal, is it necessary? Are there correlations?
- Different bins in q^2 (lower cut $q_0^2 \sim 15$ GeV^2) to test Krüger-Sehgal would be interesting
- Normalised leptonic angular observables $\bar{A}_{FB}$ and F_L from $\bar{B} \to (K + K\pi + K\pi\pi)\mu\mu$ at high-q^2 to constrain different combinations of Wilson coefficients C_9C_{10} and $C_9^2 + C_{10}^2$
- Theory uncertainty is now mainly parametric and reducible (maybe with just ~5 ab^{-1} Belle II)
  - $\bar{B} \to X_u \ell \nu$ rate (normalisation in $\mathscr{R}$)
  - $\bar{B} \to X_c \ell \nu$ moments (power correction parameters) and lattice MEs

[2205.10274] [2409.18891]