# Metamaterials for impedance optimization and sustainability

C. Zannini, L. Sito

# Overview

## Introduction

## Metamaterial insertions for lossless wave propagation

- Analytical model
- CST simulations
- Proof of concept

## Experimental progress and challenges

## Conclusions

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# Metamaterials

μετά = "to go beyond"

"Structures and composite materials that either mimic known material responses or qualitatively have new, physically realizable response functions that do not occur or may not be readily available in nature."

Nader Engheta & Richard W. Ziolkowski
"Metamaterials: Physics, Engineering and Explorations"
IEEE & Wiley Interscience Press, 2006

# Material characteristics

$\mu$

ENG Materials: artificially realizable
(& plasmas)

$\boldsymbol{\varepsilon < 0, \mu > 0}$

$\boldsymbol{\varepsilon > 0, \mu > 0}$

$\varepsilon$

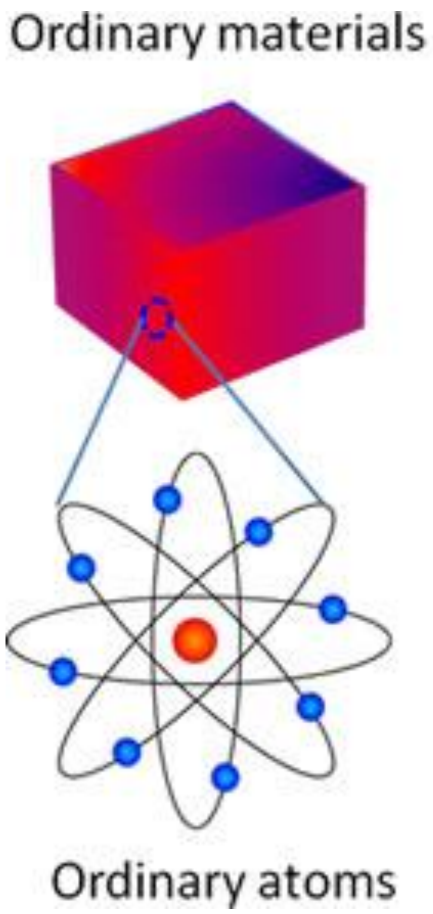
DNG Materials: artificially realizable
(not found in nature)

$\boldsymbol{\varepsilon < 0, \mu < 0}$

MNG Materials: artificially realizable
(not found in nature)

$\boldsymbol{\varepsilon > 0, \mu < 0}$

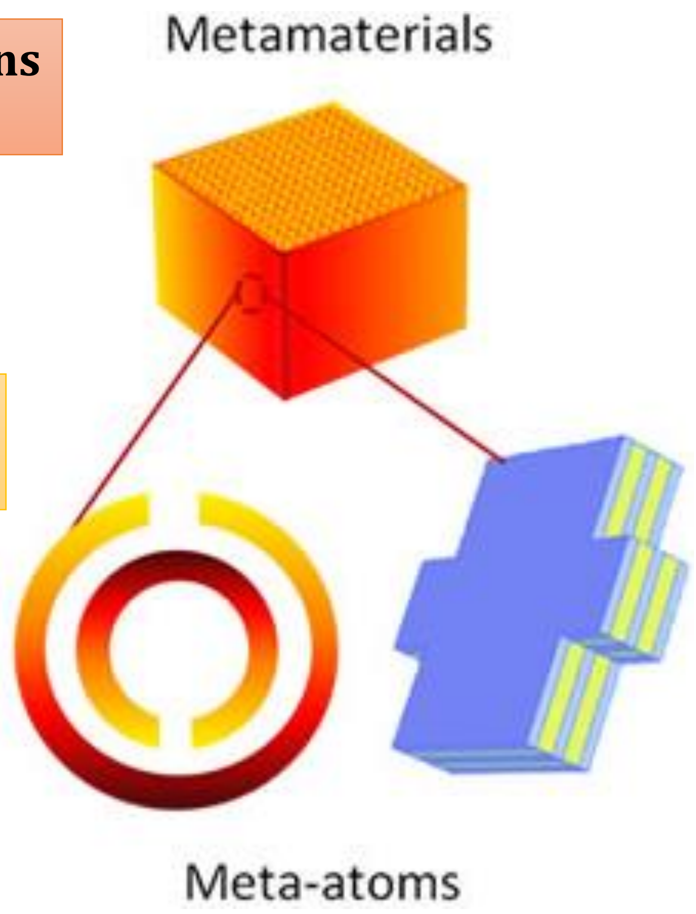
# How to produce a metamaterial?



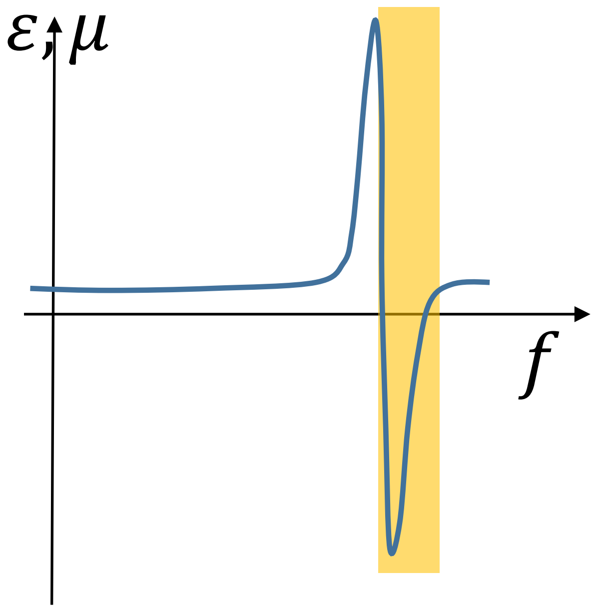

Atoms, their organization in domains and polarizations give the electromagnetic properties.

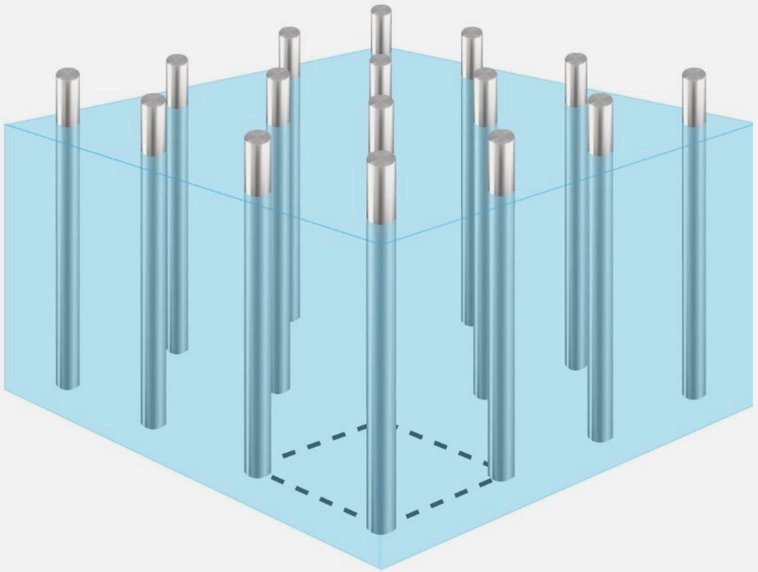
"Engineer" the material such that at certain frequencies (wavelengths) it is seen as made of "meta-atoms".

The "meta-atoms" can be engineered so as to give desired electromagnetic properties.



# ENG or **MNG** metamaterial

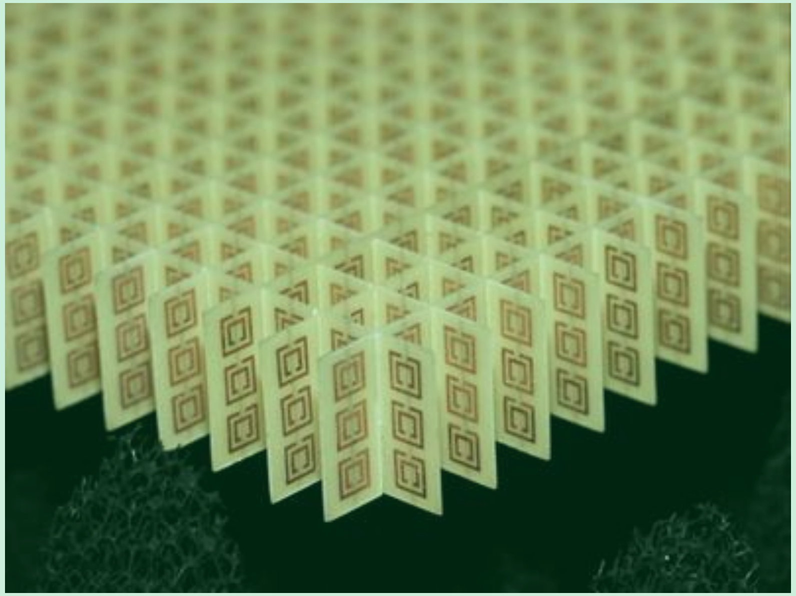


Conductive inclusions in low-ε substrates

Anisotropic material

Equivalent ε depends on concentration, $\varepsilon_{sub}, \sigma_{incl}$, orientation. ε will be a tensor.



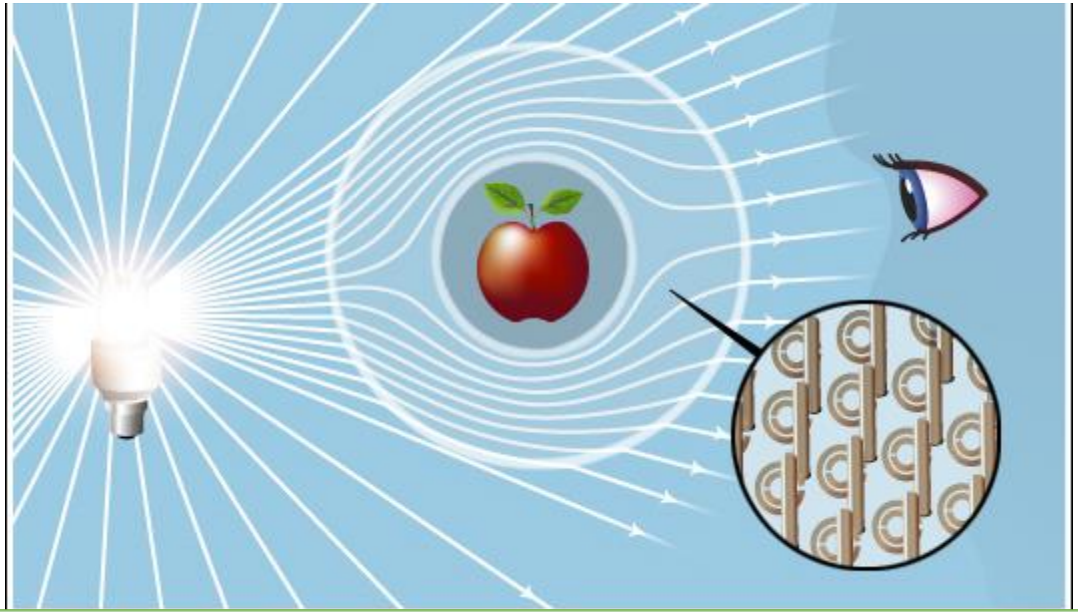
Conductive inclusions in high-ε substrates

Anisotropic material

Equivalent μ depends on concentration, $\varepsilon_{sub}, \sigma_{incl}$, orientation. μ will be a tensor.

# Metamaterials for impedance mitigation and sustainability

Using optical metamaterials one could make surrounding objects invisible to light



Metamaterials are masking the object and changing its properties

Metamaterials are artificially structured materials that allow to engineer the interaction of fields with matter.

**We are exploring the potential of electromagnetic metamaterial to mask a material and enabling lossless wave propagation**

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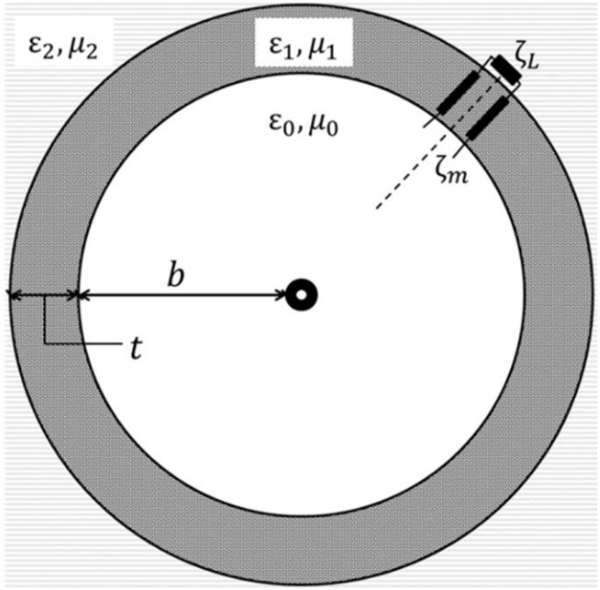
# Analytical model: the metaconductive transition




$$\zeta_L = \frac{1+j}{\sigma_2 \delta}$$

$$\zeta_1 = \sqrt{\mu_1/\varepsilon_1}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$|\varepsilon_1 \mu_1| \gg \varepsilon_0 \mu_0$$

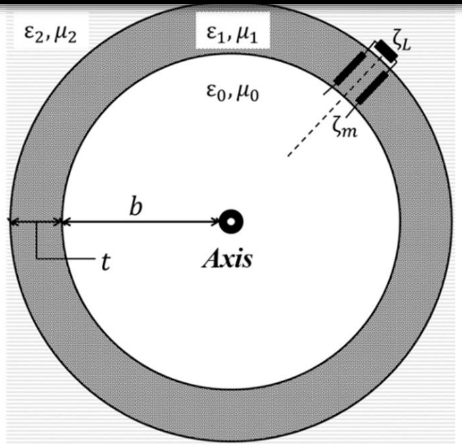
$$\delta \ll b$$

$$t \ll b$$

Surface impedance seen by the propagating wave

$$\zeta_m = \zeta_1 \cdot \frac{\zeta_L + j\zeta_1 \tan k_1 t}{\zeta_1 + j\zeta_L \tan k_1 t}$$

# Analytical model: the metaconductive transition




$$\zeta_m = \zeta_1 \cdot \frac{\zeta_L + j\zeta_1 \tan k_1 t}{\zeta_1 + j\zeta_L \tan k_1 t}$$

$$k_1 t = \beta - j\alpha$$

$$\zeta_L = a + ja$$
$$\zeta_1 = b' + jb''$$

$$\tan(\beta - j\alpha) = \frac{\tan\beta - j\tanh\alpha}{1 + j\tan\beta\tanh\alpha}$$

$$Re\{\zeta_m\} = \frac{(b'^2 + b''^2)\cdot N_1}{D} + \frac{(b'^2 - b''^2 + 2b'b'')\cdot N_2}{D} + \frac{(b'^3 + b'b''^2)\cdot N_3}{D} + \frac{(b''^3 + b'^2 b'')\cdot N_4}{D} + \frac{N_5}{D}$$

# Analytical model: the metaconductive transition

$$Re\{\zeta_m\} = \frac{\left(b'^2 + b''^2\right)\cdot N_1}{D} + \frac{\left(b'^2 - b''^2 + 2b'b''\right)\cdot N_2}{D} + \frac{\left(b'^3 + b'b''^2\right)\cdot N_3}{D} + \frac{\left(b''^3 + b'^2b''\right)\cdot N_4}{D} + \frac{N_5}{D}$$

$$D = \left|b' - b''\tan\beta\tanh\alpha - a(\tan\beta - \tanh\alpha) + j\left[a(\tan\beta + \tanh\alpha) + b'\tan\beta\tanh\alpha + b''\right]\right|^2$$

$$N_1 = a\left[1 + (\tan\beta)^2(\tanh\alpha)^2\right] + b'\tanh\alpha - b''\tan\beta$$

$$N_2 = a\left[(\tan\beta)^2 + (\tanh\alpha)^2\right]$$

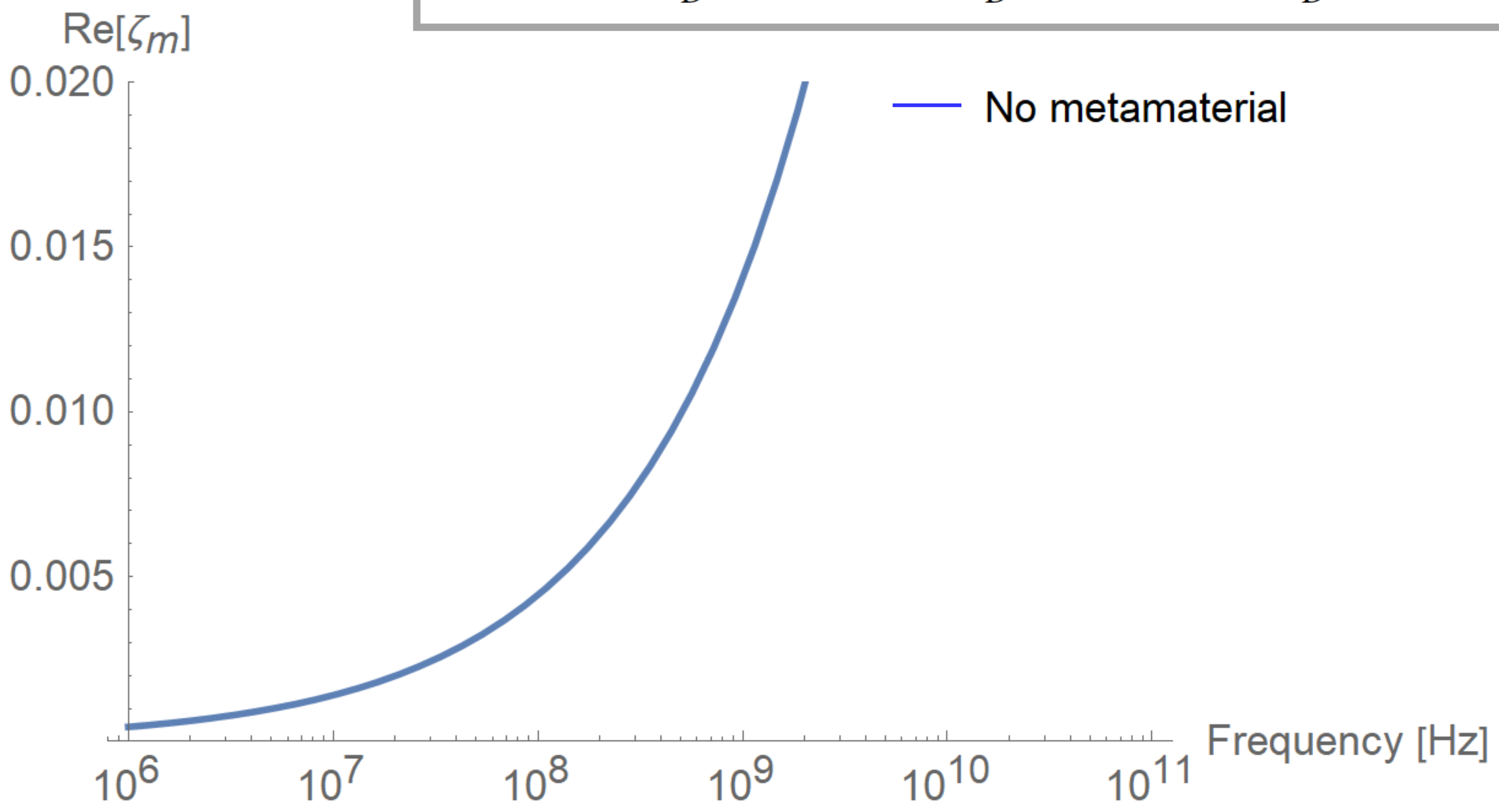
$$N_3 = \tanh\alpha(\tan\beta)^2$$

$$N_4 = \tan\beta(\tanh\alpha)^2$$

$$N_5 = 2a^2b'\tanh\alpha\left[1 + (\tan\beta)^2\right] + 2a^2b''\tan\beta\left[1 - (\tanh\alpha)^2\right]$$

# Analytical model: the metaconductive transition

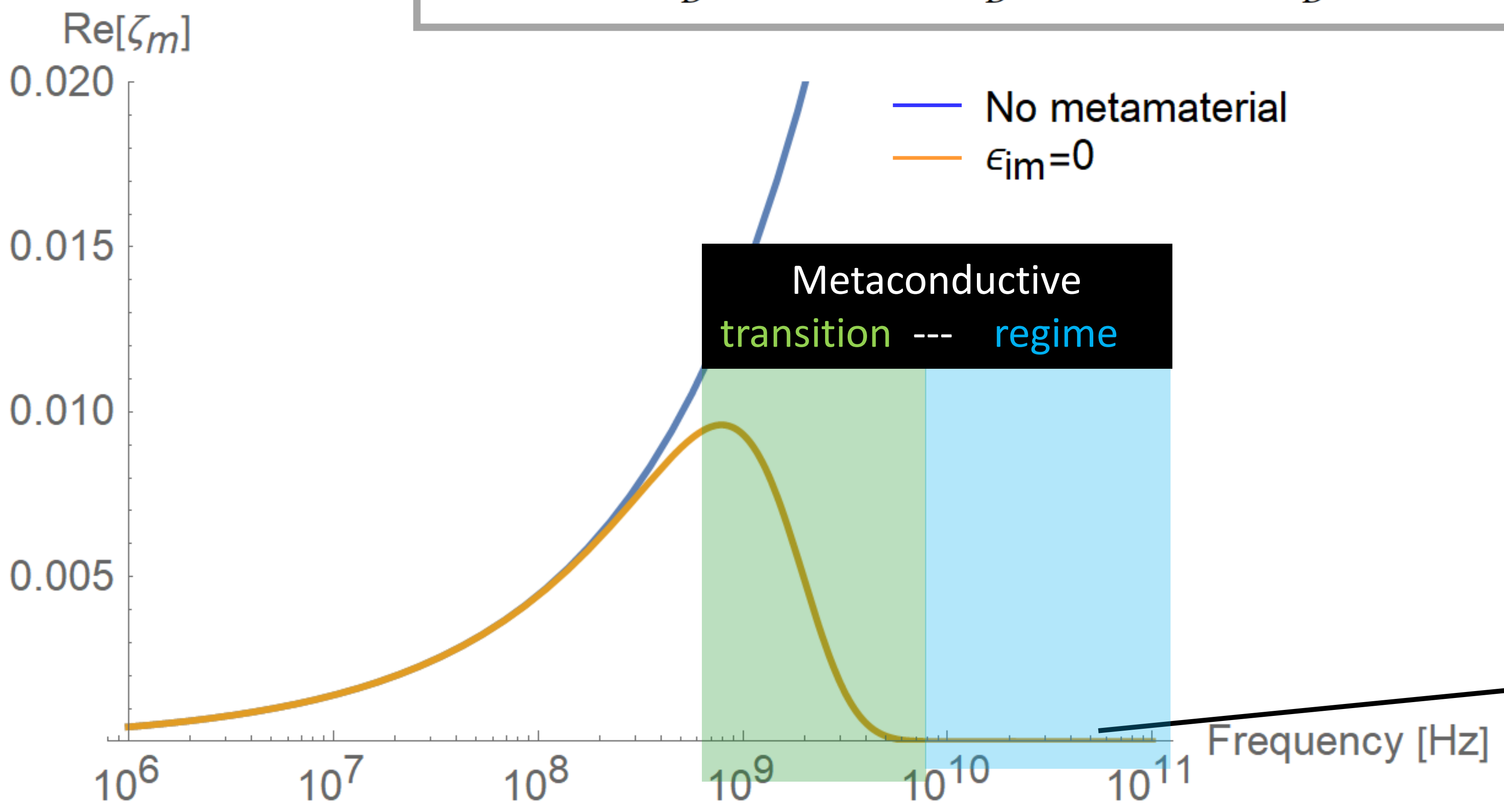
$$Re\{\zeta_m\} = \frac{\left(b'^2 + b''^2\right)\cdot N_1}{D} + \frac{\left(b'^2 - b''^2 + 2b'b''\right)\cdot N_2}{D} + \frac{\left(b'^3 + b'b''^2\right)\cdot N_3}{D} + \frac{\left(b''^3 + b'^2 b''\right)\cdot N_4}{D} + \frac{N_5}{D}$$




| Parameter | Value |
| --- | --- |
| Metal conductivity | $2 \cdot 10^7$ S/m |
| Metamaterial type | ENG (Negative electrical permittivity) |
| $\varepsilon_{r1}$ | $-10$ (lossless case);<br>$-10 + j10^{-4}$ (lossy case)<br>$-10 + j10^{-5}$ (lossy case) |
| $\mu_{r1}$ | 1 |
| Metamaterial layer thickness | 10 mm |

# Analytical model: the metaconductive transition

$$Re\{\zeta_m\} = \frac{(b'^2 + b''^2) \cdot N_1}{D} + \frac{(b'^2 - b''^2 + 2b'b'') \cdot N_2}{D} + \frac{(b'^3 + b'b''^2) \cdot N_3}{D} + \frac{(b''^3 + b'^2 b'') \cdot N_4}{D} + \frac{N_5}{D}$$

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$$\mathrm{Re}[\zeta_m] = 0$$

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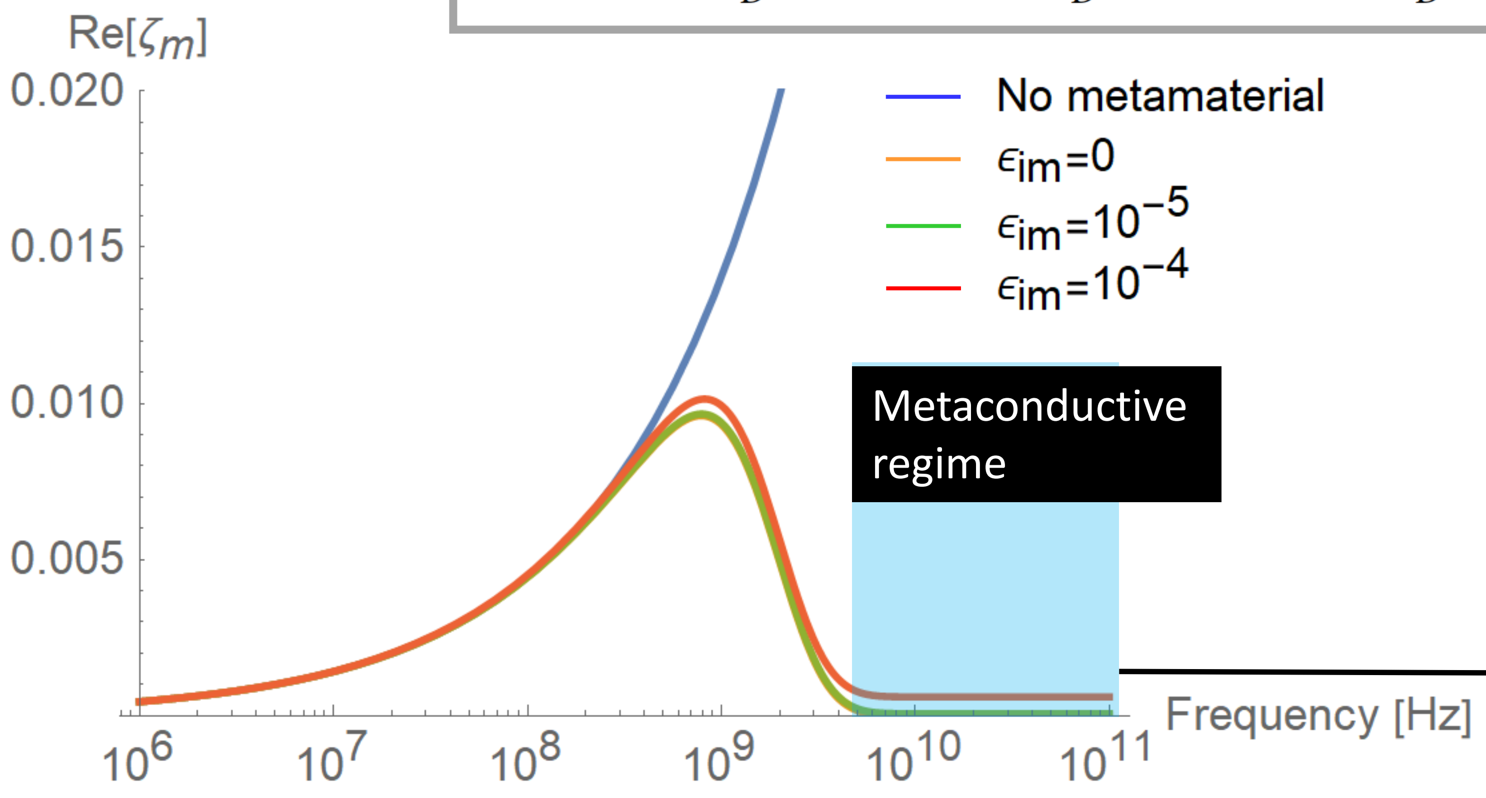


$\mathrm{Re}[\zeta_m] = b'$

| Parameter | Value |
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| Metamaterial layer thickness | 10 mm |

**The wall material has been masked and losses are only driven by the metamaterial insertion**

# A simple design equation

$$Re\{\zeta_m\} = \frac{(b'^2 + b''^2)\cdot N_1}{D} + \frac{(b'^2 - b''^2 + 2b'b'')\cdot N_2}{D} + \frac{(b'^3 + b'b''^2)\cdot N_3}{D} + \frac{(b''^3 + b'^2 b'')\cdot N_4}{D} + \frac{N_5}{D}$$

Lossless metamaterial

$$Re\{\zeta_m\} = \zeta_1 \cdot \frac{A\left[1 + (\tan k_1 t)^2\right]}{(A - \tan k_1 t)^2 + (\tan k_1 t)^2}$$

$$A = \sigma_2 \delta \zeta_1$$

$$Re\{\zeta_m\} = 0 \;\rightarrow\; \tan(k_1 t) = j \;\rightarrow$$

**It can be satisfied only by ENG or MNG metamaterials**

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

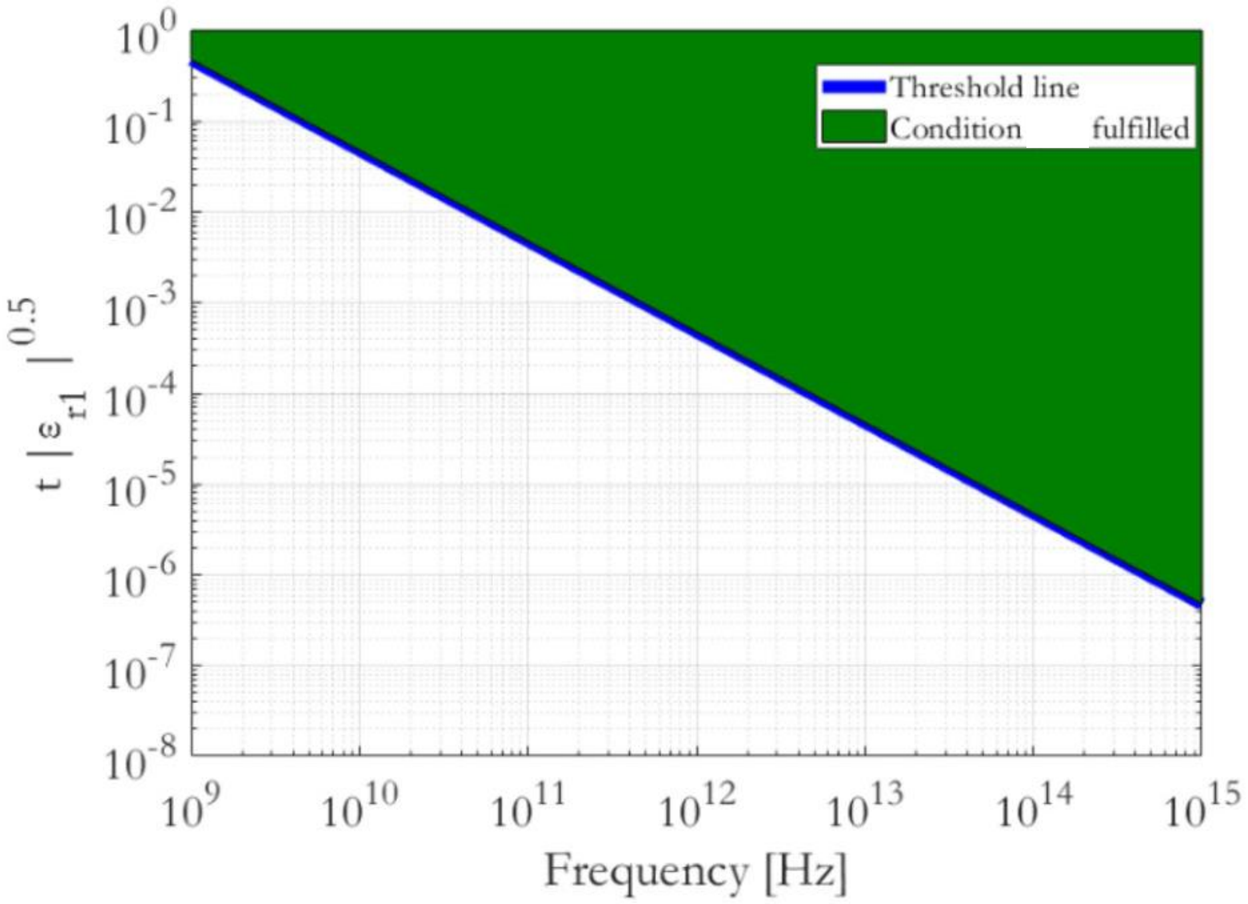
$$\tanh(|k_1 t|) = 1$$

$|k_1 t|$ → G

$$t\sqrt{|\epsilon_{r1}|} \geq \frac{Gc}{\omega_0\sqrt{|\mu_{r1}|}}$$

**Higher is G deeper we are in the metaconductive regime**

# A simple design equation




$$t\sqrt{|\epsilon_{r1}|} \geq \frac{Gc}{\omega_0\sqrt{|\mu_{r1}|}}$$

$$G = 10$$

**Knowing the frequency at which the metamaterial should operate one can estimate the required thickness and properties of the metamaterial to be in the metaconductive regime**

https://www.nature.com/articles/s41598-023-29966-2

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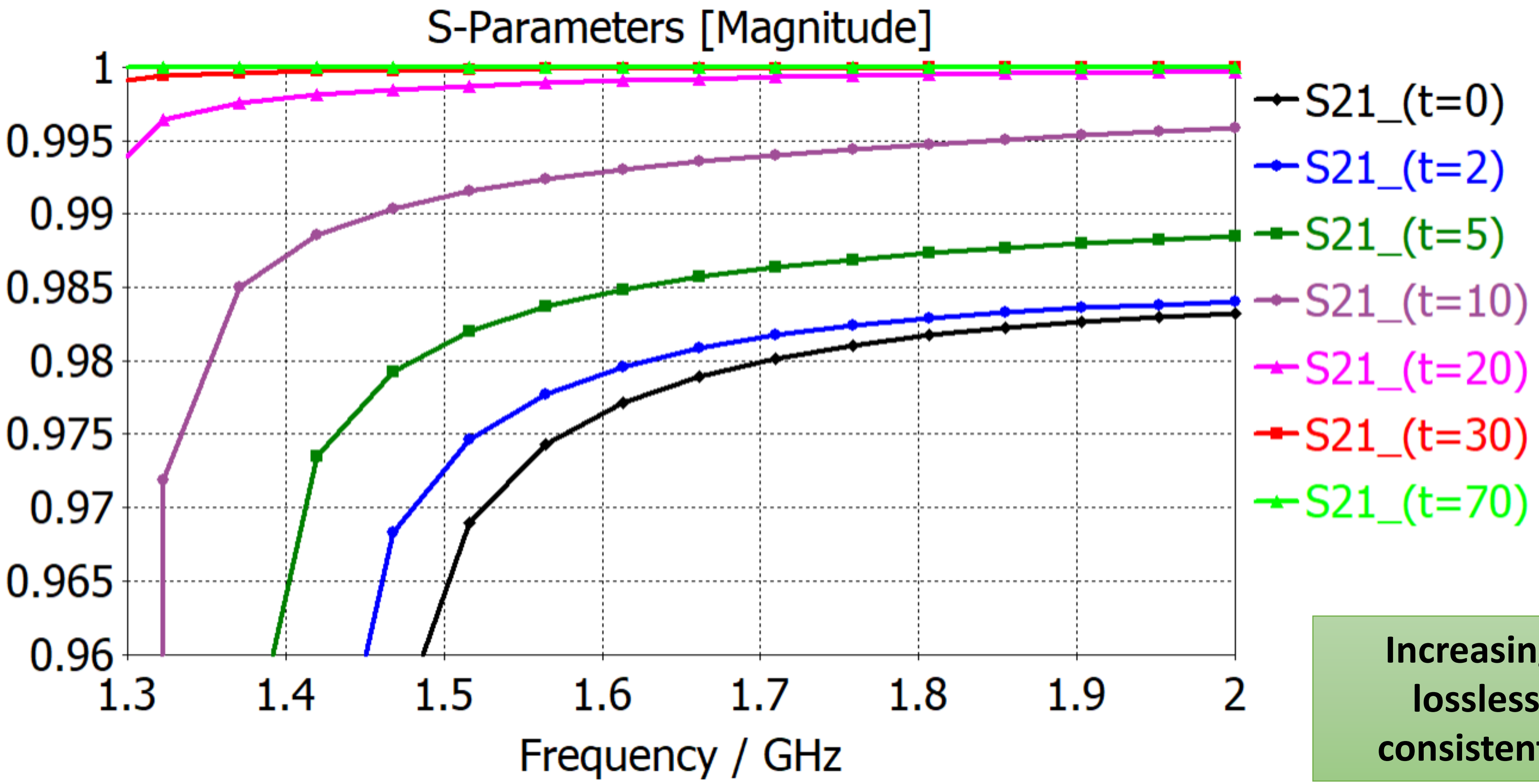
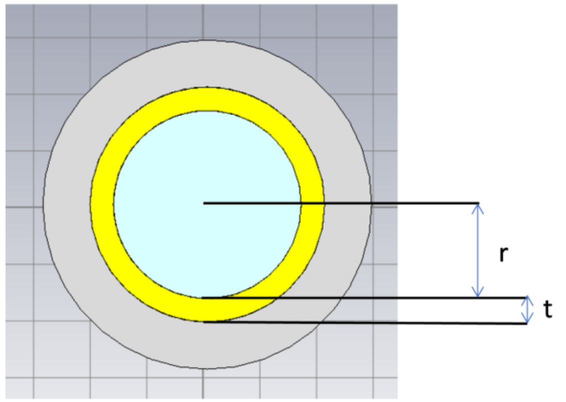
# Electromagnetic simulations

performed with **CST Studio Suite**




**Increasing the metamaterial thickness lossless propagation is approached consistently with analytical predictions**

# Electromagnetic simulations

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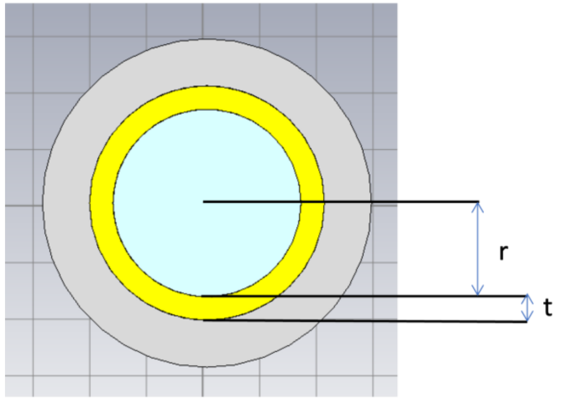


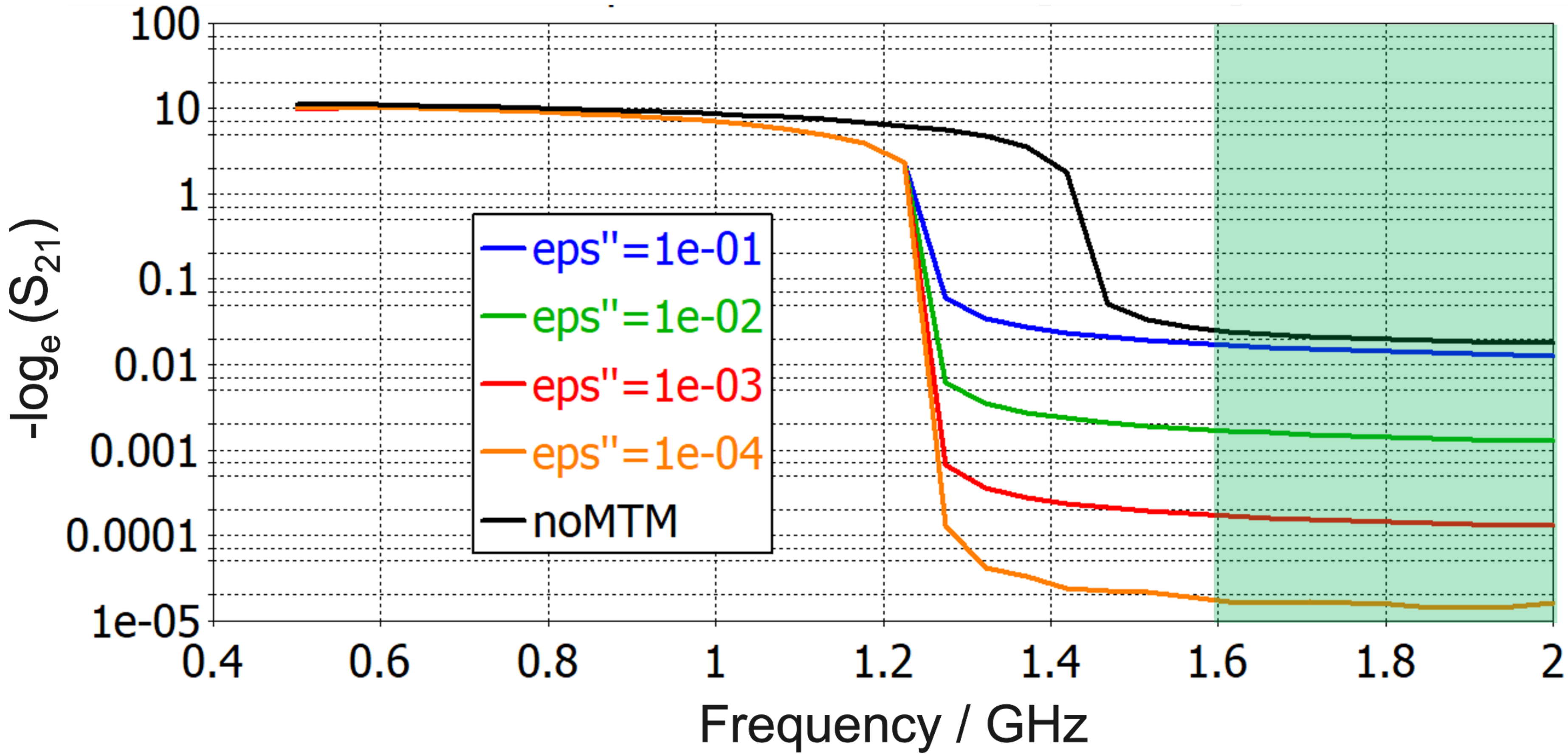


Impedance scales linearly with the metamaterial losses, as predicted analitically.

Beam coupling impedance well above cut-off frequency

$$Z(\omega) = -\,60\ log_e(S_{21})$$

C. Antuono et al. "A Wireless Method to Obtain the Impedance From Scattering Parameters," in *Proc. IPAC'22*

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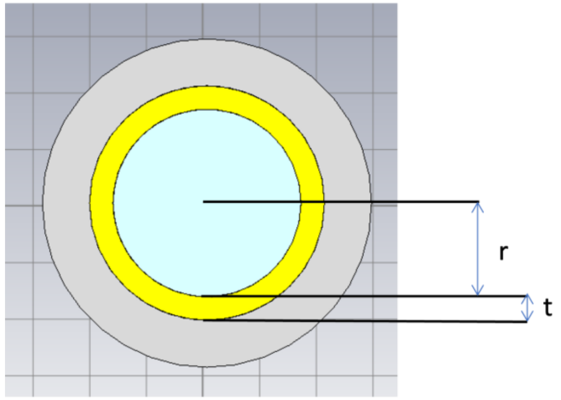
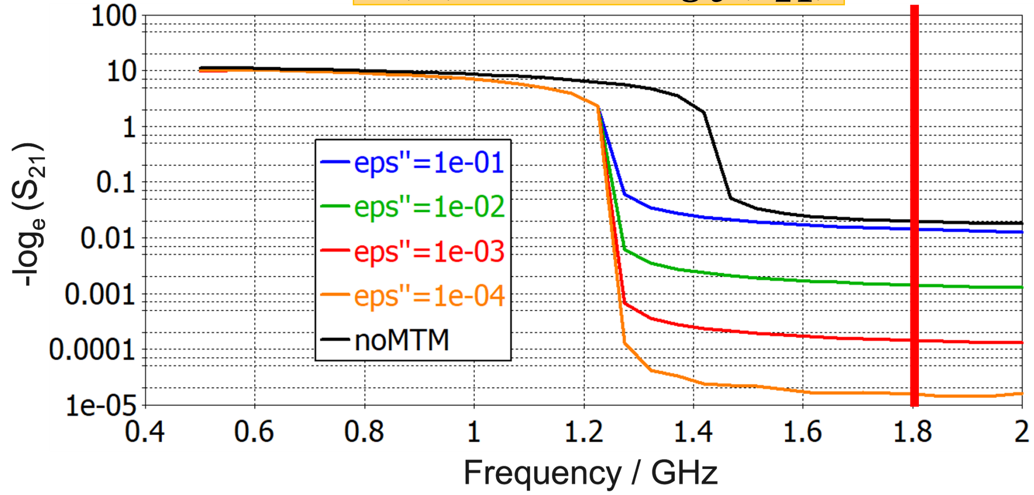
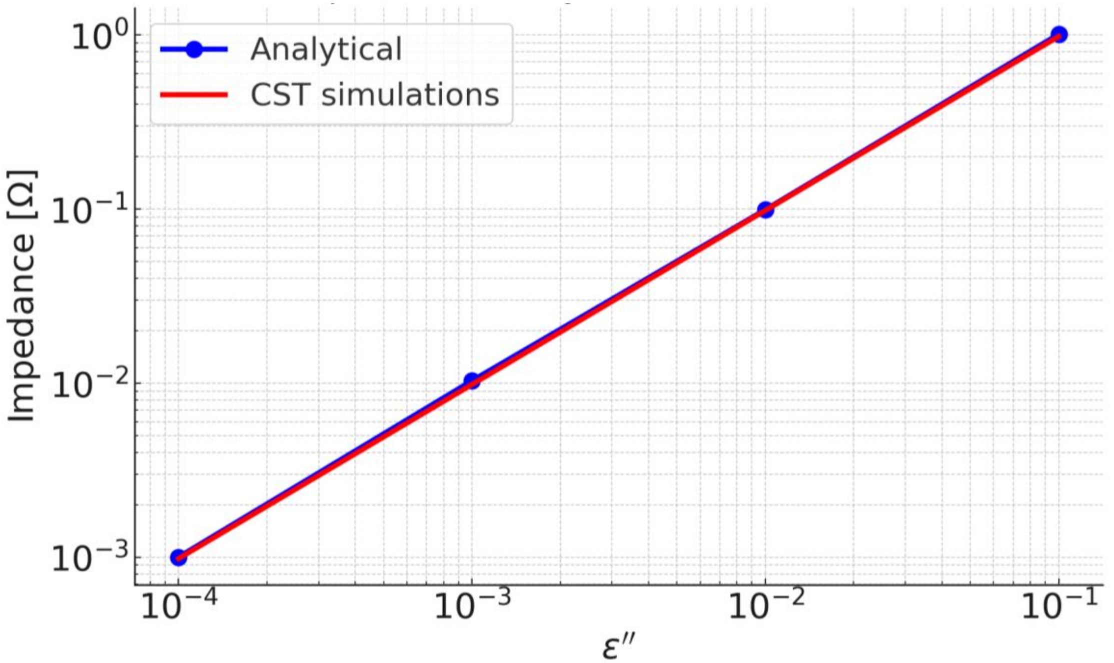



Beam coupling impedance
Well above cut-off frequency

$Z(\omega) = -60\, log_e(S_{21})$



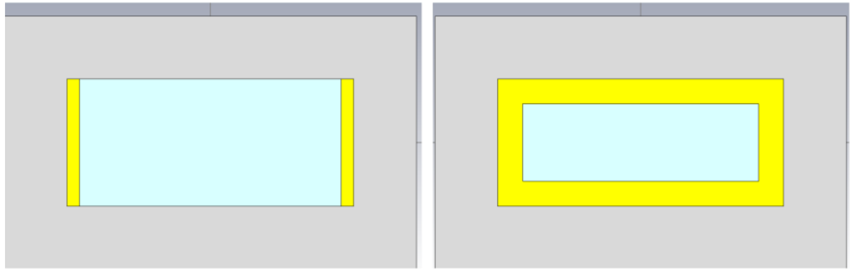

$$Re\{\zeta_m\} = \frac{(b'^2 + b''^2)\cdot N_1}{D} + \frac{(b'^2 - b''^2 + 2b'b'')\cdot N_2}{D} + \frac{(b'^3 + b'b''^2)\cdot N_3}{D} + \frac{(b''^3 + b'^2 b'')\cdot N_4}{D} + \frac{N_5}{D}$$

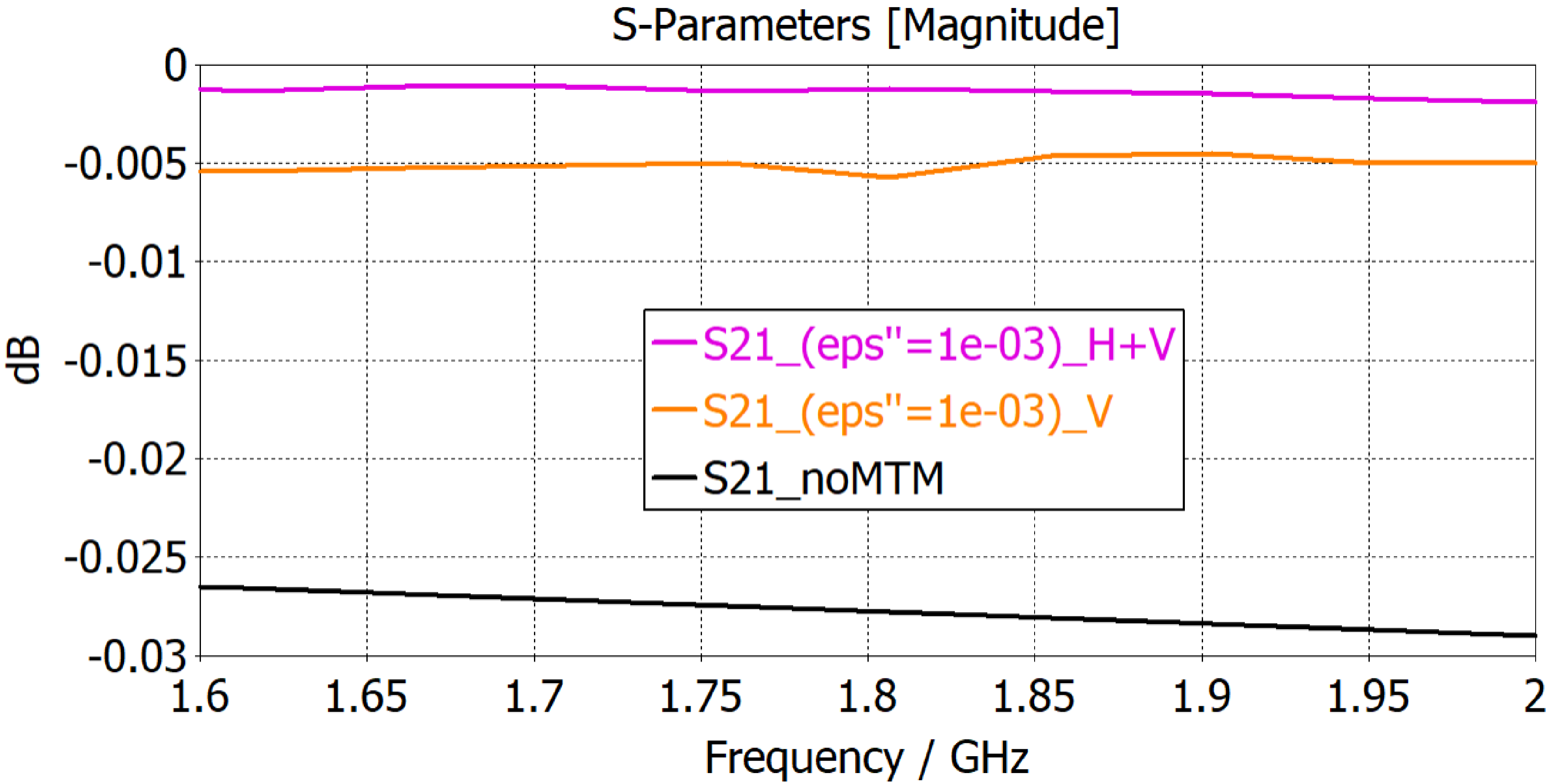
$$Z(\omega) = \frac{\zeta_m}{2\,\pi\, b}$$

**Excellent agreement in the beam coupling impedance obtained from analytical equation and CST simulations**

# Electromagnetic simulations

performed with **CST Studio Suite**

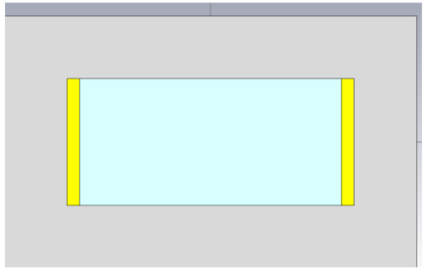


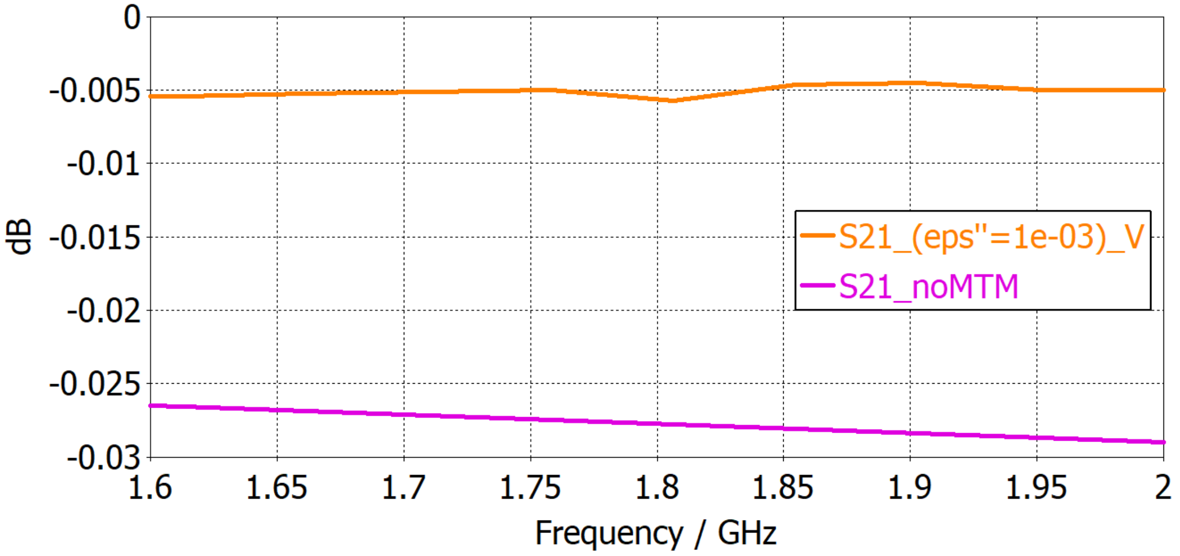


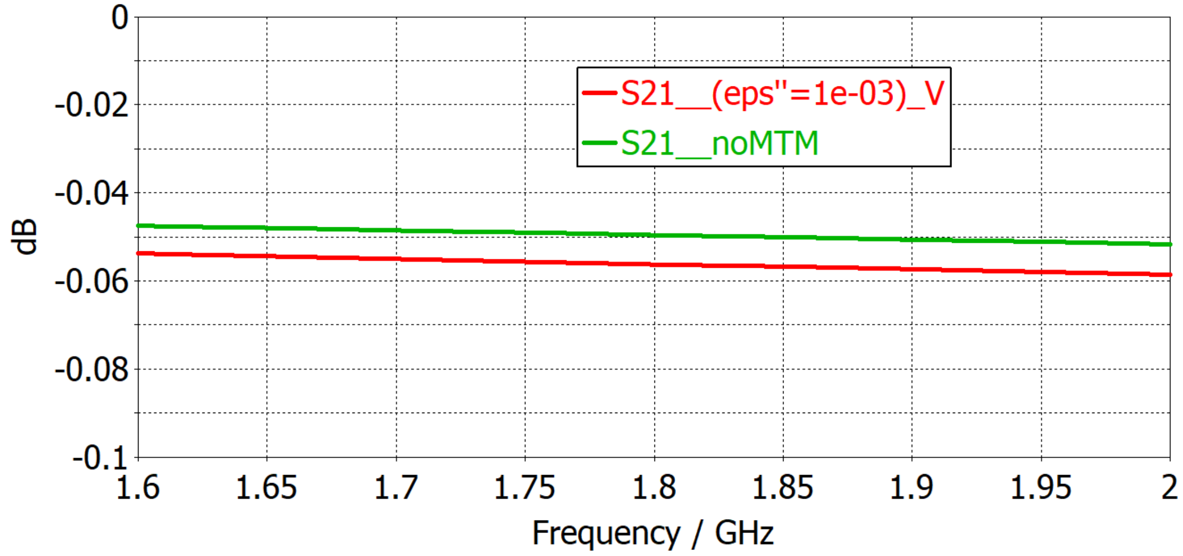

**Factor ~6 reduction** in transmission losses when only the **lateral walls** are covered, and an **additional factor** of approximately **4** is obtained for a **fully covered waveguide**

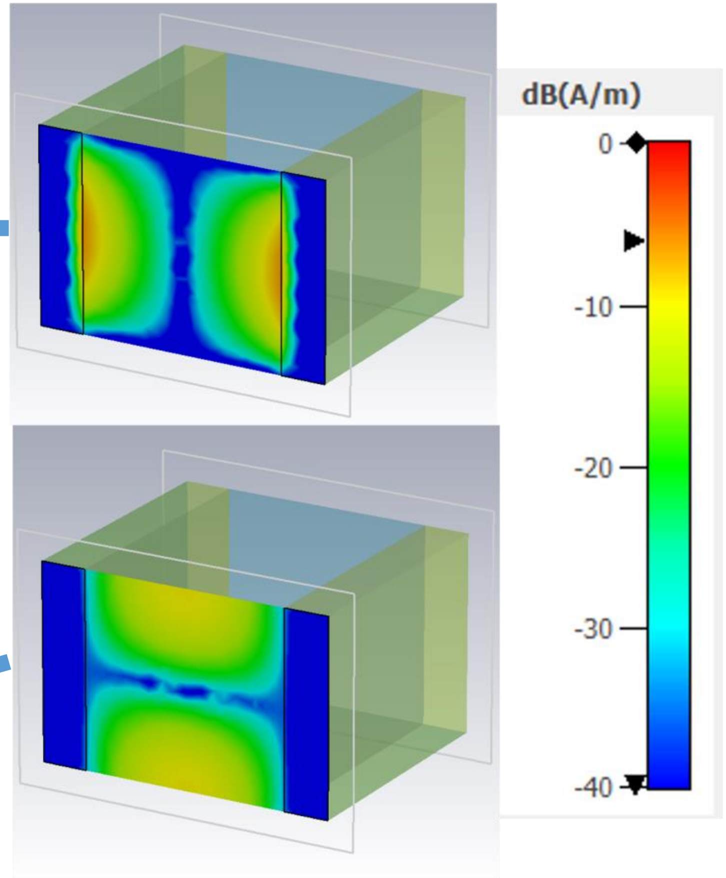
# Electromagnetic simulations

performed with **CST Studio Suite**






When wall are only **partially covered** the **impedance** is **significantly reduced** only for **modes** where **magnetic field** is mainly **concentrated on metamaterials**

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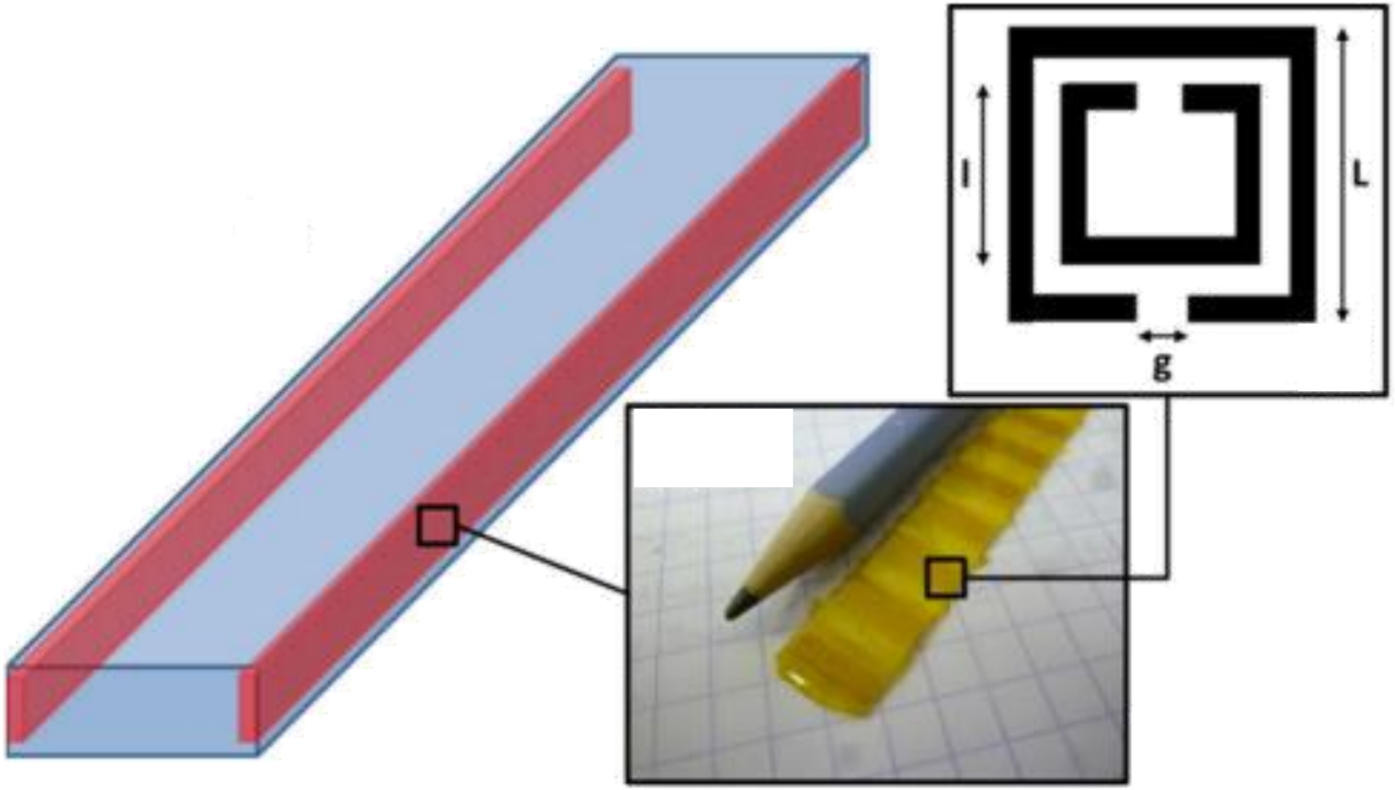
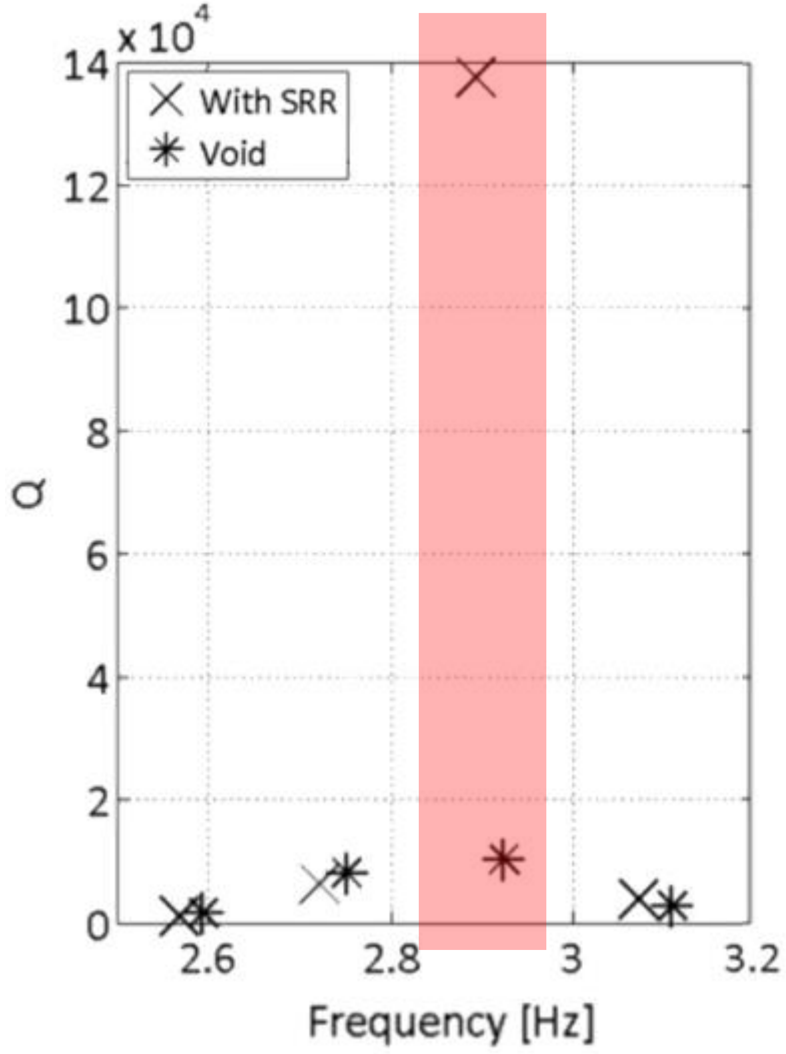
# Proof of concept

- First experiment was set in 2015 using a WR284 rectangular waveguide as a cavity.
- Waveguide is closed on a short on both sides, but with a tiny antenna on one side.
- Measurements of unloaded Q of cavity modes were done with and without metamaterial insertions




L = 5.4 mm, l = 3.4 mm, g = 1 mm.

# Proof of concept




Q increases of about 1 order of magnitude

In terms of **equivalent electrical conductivity** this corresponds to an increase of about 2 orders of magnitude

Reduction of the impedance of about 1 order of magnitude

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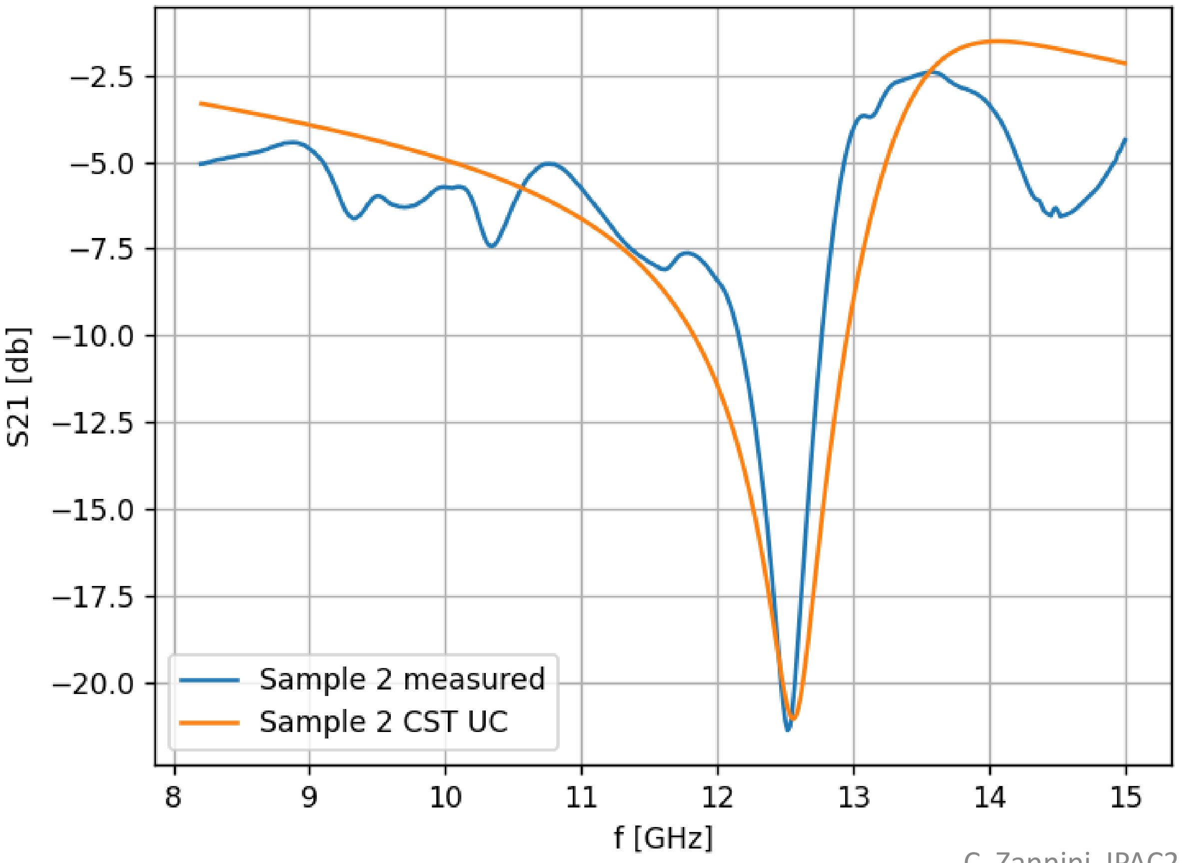
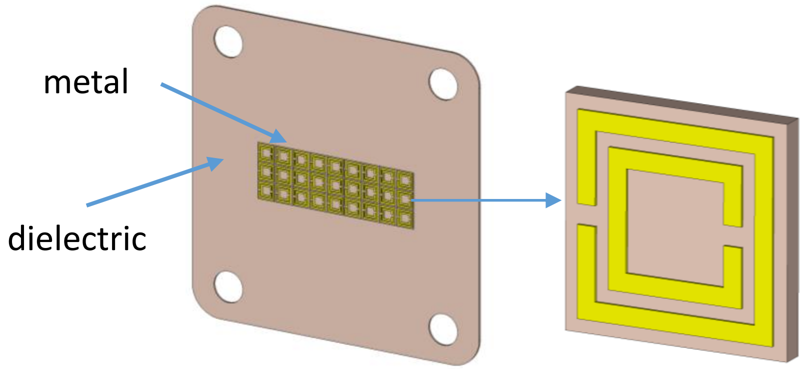
# Experimental progress and challenges

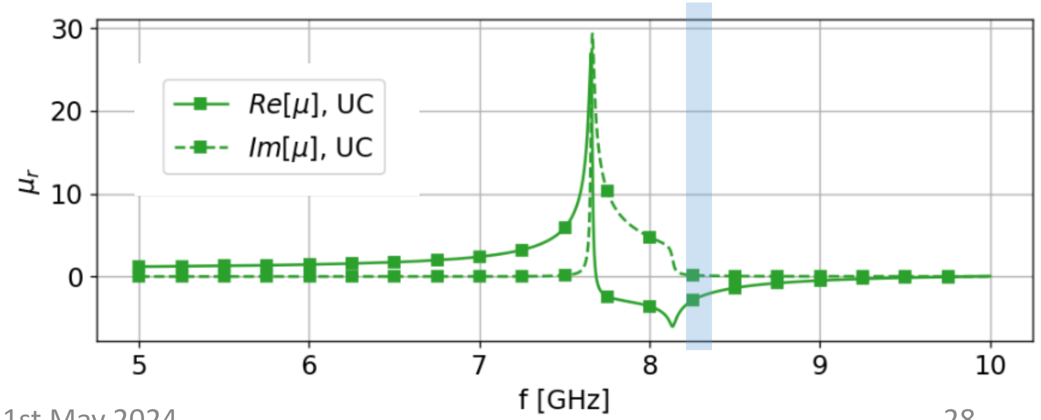
Metamaterial engineering to achieve the desired metaconductive behaviour at a given frequency requires precise control of the engineered metamaterial constitutive parameters.

Simulated and measured **frequency response are matching nicely**




Metamaterial losses need to be minimized to efficiently reduce the impedance



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- The investigation into the use of **metamaterial insertions for the emulation of perfect electrical conductive walls** has yielded promising analytical, simulations and experimental outcomes.
  - ➢ The analytical model shows the potential of metamaterials to enable an almost lossless propagation regime which was called **metaconductive regime**
  - ➢ The formulated conditions for designing appropriate metamaterial insertions have been validated and further explored with CST simulations

- This work could pave the way toward the realization of extremely low loss wave propagation or very high-Q accelerating cavities giving clear opportunities for more sustainable accelerator solutions.

- Metamaterial engineering to achieve the desired metaconductive behaviour at a given frequency requires precise control of the engineered metamaterial constitutive parameters.
  - ➢ Future research endeavours will delve into the optimization of metamaterial properties, alongside the experimental characterization of these materials.