Probing the structure of $\chi_{c1}(3872)$ with radiative decays: heavy quark symmetries at work

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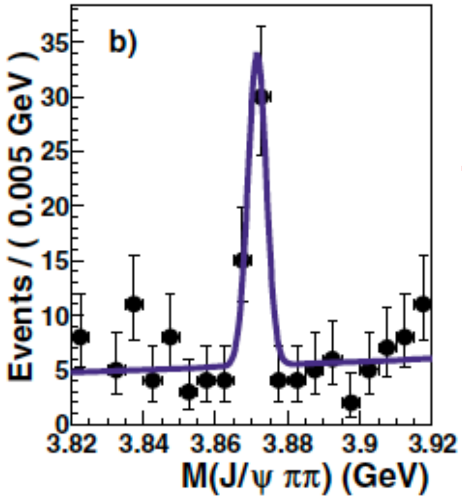
Outline

- Brief overview of χ_{c1} (3872) properties
- Heavy quark symmetries and applications to heavy-heavy mesons
- Results for heavy quarkonia radiative decays: **testing the identification of χ_{c1} (3872) with χ_{c1}(2P)**
- Other possible ways to understand if χ_{c1}(3872) is an ordinary charmonium
- Perspectives

LHCb meets Theory:
Probing the nature of the X(3872) state using radiative decays
CERN – June 27th 2024

$\chi_{c1}(3872)$

Belle Collab., PRL 91 (2003) 262001



discovered decaying to $J/\psi\ \pi^+\pi^-$

PDG 2024

Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)

$\chi_{c1}(3872)$ $\quad I^G(J^{PC}) = 0^+(1^{++})$

also known as $X(3872)$

PDG Name	Former Name	m (MeV)	Γ (MeV)	$I^G(J^{PC})$	Production	Decay	Discovery Year	Summary Table
$\chi_{c1}(3872)$	$X(3872)$	3871.64±0.06	1.19 ± 0.21	$0^+(1^{++})$	$B \to KX$ $p\bar{p} \to X...$ $pp \to X...$ $e^+e^- \to \gamma X$	$\pi^+\pi^- J/\psi(1S)$ $3\pi J/\psi(1S)$ $D^{*0}\bar{D}^0$ $\gamma J/\psi(1S)$ $\gamma\psi(2S)$ $\pi^0\chi_{c1}(1P)$	2003	YES

- discovered by Belle
- confirmed by BaBar, CDF, D0, LHCb, ATLAS, CMS, BESIII
- measurements of - production in hadron collisions
  - several decay modes
  - line shape
- search for partners

$\chi_{c1}(3872)$

- no isospin partners found
  (I=0 supported also since decay to **J/ψ** $\pi^0\pi^0$ is not observed)

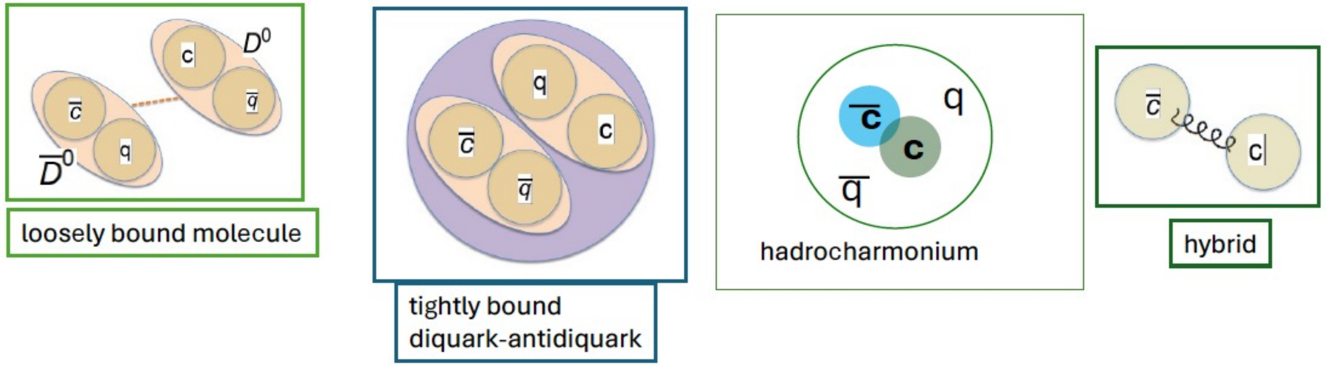
- mass close to the DD* threshold

$$m_{\chi_{c1}(3872)} - m_{D^{*0}} - m_{\bar{D}^0} = 1.1 \pm^{0.6}_{0.4} \pm^{0.1}_{0.3} \text{ MeV}$$

- decay to **J/ψ ππ π** suppressed wrt **J/ψ ππ** LHCb PRD131 (2023) L011103
  → isospin violation
  but phase space suppression also at work

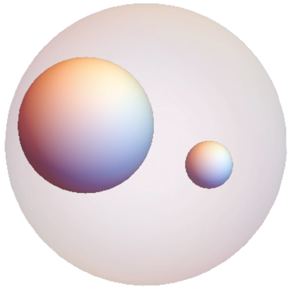
- ≈ 1500 papers devoted to χ_{c1} (3872)

- many non ordinary c-cbar interpretations



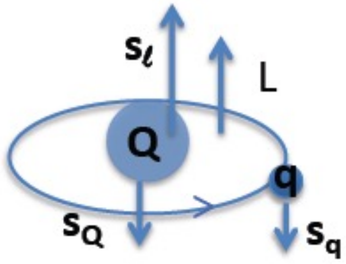
HQ systems

Heavy-light mesons



Hadrons with a single heavy quark:
exploit the heavy quark large mass limit $m_Q \to \infty$ in QCD
(Heavy Quark Effective Theory)

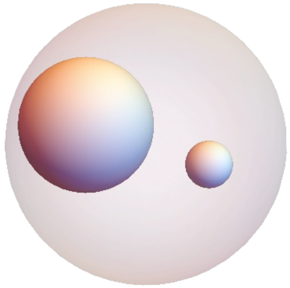
Intuitive picture



hadrons which differ only for the HQ flavour/spin
→ same configuration of the light degrees of freedom

HQ systems

Heavy-light mesons



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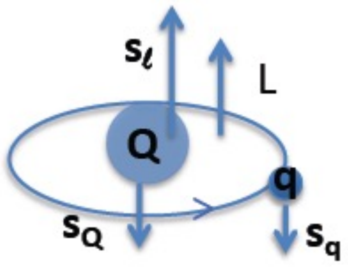


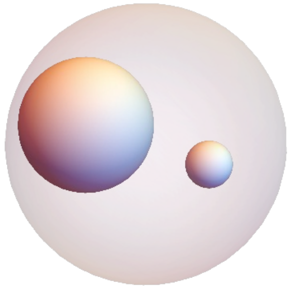
hadrons which differ only for the HQ flavour/spin
→ same configuration of the light degrees of freedom

➢ Weak matrix elements: relations among form factors in selected kinematical regions

- e.g. introduction of universal Isgur-Wise function(s)
- reduction of the uncertainty in the determination of $|V_{cb}|$
- .....

HQ systems

Heavy-light mesons



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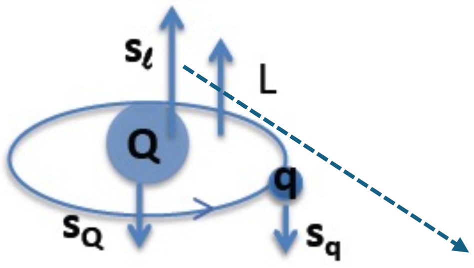


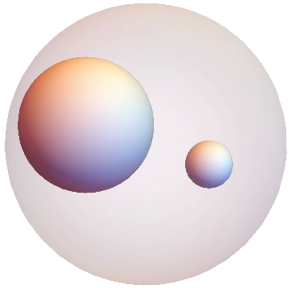
hadrons which differ only for the HQ flavour/spin
→ same configuration of the light degrees of freedom

$$\vec{s}_\ell = \vec{L} + \vec{s}_q$$

- Spectroscopic implications: $\mathbf{s_Q}$ and $\mathbf{s_\ell}$ separately conserved

HQ systems

Heavy-light mesons



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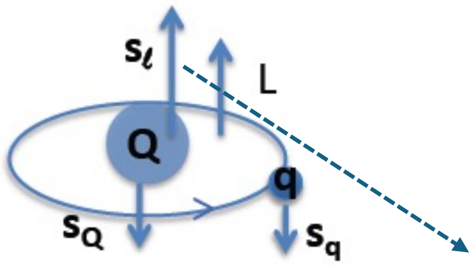


hadrons which differ only for the HQ flavour/spin
→ same configuration of the light degrees of freedom

$$\vec{s}_\ell = \vec{L} + \vec{s}_q$$

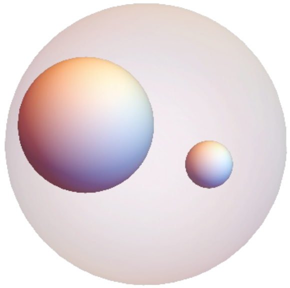
- Spectroscopic implications: $\mathbf{s_Q}$ and $\mathbf{s_\ell}$ separately conserved

Spin symmetry: mesons classified in doublets with $J = s_\ell \pm \frac{1}{2}$ $\quad P = (-1)^{L+1}$

members of the same doublet i) degenerate
ii) same total width

Flavour symmetry: - relations between charm and beauty hadron properties
- mass splittings among the doublets independent of flavour

Heavy-light mesons



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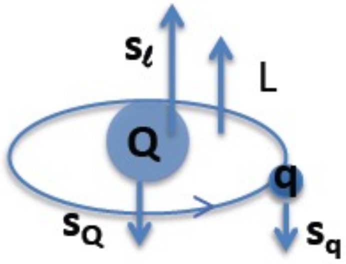


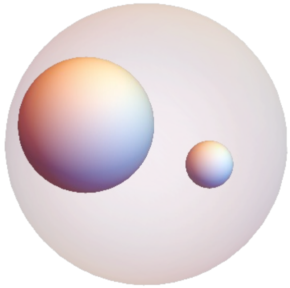
Hadrons with a single heavy quark:
exploit the heavy quark large mass limit $m_Q \to \infty$ in QCD
(Heavy Quark Effective Theory)

- $1/m_Q$ corrections can be systematically included

control over the uncertainties

HQ systems

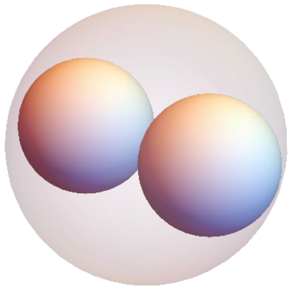
Heavy-light mesons



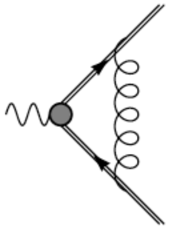
HQ limit: decoupling of the HQ

- Heavy-light mesons → HQ spin & flavour symmetry
- Heavy-heavy mesons → HQ spin symmetry

Heavy-heavy mesons



Heavy-heavy meson decays



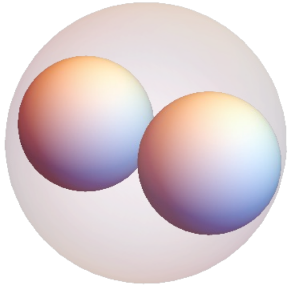
→ IR divergent for 2 HQs with the same v

- Infrared divergences regulated in the HQ limit by the kinetic energy operator O_π
- O_π breaks flavour symmetry ⇢ only spin symmetry

Thacker and Lepage, PRD43 (1991) 196

HQ systems: spectroscopy

Heavy-heavy mesons



$n^{2s+1}L_J$

- n → radial quantum number
- s = spin
- L = orbital angular momentum
- J = total angular momentum

with: $P=(-1)^{L+1}$
$C=(-1)^{L+s}$

L=0 ↔ S- wave states
L=1 ↔ P- wave states
L=2 ↔ D- wave states
.....

➢ η_c , J/ψ	1S-wave charmonia	$J^{PC}=(0^{--},1^{--})$
➢ χ_{c0}, χ_{c1}, χ_{c2} , h_c	1P-wave charmonia	$J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$
➢ χ'_{c0}, χ'_{c1}, χ'_{c2} , h'_c	2P-wave charmonia	$J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$

idem beauty partners

Multiplets for heavy quarkonia

- *L*=0 multiplet

$$J = \frac{1+ \not v}{2} [H_1^\mu \gamma_\mu - H_0 \gamma_5] \frac{1- \not v}{2}$$

H_1^μ: spin 1 state; H_0: spin 0 state

n=1: $(\eta_c(1S),\ \psi(1S))$

n=2: $(\eta_c(2S),\ \psi(2S))$

- *L*=1 multiplet

$$J^\mu = \frac{1+ \not v}{2} \left\{ H_2^{\mu\alpha} \gamma_\alpha + \frac{1}{\sqrt{2}} \epsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta H_{1\gamma} + \frac{1}{\sqrt{3}} (\gamma^\mu - v^\mu) H_0 + K_1^\mu \gamma_5 \right\} \frac{1- \not v}{2}$$

triplet: $H_2^{\mu\alpha}$ spin 2, $H_{1\gamma}$ spin 1, H_0 spin 0; singlet spin 1: K_1^μ

n=1

$[(\chi_{c0}(1P),\ \chi_{c1}(1P),\ \chi_{c2}(1P)),\ \ h_c(1P)]$

- *L*=2 multiplet

$$J^{\mu\nu} = \frac{1+ \not v}{2} \left\{ H_3^{\mu\nu\alpha} \gamma_\alpha + \frac{1}{\sqrt{6}} \left(\epsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta H_{2\gamma}^\nu + \epsilon^{\nu\alpha\beta\gamma} v_\alpha \gamma_\beta H_{2\gamma}^\mu \right) \right.$$

$$\left. + \frac{1}{2}\sqrt{\frac{3}{5}} [(\gamma^\mu - v^\mu) H_1^\nu + (\gamma^\nu - v^\nu) H_1^\mu] - \frac{1}{\sqrt{15}} (g^{\mu\nu} - v^\mu v^\nu) \gamma_\alpha H_1^\alpha + K_2^{\mu\nu} \gamma_5 \right\} \frac{1- \not v}{2}$$

triplet spin 3: $H_3^{\mu\nu\alpha}$; triplet spin 2: $H_{2\gamma}^\nu$, $H_{2\gamma}^\mu$; triplet spin 1: H_1^ν, H_1^μ, H_1^α; singlet spin 2: $K_2^{\mu\nu}$

$[(n^3D_1, n^3D_2, n^3D_3),\ n^1D_2]$

Effective Lagrangians for radiative transitions

Requirements:
Lorentz & C,P,T invariance + HQ spin symmetry

R. Casalbuoni et al., PLB 302 (1993) 95
FDF PRD 79 (2009) 054015

❖ P ←→ S

$$\mathcal{L}_{nP\leftrightarrow mS} = \delta_Q^{nPmS}\,\mathrm{Tr}[\bar{J}(mS)J_\mu(nP)]v_\nu F^{\mu\nu} + \mathrm{H.c.},$$

γ

a single constant describes all transitions
among the various members of the *P* and *S* multiplets

❖ D ←→ P

$$\mathcal{L}_{nD\leftrightarrow mP} = \delta_Q^{nDmP}\,\mathrm{Tr}[\bar{J}_\alpha(mP)J^\alpha_\mu(nD)]v_\nu F^{\mu\nu} + \mathrm{H.c.},$$

a single constant describes all transitions
among the various members of the *D* and *P* multiplets

- **reduced theoretical uncertainty**
- **model independence**

Effective Lagrangians for radiative transitions

FDF PRD 79 (2009) 054015

$$\Gamma(n^3P_J \to m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}},$$

$$\Gamma(m^3S_1 \to n^3P_J\gamma) = (2J+1)\frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}},$$

$$\Gamma(n^1P_1 \to m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}},$$

$$\Gamma(m^1S_0 \to n^1P_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},$$

$$\Gamma(m^1D_2 \to n^1P_1\gamma) = \frac{(\delta_Q^{mDnP})^2}{3\pi} k_\gamma^3 \frac{M_P}{M_D},$$

$$\Gamma(m^3D_2 \to n^3P_1\gamma) = \frac{(\delta_Q^{mDnP})^2}{4\pi} k_\gamma^3 \frac{M_P}{M_D},$$

$$\Gamma(m^3D_2 \to n^3P_2\gamma) = \frac{(\delta_Q^{mDnP})^2}{12\pi} k_\gamma^3 \frac{M_P}{M_D},$$

$$\Gamma(m^3D_1 \to n^3P_0\gamma) = \frac{5}{9}\frac{(\delta_Q^{mDnP})^2}{3\pi} k_\gamma^3 \frac{M_P}{M_D}$$

$$\Gamma(m^3D_1 \to n^3P_1\gamma) = \frac{5}{12}\frac{(\delta_Q^{mDnP})^2}{3\pi} k_\gamma^3 \frac{M_P}{M_D}$$

$$\Gamma(m^3D_1 \to n^3P_2\gamma) = \frac{1}{36}\frac{(\delta_Q^{mDnP})^2}{3\pi} k_\gamma^3 \frac{M_P}{M_D}.$$

Results

Disclaimer

- derived in 2009 in FDF PRD 79 (2009) 054015 using data available at that time
- redone for this talk using updated data (2024)
- differences will be remarked

Results: 1P to 1S transitions

input data: 2009

$$\mathcal{B}(\chi_{c0}(1P) \to J/\psi\,\gamma) = (1.28 \pm 0.11) \times 10^{-2},$$
$$\mathcal{B}(\chi_{c1}(1P) \to J/\psi\,\gamma) = (36.0 \pm 1.9) \times 10^{-2},$$
$$\mathcal{B}(\chi_{c2}(1P) \to J/\psi\,\gamma) = (20.0 \pm 1.0) \times 10^{-2},$$

2024

$$\mathcal{B}(\chi_{c0}(1P) \to J/\psi\gamma) = (1.41 \pm 0.09) \times 10^{-2}$$
$$\mathcal{B}(\chi_{c1}(1P) \to J/\psi\gamma) = (34.3 \pm 1.3) \times 10^{-2}$$
$$\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma) = (19.5 \pm 0.8) \times 10^{-2}$$

+ known $\chi_{cJ}(1P)$ total widths

$$\delta_c^{1P1S} = 0.235 \pm 0.012 \ \text{GeV}^{-1}$$
$$\delta_c^{1P1S} = 0.234 \pm 0.008 \ \text{GeV}^{-1}$$
$$\delta_c^{1P1S} = 0.231 \pm 0.008 \ \text{GeV}^{-1}$$

✓ spin symmetry well satisfied

$$\left(\delta_c^{1P1S}\right)_{\text{ave}} = 0.233 \pm 0.01 \ \text{GeV}^{-1}$$

Using (not known in 2009)

$$\mathcal{B}(h_c(1P) \to \eta_c\gamma) = (60 \pm 4) \times 10^{-2}$$
$$\Gamma(h_c(1P)) = (0.78 \pm^{0.27}_{0.24} \pm 0.12) \ \text{MeV}$$

$$\delta_c^{1P1S} = 0.205 \pm 0.04 \ \text{GeV}^{-1}$$

h_c data affected by larger uncertainty

$\Gamma(\chi_{bJ}(1P))$, $\Gamma(h_b(1P))$ not known yet

Results: 2P to 1S, 2S transitions

2009

$\mathcal{B}(\chi_{b0}(2P) \to \Upsilon(1S)\gamma) = (9 \pm 6) \times 10^{-3}$,
$\mathcal{B}(\chi_{b0}(2P) \to \Upsilon(2S)\gamma) = (4.6 \pm 2.1) \times 10^{-2}$,
$\mathcal{B}(\chi_{b1}(2P) \to \Upsilon(1S)\gamma) = (8.5 \pm 1.3) \times 10^{-2}$,
$\mathcal{B}(\chi_{b1}(2P) \to \Upsilon(2S)\gamma) = (21 \pm 4) \times 10^{-2}$,
$\mathcal{B}(\chi_{b2}(2P) \to \Upsilon(1S)\gamma) = (7.1 \pm 1.0) \times 10^{-2}$,
$\mathcal{B}(\chi_{b2}(2P) \to \Upsilon(2S)\gamma) = (16.2 \pm 2.4) \times 10^{-2}$,

2024

$\mathcal{B}(\chi_{b0}(2P) \to \Upsilon(1S)\gamma) = (3.8 \pm 1.7) \times 10^{-3}$
$^{B}(\chi_{b0}(2P) \to \Upsilon(2S)\gamma) = (1.38 \pm 0.3) \times 10^{-2}$
$\mathcal{B}(\chi_{b1}(2P) \to \Upsilon(1S)\gamma) = (9.9 \pm 1.0) \times 10^{-2}$
$\mathcal{B}(\chi_{b1}(2P) \to \Upsilon(2S)\gamma) = (18.1 \pm 1.9) \times 10^{-2}$
$\mathcal{B}(\chi_{b2}(2P) \to \Upsilon(1S)\gamma) = (6.6 \pm 0.8) \times 10^{-2}$
$\mathcal{B}(\chi_{b2}(2P) \to \Upsilon(2S)\gamma) = (8.9 \pm 1.2) \times 10^{-2}$
$\mathcal{B}((h_b(2P) \to \eta_b(1S)\gamma) = (22 \pm 5) \times 10^{-2}$
$\mathcal{B}(h_b(2P) \to \eta_b(2S)\gamma) = (48 \pm 13) \times 10^{-2}$

$$R_J^{(b)} = \frac{\Gamma(\chi_{bJ}(2P) \to \Upsilon(2S)\gamma)}{\Gamma(\chi_{bJ}(2P) \to \Upsilon(1S)\gamma)}.$$

take data and compare to theory:
widths depend on δ_b^{2P2S} or δ_b^{2P1S}

Compute $R_\delta^{(b)} = \frac{\delta_b^{2P2S}}{\delta_b^{2P1S}}$

	weighted average	naive average
2009	$\left(R_\delta^{(b)}\right)_{\text{ave}} = 8.8 \pm 0.7$	$\left(R_\delta^{(b)}\right)_{\text{ave}} = 11 \pm 4$
2024	$\left(R_\delta^{(b)}\right)_{\text{ave}} = 7.4 \pm 0.4$	$\left(R_\delta^{(b)}\right)_{\text{ave}} = 8.8 \pm 2.4$

Results: 2P to 1S, 2S transitions

Heavy-Heavy mesons: no flavour symmetry → $\delta_b^{2P1S} \neq \delta_c^{2P1S}$

reasonable assumption: the ratio stays stable → $R_\delta^{(b)} \simeq R_\delta^{(c)}$ → $\frac{\delta_b^{2P2S}}{\delta_b^{2P1S}} \simeq \frac{\delta_c^{2P2S}}{\delta_c^{2P1S}}$

predictions for charm: $R_J^{(c)} = \frac{\Gamma(\chi_{cJ}(2P) \to \psi(2S)\gamma)}{\Gamma(\chi_{cJ}(2P) \to J/\psi\gamma)}$

useful to identify $\chi_{c1}(3872)$

E.S. Swanson
Diagnostic decays of the X(3872)
PLB 598 (2004) 197

Results: 2P to 1S, 2S transitions

$$R_J^{(c)} = \frac{\Gamma(\chi_{cJ}(2P) \to \psi(2S)\gamma)}{\Gamma(\chi_{cJ}(2P) \to J/\psi\gamma)}$$

2024

$R_J^{(c)}$	weighted ave.	naive ave.
$R_0^{(c)}$	1.0 ± 0.2	1.43 ± 0.65
$R_1^{(c)}$	1.2 ± 0.2	1.62 ± 0.74
$R_2^{(c)}$	1.95 ± 0.2	2.7 ± 1.3

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prediction for $\chi_{c1}(2P)$
assigning $M(\chi_{c1}(2P))=3872$ MeV

tests the identification of $\chi_{c1}(3872)$ with $\chi_{c1}(2P)$

Results: 2P to 1S, 2S transitions

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prediction for $\chi_{c1}(2P)$
assigning $M(\chi_{c1}(2P))$=3872 MeV

tests the identification of χ_{c1} (3872) with $\chi_{c1}(2P)$

slighly smaller than the 2009 result:

$$R_1^{(c)} = \frac{\Gamma(\chi_{c1}(2P) \to \psi(2S)\gamma)}{\Gamma(\chi_{c1}(2P) \to \psi(1S)\gamma)} = 1.64 \pm 0.25.$$

Results: 2P to 1S, 2S transitions

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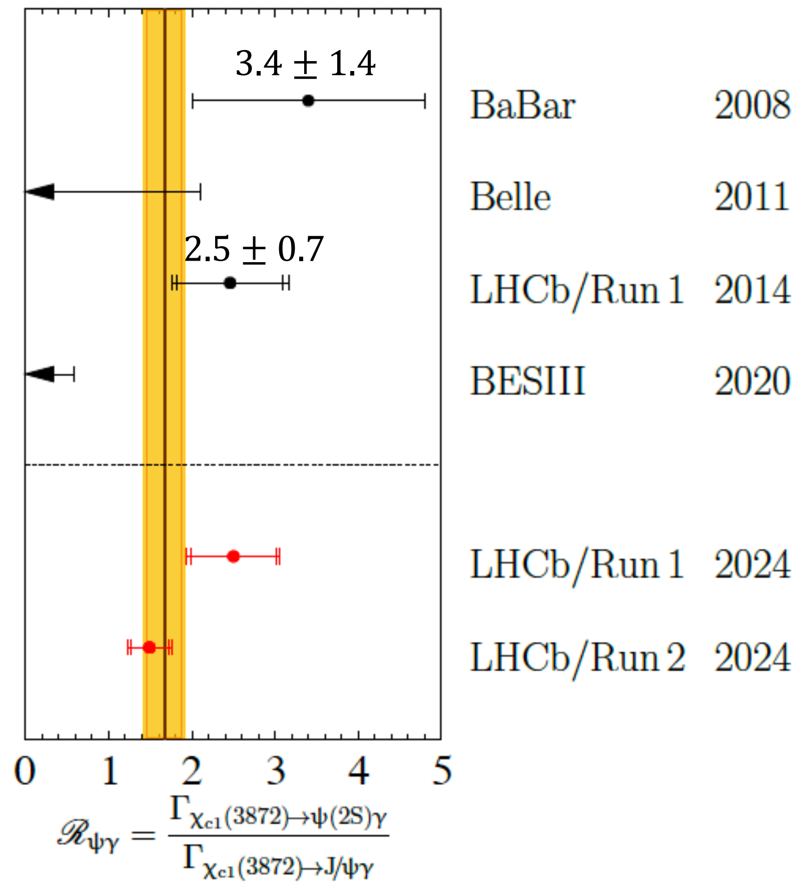


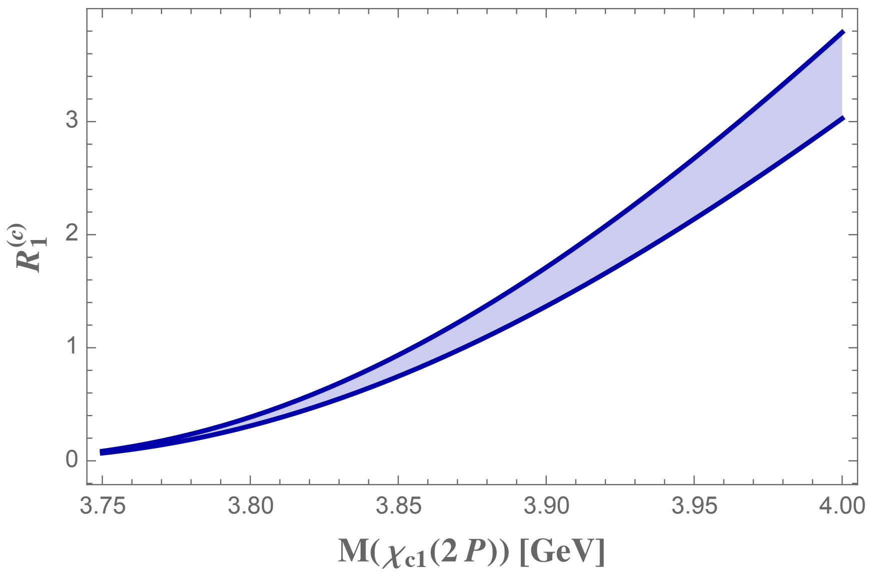
$$\mathscr{R}_{\psi\gamma} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04$$

LHCb, 2406.17006

Results: 2P to 1S, 2S transitions

$$R_J^{(c)} = \frac{\Gamma(\chi_{cJ}(2P) \to \psi(2S)\gamma)}{\Gamma(\chi_{cJ}(2P) \to J/\psi\gamma)}$$

what if $\chi_{c1}(2P)$ **is not** $\chi_{c1}(3872)$?



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probes the identification of Z(3930) with $\chi_{c2}(2P)$

Other insights on the structure of $\chi_{c1}(3872)$: semileptonic B_c decays to charmonia

- $B_c \to \eta_c$, J/ψ — 1S-wave charmonia — $J^{PC}=(0^{-+},1^{--})$
- $B_c \to \chi_{c0}, \chi_{c1}, \chi_{c2}$, h_c — 1P-wave charmonia — $J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$
- $B_c \to \chi'_{c0}, \chi'_{c1}, \chi'_{c2}$, h'_c — 2P-wave charmonia — $J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$

- Several form factors required
- Relations among them can be derived exploiting HQ symmetries + NRQCD methods

Semileptonic B_c decays to charmonium

$B_c \to \chi_{c0}$:

$$\langle \chi_{c0}(v')|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}} m_{B_c}}\,[g_+(w)(v+v')_\mu + g_-(w)(v-v')_\mu]$$
$$\langle \chi_{c0}(v')|\bar{c}\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}} m_{B_c}}\, g_P(w)$$
$$\langle \chi_{c0}(v')|\bar{c}\sigma_{\mu\nu} b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}} m_{B_c}}\, g_T(w)\,\epsilon_{\mu\nu\alpha\beta}v^\alpha v'^\beta$$

$B_c \to h_c$:

$$\langle h_c(v',\epsilon)|\bar{c}\gamma_\mu b|B_c(v)\rangle = \sqrt{m_{h_c} m_{B_c}}\Big[f_{V_1}(w)\epsilon^*_\mu + (\epsilon^*\cdot v)\big(f_{V_2}(w)(v+v')_\mu + f_{V_3}(w)(v-v')_\mu\big)\Big]$$
$$\langle h_c(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = i\sqrt{m_{h_c} m_{B_c}}\, f_A(w)\,\epsilon_{\mu\alpha\beta\sigma}\epsilon^{*\alpha}v^\beta v'^\sigma$$
$$\langle h_c(v',\epsilon)|\bar{c}b|B_c(v)\rangle = \sqrt{m_{h_c} m_{B_c}}\,(\epsilon^*\cdot v) f_S(w)$$
$$\langle h_c(v',\epsilon)|\bar{c}\sigma_{\mu\nu} b|B_c(v)\rangle = i\sqrt{m_{h_c} m_{B_c}}\Big[f_{T_1}(w)\big(\epsilon^*_\mu(v+v')_\nu - \epsilon^*_\nu(v+v')_\mu\big) + f_{T_2}(w)\big(\epsilon^*_\mu(v-v')_\nu - \epsilon^*_\nu(v-v')_\mu\big) + f_{T3}(w)\,(\epsilon^*\cdot v)\,\big(v_\mu v'_\nu - v_\nu v'_\mu\big)\Big].$$

$B_c \to \chi_{c1}$:

$$\langle \chi_{c1}(v',\epsilon)|\bar{c}\gamma_\mu b|B_c(v)\rangle = i\sqrt{m_{\chi_{c1}} m_{B_c}}\Big[g_{V_1}(w)\epsilon^*_\mu + (\epsilon^*\cdot v)[g_{V_2}(w)(v+v')_\mu + g_{V_3}(w)(v-v')_\mu]\Big]$$
$$\langle \chi_{c1}(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c1}} m_{B_c}}\, g_A(w)\,\epsilon_{\mu\alpha\beta\sigma}\epsilon^{*\alpha}v^\beta v'^\sigma$$
$$\langle \chi_{c1}(v',\epsilon)|\bar{c}b|B_c(v)\rangle = i\sqrt{m_{\chi_{c1}} m_{B_c}}\, g_S(w)\,(\epsilon^*\cdot v)$$
$$\langle \chi_{c1}(v',\epsilon)|\bar{c}\sigma_{\mu\nu} b|B_c(v)\rangle = \sqrt{m_{\chi_{c1}} m_{B_c}}\Big[g_{T_1}(w)\big(\epsilon^*_\mu(v+v')_\nu - \epsilon^*_\nu(v+v')_\mu\big) + g_{T_2}(w)\big(\epsilon^*_\mu(v-v')_\nu - \epsilon^*_\nu(v-v')_\mu\big) + g_{T_3}(w)(\epsilon^*\cdot v)(v_\mu v'_\nu - v_\nu v'_\mu)\Big]$$

$B_c \to \chi_{c2}$:

$$\langle \chi_{c2}(v',\eta)|\bar{c}\gamma_\mu b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}} m_{B_c}}\, i\, k_V(w)\,\epsilon_{\mu\alpha\beta\sigma}\eta^{*\alpha\tau}v_\tau v^\beta v'^\sigma$$
$$\langle \chi_{c2}(v',\eta)|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}} m_{B_c}}\,\big[k_{A_1}(w)\,\eta^*_{\mu\alpha}v^\alpha + \eta^*_{\alpha\beta}v^\alpha v^\beta\,\big(k_{A_2}(w)v_\mu + k_{A_3}(w)v'_\mu\big)\big]$$
$$\langle \chi_{c2}(v',\eta)|\bar{c}\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}} m_{B_c}}\, k_P(w)\,\eta^*_{\alpha\beta}v^\alpha v^\beta$$
$$\langle \chi_{c2}(v',\eta)|\bar{c}\sigma_{\mu\nu}\gamma_5 b|B_c(v)\rangle = i\sqrt{m_{\chi_{c2}} m_{B_c}}\,\big[k_{T_1}(w)(\eta^{*\alpha}_{\ \mu}v_\alpha v_\nu - \eta^{*\alpha}_{\ \nu}v_\alpha v_\mu) + k_{T_2}(w)(\eta^{*\alpha}_{\ \mu}v_\alpha v'_\nu - \eta^{*\alpha}_{\ \nu}v_\alpha v'_\mu) + k_{T_3}(w)\eta^*_{\alpha\beta}v^\alpha v^\beta(v_\mu v'_\nu - v_\nu v'_\mu)\big]$$

SM

NP

$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ Form Factors in the effective theory: relations at $O(1/m_Q)$

- relations among the form factors of the same decay mode

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- $B_c \to \chi_{c0}$

$$g_T(w) = -\frac{1}{w+1}\left[2g_-(w) + g_P(w)\right]$$

- $B_c \to \chi_{c1}$

$$g_{T_2}(w) = -\frac{1}{2}\left[g_{V_1}(w) - (1+w)g_A(w)\right]$$

$$g_{T_3}(w) = -\frac{1}{2(w-1)}\left[g_{V_1}(w) + 4g_{V_2}(w)\right] + \frac{1}{2}g_A(w) + \frac{1}{w-1}\left[g_S(w) + g_{T_1}(w)\right]$$

- $B_c \to \chi_{c2}$

$$k_{T_1}(w) = -wk_V(w) + k_{A_2}(w) + wk_{A_3}(w) + k_P(w)$$

$$k_{T_2}(w) = k_V(w) - k_{A_1}(w) - k_{A_2}(w) - wk_{A_3}(w) - k_P(w)$$

$$k_{T_3}(w) = -k_V(w) + k_{A_3}(w)$$

- $B_c \to h_c$

$$f_{T_2}(w) = \frac{1}{2}\left[f_{V_1}(w) + (1+w)f_A(w)\right]$$

$$f_{T_3}(w) = \frac{1}{2(w-1)}\left[f_{V_1}(w) + 4f_{V_2}(w)\right] + \frac{1}{2}f_A(w) - \frac{1}{w-1}\left[f_S(w) - f_{T_1}(w)\right]$$

$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ Form Factors in the effective theory: relations at LO

χ_{c0}

χ_{c1}

χ_{c2}

h_c

$$g_+(w) = 0$$

$$g_S(w) = g_{T_1}(w) = 0$$

$$k_{A_2}(w) = k_{T_3}(w) = 0$$

$$f_{V_1}(w) = f_{V_3}(w) = f_A(w) = f_{T_1}(w) = f_{T_2}(w) = 0$$

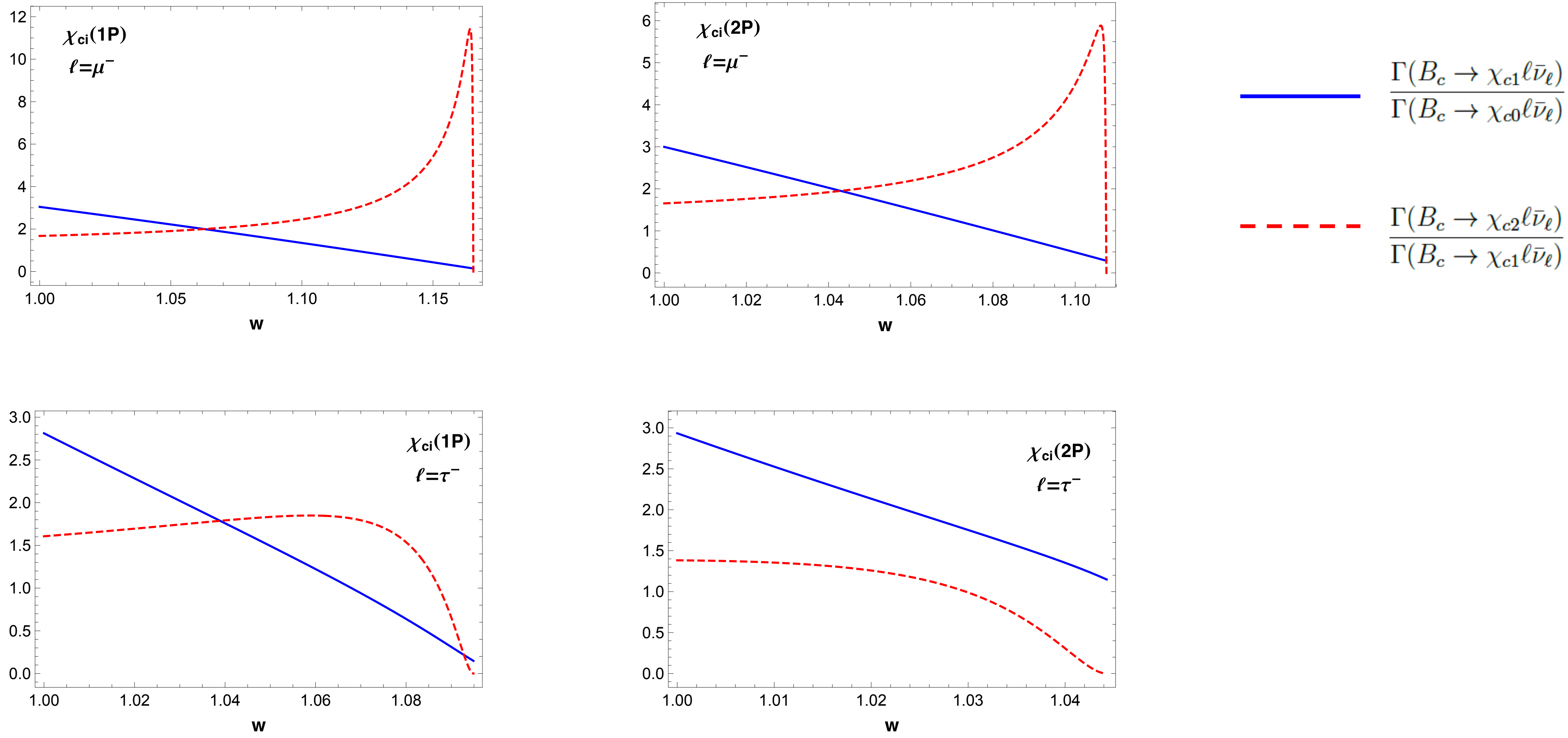
$$\begin{aligned}
\Xi(w) &= \frac{\sqrt{3}}{(w+1)} g_-(w) = -\frac{\sqrt{3}}{(w+1)} g_T(w) = \frac{\sqrt{3}}{(w^2-1)} g_P(w) \\
&= \frac{\sqrt{2}}{(w^2-1)} g_{V_1}(w) = -\frac{2\sqrt{2}}{(w-1)} g_{V_2}(w) = \frac{2\sqrt{2}}{(w+1)} g_{V_3}(w) = \frac{\sqrt{2}}{(w+1)} g_A(w) = \frac{\sqrt{2}}{(w+1)} g_{T_2}(w) \\
&= -k_V(w) = \frac{1}{w+1} k_{A_1}(w) = -k_{A_3}(w) = -k_P(w) = -k_{T_1}(w) = -k_{T_2}(w) \\
&= -f_{V_1}(w) = -f_{V_2}(w) = -\frac{1}{w+1} f_S(w) = f_{T_3}(w)
\end{aligned}$$

analogous to the IW function

$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at LO

$$\frac{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c0}\ell\bar{\nu})/dw} \qquad \frac{d\Gamma(B_c\to\chi_{c2}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}$$

→ the universal function cancels in the ratio



$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at LO

- constraint at LO both in SM and for generic NP

$$2\frac{d\Gamma}{dw}(B_c \to \chi_{c0}\ell\bar{\nu}_\ell) + \frac{d\Gamma}{dw}(B_c \to \chi_{c1}\ell\bar{\nu}_\ell) - \frac{d\Gamma}{dw}(B_c \to \chi_{c2}\ell\bar{\nu}_\ell) = 0.$$

to be satisfied by the three members of the 4-plet

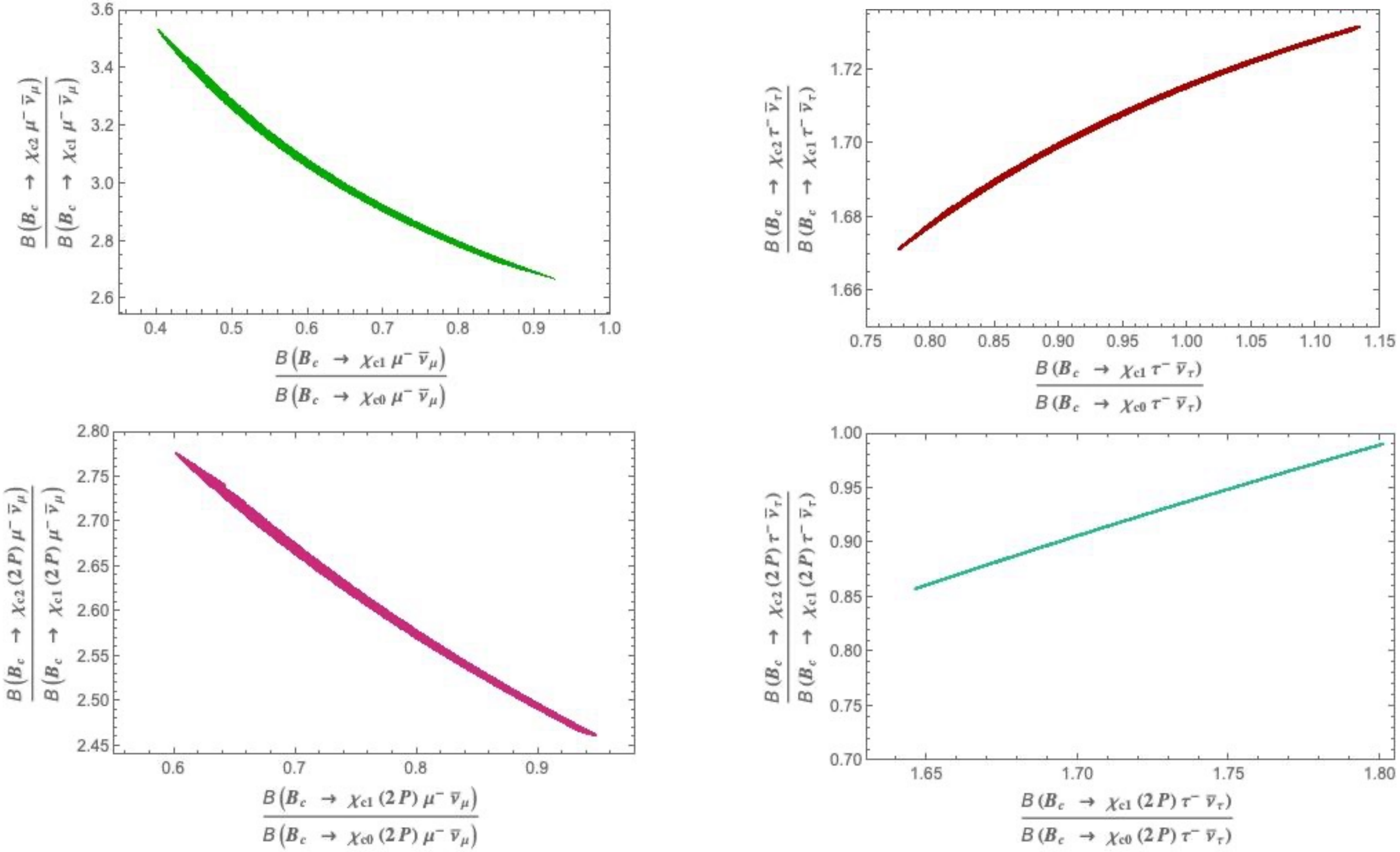
$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at LO

parametrization: $\Xi(w) = \Xi_0 + \Xi_1(w-1) + \Xi_2(w-1)^2$

$\Xi_0 \in [0.1, 1]$, $\Xi_1 \in [-1, 0]$ and $\Xi_2 \in [-1, 1]$

fulfill $\mathcal{B}(B_c^+ \to \chi_{c0}\pi^+) = (2.4^{+0.9}_{-0.8}) \times 10^{-5}$

correlations predicted:



Perspectives

Radiative decays useful to answer the question:

identifying χ_{c1} (3872) with χ_{c1}(2P) is correct?

my strategy:

- use a QCD rooted approach: HQ limit
- work out predictions for χ_{c1}(2P) assigning M(χ_{c1}(2P))=3872 MeV and check with data
- ratio of radiative decays predicted (2024)

$$R_1^{(c)} = \frac{\Gamma(\chi_{c1}(2P) \to \psi(2S)\gamma)}{\Gamma(\chi_{c1}(2P) \to J/\psi\gamma)} = 1.2 \pm 0.2 \ \ (1.62 \pm 0.74)$$

- predictions for other established ordinary states worked out with the same approach
  → check of the accuracy of the method
- another suggestion: look at semileptonic B_c decays to charmonia
- improvements foreseen: inclusion of $1/m_Q$ corrections (work in progress)