ON THE COMPOSITENESS OF THE $X(3872)$

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THE $X(3872)$ IDENTIKIT: A SHALLOW BOUND STATE (?)

$$X^0(3872) \qquad J^{PC} = 1^{++}$$

$$M = 3871.65 \pm 0.06 \text{ MeV} \simeq M_D + M_{D*}$$
$$M = 3871.65 \pm 0.06 \text{ MeV} \simeq M_{J/\psi} + M_\rho$$

$$\Gamma = 1.19 \pm 0.21 \text{ MeV}$$

$\pi^+\pi^- J/\psi$	$\mathscr{B} = 3.8 \pm 1.2\,\%$
$\omega J/\pi$	$\mathscr{B} = 4.3 \pm 2.1\,\%$
$D^0\bar{D}^0\pi^0$	$\mathscr{B} = 49^{+18}_{-20}\,\%$
$D^0\bar{D}^{*0}$	$\mathscr{B} = 37 \pm 9\,\%$
$\pi^0\chi_{c1}(1P)$	$\mathscr{B} = 3.4 \pm 1.6\,\%$

SHALLOW BOUND STATE

Near the point $\boldsymbol{E = -B}$, i.e. at very low energies, the wave funct. is $\Psi(r) = Y_{00}\,\chi(r)/r$

$$\Psi(r) = \left(\frac{2mB}{4\pi^2}\right)^{1/4} \frac{\exp(-r\sqrt{2mB})}{r}$$

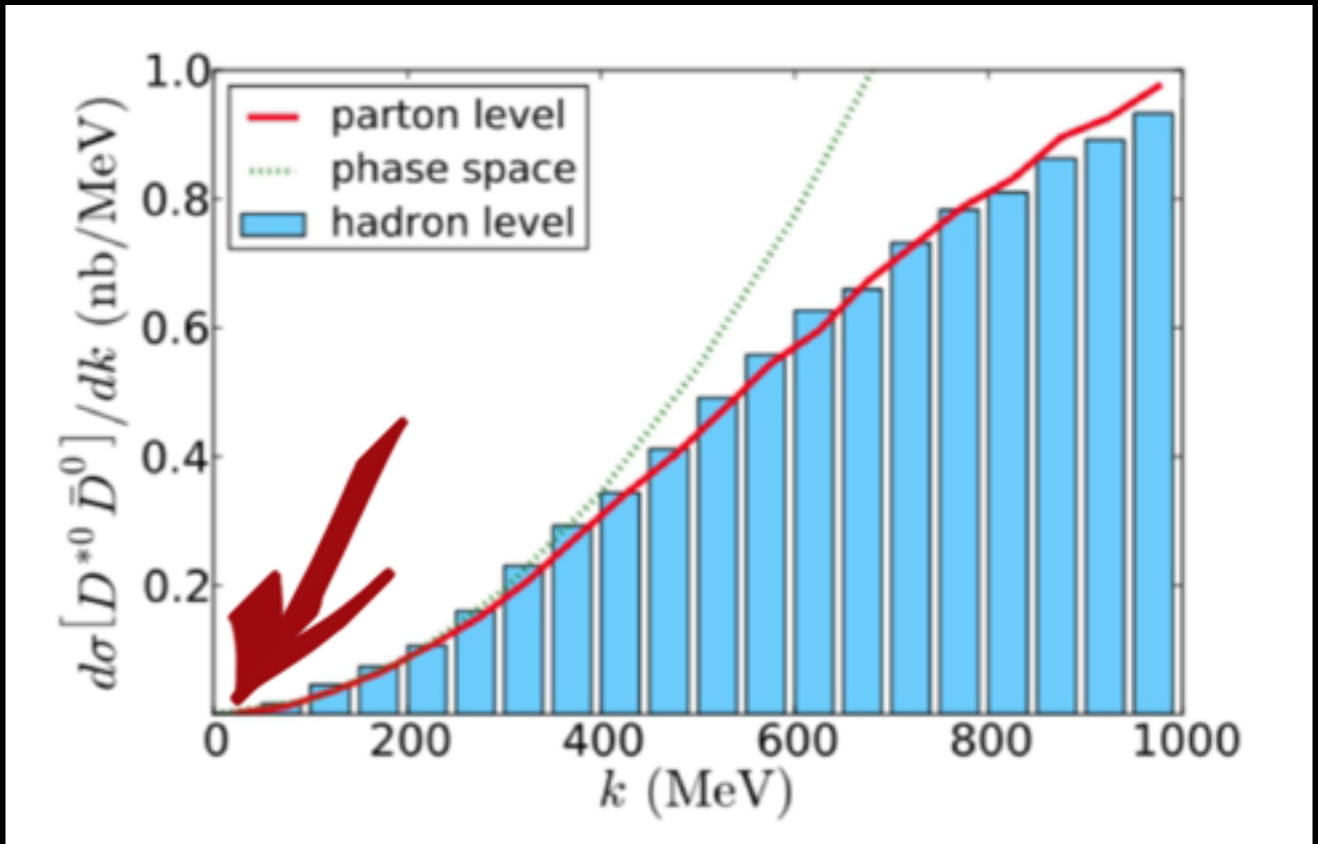
Used in the X context by Voloshin and by E. Braaten and M. Kusunoki, PRD69, 074005 (2004)

$$|\langle k^2\rangle_\Psi| = 2mB$$

$$|k| \simeq 14 \text{ MeV}$$

$X(3872)$ PROMPT PRODUCTION IN $pp(\bar{p})$ COLLISIONS

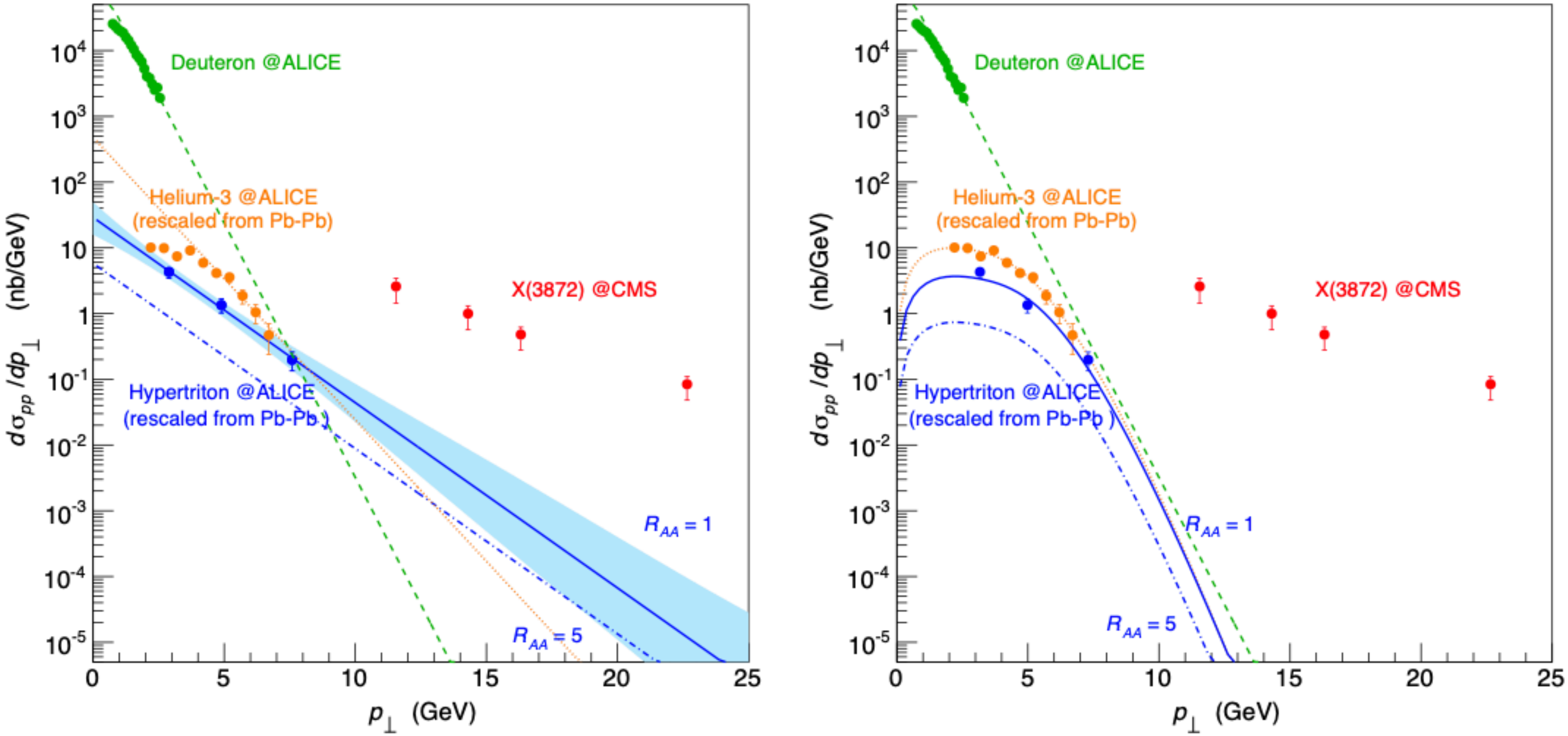
About 300 times smaller than observed (CDF), same in CMS, ATLAS.



Braaten and Artoisenet, PRD81103 (2010) 114018

Bignamini, Grinstein, Piccinini, ADP, Sabelli, PRL103 (2009) 162001

PROMPT PRODUCTION SUGGESTS A COMPACT CORE



Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, *Phys. Rev. D* 92 (2015) 3, 034028

Compact $c\bar{c}$ core suggested in Meng et al. Phys. Rev. D96, no.7, 074014 (2017)

A PARTIALLY COMPOSITE X?

Compact core $\mathfrak{X}$ + bound state $\mathfrak{B}$ $\quad |X\rangle = \sqrt{Z}\,|\mathfrak{X}\rangle + |\mathfrak{B}\rangle$

$$|\mathfrak{B}\rangle = \int C_{\boldsymbol{p}}\,\underbrace{|D\bar{D}^*(\boldsymbol{p})\rangle}_{|\alpha(\boldsymbol{p})\rangle}\,d^3p \equiv \int C_\alpha\,|\alpha\rangle\,d\alpha$$

$$H|\mathfrak{B}\rangle = (H_0 + V)|\mathfrak{B}\rangle = -B|\mathfrak{B}\rangle$$

$$H_0|\alpha\rangle = E(\alpha)|\alpha\rangle$$

$$\langle\alpha|\mathfrak{X}\rangle = 0 \Rightarrow \langle\mathfrak{X}|\mathfrak{B}\rangle = 0$$

Completeness relation $\quad 1 = |\mathfrak{X}\rangle\langle\mathfrak{X}| + \sum_\alpha |\alpha\rangle\langle\alpha|$

S. Weinberg Phys. Rev. 137, B672 (1965)

A PARTIALLY COMPOSITE X?

$$|X\rangle = \sqrt{Z}\,|\mathfrak{X}\rangle + |\mathfrak{B}\rangle$$

Impose ***X*** normalization $\langle X|X\rangle = 1$

$$1 = \langle X|\mathfrak{X}\rangle\langle\mathfrak{X}|X\rangle + \sum_\alpha \langle X|\alpha\rangle\langle\alpha|X\rangle = Z + \int |\langle\alpha|\mathfrak{B}\rangle|^2\, d\alpha$$

$$1 - Z = \int |\langle\alpha|\mathfrak{B}\rangle|^2\, d\alpha = \int \frac{|\langle\alpha|V|\mathfrak{B}\rangle|^2}{(E(\alpha)+B)^2} d\alpha$$

$$V = H_0 + V - H_0$$

S. Weinberg Phys. Rev. 137, B672 (1965)

A PARTIALLY COMPOSITE X?

$$|X\rangle = \sqrt{Z}\,|\mathfrak{X}\rangle + |\mathfrak{B}\rangle$$

$$1 - Z = g_Z^2 \int \frac{1}{(E(\alpha) + B)^2} d\alpha$$

$$g_Z^2 = |\langle D\bar{D}^* | V | \mathfrak{B}\rangle|^2 = \frac{2\pi}{m}\sqrt{\frac{2B}{m}}(1 - Z)$$

The coupling of the bound state $\mathfrak{B}$ to $D\bar{D}^*$, when the X is not a pure molecule but is partially composite. As for the compact component

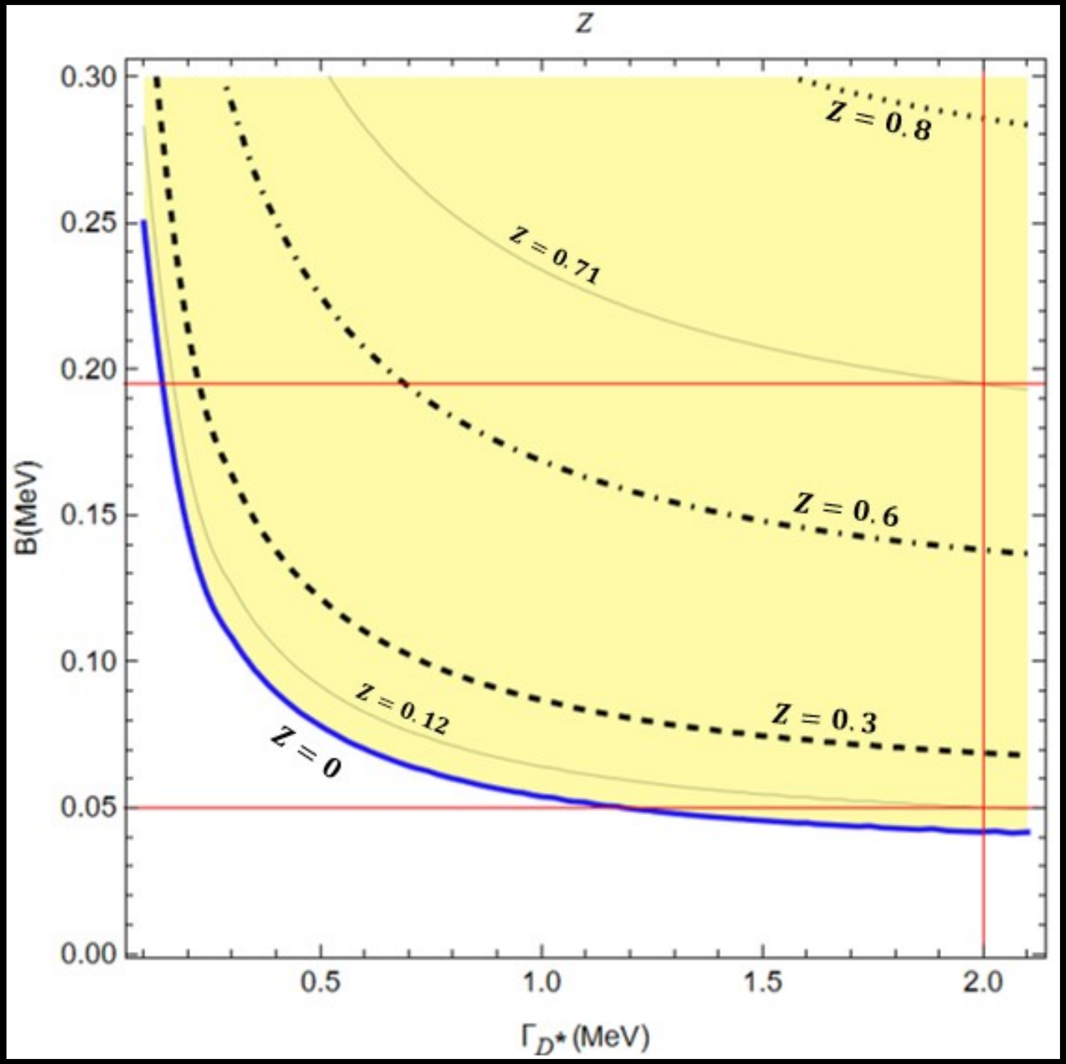
$$g_c = \langle \alpha | V | \mathfrak{X}\rangle$$

S. Weinberg Phys. Rev. 137, B672 (1965)

THE $X \to D\bar{D}\pi$ PARTIAL WIDTH — I

Let us consider the case (weak coupling of the compact component)

$$g_c \ll g_Z$$



D Germani & AD Polosa in preparation

HOW LARGE IS g_c?

The non-relativistic effective-field theory of X, D, D^* (neutral and charged) mesons with only the coupling $XD\bar{D}^*$ and **no** quartic couplings $(D\bar{D}^*)^2$ gives the formula for the $D\bar{D}^*$ scattering amplitude aka Flattè. Artoisenet, Braaten, Kang, PRD82, 014013 (2010)

In the absence of $(D\bar{D}^*)^2$, a bound state $\mathfrak{B} = D\bar{D}^*$ is not allowed. The Flattè scattering amplitude is related to the case in which the X is as elementary as D and $\bar{D}^*$ are. A compact X

$$f = -\frac{N}{E - m_F + i(g_{\rm LHCb}/2)\left(\sqrt{2mE} + \sqrt{2m_+(E-\Delta)}\right)}$$

D Germani & AD Polosa in preparation

HOW LARGE IS g_c?

$$f = -\frac{N}{E - m_F + i(g_{\rm LHCb}/2)\left(\sqrt{2mE} + \sqrt{2m_+(E-\Delta)}\right)}$$

The fact that LHCb shows a good fit to the $\boldsymbol{X}$ lineshape using $\boldsymbol{f}$

1) that the compact hypothesis fits well data
2) that the measured coupling is $\boldsymbol{g_c}$. More precisely we find

$$g_c = \frac{\pi}{m} g_{\rm LHCb}$$

$$g_c = (1.87 \pm 0.03) \times 10^{-2}\ \text{MeV}^{-1/2}$$

We will use this coupling to estimate the $\boldsymbol{X \to D\bar{D}\pi}$ width.

D Germani & AD Polosa in preparation

THE $X \to D\bar{D}\pi$ PARTIAL WIDTH — II

Let us consider the case (strong coupling of the compact component)

$$g_c \gg g_Z$$

Using g_c provided by LHCb we reproduce the PDG value

$$\Gamma(X \to D\bar{D}\pi) = (54 \pm 27)\ \text{KeV}$$

by requiring

$$\Gamma_{D^{*0}} = 105 \pm 70\ \text{KeV}$$

a value which agrees with experimental determinations for $\Gamma_{D^{*0}}$.
It looks like we are not missing any significant g_Z contribution in this calculation.

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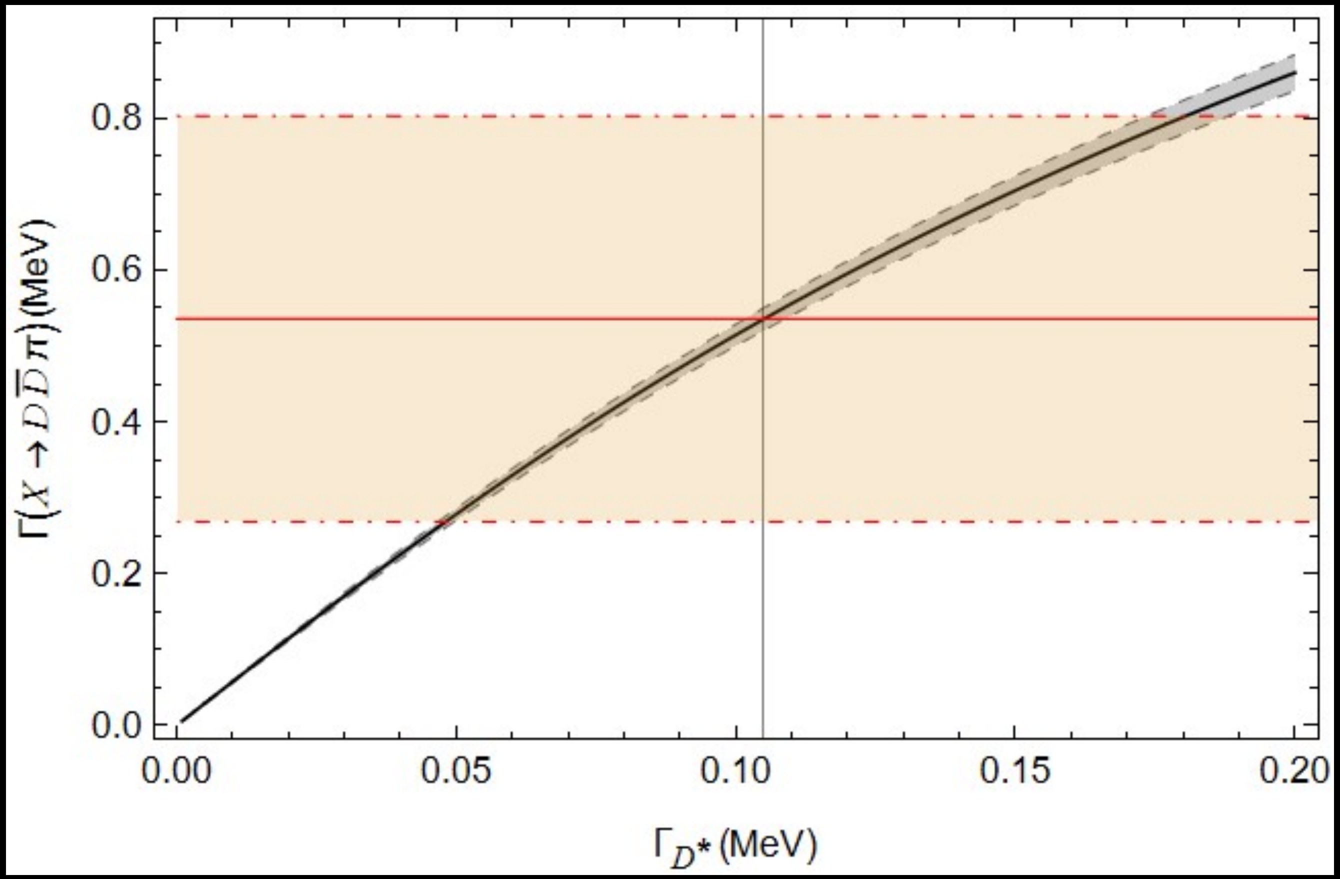
THE $X \to D\bar{D}\pi$ PARTIAL WIDTH — II

Let us consider the case (strong coupling of the compact component)

$$g_c \gg g_Z$$



D Germani & AD Polosa in preparation

THE *np* SCATTERING CASE

$$f(\alpha \to \beta) = \frac{1}{-1/a + \frac{1}{2} r_0 k^2 - ik}$$

(Bound state hypothesis!)

$$\left(-1/a + \frac{1}{2} r_0 k^2 - ik \right)_{k = i\sqrt{2mB}} = 0$$

$$f = \frac{1}{m(R_0 - r_0)} \frac{1}{E + B} \qquad (R_0 = \frac{1}{\sqrt{2mB}})$$

Assuming that $\boldsymbol{d}$ has both a bound state and an elementary component, the coupling of the bound state component to $\boldsymbol{np}$ is given by $\boldsymbol{g_Z}$ and the scattering amplitude, through the bound state, is

$$f = \frac{1}{mR_0} (1 - Z) \frac{1}{E + B}$$

THE *np* SCATTERING CASE

$$f = \frac{1}{m(R_0 - r_0)} \frac{1}{E + B}$$

$$\left(R_0 = \frac{1}{\sqrt{2mB}}\right)$$

$$f = \frac{1}{mR_0}(1 - Z)\frac{1}{E + B}$$

Comparing these two formulas we get the relation

$$r_0 = -\frac{Z}{1 - Z}R_0$$

extracted from the ***np*** scattering amplitude through the bound state deuteron assuming that the coupling of the bound state component gets reduced by $(1 - Z)$, due to the presence of an elementary component.

THE *np* SCATTERING CASE

$$f = \frac{1}{m(R_0 - r_0)} \frac{1}{E + B}$$

$$\left(R_0 = \frac{1}{\sqrt{2mB}}\right)$$

$$f = \frac{1}{mR_0}(1 - Z)\frac{1}{E + B}$$

Comparing these two formulas we get the relation

$$r_0 = -\frac{Z}{1 - Z} R_0$$

Can we really extract $\boldsymbol{Z}$, through this formula, in the case of the Flattè fit where only a compact component is included?

THE RADIATIVE DECAYS OF $X(3872)$

$$\mathcal{R} \equiv \frac{\mathcal{B}(X \to \gamma\,\psi(2S))}{\mathcal{B}(X \to \gamma\,\psi(1S))} \simeq 1.67 \pm 0.21 \quad \text{(LHCb)}$$

The phase space ratio $\Phi(2S)/\Phi(1S) \simeq 0.26$ would favor a small $\mathcal{R}$.

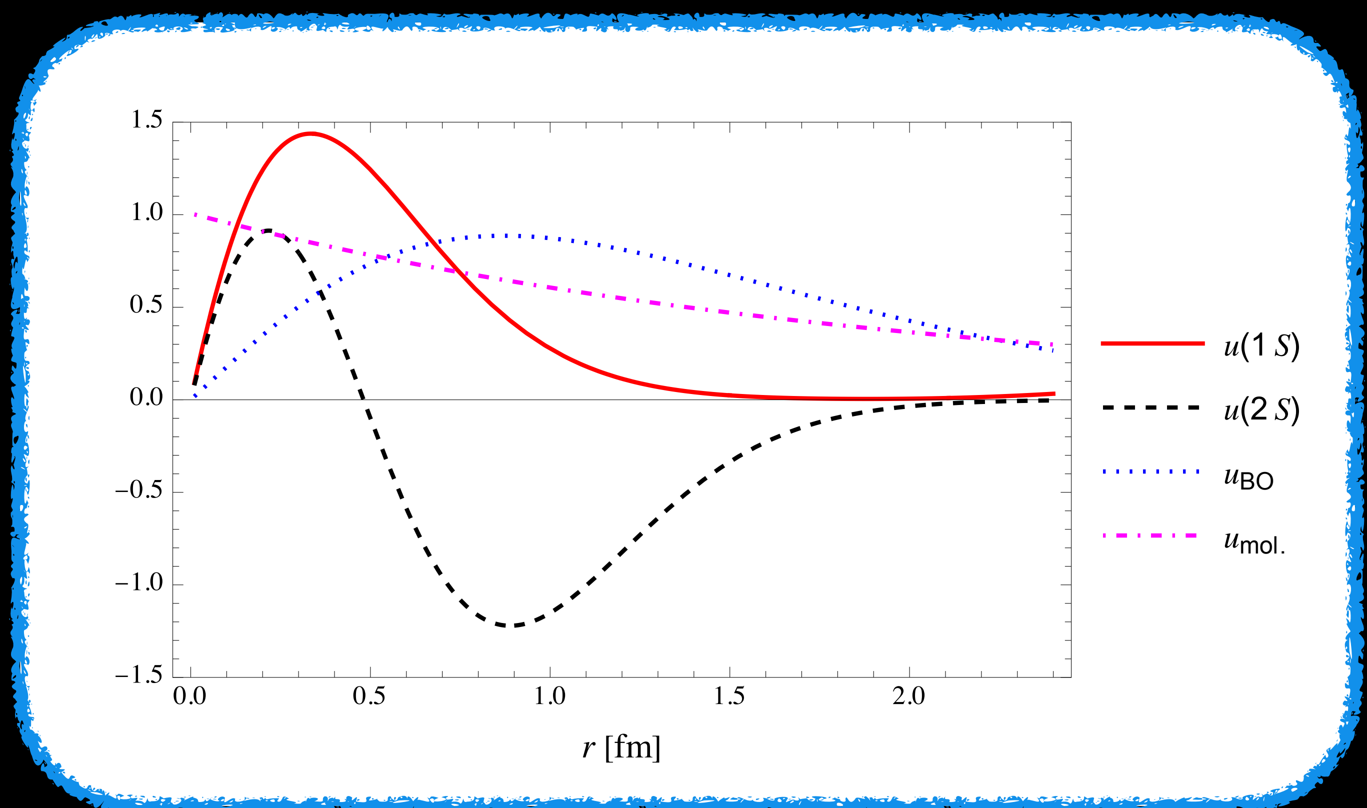
We distinguish between a **compact** $c\bar{c}q\bar{q}$ and a purely **molecular** $D\bar{D}^*$ interpretation, i.e. $Z = 0$.

We find that $\mathcal{R}$ predicted in the compact case, $\mathcal{R} \gtrsim 1$, is more than 30 times larger than that predicted for a molecule $\mathcal{R} \simeq 0.04$.

B. Grinstein, L. Maiani, A.D.P., Phys. Rev. D 109 (2024) 7, 074009

COMPARING COMPACT WF TO CHARMONIA

The universal wavefunction in the molecular picture is almost flat whereas the compact wf has a better overlap with $\psi(2S)$.

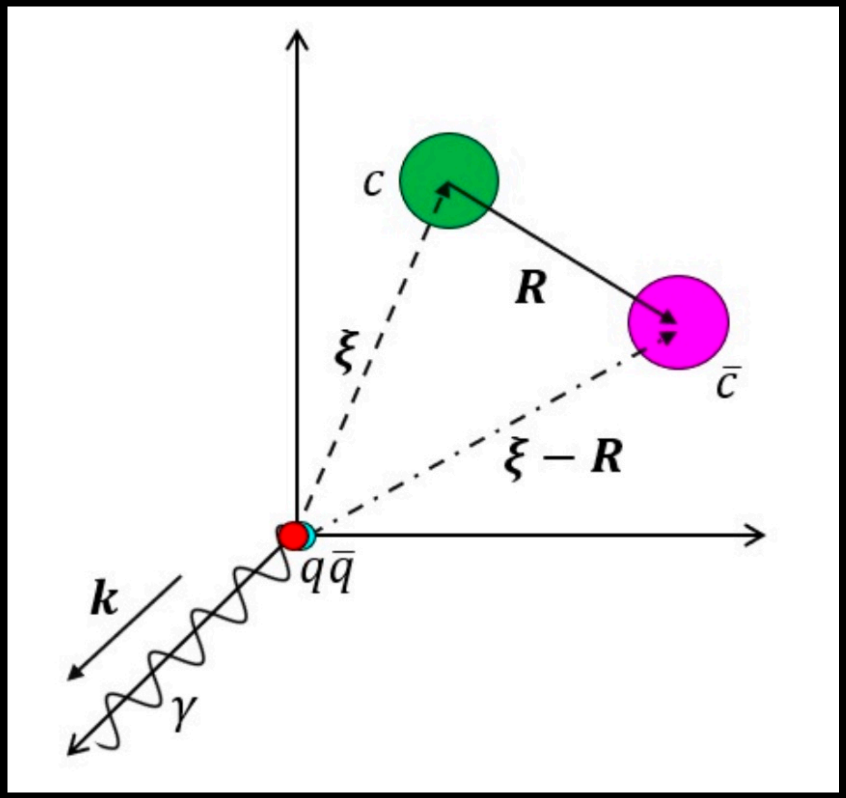


Reduced wavefunctions $u(r) = r\,R_{n0}(r)$.

B. Grinstein, L. Maiani, A.D.P., Phys. Rev. D 109 (2024) 7, 074009

RADIATIVE DECAY AMPLITUDE

$$A\left(X \to \psi^{(\prime)}\gamma\right) = \mathcal{F}\int_{\boldsymbol{R},\boldsymbol{\xi}} e^{-i\mathbf{k}\cdot\left(\boldsymbol{\xi}-\frac{\boldsymbol{R}}{2}\right)}\,\psi(|\boldsymbol{R}|)\Psi_{c\bar{c}}(|\boldsymbol{R}|)\Psi_{q\bar{q}}(|\boldsymbol{\xi}|,|\boldsymbol{\xi}-\boldsymbol{R}|)$$



B. Grinstein, L. Maiani, A.D.P., Phys. Rev. D 109 (2024) 7, 074009

RADIATIVE DECAY AMPLITUDE

$$A\left(X \to \psi^{(\prime)}\gamma\right) = \mathscr{F}\int_{\boldsymbol{R},\boldsymbol{\xi}} e^{-i\mathbf{k}\cdot\left(\boldsymbol{\xi}-\frac{\boldsymbol{R}}{2}\right)}\psi(|\boldsymbol{R}|)\Psi_{c\bar{c}}(|\boldsymbol{R}|)\Psi_{q\bar{q}}(|\boldsymbol{\xi}|,|\boldsymbol{\xi}-\boldsymbol{R}|)$$

$$\Psi_{q\bar{q}} = \chi(|\boldsymbol{\xi}|)\chi(|\boldsymbol{\xi}-\boldsymbol{R}|)$$

$$\Psi_{\rm BO}(|\boldsymbol{r}_c-\boldsymbol{r}_{\bar{c}}|)\chi_C(|\boldsymbol{r}_u-\boldsymbol{r}_c|)\chi_C(|\boldsymbol{r}_{\bar{u}}-\boldsymbol{r}_{\bar{c}}|)$$

$$\Psi_{\rm mol.}(|\boldsymbol{r}_c-\boldsymbol{r}_{\bar{c}}|)\chi_M(|\boldsymbol{r}_u-\boldsymbol{r}_{\bar{c}}|)\chi_M(|\boldsymbol{r}_{\bar{u}}-\boldsymbol{r}_c|)$$

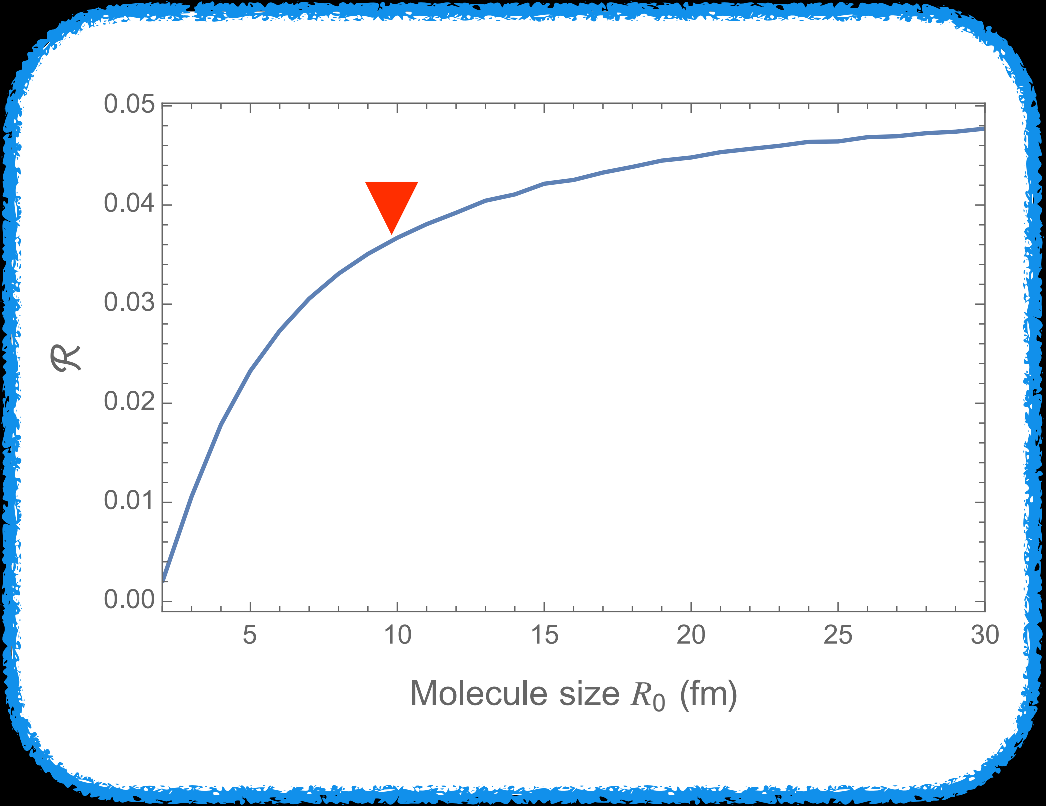
light quarks are far away

$$\mathscr{R} = \mathscr{P}\cdot\Phi\cdot\left|\frac{A\left(X\to\psi'\gamma\right)}{A\left(X\to\psi\gamma\right)}\right|^2 = 0.98\cdot 0.26\cdot\left|\frac{A\left(X\to\psi'\gamma\right)}{A\left(X\to\psi\gamma\right)}\right|^2$$

B. Grinstein, L. Maiani, A.D.P., Phys. Rev. D 109 (2024) 7, 074009

THE RADIATIVE DECAYS OF $X(3872)$

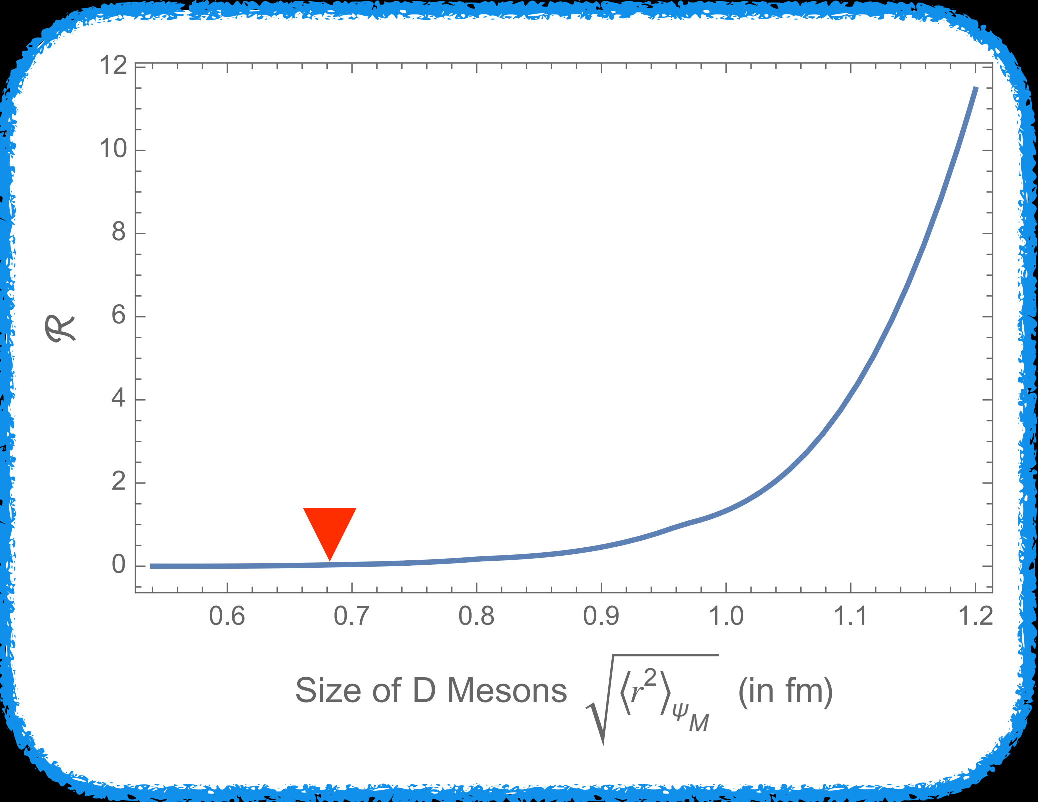
With the universal wave function for shallow bound states we find $\mathscr{R}_{\rm mol} \ll 1$



$$R_0 = 1/\sqrt{2mB}$$

B. Grinstein, L. Maiani, A.D.P., Phys. Rev. D 109 (2024) 7, 074009

To overturn the molecule situation consider that $\mathscr{R}$ is a rapidly increasing function of the size of $D^{(*)}$ (here Ψ-molecule is used).

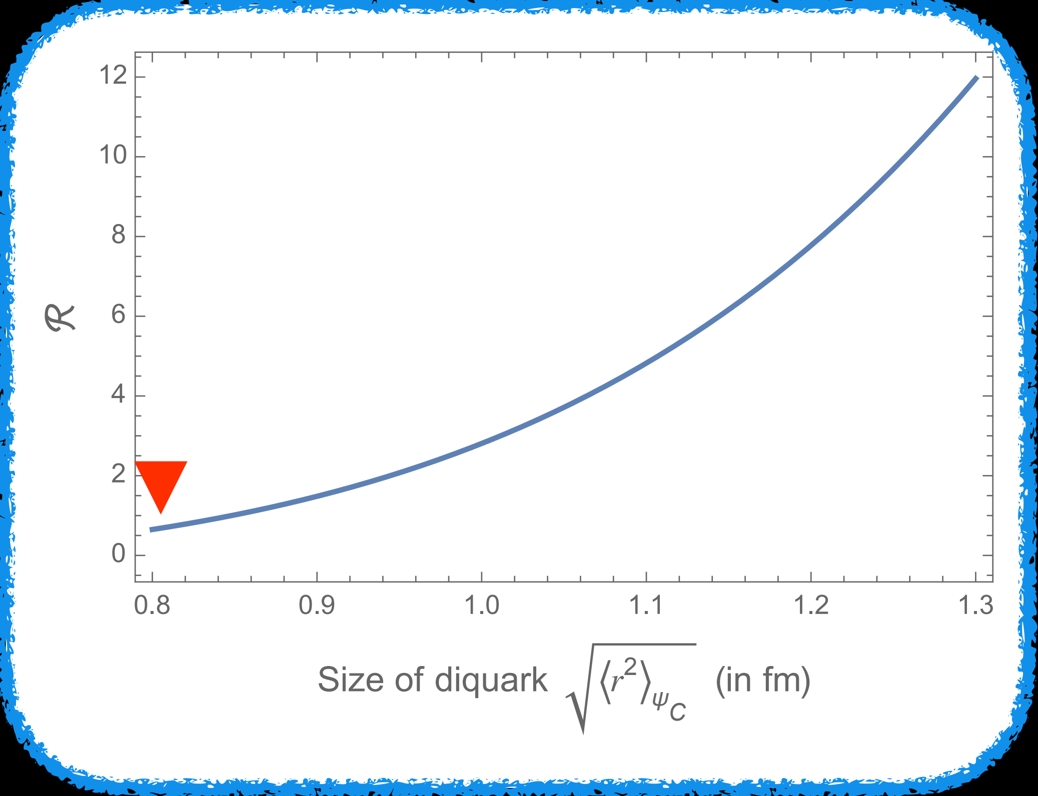


$$\chi_{\rm M}(r) = \frac{b^{3/2}}{\pi^{3/4}} e^{-\frac{1}{2}b^2 r^2}$$

B. Grinstein, L. Maiani, A.D.P., 2401.11623
Isgur, Scora, Grinstein, Wise (ISGW-model)

THE RADIATIVE DECAYS OF $X(3872)$

On the other hand the diquark in the compact tetraquarks tends to be larger than a D or a $\bar{D}^*$ meson since the binding force in the diquark is weaker! (here $\Psi_{\mathrm{BO}}(r)$ is used).



$$\chi_{\mathrm{c}}(\xi) = \sqrt{\frac{\mathscr{C}^3}{\pi}} e^{-\mathscr{C}\xi}$$

$$\mathscr{R}_{\min} = \frac{\mathscr{B}(X \to \psi'\gamma)}{\mathscr{B}(X \to \psi\gamma)} = 0.95^{+0.01}_{-0.07}$$

B. Grinstein, L. Maiani, A.D.P., Phys. Rev. D 109 (2024) 7, 074009

CORRECTIONS TO Ψ_{mol} DUE TO PION EXCHANGE

$$H = \frac{p^2}{2m} + V_s(\boldsymbol{R}) + V_w(\boldsymbol{R}) - i\frac{\Gamma}{2}$$

$$V_s(\boldsymbol{R}) = -\lambda_s \delta^3(\boldsymbol{R}) \qquad V_w(\boldsymbol{R}) = -\alpha \frac{e^{i\mu R}}{R}$$

Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

$$\mu = \sqrt{2m_\pi \delta} \simeq 43 \text{ MeV} \qquad \delta = m_{D^*} - m_D - m_\pi$$

$$\alpha = \frac{g^2\mu^2}{24\pi f_\pi^2} = 5 \times 10^{-4} \qquad f_\pi \simeq 132 \text{ MeV}$$

$$\Psi^{(1)}_{\text{mol.}} = \Psi_{\text{mol.}} + \delta_1\Psi_{\text{mol.}} = \Psi_{\text{mol.}} - \int \Psi_s \frac{\left(\Psi_s, V_w \Psi_{\text{mol.}}\right)}{B+E} \rho(E)\, dE$$

Negligible contribution!

$$\Psi_s = \frac{\sin(kR + \delta(k))}{kR} \qquad \text{with} \qquad k = \sqrt{2mE}$$

UPDATE ON BORN-OPPENHEIMER FOR TETRAQUARKS

B Grinstein, D Germani ADP, in preparation

THE $(c\bar{c})_{\mathbf{8}}$ CONFIGURATION: CHROMO-HYDROGEN

$$|(c\bar{c})_{\mathbf{8}}(q\bar{q})_{\mathbf{8}}\rangle = \sqrt{\frac{2}{3}}\,|(cq)_{\bar{\mathbf{3}}}(\bar{c}\bar{q})_{\mathbf{3}}\rangle - \sqrt{\frac{1}{3}}\,|(cq)_{\mathbf{6}}(\bar{c}\bar{q})_{\bar{\mathbf{6}}}\rangle \qquad (A)$$

$$|(c\bar{c})_{\mathbf{8}}(q\bar{q})_{\mathbf{8}}\rangle = \sqrt{\frac{8}{9}}\,|(c\bar{q})_{\mathbf{1}}(\bar{c}q)_{\mathbf{1}}\rangle - \sqrt{\frac{1}{9}}\,|(c\bar{q})_{\mathbf{8}}(\bar{c}q)_{\mathbf{8}}\rangle \qquad (B)$$

(A) diquark+antidiquark with a $\mathbf{6\bar{6}}$ component

(B) meson+antimeson witha a $\mathbf{88}$ component

(A), (B) corresponding to the two $R \to \infty$ configurations, where R is the distance between heavy quarks.

In the case of (A) there is a confining potential at large R

L. Maiani, A.D.P., V. Riquer, Phys. Rev. D 100 (2019) 1, 014002; Phys. Rev. D 100 (2019) 7, 074002

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In the one-gluon-exchange model, we can deduce the couplings

For example $\lambda_{cq} = \lambda_{\bar{c}\bar{q}} = \frac{2}{3}\frac{1}{2}(-\frac{4}{3}) + \frac{1}{3}\frac{1}{2}(\frac{10}{3} - \frac{8}{3}) = -\frac{1}{3}$

$$\lambda_{c\bar{c}} = \lambda_{q\bar{q}} = +\frac{1}{6}\alpha_s$$

$$\lambda_{cq} = \lambda_{\bar{c}\bar{q}} = -\frac{1}{3}\alpha_s$$

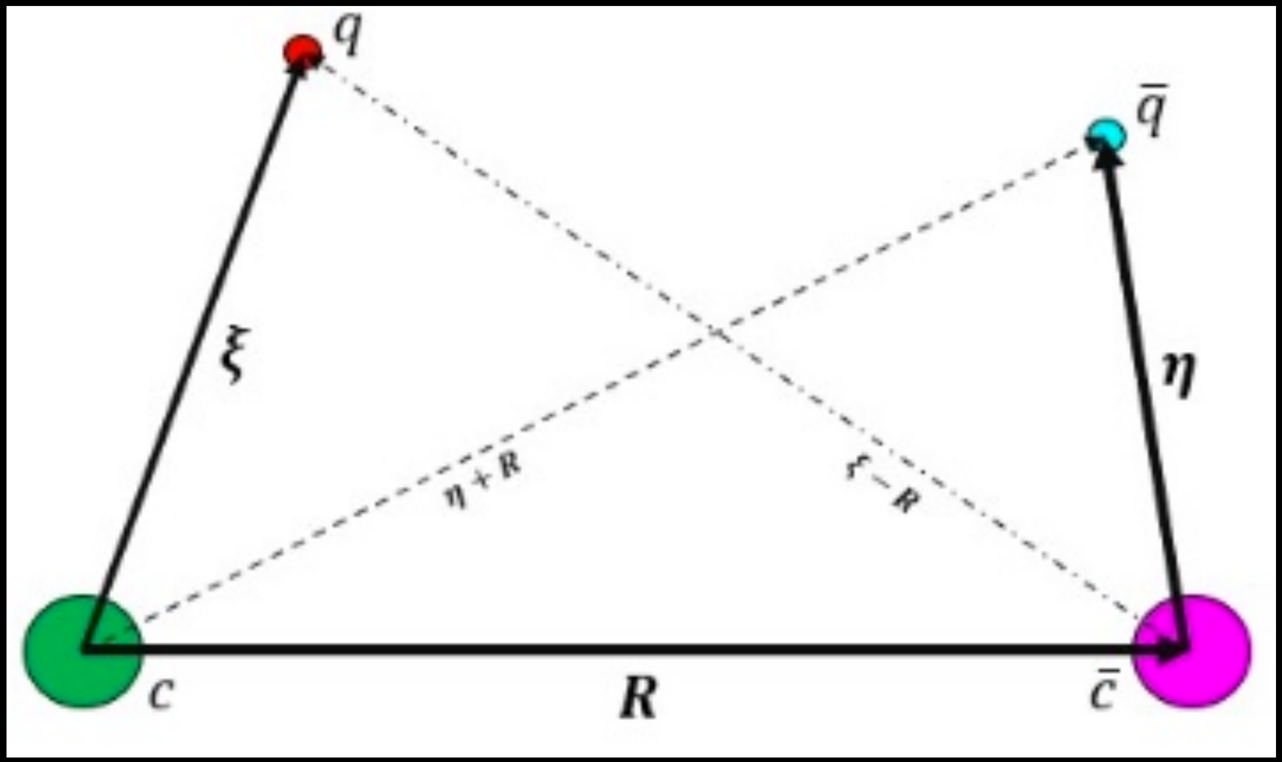
$$\lambda_{c\bar{q}} = \lambda_{\bar{c}q} = -\frac{7}{6}\alpha_s$$

L. Maiani, A.D.P., V. Riquer, Phys. Rev. D 100 (2019) 1, 014002; Phys. Rev. D 100 (2019) 7, 074002

ORBITALS

$$V_A(|\xi|) = -\frac{1}{3}\alpha_s\frac{1}{\xi} + k\,\xi \qquad (A)$$

$$V_B(|\boldsymbol{\xi} - \boldsymbol{R}|) = -\frac{7}{6}\alpha_s\frac{1}{|\boldsymbol{\xi} - \boldsymbol{R}|} + k\,|\boldsymbol{\xi} - \boldsymbol{R}| \qquad (B)$$

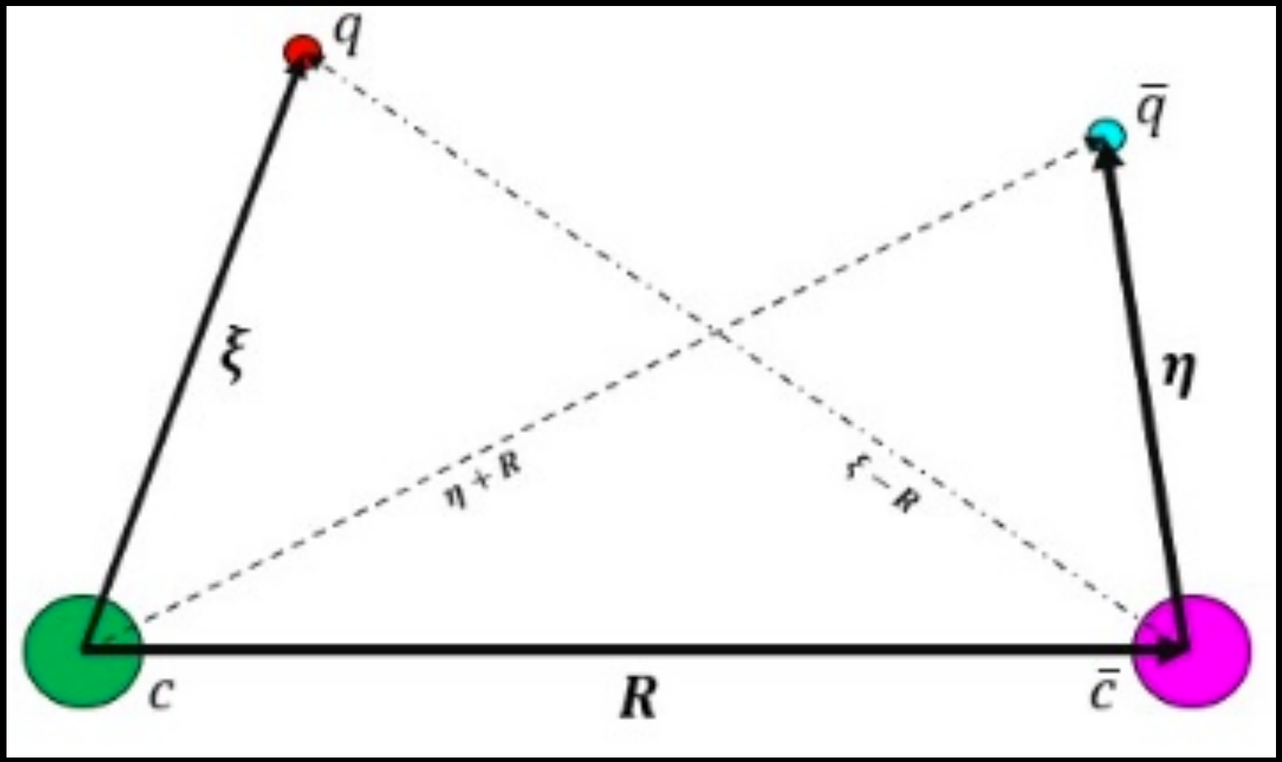


Variational test function $\psi_c(\xi) = \sqrt{\frac{\mathscr{C}^3}{\pi}}e^{-\mathscr{C}\xi}$ we will use $\mathscr{C} = A, B$

LCAO APPROXIMATION

$$\Psi_{q\bar{q}}(\boldsymbol{\xi},\boldsymbol{\eta}) = a_1\,\psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \;+\; a_2\,\psi_B(\boldsymbol{\xi}-\boldsymbol{R})\psi_B(\boldsymbol{\eta}+\boldsymbol{R})$$

Corresponding to the two possible $\boldsymbol{R} \to \infty$ configurations.



$$S_{AB}(R) = \int_{\xi} \psi_A(\boldsymbol{\xi})\psi_B(\boldsymbol{\xi}-\boldsymbol{R}) = \int_{\eta} \psi_A(\boldsymbol{\eta})\psi_B(\boldsymbol{\eta}+\boldsymbol{R})$$

B. Grinstein, L. Maiani, A.D.P., Phys. Rev. D 109 (2024) 7, 074009; $a_2 \neq 0$ **D. Germani**, ADP

TERMS IN THE MATRIX

The Hamiltonian $\boldsymbol{H}_{q\bar{q}}$ contains the kinetic terms and the potential

$$V_{q\bar{q}}(\boldsymbol{\xi},\boldsymbol{\eta};\boldsymbol{R}) = \underbrace{-\frac{1}{3}\alpha_s\left(\frac{1}{\xi}+\frac{1}{\eta}\right)}_{\Rightarrow A}\underbrace{-\frac{7}{6}\alpha_s\left(\frac{1}{|\boldsymbol{\xi}-\boldsymbol{R}|}+\frac{1}{|\boldsymbol{\eta}+\boldsymbol{R}|}\right)}_{\Rightarrow B}+$$

$$+\underbrace{\frac{1}{6}\alpha_s\frac{1}{|\boldsymbol{\xi}-\boldsymbol{\eta}-\boldsymbol{R}|}}_{q\bar{q}}$$

$$+V_{\text{conf}}+V_{q\bar{q}}^{\text{spin}}+V_0$$

$$V_{\text{conf}} = k|\boldsymbol{\xi}|+k|\boldsymbol{\eta}|+k|\boldsymbol{\xi}-\boldsymbol{R}|+k|\boldsymbol{\eta}+\boldsymbol{R}|$$

The BO potentials $\delta E(R)_\pm$ are obtained by requiring that the following matrix has *non zero determinant*. H_A has boundary condition at $R \to \infty$ given by (A); similarly for H_B.

$$\begin{pmatrix} H_A - \delta E(R) & H_{AB} - S^2_{AB}(R)\,\delta E(R) \\ H_{AB} - S^2_{AB}(R)\,\delta E(R) & H_B - \delta E(R) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0$$

(diagonal at $R \to \infty$)

$$H_A = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \rangle$$

$$H_B = \langle \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$

$$H_{AB} = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$

See e.g. Pauling & Wilson; L Covino, D Germani ADP, in preparation

MATRIX ENTRIES

$$H_A = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \rangle$$
$$H_B = \langle \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$
$$H_{AB} = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$

$$H_A(R) = 2\langle H \rangle^A_{\mathrm{min}} - 2\frac{7}{6}\alpha_s I_1^A(R) + \frac{1}{6}\alpha_s I_4^A(R) + V_A^{\mathrm{spin}}$$

where the expressions of the functions $\boldsymbol{I}$ are given by

$$I_1^A(R) = \int_{\xi} \psi_A^2(\boldsymbol{\xi}) \frac{1}{|\boldsymbol{\xi} - \boldsymbol{R}|} \int_{\eta} \psi_A^2(\boldsymbol{\eta}) = \int_{\eta} \psi_A^2(\boldsymbol{\eta}) \frac{1}{|\boldsymbol{\eta} + \boldsymbol{R}|} \int_{\xi} \psi_A^2(\boldsymbol{\xi}) = \int_{\xi} \psi_A^2(\boldsymbol{\xi}) \frac{1}{|\boldsymbol{\xi} - \boldsymbol{R}|}$$

$$I_4^A(R) = \int_{\xi,\eta} \psi_A^2(\boldsymbol{\xi})\psi_A^2(\boldsymbol{\eta}) \frac{1}{|\boldsymbol{\xi} - \boldsymbol{R} - \boldsymbol{\eta}|}$$

MATRIX ENTRIES

$$H_A = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta})\rangle$$

$$H_B = \langle \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R})\rangle$$

$$H_{AB} = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R})\rangle$$

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These integrals can be computed analytically

$$I_1^A(R) = \frac{1}{R} - A\,e^{-2AR}\left(1 + \frac{1}{AR}\right)$$

$$I_4^A(R) = A\left[\frac{1}{AR} - e^{-2AR}\left(\frac{1}{AR} + \frac{11}{8} + \frac{3}{4}AR + \frac{1}{6}A^2R^2\right)\right]$$

MATRIX ENTRIES

$$H_A = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta})\rangle$$
$$H_B = \langle \psi_B(\boldsymbol{\xi}-\boldsymbol{R})\psi_B(\boldsymbol{\eta}+\boldsymbol{R}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi}-\boldsymbol{R})\psi_B(\boldsymbol{\eta}+\boldsymbol{R})\rangle$$
$$H_{AB} = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi}-\boldsymbol{R})\psi_B(\boldsymbol{\eta}+\boldsymbol{R})\rangle$$

$$H_B(R) = 2\langle H\rangle^B_{\text{min}} - 2\frac{1}{3}\alpha_s I_1^B(R) + \frac{1}{6}\alpha_s I_4^B(R) + V_B^{\text{spin}}$$

These integrals can be computed analytically

$$I_1^A(R) = \frac{1}{R} - A\,e^{-2AR}\left(1 + \frac{1}{AR}\right)$$

$$I_4^A(R) = A\left[\frac{1}{AR} - e^{-2AR}\left(\frac{1}{AR} + \frac{11}{8} + \frac{3}{4}AR + \frac{1}{6}A^2R^2\right)\right]$$

MATRIX ENTRIES

$$H_A = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \rangle$$

$$H_B = \langle \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$

$$H_{AB} = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$

$$H_{AB}(R) = -2\frac{1}{3}\alpha_s S^{AB}(R)\, I_2^{AB}(R) + \frac{1}{6}\alpha_s I_6^{AB}(R) + V_{AB}^{\text{spin}}$$

where the expressions of the functions $\boldsymbol{I}$ are given by

$$I_2^{AB}(R) = \int_{\boldsymbol{\xi}} \psi_A(\boldsymbol{\xi})\psi_B(\boldsymbol{\xi} - \boldsymbol{R})\frac{1}{|\boldsymbol{\xi}|} = \int_{\boldsymbol{\eta}} \psi_A(\boldsymbol{\eta})\psi_B(\boldsymbol{\eta} + \boldsymbol{R})\frac{1}{|\boldsymbol{\eta}|}$$

$$I_6^{AB}(R) = \int_{\boldsymbol{\xi},\boldsymbol{\eta}} \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta})\frac{1}{|\boldsymbol{\xi} - \boldsymbol{\eta} - \boldsymbol{R}|}\psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R})$$

MATRIX ENTRIES

$$H_A = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \rangle$$

$$H_B = \langle \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$

$$H_{AB} = \langle \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta}) \,|\, H_{q\bar{q}} \,|\, \psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R}) \rangle$$

$$H_{AB}(R) = -\,2\frac{1}{3}\alpha_s\, S^{AB}(R)\, I_2^{AB}(R) + \frac{1}{6}\alpha_s I_6^{AB}(R) + V_{AB}^{\text{spin}}$$

one of the two can be computed analytically

$$I_2^{AB}(R) = 4\frac{\sqrt{A^3B^3}}{R}\left(\frac{R}{A^2 - B^2}e^{-BR} + \frac{2B}{(A^2 - B^2)^2}\left(e^{-AR} - e^{-BR}\right)\right)$$

$$I_6^{AB}(R) = \int_{\boldsymbol{\xi},\boldsymbol{\eta}} \psi_A(\boldsymbol{\xi})\psi_A(\boldsymbol{\eta})\frac{1}{|\,\boldsymbol{\xi} - \boldsymbol{\eta} - \boldsymbol{R}\,|}\psi_B(\boldsymbol{\xi} - \boldsymbol{R})\psi_B(\boldsymbol{\eta} + \boldsymbol{R})$$

L Covino, D Germani ADP, in preparation

V^{+} AND V^{-} BO POTENTIALS

Given the asymptotic behaviors

$$\Psi_{+}(\boldsymbol{\xi}, \boldsymbol{\eta}; \boldsymbol{R}) \xrightarrow[R \to +\infty]{} \psi_{A}(\boldsymbol{\xi})\psi_{A}(\boldsymbol{\eta}) \qquad (A)$$

$$\Psi_{-}(\boldsymbol{\xi}, \boldsymbol{\eta}; \boldsymbol{R}) \xrightarrow[R \to +\infty]{} \psi_{B}(\boldsymbol{\xi} - \boldsymbol{R})\psi_{B}(\boldsymbol{\eta} + \boldsymbol{R}) \qquad (B)$$

$$V_{\rm BO}^{-}(R) = \frac{1}{6}\alpha_s \frac{1}{R} + \delta E_{-}(R)$$

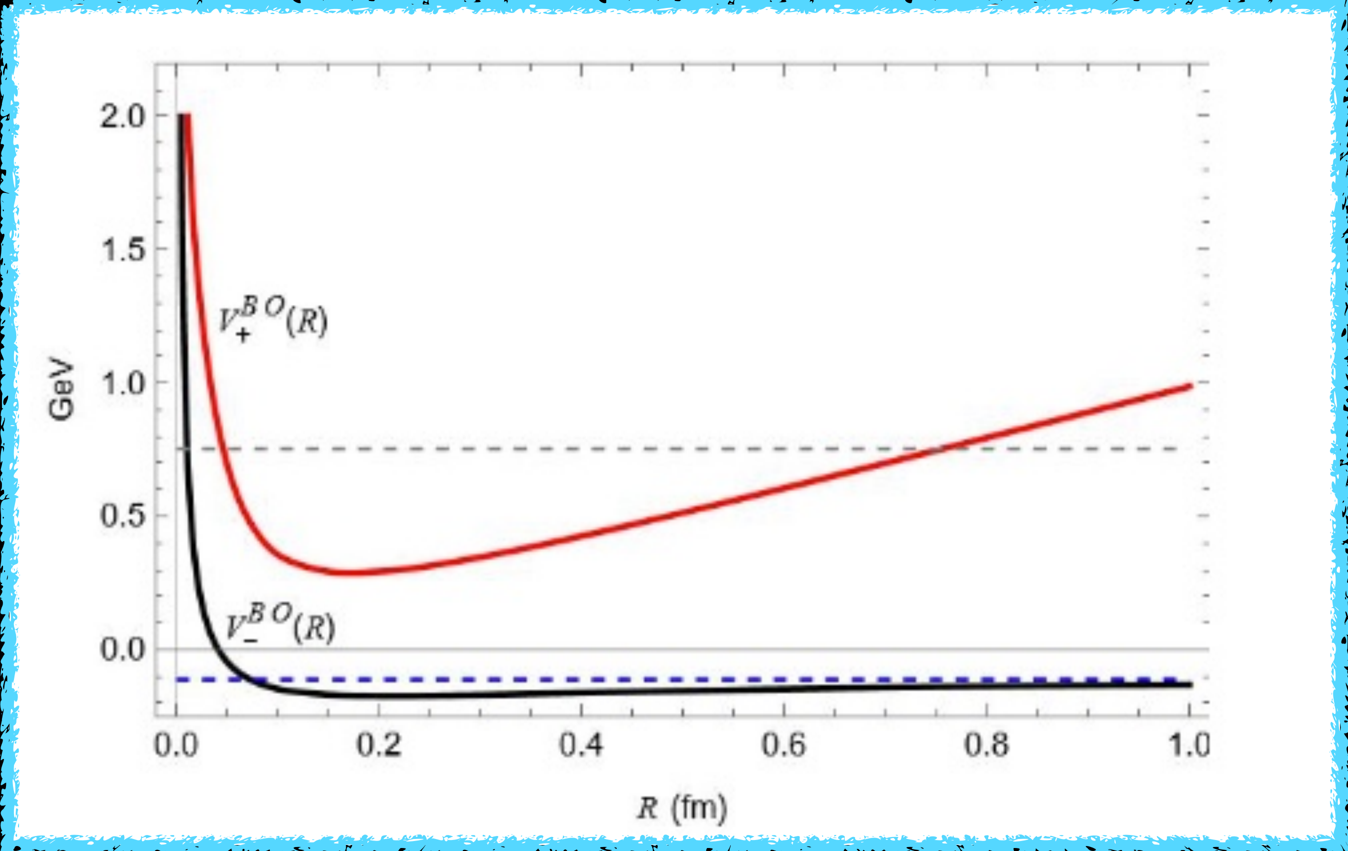
$$V_{\rm BO}^{+}(R) = \frac{1}{6}\alpha_s \frac{1}{R} + \delta E_{+}(R) + kR$$

We assume here that at large distance gluons screen the $\mathbf{8} - \mathbf{8}$ interaction

THE B.O. POTENTIALS

$$V_{\rm BO}^{-}(R) = \frac{1}{6}\alpha_s \frac{1}{R} + \delta E\_(R)$$

$$V_{\rm BO}^{+}(R) = \frac{1}{6}\alpha_s \frac{1}{R} + \delta E_{+}(R) + kR$$



~870 MeV higher

***X*(3872)**
broad w.f.

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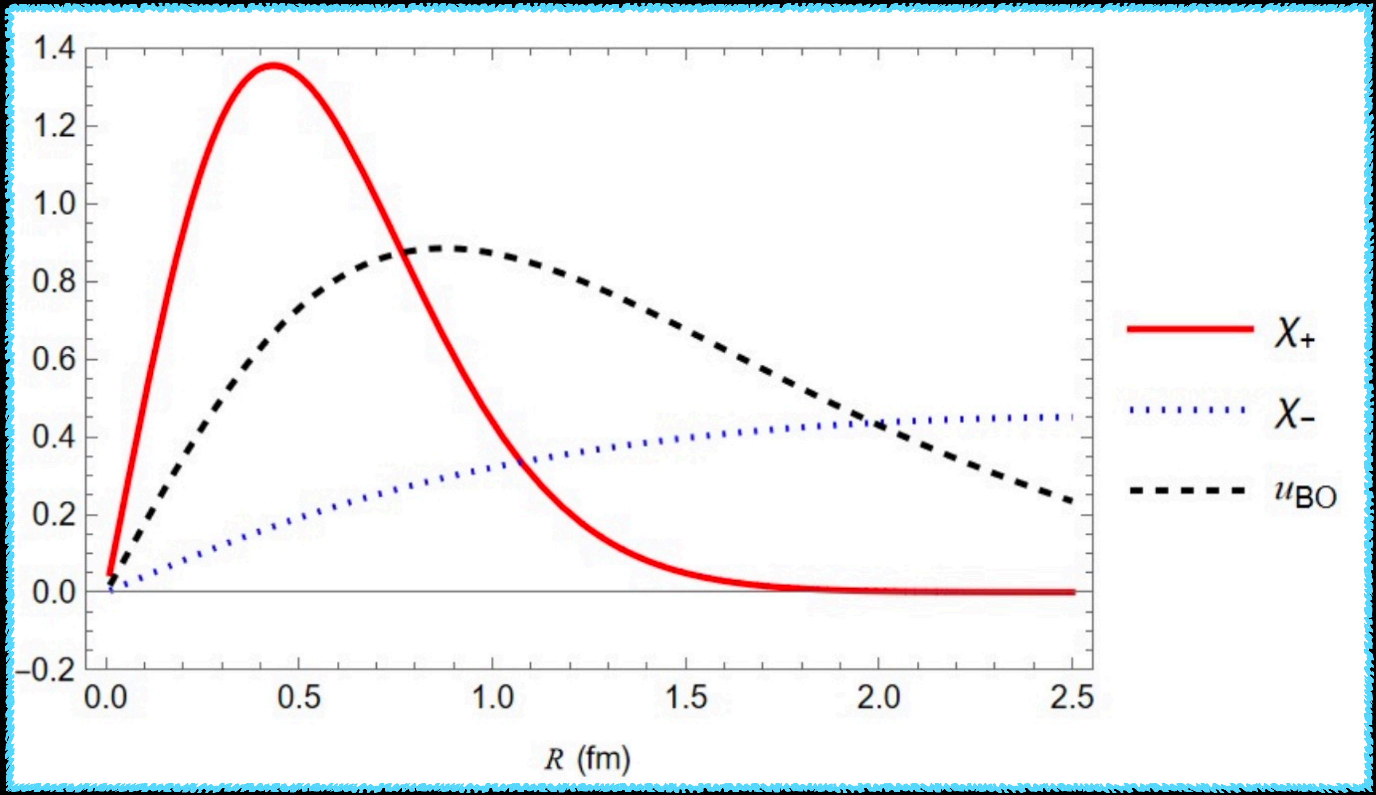
RESULTS WITH Ψ_-

The Ψ_- state is not so "compact", since $\langle R \rangle_{\Psi_-} \simeq 4$ fm. The higher one has instead a typical hadron size.



$$\mathscr{R}_- = 1.63$$

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CONCLUSIONS

- In the study of the lineshape of the $X(3872)$, LHCb tests the compact tetraquark hypothesis: the better the fit the more reliable is that hypothesis.
- Using the $g_{\rm LHCb}$ coupling (of the the compact X to $D\bar{D}^*$) one can compute $\Gamma(X \to D\bar{D}\pi)$ requiring a width of the D^{*0} in agreement with data.
- The hypothesis of a *purely molecular* $X(3872)$ does not match with radiative decays in ψ' and ψ.
- The effect of pion exchange in deforming $\Psi_{\rm mol}$ is negligible.
- We are very far from claiming that XYZ spectroscopy is all about shallow bound states.

BACKUP

POLARIZATIONS

within $\mathcal{F}$ the polarizations

$$\mathcal{S}_{\psi^{(\prime)}} = \sum_{\text{pols}} |\epsilon(\boldsymbol{e}^*_{(\psi^{(\prime)})}, \boldsymbol{e}^*_{(\gamma)}, \boldsymbol{e}_{(X)})|^2 = \epsilon_{ijk}\epsilon_{i'j'k}(\delta_{ii'} + \frac{k_i k_{i'}}{M^2_{\psi^{(\prime)}}})(\delta_{jj'} - \frac{k_j k_{j'}}{\boldsymbol{k}^2}) = 4 + 2\frac{\boldsymbol{k}^2}{M^2_{\psi^{(\prime)}}}$$

$$\mathcal{S}_{\psi'}/\mathcal{S}_{\psi} = 0.98$$

$$|\boldsymbol{k}| = \frac{(M_X^2 - M^2_{\psi^{(\prime)}})}{2M_X}$$

SOFT GLUONS

$$w^{i_1\cdots i_n}_{j_1\cdots j_m} \quad (n>m) \longrightarrow w^{i_1\cdots i_n}_{j_1\cdots j_m} A^{j_m}_{i_m} A^{j_{m-1}}_{i_{n-1}} \cdots \longrightarrow w^{i_1\cdots i_{n-m}}$$

If $\boldsymbol{n-m}$ is *odd* we can further saturate with soft gluons as in

$$w^{i_1\cdots i_{n-m}} \longrightarrow w^{i_1\cdots i_{n-m}} A^{rs}_{i_1 i_2}\epsilon_{rsi_3}\cdots \qquad (\in \overline{\mathbf{10}})$$

reducing to a $\bar{3}$ – lower index $\boldsymbol{i_3}$. So we conclude that

$$\bar{\mathbf{6}}\otimes\mathbf{6}\to\mathbf{3}\otimes\bar{\mathbf{3}}$$
$$\mathbf{8}\otimes\mathbf{8}\to\mathbf{1}\otimes\mathbf{1}$$

RECOIL FACTOR

$$\psi(\boldsymbol{x}) = (\Phi_{\boldsymbol{x}}, \Psi)$$

The final $c\bar{c}$ recoils against the photon with (non-relativistic) velocity $V = k/2M$, since $|\boldsymbol{k}| = (M_X^2 - M_{\psi^{(\prime)}}^2)/2M_X$.

$$\psi \to (\Phi_{\boldsymbol{x}}, e^{-iK\cdot V}\Psi) = \int_{\boldsymbol{p}} (\Phi_{\boldsymbol{x}}, \chi_{\boldsymbol{p}})(\chi_{\boldsymbol{p}}, e^{-iK\cdot V}\Psi) = \int_{\boldsymbol{p}} (\Phi_{\boldsymbol{x}}, \chi_{\boldsymbol{p}})(\chi_{\boldsymbol{p}-2MV}, \Psi) =$$

$$= \int_{\boldsymbol{p}} (\Phi_{\boldsymbol{x}}, \chi_{\boldsymbol{p}+2MV})(\chi_{\boldsymbol{p}}, \Psi) = e^{ik\cdot \boldsymbol{x}}\,\psi$$

Refer $\boldsymbol{x}$ to $\boldsymbol{x}_{\rm com} = \dfrac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2}$ and $\boldsymbol{\eta} = \boldsymbol{\xi} - \boldsymbol{R}$

B. Grinstein, L. Maiani, A.D.P., 2401.11623

DATA ON X: LHCB ANALYSIS

arXiv:2005.13419

For small kinetic energies – Flatté parametrization for the coupled channel decay $X \to D^0 \bar{D}^{*0}$ and $X \to D^+ \bar{D}^{*-}$.

$$f(X \to J/\psi\pi\pi) = -\frac{N}{(E - m_X^0) + \frac{i}{2} g(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)}) + \frac{i}{2}(\Gamma_\rho + \Gamma_\omega + \ldots)}$$

$\delta = (m_{D^{*-}} + m_{D^+}) - (m_{\bar{D}^{*0}} + m_{D^0}) =$ Isospin splitting

$$E = m_{J/\psi\pi\pi} - m_D - m_{\bar{D}*}$$

$\mu_+, \mu =$ reduced mass of the charged/neutal $D\bar{D}*$ pair

$m_X^0 =$ a bare parameter for the mass of the X (stable determination)

If the fit to the Flatté lineshape works, it means only that the compact hypothesis for the X works!

A consistent r_0 should be negative here!

DATA ON X: LHCB ANALYSIS

arXiv:2005.13419

The single channel analysis is obtained by $\Gamma_V, \ldots = 0$

$$f(X \to J/\psi\pi\pi) = -\frac{(2N/g)}{(2/g)(E - m_X^0) - \sqrt{2\mu_+\delta} + E\sqrt{\mu_+/2\delta} + ik}$$

$$-\frac{1}{a} = \frac{2m_X^0}{g} + \sqrt{2\mu_+\delta} \simeq -6.92 \text{ MeV} \quad \text{positive } a$$

$$r_0 = -\frac{2}{\mu g} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm} \quad \text{negative } r_0$$

using $E = k^2/2\mu$, μ being the reduced mass of the neutral $D\bar{D}^*$ pair, and taking g (LHCb) and m_X^0 (stable determination) from the experimental analysis. Since g can be larger, $r_0 \leq -2$ fm.

DETERMINATION OF Z

Neglect for the moment $O(1/\Lambda)$ corrections

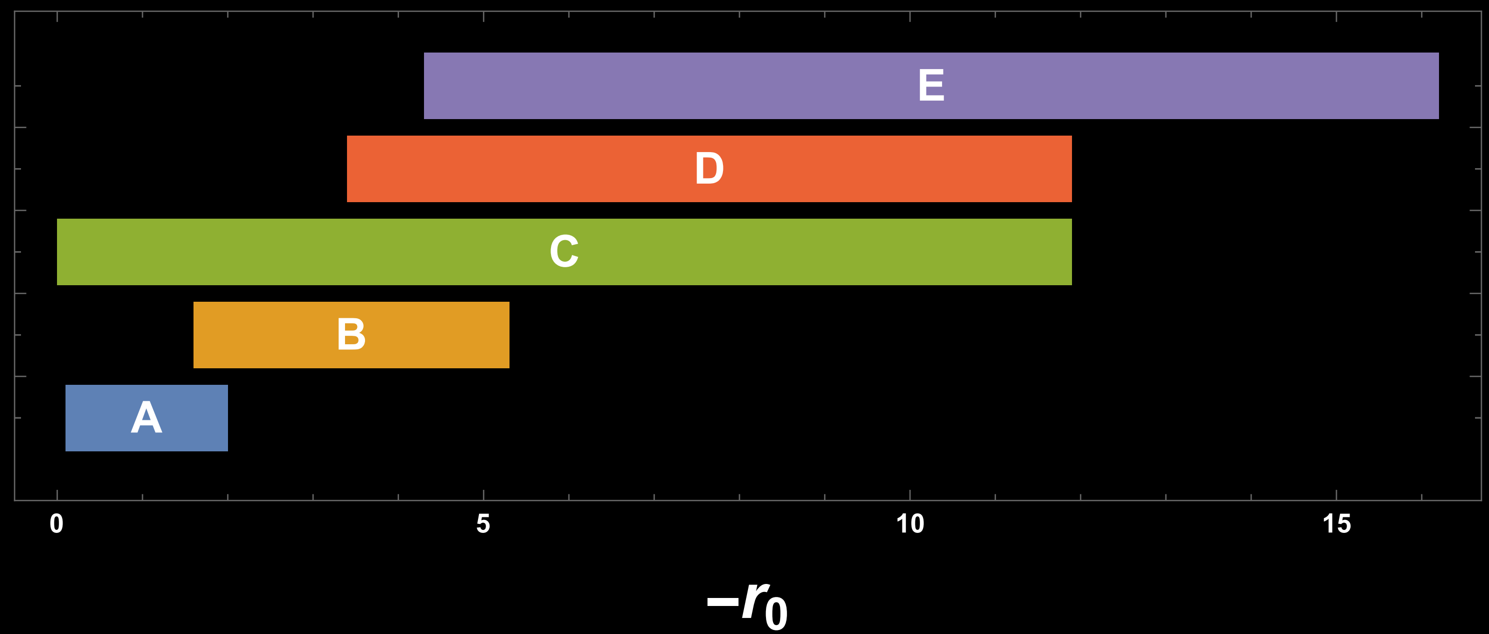
$$r_0 = -\frac{Z}{1-Z}R_0 = -5.34 \text{ fm}$$
$$a = \frac{2(1-Z)}{2-Z}R_0 = 197/6.92 \text{ fm}$$

Gives $Z = 0.15 \neq 0!$ and $B = 20$ keV

Including ± 5 fm makes quite a difference depending on the sign.
In the case of -5 fm we might have $Z = 0$ even with $r_0^{\text{exp}} = -5.32$ fm!

However we shall see that in the molecular case $O(1/\Lambda) \to -0.2$ MeV

$(-r_0)$ ACCORDING TO SOME ESTIMATES



A: Baru et al., 2110.07484

B: Esposito et al., 2108.11413

C: LHCb, 2109.01056

D: Maiani & Pilloni GGI-Lects

E: Mikhasenko $T_{cc'}$, 2203.04622

H. Xu, N. Yu and Z. Zhang 2401.00411: $r_0 \approx -14$ fm combining LHCb and Belle data (for the X).

r_0 FROM LATTICE

M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 2202.10110

Applying the lattice Lüscher method, the authors study the $D\bar{D}^*$ scattering amplitude and make a determination of the scattering length and of the effective range for $\mathcal{T}_{cc}$

$$a = -1.04(29)\ \text{fm}$$
$$r_0 = +0.96^{+0.18}_{-0.20}\ \text{fm}$$

The mass of the pion is $m_\pi = 280$ MeV, to keep the D^* stable. This result, for the moment, is compatible with a *virtual state* because of the negative a – like the singlet deuteron. As for LHCb (2109.01056 p.12)

$$a = +7.16\ \text{fm}$$
$$-11.9 \leq r_0 \leq 0\ \text{fm}$$