Tetraquarks: Successes & Mysteries

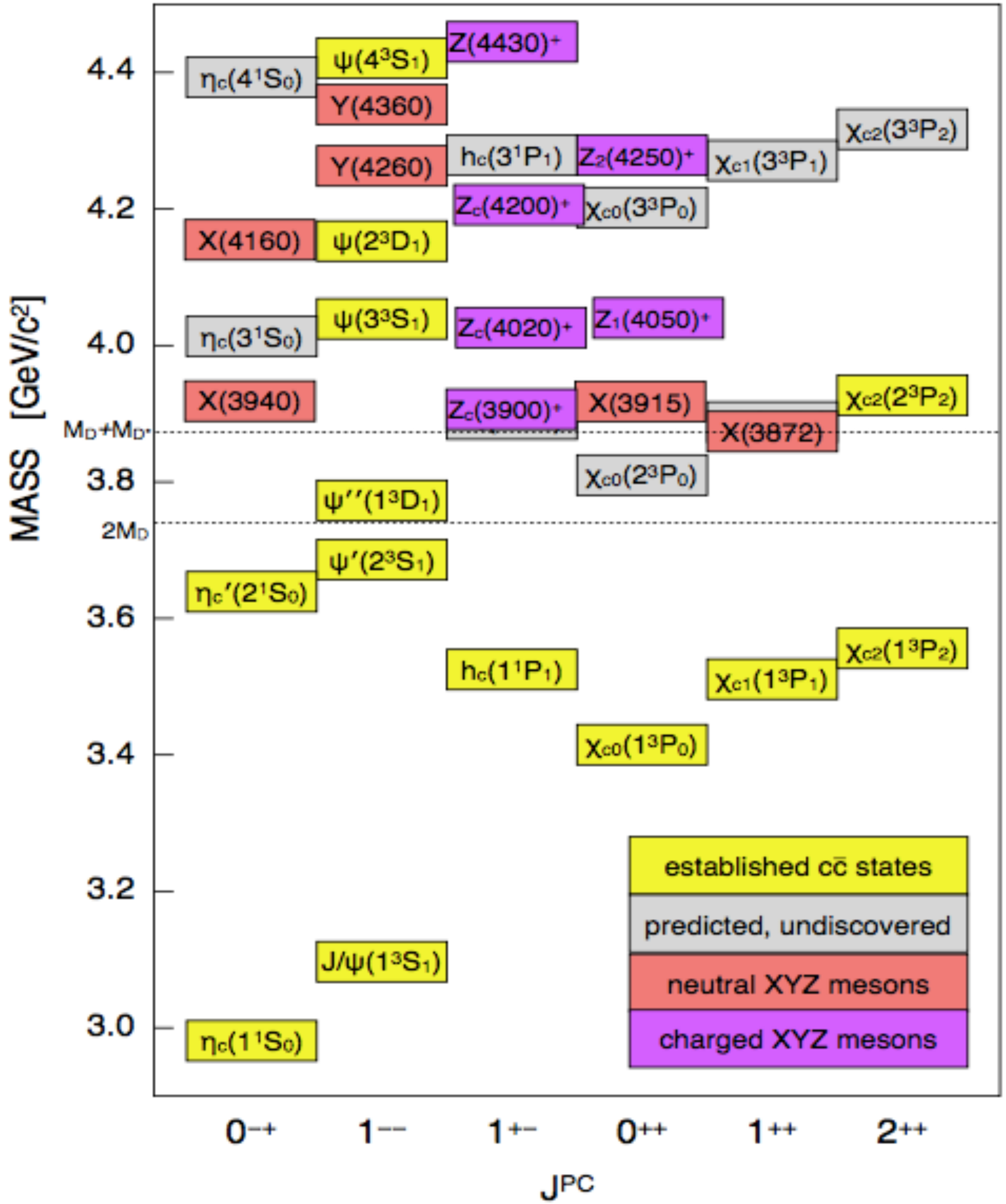
Luciano Maiani, CERN

starring:

- LHCb, BES II/III
- (not many) theorists aficionados to hadron spectrocopy

1. Expected and Unexpected Charmonia

figure by:
S. L. Olsen (2015)
arXiv:1511.01589



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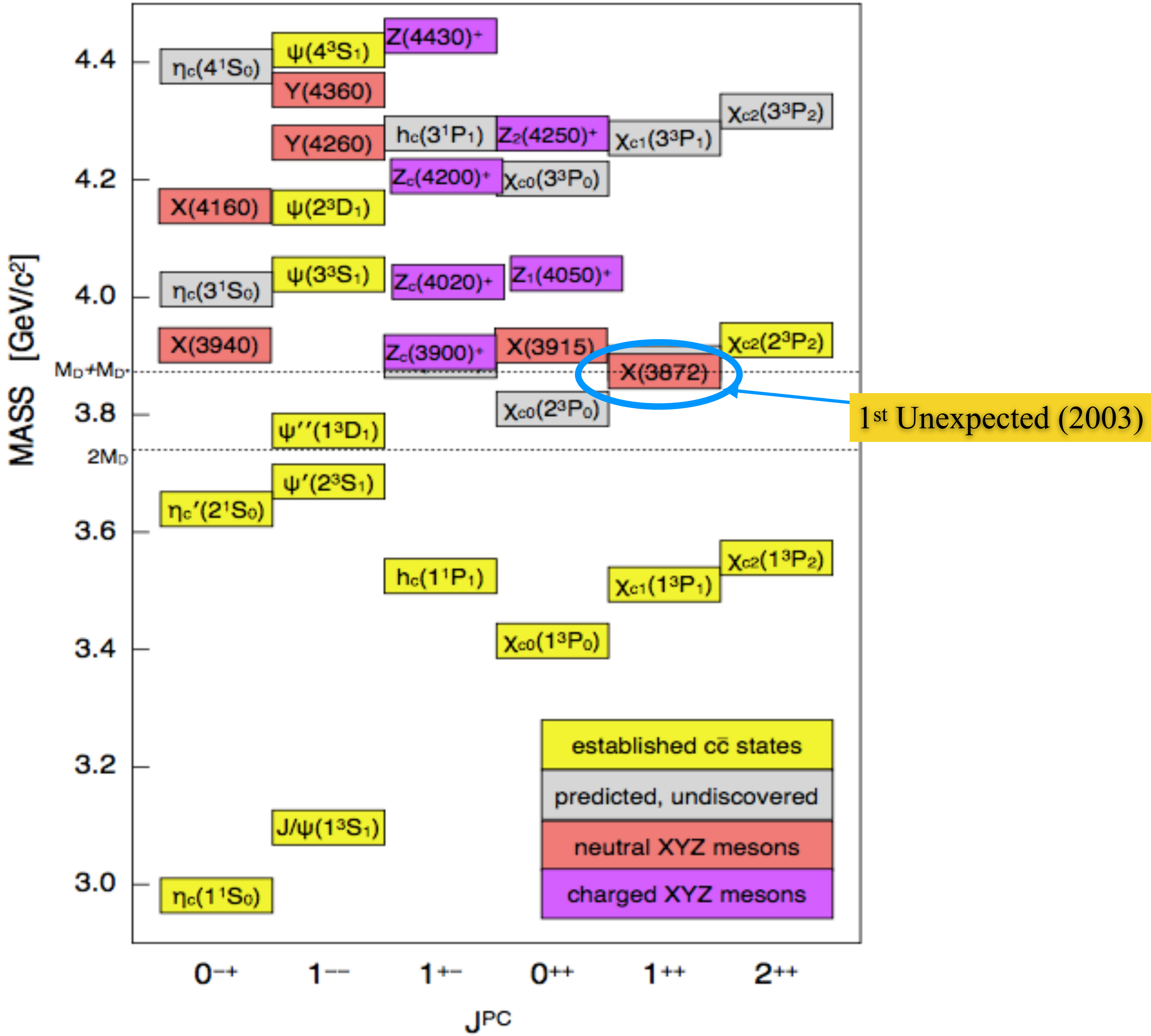

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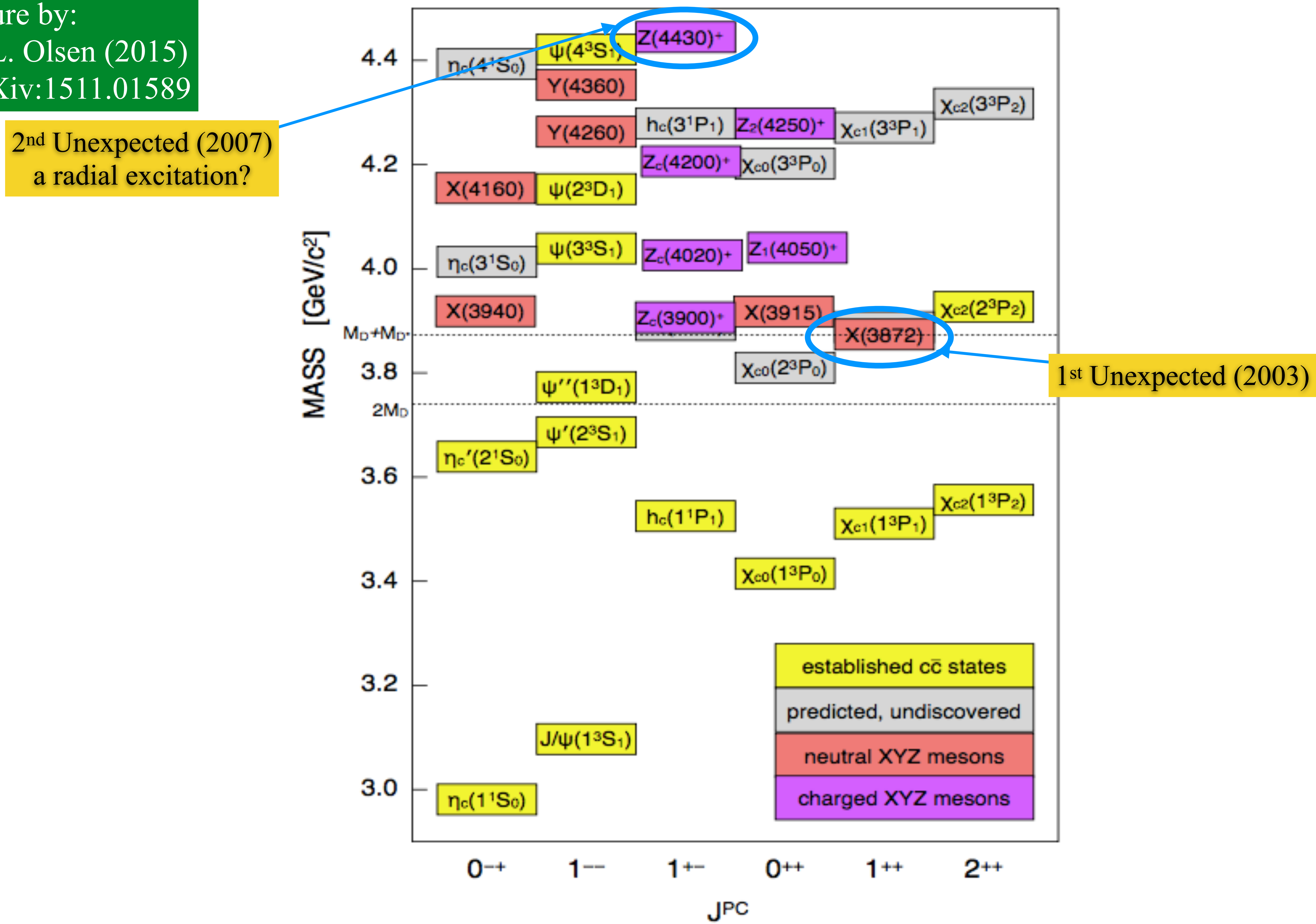

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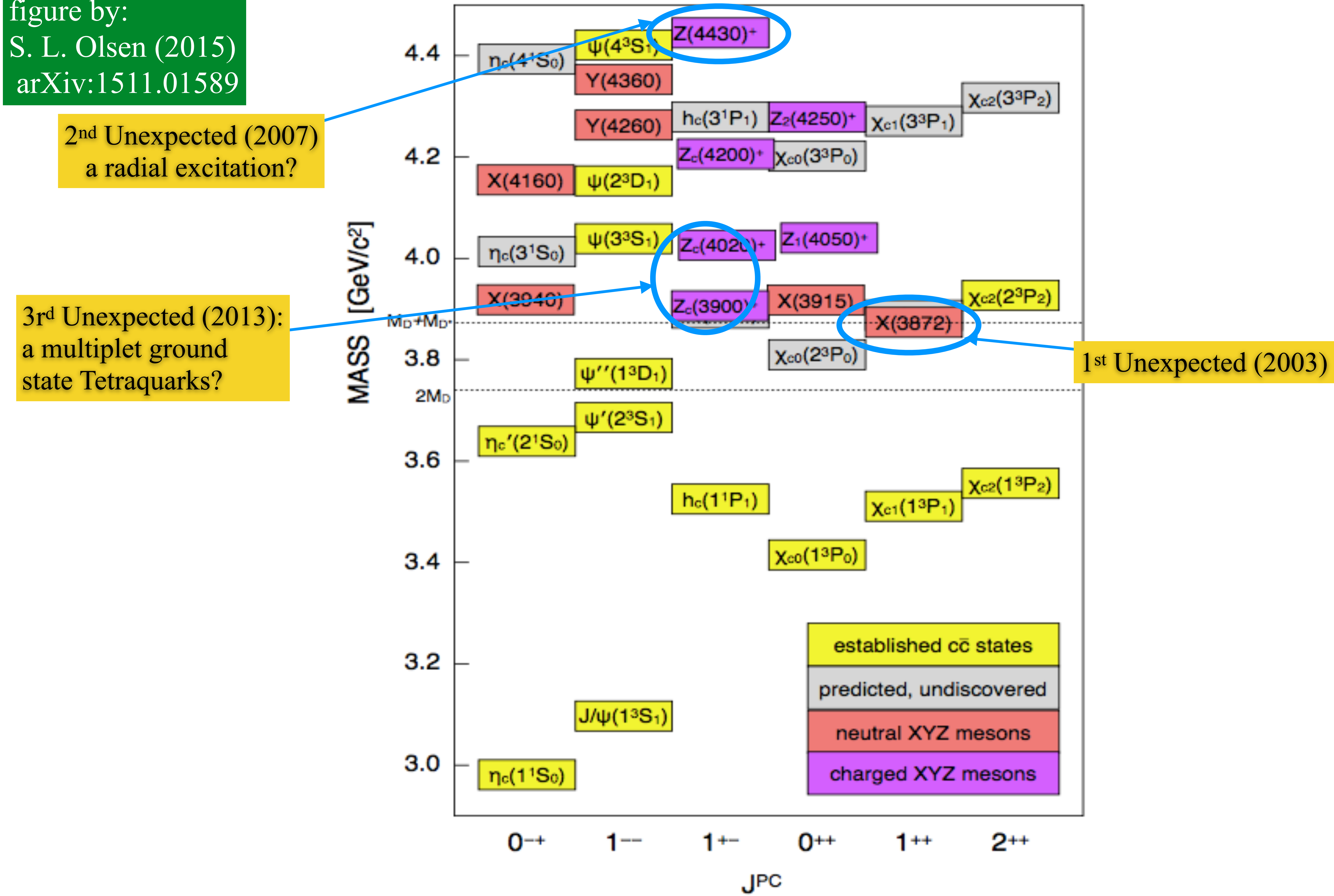

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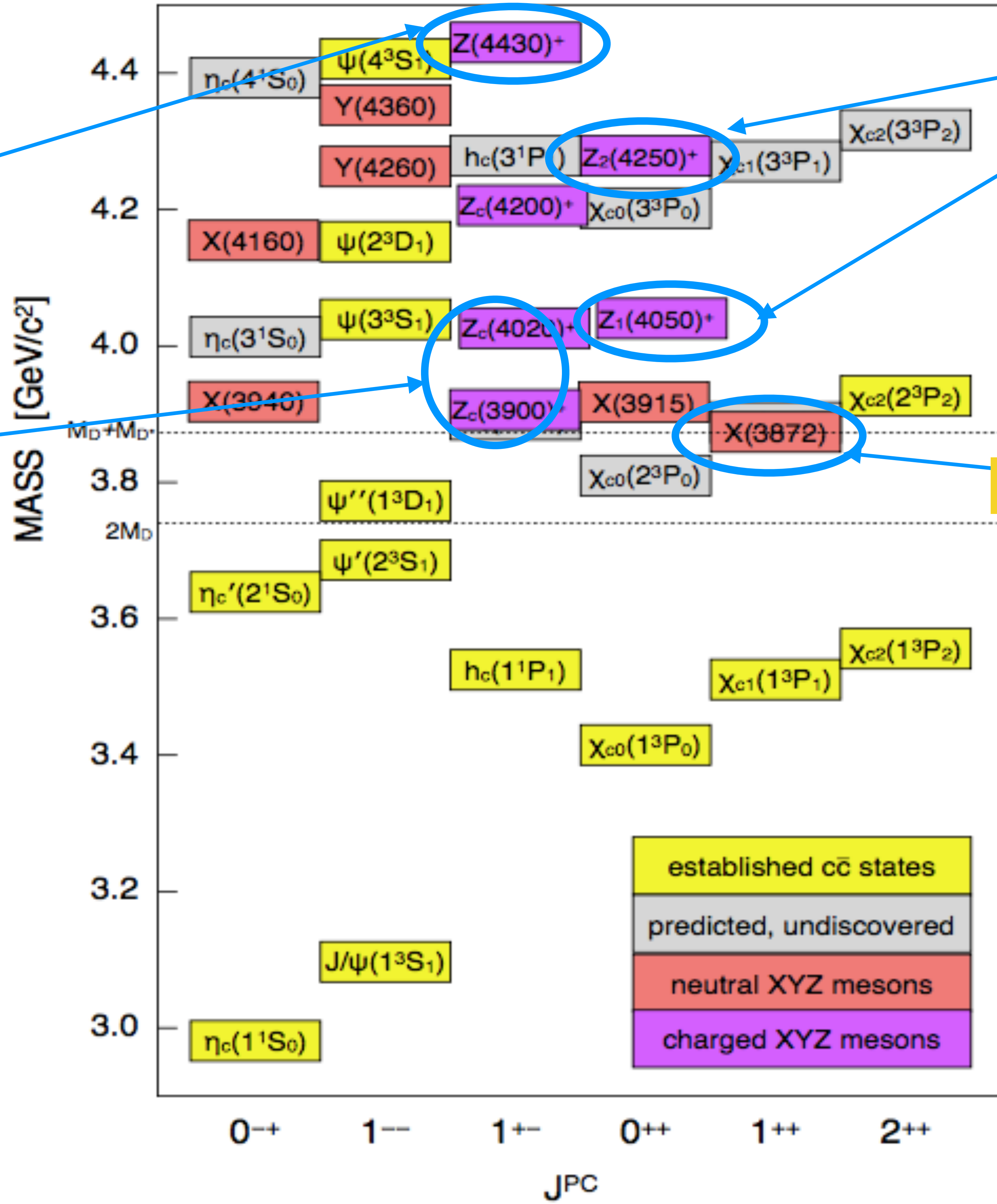
recent additions:
more than coincidence?
or
an almost filled multiplet?

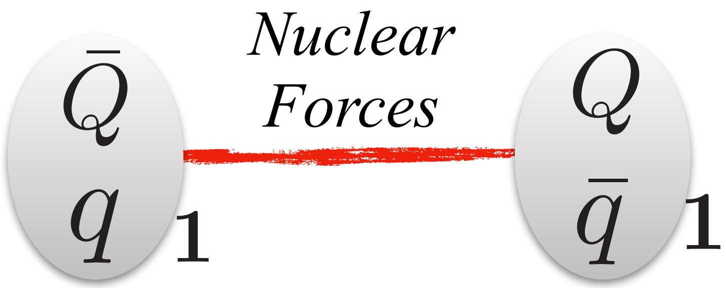
2nd Unexpected (2007)
a radial excitation?

3rd Unexpected (2013):
a multiplet ground
state Tetraquarks?

1st Unexpected (2003)

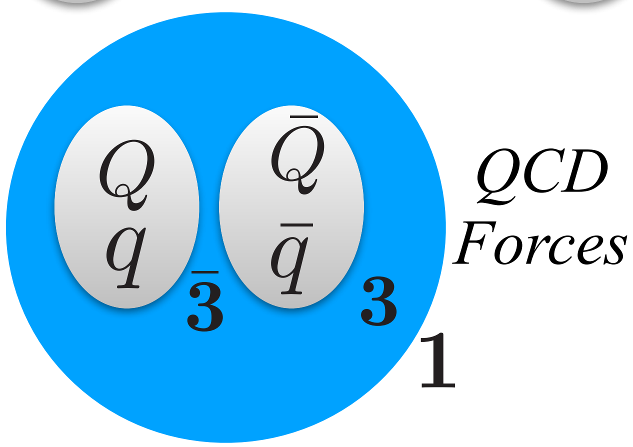


Twenty years after X(3872) discovery, there is no consensus yet on the internal structure of Exotic hadrons



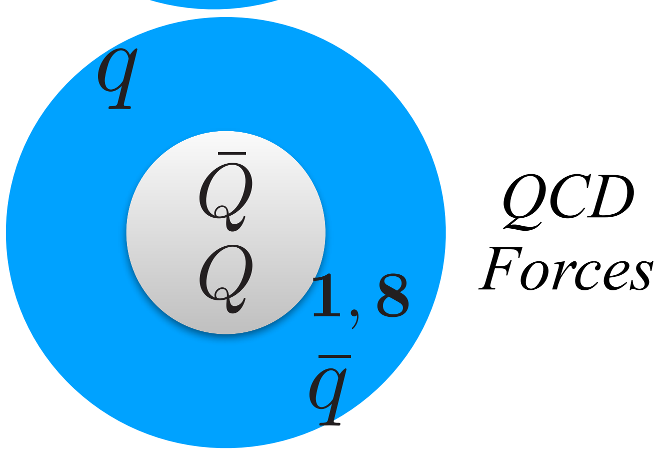
Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005) 014028; **D 89** (2014) 114010.



HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B.Voloshin, Phys. Lett. **B 666**,(2008) 344.

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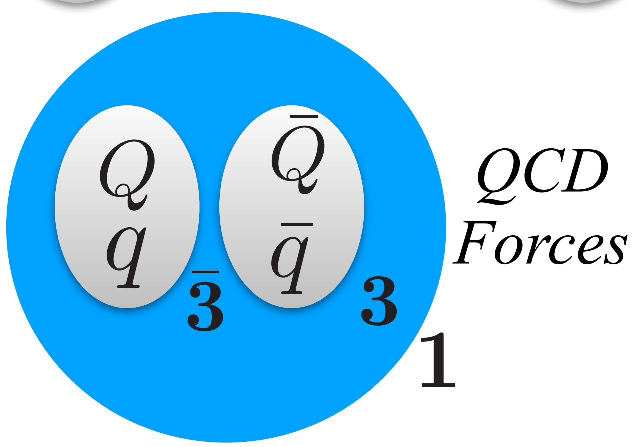

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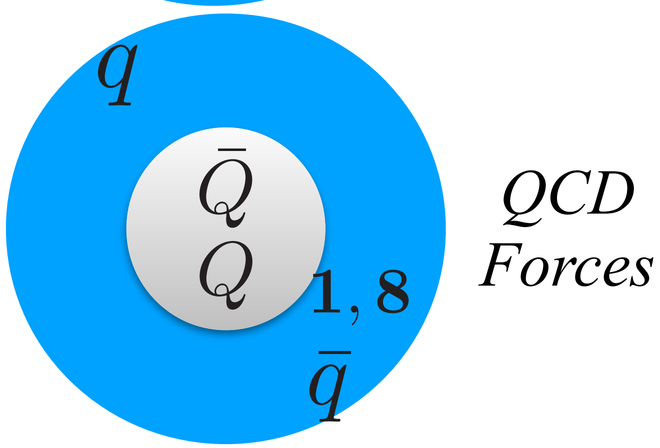

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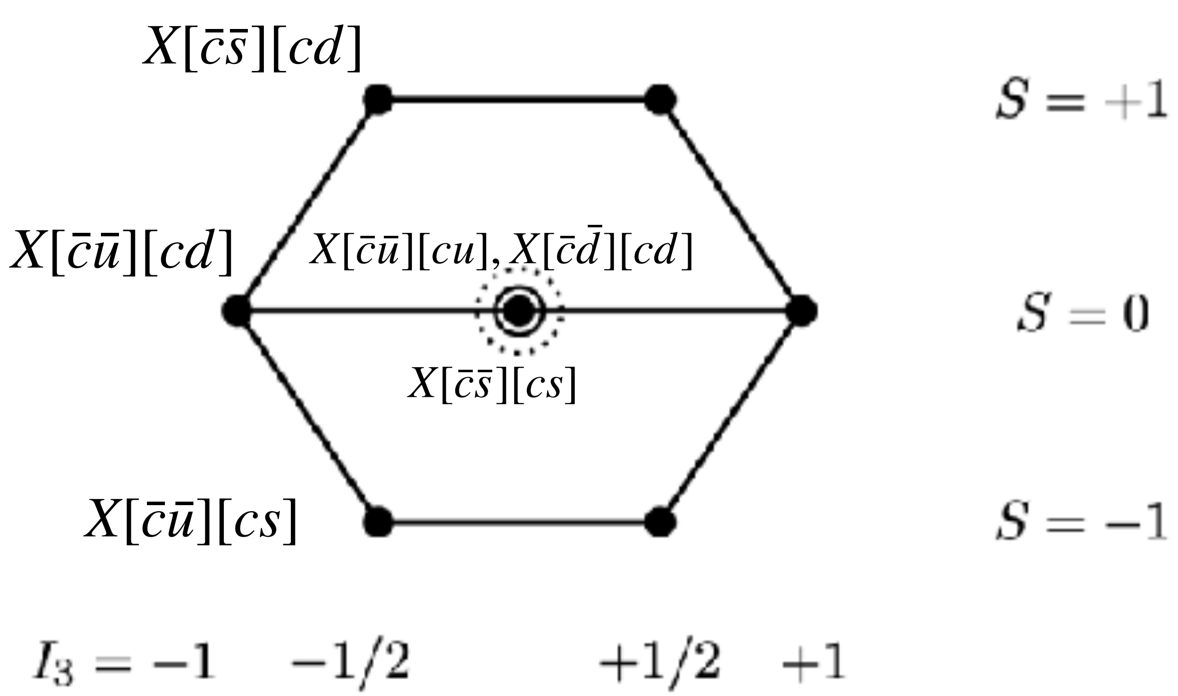
- The compact diquark-antidiquark model makes ***a firm quantitative prediction***: tetraquarks must form ***complete multiplets of flavor SU(3)*** with mass differences determined by the quark mass difference $m_s - m_{u,d}$.
- Comparing tetraquarks with different strangeness ***the rule can be tested and the structure can be deciphered***. A definite shopping list of the missing particles is today available.

2. Hidden Charm Tetraquarks

Hidden Charm Tetraquarks form ***nonets of flavor SU(3)*** with mass differences determined by the quark mass difference $m_s - m_{u,d}$

with Z_{cs}(3082), Z_{cs}(4003), Z_{cs}(4220) we can almost fill three tetraquark nonets with the expected scale of mass differences



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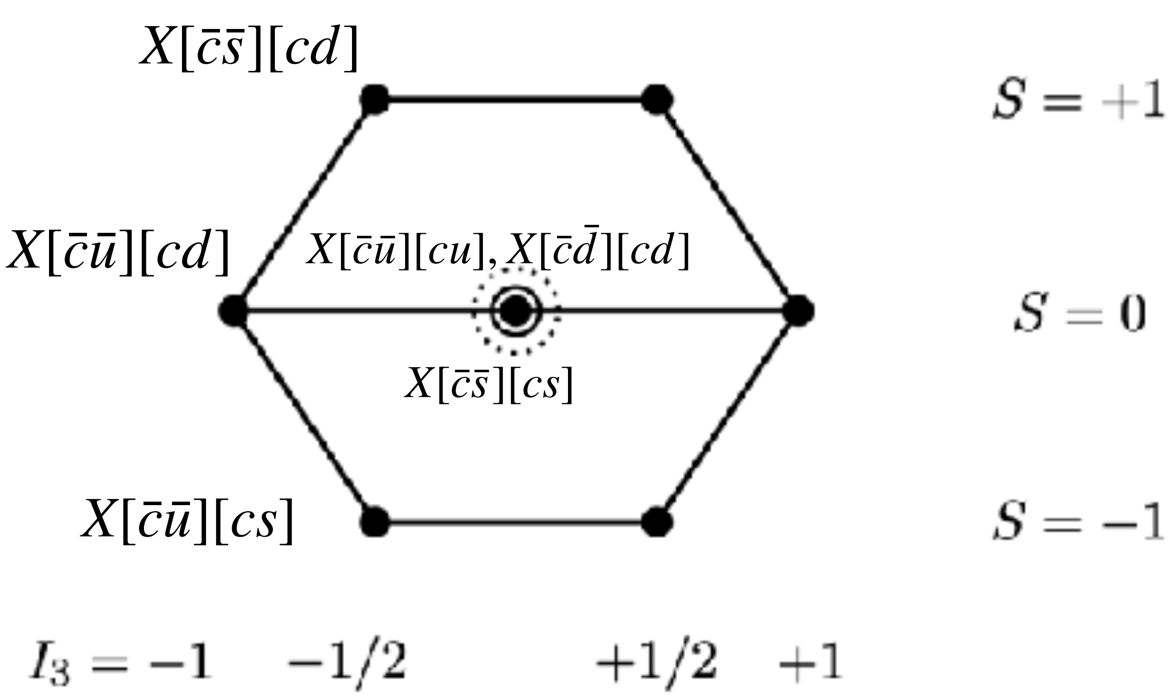
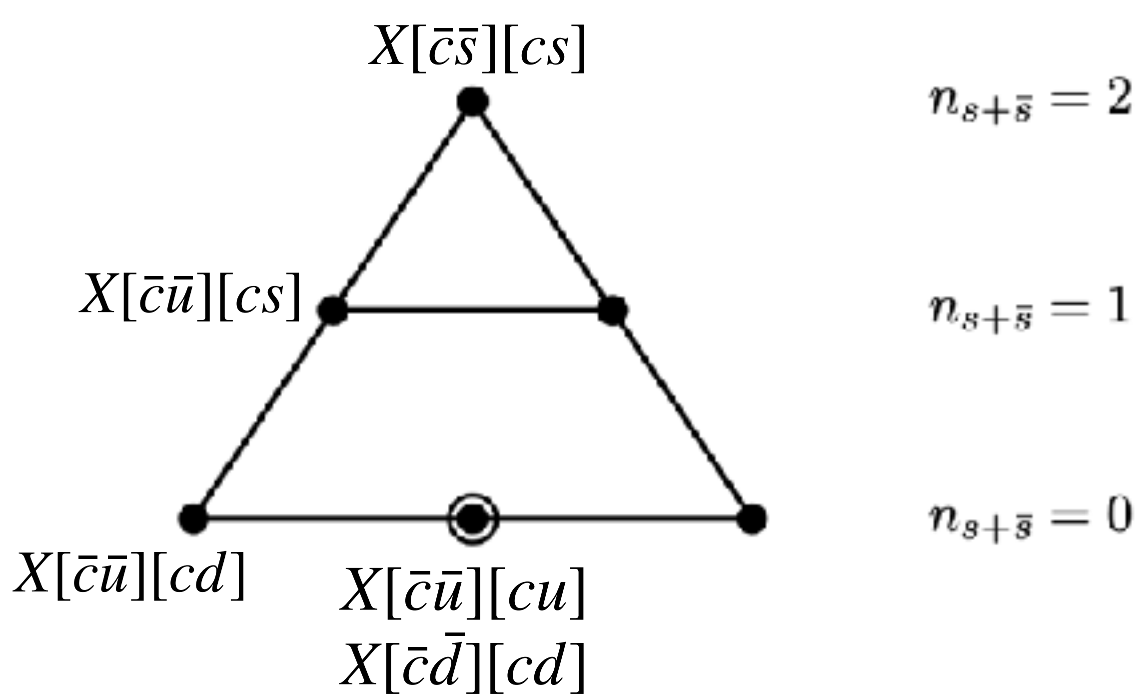
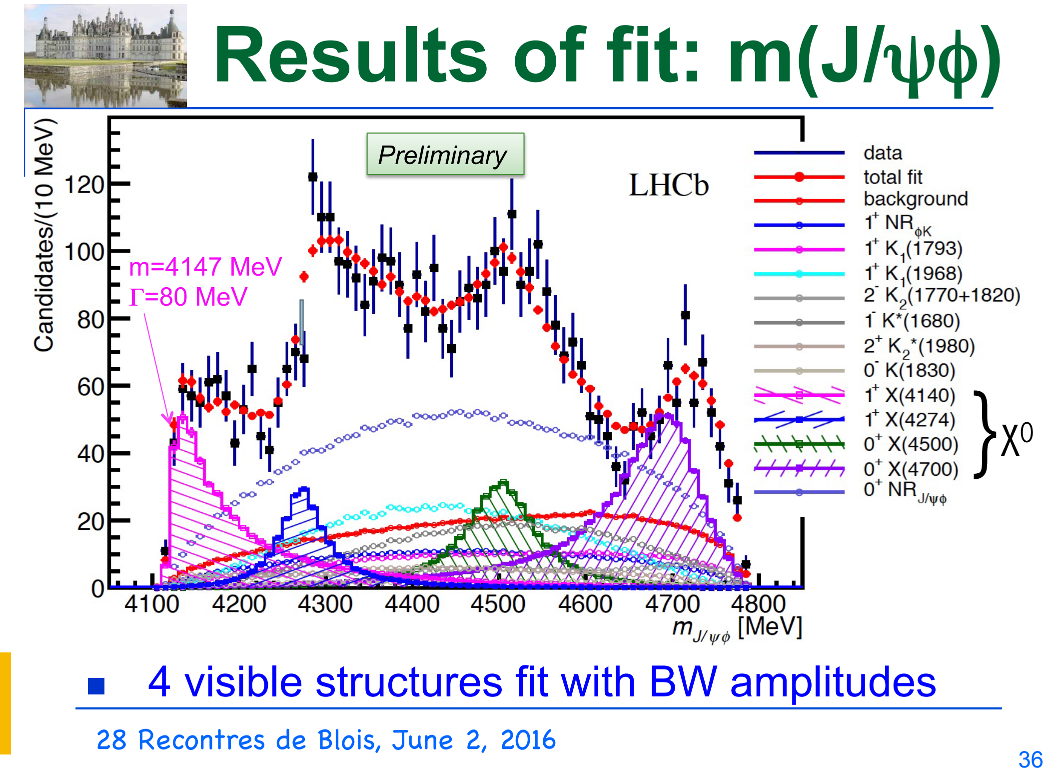




- Octet particles can be also represented in function of the total number of s or $\bar{s}$ quarks;
- octet breaking implies ***the equal spacing rule*** of the masses in the ladder.

Charm-strange exotics

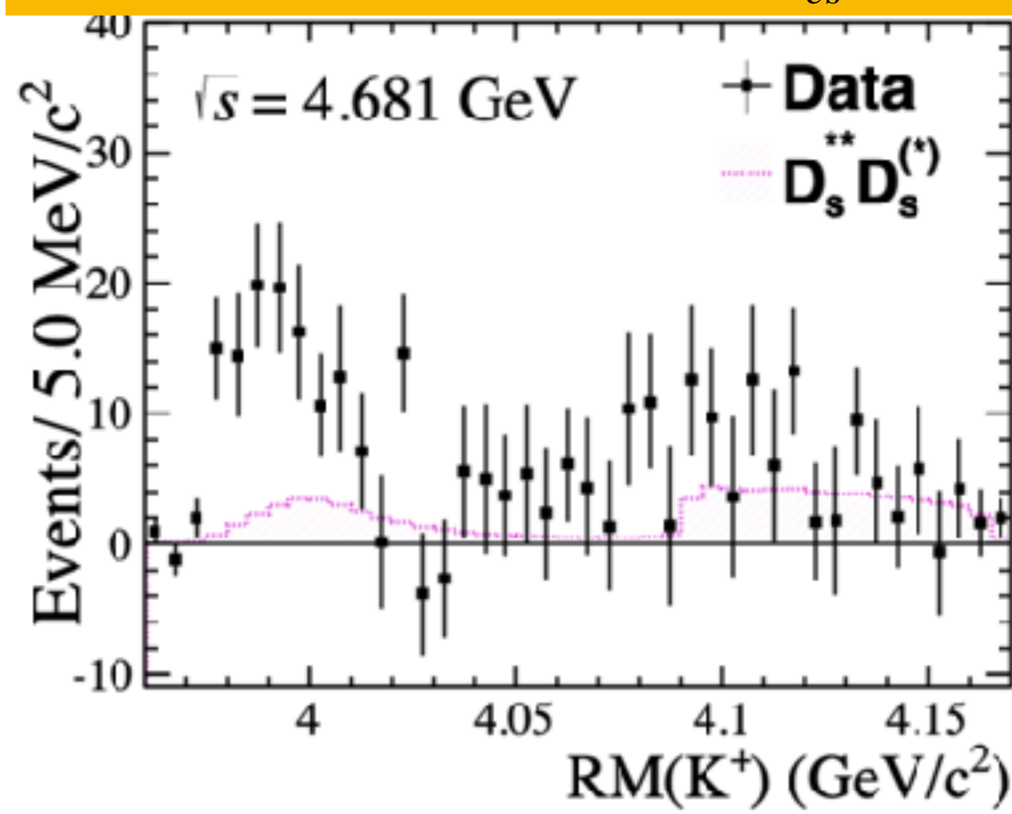
Hidden charm and strangeness and charm-strange tetraquarks have provided crucial steps forward.



LHCb (2016): Ψ ϕ resonances (***2016***)

$B^+ \to K^+ + X(4140) \to K^+ + \phi\ \Psi$, etc .

BES III (***2021***): $e^+e^- \to K^+ + Z_{cs}^-(3985) \to K^+(D_s^* D^0 + D_s^- D^{*0})$



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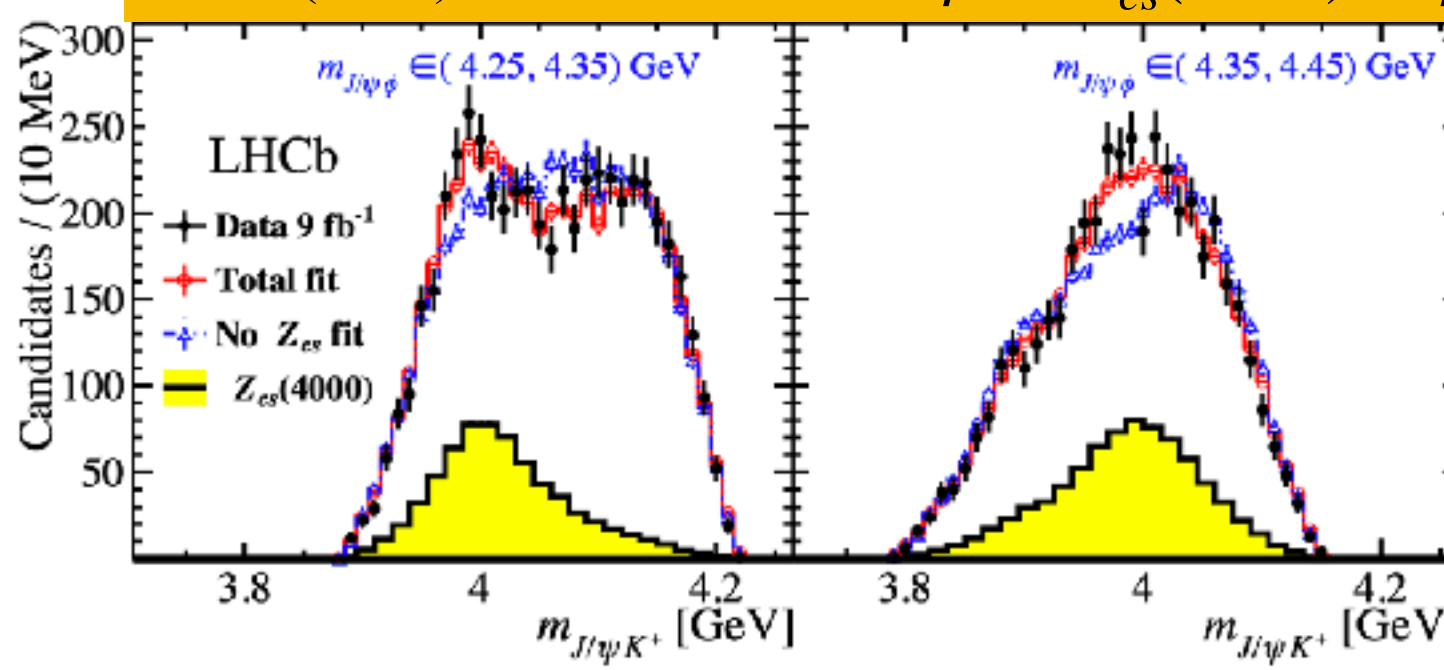

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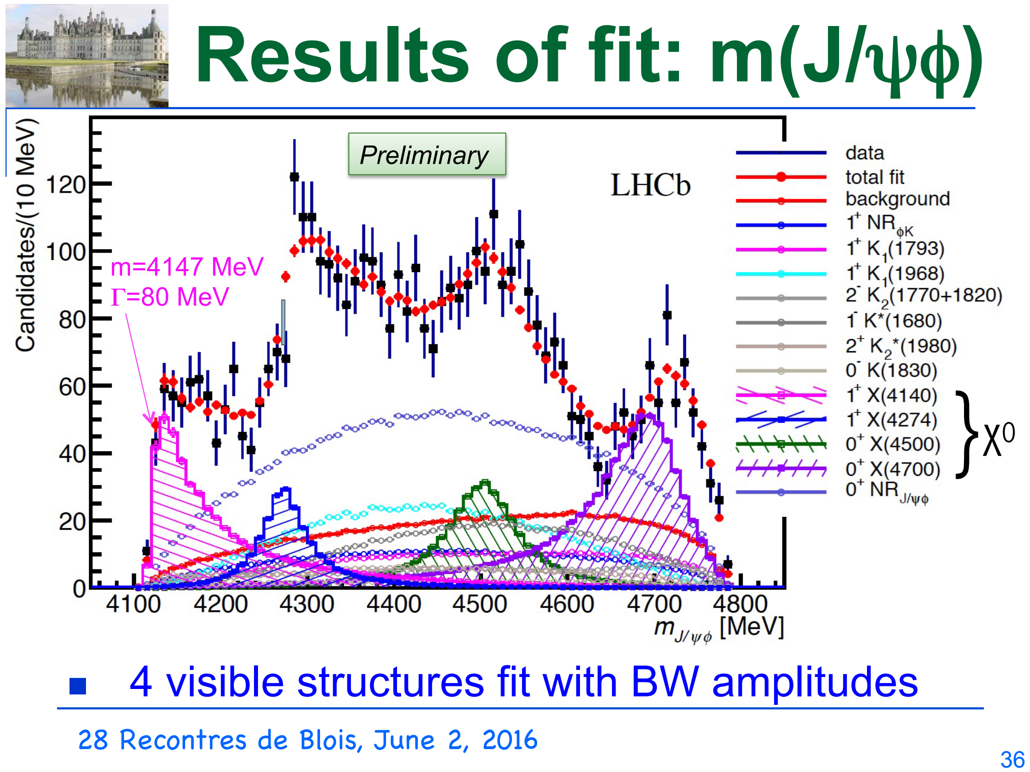

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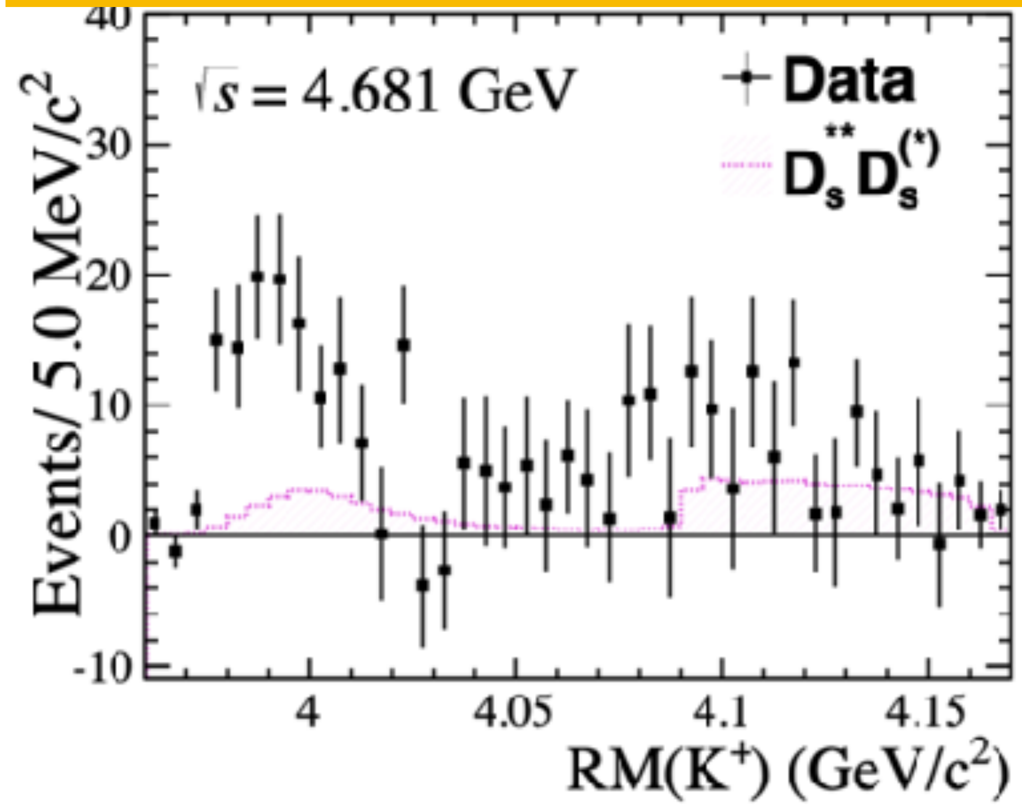

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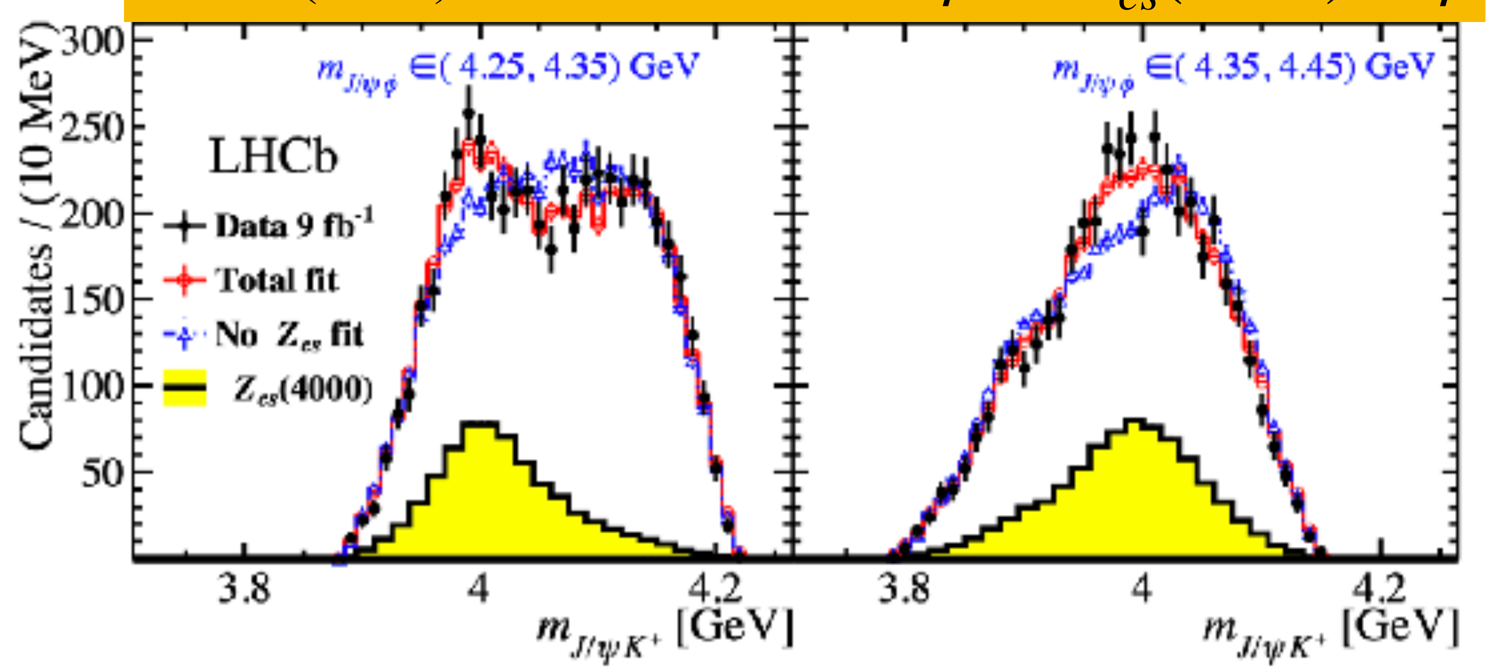


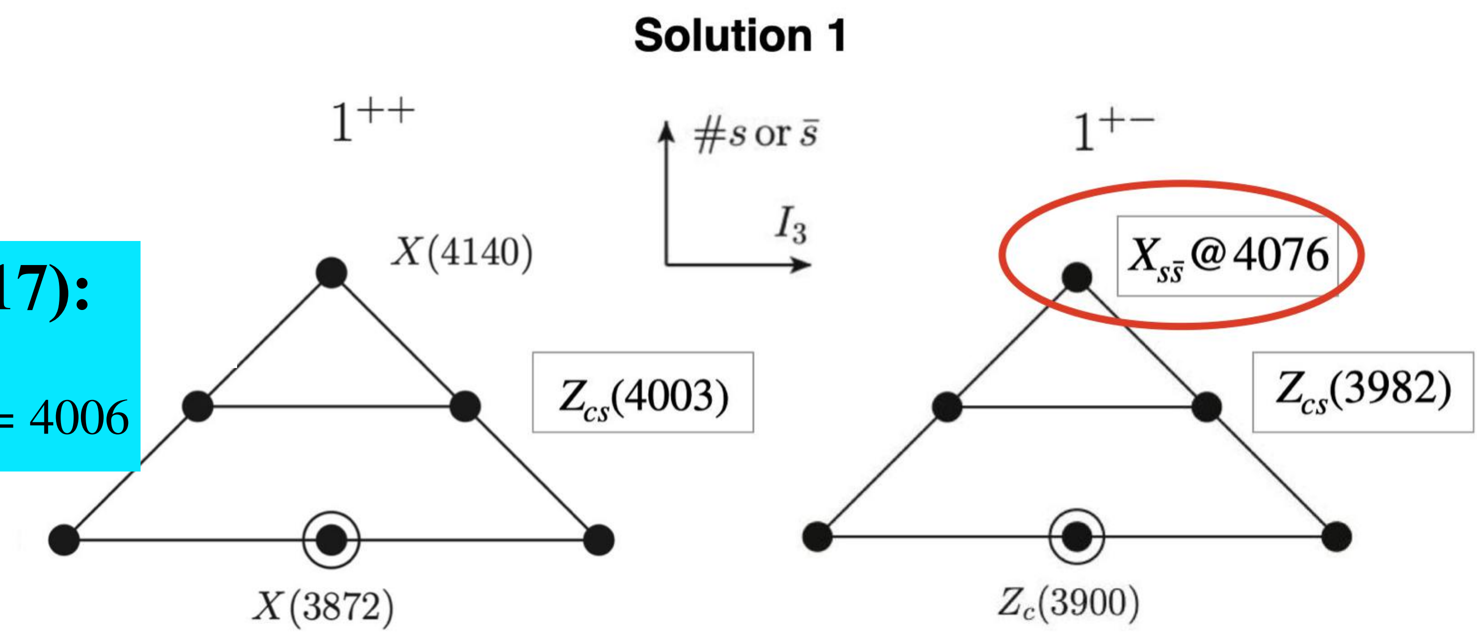
PdG: The incompatible values for the widths reported by AAIJ 2021E and ABLIKIM 2021G could either indicate the existence of two separate states or possibly be explained in a coupled channel model (see ORTEGA 2021

Two nonets: Solution 1 (preferred)

L. Maiani, A. D. Polosa and V. Riquer, Sci. Bull. **66** (2021), 1616, arXiv:2103.08331

Predicted (2017):

$$\frac{X(4140) + X(3872)}{2} = 4006$$



- There is a ***third nonet associated with*** $Z_c(4020)$, $J^{PC} = 1^{+-}$: a third Z_{cs} is required, with Mass=4150 - 4170
- LHCb sees a $Z_{cs}(4220)$, $J^P = 1^+$***: is it too heavy ?***
- If $Z_{cs}(4220)$ is $J^P = 1^+$, we have a bold proposal: ***two nonets mixing***

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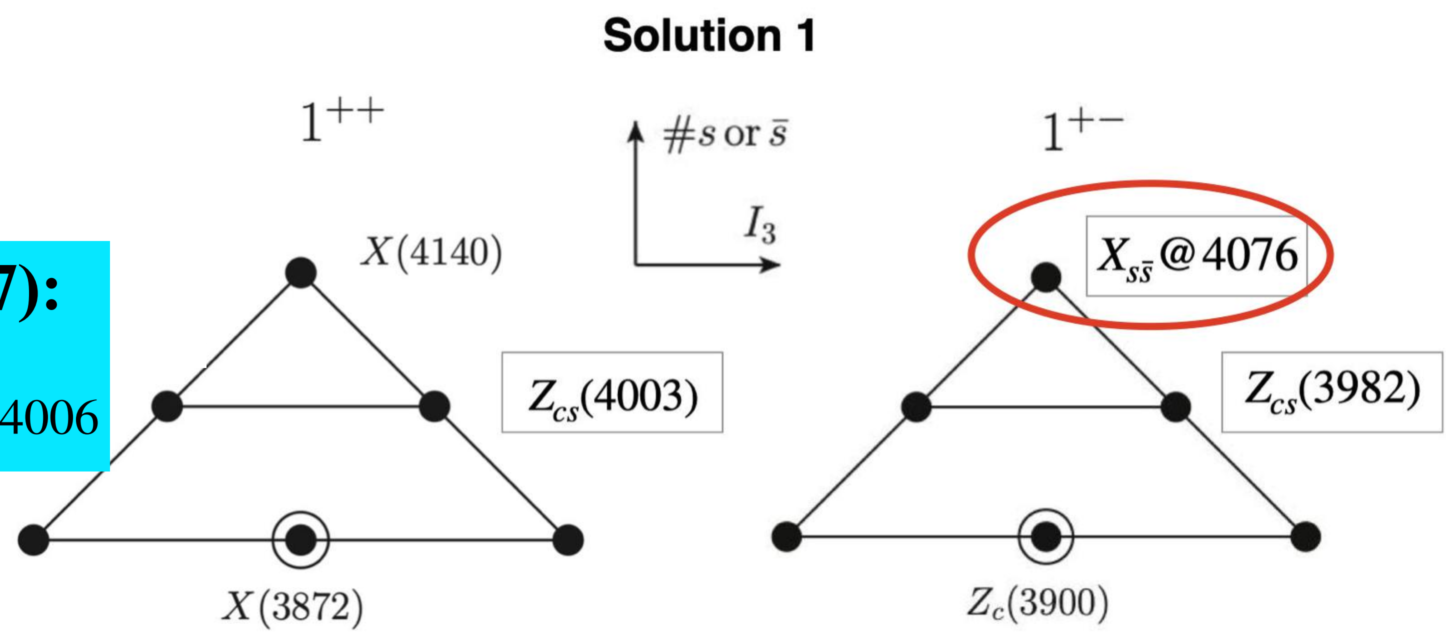

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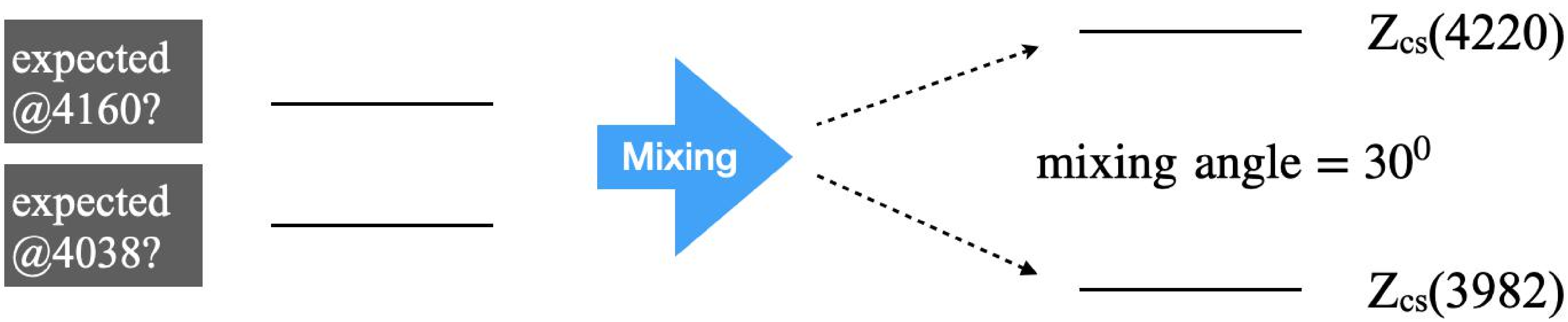
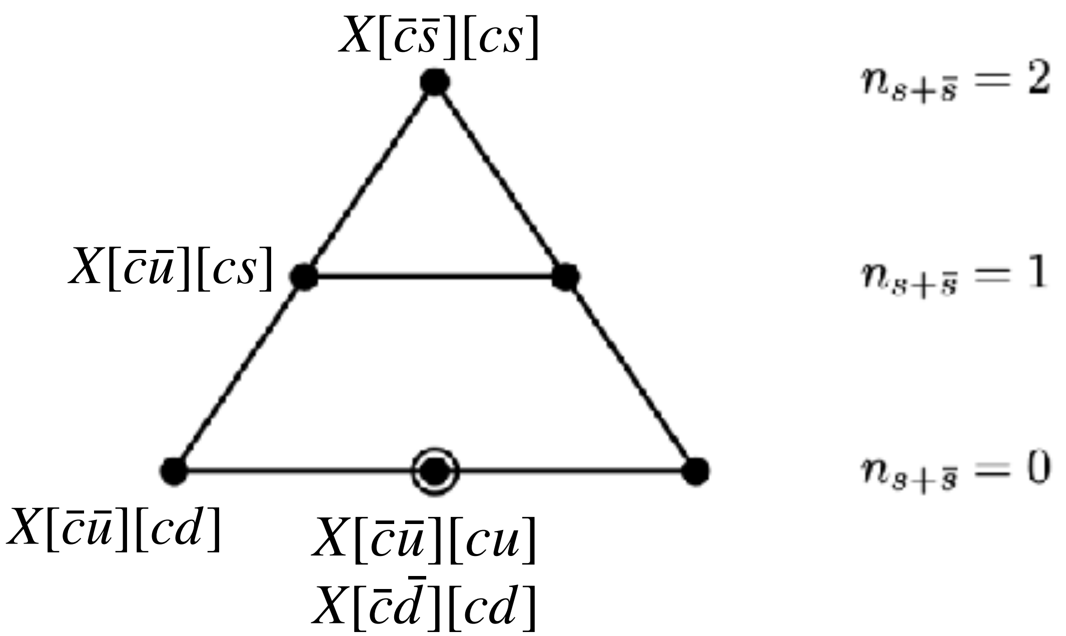


Missing particles to complete the hidden charm nonets, $J^P = 1^+$

The shopping list towards completion of the ***hidden charm nonets***:



- two $X_{[\bar{c}\bar{s}][cs]}$, expected at $M \sim 4076$ for $Z_c(3900)$ and $M \sim 4300$ for $Z_c(4020)$

decays: $\eta\psi$, $\eta_c\ \phi$, $D_s^* \bar{D}_s$ (threshold: 4080 MeV)

- the I=1 partner of X(3872), with decay:

$X^+ \to J/\psi\ \rho^\pm \to J/\psi\ \pi^+\pi^0$ with the bounds:

$$0.057 < R_{2\pi}^{(0+,00)} = \frac{\Gamma(B^0 \to K^+X^- \to K^+\psi\ \pi^0\pi^-)}{\Gamma(B^0 \to K^0X(3872) \to K^0\psi\ \pi^+\pi^-)} < 0.50$$

L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. D **102** (2020) 034017

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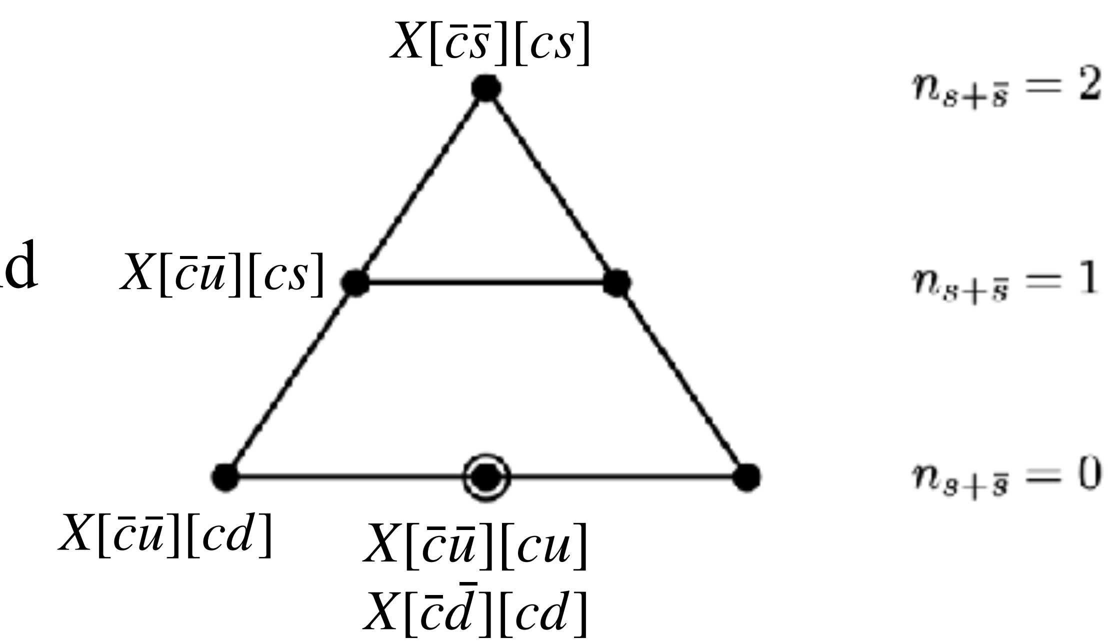

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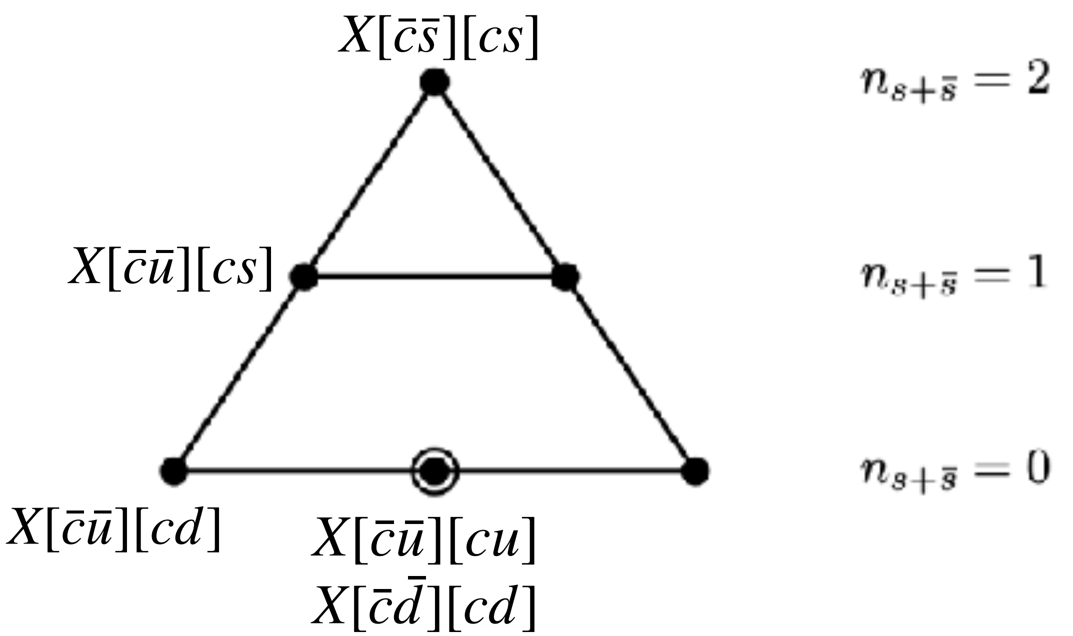

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Only few particles are missing, in well defined mass regions and with identified decay modes.

3. Single charm tetraquarks, with three SU(3)-flavour light mesons: the case of $J^P = 0^+$

L.Maiani, A. Polosa, V.Riquer, ArXiv:2405.08545

- In a recent lattice QCD calculation the $SU(3)_{flavor}$ configurations of possible bound states in the $\bar{D}K, J^P = 0^+$, channel are studied;
- the allowed $SU(3)_{flavor}$ channels are those appearing as irreducible components of the tensor product

$$\bar{D}K = \mathbf{3} \otimes \mathbf{8} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$$

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J. D. E.Yeo *et al.*, [arXiv:2403.10498]

- Consider now four-quark mesons in the simplest diquark-antidiquark model restricting to all spin zero case:

$$[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}}[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}}$$

$$S_{c3} = S_{12} = 0; \;\; J^P = S_{c3} + S_{12} = 0^+$$

- The product $[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}=0}$ is antisymmetric in spin (to get total spin 0) and color (to obtain a $\bar{\mathbf{3}}_c$).

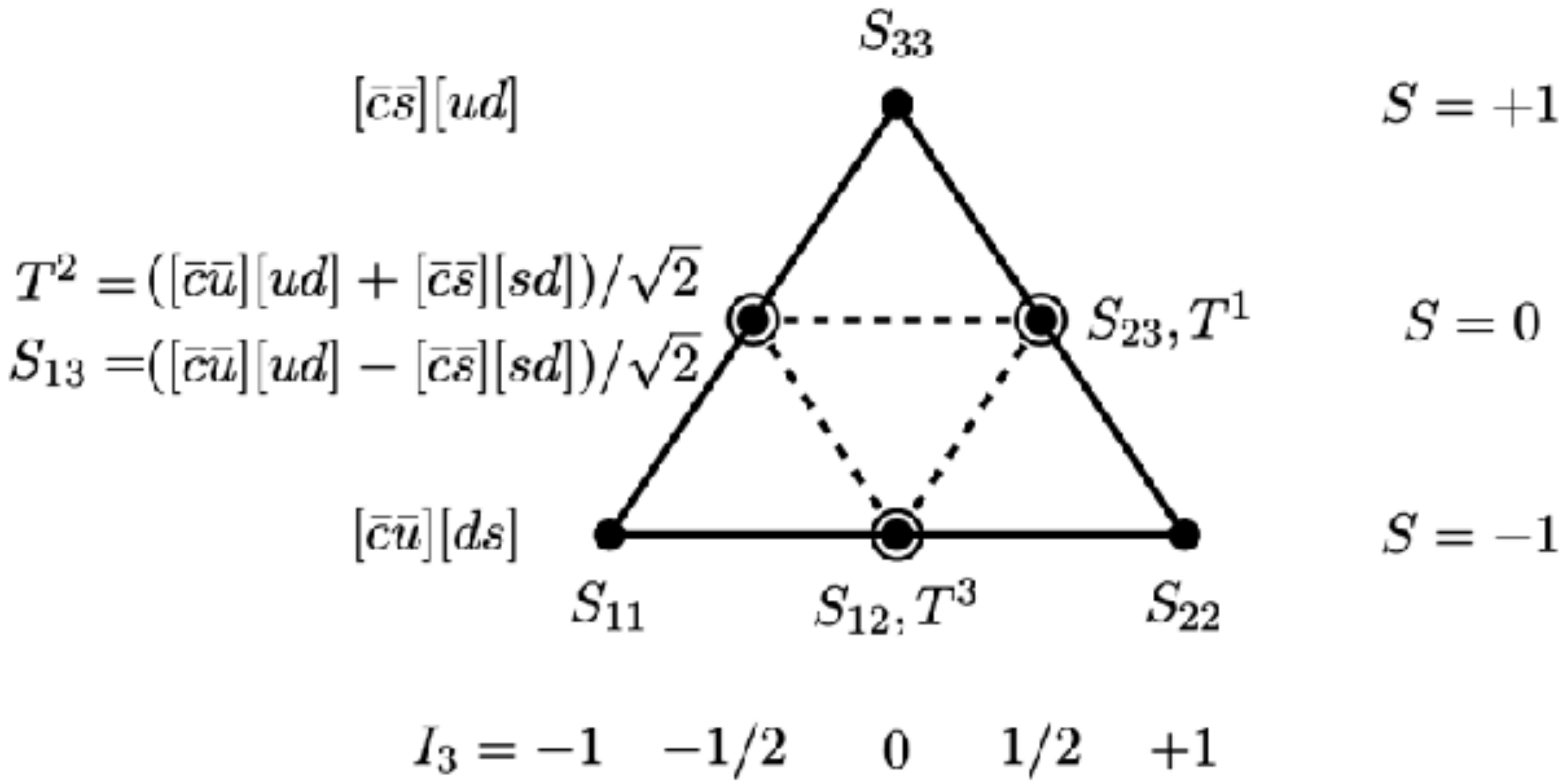
- Fermi statistics: *quarks in the light diquark must be antisymmetric in flavour, i.e must be in a* $\bar{\mathbf{3}}$ *of* $SU(3)_{flavor}$.
- combining with the light antiquark $\bar{q} \propto \bar{\mathbf{3}}$, the tetraquark must be in a $SU(3)_{flavor}$ multiplet:

$$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}} \text{ , no } \mathbf{15}$$

in agreement with the lattice indication.

Quantum numbers and states of $[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}=0}[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}=0}$ (cont'd)

The $\mathbf{3} \oplus \bar{\mathbf{6}}$ representation in the $I_3 - S$ plane:

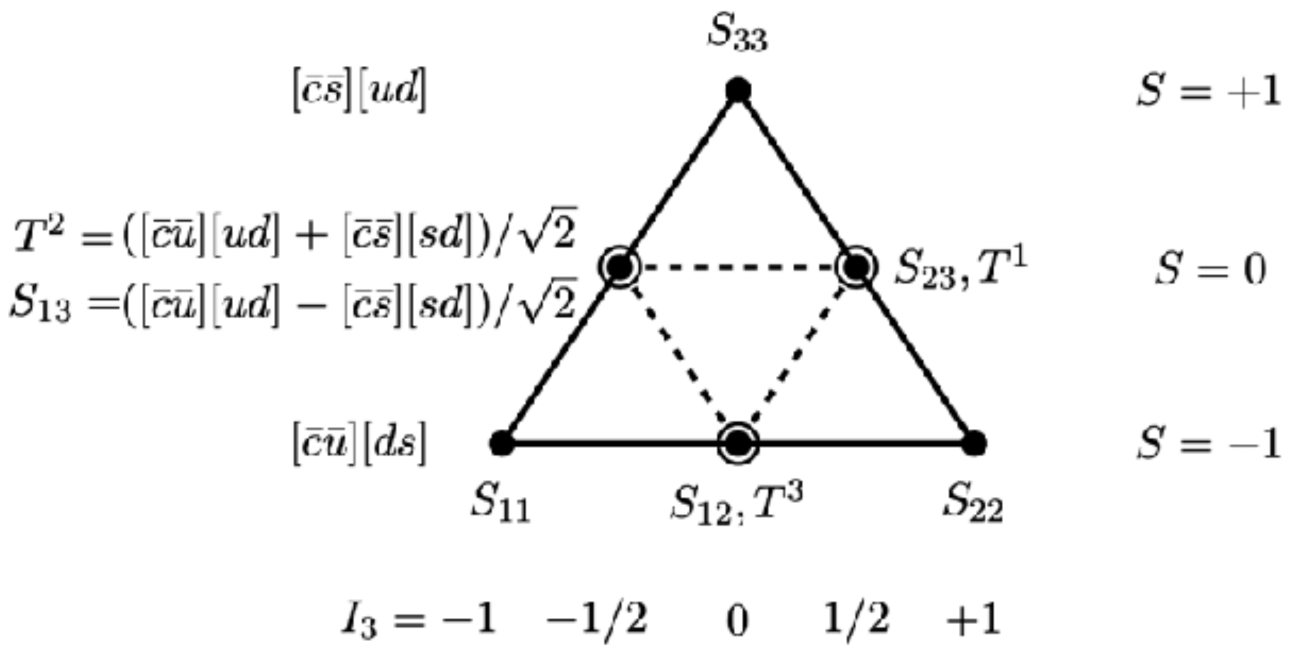


Mass formulae with octet symmetry breaking of $SU(3)_{flavor}$

- $\mathbf{6} \otimes \bar{\mathbf{6}}$ contains rep. **8** only once and the same for $\bar{\mathbf{3}} \otimes \mathbf{3}$:
- in both representations, octet symmetry breaking is proportional to Strangeness and the mass formulae are:

$$M_{\bar{\mathbf{6}}} = m_{\bar{\mathbf{6}}} + \alpha(S+1);$$

$$M_{\mathbf{3}} = m_{\mathbf{3}} + \beta(S+1);$$



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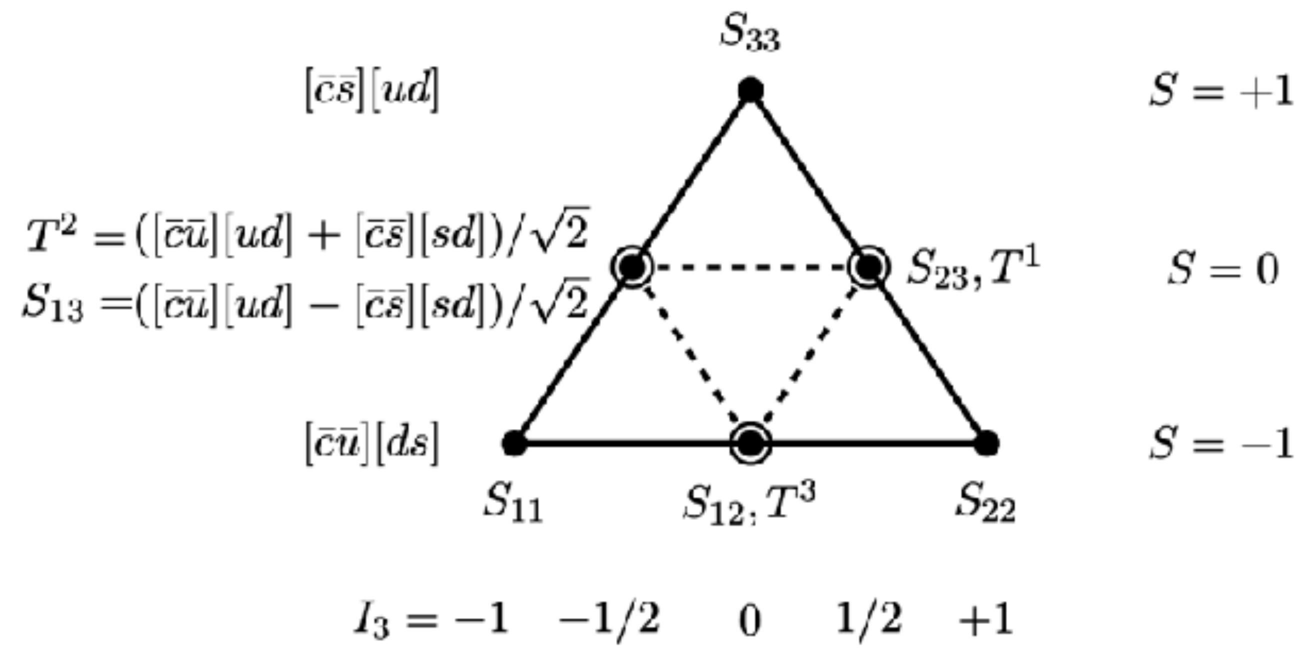

- octet breaking produces a mixing $S_{13} - T^2$ with a matrix: $\mathcal{M} = \begin{pmatrix} m_{\mathbf{3}} + \beta & \delta \\ \delta & m_{\bar{\mathbf{6}}} + \alpha \end{pmatrix}$
- and the same matrix for $S_{23} - T^1$ mixing.

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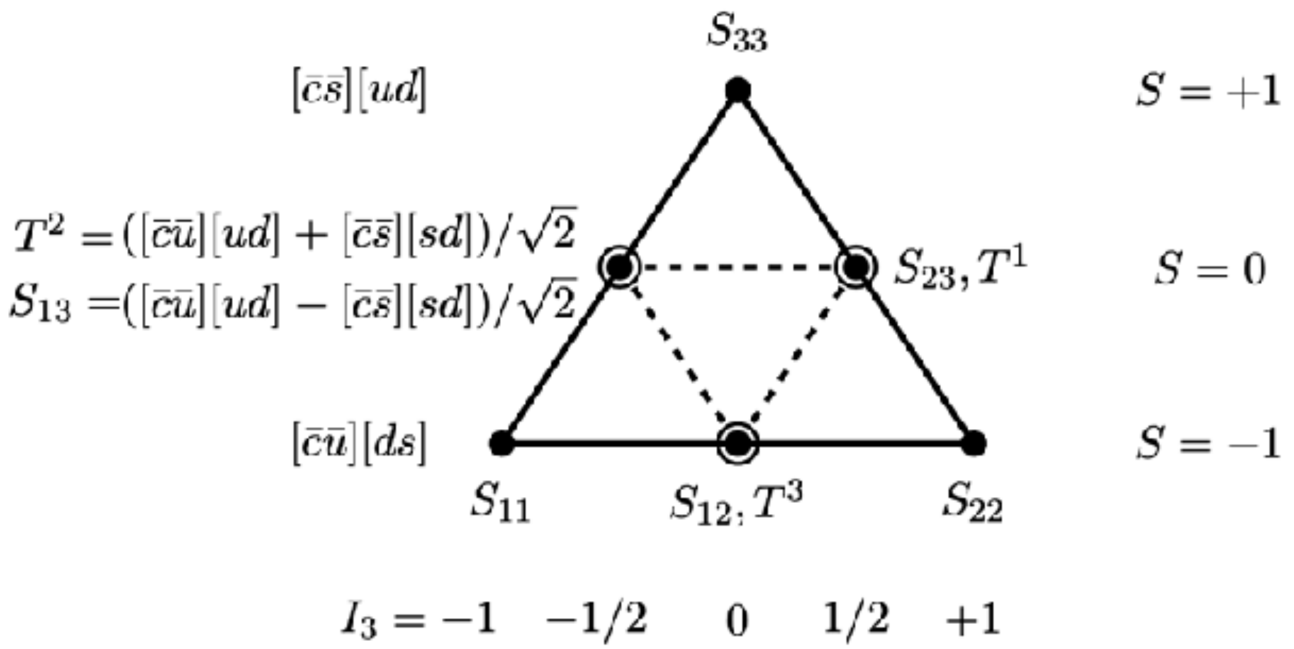

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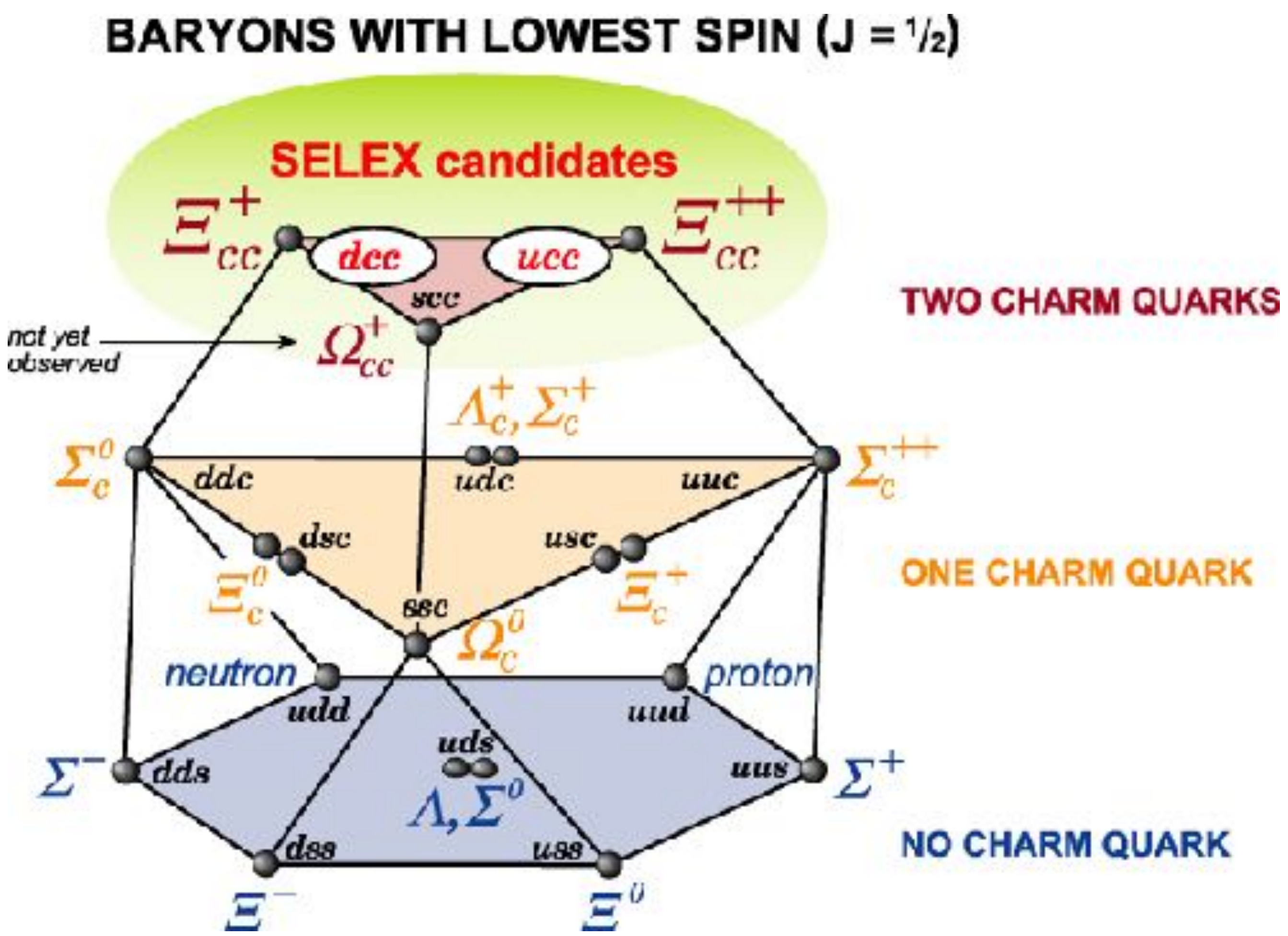
- Quark model requires:

(i) all S=-1 states to have about the same mass ($m_{\bar{\mathbf{6}}} = m_{\mathbf{3}} = M$): they have the same quark composition (like $\rho - \omega$ mesons)
(ii) $\alpha = \beta$: for the mixing matrix to be diagonalized by $(S_{13} \pm T^2)/\sqrt{2}$ (analogous to $\omega - \phi$ mixing)

Single Charm Baryons & Fermi Statistics



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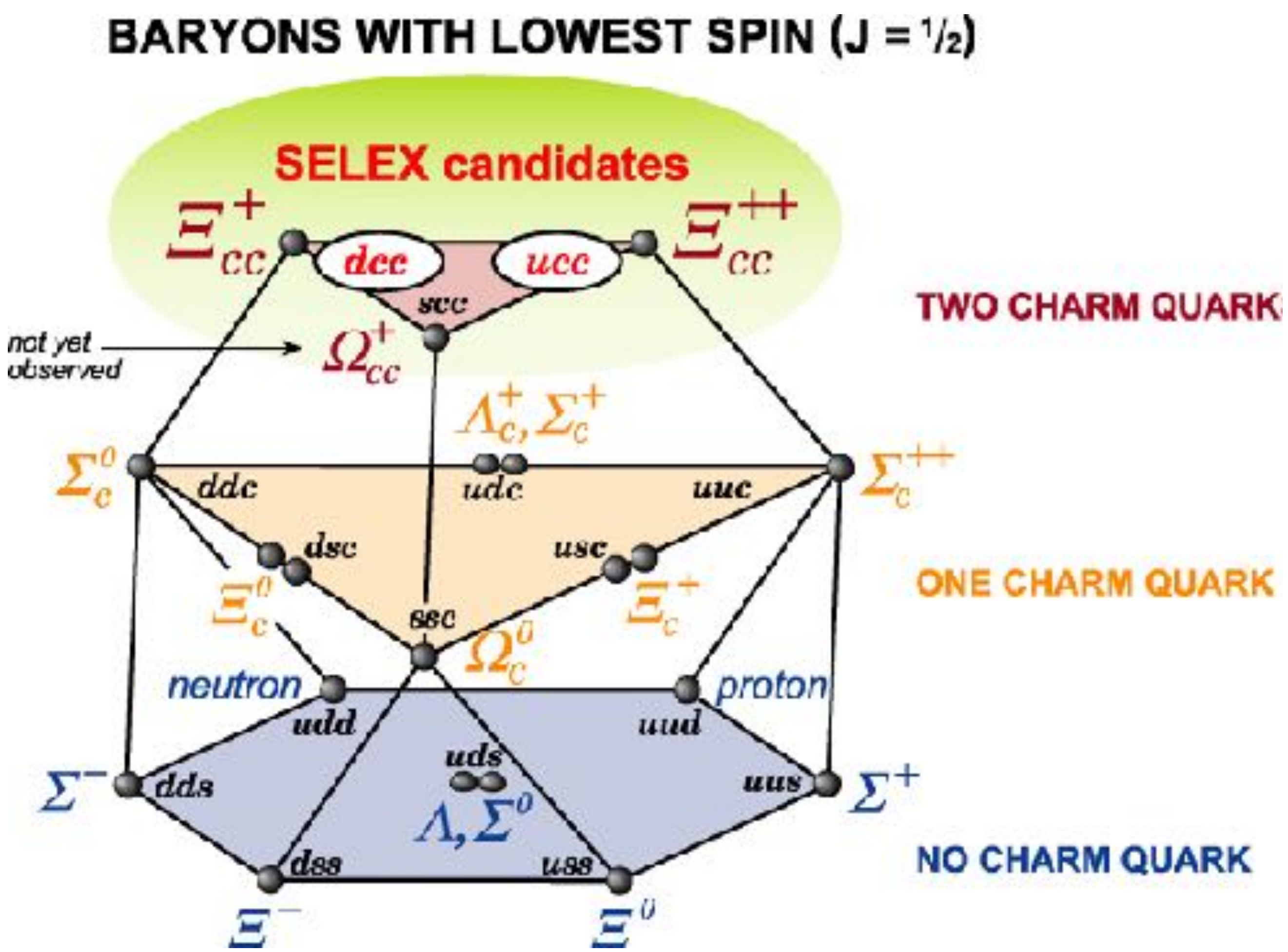


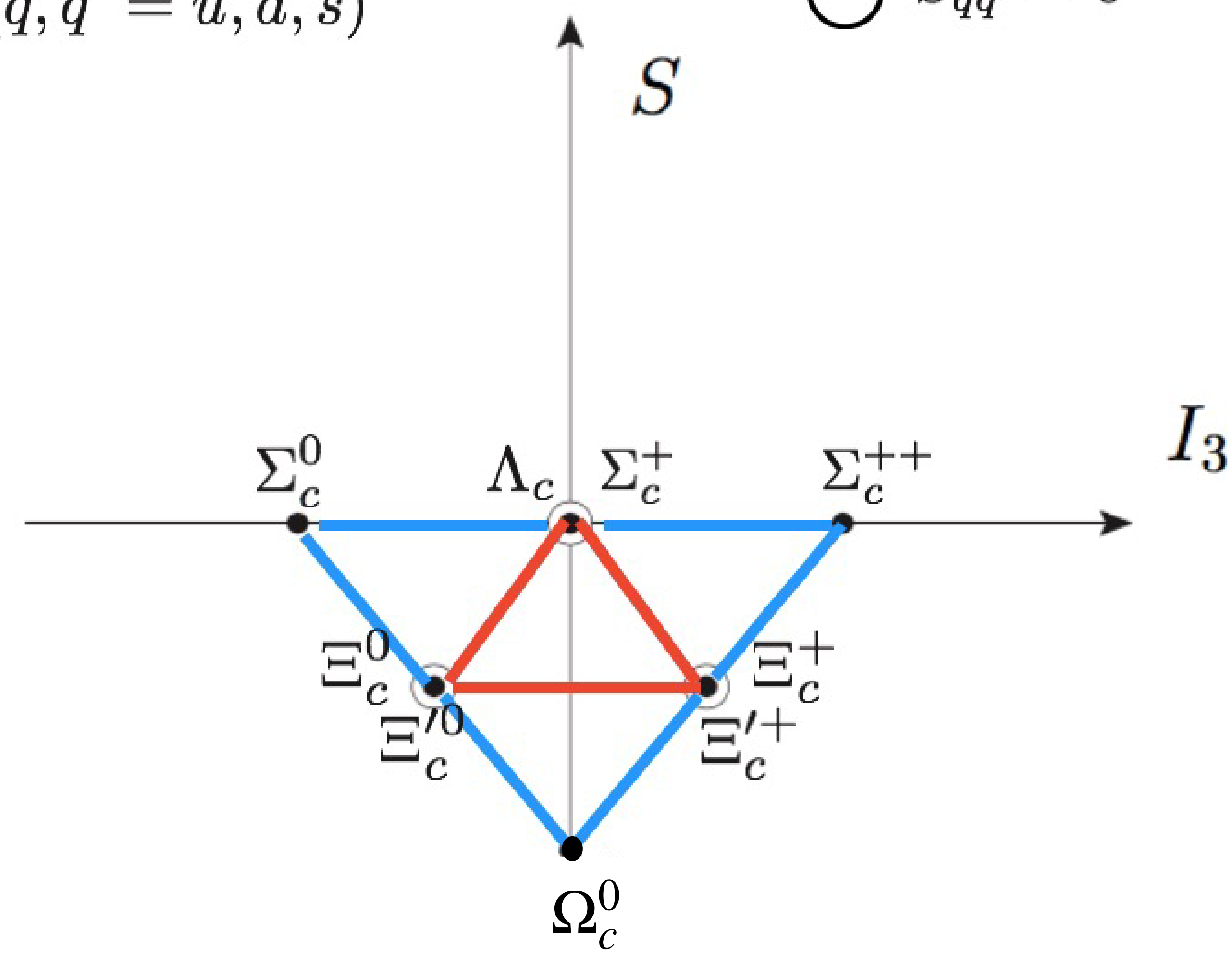
$q,\ q'$: color $= \mathbf{3} \otimes \mathbf{3} \to \bar{\mathbf{3}}$

$\mathbf{J} = 1/2:\ \mathbf{6} \oplus \bar{\mathbf{3}}$

$(q, q' = u, d, s)$

● $S_{qq'} = 1$

○ $S_{qq'} = 0$



$$I = 1 \ \leftrightarrow \ spin = 1,\ (\Sigma_c^{0,+,++})$$

$$I = 0 \ \leftrightarrow \ spin = 0 \ \ (\Lambda_c^+)$$

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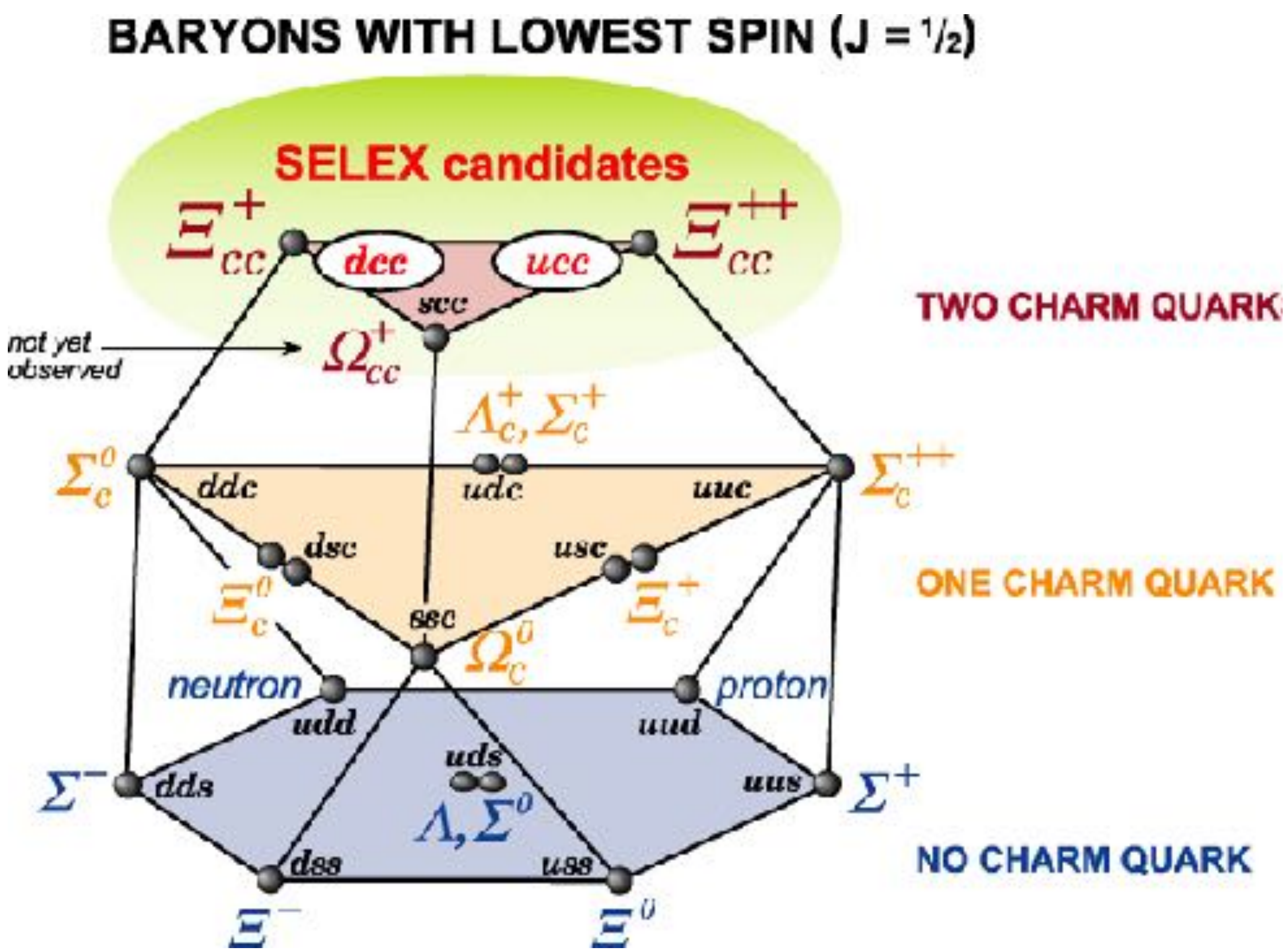

The masses of spin 1 diquarks in the **6** increase linearly with Strangeness, i.e. with the number of strange quarks: $M(\Omega_c) - M(\Sigma_c) \simeq 270$ MeV

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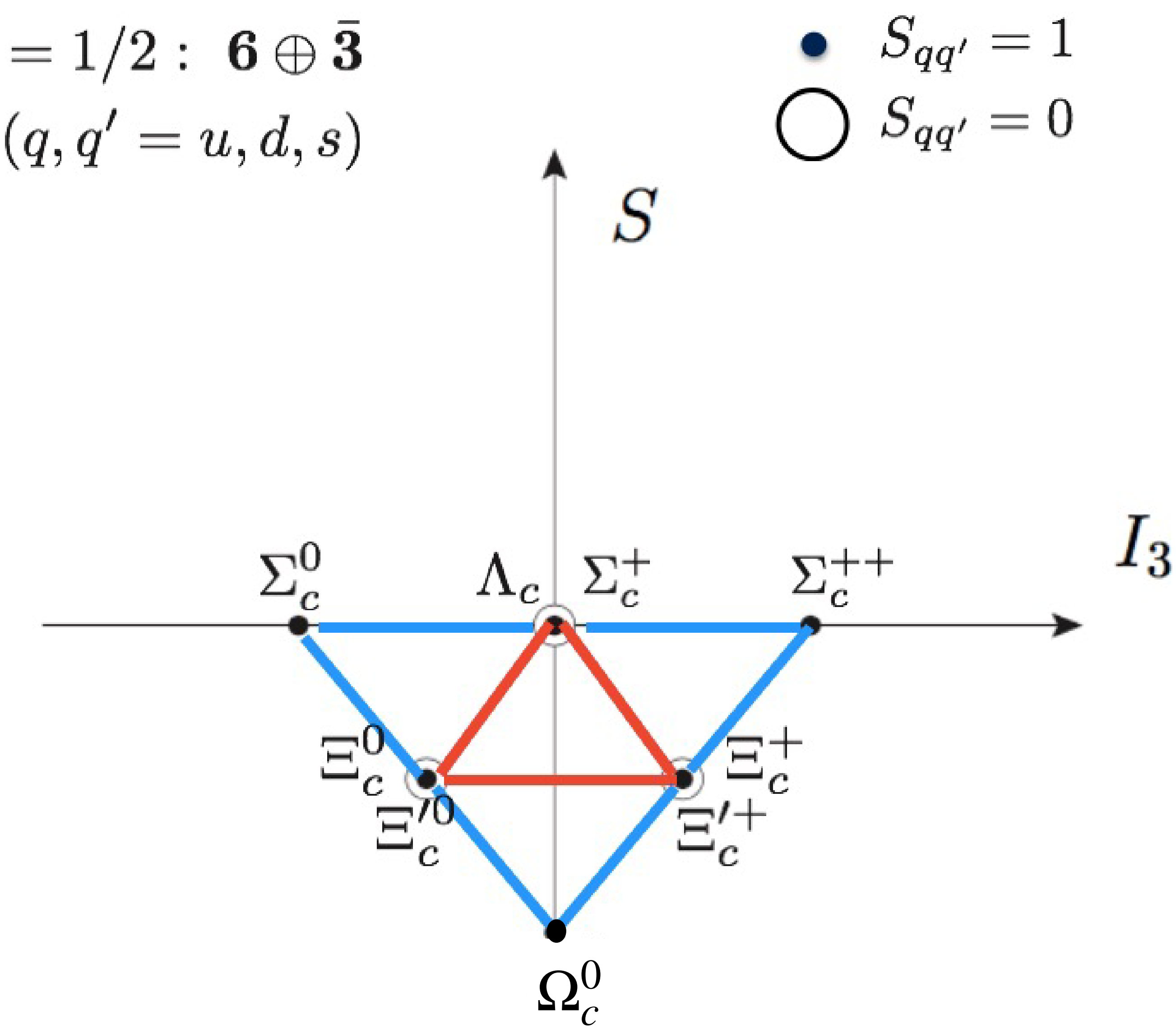
$\mathbf{J} = 1/2 :\ \mathbf{6} \oplus \bar{\mathbf{3}}$

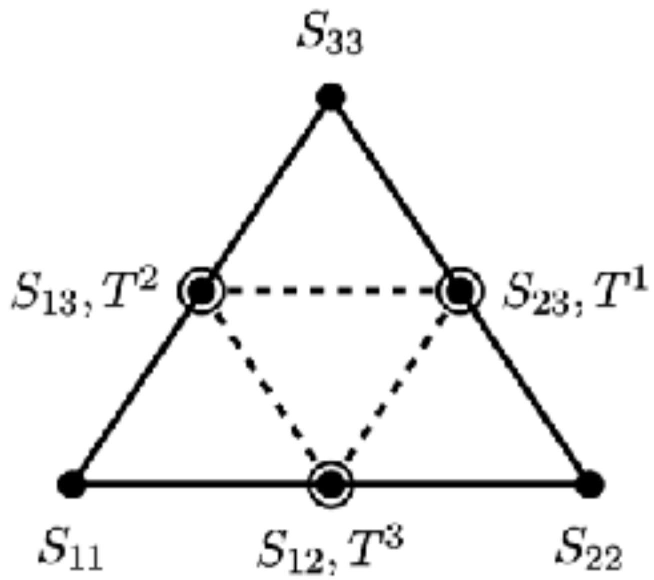
$(q, q' = u, d, s)$



$$I = 1 \ \leftrightarrow \ spin = 1,\ (\Sigma_c^{0,+,++})$$

$$I = 0 \ \leftrightarrow \ spin = 0 \ \ (\Lambda_c^+)$$

A remarkable regularity



Like the masses of single charm mesons, the masses of single charm tetraquarks are equally spaced in Strangeness, with a slope given by the parameter α.

However, ***unlike charmed baryons, the lower indices in*** S_{11} ***correspond to the quark-diquark antisymmetric configuration*** $\bar{u} \otimes [ds]_A$***, while the lower indices in*** S_{33} ***correspond to*** $\bar{s} \otimes [ud]_A$***,***

which have obviously the same content in quark masses, two light and one heavy.

- Exact equality $M(S_{33}) = M(S_{11})$ corresponds to $\alpha \sim 0$: same masses at the upper vertex and lower corners of the triangle in the figure.
- In this case, symmetry breaking is restricted to the mass difference between the two $S = 0$, $I = 1/2$ multiplets induced by the $\mathbf{3} - \bar{\mathbf{6}}$ mixing and of order $\delta \sim 2(m_s - m_q)$, with all other masses degenerate at M.
- A small, non vanishing value of α arises from differences in the hyperfine interactions, which are between different pairs in the two cases.

Note: *In charmed baryons, two light quarks in spin one are also in a* **6** *representation. However indices 1 or 3 correspond univocally to u or s quarks. Group theory disentangles efficiently the ambiguity in these two* **6** *representations* ***making use of the parameter allowed by the Wigner-Eckart theorem***.

A "Constituent Diquark-Antidiquark Model"

- We define ``complete diquark masses" which include the hyperfine interaction appropriate to diquarks with spin =0, e.g.

$$\overline{M}_{cq} = M_{cq} - \frac{3}{2}\kappa_{cq}, \text{ etc.}; \quad \overline{M}_{cq} = \overline{M}_{\bar{c}\bar{q}}$$

- Charmed diquark masses and hyperfine interactions are taken from the masses of hidden charm (X(3872), etc.), hidden charm and strangeness (X(4140), etc.)

L. Maiani et al., Phys. Rev. D 89 (2014), 114010 [arXiv:1405.1551 [hep-ph]]; Phys. Rev. D 94 (2016) , 054026 [arXiv:1607.02405 [hep-ph]].

- and the masses of uncharmed, spin 0 diquarks from the, not so well determined, masses of the light scalar mesons $f_0(500)$ and $f_0(980)$ (see errors in Tab).

R. L. Jaffe, Phys. Rev. D 15 (1977), 281

quark	q	s	c
q	300 ± 100	490 ± 10	1877
s	—–-	—	2035
c	—–-	—–-	—

Table 1: Complete diquark masses, $\overline{M}_{ij}$, in MeV.

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$$M(S_{12}) = M(T_3) = \overline{M}_{cu} + \overline{M}_{sd} = 2367 \pm 10 \text{ MeV} \longleftrightarrow D^*_{cs0}(2317)$$

$$M(T_-) = \overline{M}_{cu} + \overline{M}_{ud} = 2177 \pm 100 \text{ MeV}$$

$$M(T_+) = \overline{M}_{cs} + \overline{M}_{sd} = 2525 \pm 10 \text{ MeV}$$

$$M(S_{33}) = \overline{M}_{cs} + \overline{M}_{ud} = 2335 \pm 100 \text{ MeV}$$

Note: $M(S_{33}) \sim M(S_{12})$

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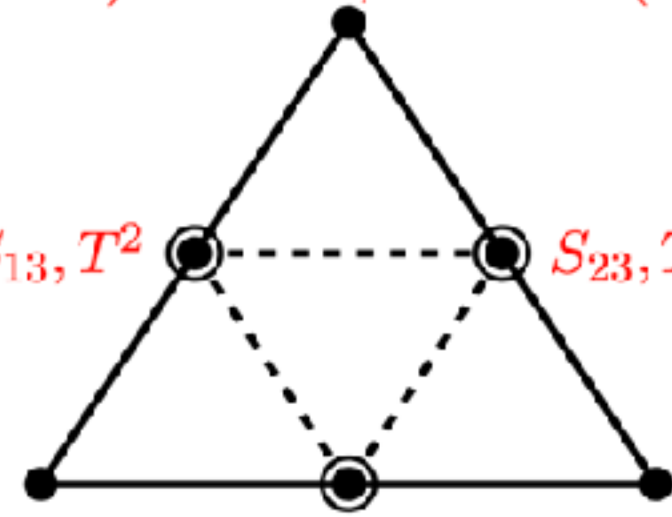
Table 1: Complete diquark masses, $\overline{M}_{ij}$, in MeV.

The missing partners of the charm-strange meson : $D^*_{cs0}(2317) \to \bar{D}^-_s \pi^0$

In red the missing S=0 states and their estimated masses. Expected decay modes:

$[\bar{c}\bar{s}][su](2525 \pm 10) \to \bar{D}^-_s K^0, \bar{D}^0 \eta$

$[\bar{c}\bar{d}][ud](2177 \pm 100) \to \bar{D}\pi$



- $D^{*-}_{s0}(2317)$ has I=0 (PdG) and it should decay into $\bar{D}^-_s \eta$, which however is forbidden by phase space.
- There are two independent mechanisms for the observed $\bar{D}^-_s \pi^0$decay, both related to the $m_d - m_u \sim 5$ MeV mass difference: mixing $T^3 - S_{12}$, or $\eta - \pi^0$mixing.
- ***Interesting*** to observe the decay $D^*_{s0} \to \bar{D}_s \gamma\gamma$, quoted in PdG with the upper bound $B(\gamma\gamma) < 0.18$, to compare with $D^*_{s0}(2317) \to D^-_s \eta^* \to D^-_s \gamma\gamma$ via a virtual η.

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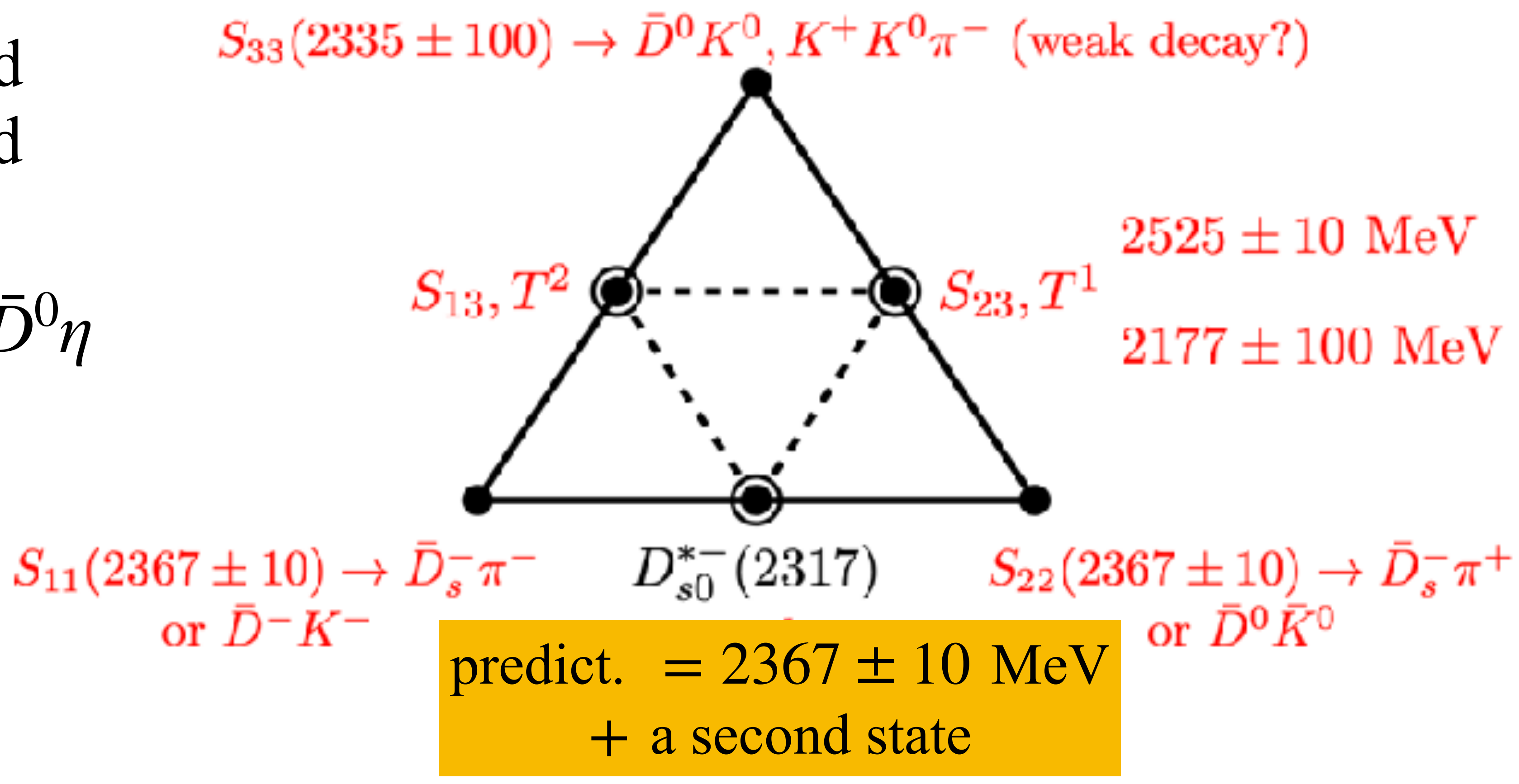

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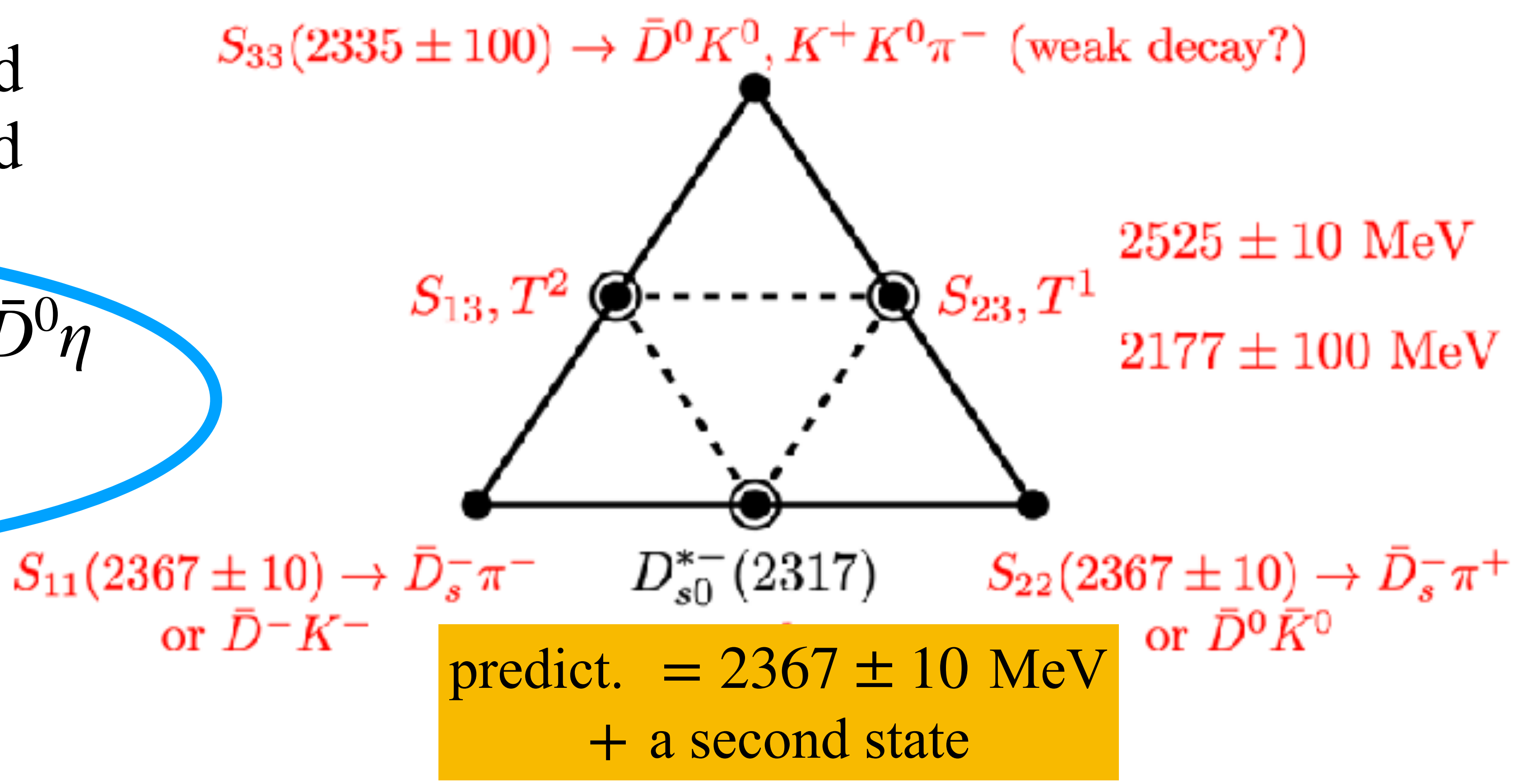

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4. The $\mathbf{3} \oplus \bar{\mathbf{6}}$ (n=2) radially excited multiplet

- $X_0(2900)$ and $D^0_{s0}(2900)$, $D^{++}_{s0}(2900)$ observed by LHCb

$X_0(2900)$ R. Aaij et al. [LHCb], Phys. Rev. D 102 (2020), 112003 | $D^{0,++}_{s0}(2900)$ R. Aaij et al. [LHCb], Phys. Rev. Lett. 131 (2023) 041902;

are too heavy to be included in the basic $\mathbf{3} \oplus \bar{\mathbf{6}}$ multiplet of $D^*_{s0}(2317)$.

- The mass difference: $M(2900) - M(2317) = 583$ MeV is similar to the mass gaps

$$M(J/\Psi) - M(J/\Psi') = 590 \text{ MeV},\ M(X(3872) - M(Z(4430)) = 558 \text{ MeV}$$

- we interpret the LHCb resonances as the ***first radial excitations*** ($n = 2$) of the basic $D^*_{s0}(2317)$ multiplet.

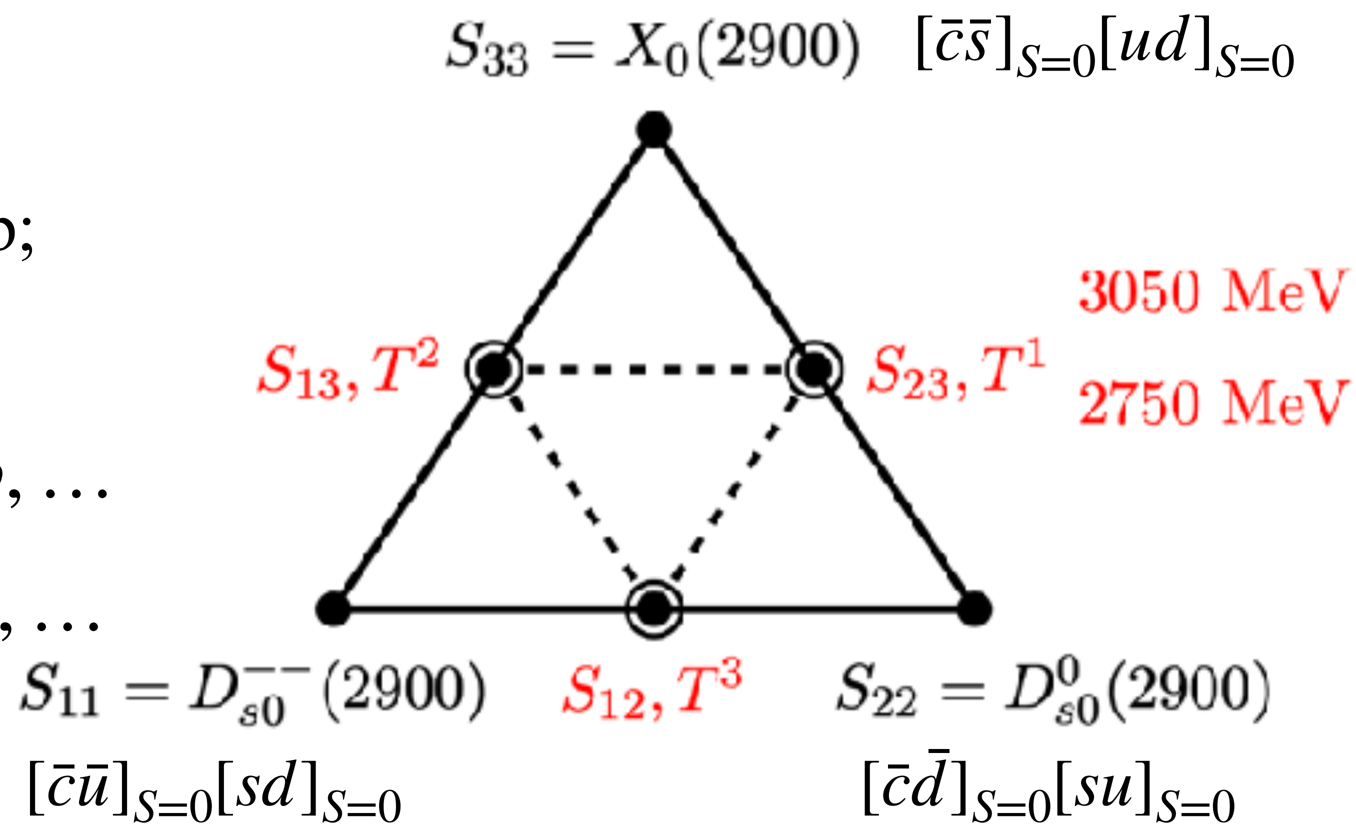
The n=2 multiplet:

(i) in black the resonances observed by LHCb;

(ii) in red the missing S=0 states and their estimated masses. Expected decay modes:

$$[\bar{c}\bar{s}][sd]_{(n=2)}(3050) \to \bar{D}^-\eta,\ \bar{D}^-_s K^0,\ \bar{D}^{*-}\phi, \dots$$

$$[\bar{c}\bar{u}][ud]_{(n=2)}(2750),\ [\bar{c}\bar{d}][ud]_{(n=2)} \to \bar{D}\pi, \dots$$



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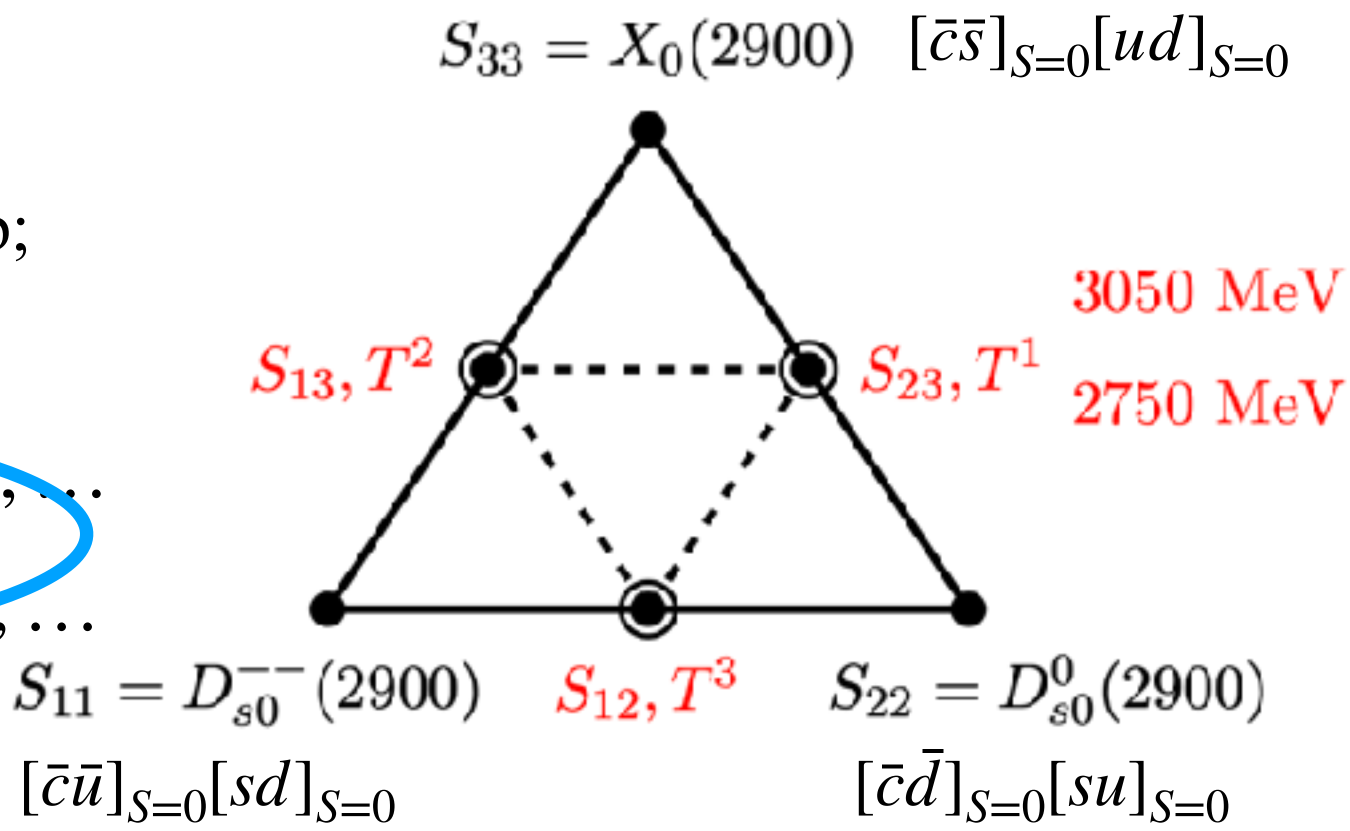

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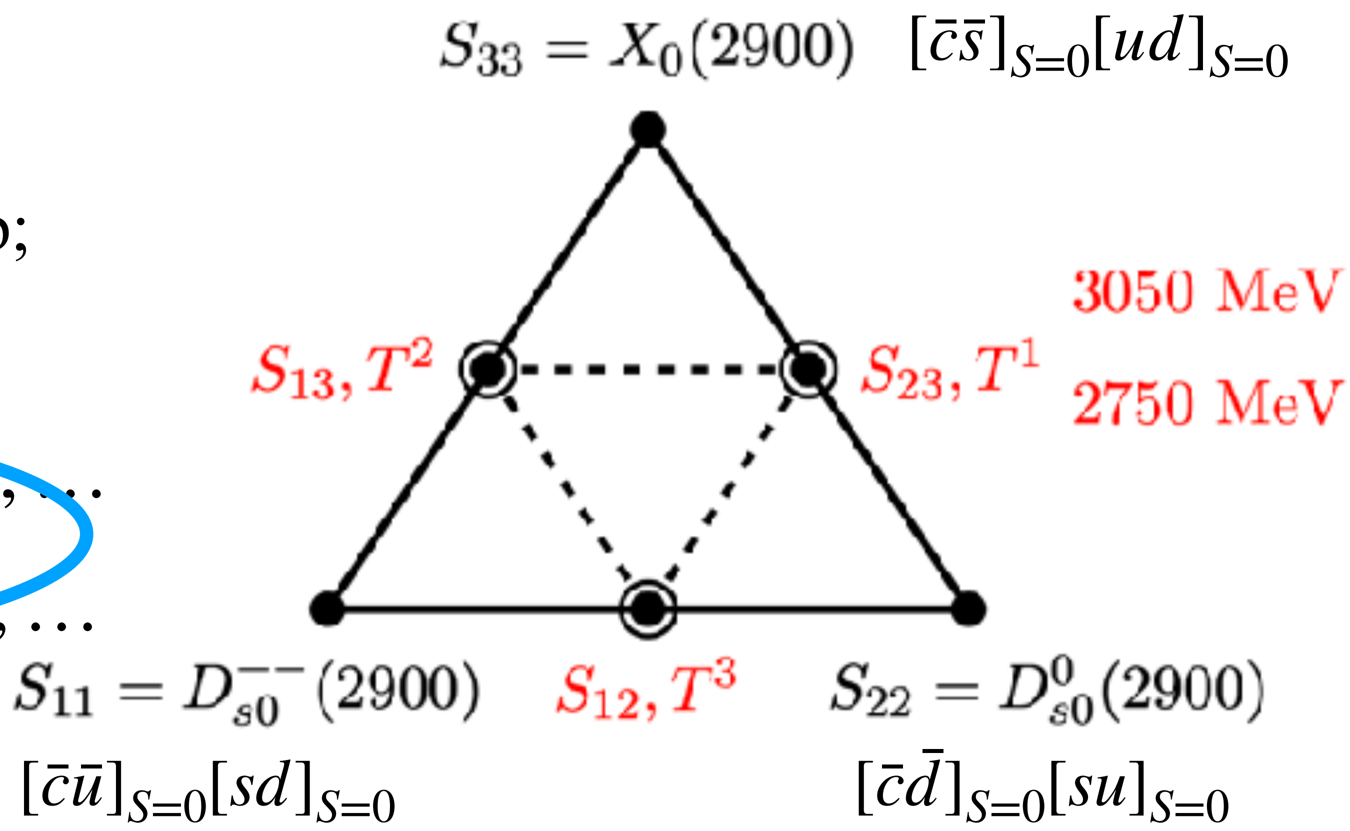

- ***The mass degeneracy between*** $\mathbf{X_0(2900)}$ ***(S=+1) and*** $\mathbf{D^{--,0}_{s0}(2900)}$ ***(S=-1) is the footprint of the tetraquark compositions:***

$$[\bar{c}\bar{s}]_0[ud]_0 \text{ and } [\bar{c}\bar{u}]_0[sd]_0$$

The n=2 multiplet (cont'd)

- The resonances $D_{s0}^{--,0}(2900)$ and $X_0(2900)$ recently discovered by LHCb nicely fit in a $\bar{\mathbf{6}} \oplus \mathbf{3}$ representation of $SU(3)_{flavor}$.
- ***Missing states***:

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(ii) its (almost degenerate) $(I_3 = 0,\ I = 0)$ partner;

(iii) two other I=1/2 tetraquark multiplets of composition:

$([\bar{c}\bar{s}][sd] + d \to u)$ and $([\bar{c}\bar{u}][ud] + \bar{u} \to \bar{d})$ of predicted masses and decay modes.

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- The observation that

$$M(2900) - M(2317) = 583\ \text{MeV} \simeq M(\psi(2S)) - M(\psi(1S))$$

suggests that the multiplet we discuss could be the radial excitation of the lower multiplet containing the $D_{s0}^{*}(2317)$, in a similar way in which $Z(4430)$ is interpreted as a radial excitation of $X(3872)$.

- Using $SU(3)_{flavor}$ symmetry breaking we obtain mass predictions for the missing partners of $D_{s0}^{*}(2317)$.

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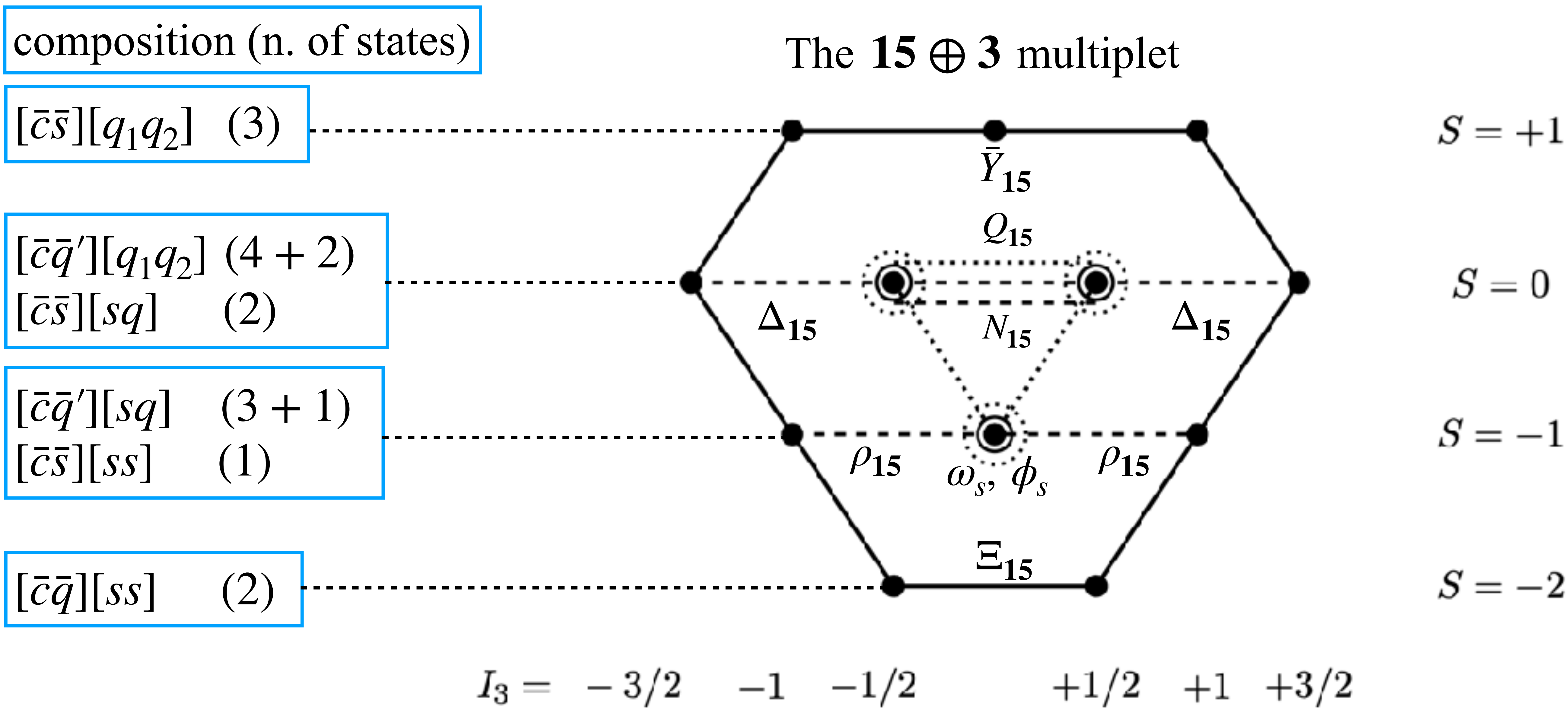
- Using $SU(3)_{flavor}$ symmetry breaking we obtain mass predictions for the missing partners of $D_{s0}^{*}(2317)$.
- Our results are in agreement with a recent lattice calculation showing that in the $\bar{D}K$ scattering there is no attraction in the **15** representation, something which is expected in the quark model description we present here.

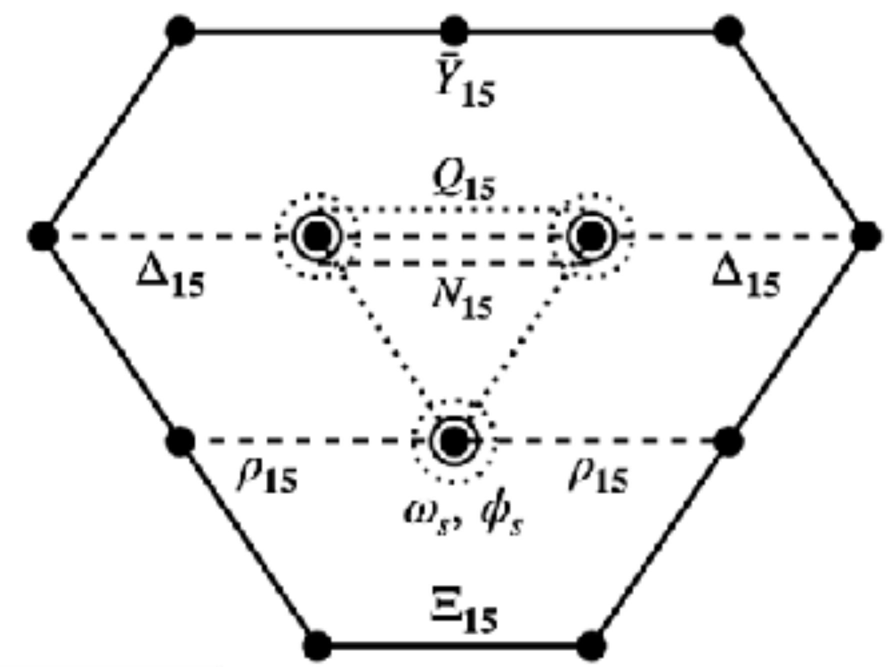
5. The SU(3)-flavour $\mathbf{15} \oplus \mathbf{3}$ multiplet of single charm tetraquarks: $S_{\bar{c}\bar{q}'} = 0,\ S_{q_1 q_2} = 1,\ J^P = 1^+$ (unpublished)

- We consider tetraquarks of the form $[\bar{c}\bar{q}']^{\mathbf{3}}_{S_{\bar{c}\bar{q}'}=0}\ [q_1 q_2]^{\bar{\mathbf{3}}}_{S_{q_1q_2}=1},\ J^P = 1^+$
- Fermi Statistics requires $q_1 q_2$ in the flavour symmetric **6**, the three light quarks are in $\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{15} \oplus \mathbf{3}$



Mass estimates and decay modes of: $[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}=0}[q_1 q_2]^{\bar{\mathbf{3}}}_{S_{12}=1}$, $J^P = 1^+$

$\bar{Y}^-_{\mathbf{15}}$	$\Delta^{--}_{\mathbf{15}}$	$N^-_{\mathbf{15}}$	$Q^-_{\mathbf{15}}$
$[\bar{c}\bar{s}][dd]$	$[\bar{c}\bar{u}][dd]$	$[\bar{c}\bar{d}][dd]+[\bar{c}\bar{u}][ud]$	$[\bar{c}\bar{s}][ds]$
δm	0	0	$2\ \delta m$
2535 ± 100	2377 ± 100	2377 ± 100	2725 ± 10
$D^{*-}\bar{K}^0$	$D^{*-}\pi^-$	$D^{*-}\eta$	$D^-\phi, D_s^{*-}\bar{K}^0$



$\rho^{--}_{\mathbf{15}}$	ω^-_s	ϕ^-_s	$\Xi^{--}_{\mathbf{15}}$
$[\bar{c}\bar{u}][ds]$	$[\bar{c}\bar{d}][ds]+[\bar{c}\bar{u}][us]$	$[\bar{c}\bar{s}][ss]$	$[\bar{c}\bar{u}][ss]$
δm	δm	$3\ \delta m$	$2\ \delta m$
2567 ± 10	2567 ± 10	2951 (no err.)	2793 (no err.)
$D^{*-}K^-, D_s^{*-}\pi^-$	$D_s^-\omega, D_s^{*-}\eta/\eta'$	$D_s^-\phi, D_s^{*-}\eta/\eta'$	$D_s^{*-}K^-, D_s^-K^{*-}$

Particles in the $\mathbf{15} \oplus \mathbf{3}$ multiplet.
Second row: diquark-antidiquark composition.
Third row: mass breaking to be added to the symmetric mass M in each isospin multiplet, excluding hyperfine interactions, with: $\delta m = m_s - m_q$.
Fourth row: masses from the constituent diquark-antidiquark model.
Fifth row: decay modes.

Mass estimates and decay modes of: $[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}=0}[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}=1}$, $J^P = 1^+$

$\bar{Y}^-_{\mathbf{15}}$	$\Delta^{--}_{\mathbf{15}}$	$N^-_{\mathbf{15}}$	$Q^-_{\mathbf{15}}$
$[\bar{c}\bar{s}][dd]$	$[\bar{c}\bar{u}][dd]$	$[\bar{c}\bar{d}][dd]+[\bar{c}\bar{u}][ud]$	$[\bar{c}\bar{s}][ds]$
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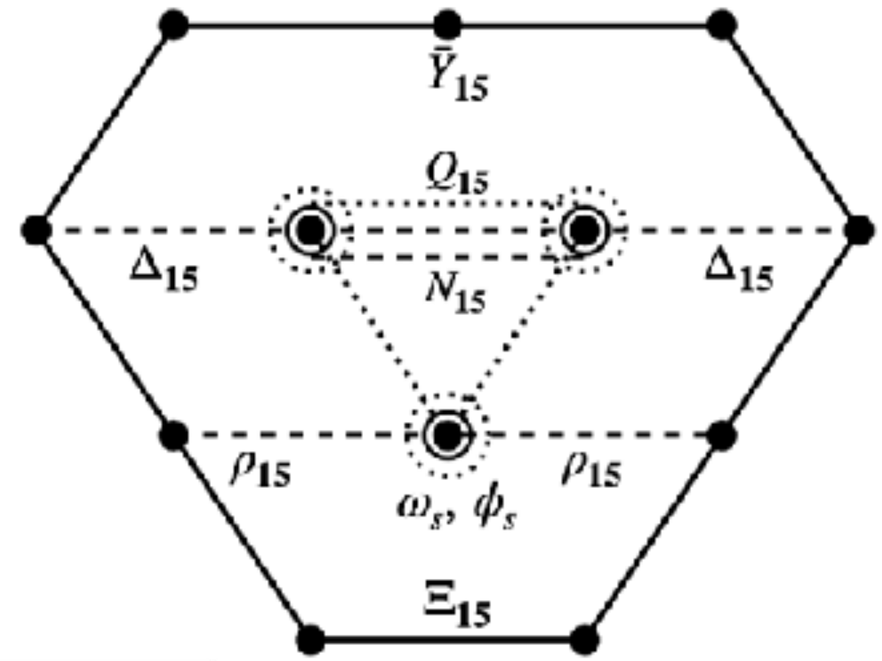

$\rho^{--}_{\mathbf{15}}$	ω^-_s	ϕ^-_s	$\Xi^{--}_{\mathbf{15}}$
$[\bar{c}\bar{u}][ds]$	$[\bar{c}\bar{d}][ds]+[\bar{c}\bar{u}][us]$	$[\bar{c}\bar{s}][ss]$	$[\bar{c}\bar{u}][ss]$
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6. Heavy particle spin conservation and Fermi Statistics of light quark pairs: QCD tetraquarks vs hadron molecules

- For molecular tetraquarks, in the limit of very massive charm quark, the light quark total spin is a separatly conserved quantity (this is the *light quark spin symmetry in the static quark approximation* introduced by Isgur and Wise)
- For hidden charm molecules $(\bar{c}q)(\bar{q}'c)$, flavour symmetry, e.g. Isospin, is also an independent (commuting) conserved quantity. The possible combinations of light and heavy spin generate six states with definite Isospin, total angular momentum and charge conjugation:

Z. H. Zhang *et al*., arXiv:2404.11215 [hep-ph]

$$J_I^{PC} = 0_I^{++},\ 1_I^{+-},\ 1_I^{\prime +-},\ 1^{++},\ 0_I^{\prime ++},\ 2_I^{++}.$$

- These are the same J_I^{PC} states predicted long ago by diquark-antidiquark, color singlet tetraquarks of the form $[cq]^{\bar{\mathbf{3}}}[\bar{c}\bar{q}']^{\mathbf{3}}$. Noticeably, they include the I=1 partner of $X(3872)$, i.e. X^+.

L. Maiani et al., Phys. Rev. D 89 (2014), 114010; Phys. Rev. D 94 (2016) , 054026].

- Concerning Fermi Statistics, the situation is different for the molecular structure $(\bar{c}q_1)(\bar{s}q_2)$ with respect to diquark-antidiquark.
- q_1 and q_2, sit in different color singlets and the color of the pair $q_1 \otimes q_2$ is not determined (in fact it is a superposition of color $\bar{\mathbf{3}}$ and $\mathbf{6}$): there is no definite restriction to their behaviour under flavor exchange and no forbidden **15**.

Final questions (to LHCb /BESIII)

- Are $Z_{cs}(3986)$ and $Z(4003)$ two different states?
- Can X^+ near $X(3872)$ be found in B decays?
- can we find the missing partners of the $\bar{\mathbf{6}} \oplus \mathbf{3}$, (n=2) multiplet:

$$[\bar{c}\bar{s}][sd]_{(n=2)}(3050) \to \bar{D}^- \eta,\ \bar{D}_s^- K^0,\ \bar{D}^{*-}\phi, \ldots$$
$$[\bar{c}\bar{u}][ud]_{(n=2)}(2750),\ [\bar{c}\bar{d}][ud]_{(n=2)} \to \bar{D}\pi, \ldots$$

- LHCb has used efficiently the channel $B \to (J/\Psi)\phi K + \ldots$ to study $X_{ss}(4140)$ etc.., and Z_{cs} etc. of SU(3) nonet tetraquarks…
- Can the study of $D_s^-\phi$ channel be similarly used to study single charm $[\bar{c}\bar{s}]_{S=1}[ss]_{S=0}$, $J^P = 1^+$ tetraquarks of the similarly interesting $\mathbf{15} \oplus \mathbf{3}$, $J^P = 1^+$ multiplet ?

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Many Thanks !!