$X(3872)$ radiative decays: View from molecular side

F.-K. Guo, C. Hanhart, A. Nefediev

What would we like to learn about $X(3872)$?

- Which measured properties of the $X(3872)$ are sensitive to the short-range core and which “feel” the long-range tail?
- What is the ratio between $D\bar{D}^*$ and compact parts of the $X(3872)$ w.f. compatible with data?
- What can we conclude from the measured ratio

$$R = \frac{\mathrm{Br}(X(3872) \to \gamma\psi')}{\mathrm{Br}(X(3872) \to \gamma J/\psi)} \sim 1\ ?$$

BaBaR’09	Belle’11	LHCb’14	BESIII’20	LHCb’24
3.4 ± 1.4	< 2.1	2.46 ± 0.7	< 0.59	1.67 ± 0.25

To loop or not to loop?

- Positronium — a false hadronic physicist's friend
  - Textbook knowledge $\Longrightarrow$ $\mathcal{M}(e^+e^-(^1S_0) \to \gamma\gamma) \propto \psi(0)$
  - Naive conclusion $\Longrightarrow$ Hadronic molecules decay "via" $\psi(0)$
  - Immediate problem $\Longrightarrow$ Model estimates for $\psi(0)$ differ drastically
  - Important note $\Longrightarrow$ "$\psi(0)$" formula implies particular scales hierarchy

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  - Tex
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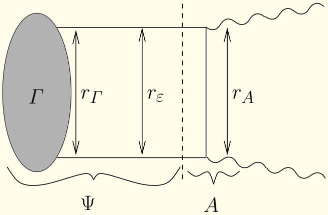

Hanhart et al., Phys.Rev.D 75 (2007) 074015

Positronium:

$$r_\Gamma \sim \frac{1}{\alpha m_e} \gg r_A \sim \frac{1}{m_e}$$

Point-like limit:

$$r_\Gamma \sim \frac{1}{\beta} \ll r_A \sim \frac{1}{m_{\rm ex}}$$

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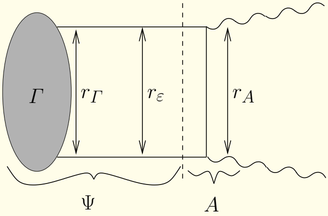
  - Conclusion $\Longrightarrow$ Employ w.f. in hadronic physics with a lot of caution!



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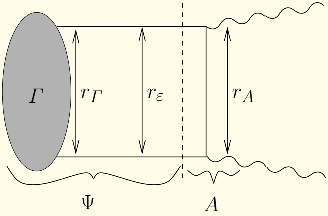

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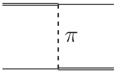
- Loop amplitude — a true hadronic physicist's friend
  - QFT loop integral $\Longrightarrow$ Most general treatment
  - "Positronium" decay $\Longrightarrow$ Limiting case of general formula
  - Conclusion $\Longrightarrow$ Use of loop amplitudes is safe and preferred

$D\bar{D}^*$ interaction in $X(3872)$

- Pion exchange

$V_{\text{OPE}} =$



$$\sim \frac{q_i q_j}{q^2 - m_\pi^2}$$

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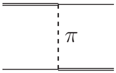

$$\sim \frac{q_i q_j}{q^2 - m_\pi^2} \sim \overbrace{-1}^{\text{Short-range}} + \underbrace{\frac{\mu_\pi^2}{\boldsymbol{q}^2 + \mu_\pi^2}}_{\text{Long-range}}$$

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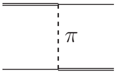
$V_{\text{OPE}} =$ 

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- Further (not quantified) short-range interactions need to be included
- Naive approach to processes sensitive to short-range physics
  - Contact theory $\implies$ $\psi(r) \sim \frac{1}{r} e^{-\gamma r}$ with $\gamma \sim \sqrt{E_B}$
  - "Positronium"-like prescription $\implies$ $\mathcal{M} \propto \psi(0)$
  - Infinite (regulator-dependent) probability $\implies$ Confusion

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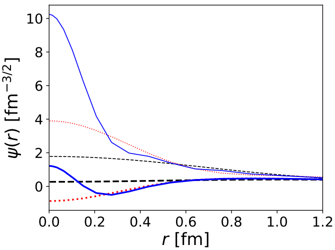
Deuteron w.f. for different regulators



Albaladejo et al., Chin.Phys. C **41** (2017) 12, 121001

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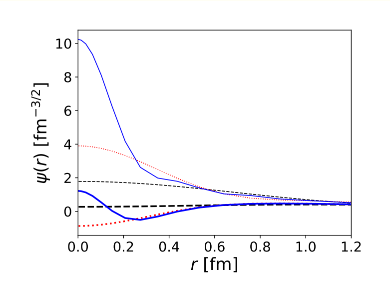

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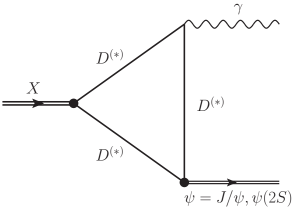
- If short-ranged dynamics is important, using only long-range tail of w.f. as the full w.f. is incorrect
- Loop amplitudes incorporate short- and long-range dynamics properly $(\Gamma[S \to \gamma\gamma] \sim \alpha^2\sqrt{mE_B} + \mathcal{O}(mE_B/\beta^2))$

$X(3872)$ radiative decay via D-meson loops



F.-K.Guo et al., Phys.Lett. B 742 (2015) 394

- No $q\bar{q} \to \gamma$ vertex — the photon emerges from (well-known) electric and magnetic $D^{(*)} \to D^{(*)}\gamma$ vertices
- Known (numerically uncertain) $X(3872) \to D\bar{D}^*$ coupling (Landau, Weinberg works dated 1960's) — cancels in the ratio R
- Unknown $J/\psi, \psi(2S) \to D^{(*)}D^{(*)}$ couplings but
  - Natural to expect $g_2'(\psi(2S) \to D^{(*)}D^{(*)}) \gtrsim g_2(J/\psi \to D^{(*)}D^{(*)})$
  - These couplings largely cancel in the ratio R

$X(3872)$ radiative decay via D-meson loops

- Good news:
  - A straightforward calculation of loop amplitudes is possible
  - For a fixed regularisation, the ratio R depends on a single unknown ratio of couplings g_2'/g_2
- Bad news:
  - No straightforward way to estimate unknown couplings g_2' & g_2
  - Contribution from the short-range component may be sizeable $\Longrightarrow$ Potentially strong scheme dependence
- Dilemma:
  - Loops diverge $\Longrightarrow$ Need input to fix the subtracted amplitude
  - Use R to fix the theory $\Longrightarrow$ No further data to compare with
  - Employ not renormalised theory $\Longrightarrow$ No reliable quantitative conclusion are possible

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Strategy: employ renormalisation group equation

$$\frac{\partial R}{\partial \mu_{\rm ren}} = 0$$

to assess the role of the short-range part of w.f. and qualitative compatibility of $D\bar{D}^*$ molecule with $R_{\rm exp} \sim 1$

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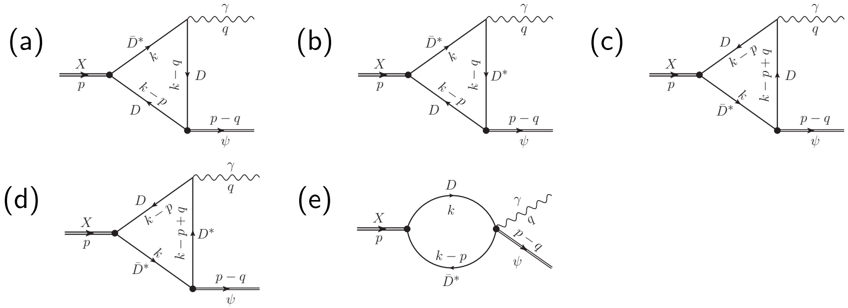

	$\mu_{\rm ren} = m_X/2$	$\mu_{\rm ren} = m_X$	$\mu_{\rm ren} = 2m_X$
R	$0.39(g_2'/g_2)^2$	$0.21(g_2'/g_2)^2$	$0.14(g_2'/g_2)^2$

Update: P.-P. Shi et al., Phys.Lett. B 843 (2023) 137987

Similar analysis: D.A.S. Molnar et al., 1601.03366

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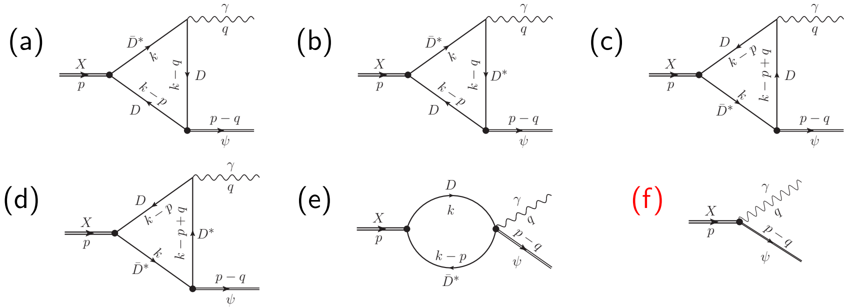

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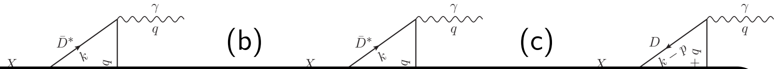
(a) (b) (c)



Conclusions:

- For naturally expected $g_2'/g_2 \simeq 1..2 \quad \Longrightarrow \quad R_{\rm mol} \sim 1$
- Radiative decays of the $X(3872)$ are sensitive to the short-ranged (compact) part of the w.f.

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Nature of $X(3872)$ from radiative decays?

Very naive hand waving consideration (to emphasise problem, not solve it)

$$R_{\rm comp} \gtrsim \Delta R_{\rm comp}(\mu_{\rm ren}) \simeq \Delta R_{\rm mol}(\mu_{\rm ren}) \simeq R_{\rm mol} \sim 1$$

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Up to the phase space factors,

$$R \simeq \frac{\left|\sqrt{Z}\mathcal{M}_{\rm comp}(X \to \gamma\psi(2S)) + \sqrt{1-Z}\mathcal{M}_{\rm mol}(X \to \gamma\psi(2S))\right|^2}{\left|\sqrt{Z}\mathcal{M}_{\rm comp}(X \to \gamma J/\psi) + \sqrt{1-Z}\mathcal{M}_{\rm mol}(X \to \gamma J/\psi)\right|^2}$$

or, after straightforward manipulations,

$$R \simeq \left|\frac{\xi}{1+\xi}\sqrt{R_{\rm comp}} + \frac{1}{1+\xi}\sqrt{R_{\rm mol}}\right|^2$$

with

$$\xi = \sqrt{\frac{Z}{1-Z}}\frac{\mathcal{M}_{\rm comp}(X \to \gamma J/\psi)}{\mathcal{M}_{\rm mol}(X \to \gamma J/\psi)}$$

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- No estimate of Z is possible without model calculations
- For $R_{\rm comp} \simeq R_{\rm mol} \simeq 1 \quad \Longrightarrow \quad R \simeq 1$

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Conclusions

- Experimental situation with the $X(3872)$ radiative decays gets clearer
- The ratio R is measured to be of the order of unity
- Calculations based on w.f. overlaps need to be treated with caution
- Conclusions from calculation of D-meson loop amplitudes:
  - $R_{\rm mol} \simeq 1$ for natural couplings of J/ψ and $\psi(2S)$ to D-mesons
  - Further efforts are needed to evaluate the ratio of couplings g_2'/g_2 theoretically and/or phenomenologically
  - Radiative decays of X are sensitive to short-range component of w.f. that are out of control in hadronic theory
- Under these circumstances, the measured ratio $R_{\rm exp} \simeq 1$ is not decisive in discriminating between contributions to the $X(3872)$ w.f.