

# Conventional and exotic charm meson spectroscopy (experiment)

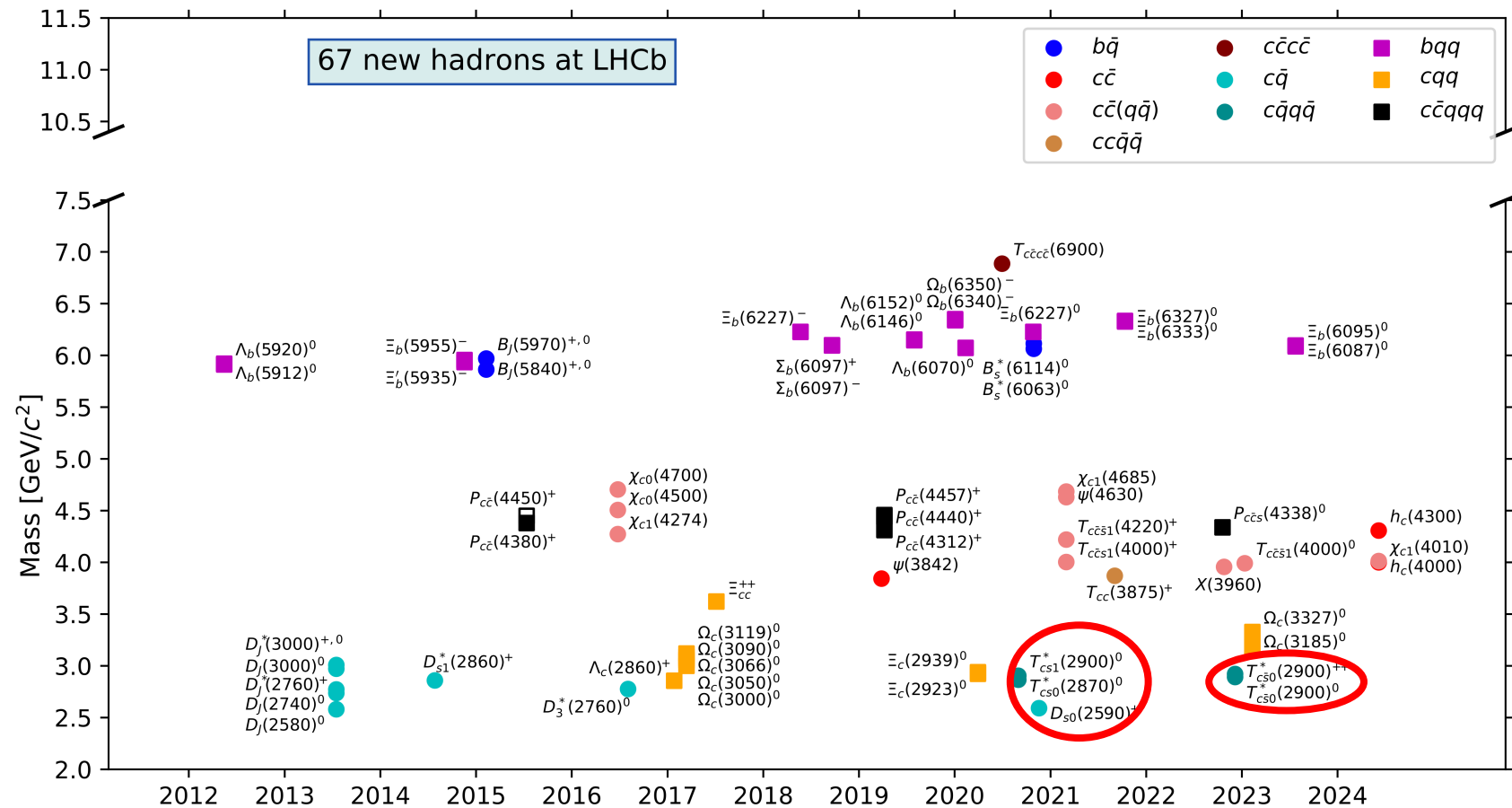
Linxuan Zhu (UCAS)  
on behalf of LHCb Collaboration



中国科学院大学  
University of Chinese Academy of Sciences

# Hadrons discovered at LHCb experiment

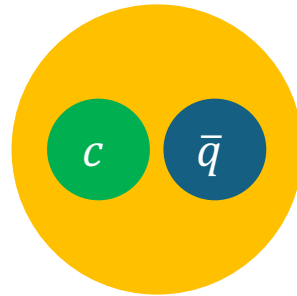
- [Link](#)
- Mass vs discovery date
- Focus on  $c\bar{q}$  and  $c\bar{q}q\bar{q}$
- Discovered through  $B \rightarrow DDX$  decays
- Full reconstruction of all final-state particles



# Outline

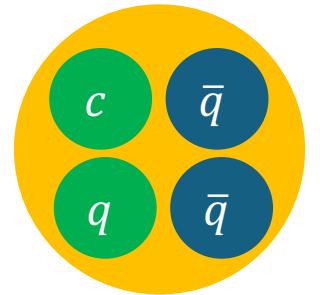
- Conventional charm-strange meson

- $D_{s0}(2590)^+$



- Exotic charm-strange meson

- $T_{cs1}^*(2900)^0$
- $T_{cs0}^*(2870)^0$
- $T_{c\bar{s}0}^*(2900)^{++}$
- $T_{c\bar{s}0}^*(2900)^0$

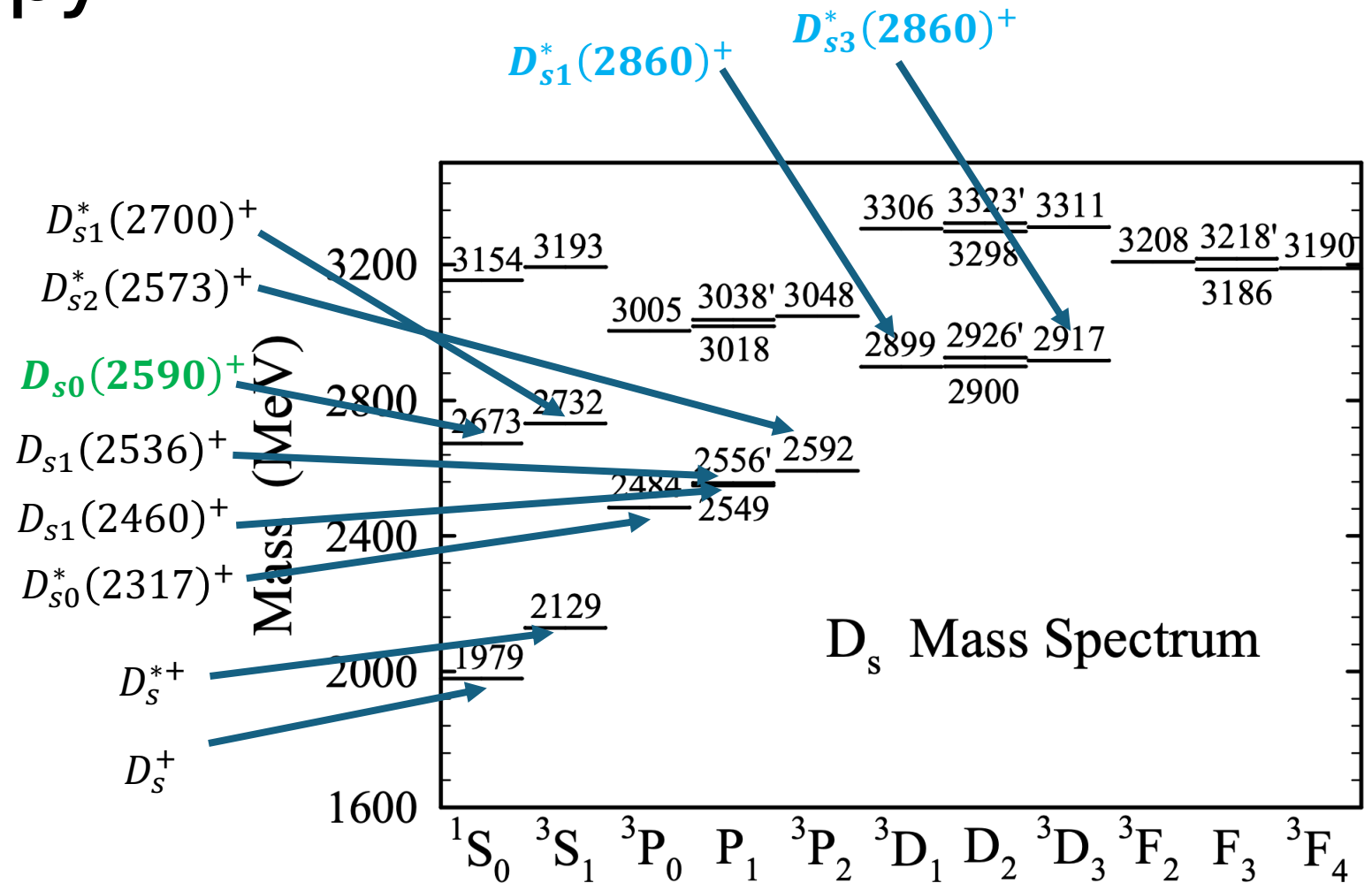


- Controversy on  $D_{s0}^*(2317)$  and  $D_{s1}(2460)^+$
- Will focus on the most recent results
- Inclusion of charge-conjugate processes is implied throughout the talk

# $D_S^{**}$ spectroscopy

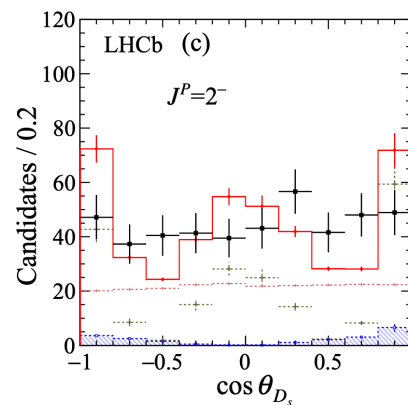
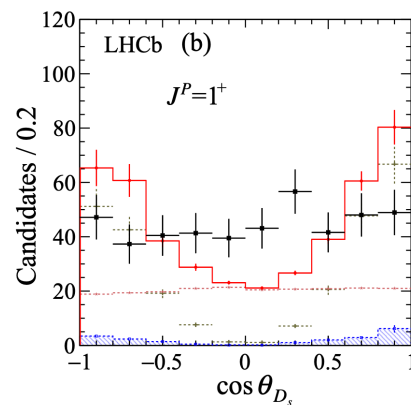
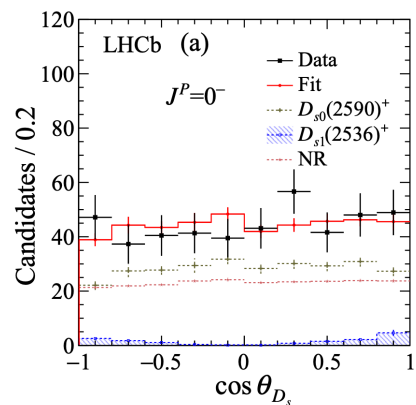
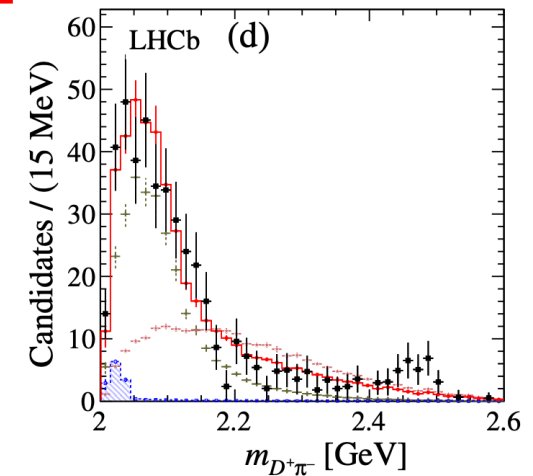
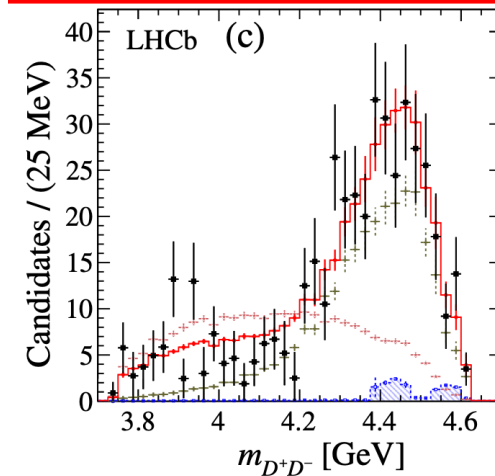
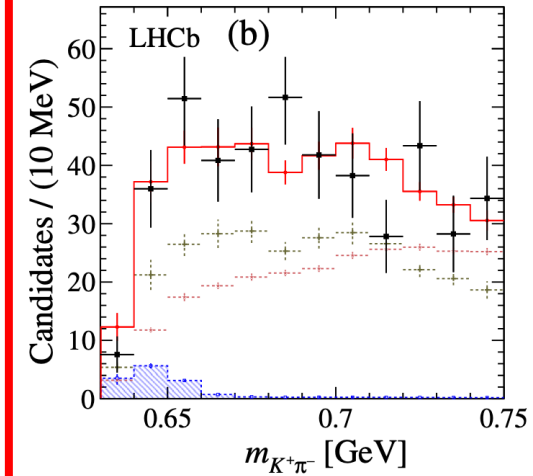
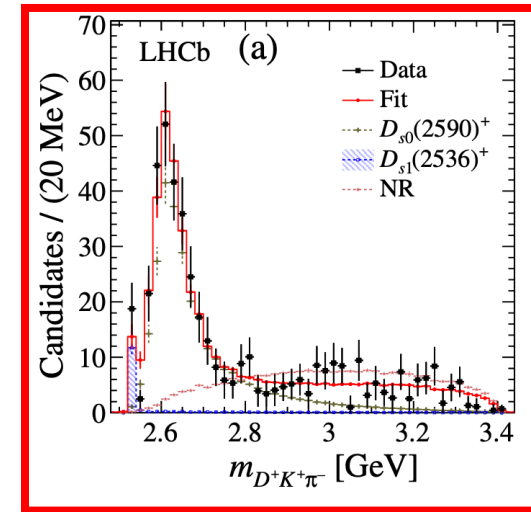
- Relativistic quark model
- Some discrepancies between predicted and measured masses
- Blue: states discovered in LHCb
- Green: states discovered in LHCb and to be introduced in the talk

[Phys. Rev. D 89, 074023](#)



# $D_{S0}(2590)^+$ : strong candidate for $D_S^+(2^1S_0)$

- $5.4 \text{ fb}^{-1}$
- $B^0 \rightarrow D^- D^+ K^+ \pi^-$ ;  $D_{S0}(2590)^+ \rightarrow D^+ K^+ \pi^-$
- $m(K^+ \pi^-) < 750 \text{ MeV}$ : dominantly S-wave ( $J^P = 0^+$ ), any  $R(D^+ K^+ \pi)$  should have unnatural  $J^P = (-1)^{J+1}$
- Pole mass and pole width
  - $m_R = 2591 \pm 6 \pm 7 \text{ MeV}$ ;  $\Gamma_R = 89 \pm 16 \pm 12 \text{ MeV}$
- $J^P = 0^- > 10 \sigma$
- $\theta_{D_S}$ : Angle between  $D^+$  and the opposite direction of  $B^0$  in the  $D_S^+$  rest frame



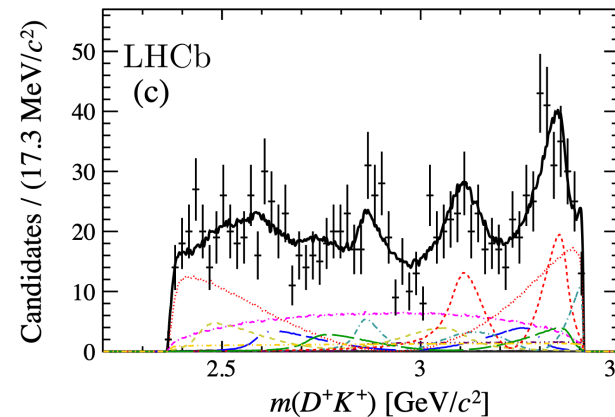
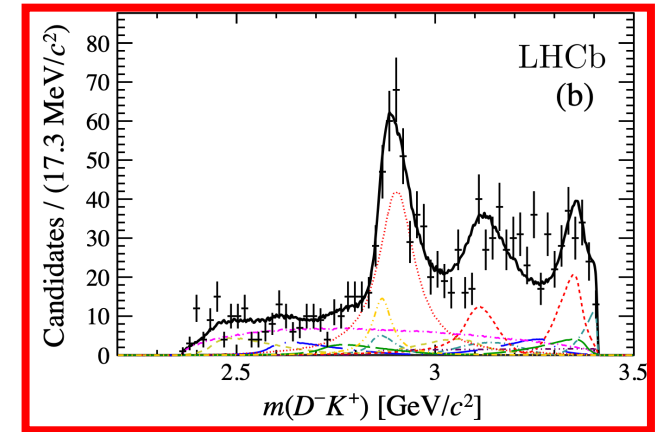
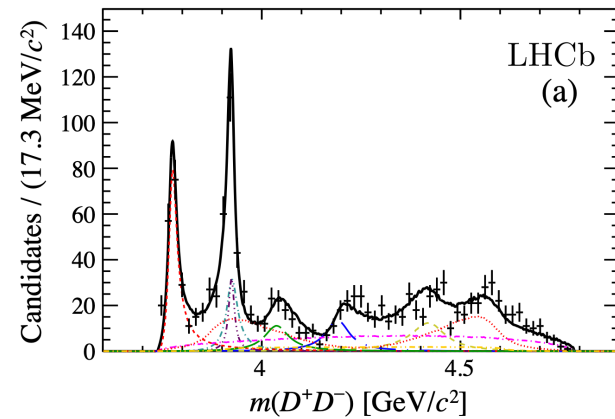
[Phys. Rev. Lett. 126 \(2021\) 122002](#)

# Exotic charm-strange mesons

States	Quark component	Mass (MeV)	Width (MeV)	Discovery channel
$T_{cs0}^*(2870)^0$	$\bar{c}d\bar{s}u$	$2866 \pm 7$	$57 \pm 13$	$B^+ \rightarrow D^+ D^- K^+$ $B^- \rightarrow D^- D^0 K_S^0$ $B^+ \rightarrow D^{*+} D^- K^+$
$T_{cs1}^*(2900)^0$	$\bar{c}d\bar{s}u$	$2904 \pm 5$	$110 \pm 12$	$B^+ \rightarrow D^+ D^- K^+$ $B^+ \rightarrow D^{*+} D^- K^+$
$T_{c\bar{s}0}^*(2900)^{++}$	$c\bar{s}u\bar{d}$	$2921 \pm 26$	$140 \pm 40$	$B^+ \rightarrow D^- D_S^+ \pi^+$
$T_{c\bar{s}0}^*(2900)^0$	$c\bar{s}\bar{u}d$	$2892 \pm 21$	$119 \pm 29$	$B^0 \rightarrow \bar{D}^0 D_S^+ \pi^-$

# Discovery of $T_{cs0}^*(2870)^0$ and $T_{cs1}^*(2900)^0$

- $9.0 \text{ fb}^{-1}$
- $B^+ \rightarrow D^+ D^- K^+$
- Two new tetraquark states:  
 $T_{cs0}^*(2870)^0$  and  $T_{cs1}^*(2900)^0$ 
  - $\bar{c}d\bar{s}u$
  - $m = 2866 \pm 7 \pm 2 \text{ MeV}$
  - $\Gamma = 57 \pm 12 \pm 4 \text{ MeV}$
  - $m = 2904 \pm 5 \pm 1 \text{ MeV}$
  - $\Gamma = 110 \pm 11 \pm 4 \text{ MeV}$
- One new state:  $\chi_{c0}(3915)$



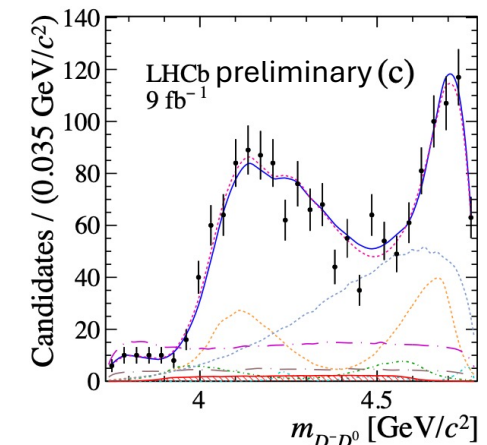
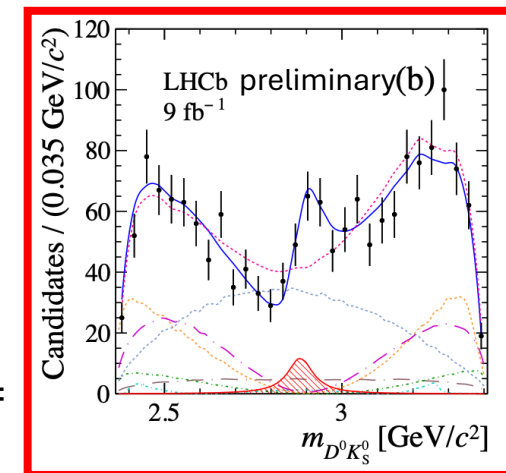
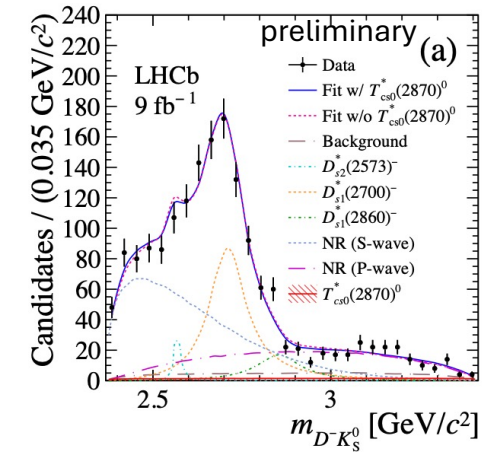
- $\psi(3770) \rightarrow D^+ D^-$
- $\chi_{c0}(3930) \rightarrow D^+ D^-$   $\chi_{c0}(3915): 0^+$
- $\chi_{c2}(3930) \rightarrow D^+ D^-$
- $\psi(4040) \rightarrow D^+ D^-$
- $\psi(4160) \rightarrow D^+ D^-$
- $\psi(4415) \rightarrow D^+ D^-$
- $X_0(2900) \rightarrow D^- K^+$   $T_{cs0}^*(2870)^0: 0^+$
- $X_1(2900) \rightarrow D^- K^+$   $T_{cs1}^*(2900)^0: 1^-$
- Nonresonant

[Phys. Rev. D102 \(2020\) 112003](#)

# $T_{CS0}^*(2870)^0$ in $B^- \rightarrow D^- D^0 K_S^0$

preliminary

- $9.0 \text{ fb}^{-1}$
- $T_{CS0}^*(2870)^0$  (Significance of  $5.3 \sigma$ )
  - $m = 2883 \pm 11 \pm 6 \text{ MeV}$
  - $\Gamma = 87_{-47}^{+22} \pm 6 \text{ MeV}$
- Use  $D_{SJ}$  states to model  $m(D^- K_S^0)$ 
  - K-matrix: overlap between  $D_{S1}^*(2700)$  and  $D_{S1}(2860)$
  - Higher spin  $D_{S3}^*(2860)$
- $T_{CS1}^*(2900)^0$  not significant
- $\frac{\Gamma(T_{CS0}^*(2870)^0 \rightarrow D^0 \bar{K}^0)}{\Gamma(T_{CS0}^*(2870)^0 \rightarrow D^+ K^-)} = 3.3 \pm 1.9$
- $\frac{\Gamma(T_{CS1}^*(2900)^0 \rightarrow D^0 \bar{K}^0)}{\Gamma(T_{CS1}^*(2900)^0 \rightarrow D^+ K^-)} = 0.15 \pm 0.16$
- [Phys. Rev. D103 \(2021\) 014004](#)
  - $T_{CS1}^*(2900)^0$  is caused by triangle singularities of  $B^- \rightarrow D^{(*)0} D_{SJ}^{(*)-} (\rightarrow D^- K^{(*)0})$  decays with final-state rescattering
  - The state does not have a definite isospin



LHCB-PAPER-2024-040



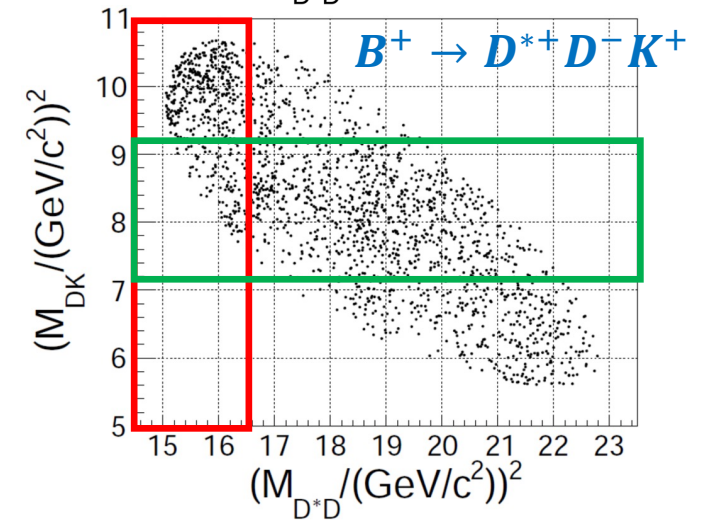
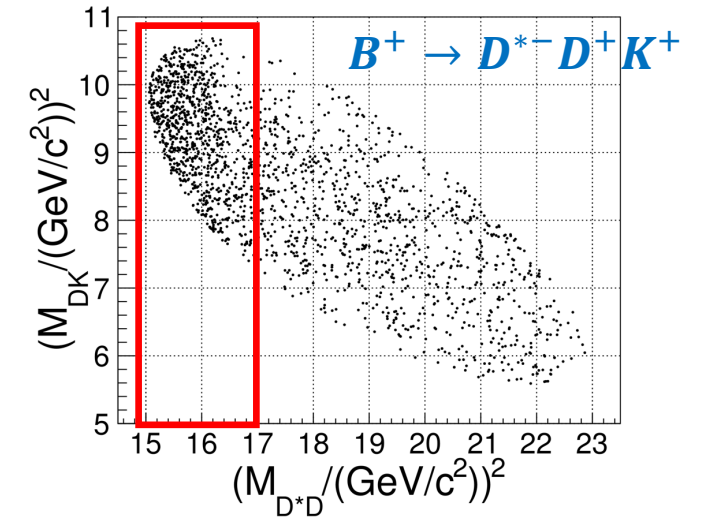
# $T_{cs0}^*(2870)^0$ and $T_{cs1}^*(2900)^0$ in $B^+ \rightarrow D^{*\pm} D^{\mp} K^+$

- $9.0 \text{ fb}^{-1}$
- $B^+ \rightarrow D^{*+} D^- K^+$  and  $B^+ \rightarrow D^{*-} D^+ K^+$
- $B^+ \rightarrow R(D^{*\pm} D^{\mp}) K^+$  The amplitude is related by C parity

$$\left. \begin{array}{l} B^+ \rightarrow D^{*+} D^- K^+ \\ B^+ \rightarrow D^{*-} D^+ K^+ \end{array} \right\} B^+ \rightarrow X K^+, X \rightarrow \begin{cases} D^{*+} D^- \\ D^{*-} D^+ \end{cases}$$

- $A(x) = \frac{1+d}{2} [c_j A_j(x) + c_k A_k(x)] + \frac{1-d}{2} [C_j c_j A_j(x) + c_l A_l(x)]$
- It is the first time that amplitude analysis can determine the C-parity of the resonances
- Clear difference due to interference of different C-parities
- At least three new states:  $h_c(4000)$ ,  $\chi_{c1}(4010)$ ,  $h_c(4300)$

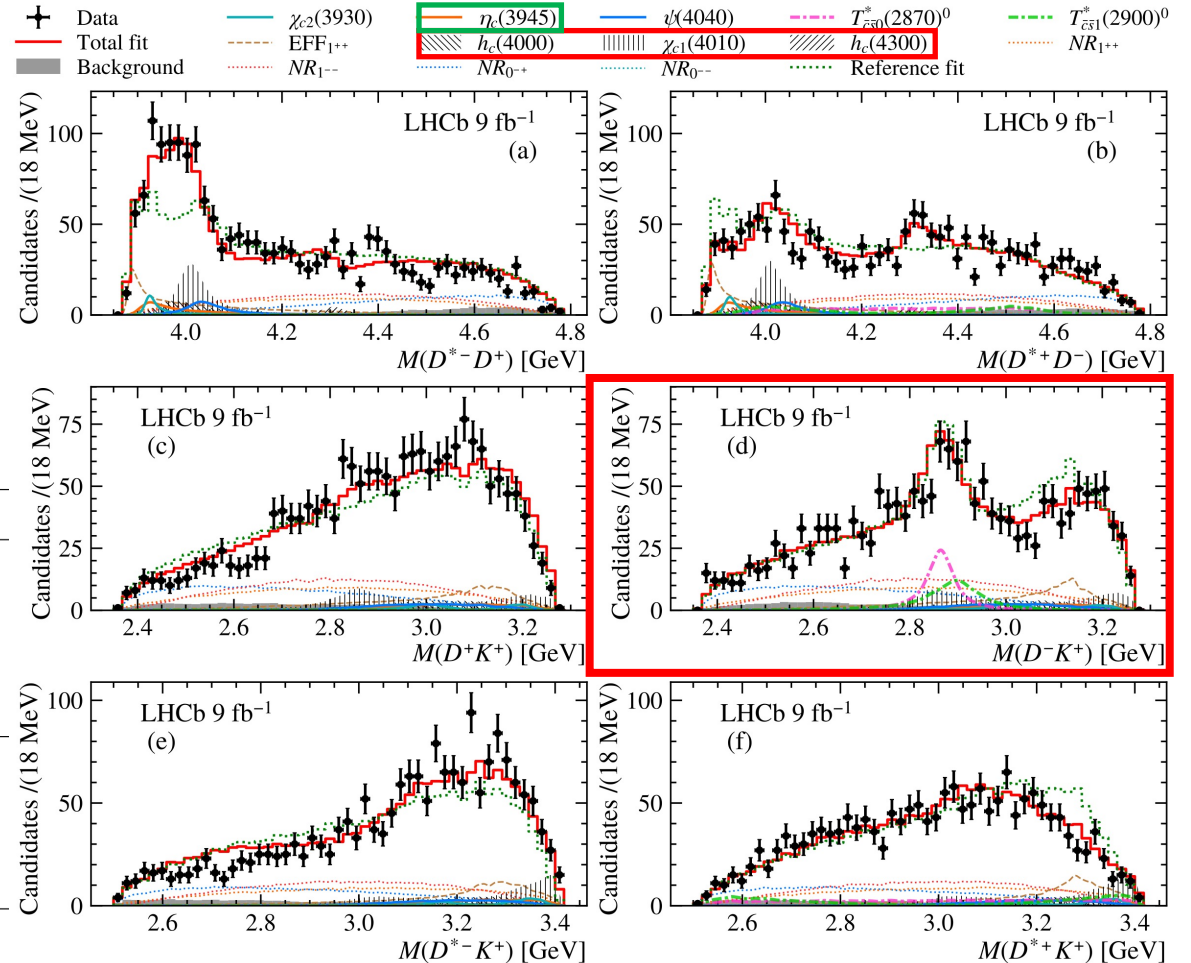
[Phys. Rev. Lett. 133 \(2024\) 131902](#)



# $T_{c\bar{s}0}^*(2870)^0$ and $T_{c\bar{s}1}^*(2900)^0$ in $B^+ \rightarrow D^{*\pm} D^{\mp} K^+$

- Contribution from  $T_{c\bar{s}}^*$  seen in one channel
- Some tension in the mass, width and fractions
- $T_{c\bar{s}0}^*(2870)^0 \rightarrow D^{*-} K^+$  is forbidden
- The processes  $B^+ \rightarrow T_{c\bar{s}1}^*(2900)^0 D^+$ ,  $T_{c\bar{s}1}^*(2900)^0 \rightarrow D^{*-} K^+$  and  $B^+ \rightarrow T_{c\bar{s}1}^*(2900)^{++} D^{*-}$ ,  $T_{c\bar{s}1}^*(2900)^{++} \rightarrow D^+ K^+$  are not obvious

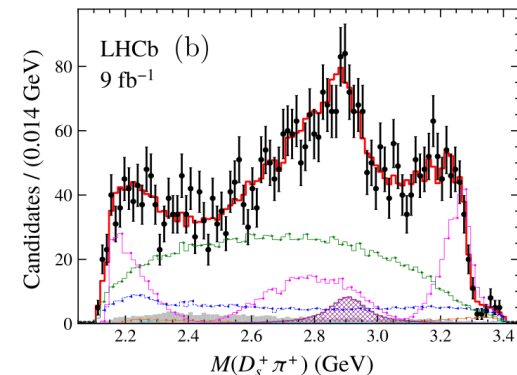
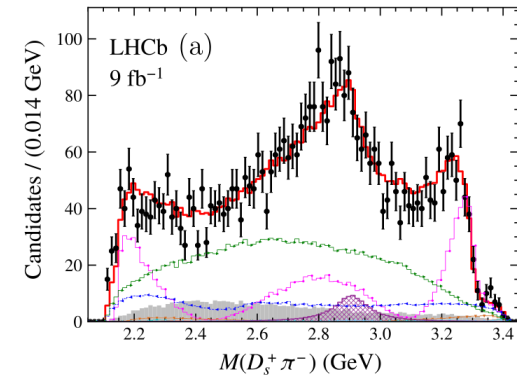
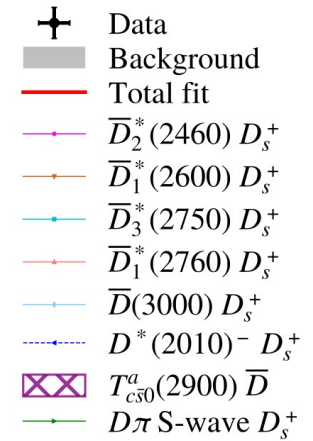
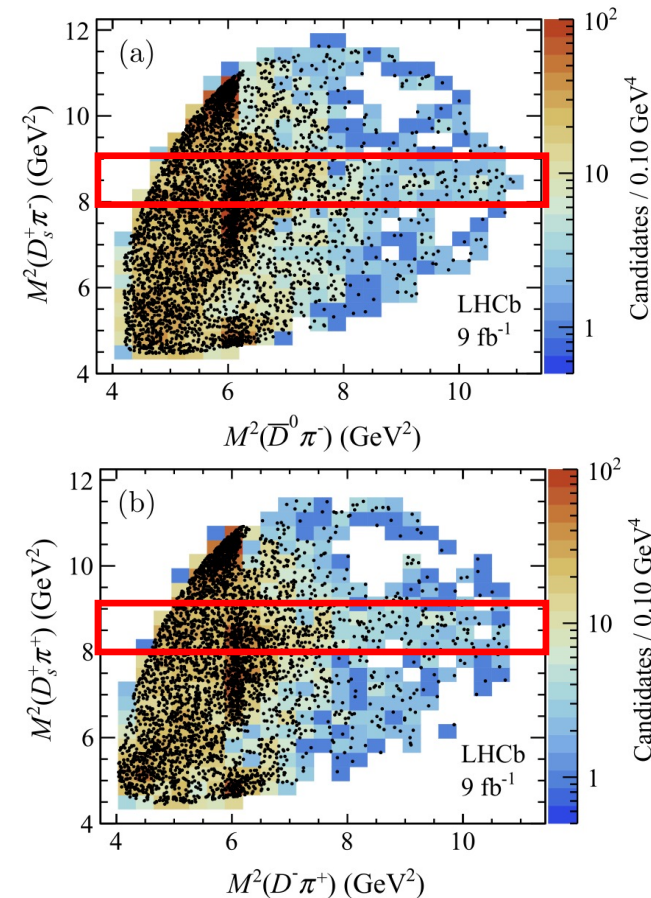
Property	This work	Previous work
$T_{c\bar{s}0}^*(2870)^0$ mass [MeV]	$2914 \pm 11 \pm 15$	$2866 \pm 7$
$T_{c\bar{s}0}^*(2870)^0$ width [MeV]	$128 \pm 22 \pm 23$	$57 \pm 13$
$T_{c\bar{s}1}^*(2900)^0$ mass [MeV]	$2887 \pm 8 \pm 6$	$2904 \pm 5$
$T_{c\bar{s}1}^*(2900)^0$ width [MeV]	$92 \pm 16 \pm 16$	$110 \pm 12$
$\mathcal{B}(B^+ \rightarrow T_{c\bar{s}0}^*(2870)^0 D^{(*)+})$	$(4.5^{+0.6+0.9}_{-0.8-1.0} \pm 0.4) \times 10^{-5}$	$(1.2 \pm 0.5) \times 10^{-5}$
$\mathcal{B}(B^+ \rightarrow T_{c\bar{s}1}^*(2900)^0 D^{(*)+})$	$(3.8^{+0.7+1.6}_{-1.0-1.1} \pm 0.3) \times 10^{-5}$	$(6.7 \pm 2.3) \times 10^{-5}$
$\frac{\mathcal{B}(B^+ \rightarrow T_{c\bar{s}0}^*(2870)^0 D^{(*)+})}{\mathcal{B}(B^+ \rightarrow T_{c\bar{s}1}^*(2900)^0 D^{(*)+})}$	$1.17 \pm 0.31 \pm 0.48$	$0.18 \pm 0.05$



# $T_{c\bar{s}0}^*(2900)$ in $B \rightarrow DD_s\pi$

- $9.0 \text{ fb}^{-1}$
- $B^0 \rightarrow \bar{D}^0 D_s^+ \pi^-$  and  $B^+ \rightarrow D^- D_s^+ \pi^+$
- Isospin symmetry
- $T_{c\bar{s}0}^*(2900)^{++}: c\bar{s}u\bar{d}$
- $T_{c\bar{s}0}^*(2900)^0: c\bar{s}\bar{u}d$
- $J^P = 0^+$
- $m = 2908 \pm 11 \pm 20 \text{ MeV}$
- $\Gamma = 136 \pm 23 \pm 13 \text{ MeV}$
- Might belong to an isospin triplet

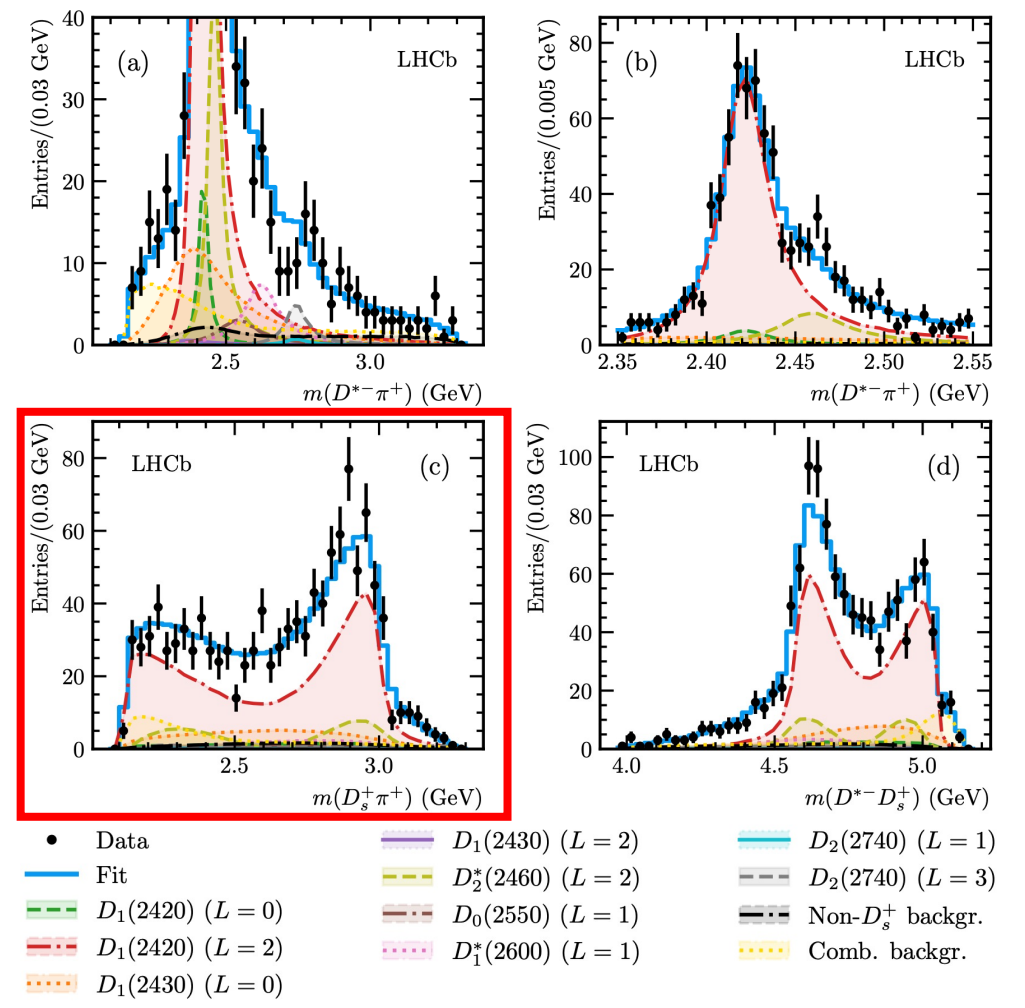
[Phys. Rev. Lett. 131 \(2023\) 041902](#)



# Search for $T_{c\bar{s}0}^*(2900)^{++}$ in $B^+ \rightarrow D^{*-} D_s^+ \pi^+$

- $9.0 \text{ fb}^{-1}$
- $B^+ \rightarrow D^{*-} D_s^+ \pi^+$
- Main contribution from excited charm meson
- No strong evidence of  $T_{c\bar{s}0}^*(2900)^{++}$ , an upper limit is set 2.5% @ 90% CL
- The statistics is limited

[JHEP08 \(2024\) 165](#)



# $D_{s1}(2460)^+$ in $B$ decays

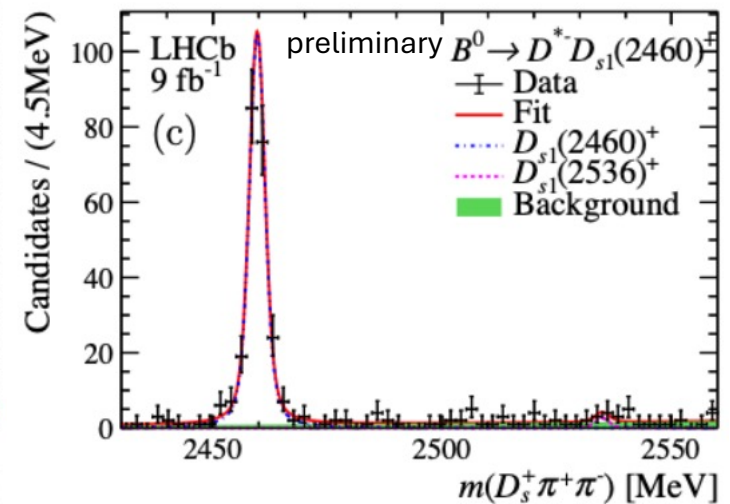
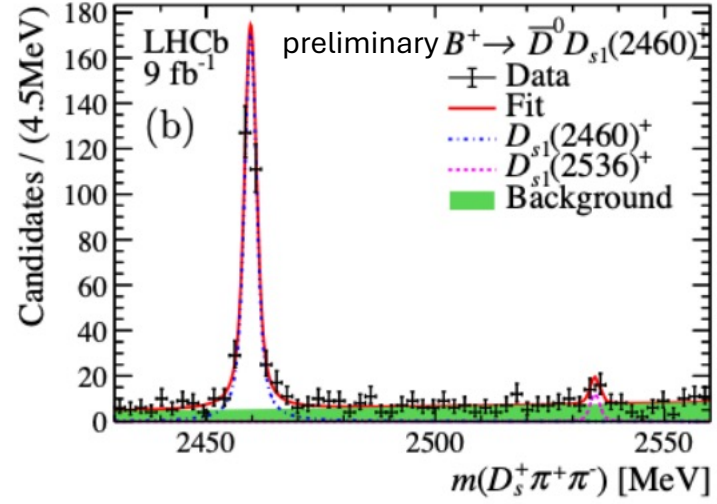
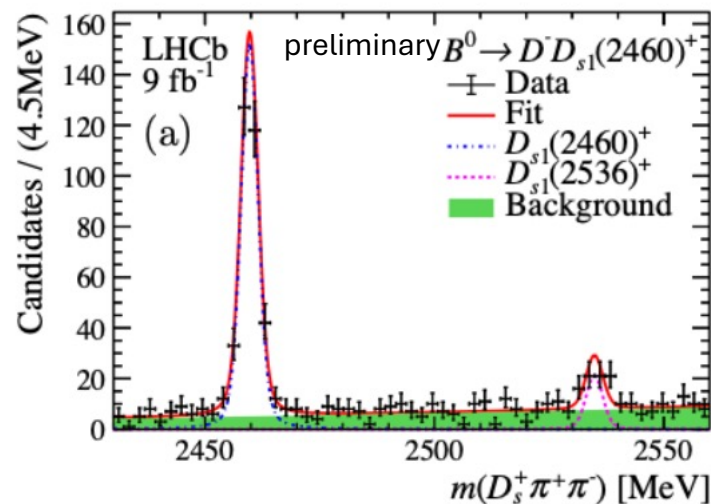
LHCB-PAPER-2024-033

- The nature of  $D_{s0}^*(2317)^+$  and  $D_{s1}(2460)^+$  has been discussed extensively without a firm conclusion
- Their masses are 100 MeV lower than the quark model expectation
- Lower mass makes decaying to  $D^*K$  impossible, dominant isospin violating decay of  $D_S^{(*)+}\pi^0$
- Isospin conserving decay  $D_{s1}(2460)^+ \rightarrow D_S^+\pi^+\pi^-$  at a sizable rate
- Double-bump lineshape in  $m(\pi\pi)$  if  $D_{s1}(2460)^+$  is a  $D^*K$  hadronic molecule
  - [Commun. Theor. Phys. 75 055203](#)
  - The multiplet including  $T_{c\bar{s}}(2900)^{++}$ ,  $T_{c\bar{s}}(2900)^0$ , and  $T_{cs0}(2900)^0$  could be the radial excitation of a lighter multiplet containing  $D_{s0}^*(2317)^+$
  - [Phys. Rev. D \*\*110\*\*, 034014](#)

# $D_{s1}(2460)^+$ in $B$ decays

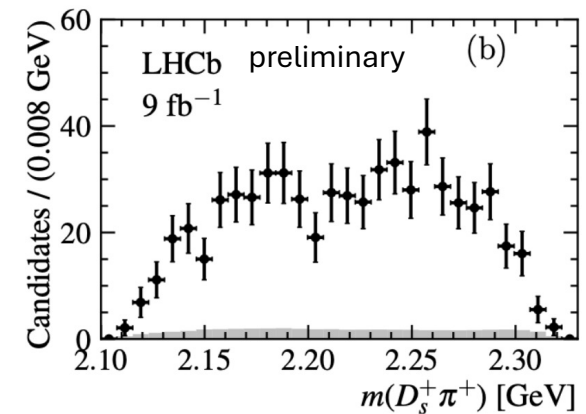
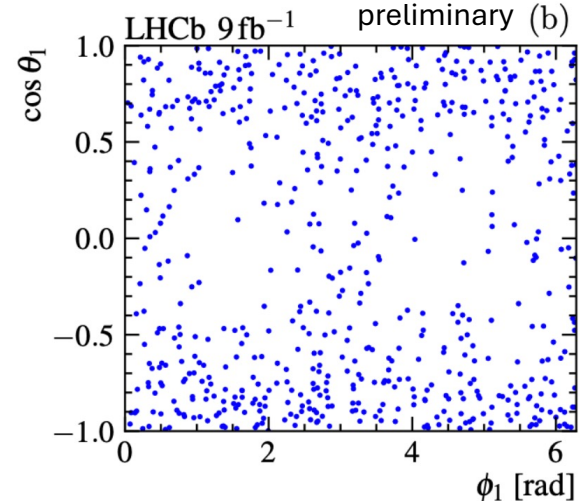
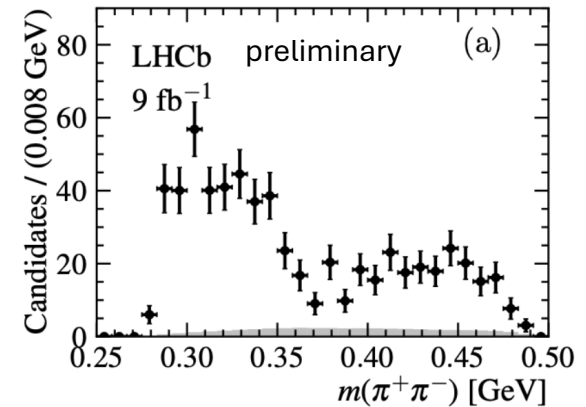
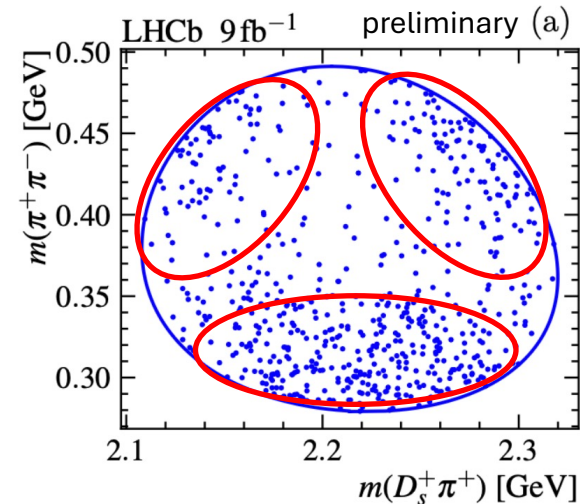
preliminary

- $9.0 \text{ fb}^{-1}$
- $D_{s1}(2460)^+ \rightarrow D_S^+ \pi^+ \pi^-$  in  $B \rightarrow \bar{D}^{(*)} D_S^+ \pi^+ \pi^-$
- $B^0 \rightarrow D^- D_{s1}(2460)^+$ ,  $B^+ \rightarrow \bar{D}^0 D_{s1}(2460)^+$ , and  $B^0 \rightarrow D^{*-} D_{s1}(2460)^+$



# $D_{s1}(2460)^+$ in $B$ decays

- Double-bump structure in  $m(\pi\pi)$
- Amplitude fit
  - Isobar approach
  - TF-PWA software [link](#)
- The models  $f_0(500) + f_0(980)$  and  $\pi\pi$  K-matrix cannot describe the data well
- The model in paper [Commun. Theor. Phys. 75 055203](#) also cannot describe the data well



# $D_{s1}(2460)^+$ in $B$ decays

- Two models can describe the data well
- One w/o exotic contribution
  - $f_0(500) + f_0(980) + f_2(1270)$ 
    - $f_0(500)$ : relativistic Breit-Wigner (RBW)
    - $f_0(980)$ : Flatte model
    - $f_2(1270)$ : RBW w/ mass and width fixed
- $\rho$  contribution is not significant
- One w/ exotic contribution
  - $f_0(500) + T_{c\bar{s}}^{++} + T_{c\bar{s}}^0$
  - $T_{c\bar{s}}$  tested with two models
    - RBW
    - K-matrix (scattering length approximation)
    - Describes the rescattering between  $D_s\pi$  and  $DK$  channel
    - Natural parametrisation of the  $DK$  molecular state

$$K = \begin{pmatrix} \gamma & \beta \\ \beta & \gamma_2 \end{pmatrix}$$

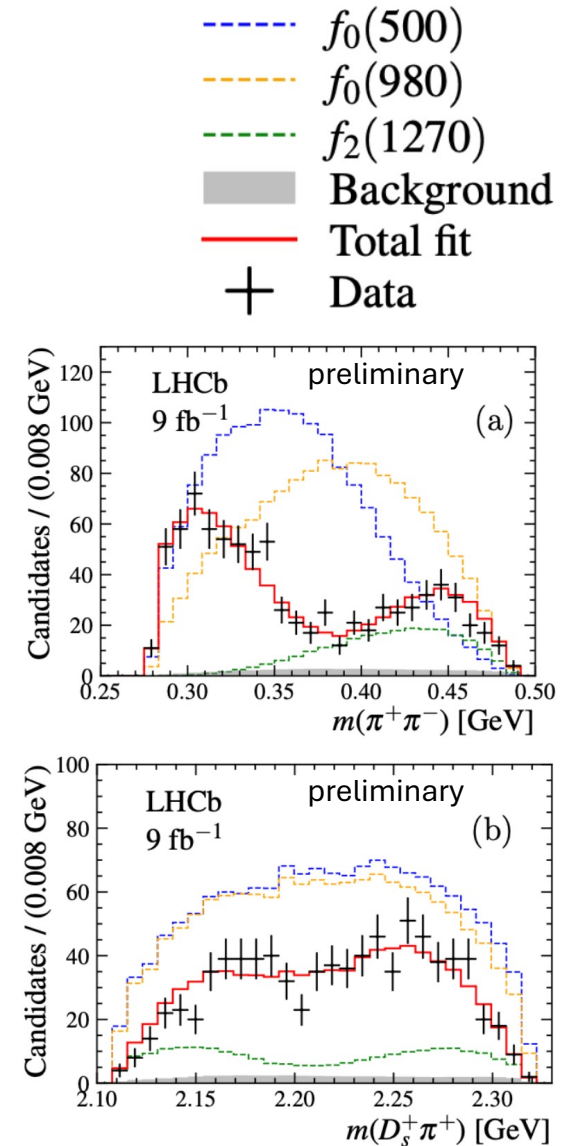
Elastic  $DK$  (red arrow pointing to  $\gamma$ )  
 $DK \leftrightarrow D_s\pi$  (green arrow pointing to  $\beta$ )  
 Elastic  $D_s\pi$  (purple arrow pointing to  $\gamma_2$ )



# $D_{s1}(2460)^+$ in $B$ decays

- $f_0(500) + f_0(980) + f_2(1270)$ 
  - The large contribution from  $f_0(980)$  and  $f_2(1270)$
  - The large interference between  $f_0(500)$  and  $f_0(980)$  forming the double bump lineshape in  $m(\pi\pi)$
  - The mass and width of  $f_0(500)$  are different from the known values

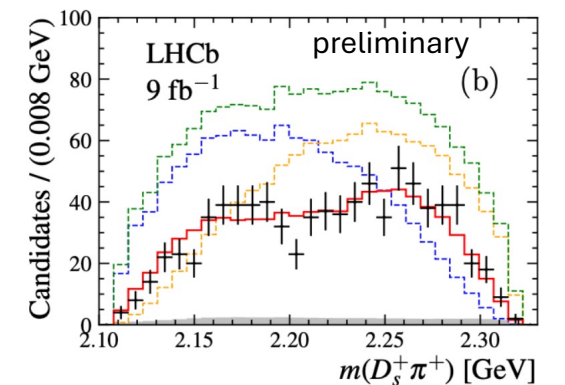
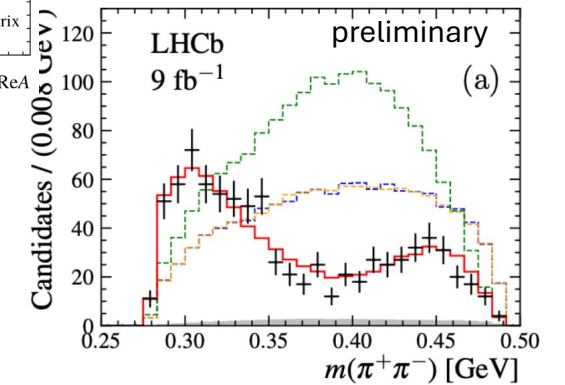
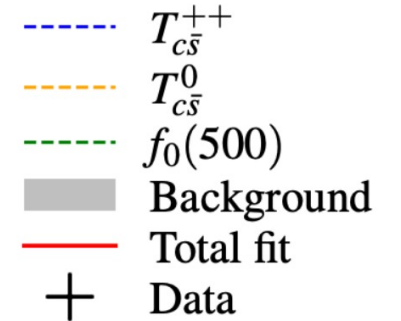
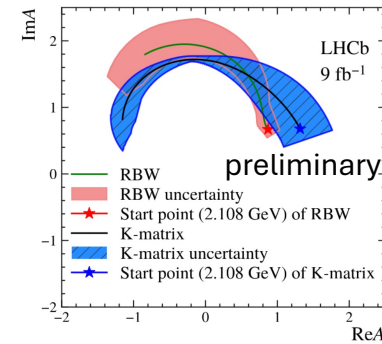
Resonance	Mass (MeV)	Width (MeV)	FF (%)
$f_0(500)$	$376 \pm 9 \pm 16$	$175 \pm 23 \pm 16$	$197 \pm 35 \pm 23$
$f_0(980)$	945.5	167	$187 \pm 38 \pm 43$
$f_2(1270)$	1275.4	186.6	$29 \pm 2 \pm 1$



# $D_{s1}(2460)^+$ in $B$ decays

- $f_0(500) + T_{c\bar{s}}^{++} + T_{c\bar{s}}^0$ 
  - Pole mass just below  $DK$  threshold
  - Isospin symmetry is conserved
  - $J^P$  favours  $0^+$
  - Significance over  $f_0(500) + f_0(980)$  model is larger than  $10\sigma$
  - Consistent results obtained w/ RBW and K-matrix model except for the width
  - Assign large systematic uncertainty for the width

Resonance	Mass (MeV)	Width (MeV)	FF (%)
$f_0(500)$	$472 \pm 32 \pm 19$	$226 \pm 24 \pm 18$	$237^{+51}_{-43} \pm 42$
$T_{c\bar{s}}$	$2328 \pm 12 \pm 12$	$96 \pm 16^{+170}_{-23}$	$151^{+31}_{-33} \pm 25$



# $D_{s1}(2460)^+$ in $B$ decays

- Some discussion
  - Large interference is inevitable since narrow phase space
  - Two models could describe the data well
  - The first one only with  $R(\pi\pi)$  contribution
    - Large contribution from  $f_0(980)$  and  $f_2(1270)$  far away from the upper limit of  $m(\pi\pi)$
    - The mass and width for  $f_0(500)$  not consistent with PDG
    - We cannot fully reject this model, but we find it implausible.
  - The second one with exotic charm meson contribution
    - $T_{c\bar{s}}^{++}$  and its isopin partner  $T_{c\bar{s}}^0$

# Summary

- LHCb has discovered multiple conventional and exotic charm meson over the past few years
- Need more data to understand the production mechanism of newly observed tetraquark states and find their isospin partners
- New results on the  $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$  decay shed light on the open question regarding the structure of the  $D_{sJ}$  states.

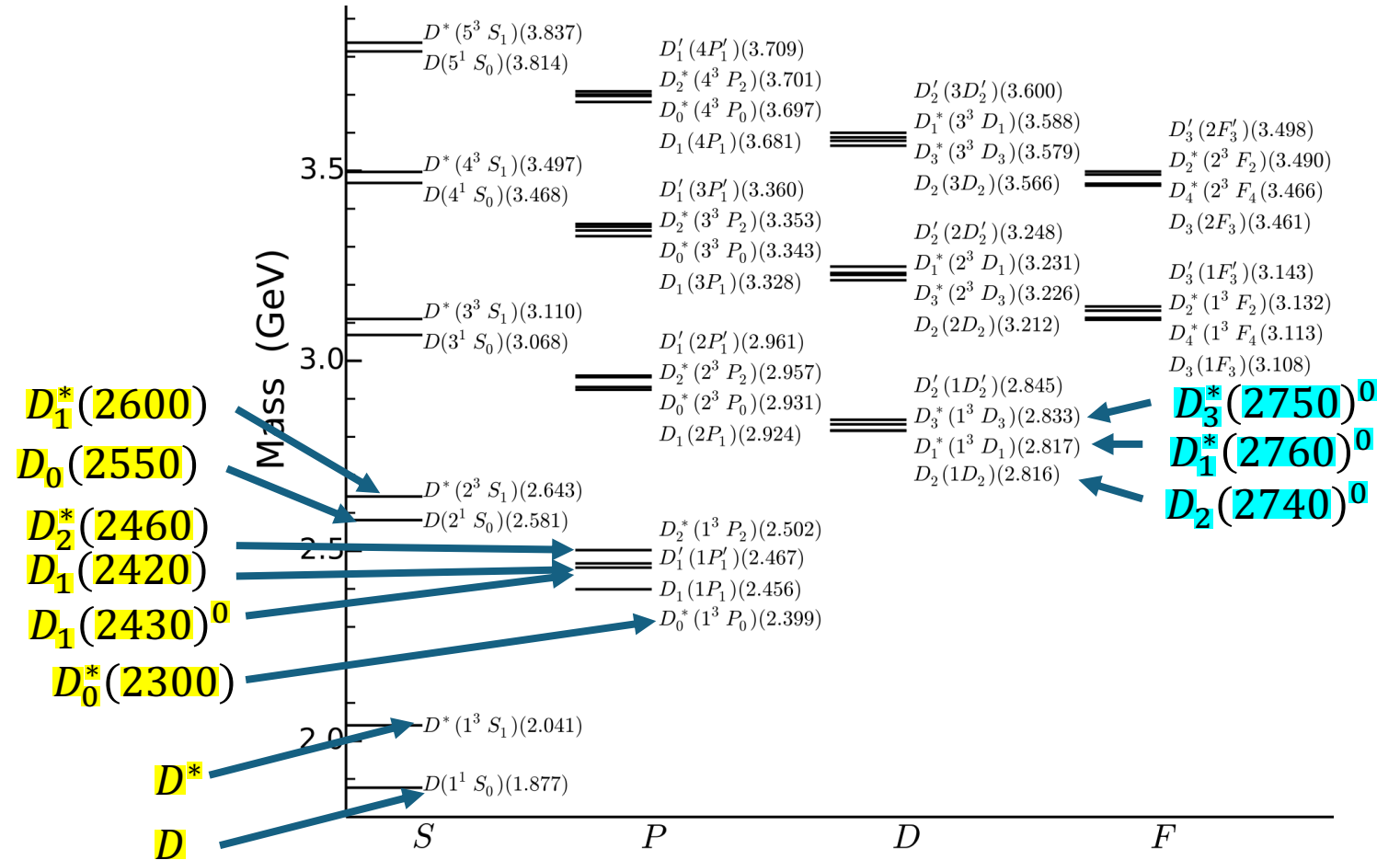
Thank you!

# Backup slides

# $D^{**}$ spectroscopy

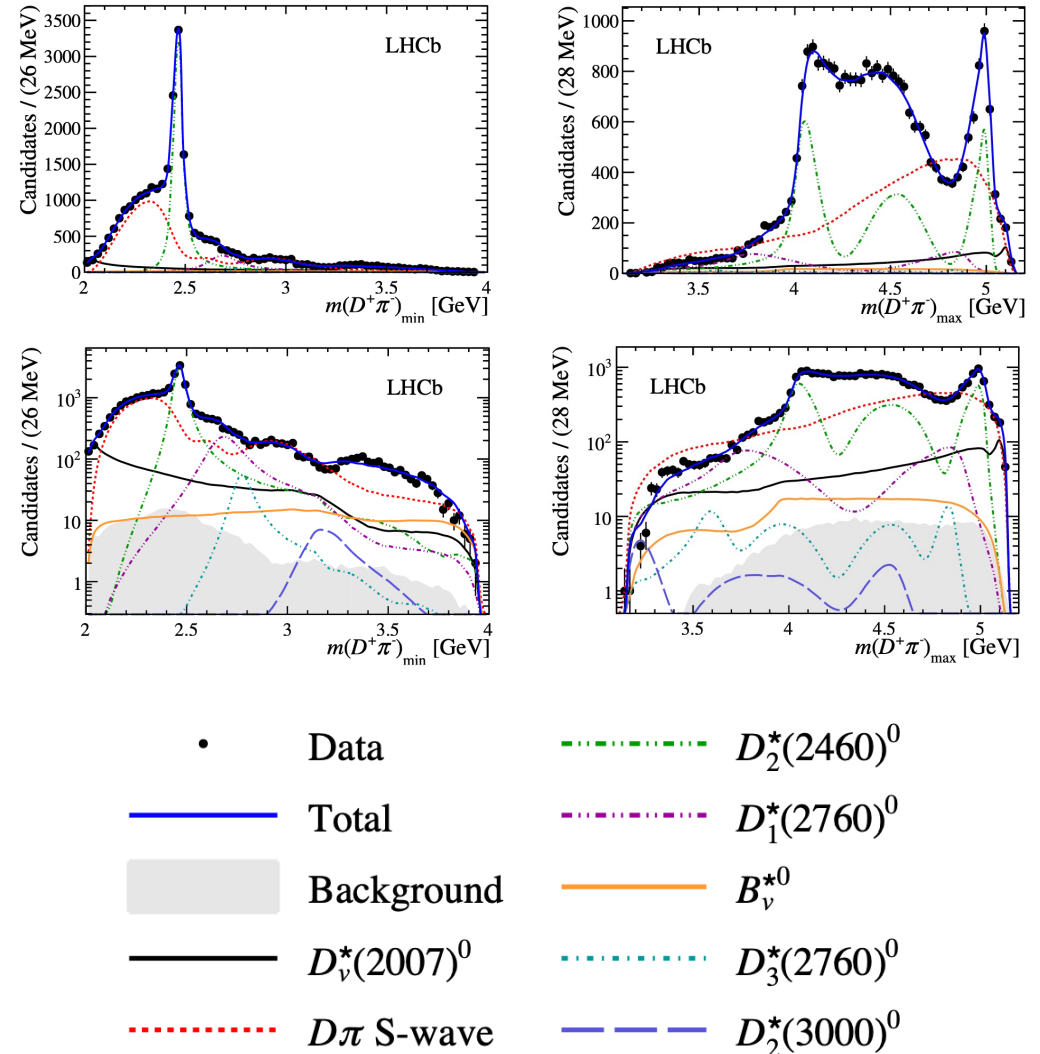
- Relativistic quark model
- Some discrepancies between predicted and measured masses
- Blue: states discovered in LHCb
- [Link](#)

$D^*(2640)^\pm?$   
 $D(3000)^0?$



# Observation of $D_3^*(2750)^0$ , $D(3000)^0$

- $3.0 \text{ fb}^{-1}$
- Amplitude analysis of  $B^+ \rightarrow D^- \pi^+ \pi^+$
- Significant contribution from  $D_2^*(2460)^0$ ,  $D_1^*(2680)^0$ ,  $D_3^*(2750)^0$ ,  $D(3000)^0$
- $D_1^*(2680)^0$ : Both 2S and 1D states with spin-parity  $J^P = 1^-$  are expected in this region
- $D_3^*(2750)^0$ : a member of 1D family
- $D(3000)^0$ : a member of 2P or 1F family
- No spin-parity preference

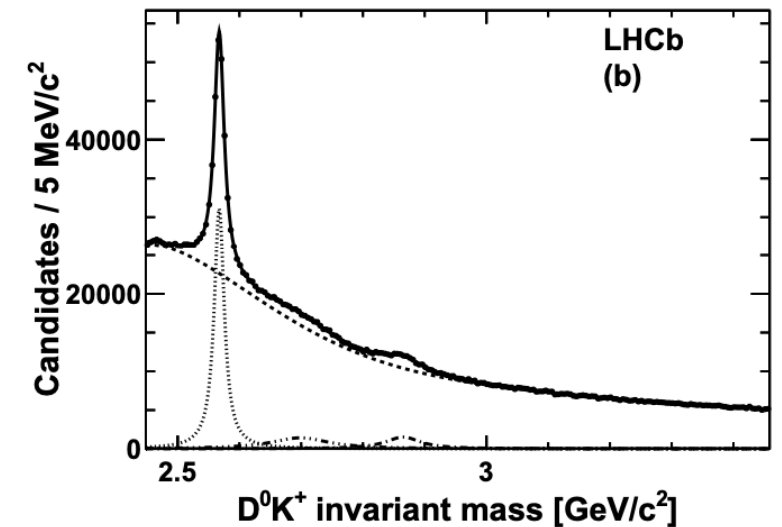
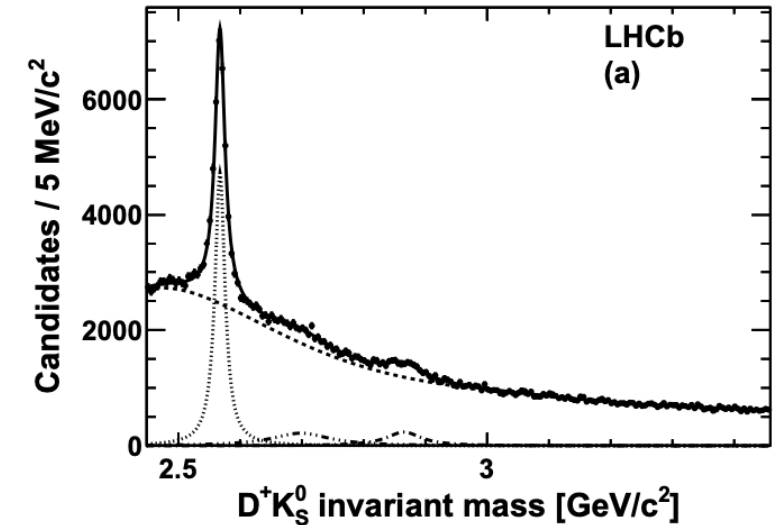


[Phys. Rev. D94 \(2016\) 072001](https://arxiv.org/abs/1512.07201)

# Confirmation of $D_{S1}^*(2700)^+$ and $D_{SJ}^*(2860)^+$

- $3.0 \text{ fb}^{-1}$
- $pp \rightarrow D^+ K_S^0 + X$  and  $pp \rightarrow D^0 K^+ + X$
- $m(D_{S1}^*(2700)^+) = 2709.2 \pm 1.9 \pm 4.5 \text{ MeV}$
- $\Gamma(D_{S1}^*(2700)^+) = 115.8 \pm 7.3 \pm 12.1 \text{ MeV}$
- $m(D_{SJ}^*(2860)^+) = 2866.1 \pm 1.0 \pm 6.3 \text{ MeV}$
- $\Gamma(D_{SJ}^*(2860)^+) = 69.9 \pm 3.2 \pm 6.6 \text{ MeV}$

[JHEP 10 \(2012\) 151](#)

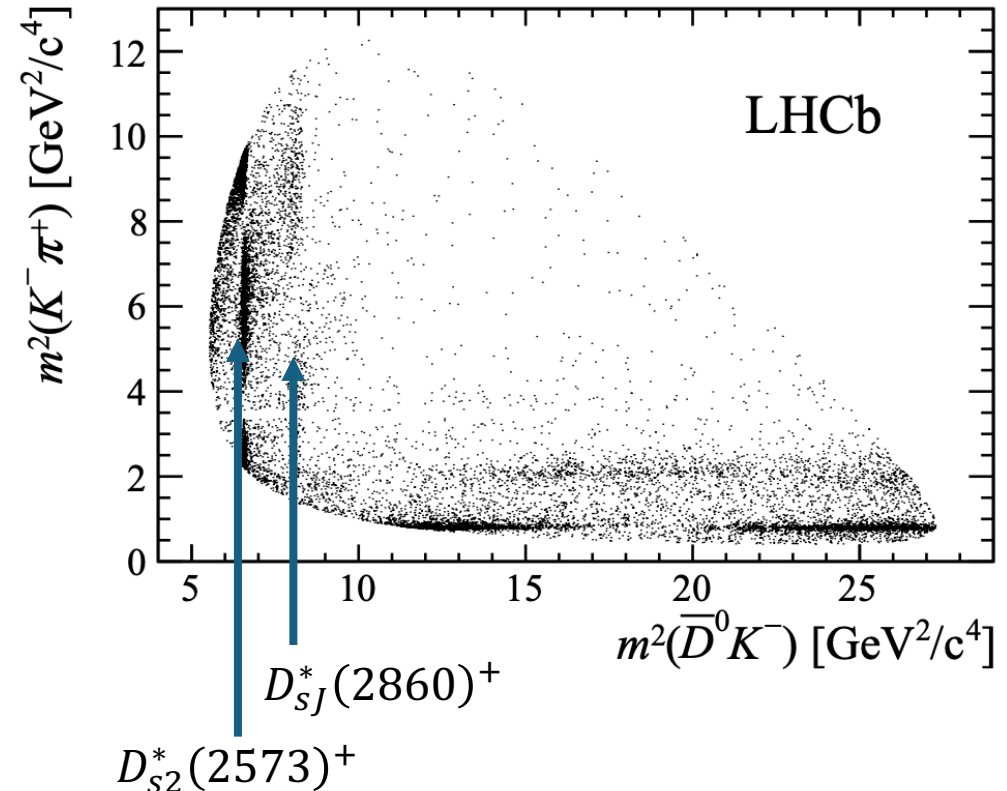




# Resolve the $J^P$ of $D_{s1}^*(2860)^+$ and $D_{s3}^*(2860)^+$

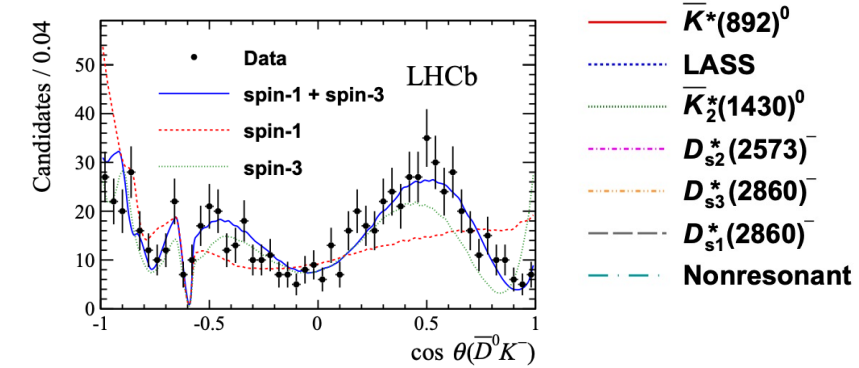
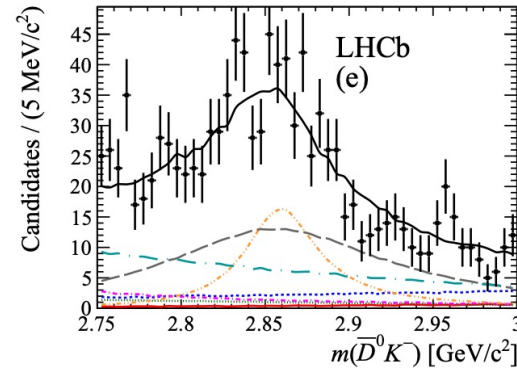
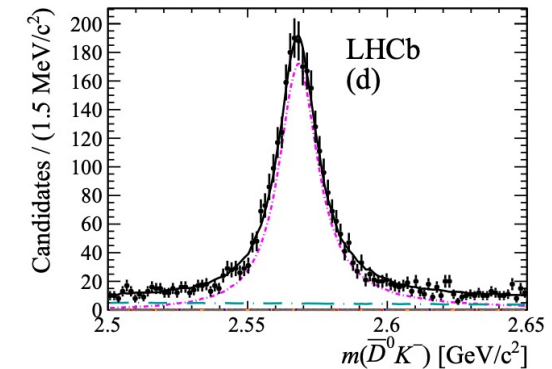
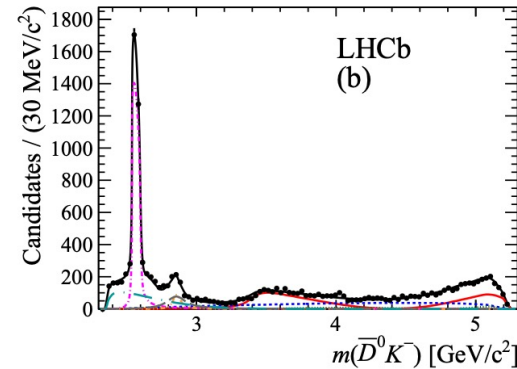
- $3.0 \text{ fb}^{-1}$
- $B_s^0 \rightarrow \bar{D}^0 K^- \pi^+$
- Dalitz plot distribution in the signal region

[Phys. Rev. Lett. 113 \(2014\) 162001](#)



# Resolve the $J^P$ of $D_{s1}^*(2860)^+$ and $D_{s3}^*(2860)^+$

- $m(D_{s2}^*(2573)^+) = 2568.39 \pm 0.29 \pm 0.19 \pm 0.18 \text{ MeV}$
- $\Gamma(D_{s2}^*(2573)^+) = 16.9 \pm 0.5 \pm 0.4 \pm 0.4 \text{ MeV}$
- $m(D_{s1}^*(2860)^+) = 2859 \pm 12 \pm 6 \pm 23 \text{ MeV}$
- $\Gamma(D_{s1}^*(2860)^+) = 159 \pm 23 \pm 27 \pm 72 \text{ MeV}$
- $m(D_{s2}^*(2573)^+) = 2860.5 \pm 2.6 \pm 2.5 \pm 6.0 \text{ MeV}$
- $\Gamma(D_{s2}^*(2573)^+) = 53 \pm 7 \pm 4 \pm 6 \text{ MeV}$
- The angle between  $\pi^+$  and  $\bar{D}^0$  meson momenta in the  $\bar{D}^0 K^-$  rest frame

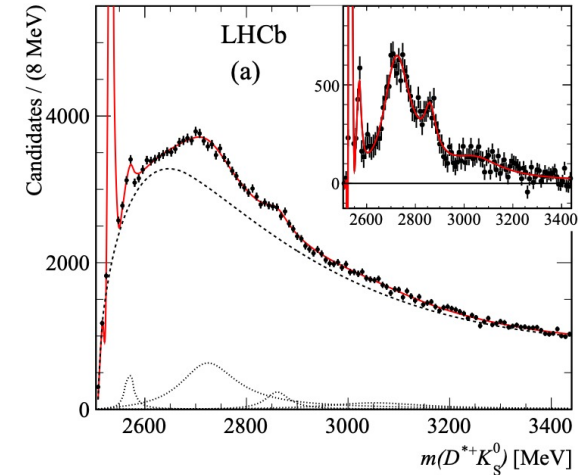


- $\bar{K}^*(892)^0$
- LASS
- $\bar{K}_2^*(1430)^0$
- $D_{s2}^*(2573)^-$
- $D_{s3}^*(2860)^-$
- $D_{s1}^*(2860)^-$
- Nonresonant

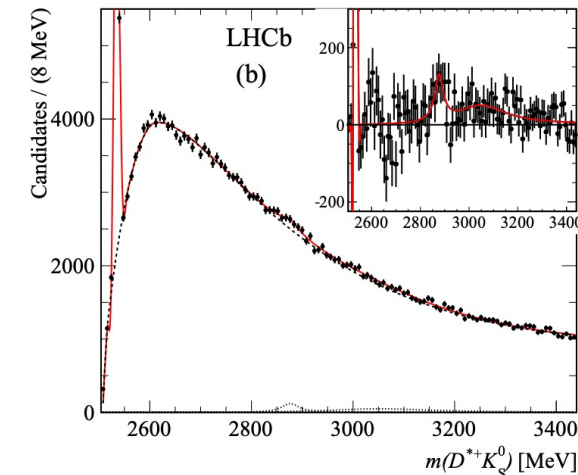
# Confirmation of $D_{S1}^*(2700)^+$ and $D_{S3}^*(2860)^+$

- $3.0 \text{ fb}^{-1}$
- $pp \rightarrow D^{*+} K_S^0 + X$  and  $pp \rightarrow D^{*0} K^+ + X$
- Significant  $D_{S1}(2573)^+$
- Hint of  $D_{SJ}(3040)^+$
- $m(D_{S1}^*(2700)^+) = 2732.3 \pm 4.3 \pm 5.8 \text{ MeV}$
- $\Gamma(D_{S1}^*(2700)^+) = 136 \pm 19 \pm 24 \text{ MeV}$
- $m(D_{S3}^*(2860)^+) = 2867.1 \pm 4.3 \pm 1.9 \text{ MeV}$
- $\Gamma(D_{S3}^*(2860)^+) = 50 \pm 11 \pm 13 \text{ MeV}$
- $\theta_H$ : angle between  $\pi$  and  $K_S^0$  in the  $D^{*+} K_S^0$  rest frame

[JHEP 02 \(2016\) 133](#)



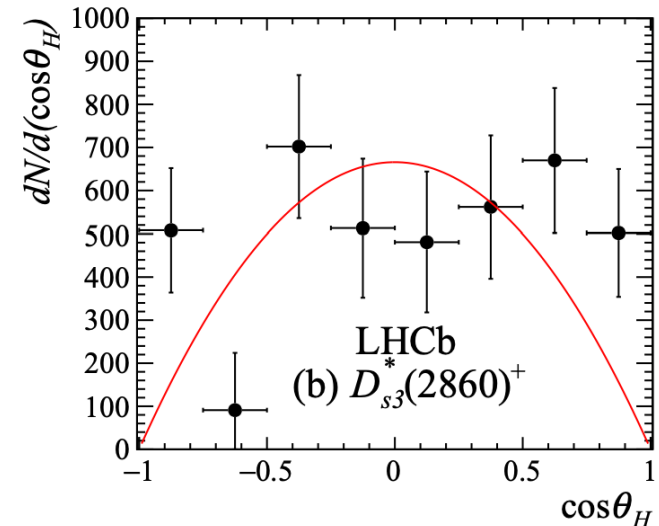
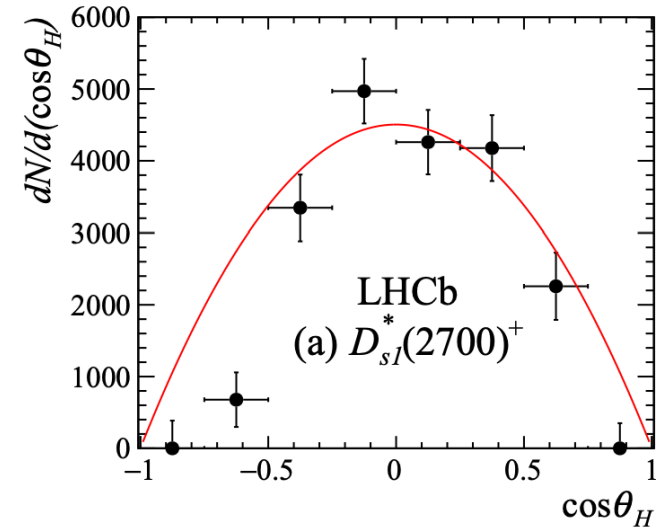
NP sample  
 $|\cos \theta_H| < 0.5$



UP sample  
 $|\cos \theta_H| > 0.5$

# $D_{s1}^*(2700)^+$ and $D_{s3}^*(2860)^+$

- Spin-parity analysis
- $J^P = 1^-$ :  $D_{s1}^*(2700)^+$
- $J^P = 3^-$ :  $D_{s3}^*(2860)^+$
- Mainly  $D_{s3}^*(2860)^+$  contribution



# $D_{s0}(2590)^+$ : strong candidate for $D_s(2^1S_0)$

- $\Gamma^{D_{sJ}}(m_{DK\pi}) = \Gamma^{D_{sJ} \rightarrow D^*K}(m_{DK\pi}) + \Gamma^{D_{sJ} \rightarrow DK\pi}(m_{DK\pi})$
- Two-body mass-dependent width
- $\Gamma^{D_{sJ} \rightarrow D^*K}(m_{DK\pi}) = \Gamma^{D_{sJ} \rightarrow D^*K}(m_0) \cdot \left(\frac{q}{q_0}\right)^{2L+1} \cdot \frac{m_0}{m_{DK\pi}} \cdot B'_L(q, q_0, d)^2$
- Constant
- $r = \Gamma^{D_{sJ} \rightarrow DK\pi}(m_0) / \Gamma^{D_{sJ}}(m_0)$
- Almost equally good fit quality and the same  $D^+K^+\pi^-$  mass lineshape are found for different width fractions  $r$  in the range 0 to 1
- $r$  cannot be determined with the current data, and is fixed to 0.5 in the fit

# K-matrix for $T_{c\bar{s}}$ model

- $\begin{pmatrix} \gamma & \beta \\ \beta & \gamma_2 \end{pmatrix}$
- Lineshape
  - $\frac{\beta^2 \rho_{DK} + i\gamma_2 (i\gamma \rho_{DK} - 1)}{\beta^2 \rho_{DK} \rho_{D_s \pi} + (i\gamma \rho_{DK} - 1)(i\gamma_2 \rho_{D_s \pi} - 1)}$
- Scattering length
  - $a = \frac{1}{8\pi\sqrt{s_{\text{thr}}}} \left( \gamma + i\beta^2 \rho_{D_s \pi}(s_{\text{thr}}) \right)$