The Muon Shot — Combining energy with precision

'This is our Muon Shot'

The US physics community dreams of building a muon collider.

T. Han/W. Kilian/N. Kreher/Y. Ma/JRR/T. Striegl/K. Xie arXiv: 2108.05362 [JHEP]

Celada/Han/Kilian/Kreher/Ma/Maltoni/Pagani/JRR/Striegl/Xie, arXiv:2312.13082 [JHEP]

P. Bredt, W. Kilian, JRR, P. Stienemeier arXiv: 2208.09438 [JHEP]

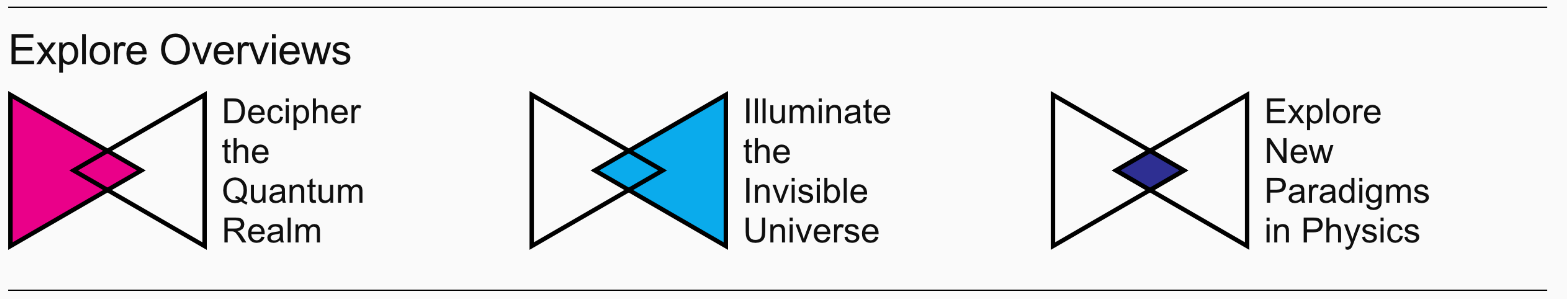
K. Mękała/JRR/A.F. Żarnecki, arXiv: 2301.02602 [PLB] + arXiv:2312.05223 [JHEP]

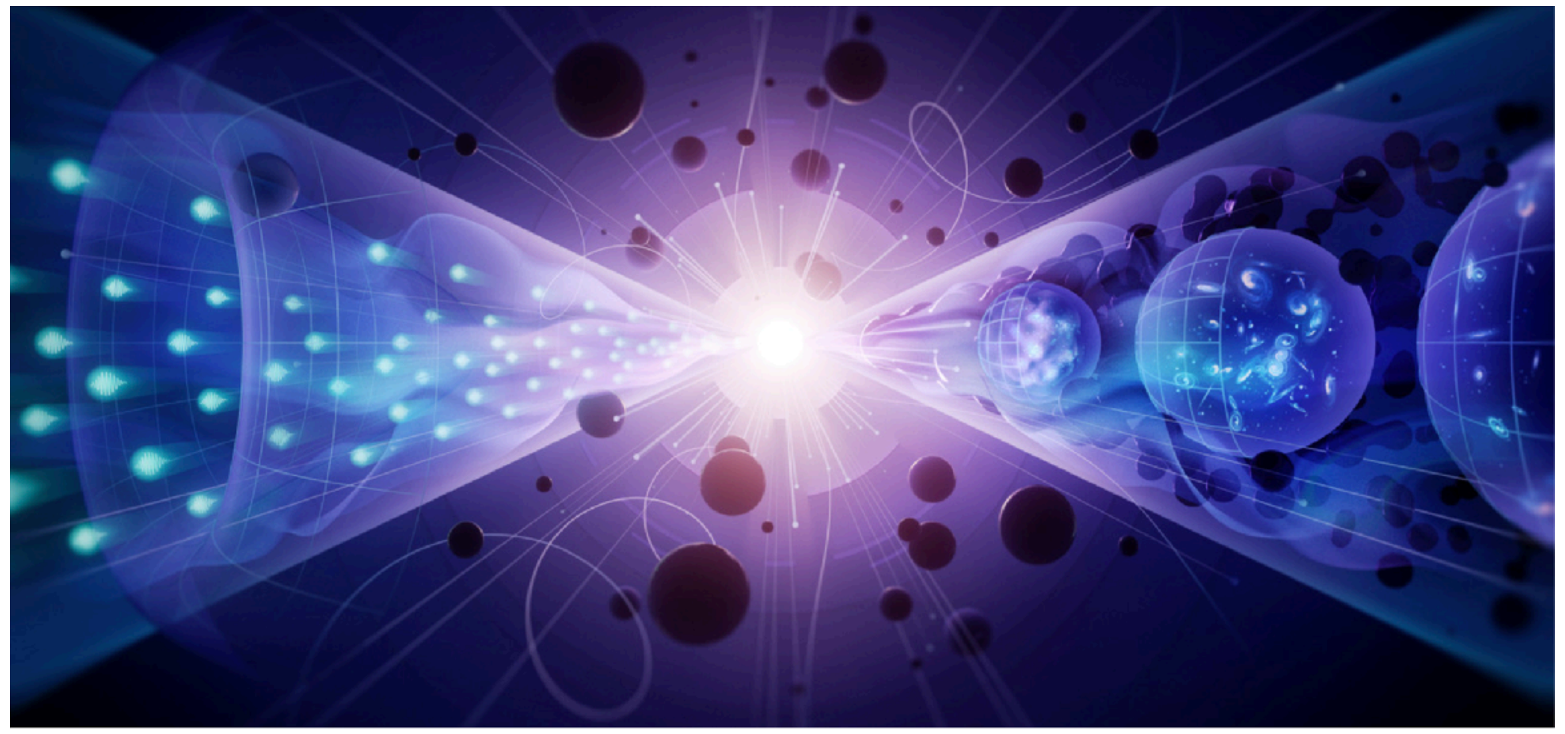
K. Korshynska, M. Löschner, M. Marinichenko, K. Mękała/JRR, arXiv: 2402.18460 [EPJC]

Jürgen R. Reuter

The Muon Shot

- EPPSU 2020: MuC R&D (accelerator roadmap) $\Longrightarrow$ start of IMCC





- US Snowmass 2021 Summer Study: great enthusiasm for high-energy Muon Colliders (MuC)
- Road map in P5 (Particle Physics Projects Prioritization Panel) report: the Muon Shot ↪ P5 report

The Muon Shot

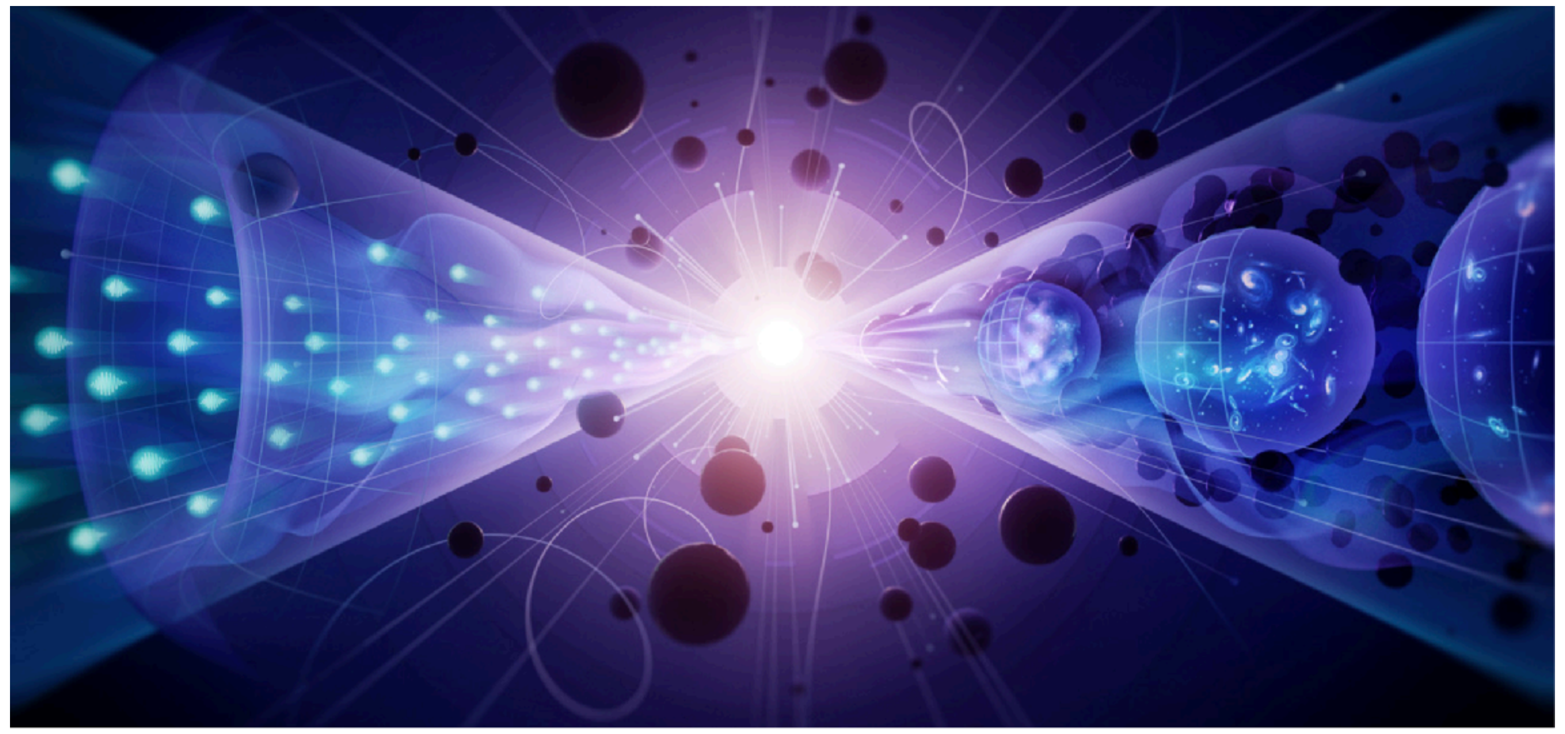
- EPPSU 2020: MuC R&D (accelerator roadmap) $\Longrightarrow$ start of IMCC

Explore Overviews

Decipher the Quantum Realm

Illuminate the Invisible Universe

Explore New Paradigms in Physics



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> A 10 TeV pCM collider (muon collider, FCC-hh, or possible wakefield collider) will provide the most comprehensive increase in BSM discovery potential (Recommendation 4a). Dramatic increases in sensitivity are expected for both model-dependent and model-independent searches. Such a collider will be able to reach the thermal WIMP target for minimal WIMP candidates and hence will play a critical role in providing a definitive test for this class of models.

> For example, a muon collider, if technologically achievable and affordable, presents a great opportunity to bring a new collider to US soil. A 10 TeV collider fits on the Fermilab site and is a good match with Fermilab's strengths. Its development has synergies with the neutrino program beyond

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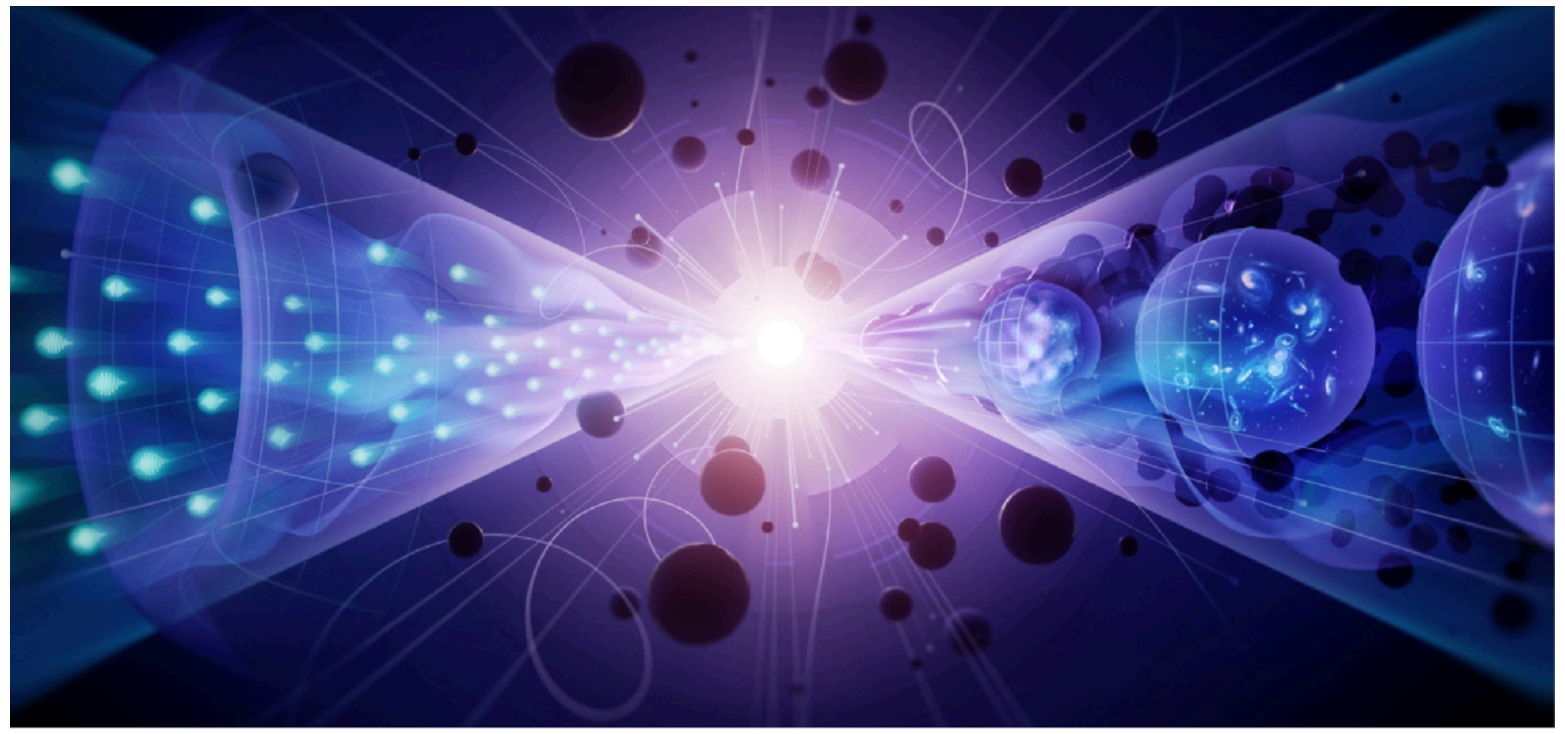

Explore Overviews

Decipher the Quantum Realm	Illuminate the Invisible Universe	Explore New Paradigms in Physics

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$$m_\mu = 0.1056\,\mathrm{GeV} \approx 207 \cdot m_e$$

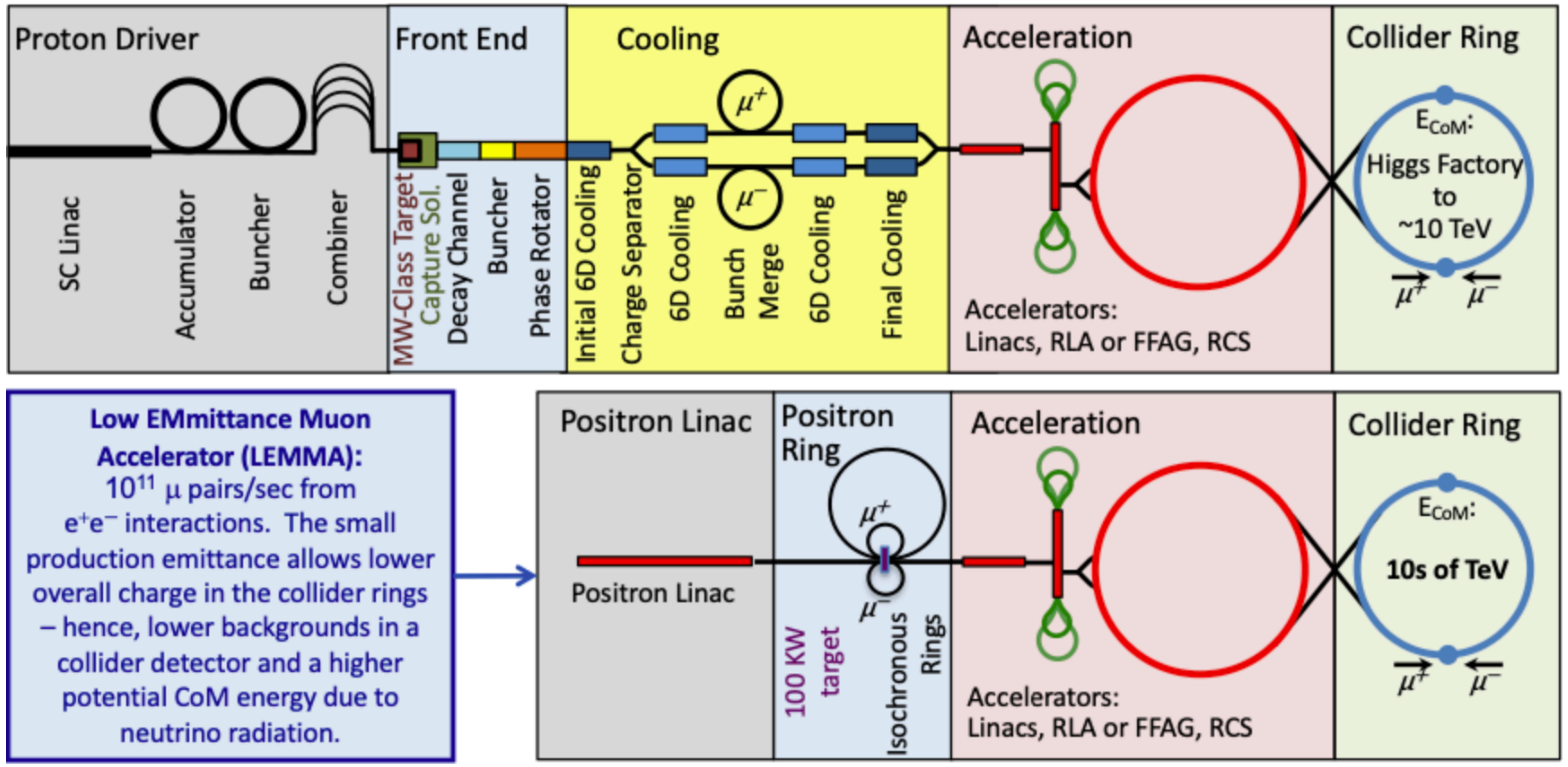
$$\Gamma_\mu = 3 \cdot 10^{-19}\,\mathrm{GeV} \qquad \tau_\mu = 2.2\,\mu\mathrm{s}$$

$$c\tau_\mu \approx 660\,\mathrm{m}$$

The glory of a muon collider

- ☑ Muons pointlike objects: cleaner environment than hh
- ☑ Much less synchrotron radiation than electrons
- ☑ Much smaller beam energy spread: $\Delta E \approx 0.1 - 0.001\%$

- ☐ Short lifetime: difficult to get high-quality/lumi beams
- ☐ Difficult cooling of beams
  considerable progress: MICE collaboration
- ☐ Beam-induced bkgds (BIP) from decay @ IP
- ☐ Radiation hazard from beam dump (neutrinos)



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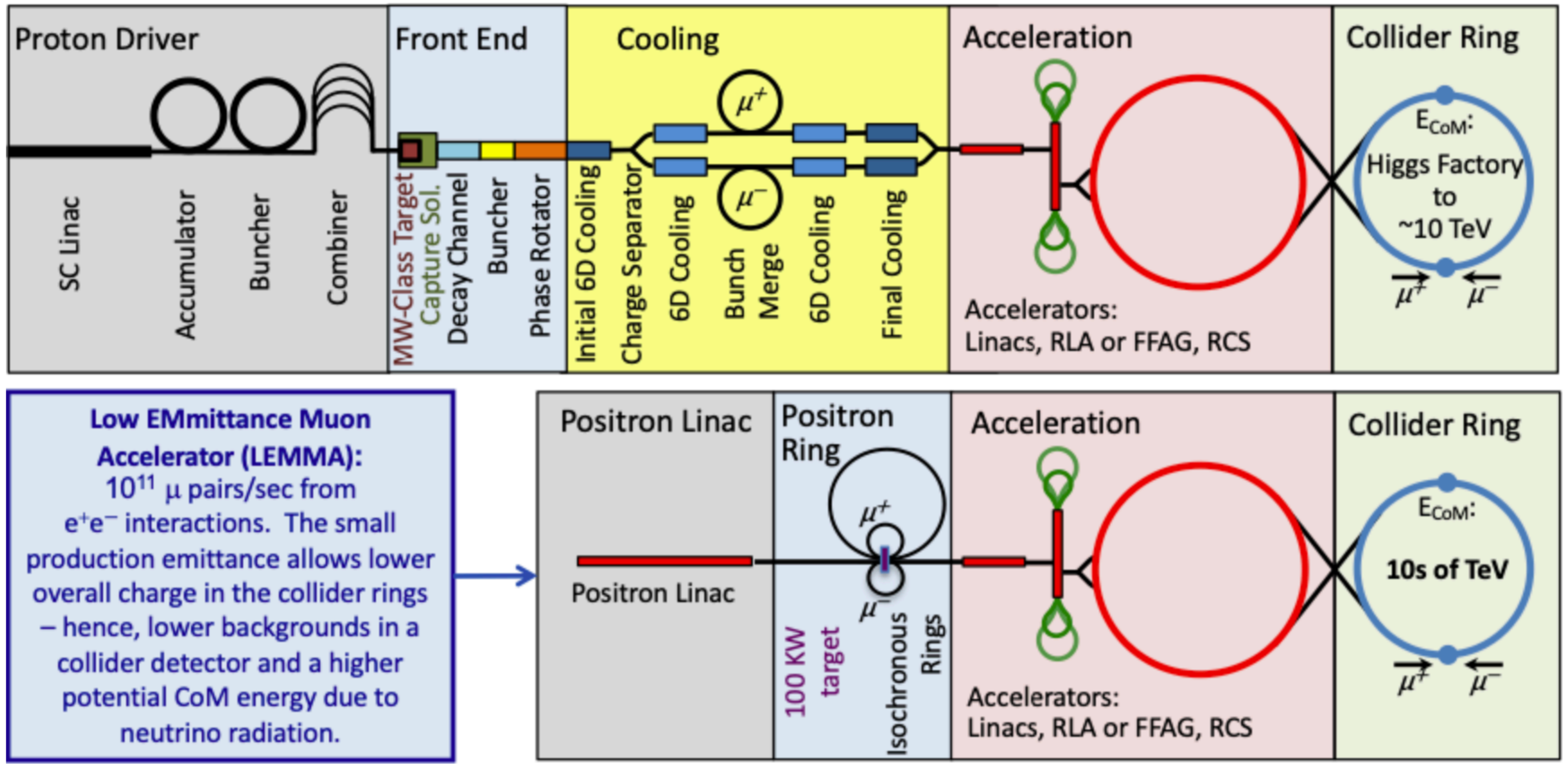

$\sqrt{s}$	$\int \mathcal{L} dt$
3 TeV	1 ab^{-1}
10 TeV	10 ab^{-1}
14 TeV	20 ab^{-1}

1901.06150; 2001.04431;
PoS(ICHEP2020)703; Nat.Phys.17, 289-292;
IMCC study group

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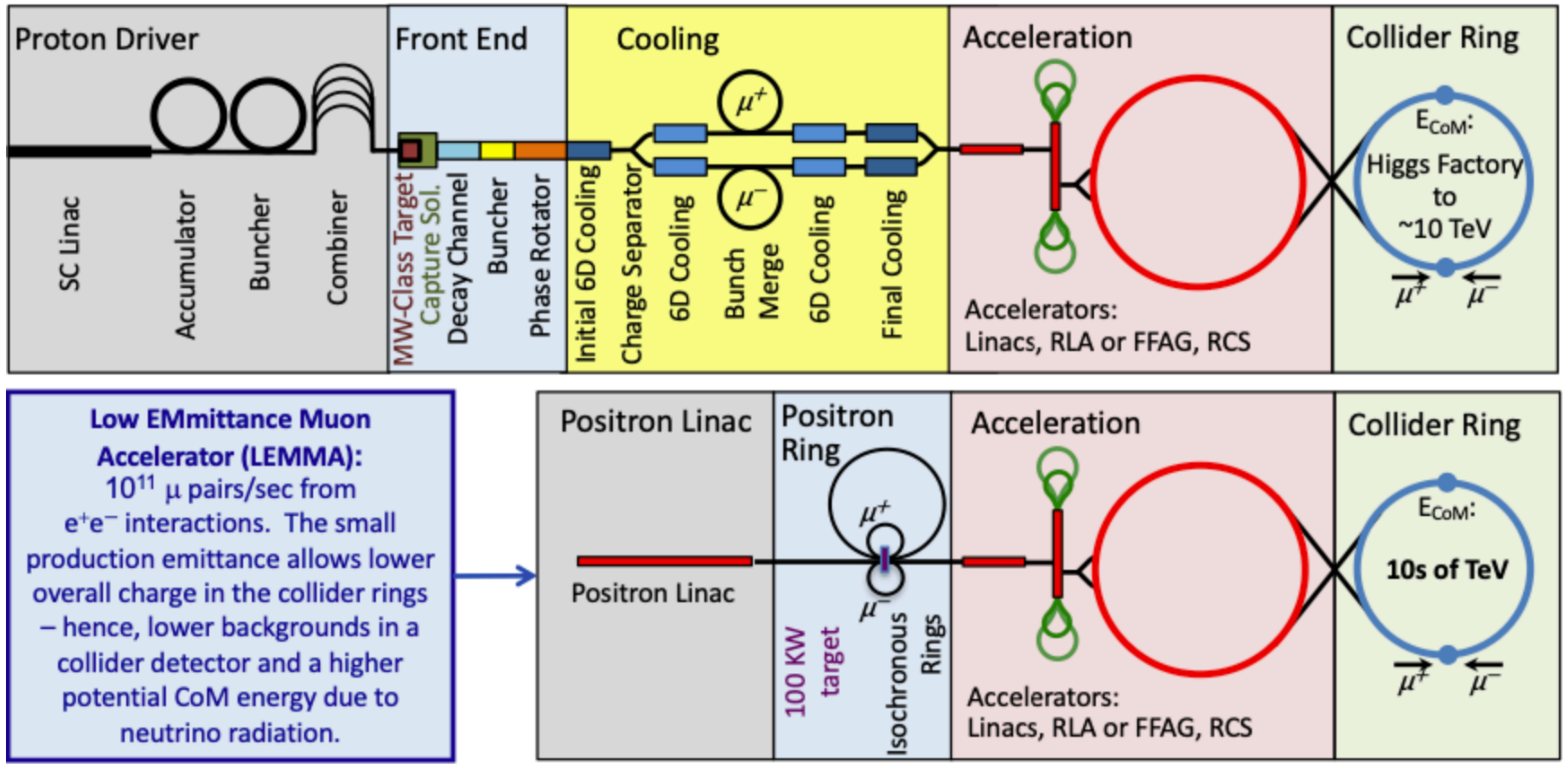

$\sqrt{s}$	$\int \mathcal{L}dt$
3 TeV	1 ab^{-1}
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14 TeV	20 ab^{-1}

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IMCC study group

credit: A. Wulzer

The (high-energy) muon collider

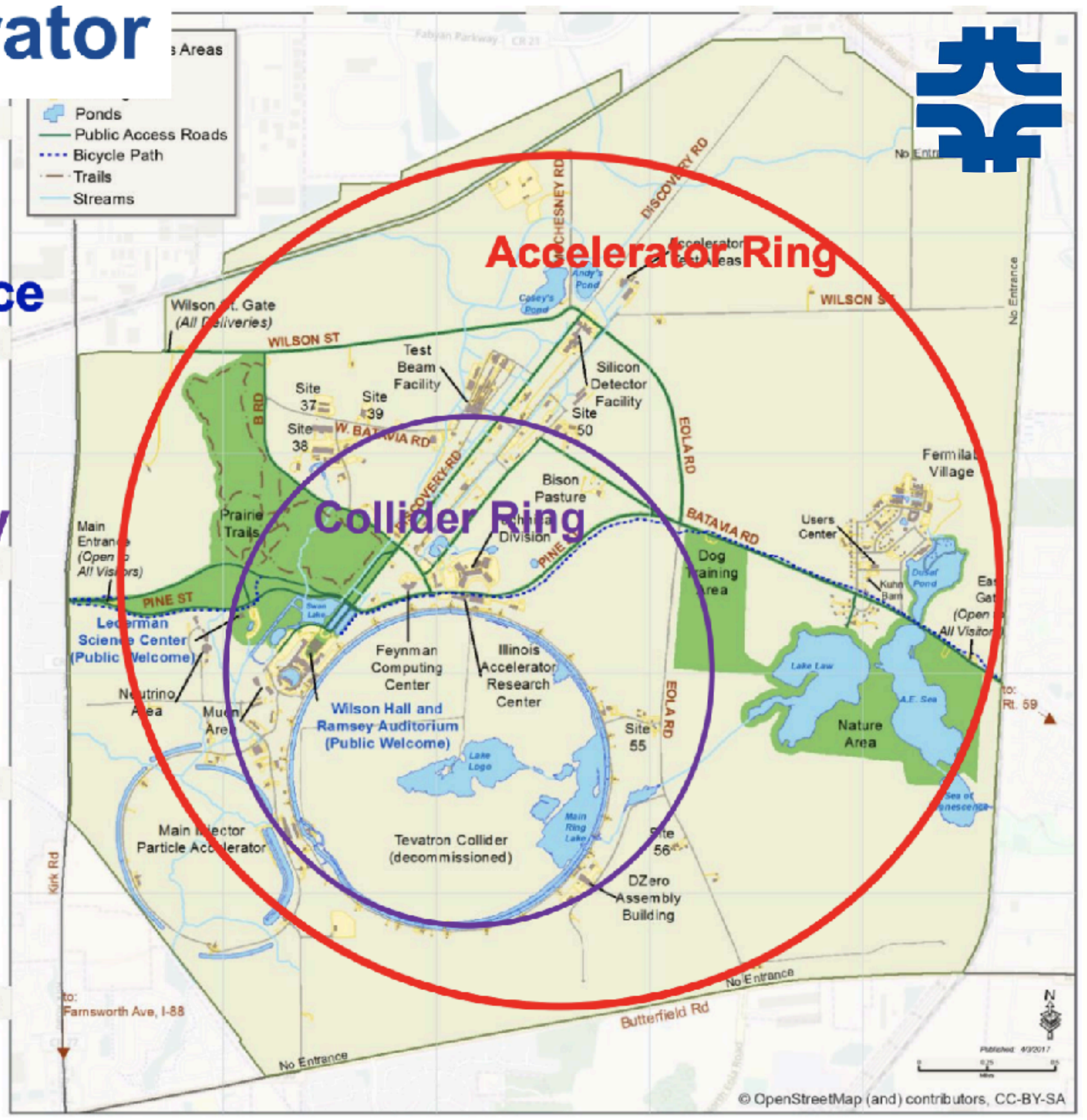
Site filler Accelerator

- ➢**Largest Radius is ~2.65 km**
  - **~16.5 km Circumference**
    - **~2/3 LHC**

~RCS accelerator
If B_{ave}= 3 T → E_μ= 2.4 TeV
(B_{max} = 8T, B_{pulse} =±2T)

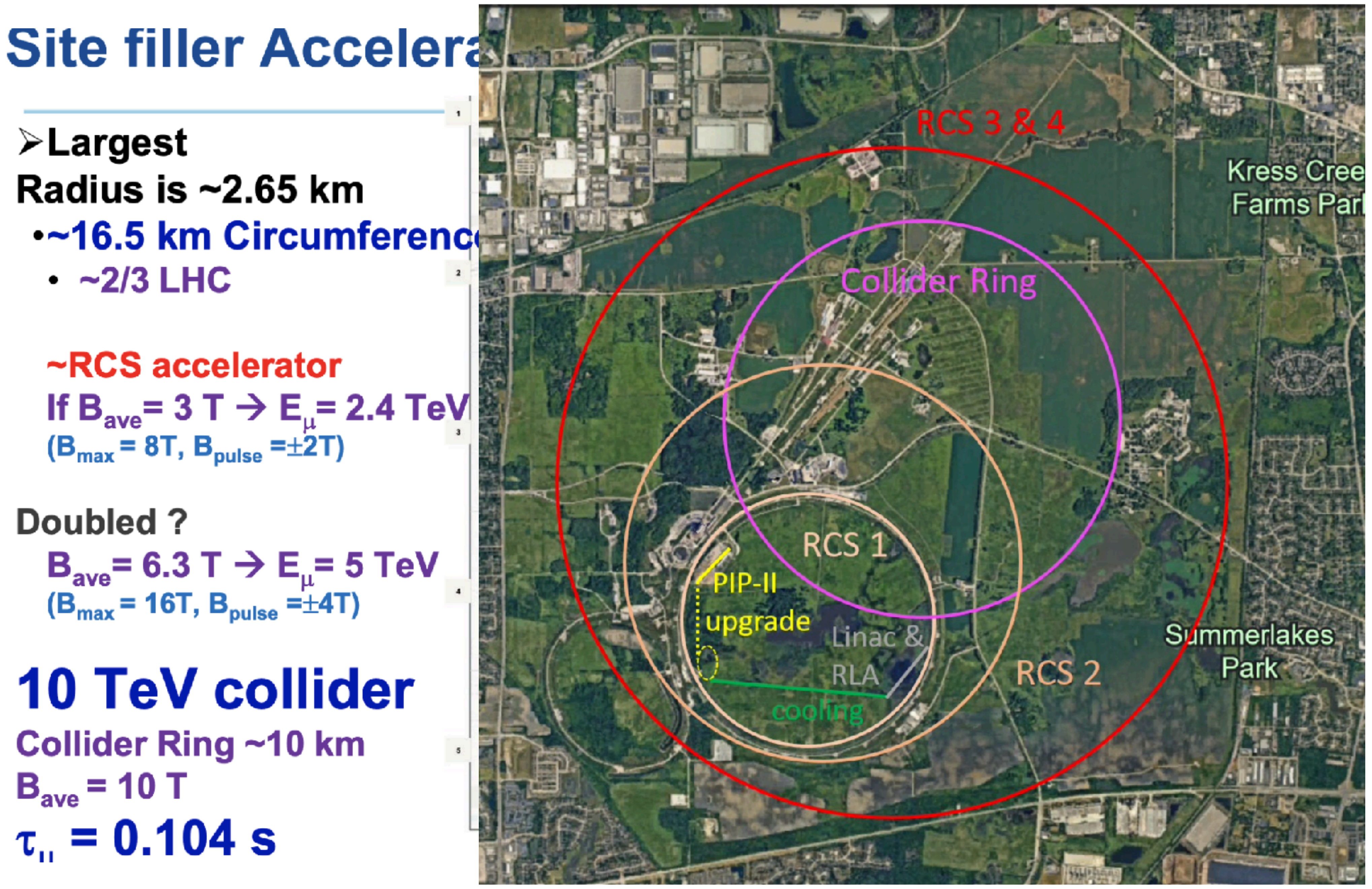
Doubled ?
B_{ave}= 6.3 T → E_μ= 5 TeV
(B_{max} = 16T, B_{pulse} =±4T)

10 TeV collider
Collider Ring ~10 km
B_{ave} = 10 T
τ_μ = 0.104 s



Siting at FNAL

The (high-energy) muon collider



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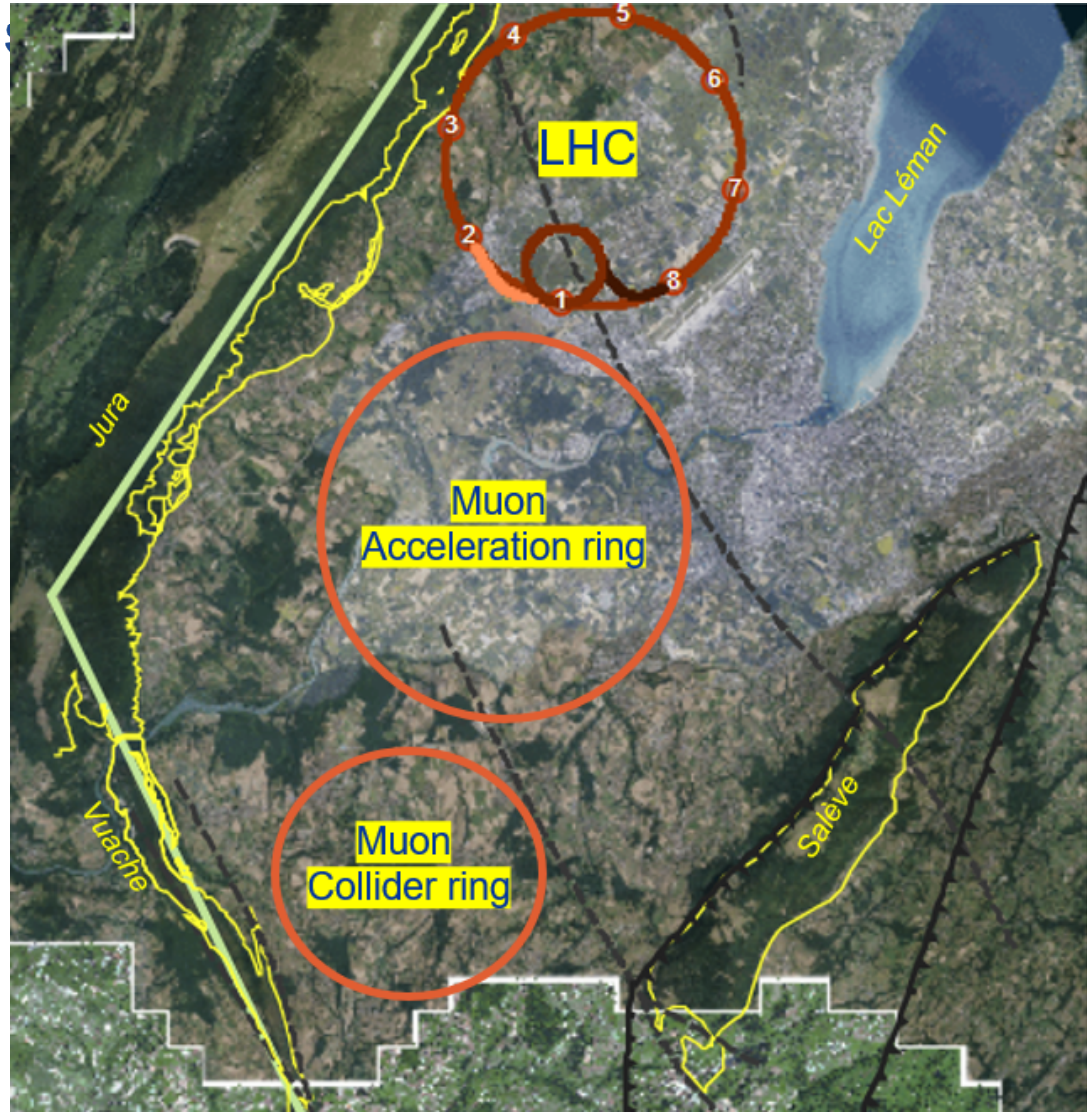
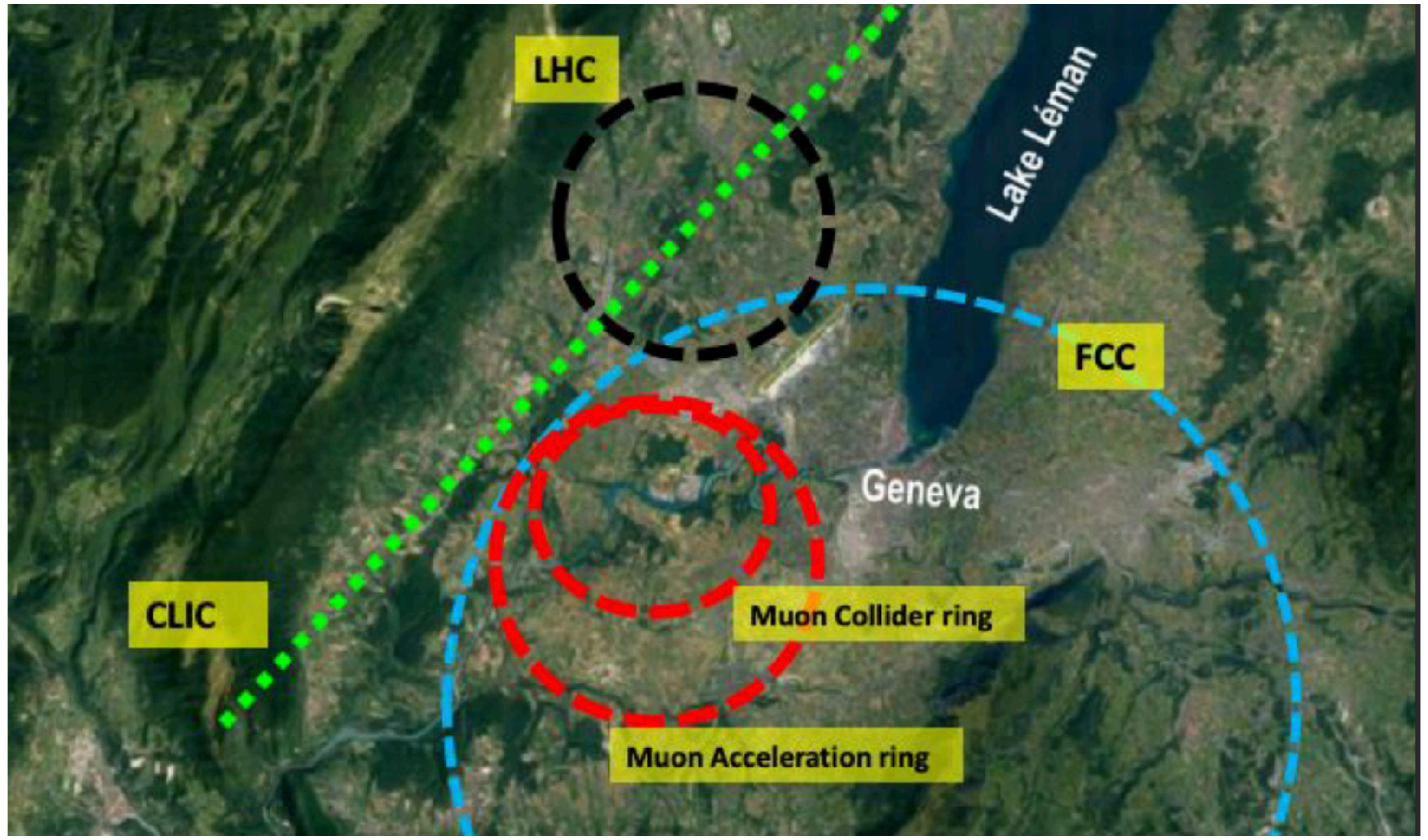
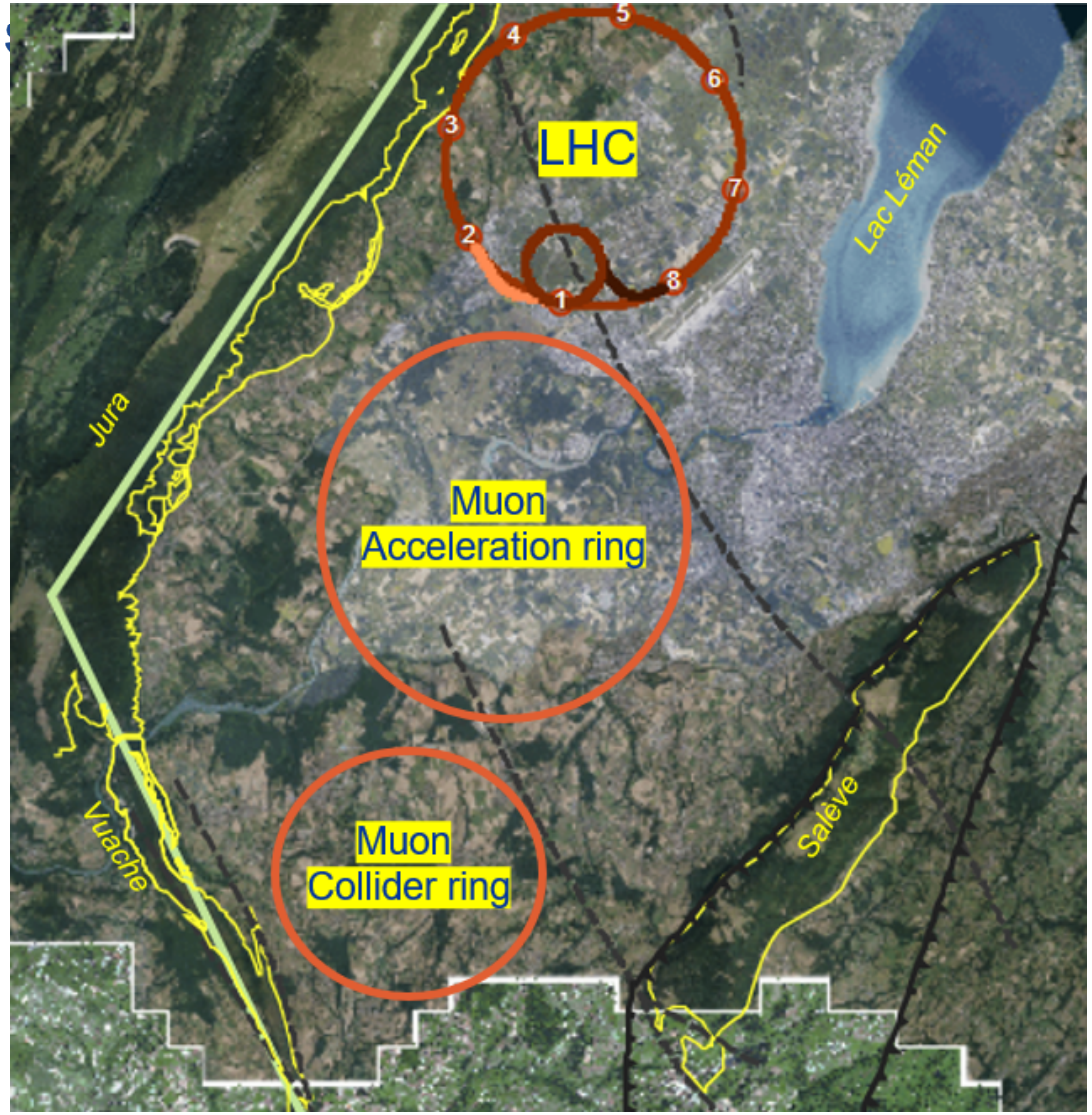


Siting at CERN



The (high-energy) muon collider



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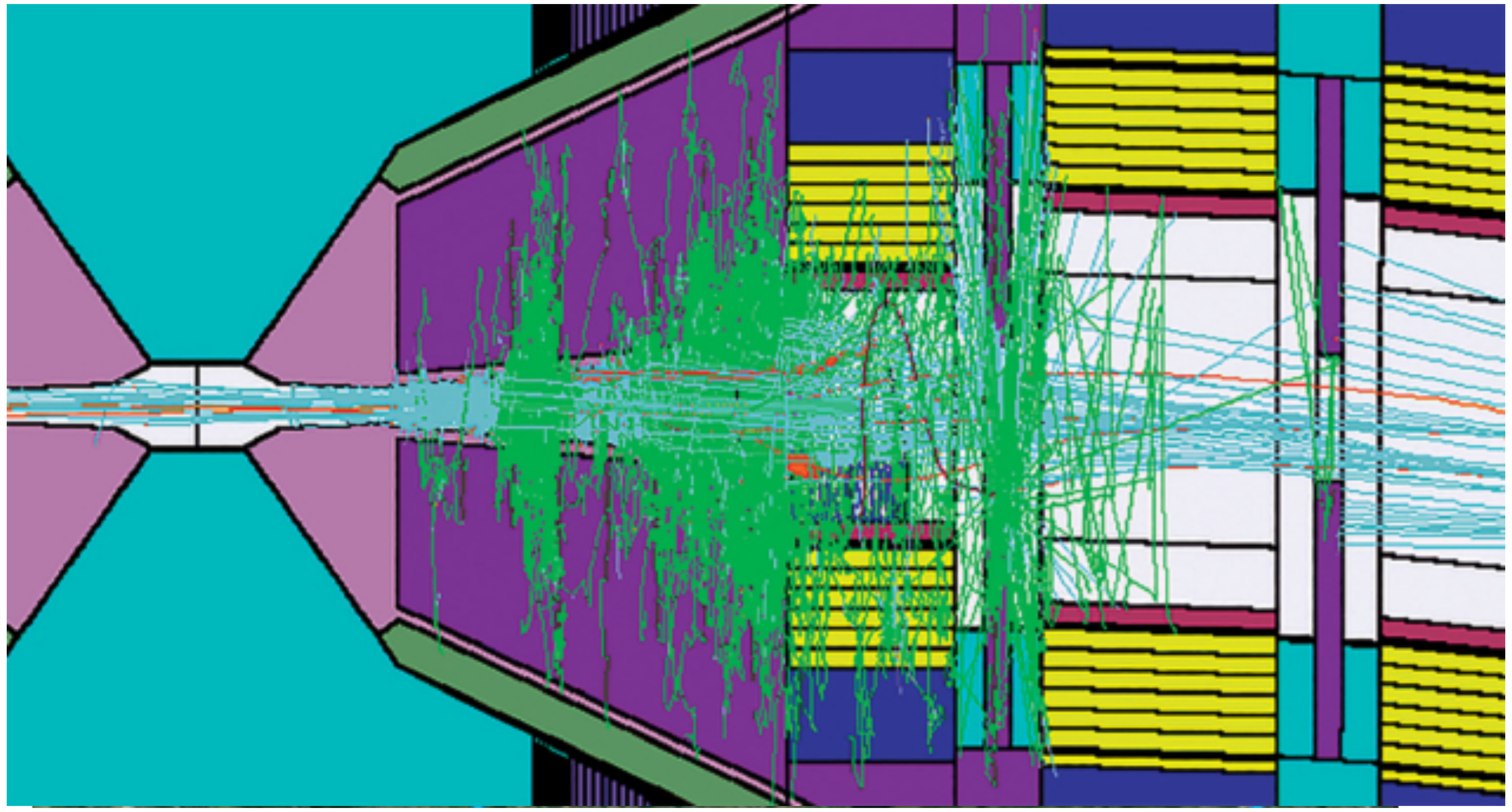
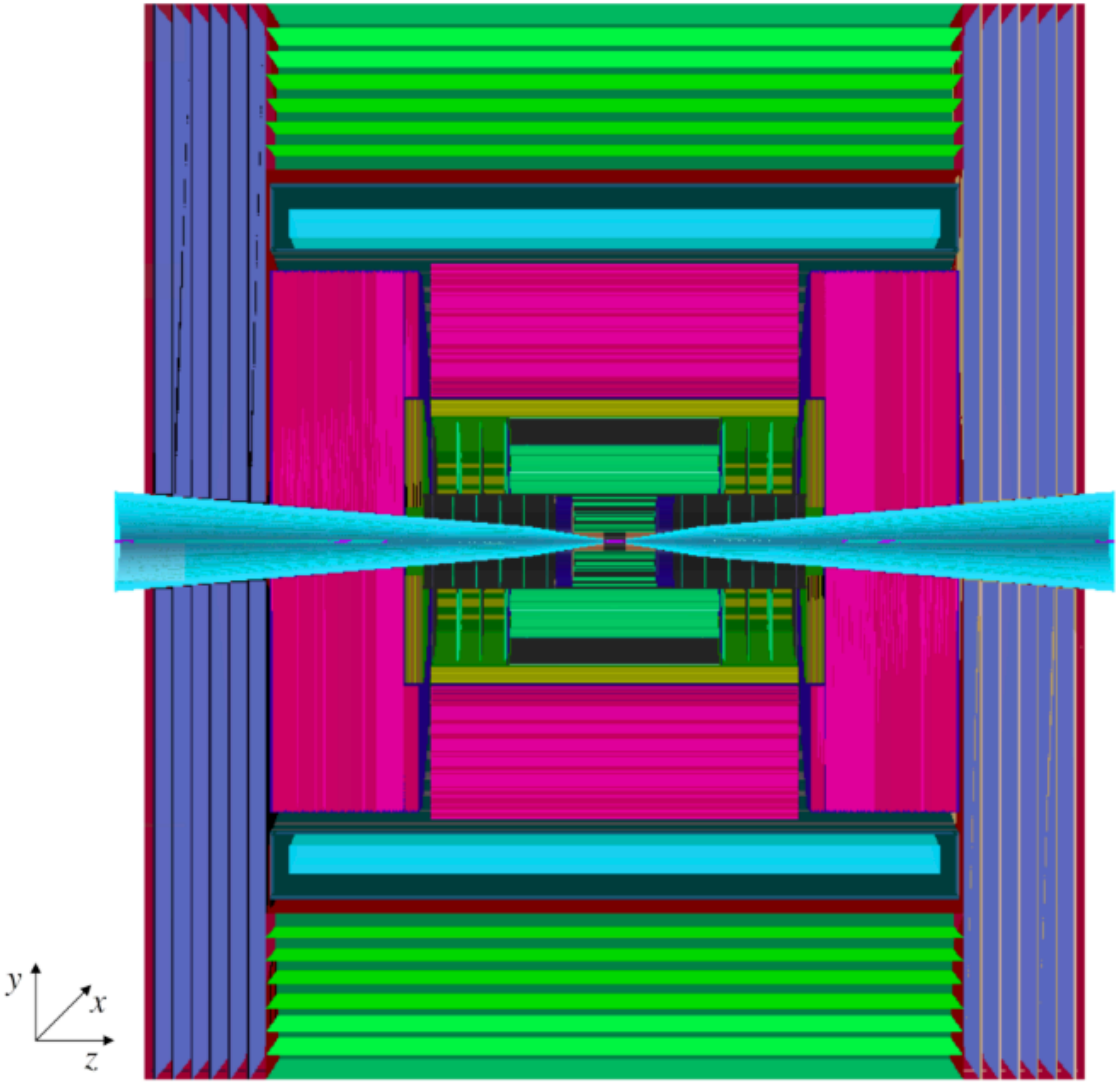
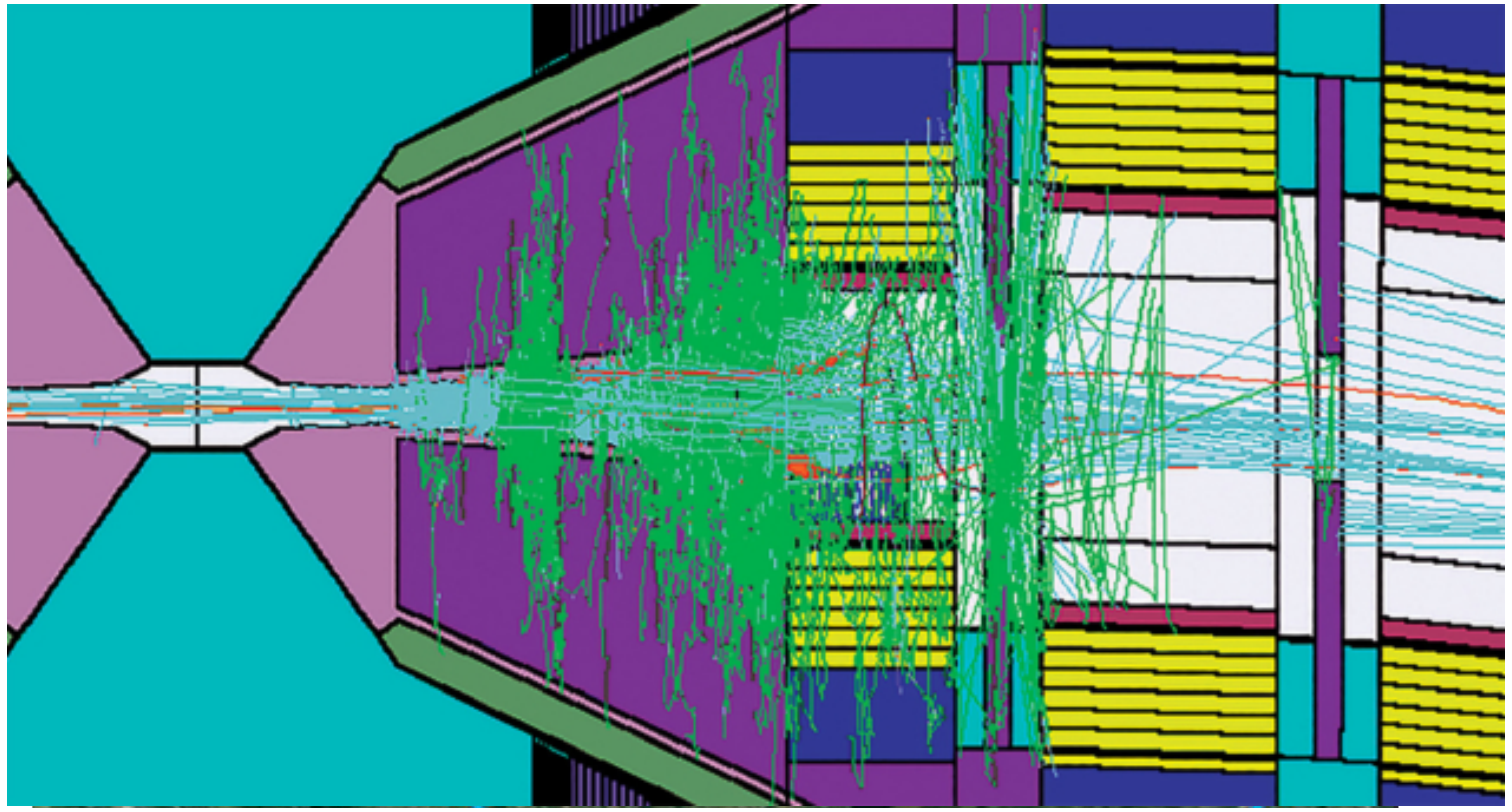


Beam-induced background for the machine-detector interface (MDI)

The (high-energy) muon collider





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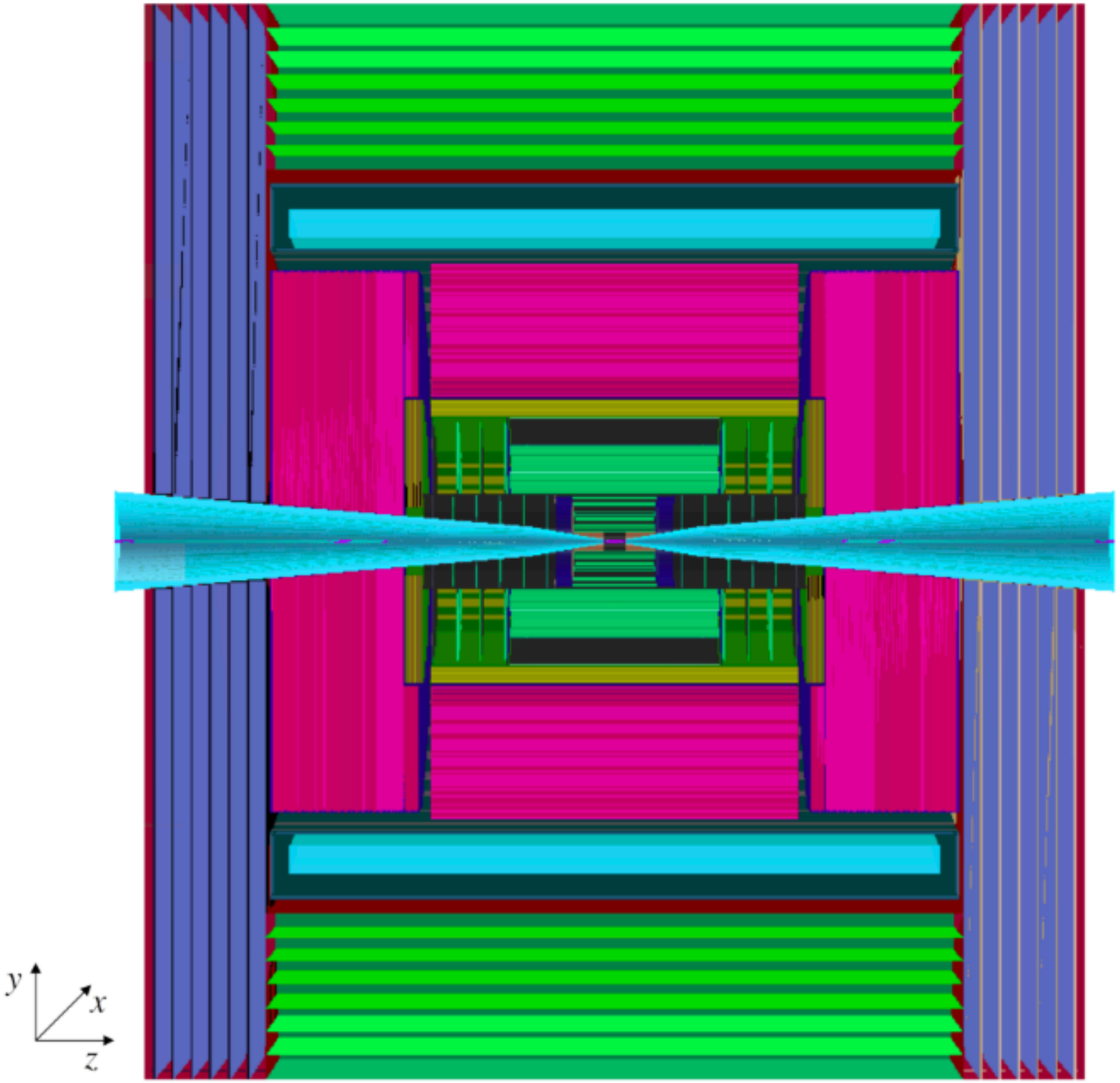

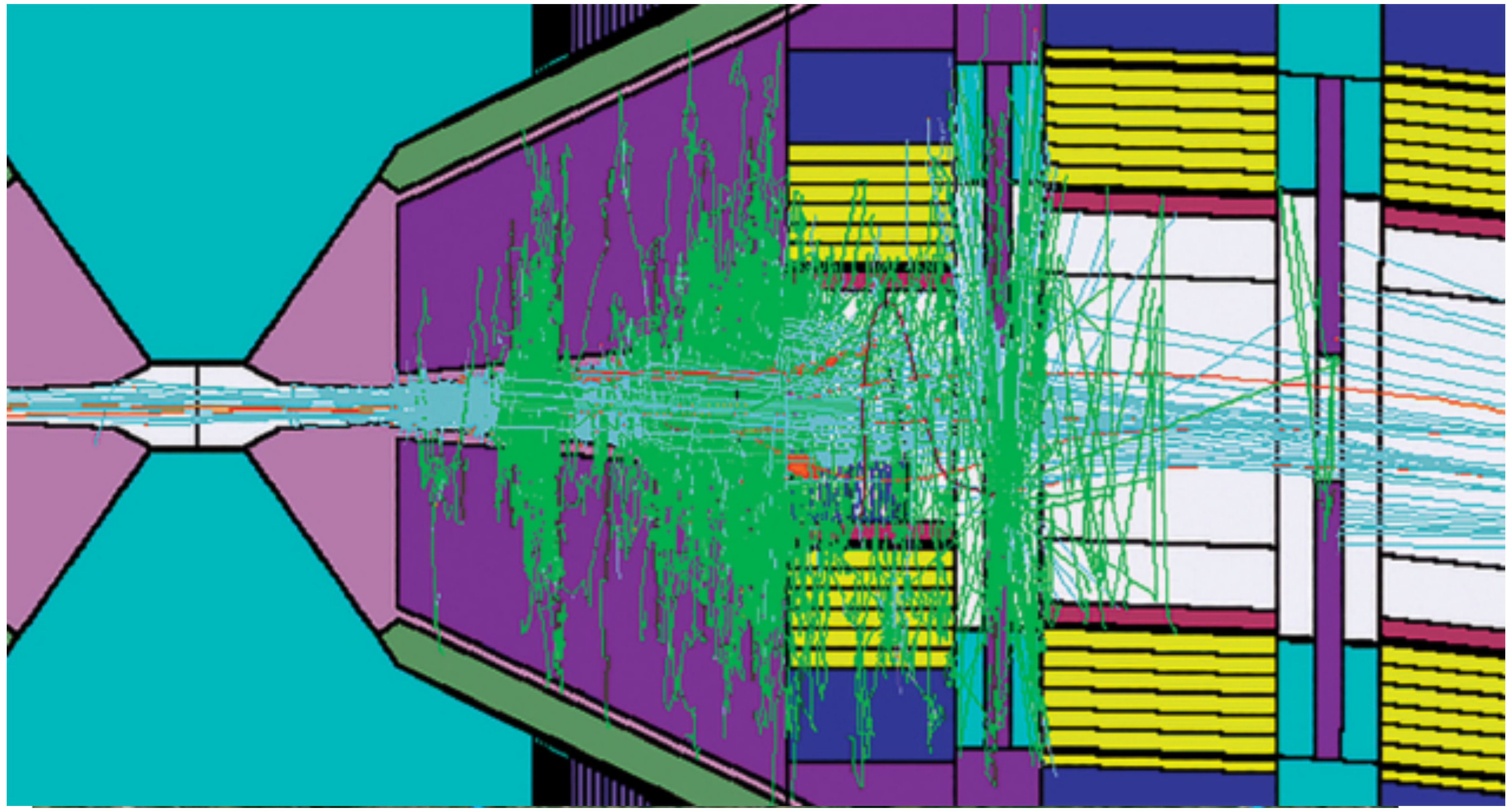

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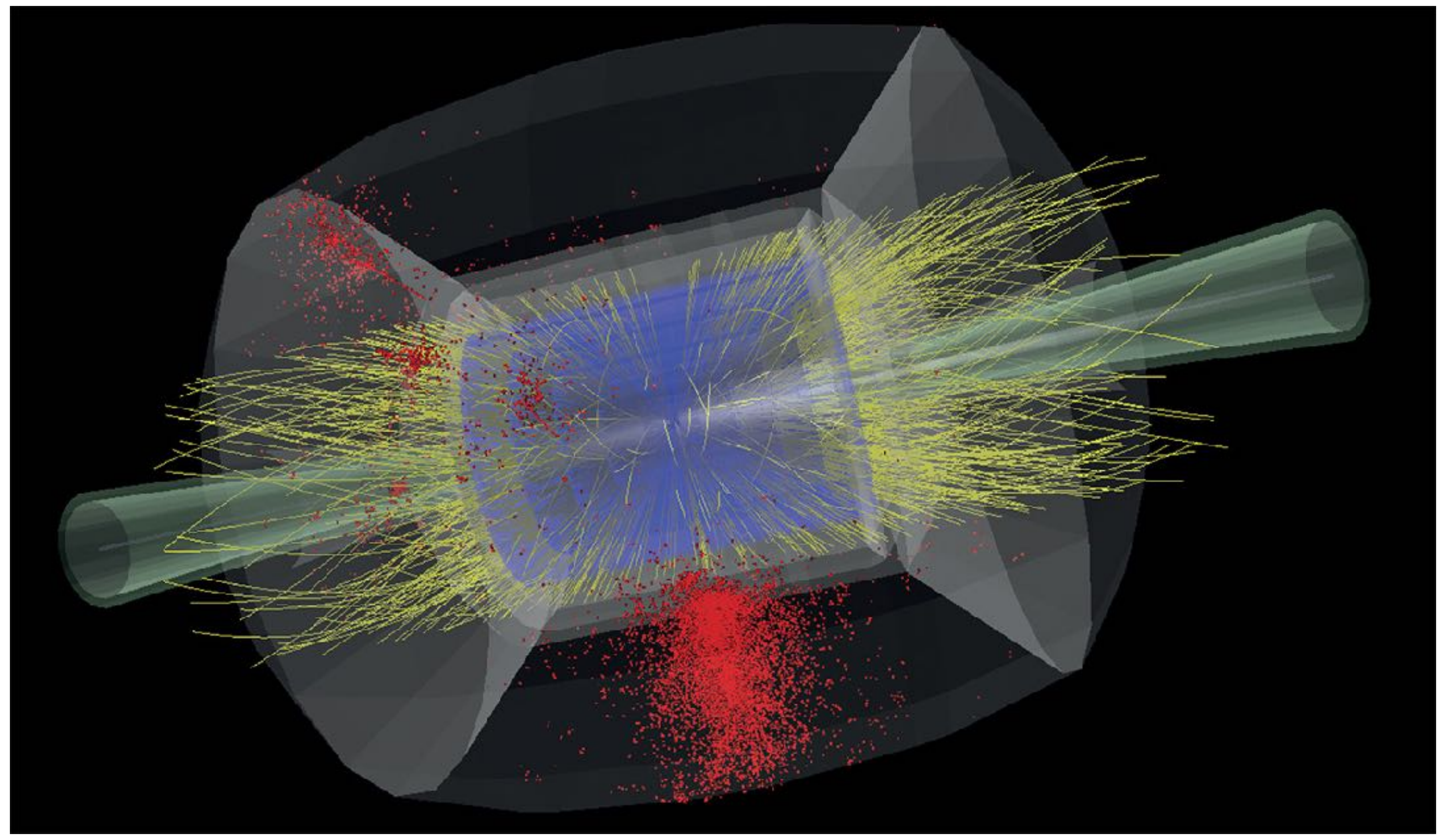


VBF Higgs

Three lamppost BSM searches

Scrutiny of 2nd generation Yukawa couplings

Motivation: Mass generation to be tested for *all* particles

Celada/Han/Kilian/Kreher/Ma/Maltoni/Pagani/JRR/Striegl/Xie, arXiv: 2108.05362 [JHEP] + arXiv:2312.13082 [JHEP]

Search for Heavy Neutral Currents

Motivation: GUTs, B—L , difficulty of global symmetries

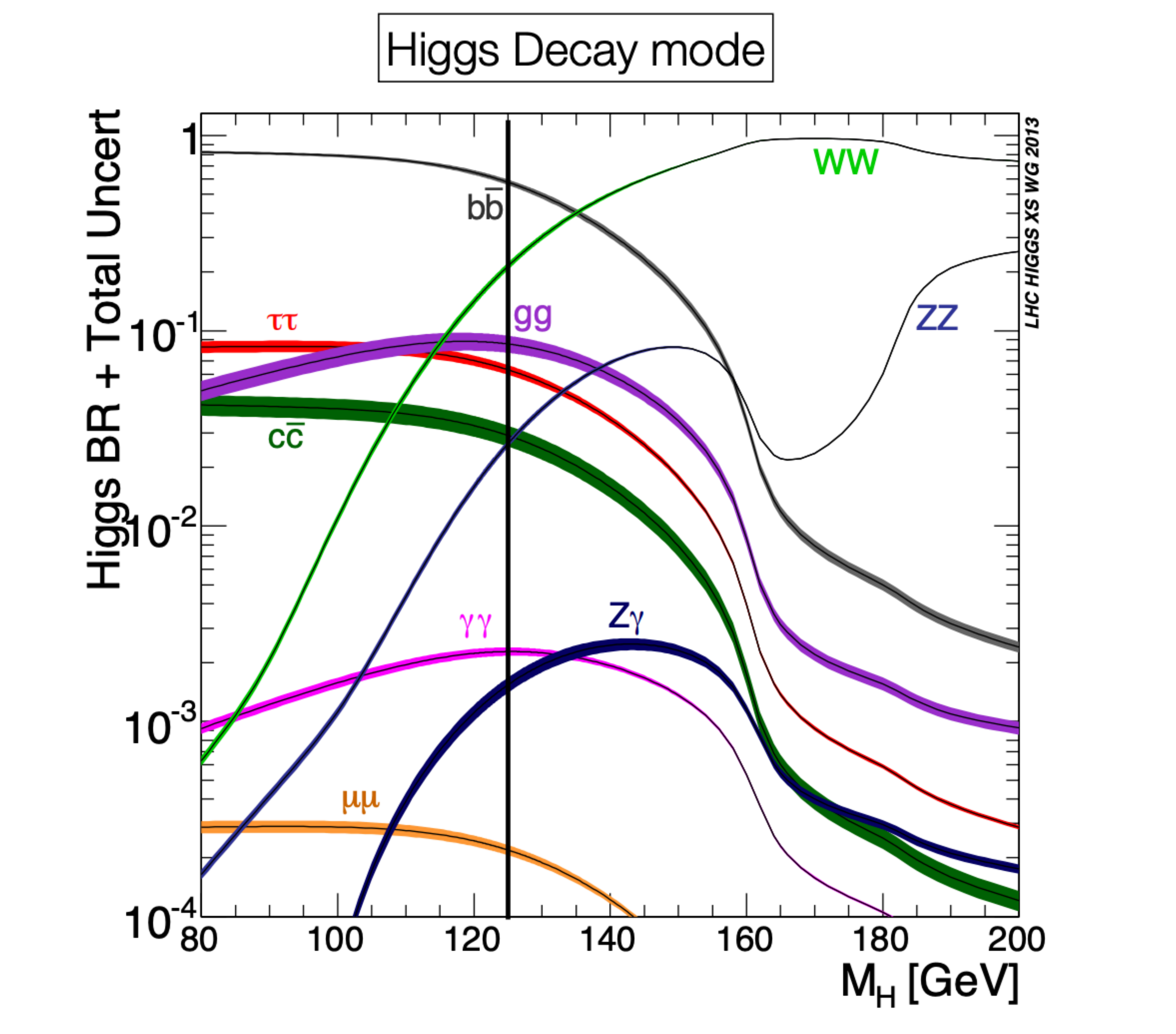
K. Korshynska, M. Löschner, M. Marinichenko, K. Mękała/JRR, arXiv: 2402.18460 [EPJC]

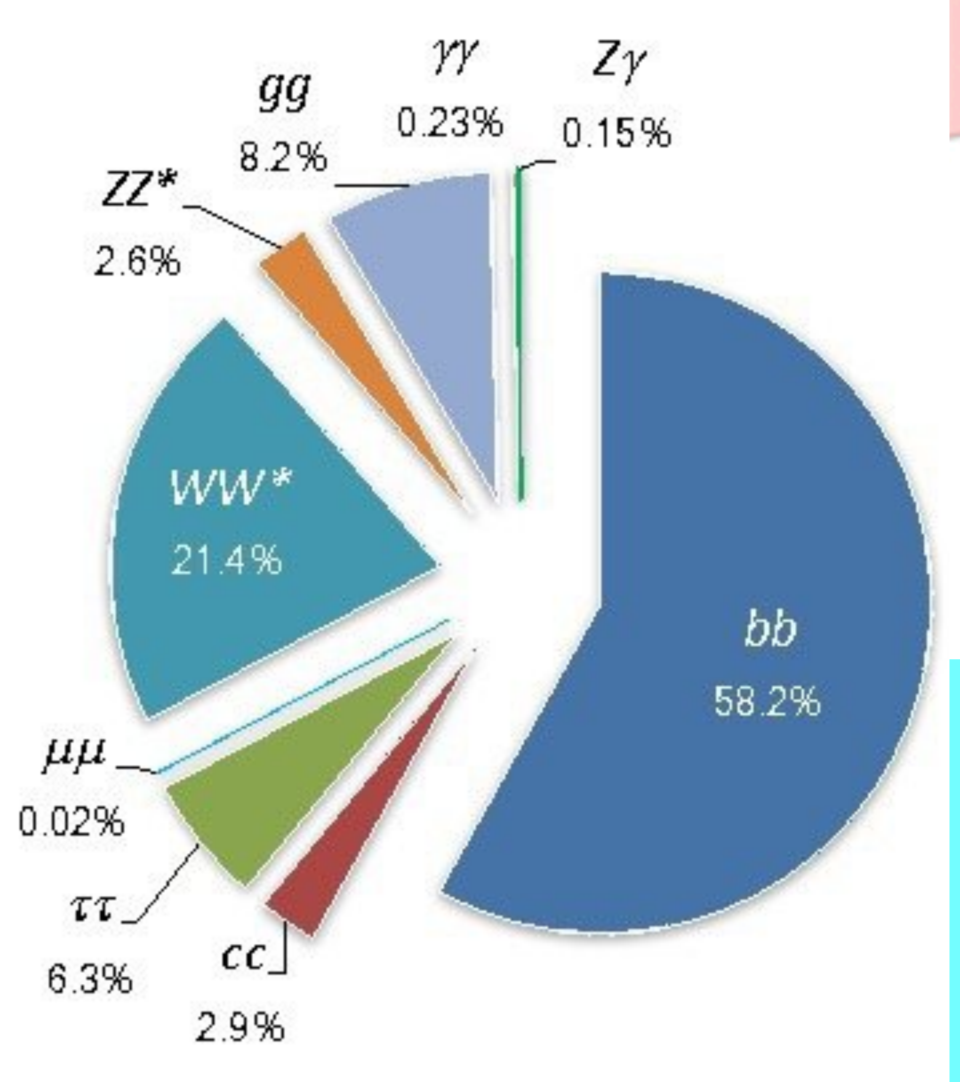
Search for Heavy Neutral Leptons

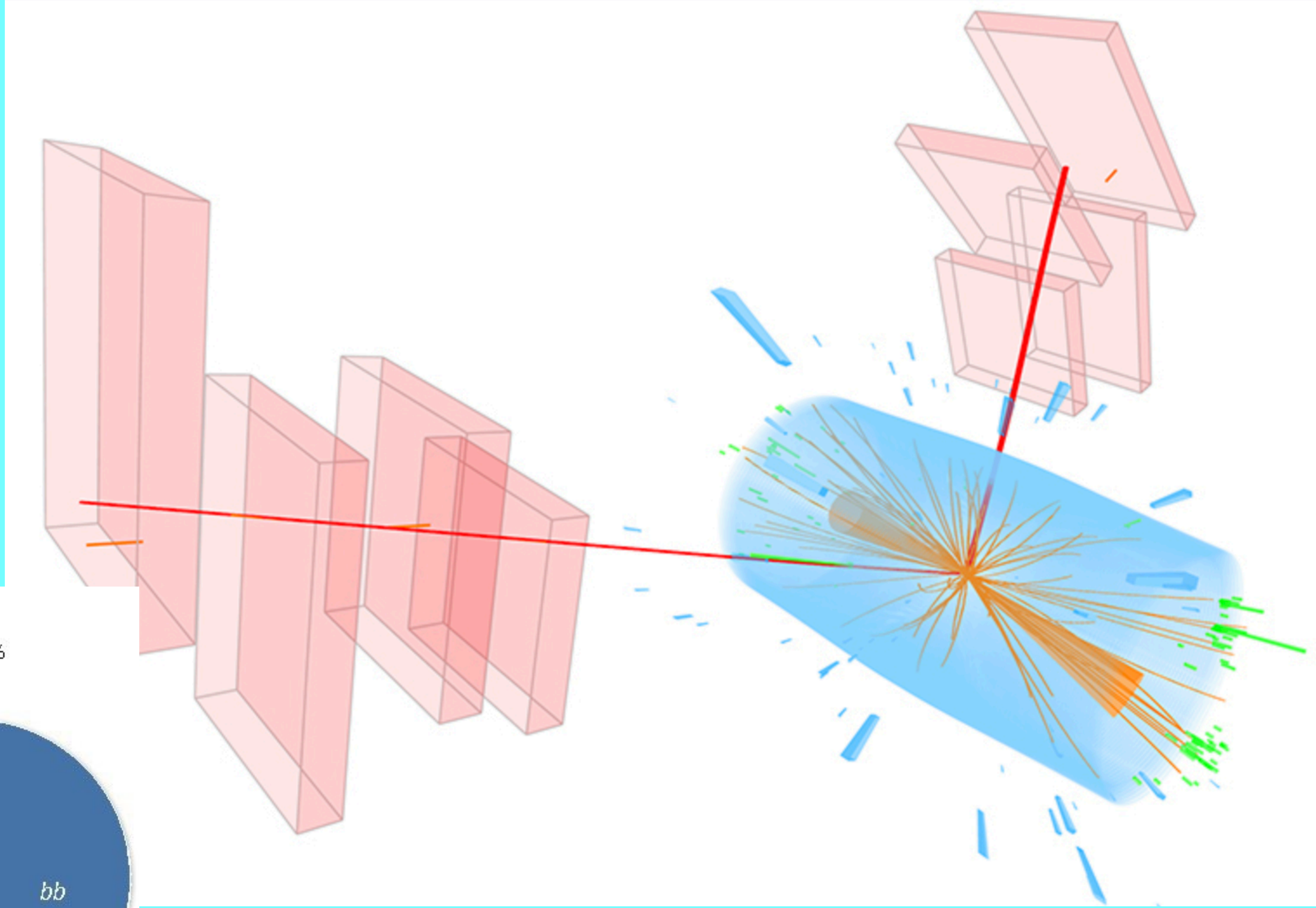
Motivation: neutrino masses, LFV, CP violation, GUTs

K. Mękała/JRR/A.F. Żarnecki, arXiv: 2301.02602 [PLB] + arXiv:2312.05223 [JHEP]

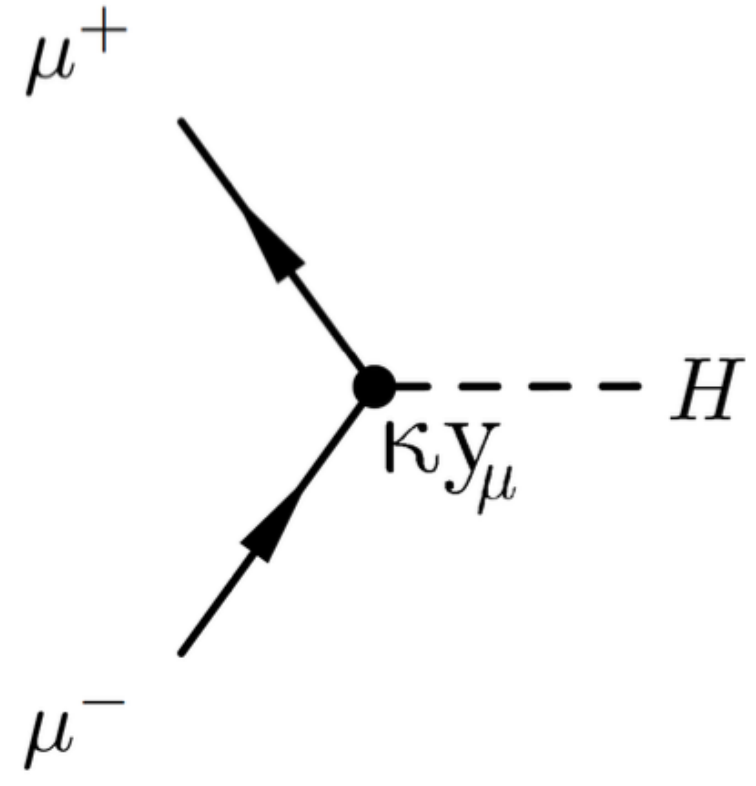
Multi-Bosons: Elusive couplings







EFT Modelling of SM μ-H coupling deviations

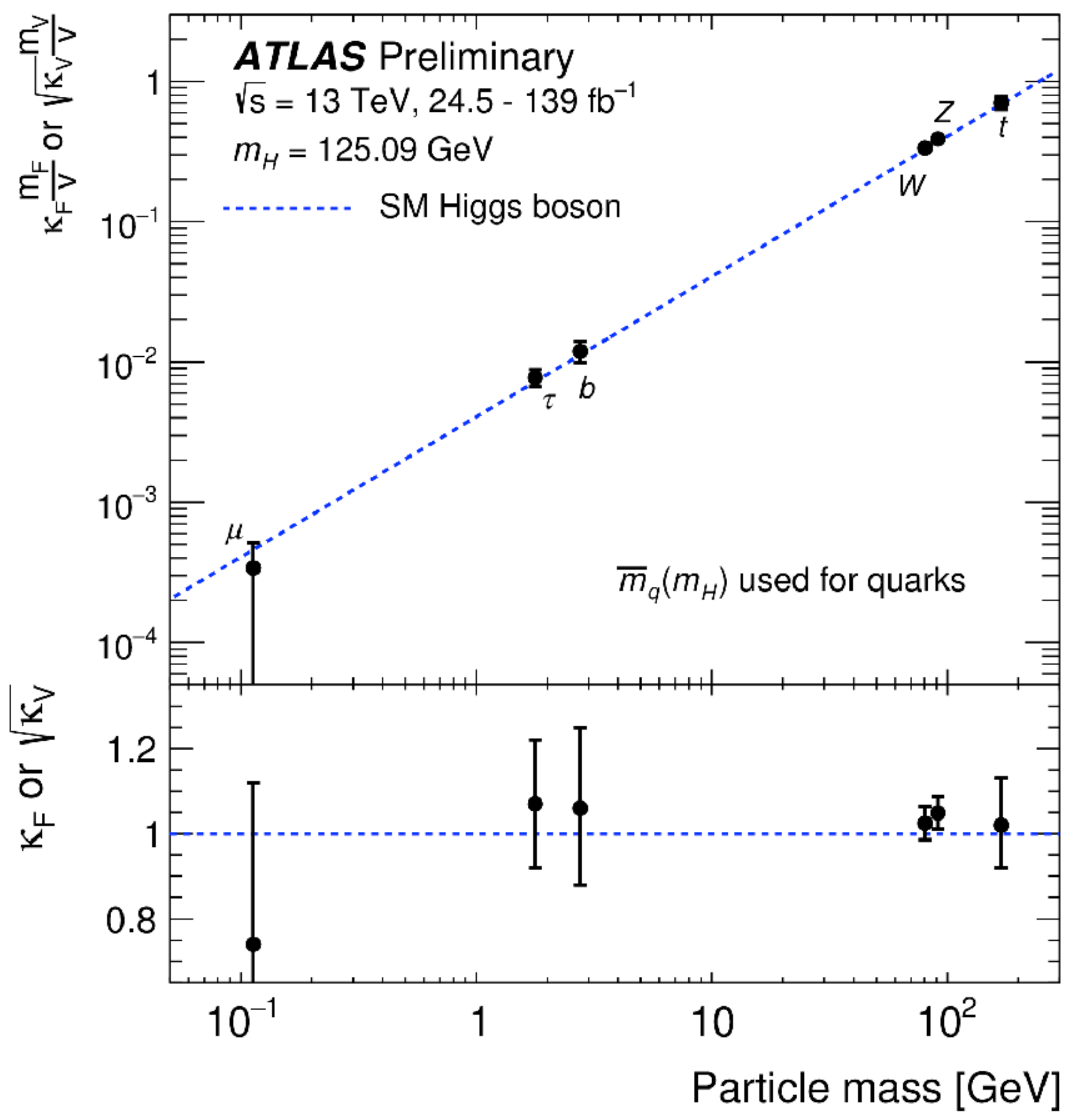


SM: $\kappa = 1$

or $\Delta\kappa = 0$

- Evidence for muon Yukawa coupling at LHC (not yet 5σ) [ATLAS: 2007.07830 ; CMS: 2009.04363]
- Projections for the high-luminosity LHC (HL-LHC): (model-dependent) sensitivity with precision of 5-10% [ATLAS-PHYS-PUB-2014-016]

- Higgs muon Yukawa coupling — connected to muon mass [in the SM!]
- Model-independent test for this coupling; directly, not relying on decays
- Sensitivity to the sign (and maybe phase) of coupling



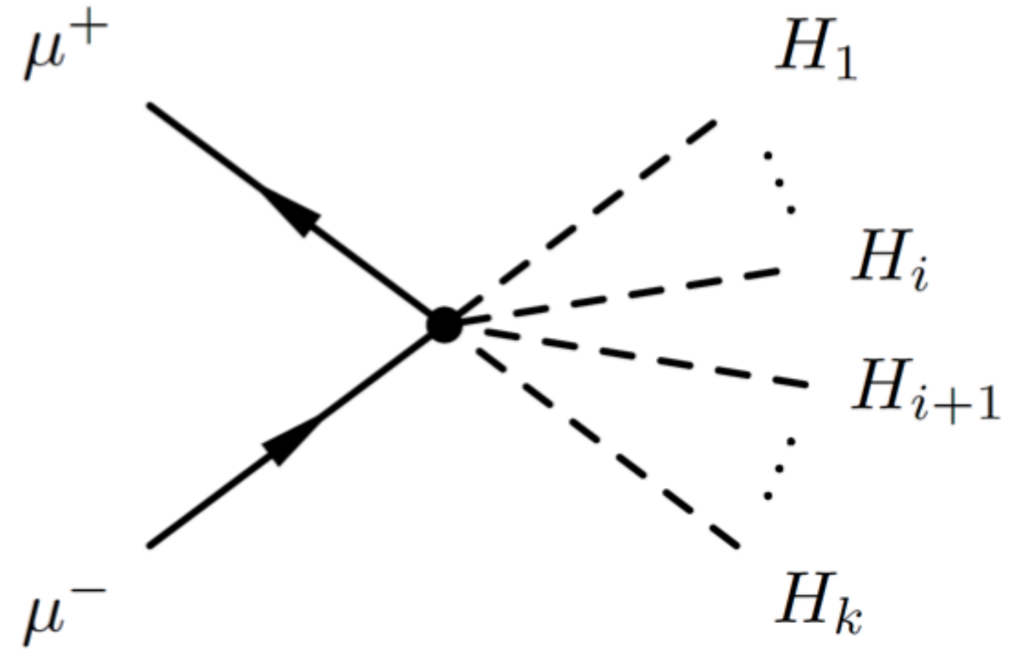
Non-linear representation (HEFT) vs. Linear representation ([truncated] SMEFT)

H doublet $\quad \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi^+ \\ v + H + i\phi^0 \end{pmatrix}$

$$\mathcal{L}_\varphi = \left[-\bar{\mu}_L y_\mu \varphi \mu_R + \sum_{n=1}^{N} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} (\varphi^\dagger \varphi)^n \bar{\mu}_L \varphi \mu_R + \text{h.c.} \right]$$

Generalized (μ) Yukawa sector

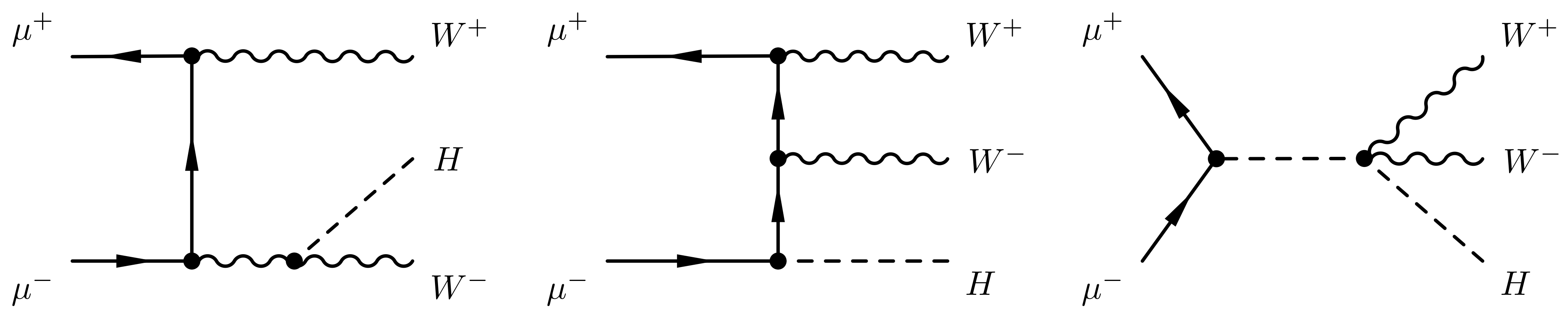
$$-i \frac{k!}{\sqrt{2}} \left[Y_\ell \delta_{k,1} - \sum_{n=n_k}^{M-1} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \binom{2n+1}{k} \frac{v^{2n+1-k}}{2^n} \right] = 0 \quad =$$



Benchmark scenario: "matched" case

Multi-boson final states

- Subtle cancellation between Yukawa coupling and multi-boson final states [hep-ph/0106281]



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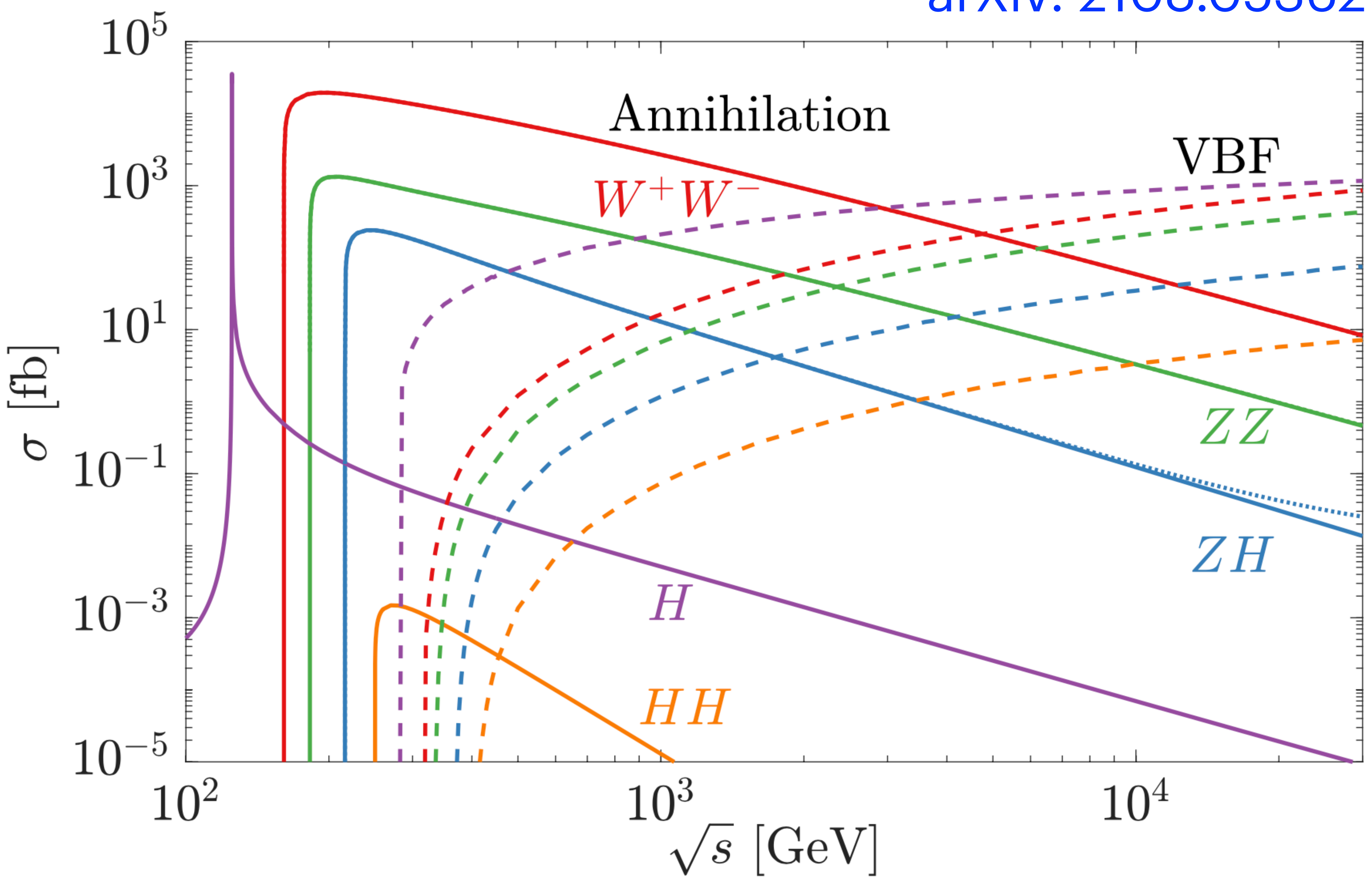
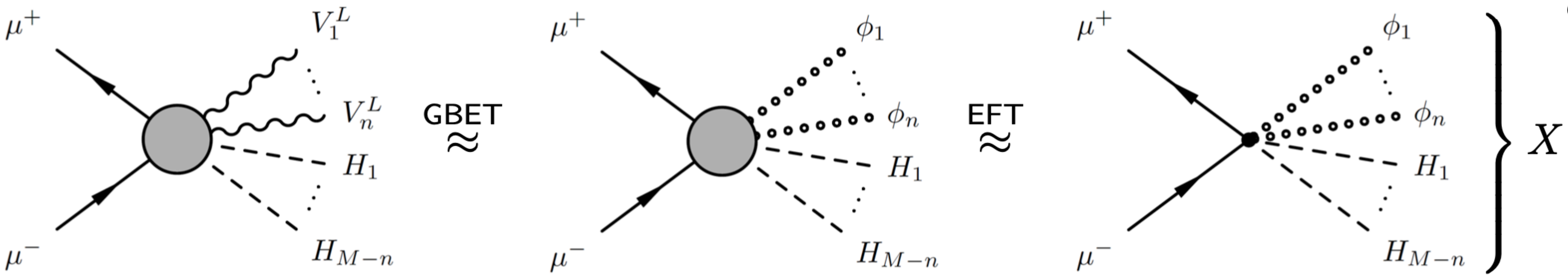


- (Multi-) boson final states: longitudal polarizations dominate high energies
- Analytic calculations can be approximated by Goldstone-boson Equivalence Theorem (GBET) [NPB261(1985) 379; PRD34(1986) 379]
- New physics parameterized by EFT operator insertions (Wilson coeff. C_X)



$$\sigma_X \approx \frac{1}{4}\left(\frac{\pi}{2(2\pi)^4}\right)^{M-1} \frac{s^{M-2}}{\Gamma(M)\Gamma(M-1)}|C_X|^2 \left(\prod_{j\in J_X}\frac{1}{n_j!}\right)$$

Cross section ratios:

$$R = \frac{\sigma_X}{\sigma_Y} \approx \frac{|C_X|^2 \left(\prod_{j\in J_X}\frac{1}{n_j!}\right)}{|C_Y|^2 \left(\prod_{j\in J_Y}\frac{1}{n_j!}\right)}$$

Simulation, Consistency, Unitarity & Cross Sections

- Analytical calculations checked independently by 3 groups
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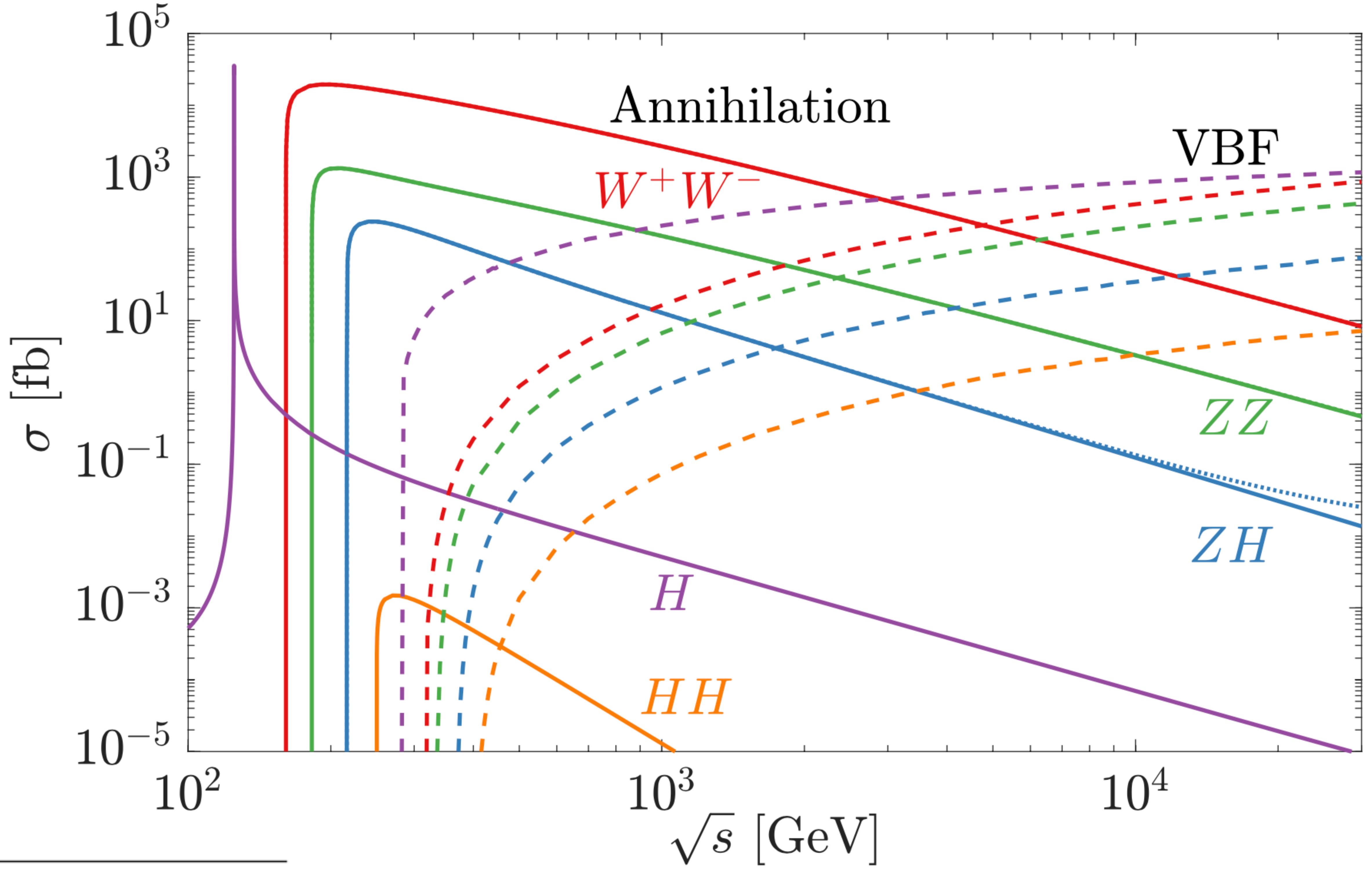
- Final simulation: using UFO files in `WHIZARD`

States with multiplicity 2

- Different cases: dim 6 alone, dim 8 alone, dim 6+8 combined
- Matched case: combination such that Yukawa coupling is zero



	$\Delta\sigma^X/\Delta\sigma^{W^+W^-}$					
	SMEFT				HEFT	
X	$\dim_6$	$\dim_8$	$\dim_{6,8}$	$\dim_{6,8}^{\text{matched}}$	$\dim_\infty$	$\dim_\infty^{\text{matched}}$
W^+W^-	1	1	1	1	1	1
ZZ	1/2	1/2	1/2	1/2	1/2	1/2
ZH	1	1/2	1	1	$R^{\text{HEFT}}_{(2),1}$	1
HH	9/2	25/2	$R^{\text{SMEFT}}_{(2),1}/2$	0	$2\,R^{\text{HEFT}}_{(2),2}$	0

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States with multiplicity 3

- Different cases: dim 6 alone, dim 8 alone, dim 6+8 combined
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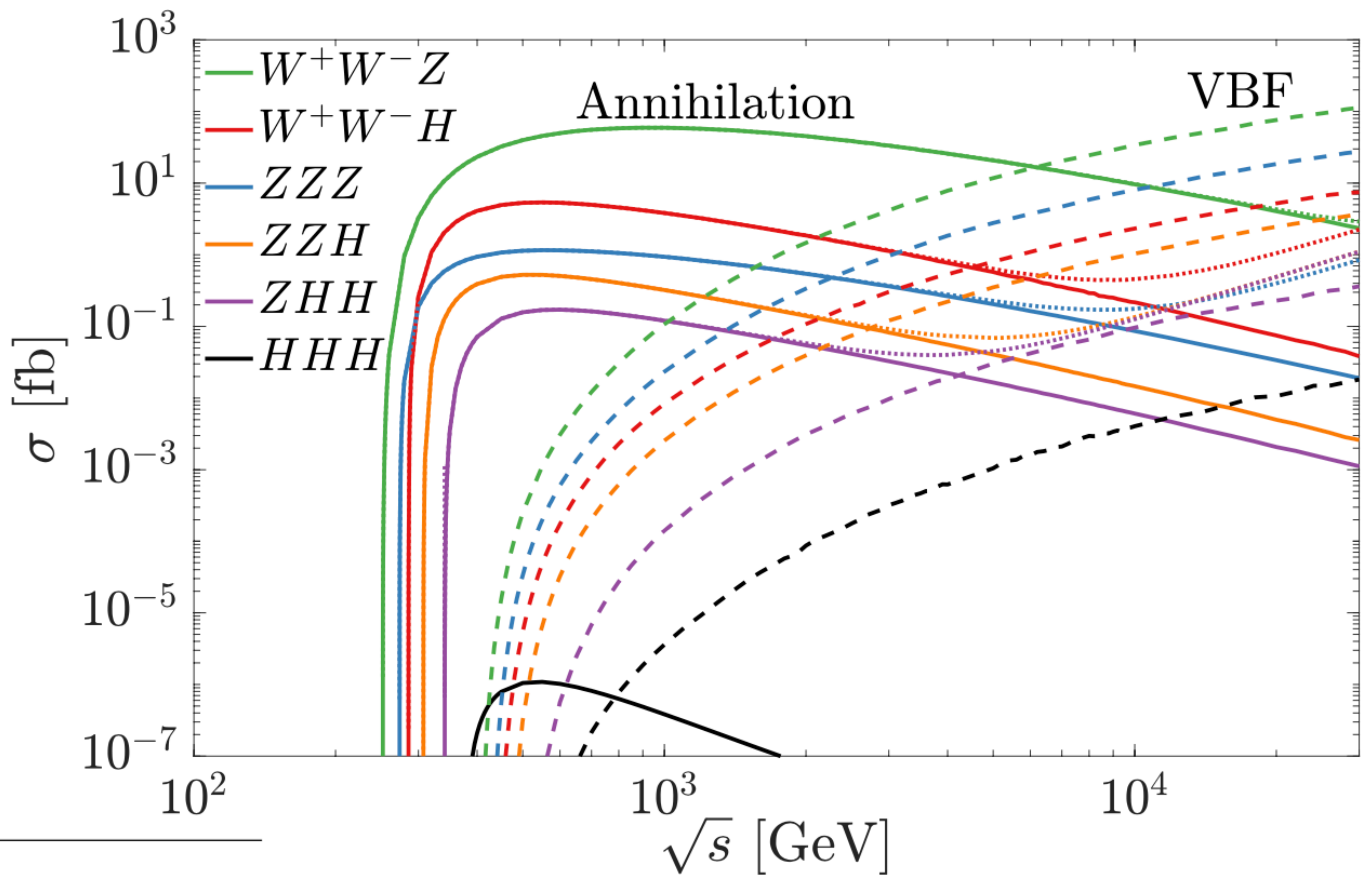

	$\Delta\sigma^X/\Delta\sigma^{W^+W^-H}$					
	SMEFT				HEFT	
$\mu^+\mu^- \to X$	$\dim_6$	$\dim_8$	$\dim_{6,8}$	$\dim_{6,8}^{\text{matched}}$	$\dim_\infty$	$\dim_\infty^{\text{matched}}$
WWZ	1	1/9	$R^{\text{SMEFT}}_{(3),1}$	1/4	$R^{\text{HEFT}}_{(3),1}/9$	1/4
ZZZ	3/2	1/6	$3\,R^{\text{SMEFT}}_{(3),1}/2$	3/8	$R^{\text{HEFT}}_{(3),1}/6$	3/8
WWH	1	1	1	1	1	1
ZZH	1/2	1/2	1/2	1/2	1/2	1/2
ZHH	1/2	1/2	1/2	1/2	$2\,R^{\text{HEFT}}_{(3),2}$	1/2
HHH	3/2	25/6	$3\,R^{\text{SMEFT}}_{(3),2}/2$	75/8	$6\,R^{\text{HEFT}}_{(3),3}$	0

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States with multiplicity 4

- Different cases: dim 6 alone, dim 8 alone, dim 6+8 combined
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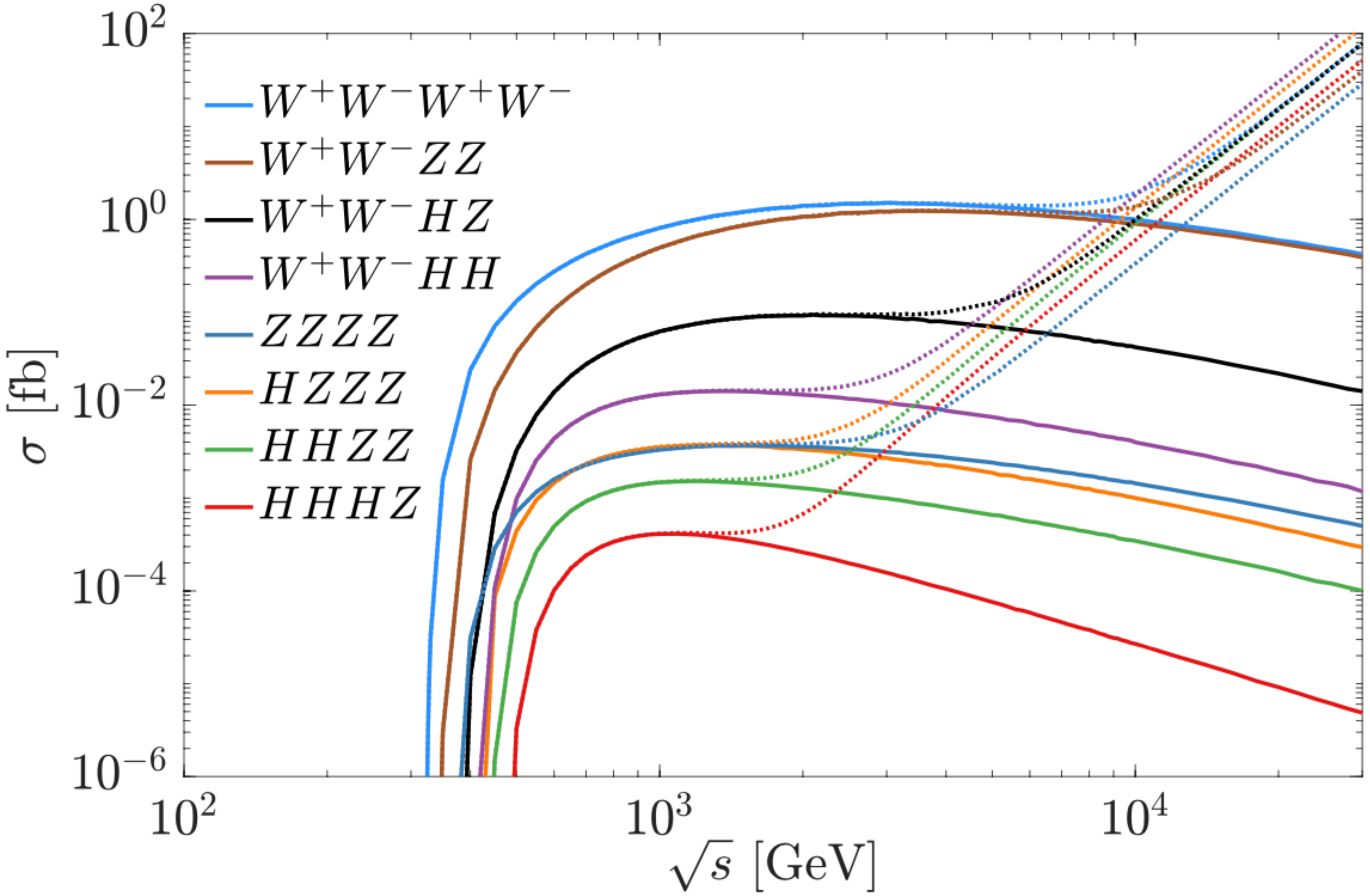

	SMEFT				HEFT	
$\mu^+\mu^- \to X$	$\text{dim}_{6,8}$	dim_{10}	$\text{dim}_{6,8,10}$	$\text{dim}^{\text{matched}}_{6,8,10}$	dim_∞	$\text{dim}^{\text{matched}}_\infty$
$WWWW$	$2/9$	$2/25$	$2\,R^{\text{SMEFT}}_{(4),1}/9$	$1/2$	$R^{\text{HEFT}}_{(4),1}/18$	$1/2$
$WWZZ$	$1/9$	$1/25$	$R^{\text{SMEFT}}_{(4),1}/9$	$1/4$	$R^{\text{HEFT}}_{(4),1}/36$	$1/4$
$ZZZZ$	$1/12$	$3/100$	$R^{\text{SMEFT}}_{(4),1}/12$	$3/16$	$R^{\text{HEFT}}_{(4),1}/48$	$3/16$
$WWZH$	$2/9$	$2/25$	$2\,R^{\text{SMEFT}}_{(4),1}/9$	$1/2$	$R^{\text{HEFT}}_{(4),2}/8$	$1/2$
$WWHH$	1	1	1	1	1	1
$ZZZH$	$1/3$	$3/25$	$R^{\text{SMEFT}}_{(4),1}/3$	$3/4$	$R^{\text{HEFT}}_{(4),2}/12$	$3/4$
$ZZHH$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$
$ZHHH$	$1/3$	$1/3$	$1/3$	$1/3$	$3\,R^{\text{HEFT}}_{(4),3}$	$1/3$
$HHHH$	$25/12$	$49/12$	$25\,R^{\text{SMEFT}}_{(4),2}/12$	$1225/48$	$12\,R^{\text{HEFT}}_{(4),4}$	0

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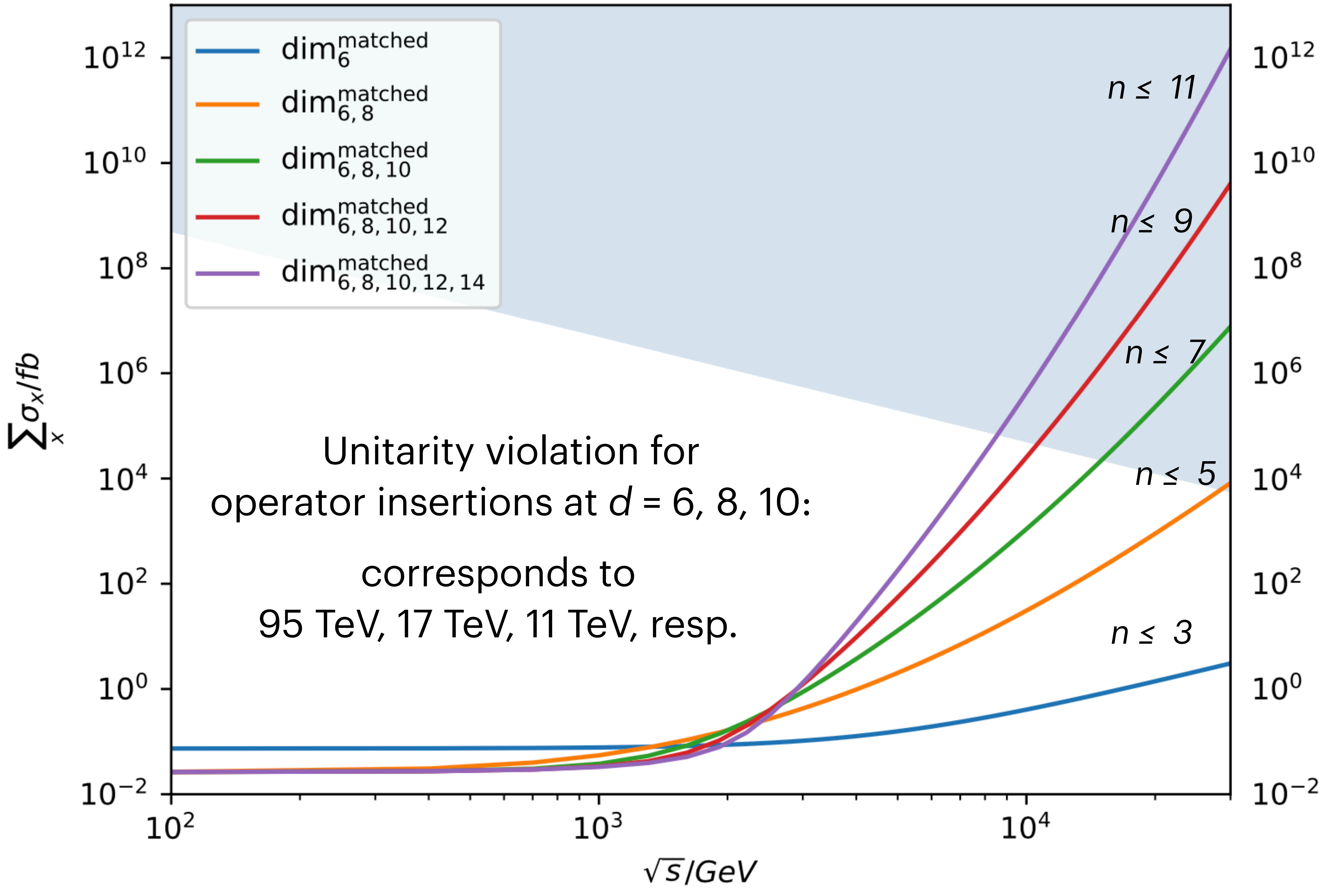


$\mu^+\mu^- \to X$	SMEFT $\mathrm{dim}_{6,8}$	dim_{10}	$\mathrm{dim}_{6,8,10}$	$\mathrm{dim}^{\mathrm{matched}}_{6,8,10}$		
$WWWW$	2/9	2/25	$2\,R^{\mathrm{SMEFT}}_{(4),1}/9$	1/2	R [illegible]	[illegible]
$WWZZ$	1/9	1/25	$R^{\mathrm{SMEFT}}_{(4),1}/9$	1/4	R [illegible]	[illegible]
$ZZZZ$	1/12	3/100	$R^{\mathrm{SMEFT}}_{(4),1}/12$	3/16	$R^{\mathrm{HEFT}}_{(4),1}/48$	3/16
$WWZH$	2/9	2/25	$2\,R^{\mathrm{SMEFT}}_{(4),1}/9$	1/2	$R^{\mathrm{HEFT}}_{(4),2}/8$	1/2
$WWHH$	1	1	1	1	1	1
$ZZZH$	1/3	3/25	$R^{\mathrm{SMEFT}}_{(4),1}/3$	3/4	$R^{\mathrm{HEFT}}_{(4),2}/12$	3/4
$ZZHH$	1/2	1/2	1/2	1/2	1/2	1/2
$ZHHH$	1/3	1/3	1/3	1/3	$3\,R^{\mathrm{HEFT}}_{(4),3}$	1/3
$HHHH$	25/12	49/12	$25\,R^{\mathrm{SMEFT}}_{(4),2}/12$	1225/48	$12\,R^{\mathrm{HEFT}}_{(4),4}$	0

Unitarity bound for final states $X \neq \mu\mu$:

$$\sum_X \sigma_{\mu^+\mu^-\to X}(s) \leq \frac{4\pi}{s}$$

hep-ph/0106281

Kinematic separation of signal

$\mu^+\mu^- \to W^+W^-H$

Kinematic separation between multi-boson direct production and VBF, e.g. 10 TeV: arXiv: 2108.05362

- *WWZ* largest cross section, but small deviation
- *WWH* large cross section and considerable deviation
- *ZZH* smaller/-ish cross section, but largest (relative) deviation
- Direct production has almost full energy (except for ISR) $\Longrightarrow$ M_{3B}
- VBF generates mostly forward bosons $\Longrightarrow$ Θ_B
- Separation criterion for final state bosons $\Longrightarrow$ ΔR_{BB}

Cut flow	$\kappa_\mu = 1$	w/o ISR	$\kappa_\mu = 0$ (2)	CVBF	NVBF
σ [fb]			WWH		
No cut	0.24	0.21	0.47	2.3	7.2
$M_{3B} > 0.8\sqrt{s}$	0.20	0.21	0.42	$5.5 \cdot 10^{-3}$	$3.7 \cdot 10^{-2}$
$10° < \theta_B < 170°$	0.092	0.096	0.30	$2.5 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
$\Delta R_{BB} > 0.4$	0.074	0.077	0.28	$2.1 \cdot 10^{-4}$	$2.4 \cdot 10^{-4}$
# of events	740	770	2800	2.1	2.4
S/B			2.8		

Results and final projections

Muon collider with energy range $1 < \sqrt{s} < 30 \text{ TeV}$ and luminosity $\mathcal{L} = \left(\frac{\sqrt{s}}{10 \text{ TeV}}\right)^2 10 \text{ ab}^{-1}$

1901.06150; 2001.04431; PoS(ICHEP2020)703; Nat.Phys.17, 289-292

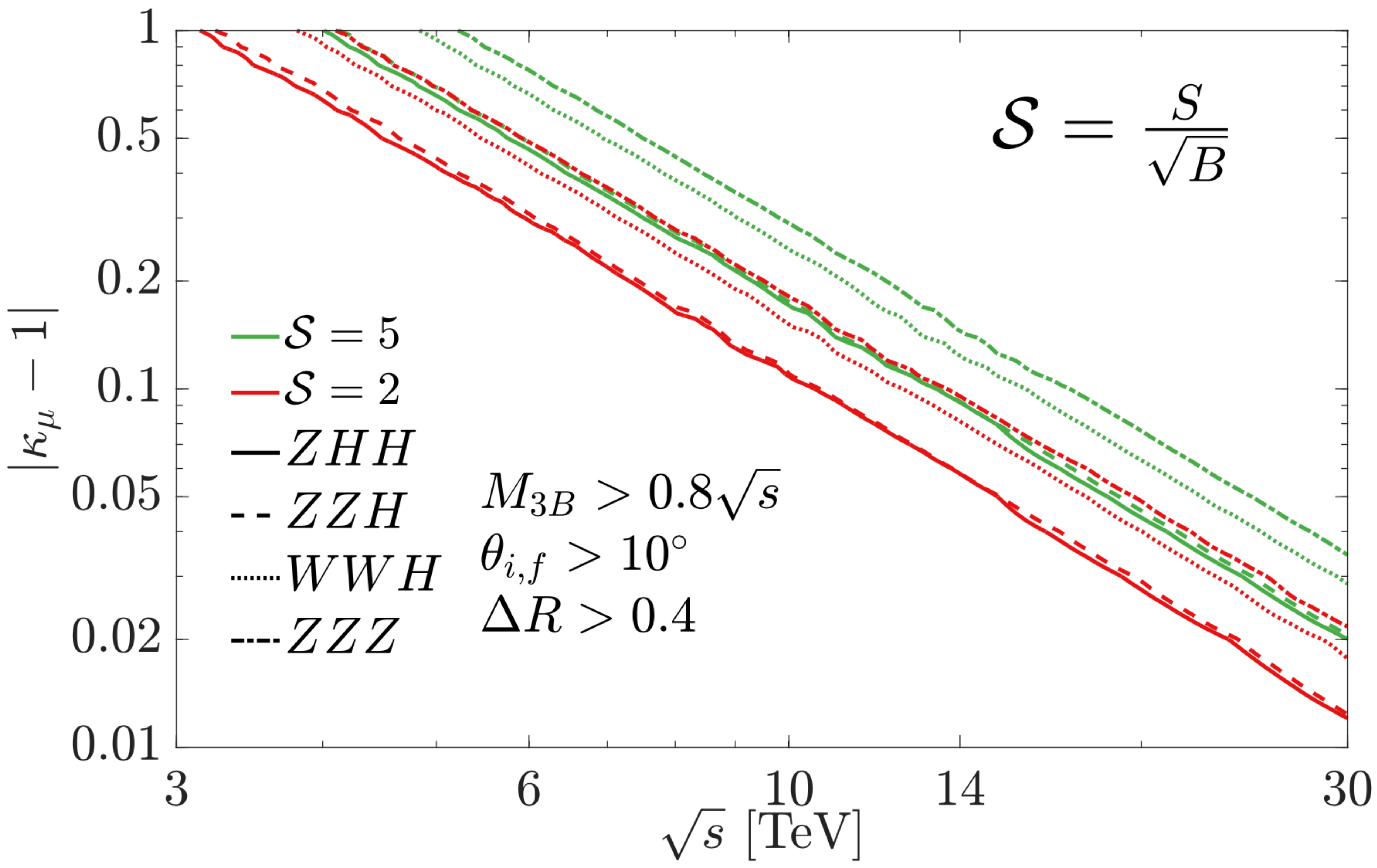
- Sensitivity to (deviations of) the muon Yukawa coupling
- Definition of # signal events: $S = N_{\kappa_\mu} - N_{\kappa_\mu=1}$
- Definition of # background events: $B = N_{\kappa_\mu=1} + N_{\text{VBF}}$
- Statistical significance of anom. muon Yukawa couplings:

$$\mathcal{S} = \frac{S}{\sqrt{B}}$$ (note that always: $N_{\kappa_\mu} \geq N_{\kappa_\mu=1}$)

$$\sigma|_{\kappa_\mu=1+\delta} = \sigma|_{\kappa_\mu=1-\delta} \quad \Longrightarrow \quad \mathcal{S}|_{\kappa_\mu=1+\delta} = \mathcal{S}|_{\kappa_\mu=1-\delta}$$

- 5σ sensitivity to 20% @ 10 TeV .... 2% @ 30 TeV
- Sensitivity to κ translates to new physics scale Λ

$$\Lambda > 10 \text{ TeV} \sqrt{\frac{g}{\Delta\kappa_\mu}}$$



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Multi-Higgs/V processes

Celada/Han/Kilian/Kreher/Ma/Maltoni/Pagani/JRR/Striegl/Xie, arXiv:2312.13082

- EFT setup generating multi-boson vertices of higher multiplicity
- Paradigmatic BSM implementations: scalar singlet *S* / vector-like fermions $E_{L/R}$
- Vertex parameterizations (can be expressed by HEFT or SMEFT operators):

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$$\mathcal{L} \supset -\frac{m_H^2}{2}H^2 - m_\mu \bar{\mu}\mu - \sum_{n=3}^{\infty} \beta_n \frac{\lambda}{v^{n-4}} H^n - \sum_{n=1}^{\infty} \alpha_n \frac{m_\mu}{v^n} \bar{\mu}\mu H^n.$$

$$y_{\mu,n} = \frac{\sqrt{2}m_\mu}{v}\alpha_n, \qquad f_{V,n} = \beta_n \lambda$$

$$\alpha_1 = \frac{v}{\sqrt{2}m_\mu} y_{l,1} = 1 + \frac{v^3}{\sqrt{2}m_\mu}\frac{c_{\ell\phi}^{(6)}}{\Lambda^2} + \frac{v^5}{\sqrt{2}m_\mu}\frac{c_{\ell\phi}^{(8)}}{\Lambda^4} + \frac{3v^7}{4\sqrt{2}m_\mu}\frac{c_{\ell\phi}^{(10)}}{\Lambda^6},$$

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H \ V	0	1	2	3	4	5
0	-	Z	Z^2,W^2	Z^3 W^2Z	Z^4, W^4 W^2Z^2	Z^5,W^2Z^3 W^4Z
1	H	ZH	W^2H Z^2H	W^2ZH Z^3H	W^4H, Z^4H W^2Z^2H	-
2	H^2	ZH^2	W^2H^2 Z^2H^2	W^2ZH^2 Z^3H^2	-	-
3	H^3	ZH^3	W^2H^3 Z^2H^3	-	-	-
4	H^4	ZH^4	-	-	-	-
5	H^5	-	-	-	-	-

α_1

$\alpha_{1,2}$

$\alpha_{1,2,3}$

$\alpha_{1\ldots4}$

$\alpha_{1\ldots5}$

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2	H^2	ZH^2	W^2H^2 Z^2H^2	W^2ZH^2 Z^3H^2	-	-
3	H^3	ZH^3	W^2H^3 Z^2H^3	-	-	-
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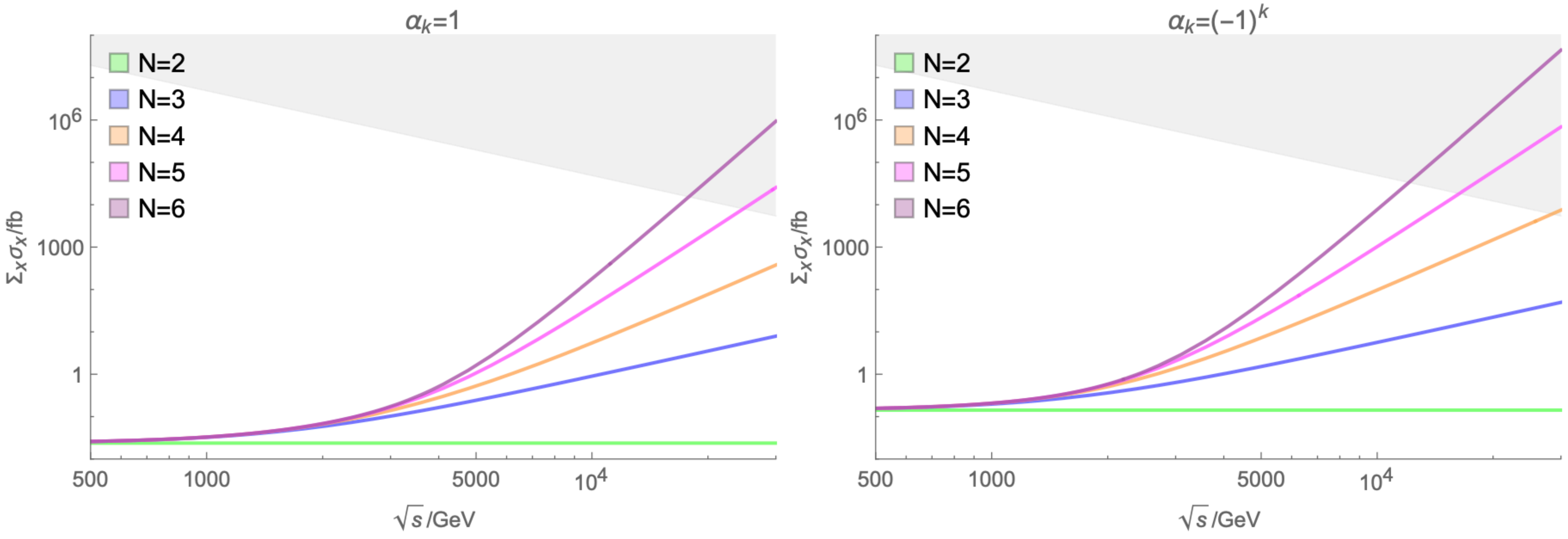
α_1 / $\alpha_{1,2}$ / $\alpha_{1,2,3}$ / $\alpha_{1\ldots4}$ / $\alpha_{1\ldots5}$

Perturbative Unitarity bound



Results for $\mu^+\mu^- \to V^k H^l$

Cut flow: signal and background events

$\sqrt{s}$	3 TeV				10 TeV			
	$\alpha_{2(3)} = 1^\dagger$	SM LO	Loop	VBF	$\alpha_{2(3)} = 1^\dagger$	SM LO	Loop	VBF
σ [fb]	$2H$							
No cut	$2.4 \cdot 10^{-2}$	$1.6 \cdot 10^{-7}$	$2.6 \cdot 10^{-3}$	0.951	$2.4 \cdot 10^{-2}$	$1.3 \cdot 10^{-9}$	$4.2 \cdot 10^{-4}$	3.80
$M_F > 0.8\sqrt{s}$	$2.4 \cdot 10^{-2}$	$1.6 \cdot 10^{-7}$	$2.6 \cdot 10^{-3}$	$6.12 \cdot 10^{-4}$	$2.4 \cdot 10^{-2}$	$1.3 \cdot 10^{-9}$	$4.2 \cdot 10^{-4}$	$6.50 \cdot 10^{-4}$
$\lvert\theta_{iB}\rvert > 10^\circ$	$2.3 \cdot 10^{-2}$	$1.6 \cdot 10^{-7}$	$2.6 \cdot 10^{-3}$	$1.18 \cdot 10^{-4}$	$2.3 \cdot 10^{-2}$	$1.3 \cdot 10^{-9}$	$4.1 \cdot 10^{-4}$	$3.46 \cdot 10^{-5}$
event #	23	–	2.6	0.12	230	–	4.1	0.3
σ [fb]	$3H$							
No cut	$3.1 \cdot 10^{-2}$	$3.0 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$3.69 \cdot 10^{-4}$	$3.7 \cdot 10^{-1}$	$2.3 \cdot 10^{-9}$	$1.7 \cdot 10^{-6}$	$5.52 \cdot 10^{-3}$
$M_F > 0.8\sqrt{s}$	$3.1 \cdot 10^{-2}$	$3.0 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$2.84 \cdot 10^{-6}$	$3.7 \cdot 10^{-1}$	$2.3 \cdot 10^{-9}$	$1.7 \cdot 10^{-6}$	$7.85 \cdot 10^{-5}$
$\lvert\theta_{iB}\rvert > 10^\circ$	$3.0 \cdot 10^{-2}$	$2.8 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$6.82 \cdot 10^{-7}$	$3.5 \cdot 10^{-1}$	$2.2 \cdot 10^{-9}$	$1.7 \cdot 10^{-6}$	$7.37 \cdot 10^{-5}$
$\Delta R_{BB} > 0.4$	$2.9 \cdot 10^{-2}$	$2.7 \cdot 10^{-8}$	$8.1 \cdot 10^{-6}$	$6.07 \cdot 10^{-7}$	$3.4 \cdot 10^{-1}$	$2.1 \cdot 10^{-9}$	$6.8 \cdot 10^{-7}$	$7.22 \cdot 10^{-5}$
event #	29	–	–	–	3400	–	–	0.7

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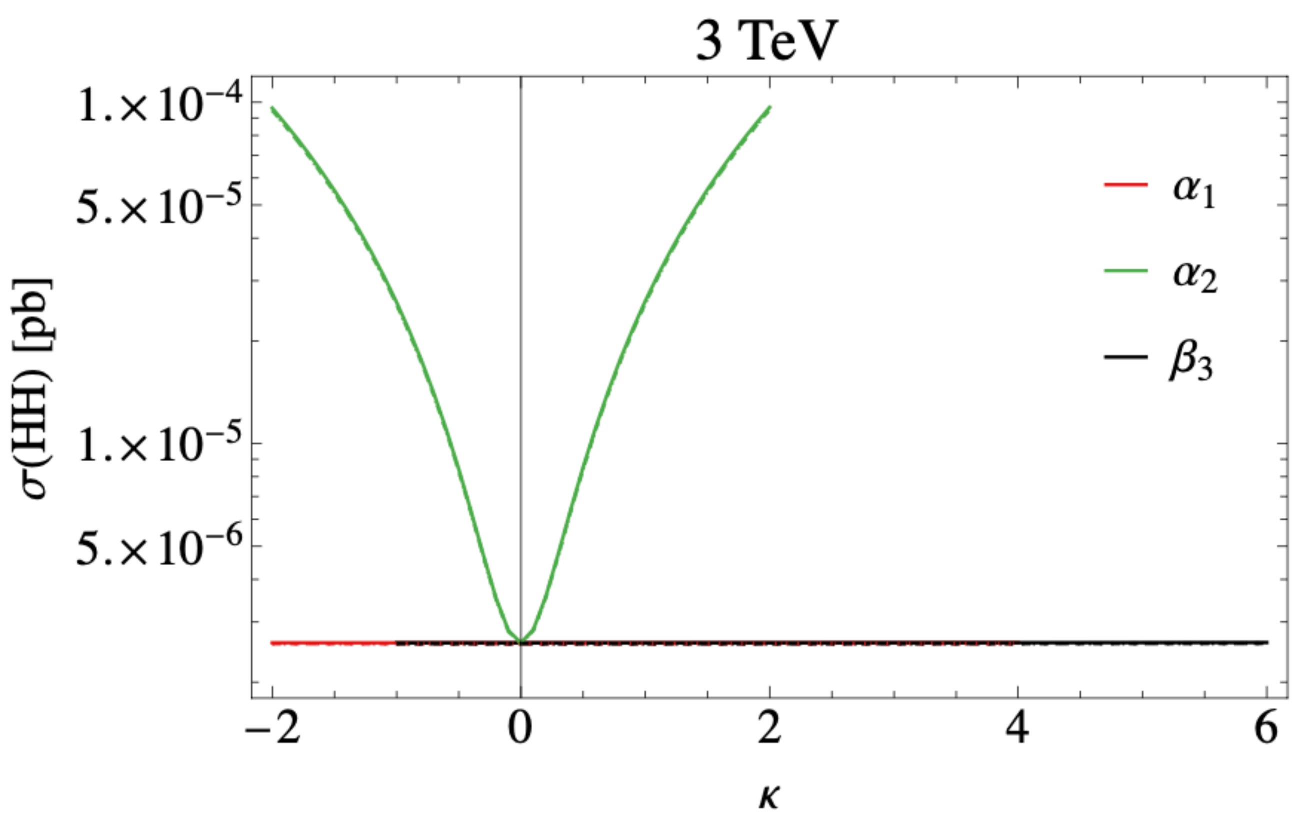

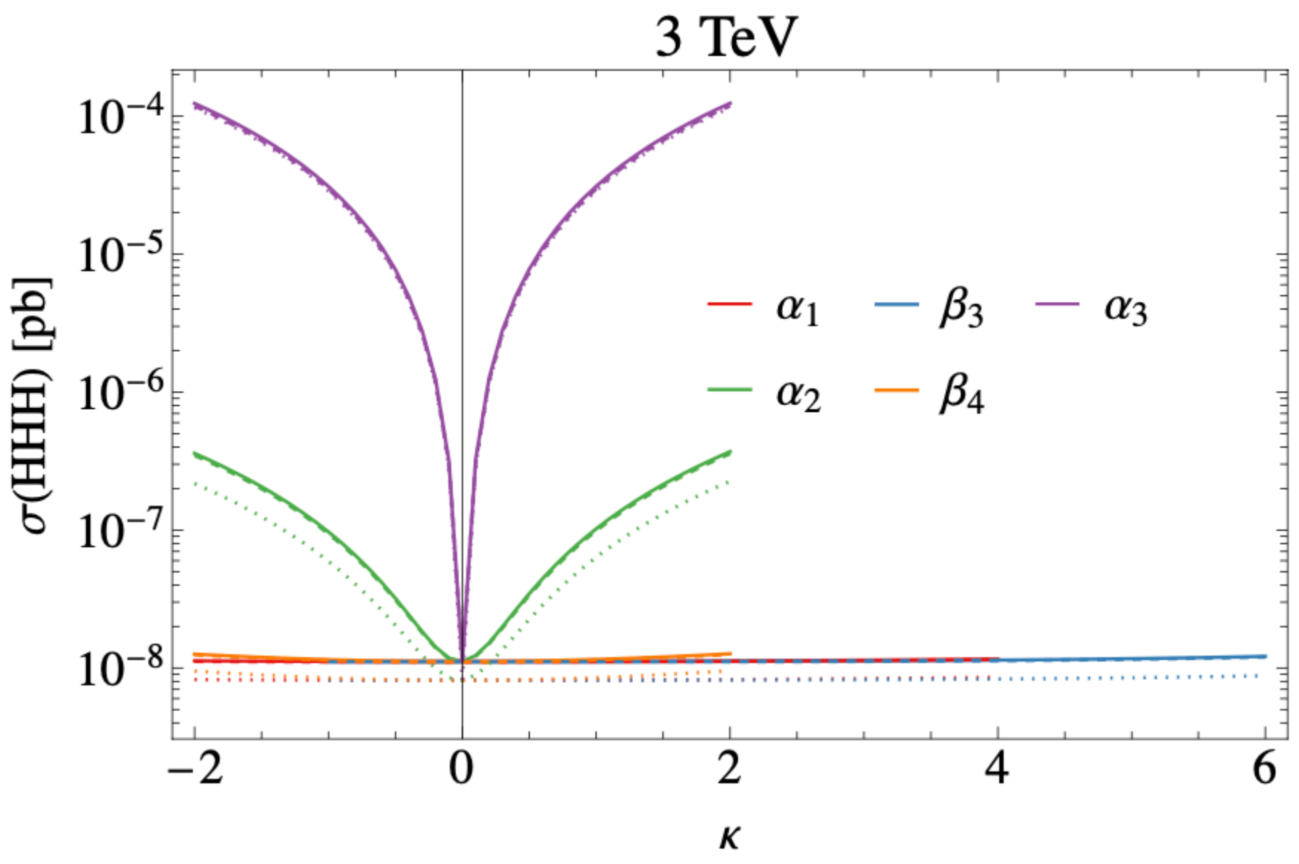

Results for $\mu^+\mu^- \to V^k H^l$

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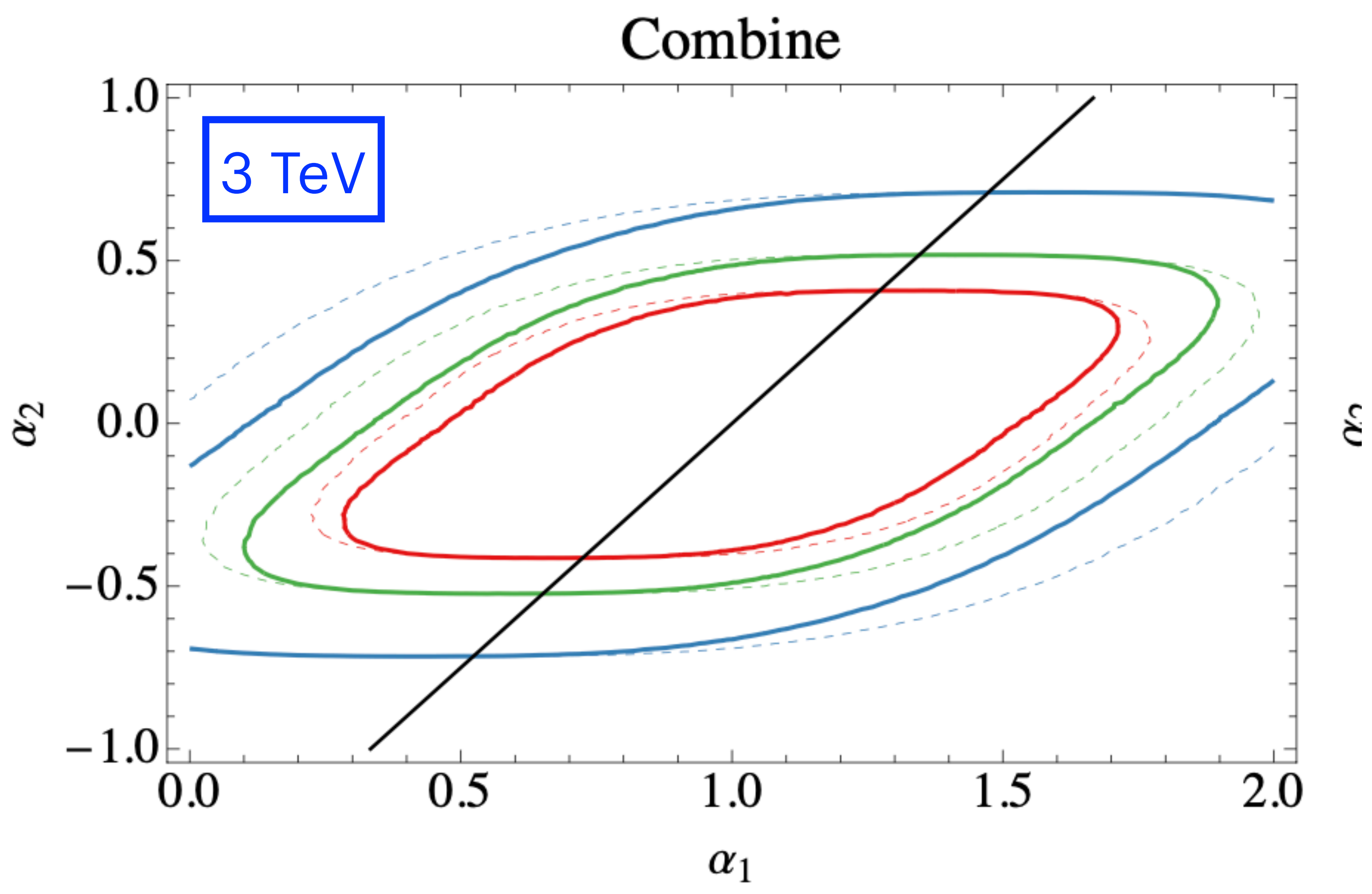
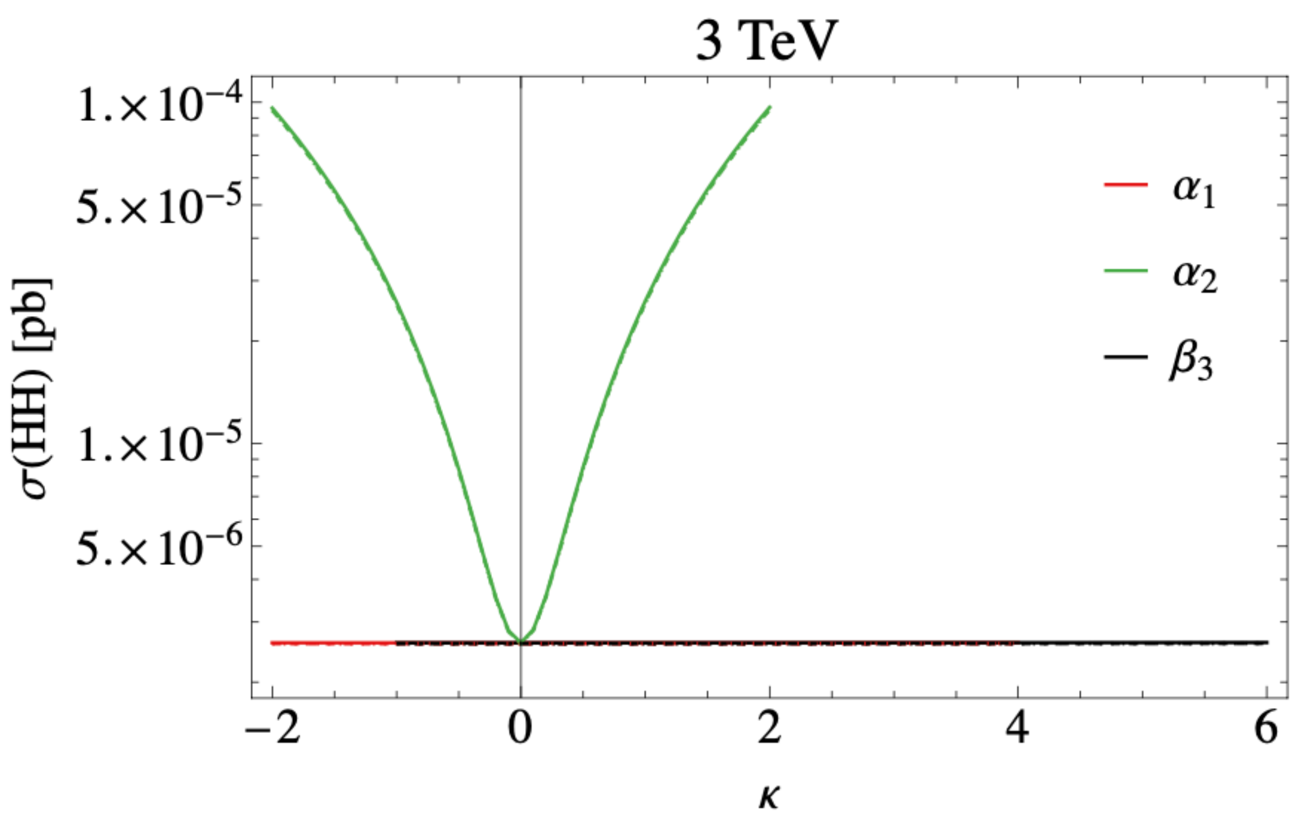
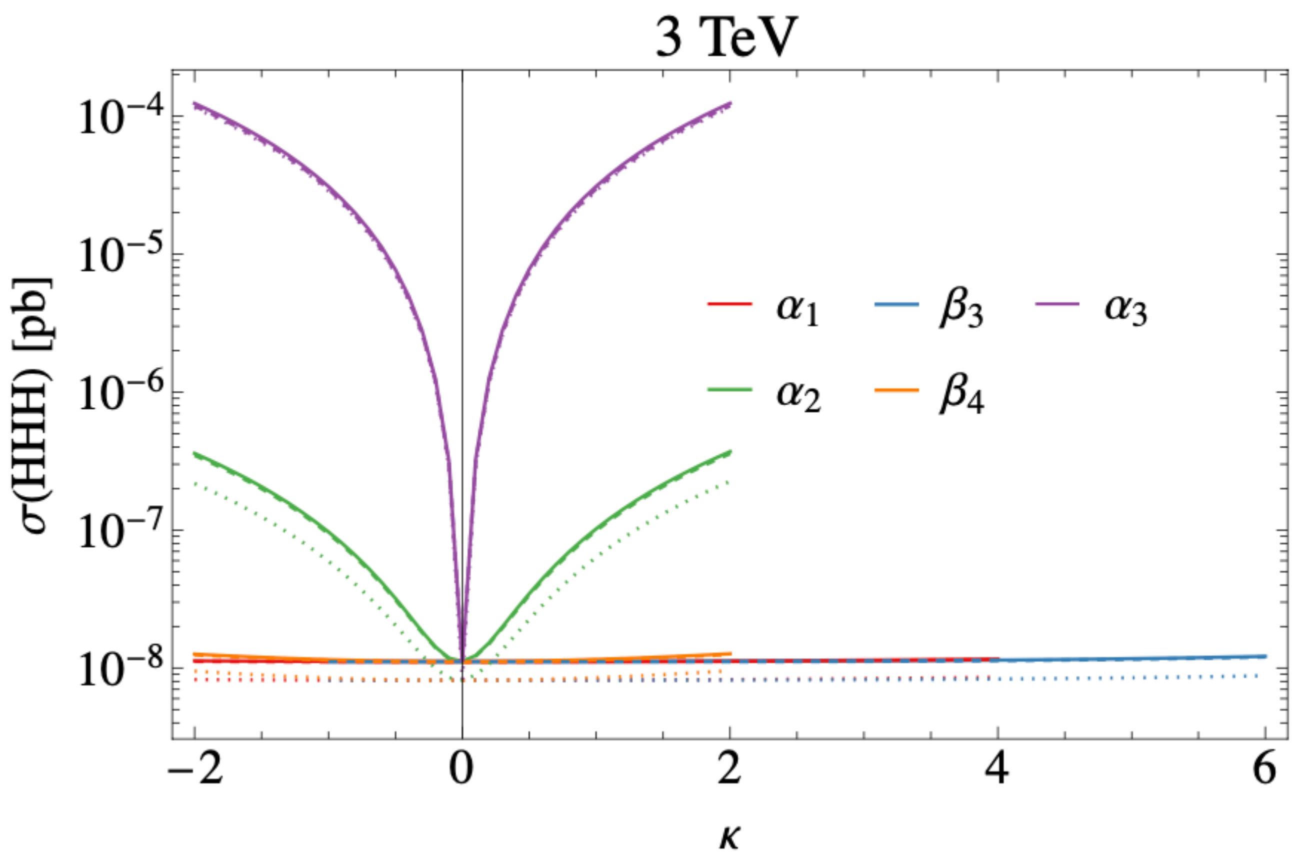
Combination of $\mu\mu \to HH, HVV, V^k$







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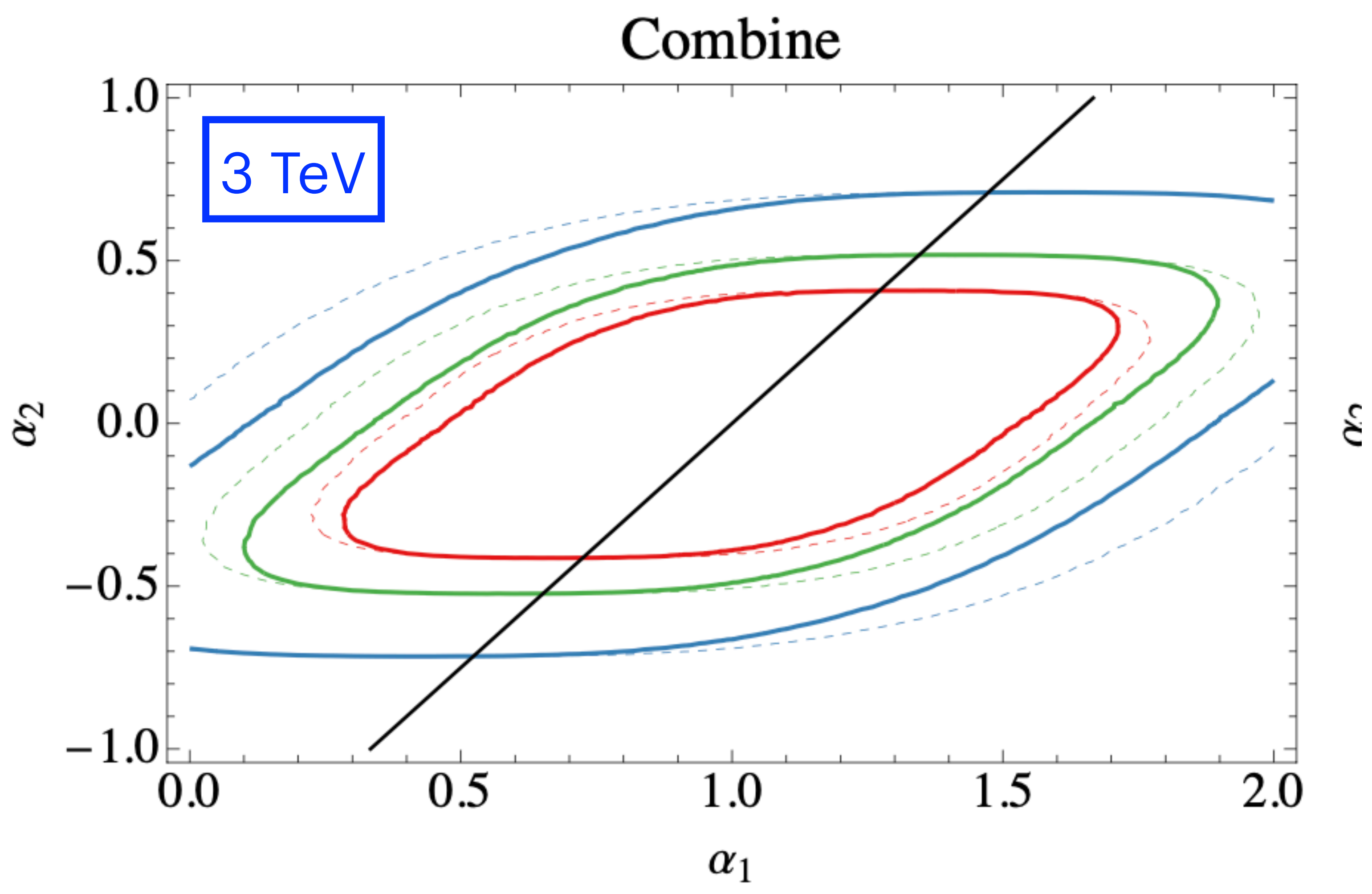
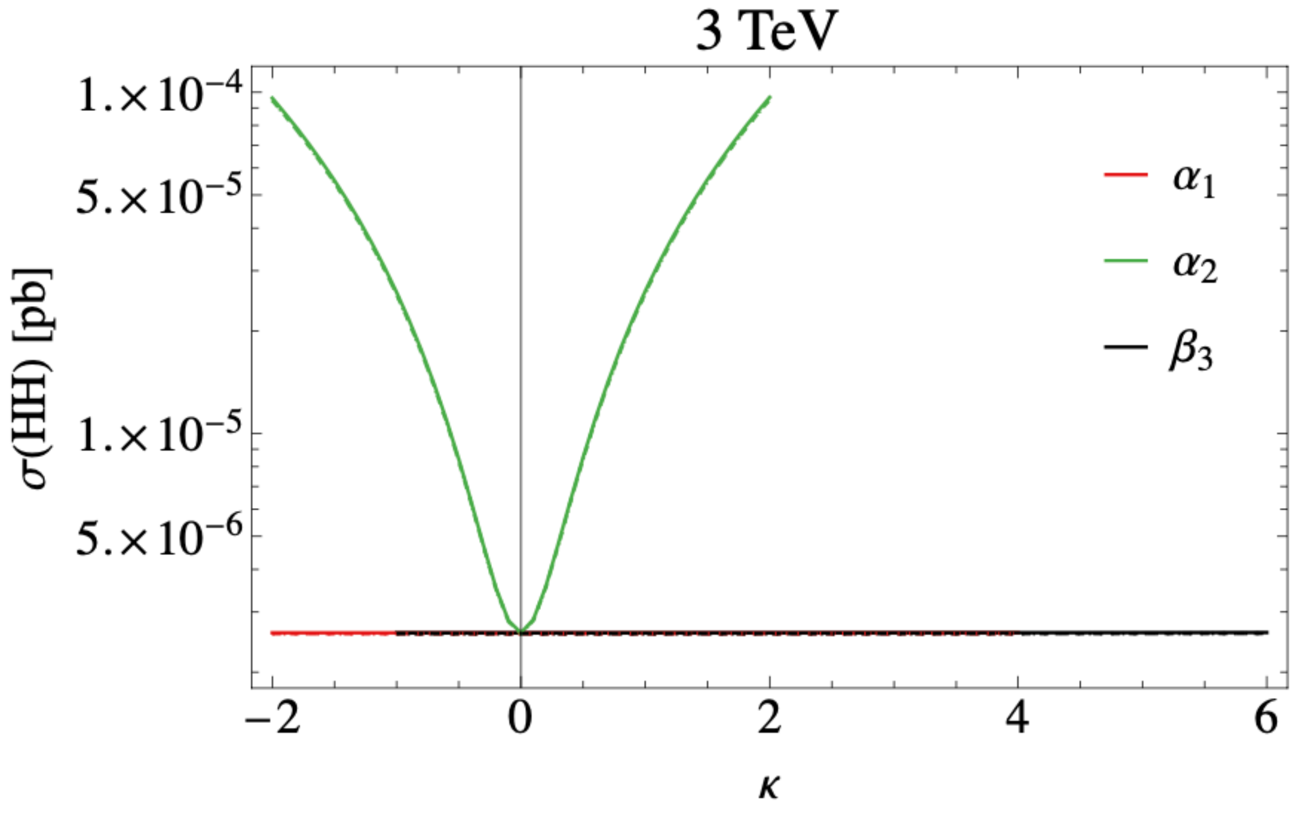
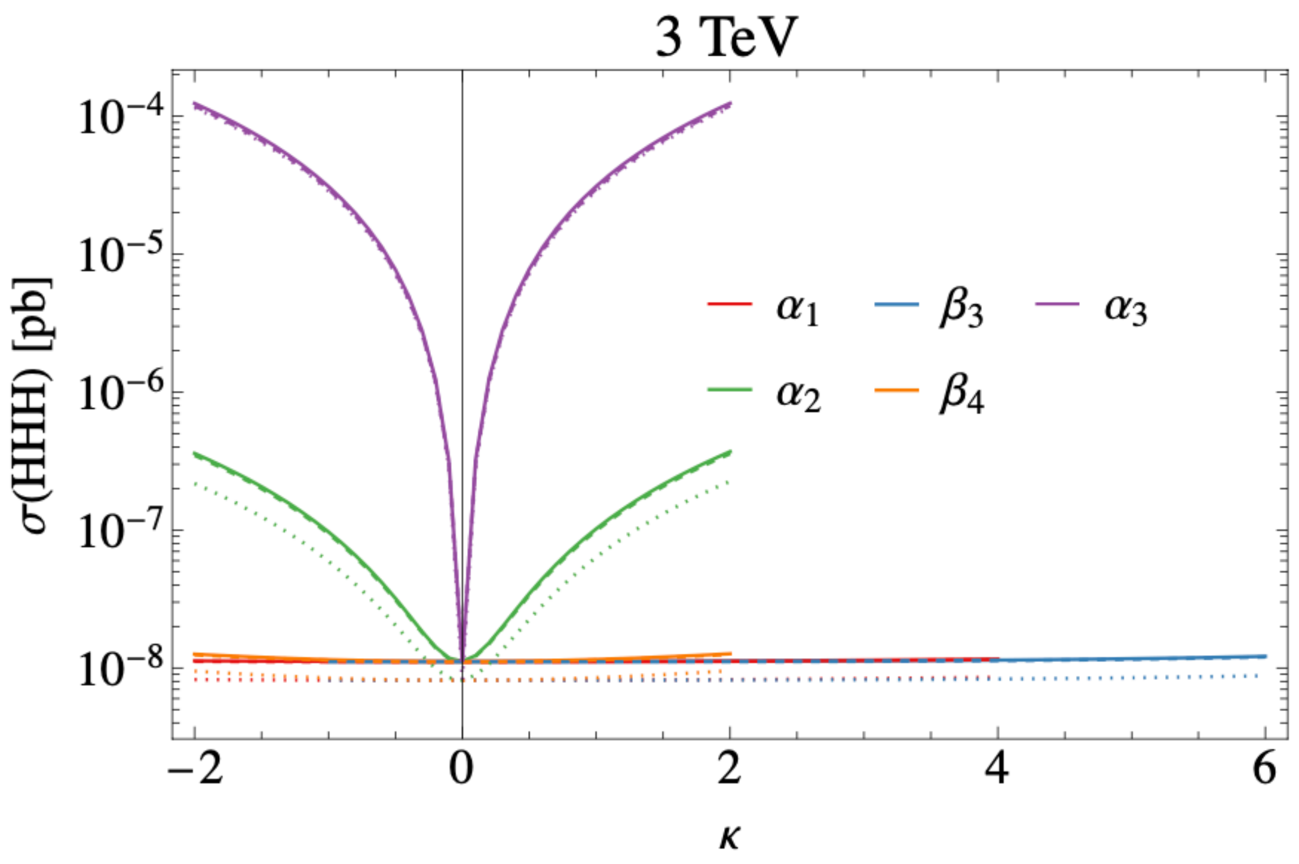
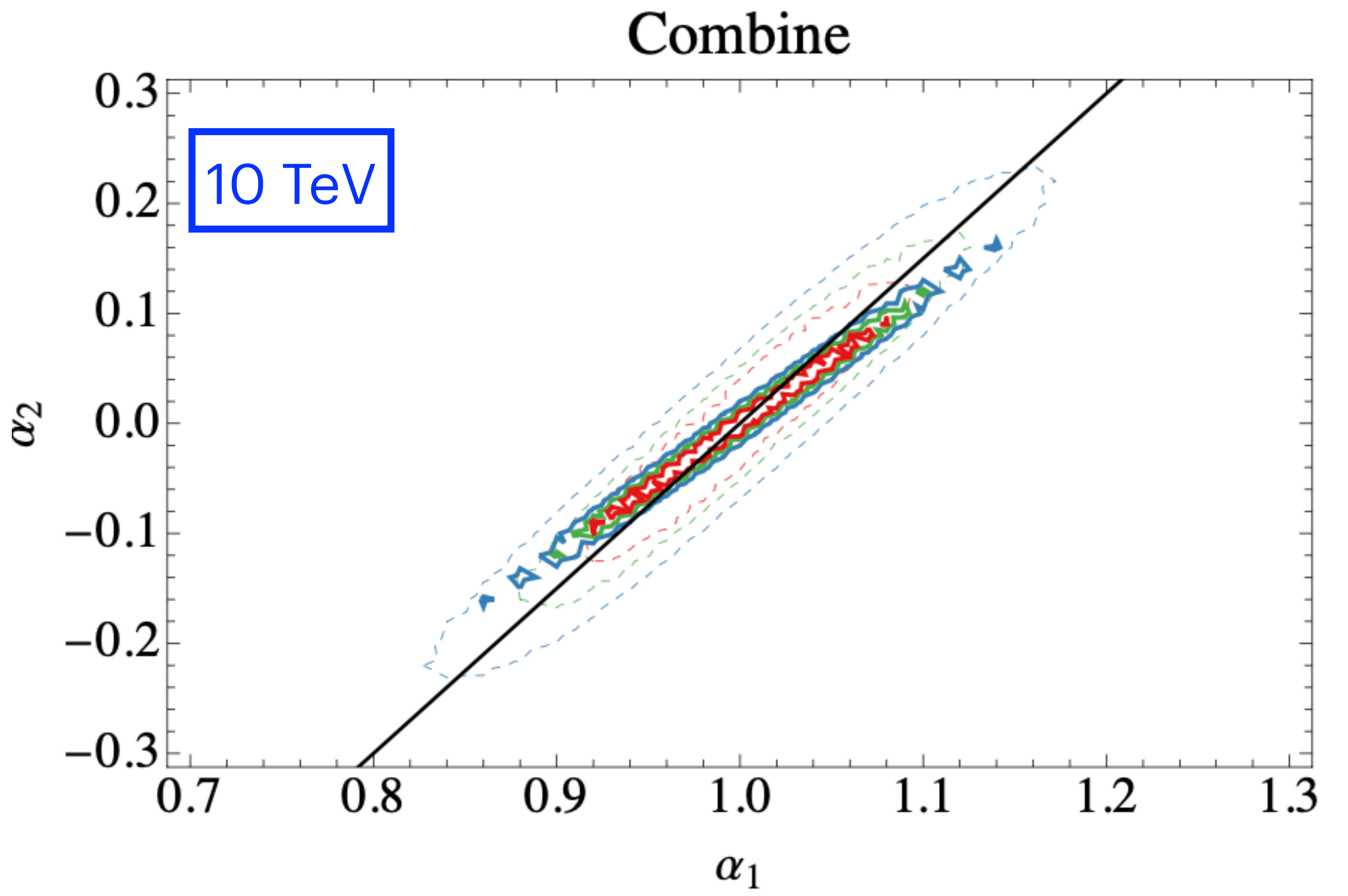
Combination of $\mu\mu \to HH, HVV, V^k$

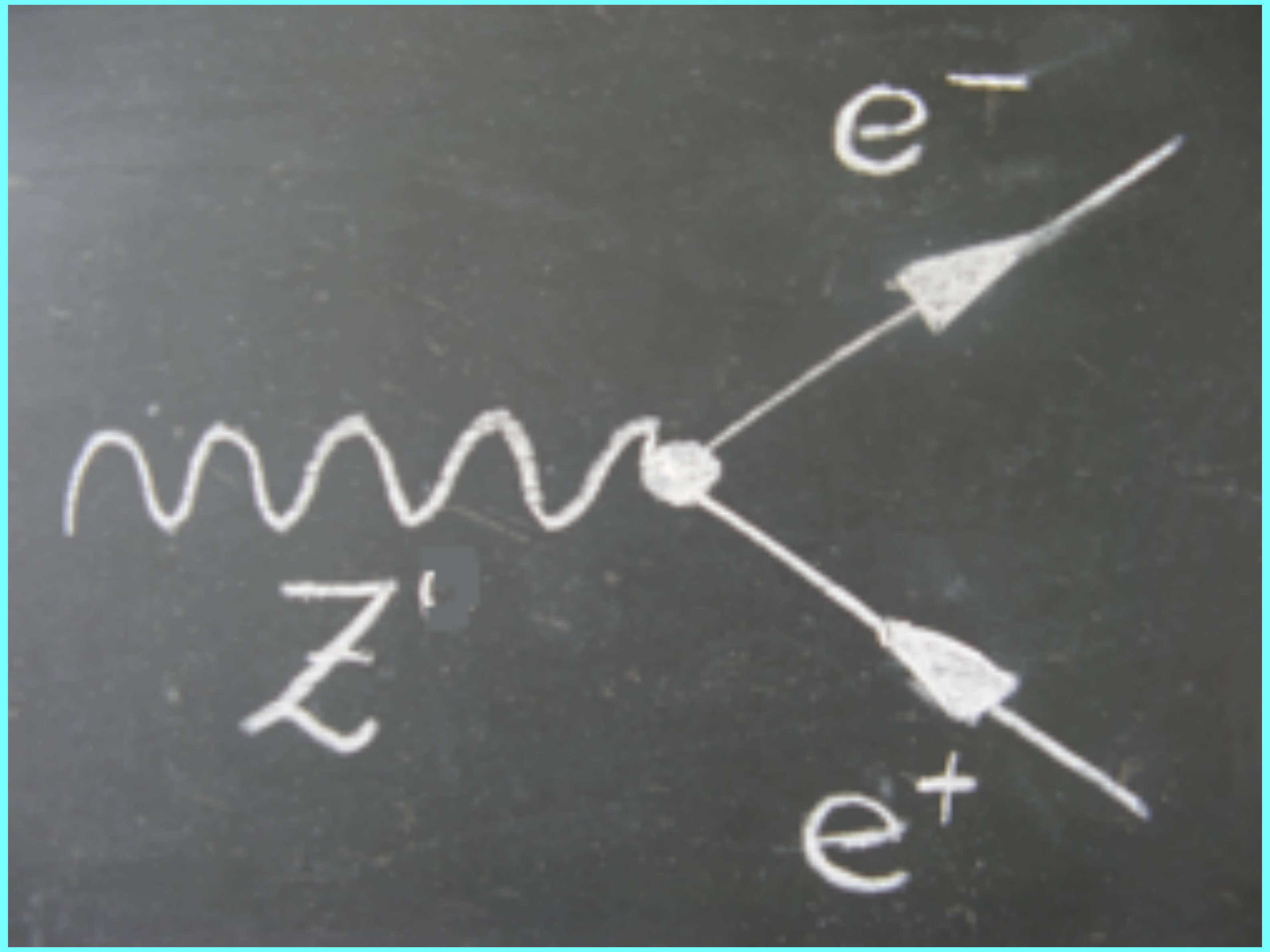


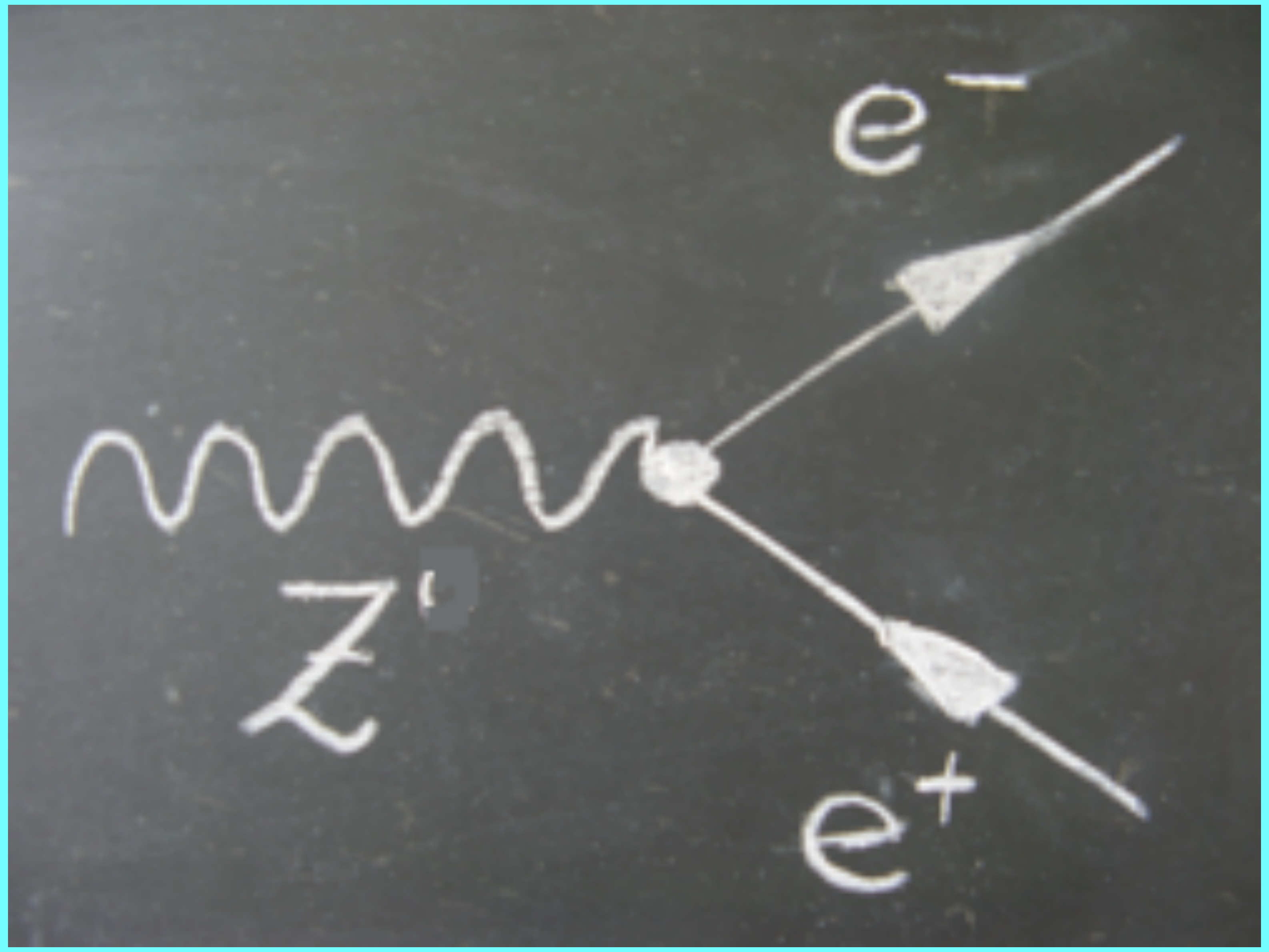






Search for Heavy Neutral Currents





Rationale and search processes

- Many different motivations for Z': GUTs, gauge composite models, gauged flavor symmetries
- (Remember: global symmetries are difficult in string theory)
- Most basic processes: $\mu^+\mu^- \to f\bar{f} \qquad f = e, \mu, \tau, j, c, b, (t)$
- Very simple event topologies
- Discovery & discrimination of models, many observables:

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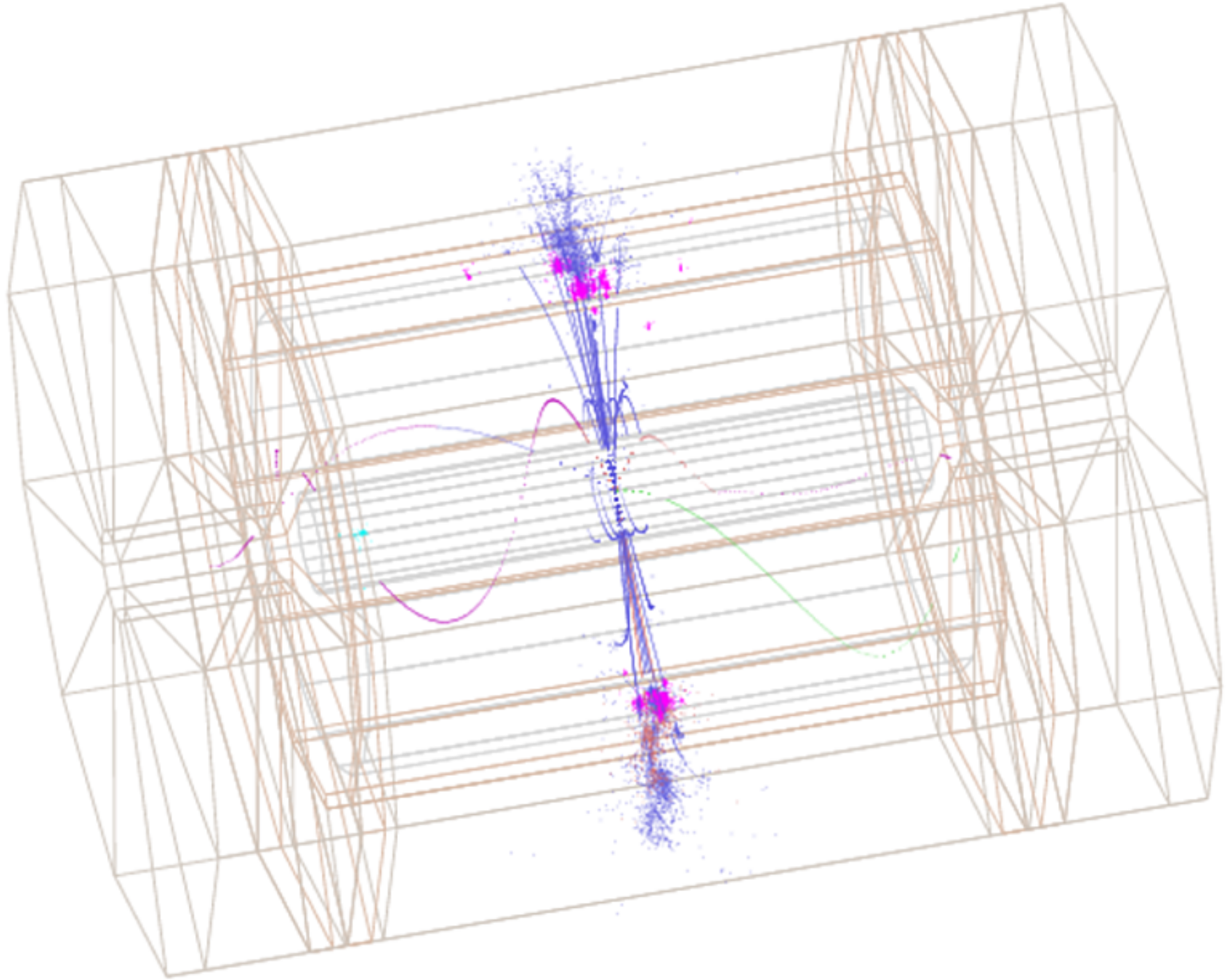

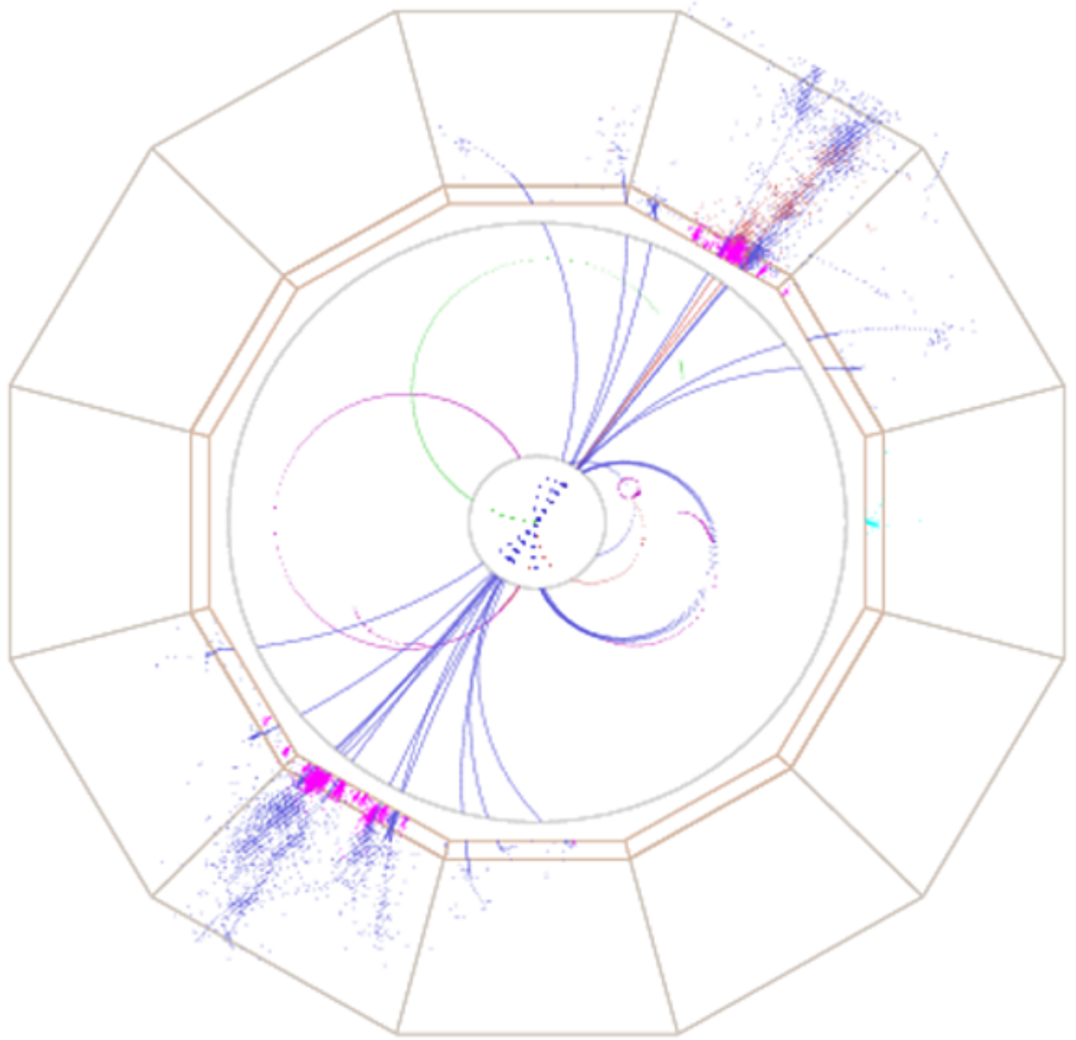

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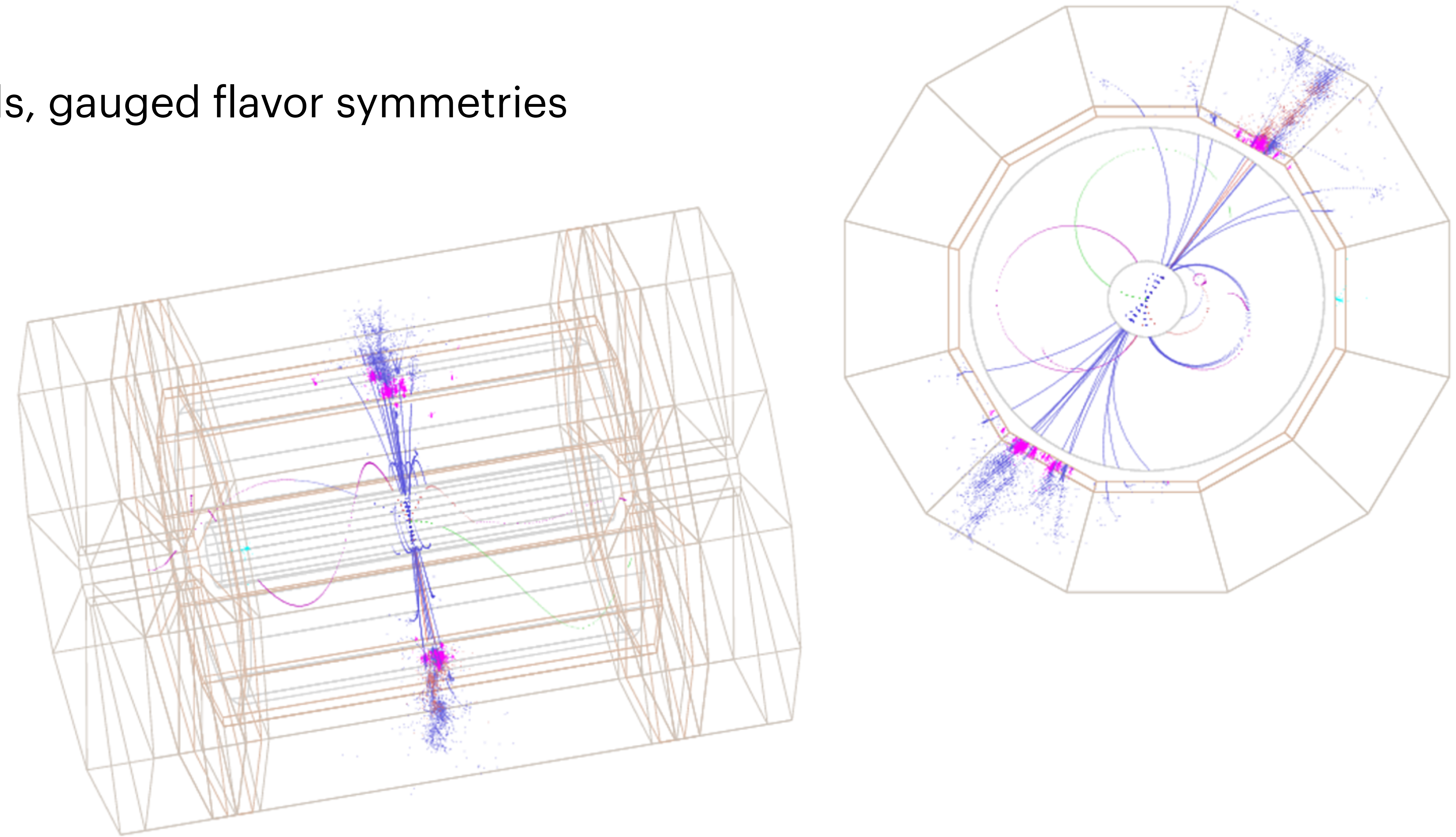

- [x] Forward-backward asymmetries: $A^f_{FB}(\mu^+\mu^- \to f\bar{f})$
- [x] Tau polarization asymmetries: $A^\tau_{pol.}(\mu^+\mu^- \to \tau^+\tau^-)$
- [x] Binned angular distributions
- [x] Left-right asymmetries: $A^f_{LR}(\mu^+\mu^- \to f\bar{f})$ (needs beam polarization)
- [ ] Spin-sensitive processes $(\mu^+\mu^- \to W^+W^-, t\bar{t})$

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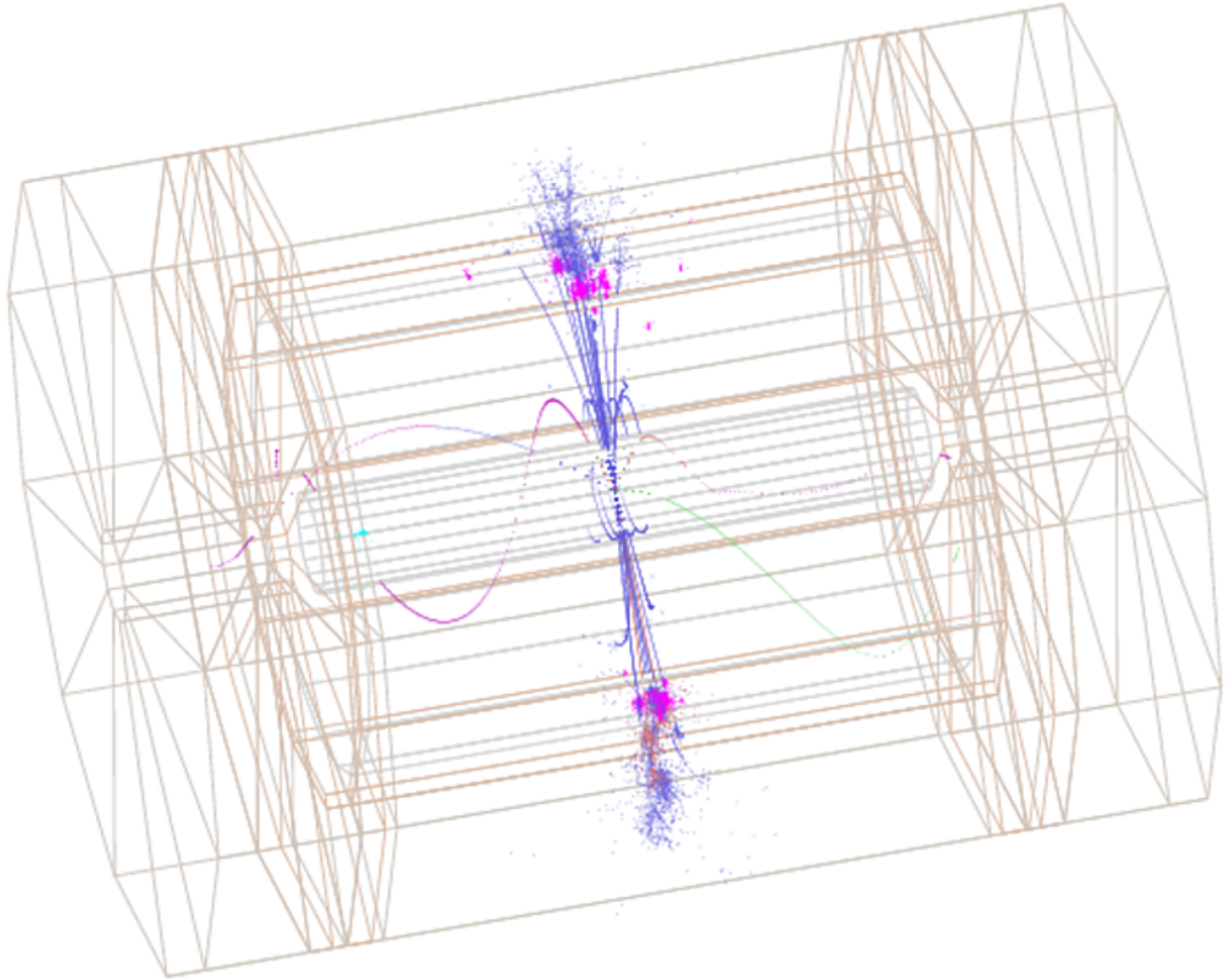

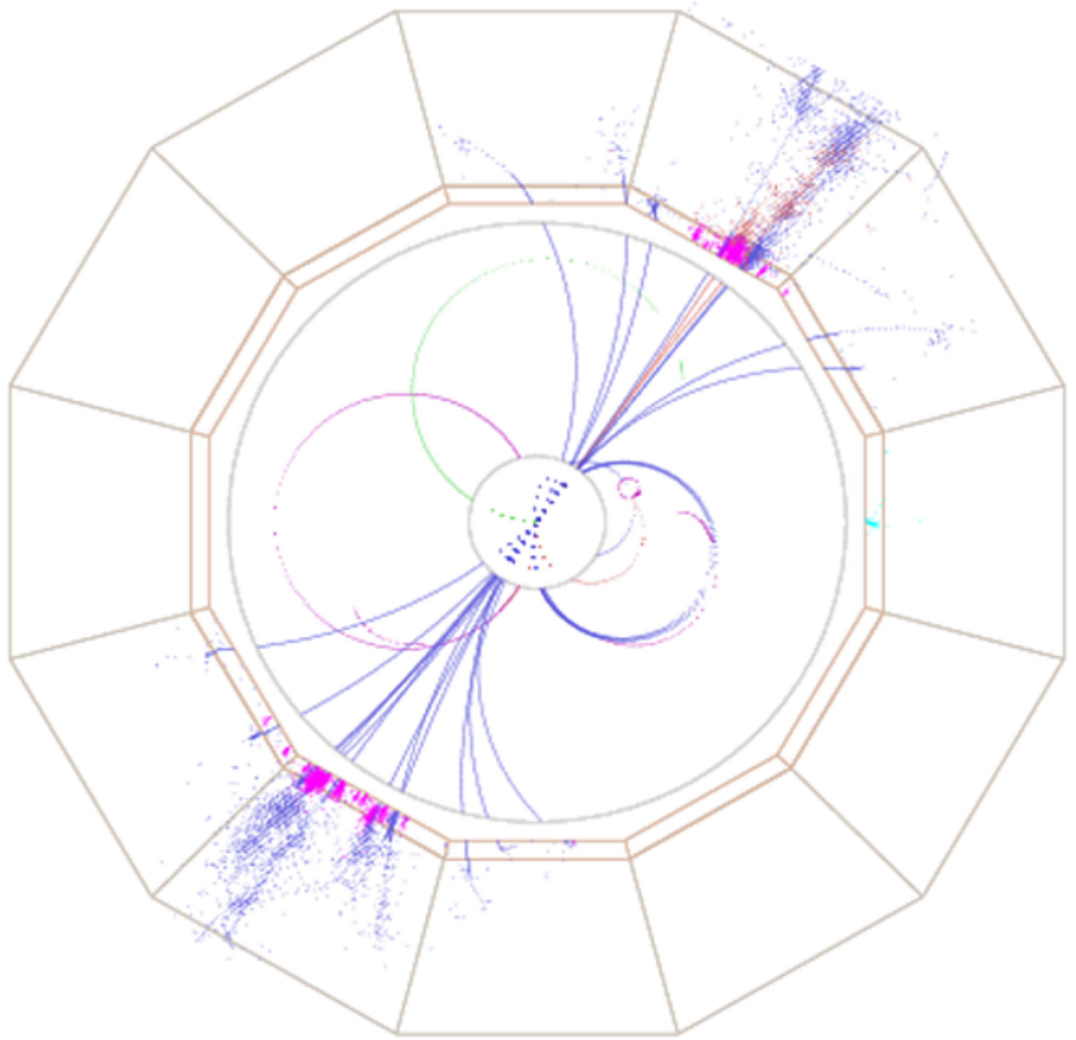

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Investigated Z' models:
(1) Sequential SM (SSM)
(2) E_6, $SU(2)_L \otimes SU(2)_R$ (LR)
(3) Littlest Higgs (LH), Simplest Little Higgs (SLH)
(4) $U(1)_X$ model
(5) many more

Resolving power for Z'

Normalization of couplings:

	$g_{Z'}$
SSM	$e/(s_W c_W)$
E_6, LR	e/c_W
ALR	$e/(s_W\, c_W \sqrt{1-2s_W^2})$
LH	e/s_W
USLH, AFSLH	$e/(c_W \sqrt{3-4s_W^2})$
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- Model parameters to be determined: vector and axial vector couplings *v, a*
- Observables: $\mathcal{O}_i \in \left\{\sigma_{tot}, A_{FB}^f, A_{LR}^f\right\}$
- Resolving power: $\chi^2_{model} = \sum_{i=1}^{n_{ob}} \left(\frac{\mathcal{O}_i^{model} - \mathcal{O}_i(v,a)}{\Delta\mathcal{O}_i(v,a)} \right)^2 < 5.99$ for 95% CL
- Statistical uncertainties: $\frac{\Delta\sigma_{tot}}{\sigma_{tot}} = \frac{1}{\sqrt{N}}, \qquad \Delta A_{FB} = \sqrt{\frac{1-A_{FB}^2}{N}}, \qquad \Delta A_{LR} = \sqrt{\frac{1-(PA_{LR})^2}{NP^2}}$
- Collider luminosity: $\mathcal{L}_{int}(E_{CM}) = 10\ \text{ab}^{-1} \cdot E_{CM}/10$ TeV

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f	ν	e	u	d
		SSM		
$2v'_f$	$\frac{1}{2}$	$2s_W^2 - \frac{1}{2}$	$\frac{1}{2} - \frac{4}{3}s_W^2$	$\frac{2}{3}s_W^2 - \frac{1}{2}$
$2a'_f$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
		E_6		
$2v'_f$	$3A+B$	$4A$	0	$-4A$
$2a'_f$	$3A+B$	$2(A+B)$	$2(B-A)$	$2(A+B)$
		LR		
$2v'_f$	$\frac{1}{2\alpha}$	$\frac{1}{\alpha} - \frac{\alpha}{2}$	$\frac{\alpha}{2} - \frac{1}{3\alpha}$	$-\frac{1}{3\alpha} - \frac{\alpha}{2}$
$2a'_f$	$\frac{1}{2\alpha}$	$\frac{\alpha}{2}$	$-\frac{\alpha}{2}$	$\frac{\alpha}{2}$

		ALR		
$2v'_f$	$s_W^2 - \frac{1}{2}$	$\frac{5}{2}s_W^2 - 1$	$\frac{1}{2} - \frac{4}{3}s_W^2$	$\frac{1}{6}s_W^2$
$2a'_f$	$s_W^2 - \frac{1}{2}$	$-\frac{1}{2}s_W^2$	$s_W^2 - \frac{1}{2}$	$-\frac{1}{2}s_W^2$
		LH		
$2v'_f$	$\frac{c}{4s}$	$-\frac{c}{4s}$	$\frac{c}{4s}$	$-\frac{c}{4s}$
$2a'_f$	$\frac{c}{4s}$	$-\frac{c}{4s}$	$\frac{c}{4s}$	$-\frac{c}{4s}$
		USLH		
$2v'_f$	$\frac{1}{2} - s_W^2$	$\frac{1}{2} - 2s_W^2$	$\frac{1}{2} + \frac{1}{3}s_W^2$	$\frac{1}{2} - \frac{2}{3}s_W^2$
$2a'_f$	$\frac{1}{2} - s_W^2$	$\frac{1}{2}$	$\frac{1}{2} - s_W^2$	$\frac{1}{2}$

		AFSLH		
$2v'_f$	$\frac{1}{2} - s_W^2$	$\frac{1}{2} - 2s_W^2$	$-\frac{1}{2} + \frac{4}{3}s_W^2$	$\frac{1}{3}s_W^2 - \frac{1}{2}$
$2a'_f$	$\frac{1}{2} - s_W^2$	$\frac{1}{2}$	$-\frac{1}{2}$	$s_W^2 - \frac{1}{2}$
		$U(1)_X$		
$2v'_f$	$-x_H - x_\Phi$	$-3x_H - x_\Phi$	$\frac{5}{3}x_H + \frac{1}{3}x_\Phi$	$-\frac{1}{3}x_H + \frac{1}{3}x_\Phi$
$2a'_f$	$-x_H$	x_H	$-x_H$	x_H

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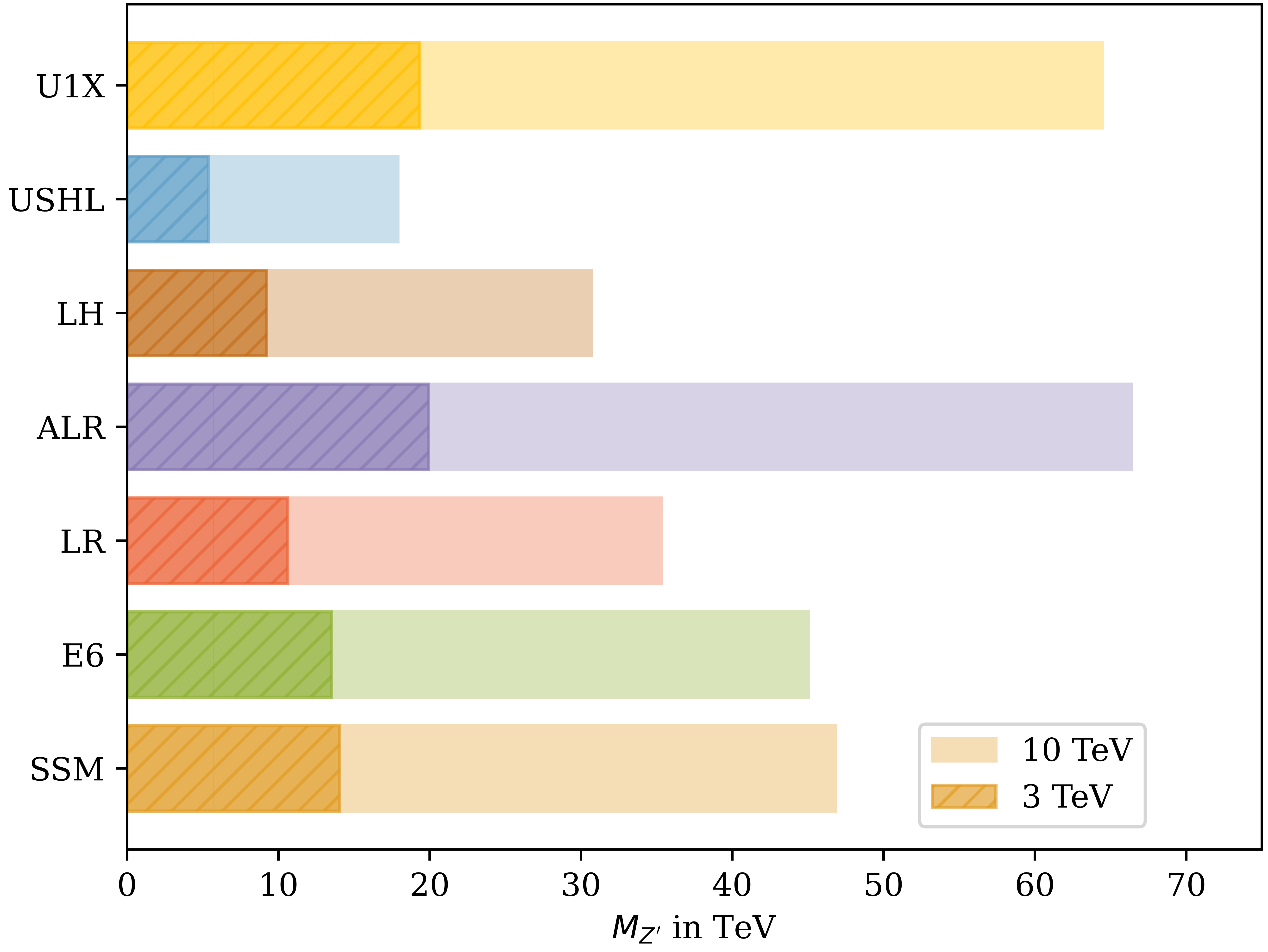
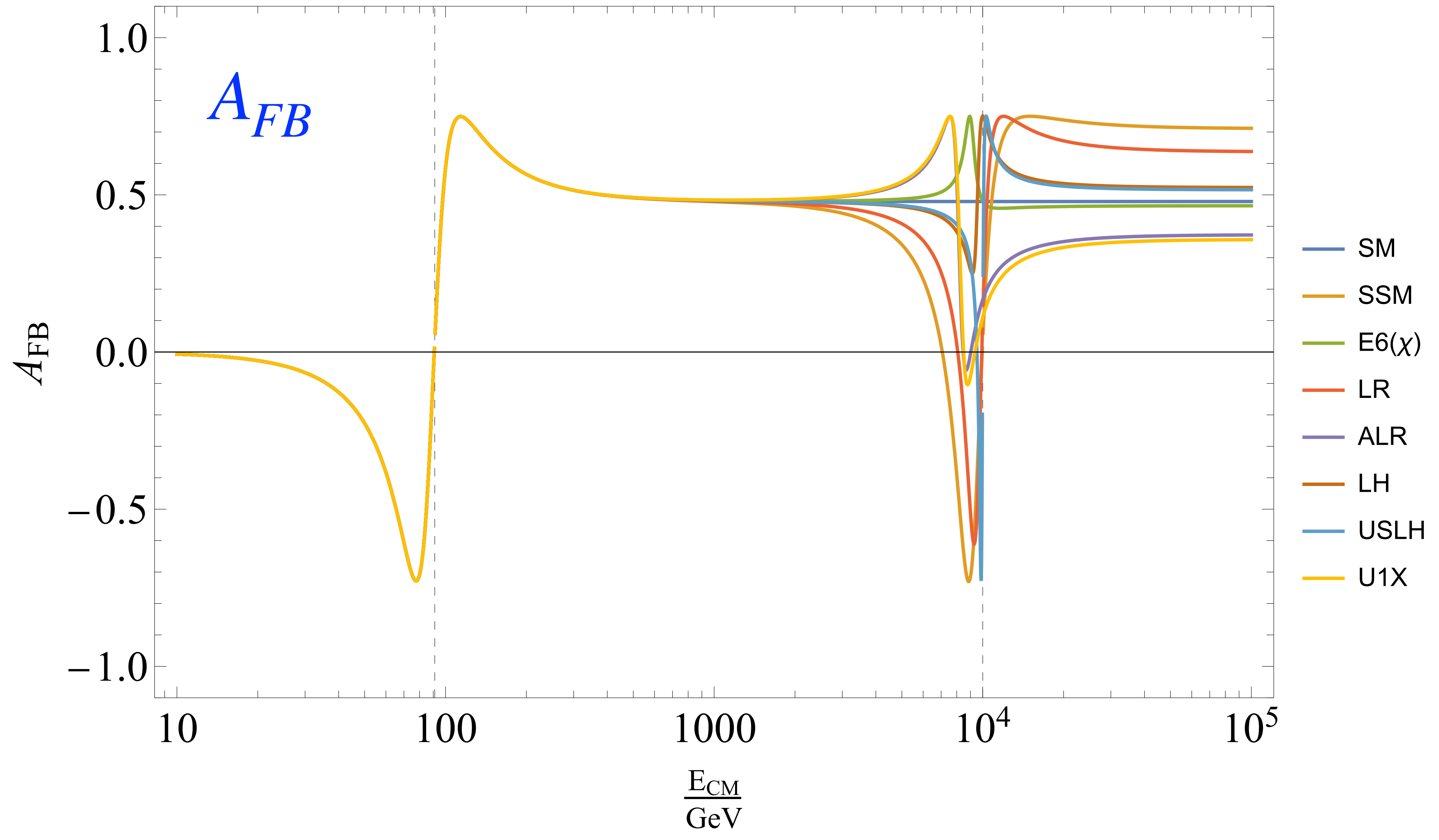
Mass reach and discrimination





- HL-LHC between 7-8 TeV (15 TeV for SSM)
- MuC-10 covers 20—70 TeV
- very conservative: no hadronic observables, no top final states, no optimization

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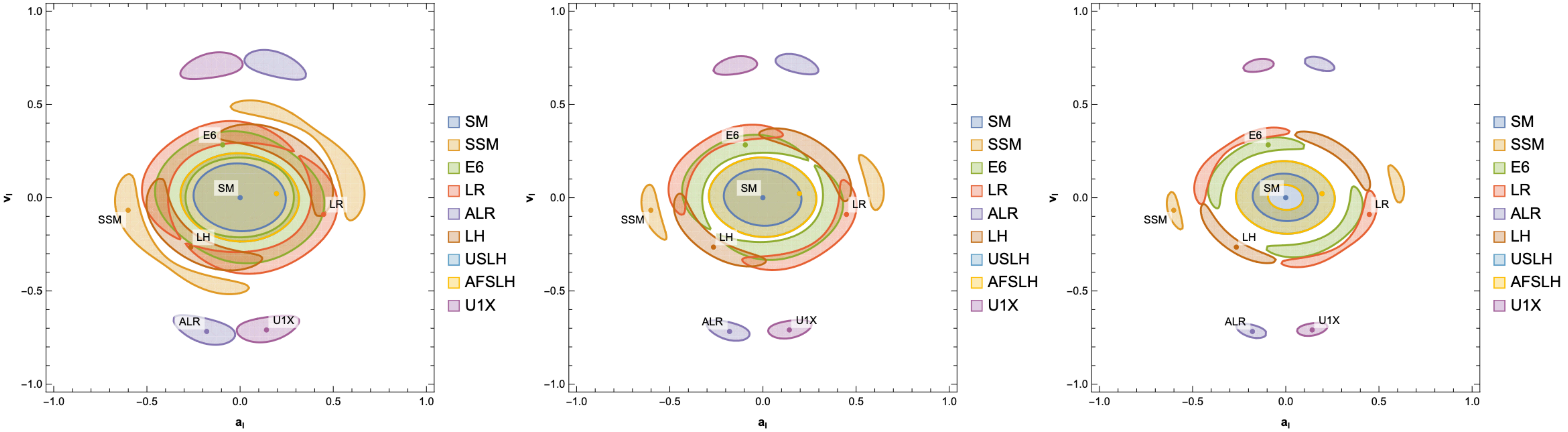
Results for discrimination



(a) $L_{\mathrm{int}} = 5\,\mathrm{ab}^{-1}$ (b) $L_{\mathrm{int}} = 10\,\mathrm{ab}^{-1}$ (c) $L_{\mathrm{int}} = 20\,\mathrm{ab}^{-1}$

$E_{CM} = 10$ TeV, $P = 1$, $M_{Z'} = 30$ TeV

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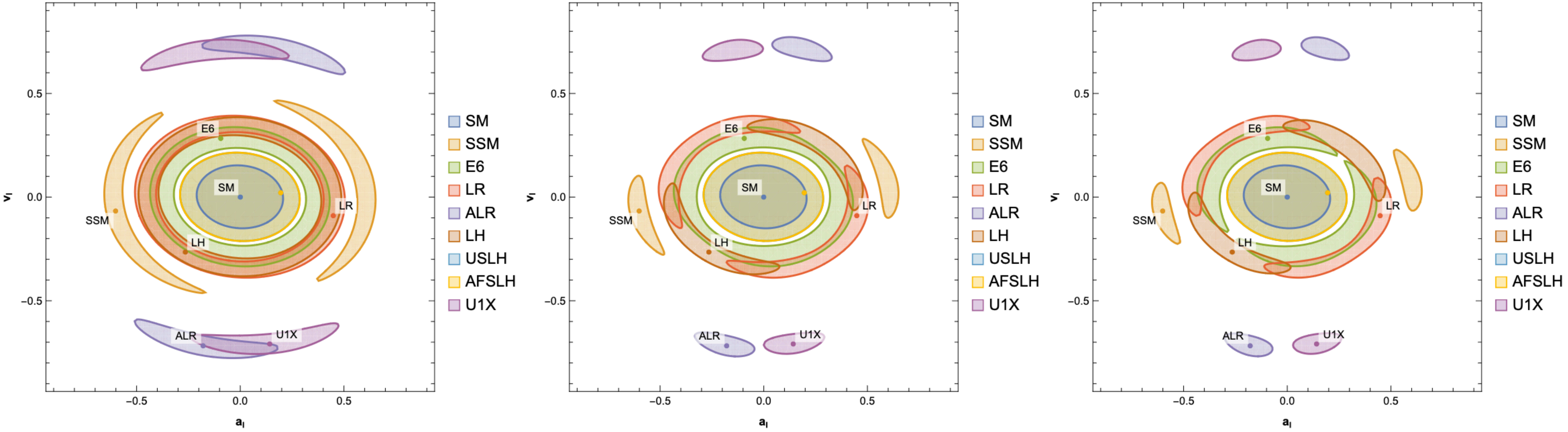

(a) $P = 0.3$ (b) $P = 0.8$ (c) $P = 1$

$E_{CM} = 10\,\text{TeV}, \quad \mathscr{L} = 10\,\text{ab}^{-1}, \quad M_{Z'} = 30\,\text{TeV}$

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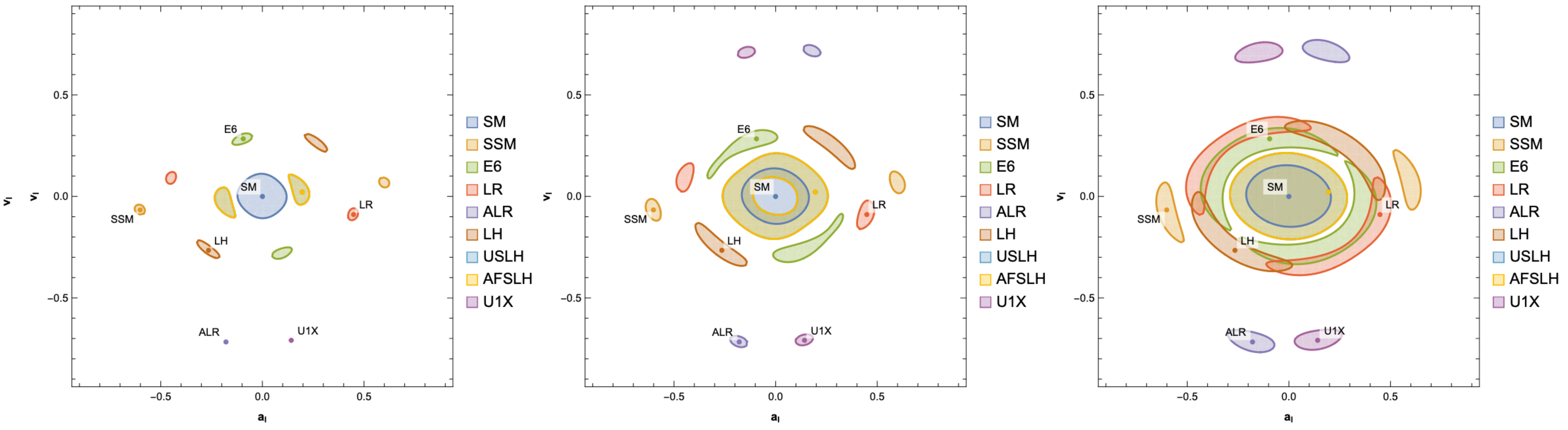

(a) $M_{Z'} = 15\,\mathrm{TeV}$ (b) $M_{Z'} = 20\,\mathrm{TeV}$ (c) $M_{Z'} = 30\,\mathrm{TeV}$

$E_{CM} = 10\,\text{TeV}, \quad \mathscr{L} = 10\,\text{ab}^{-1}, \quad P = 1$

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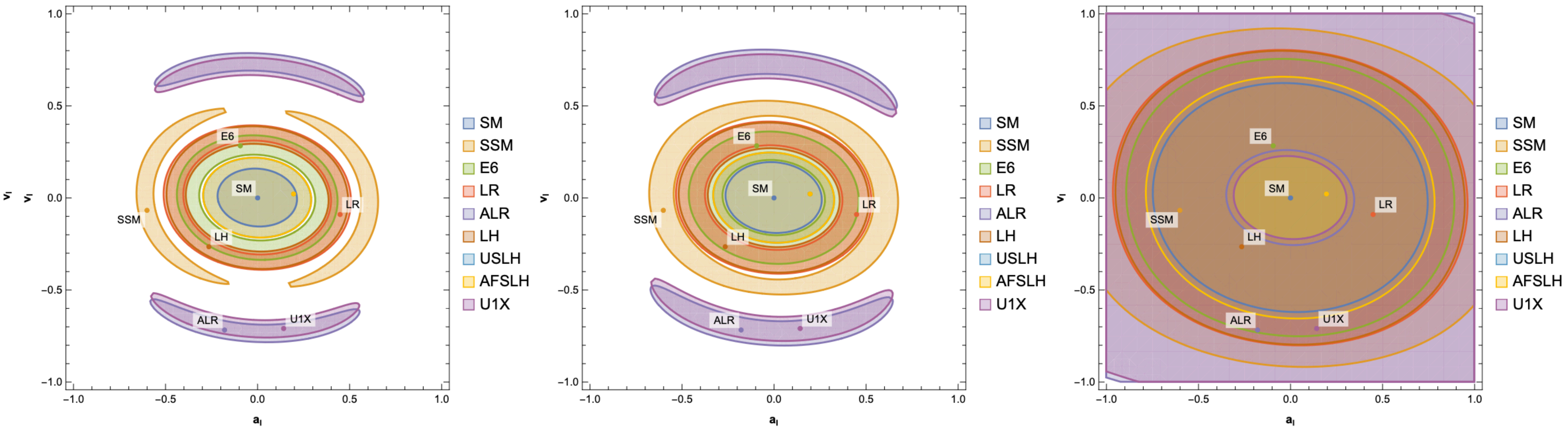
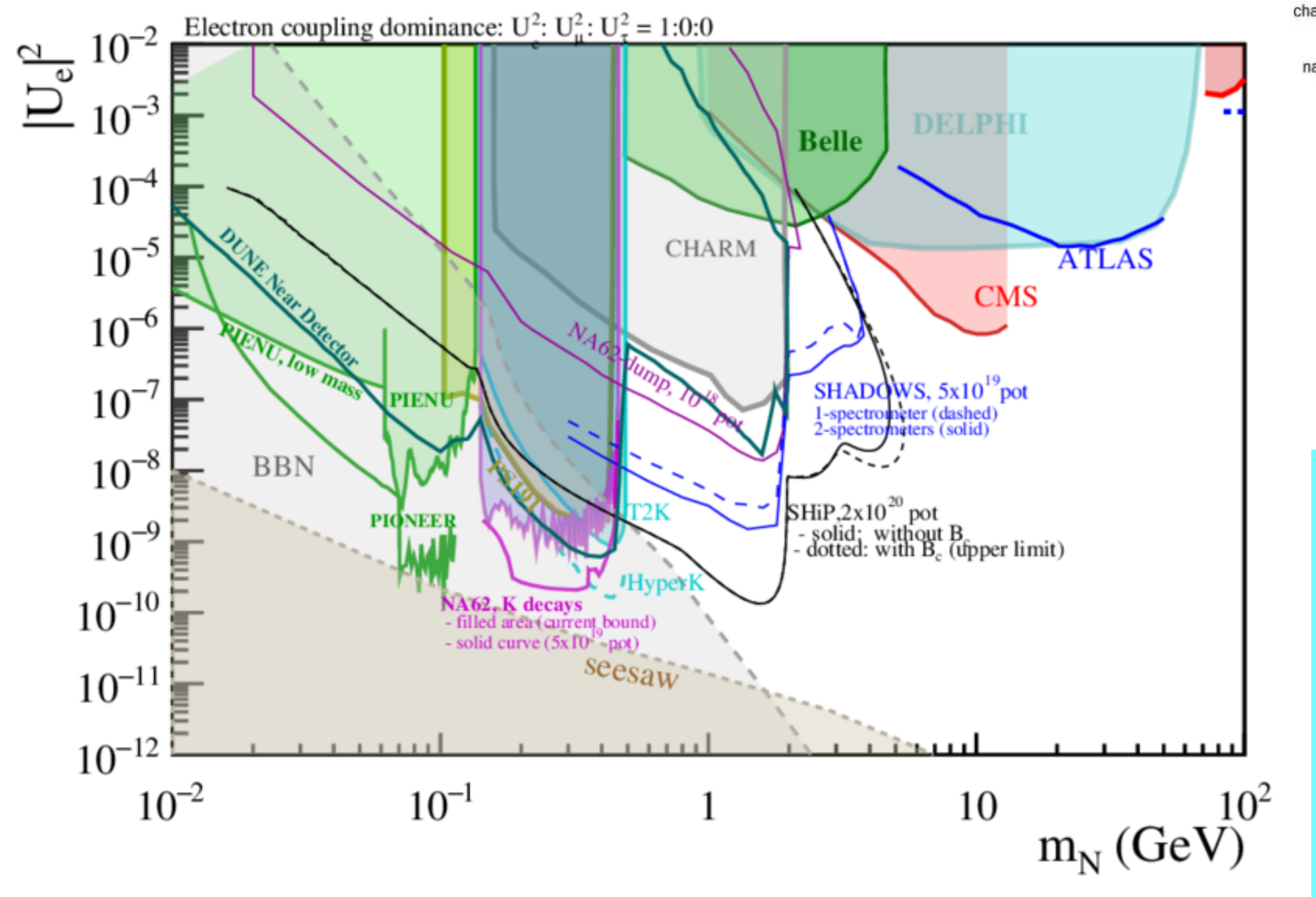
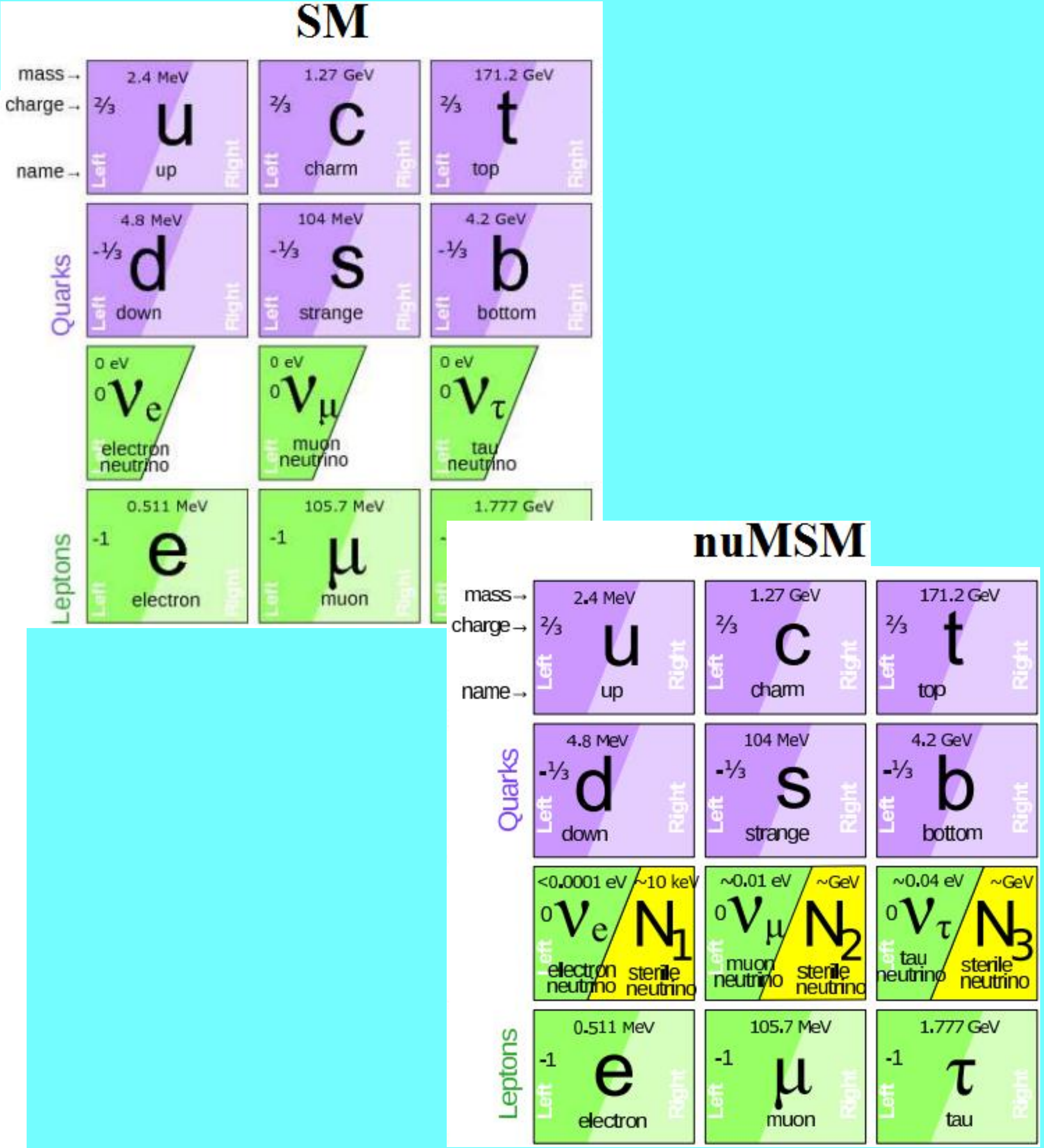


(a) $\Delta O_{\text{sys}} = 0.1\%$ (b) $\Delta O_{\text{sys}} = 1\%$ (c) $\Delta O_{\text{sys}} = 10\%$

Systematic uncertainties should be tuned down to the level of ~ 1 per cent

Search for Heavy Neutral Leptons (HNL)

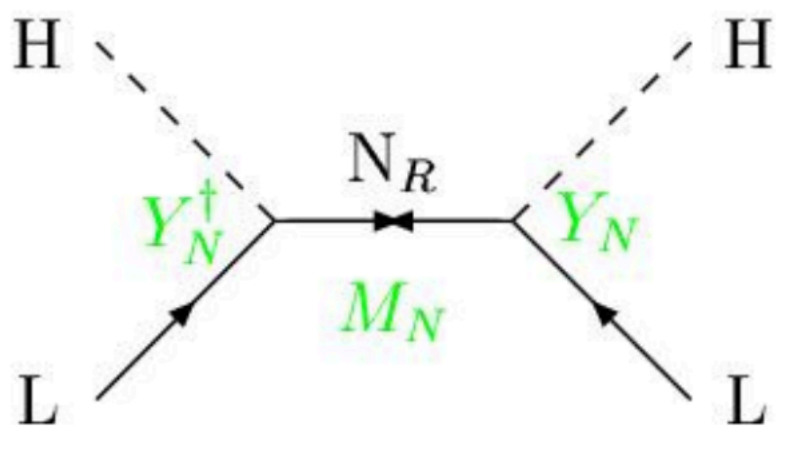




The neutrino mystery

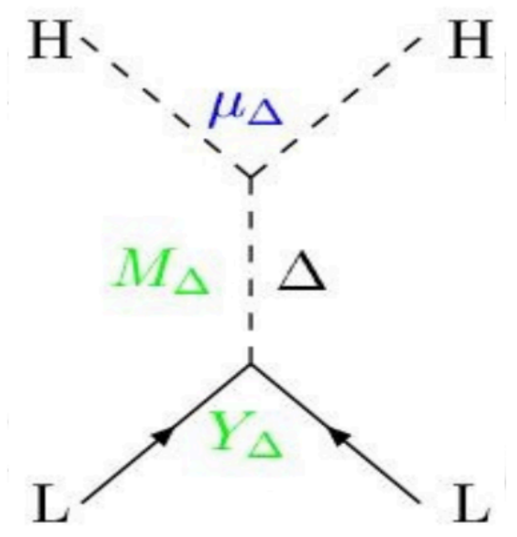
- Neutrinos masses is already physics beyond the standard model
- Simple extension of SM: just add ν_R and Yukawa couplings $\nu_R = (\mathbf{1},\mathbf{1},1)$ $\quad -m_\nu(\overline{\nu}_L \nu_R + h.c.)\left(1+\frac{h}{v}\right)$
- Singlet allows for a Majorana mass term: $-M_\nu \overline{\nu^C}\nu$ [Minkowski, 1977; Mohapatra/Senjanovic, 1980; Yanagida, 1981]
- Dedicated “seesaw” models for neutrino physics: type I (singlet fermion), type II (triplet scalar), type III (triplet fermion)

Right-handed singlet: (type-I seesaw)



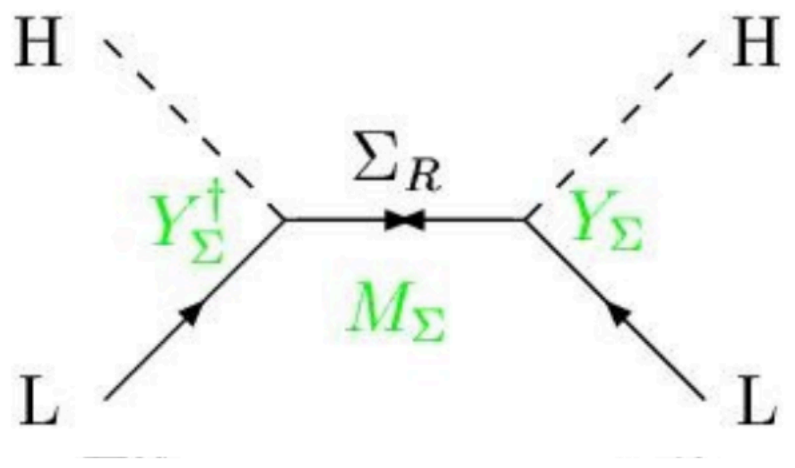
$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Scalar triplet: (type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Fermion triplet: (type-III seesaw)



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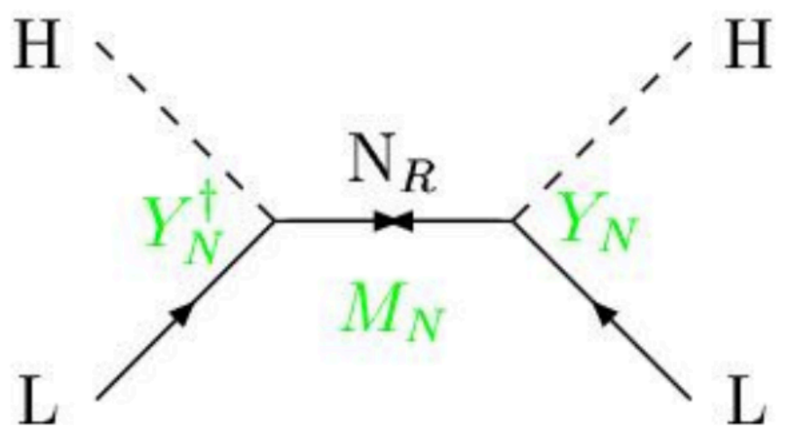

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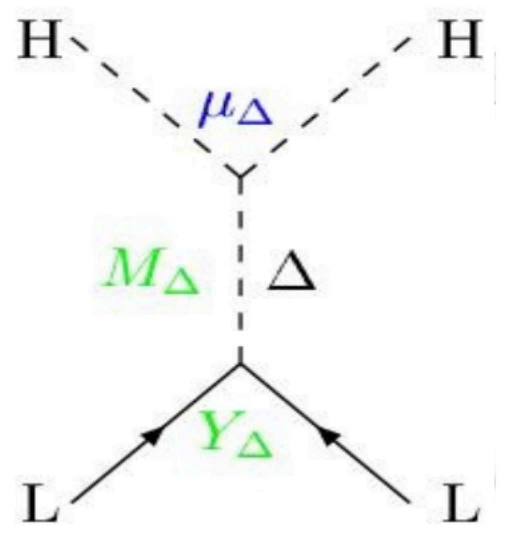

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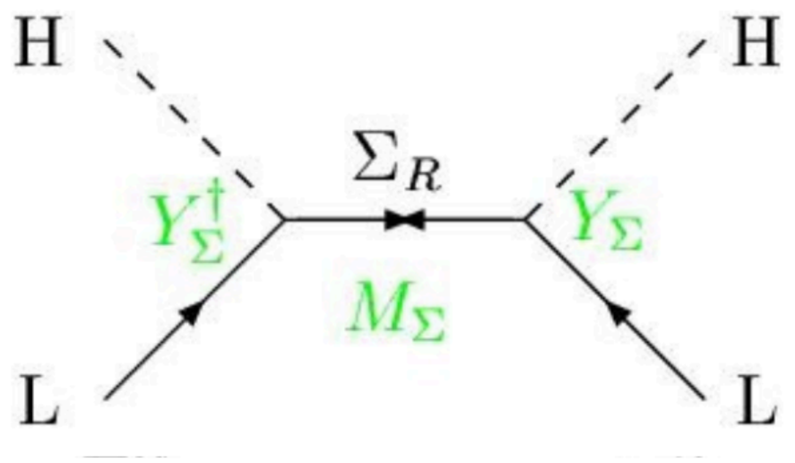


$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

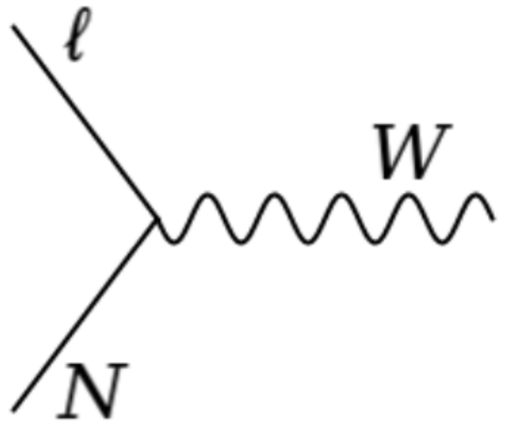
- Pheno of neutrino oscillations, flavor etc.
- Connections to Dark Matter (DM) (?)
- Lepton sector CP violation (?)
- Leptogenesis / Baryogenesis / Baryon Asymmetry of Universe (BAU)
- Lepton Flavor/Number Violation
- Fundamental Majorana Particles (?)

Simplified neutrino model

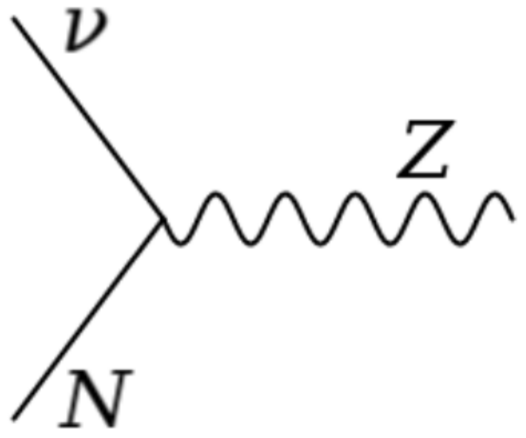
- Simplified model with right-handed (νSM) and sterile neutrinos
- After EWSB heavy (sterile) neutrinos do mix with νSM neutrinos
- Lagrangian: $\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_N + \mathcal{L}_{WN\ell} + \mathcal{L}_{ZN\nu} + \mathcal{L}_{HN\nu}$

$$\mathcal{L}_N = \xi_\nu \cdot \left(\bar{N}_k i \partial\!\!\!/ N_k - m_{N_k} \bar{N}_k N_k \right) \qquad \text{for } k = 1,\, 2,\, 3$$

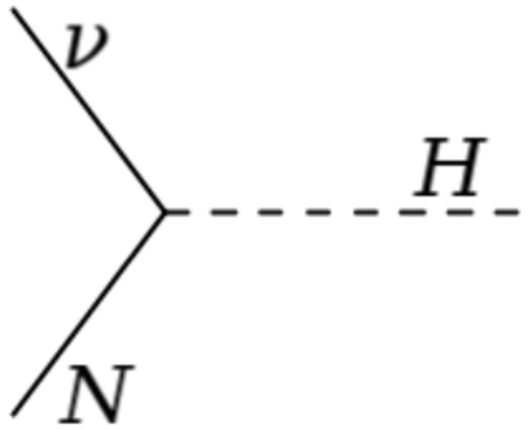
$$\mathcal{L}_{WN\ell} = -\frac{g}{\sqrt{2}} W^+_\mu \sum_{k=1}^{3} \sum_{l=e}^{\tau} \bar{N}_k V^*_{lk} \gamma^\mu P_L \ell^- + \text{ h.c.},$$



$$\mathcal{L}_{ZN\nu} = -\frac{g}{2\cos\theta_W} Z_\mu \sum_{k=1}^{3} \sum_{l=e}^{\tau} \bar{N}_k V^*_{lk} \gamma^\mu P_L \nu_l + \text{ h.c.}$$



$$\mathcal{L}_{HN\nu} = -\frac{g m_N}{2 M_W} h \sum_{k=1}^{3} \sum_{l=e}^{\tau} \bar{N}_k V^*_{lk} P_L \nu_l + \text{ h.c.}$$



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Incomplete literature:

Aguilar-Saavedra ea., hep-ph/0502189; hep-ph/0503026; Shaposhnikov, 0804.4542; Das/Okada, 1207.3734; Banerjee ea., 1503.05491; Antusch, Cazzato, Fischer, 1612.0272; Cai, Han, Li, Ruiz, 1711.02180; Pascoli, Ruiz, Weiland, 1812.08750

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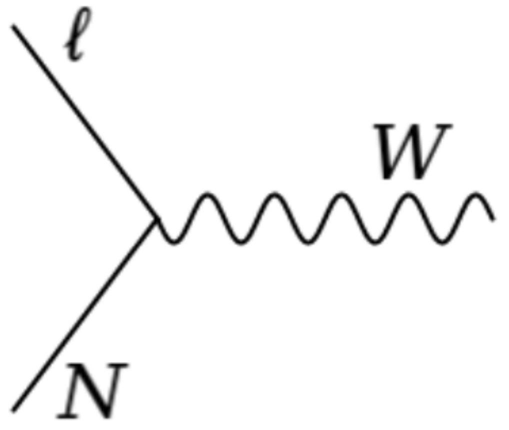

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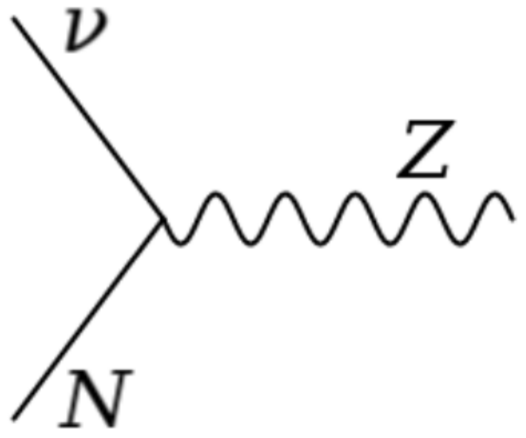

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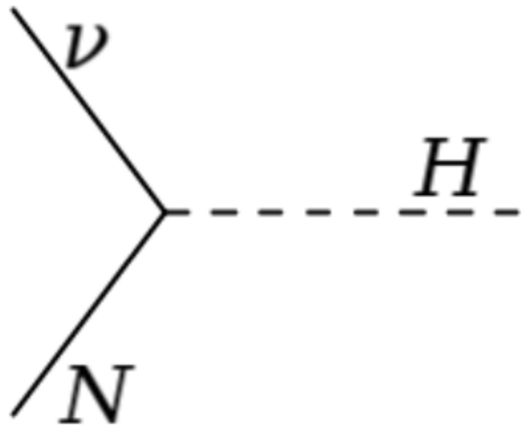

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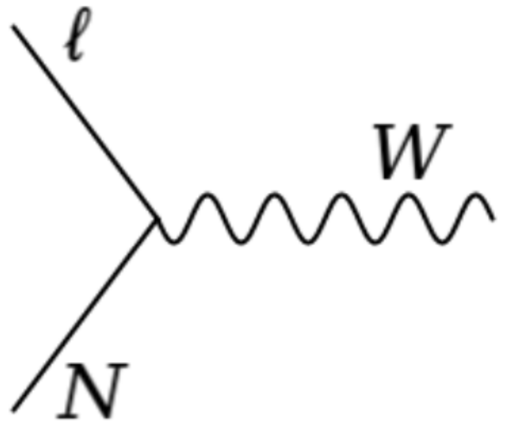
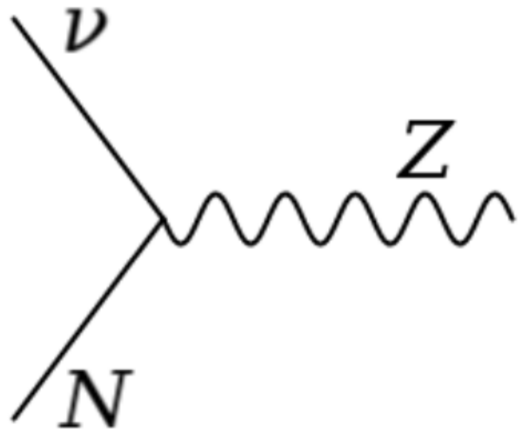
$$\mathcal{L}_N = \xi_\nu \cdot \left(\bar{N}_k i\partial\!\!\!/ N_k - m_{N_k}\bar{N}_k N_k\right) \qquad \text{for } k = 1,\, 2,\, 3$$

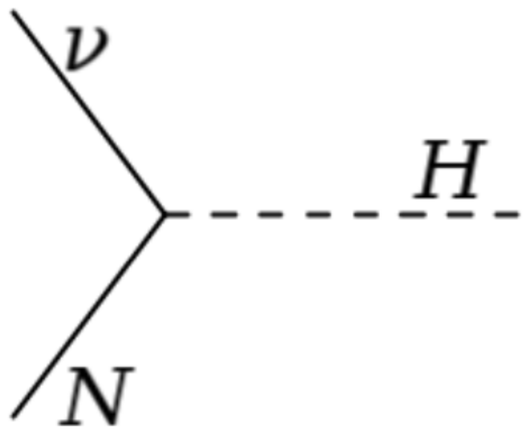
$$\mathcal{L}_{WN\ell} = -\frac{g}{\sqrt{2}} W_\mu^+ \sum_{k=1}^{3}\sum_{l=e}^{\tau} \bar{N}_k V_{lk}^* \gamma^\mu P_L \ell^- + \text{ h.c.},$$



$$\mathcal{L}_{ZN\nu} = -\frac{g}{2\cos\theta_W} Z_\mu \sum_{k=1}^{3}\sum_{l=e}^{\tau} \bar{N}_k V_{lk}^* \gamma^\mu P_L \nu_l + \text{ h.c.}$$



$$\mathcal{L}_{HN\nu} = -\frac{g m_N}{2M_W} h \sum_{k=1}^{3}\sum_{l=e}^{\tau} \bar{N}_k V_{lk}^* P_L \nu_l + \text{ h.c.}$$



Incomplete literature:

Aguilar-Saavedra ea., hep-ph/0502189; hep-ph/0503026; Shaposhnikov, 0804.4542; Das/Okada, 1207.3734; Banerjee ea., 1503.05491; Antusch, Cazzato, Fischer, 1612.0272; Cai, Han, Li, Ruiz, 1711.02180; Pascoli, Ruiz, Weiland, 1812.08750

- ☑ At lepton colliders, single production possible
- ☑ Associated production: $\ell^+\ell^- \to \nu N$
- ☑ Vector boson fusion: $\ell^+\ell^- \to \bar{\nu}\nu N + \ell^+\ell^- N$
- ☑ Three neutrino masses: $M_{N_1}, M_{N_2}, M_{N_3}$
- ☑ Nine real mixing parameters: $V_{\ell k}$, $\ell = e, \mu\, \tau, k = N_1, N_2, N_3$
- ☑ Three neutrino widths: $\Gamma_{N_1}, \Gamma_{N_2}, \Gamma_{N_3}$

Signal, simulation, selection

- At lepton colliders: optimal channel single production with decay to $N \to jj\ell$
- In that case: full reconstruction of *N* (incl. mass peak) possible
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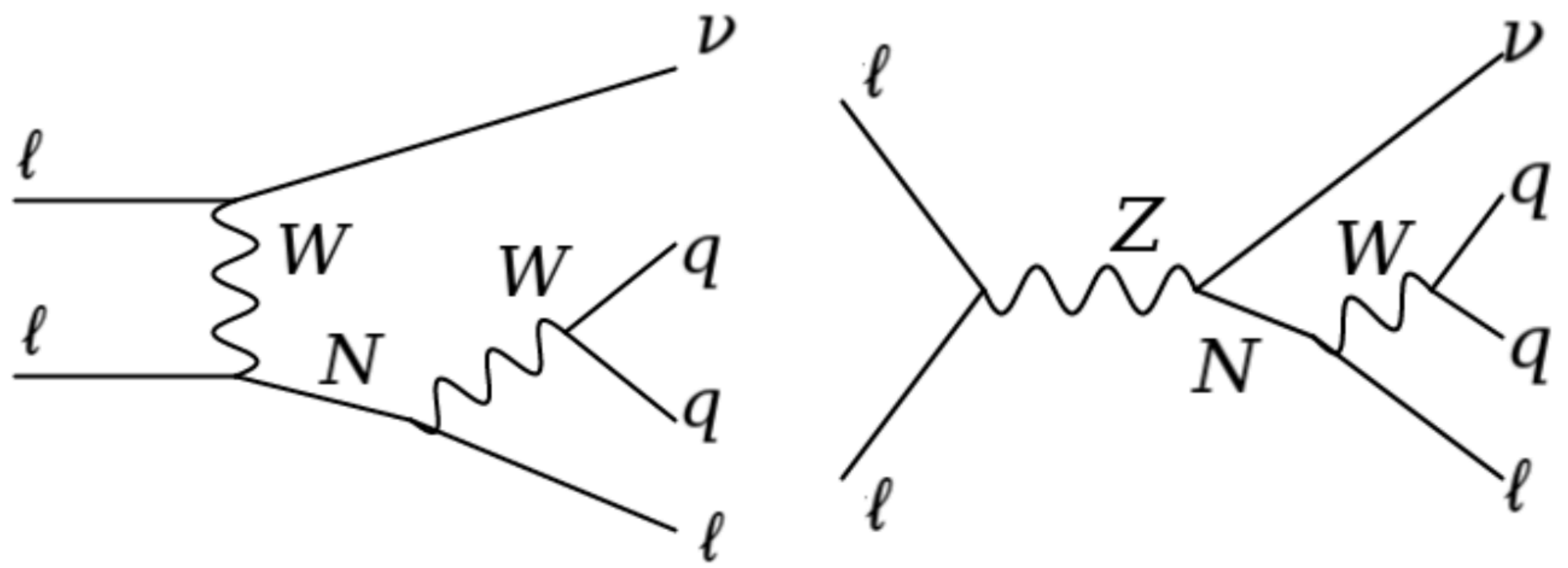

K. Mękała/JRR/A.F. Żarnecki, 2202.06703; 2301.02602

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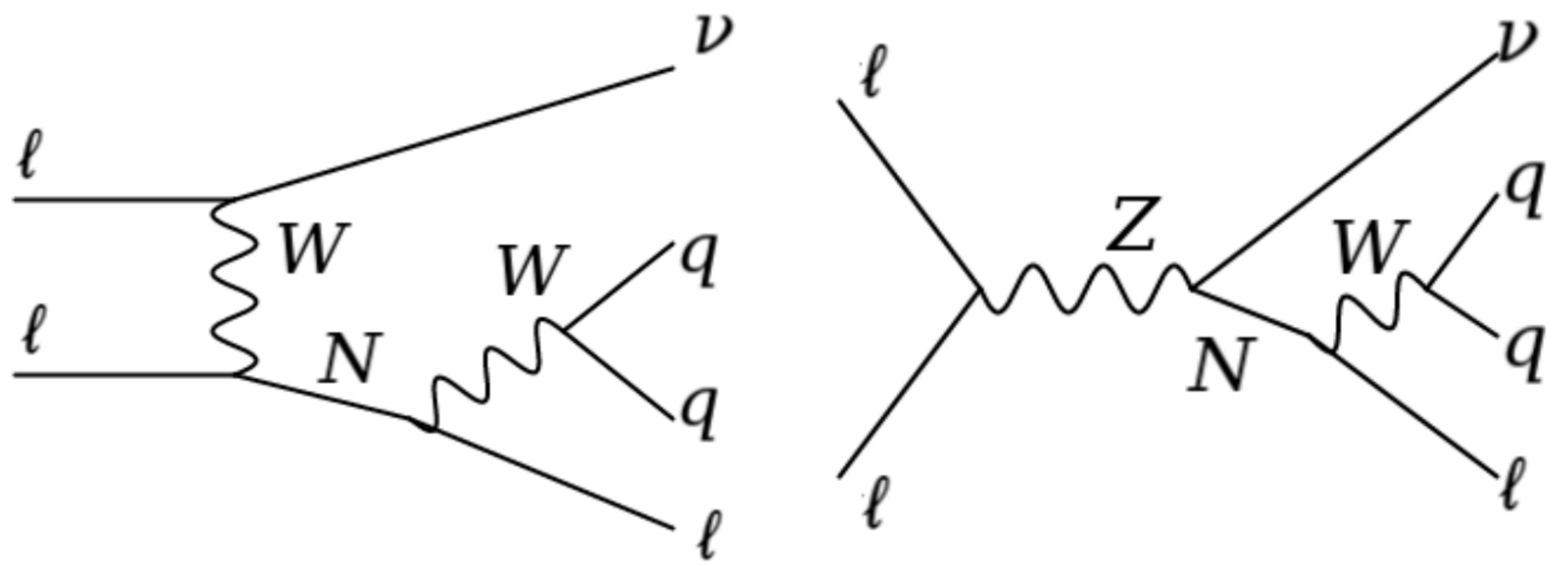

K. Mękała/JRR/A.F. Żarnecki, 2202.06703; 2301.02602

- ☑ Assumption on couplings: $|V_{eN_1}|^2 = |V_{\mu N_1}|^2 + |V_{\tau N_1}|^2 \equiv |V_{\ell N_1}|^2$
- ☑ Neutrinos masses: 100 GeV $\leq M_{M_1} \leq 10.5$ TeV, $M_{N_{2,3}} = 10^{10}$ GeV
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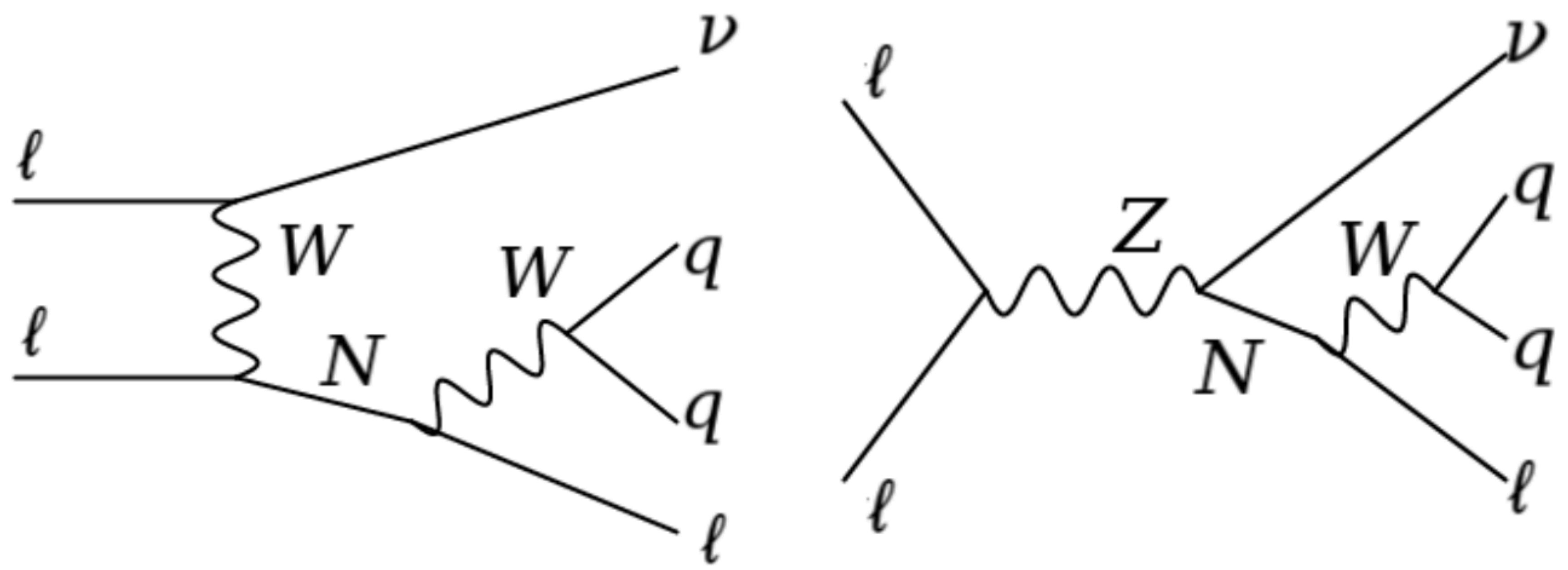

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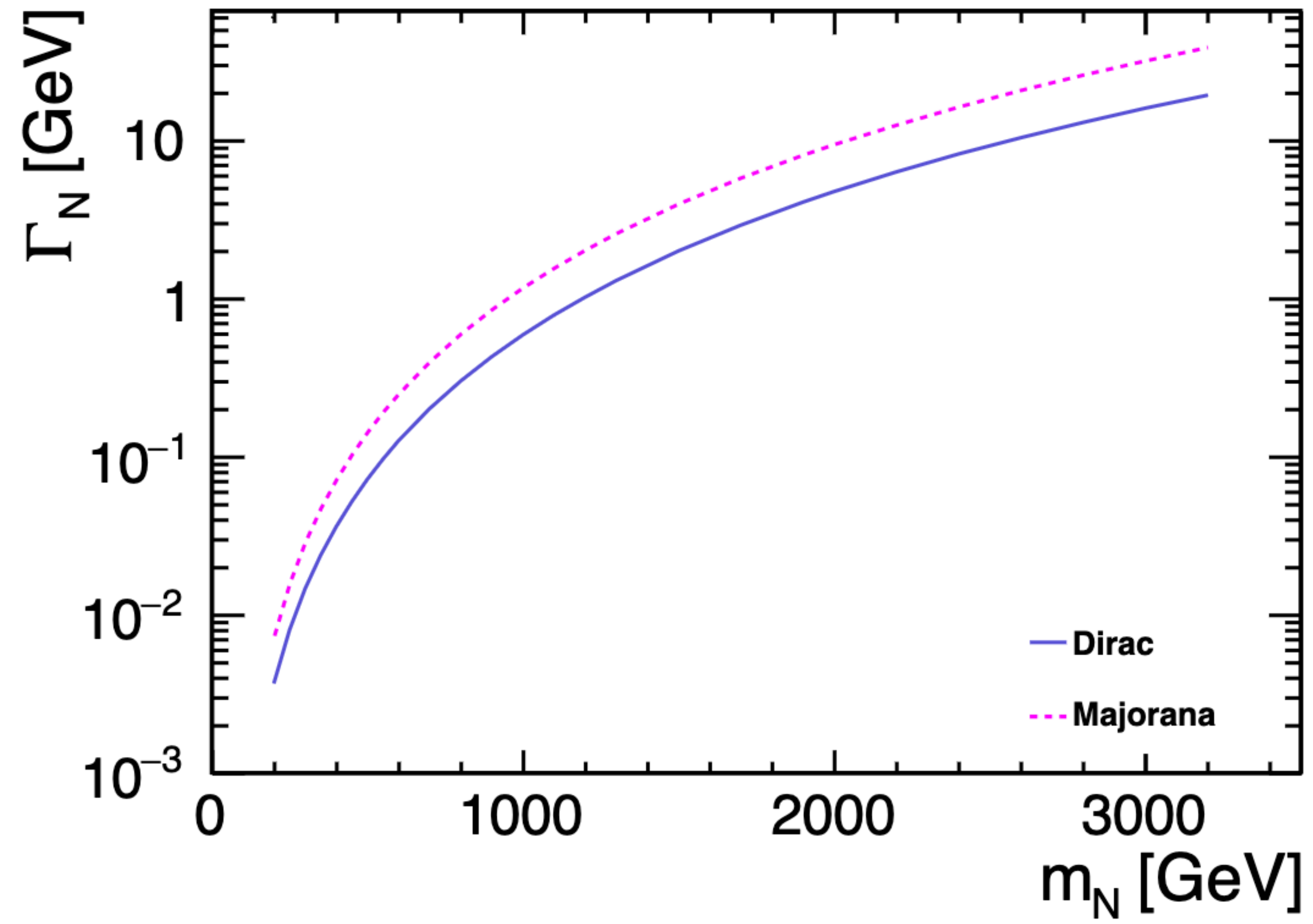

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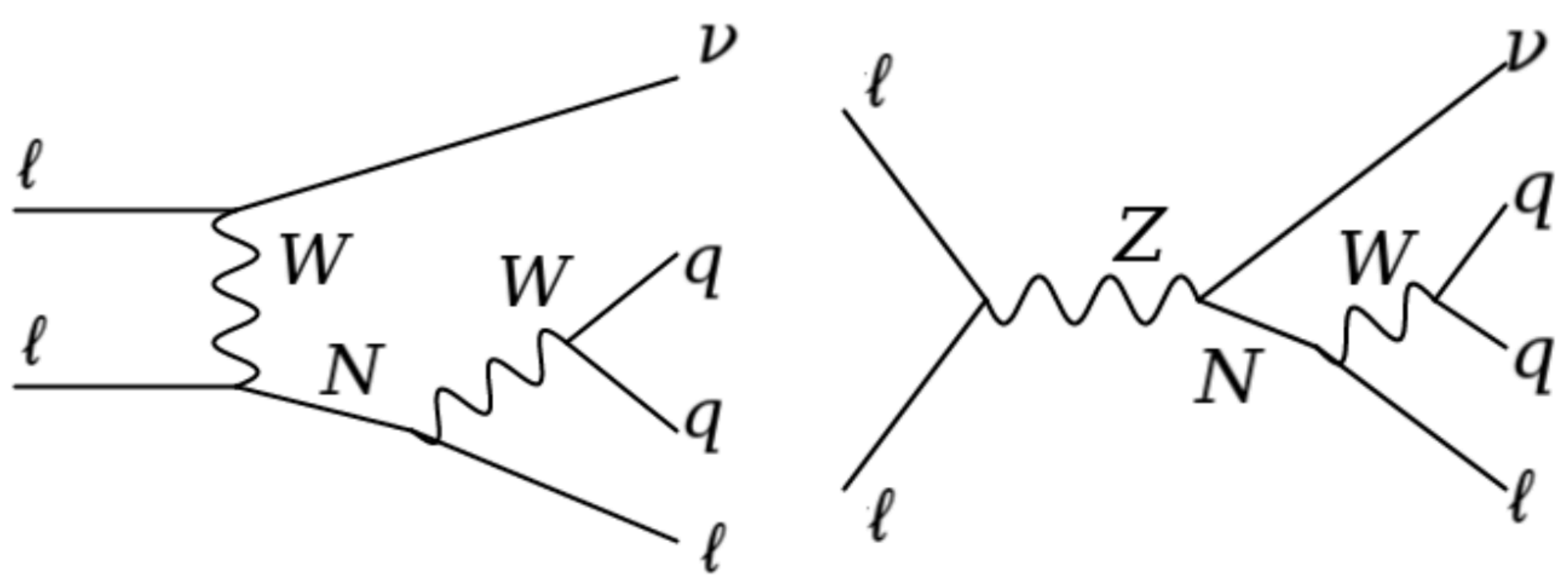

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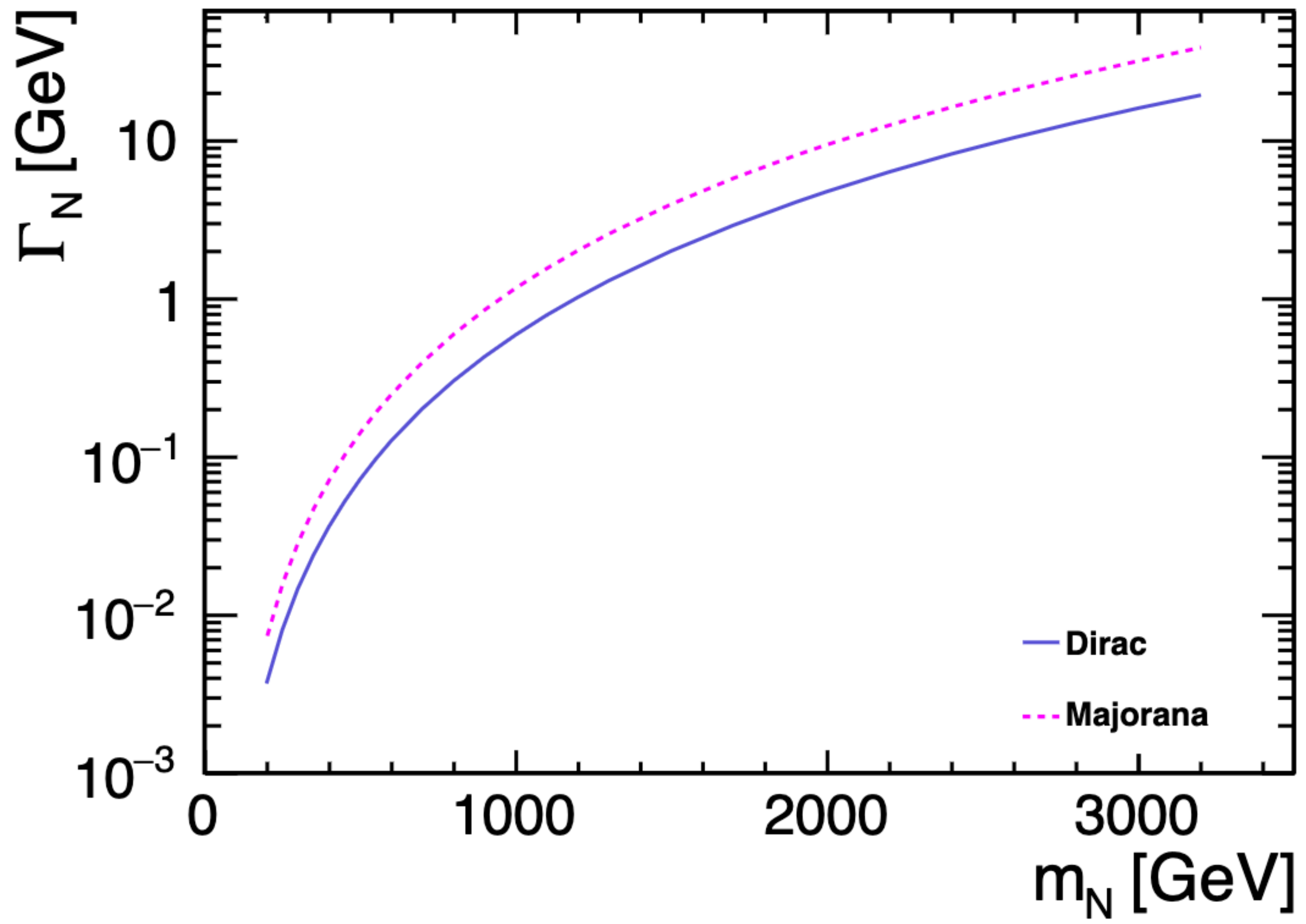
  - Background simulation: without N propagators ("background")
  - Signal simulation: $\ell\ell \to N\nu \to \ell jj\nu$ ("signal")
  - $S/B \sim 10^{-3}$ e.g. ILC500: $jj\ell\nu$ bkgd. ~ 10 pb, signal ~ 10 fb
  - Preselection on signal topology: exactly 1 lepton and 2 jets
  - BDT training; CLs method to get final results



Event selection & analysis

Bkgd processes with at least one lepton

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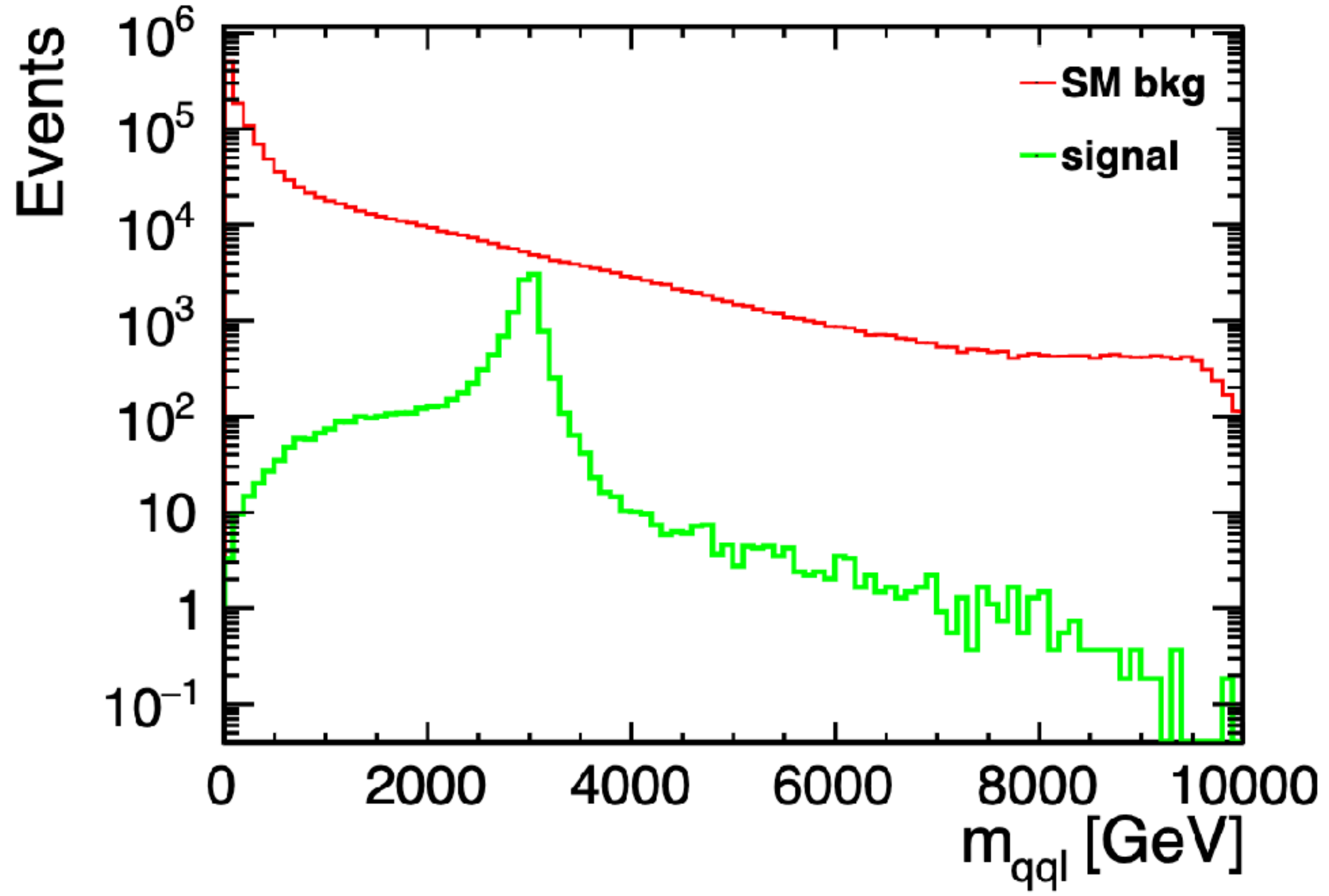

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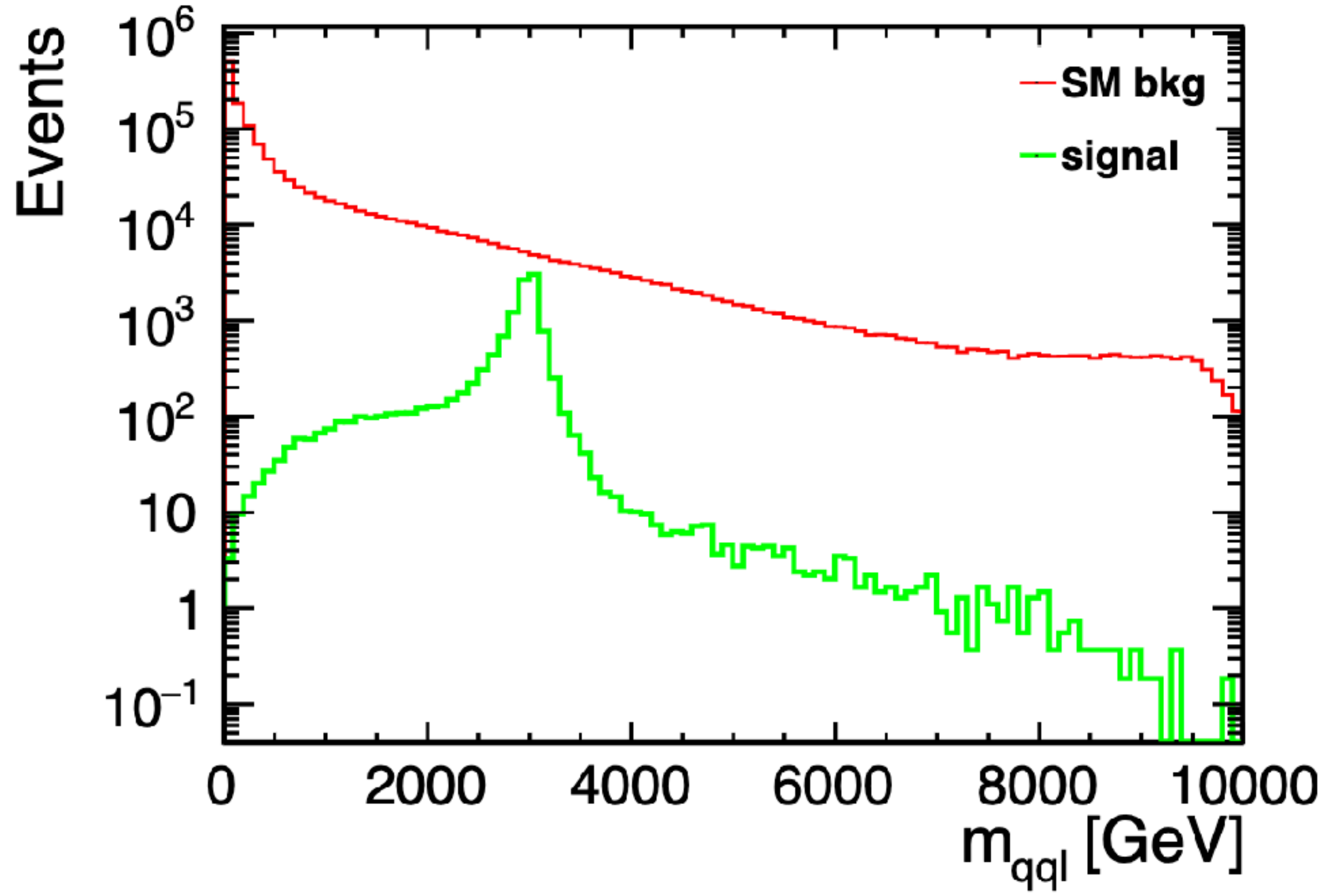

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- ☑ QED-ISR/beamstrahlung: CLIC-3 vs. MuC-3
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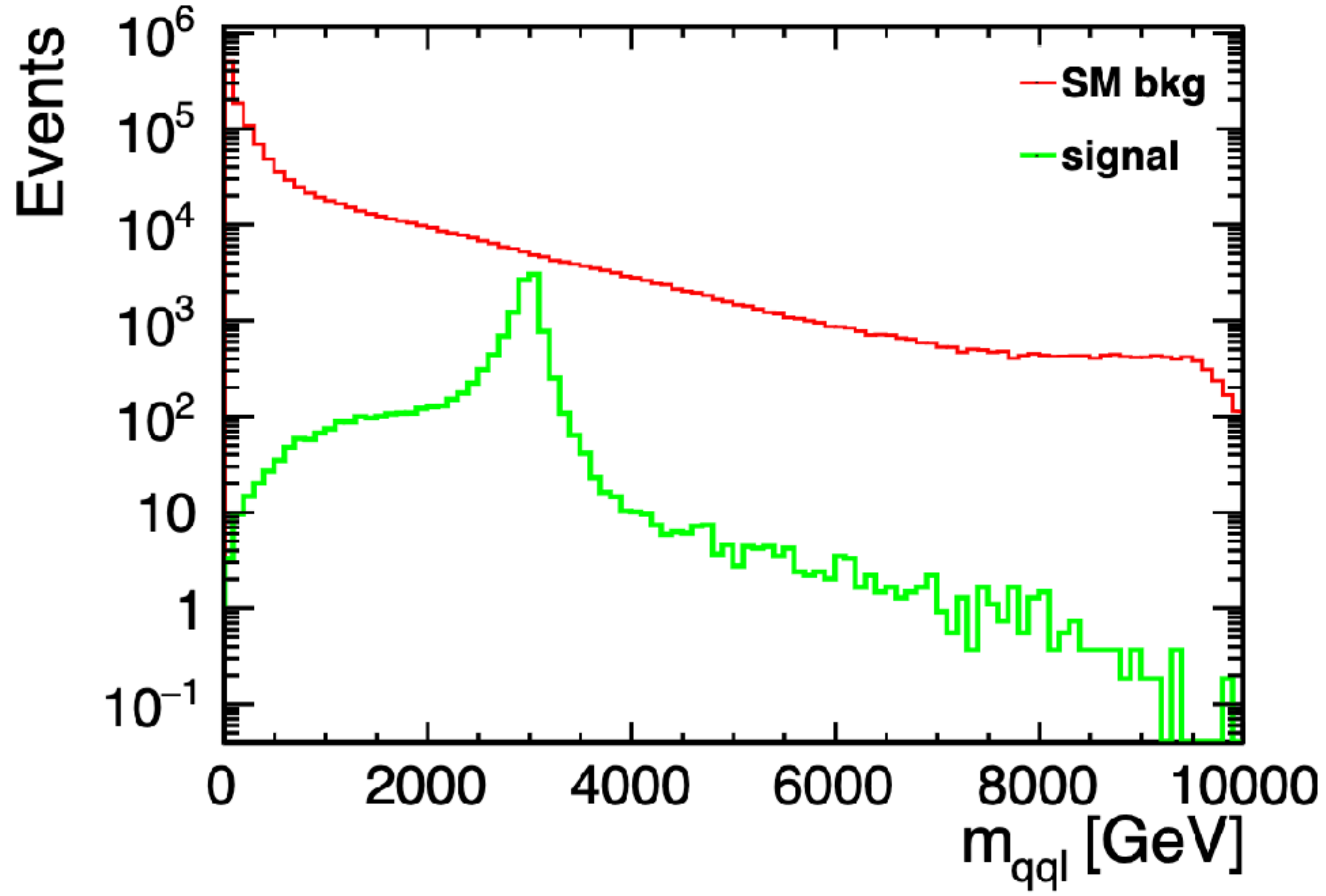

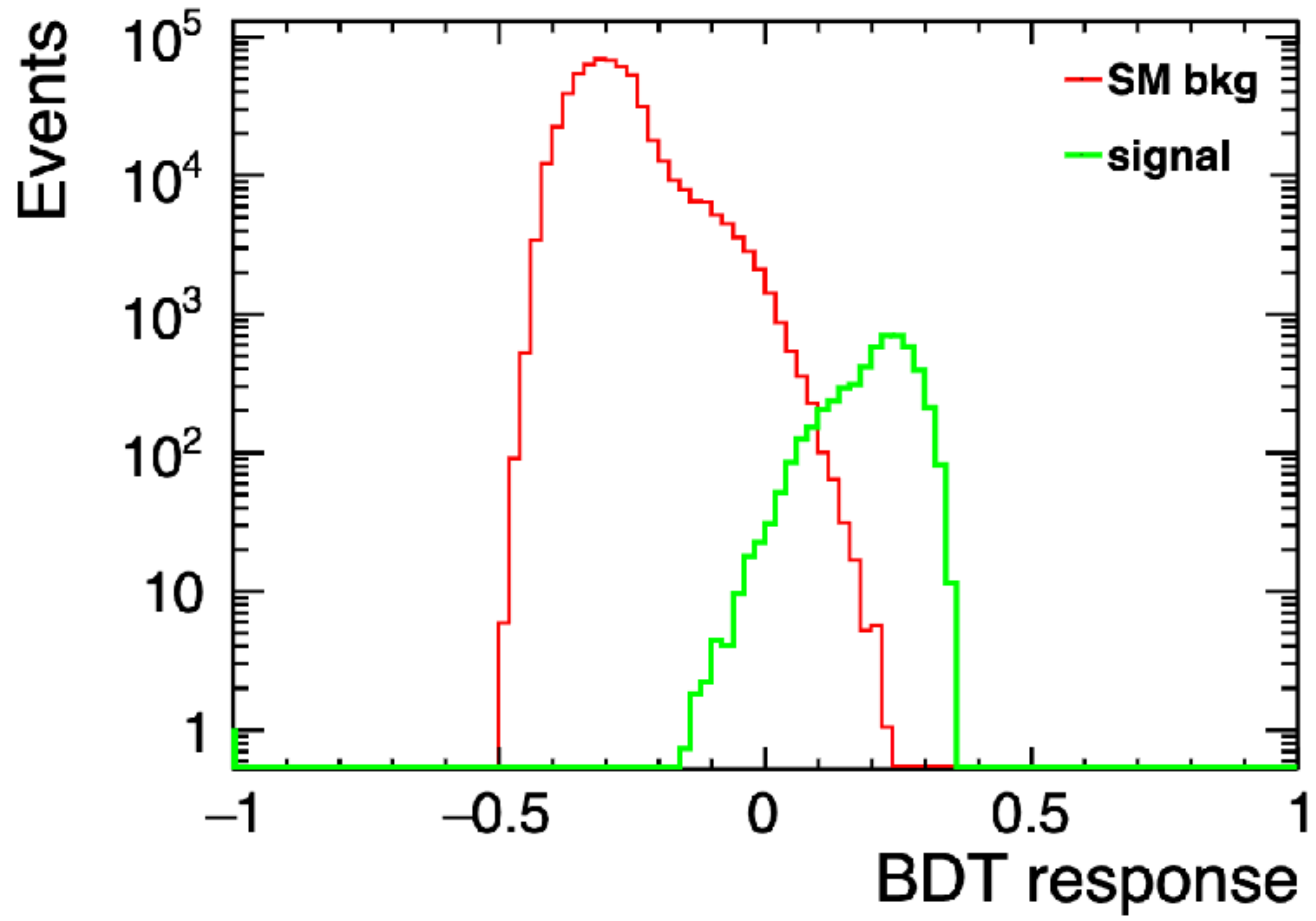

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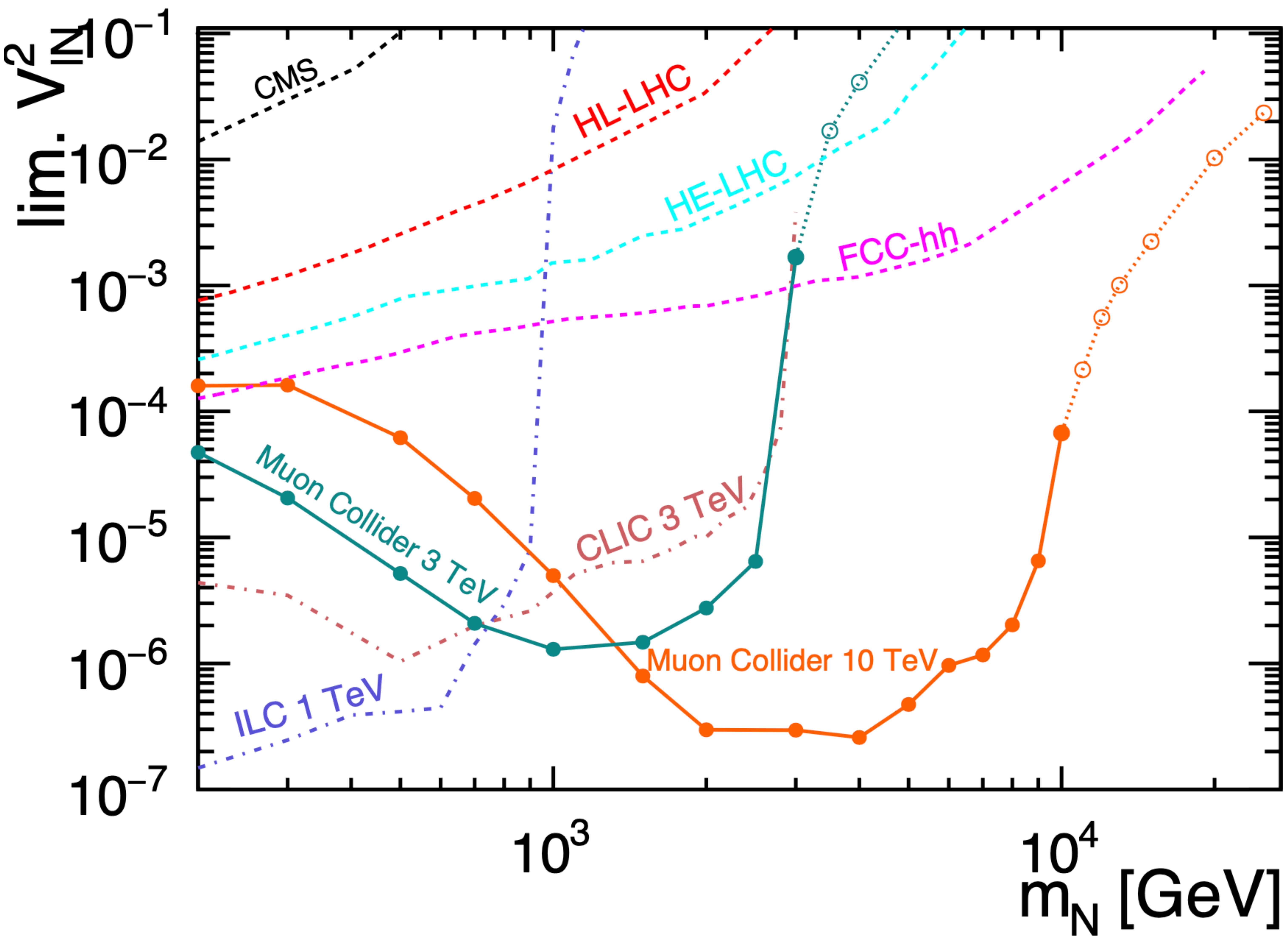
BDT response for model in `RooStats`, CLs method to set cross section limits

Combination of $e^\pm$ and $\mu^\pm$ channels

8 variables considered in BDT

- $m_{qq\ell}$ – invariant mass of the dijet-lepton system,
- α – angle between the dijet-system and the lepton,
- α_{qq} – angle between the two jets,
- E_ℓ – lepton energy,
- $E_{qq\ell}$ – energy of the dijet-lepton system,
- p^T_ℓ – lepton transverse momentum,
- p^T_{qq} – dijet transverse momentum,
- $p^T_{qq\ell}$ – transverse momentum of the dijet-lepton system.

Reach for HNLs



K. Mękała/JRR/A.F. Żarnecki, 2301.02602

LHC analysis [1812.08750],
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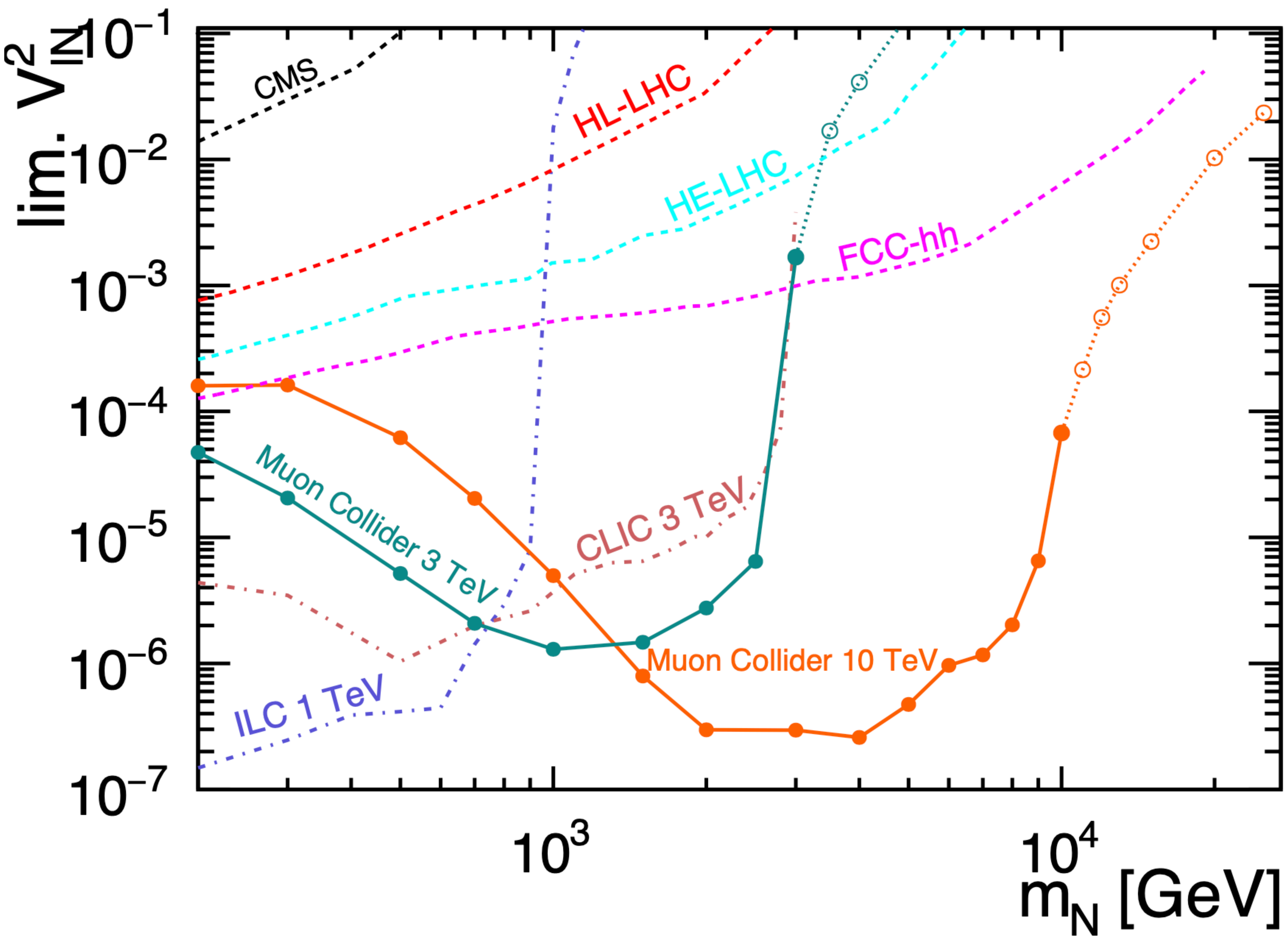

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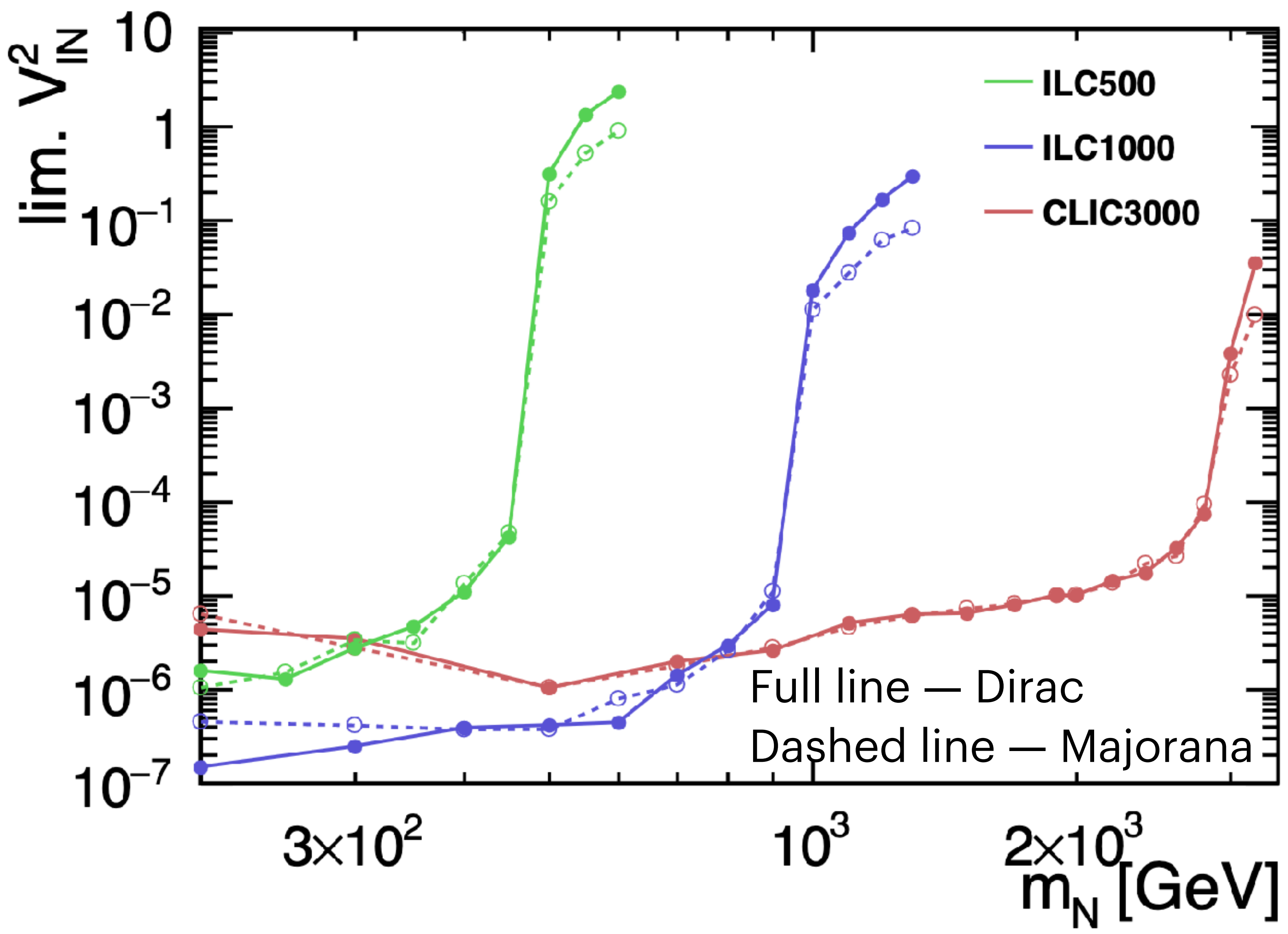
MuC-10 outperforms FCC-hh-100
over the whole mass range!

Discrimination of Dirac vs. Majorana

- Exclusion limit very similar for Dirac & Majorana neutrino (except: off-shell production)
- Possible discriminant: lepton emission angle in N rest frame



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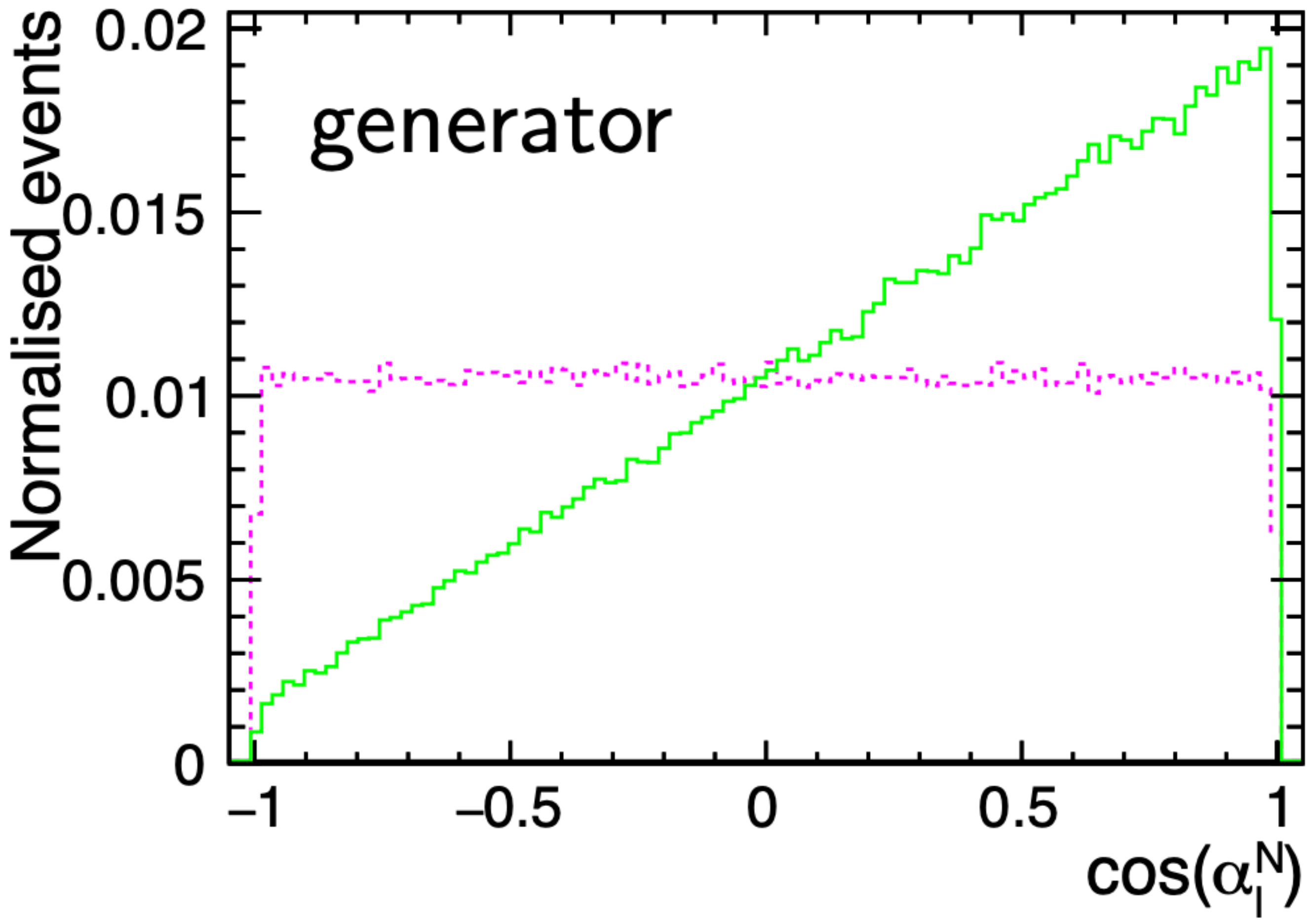

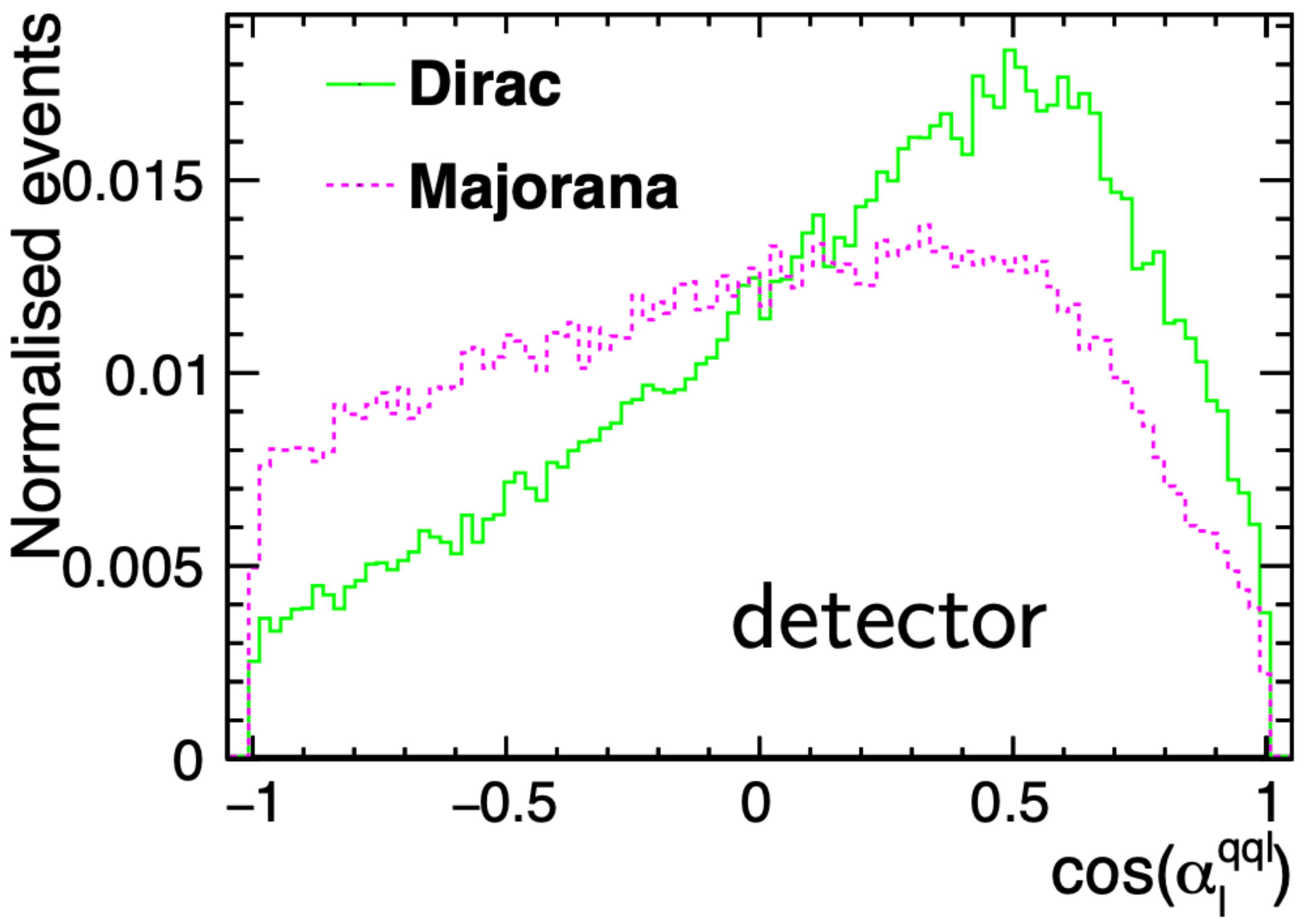

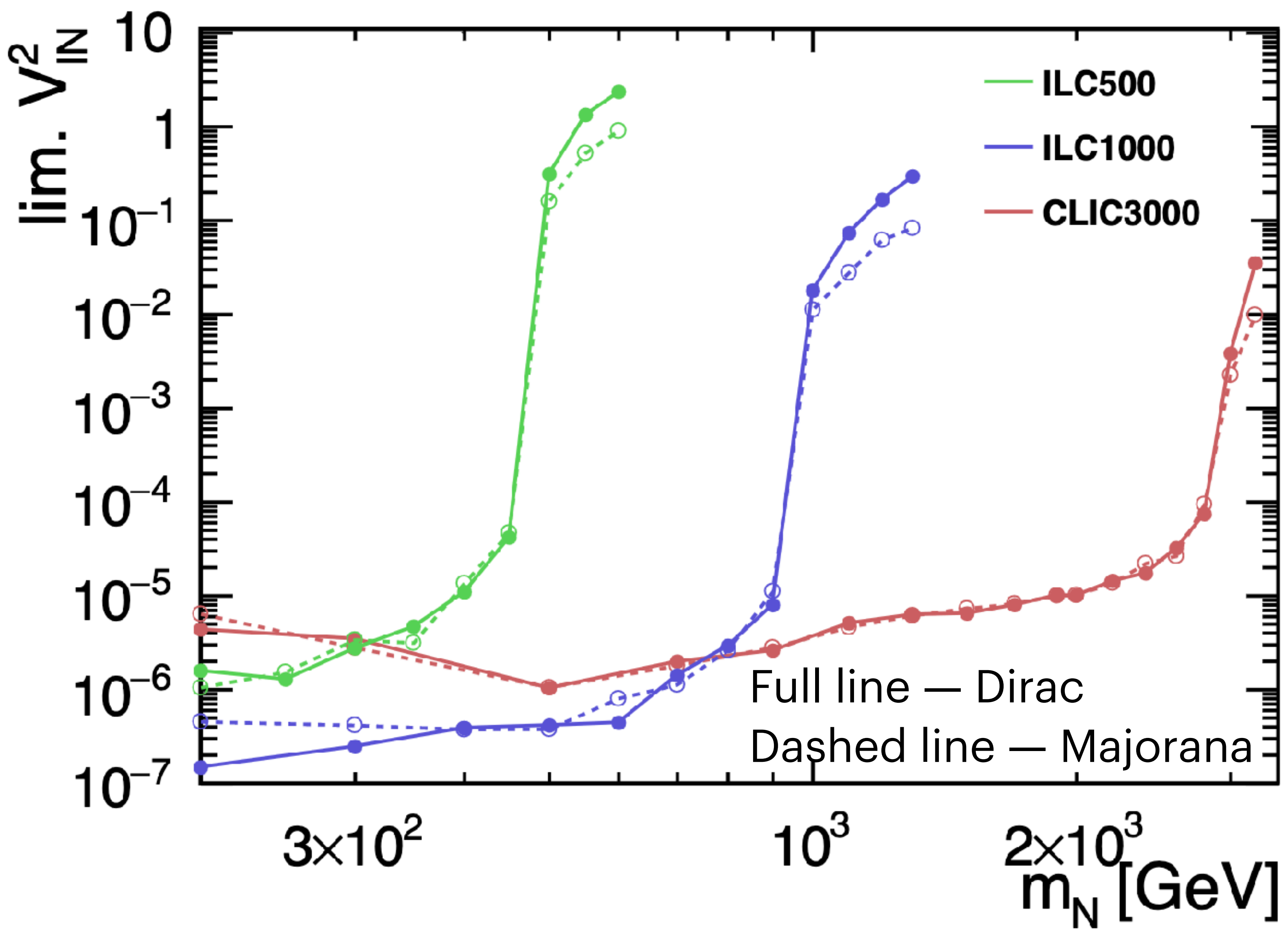

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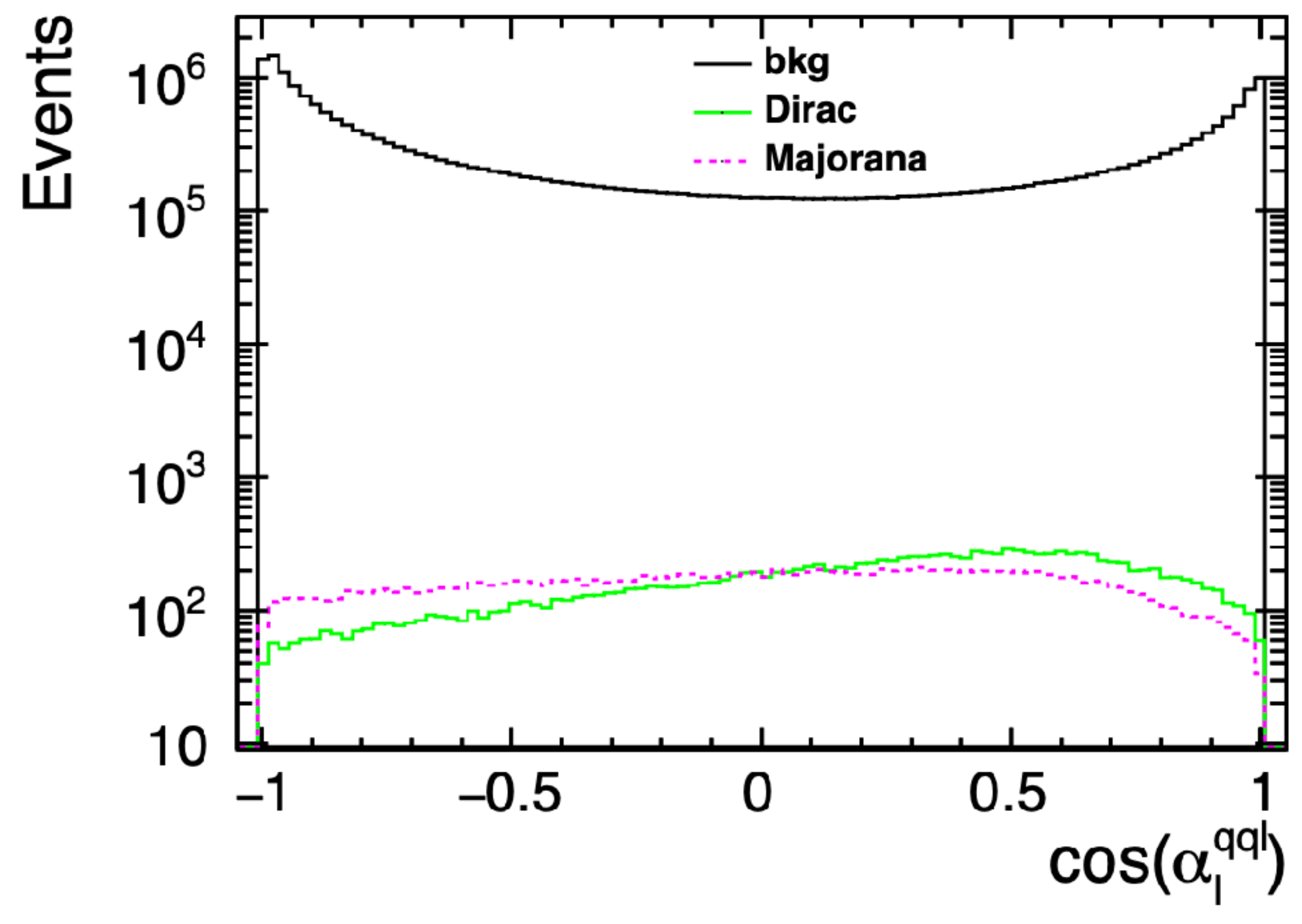

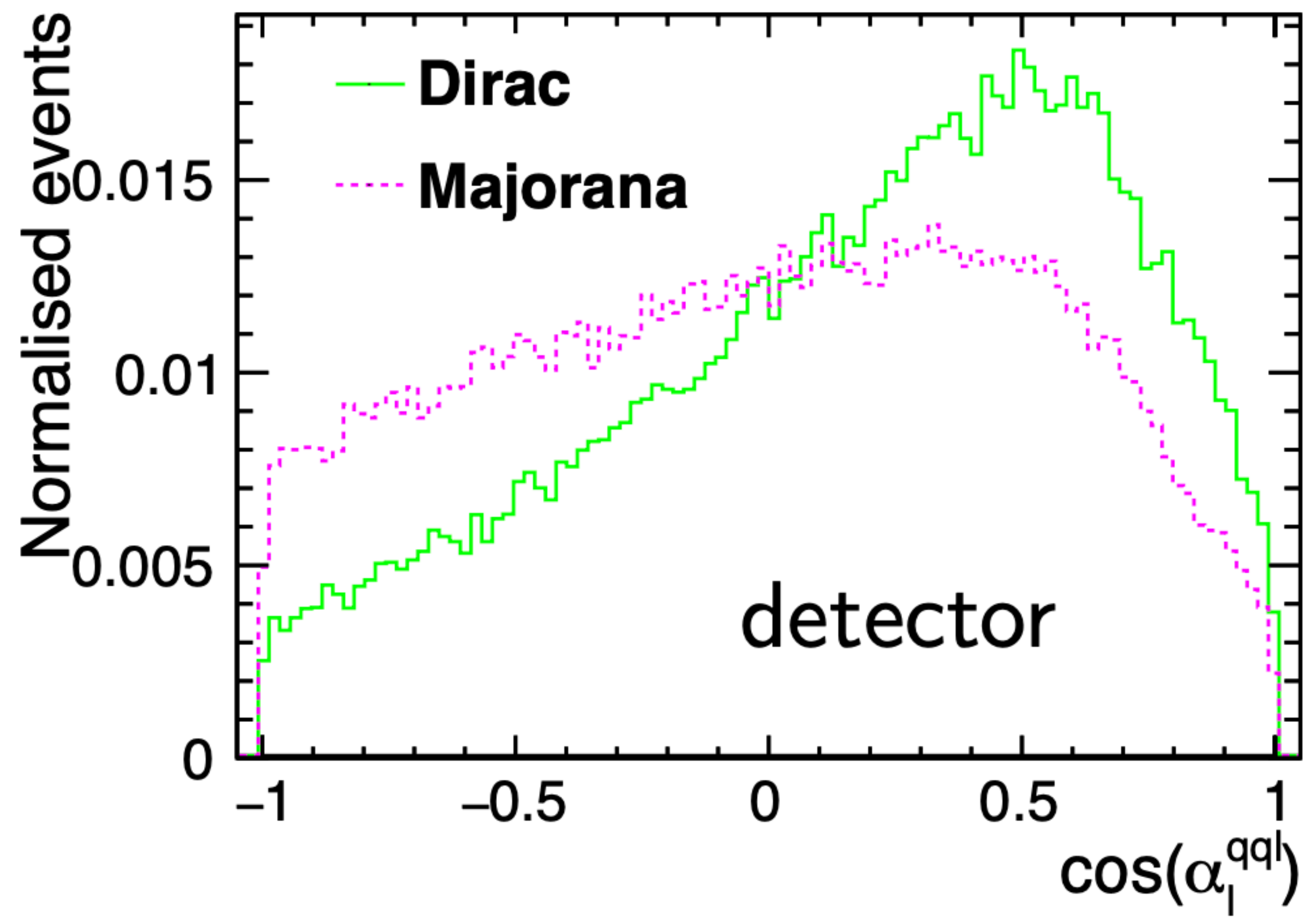

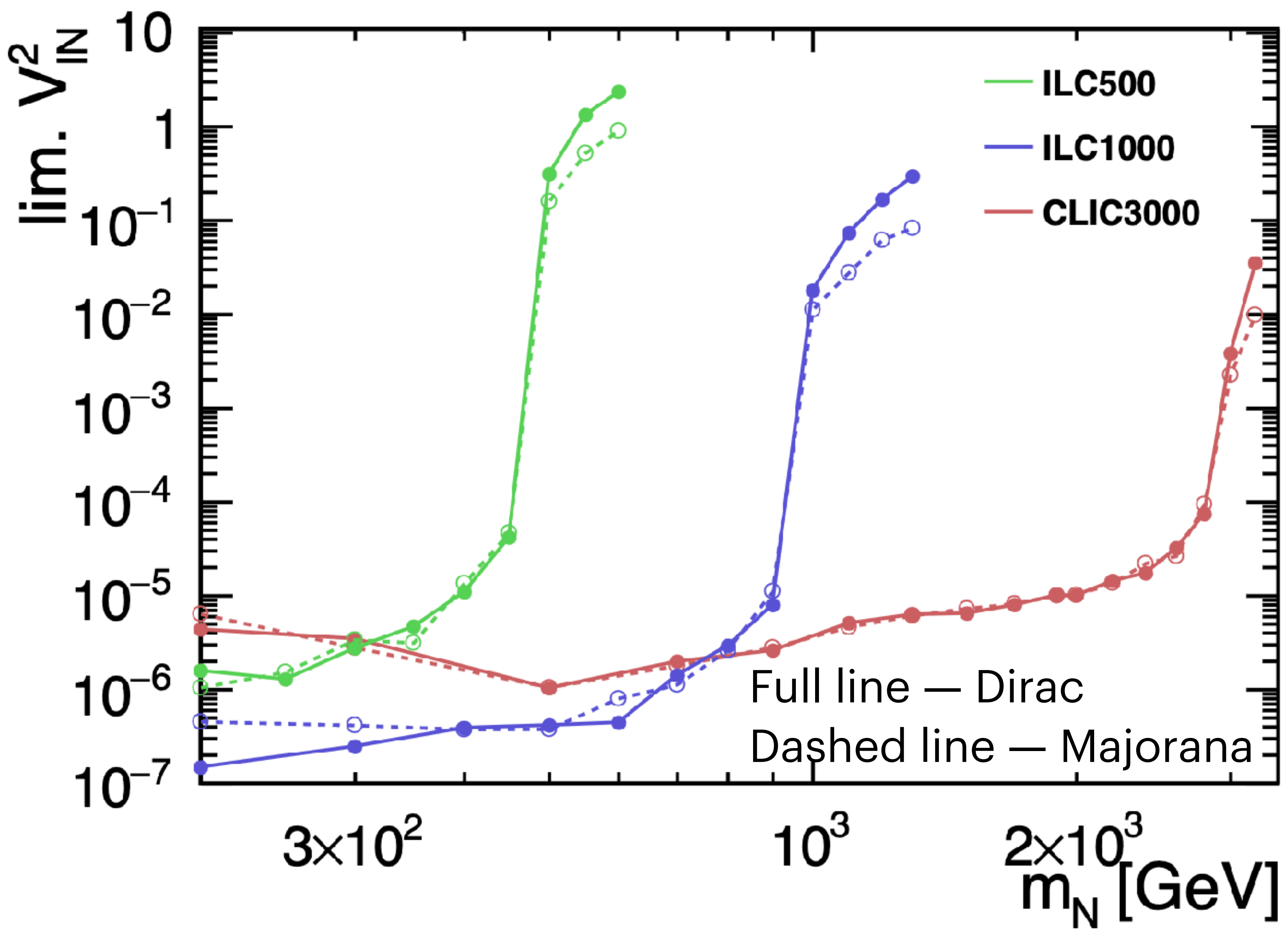

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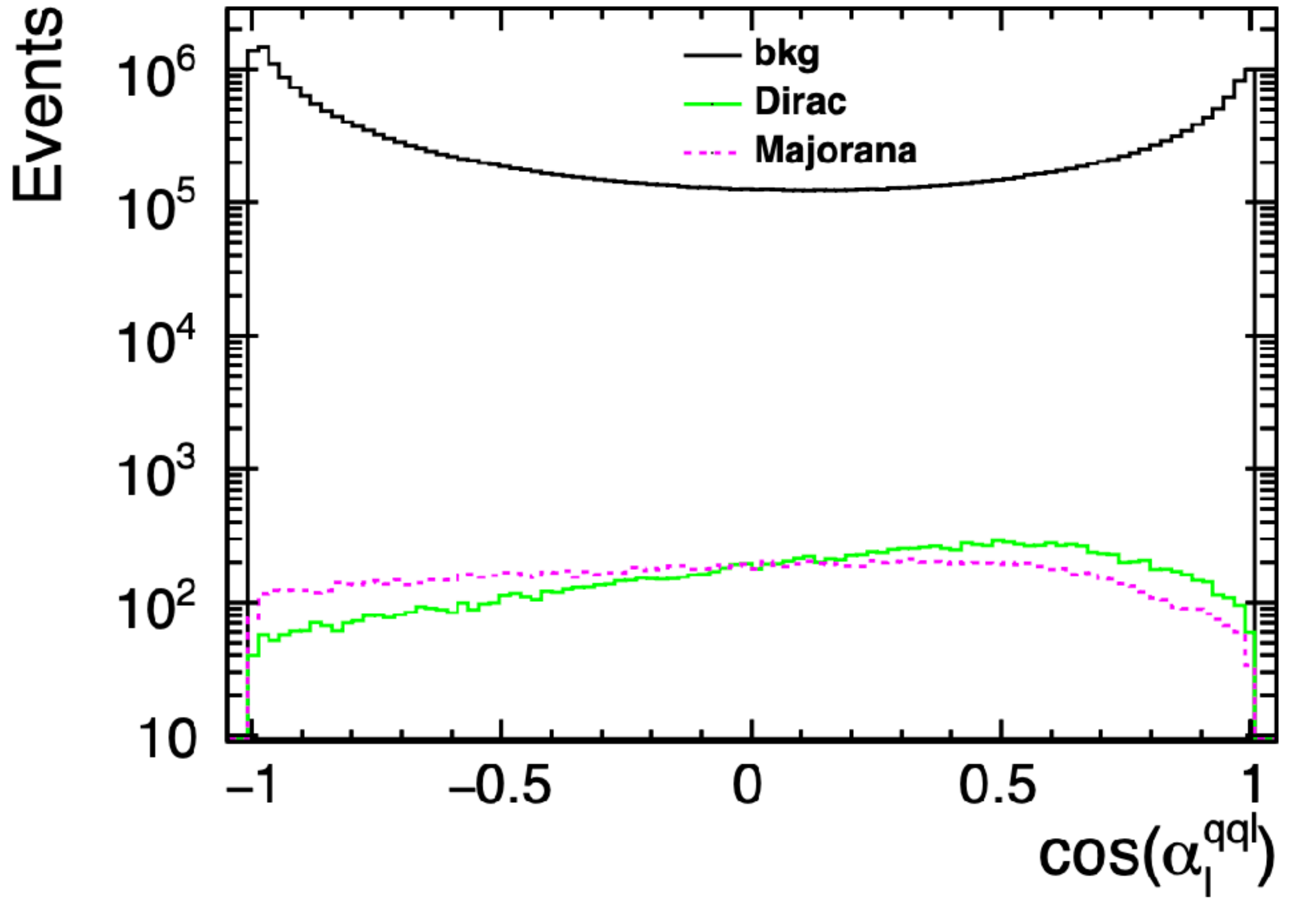
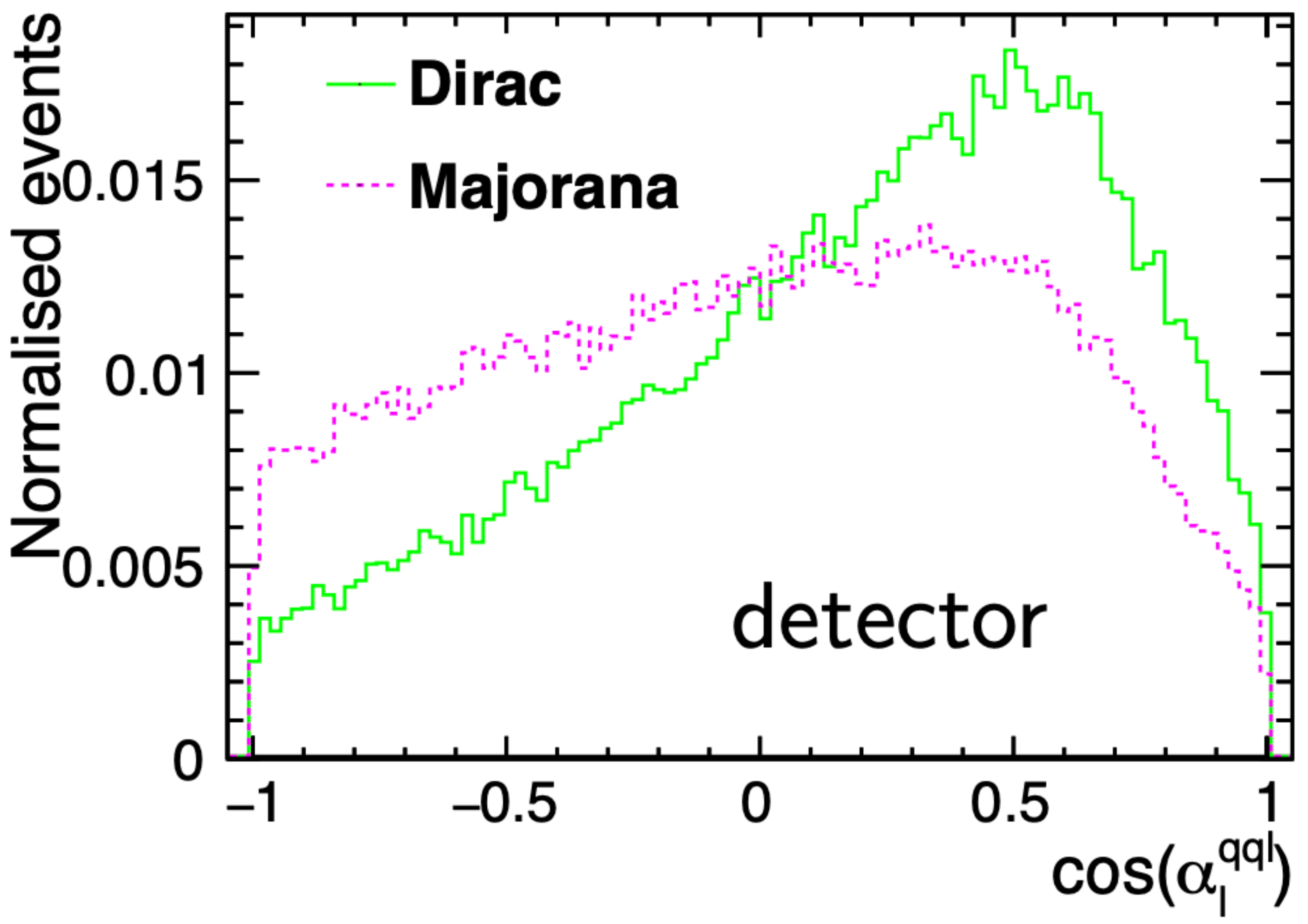
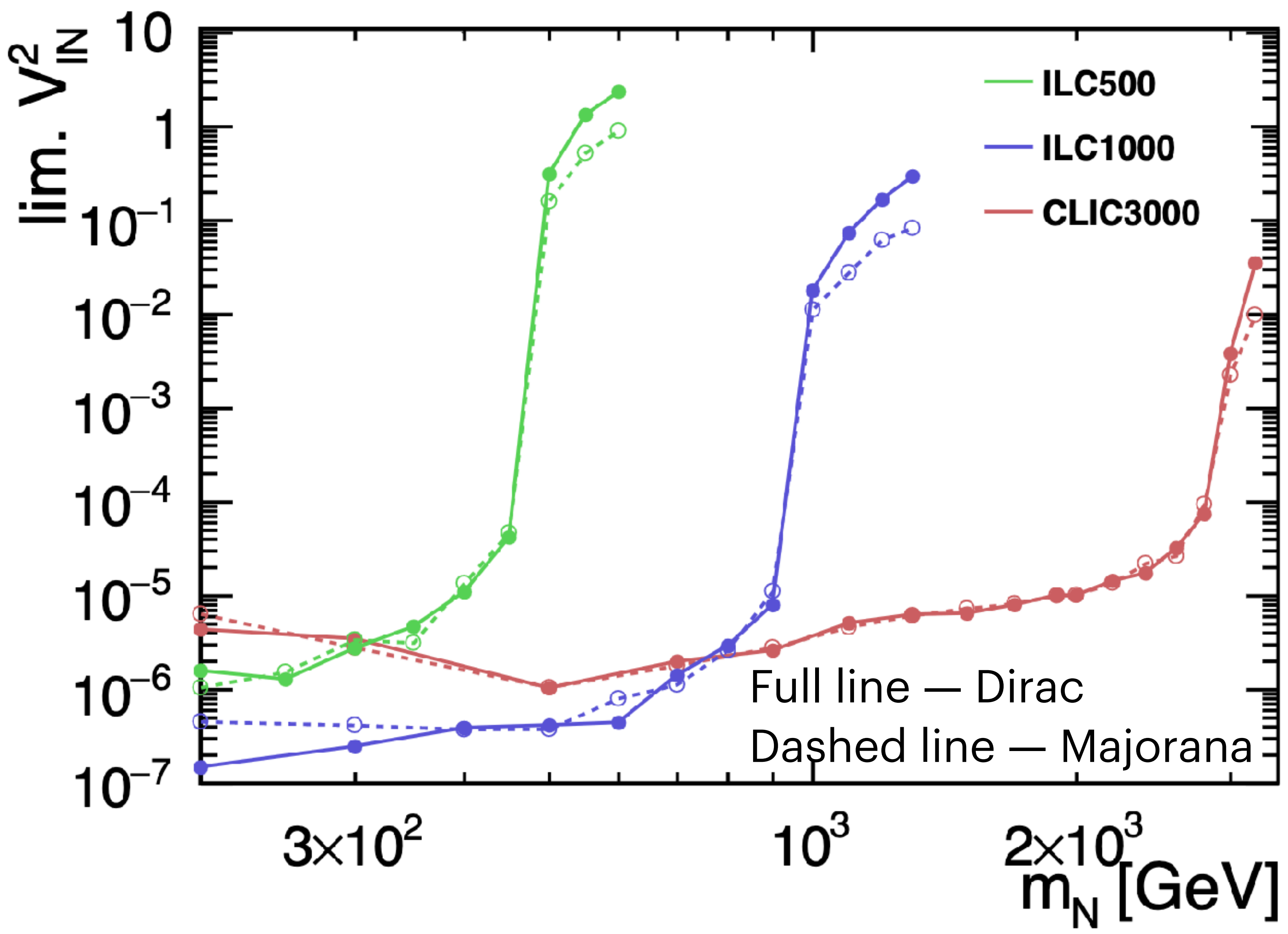
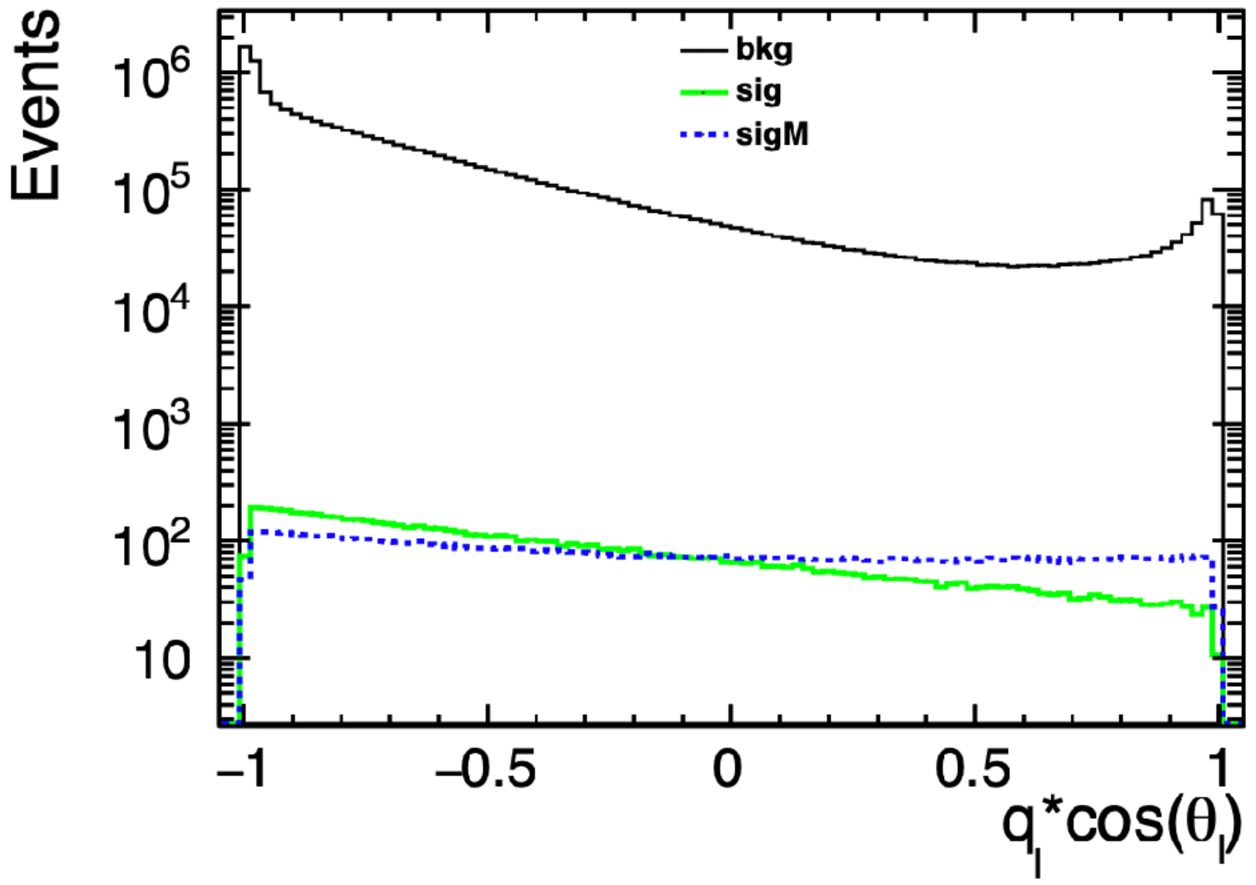


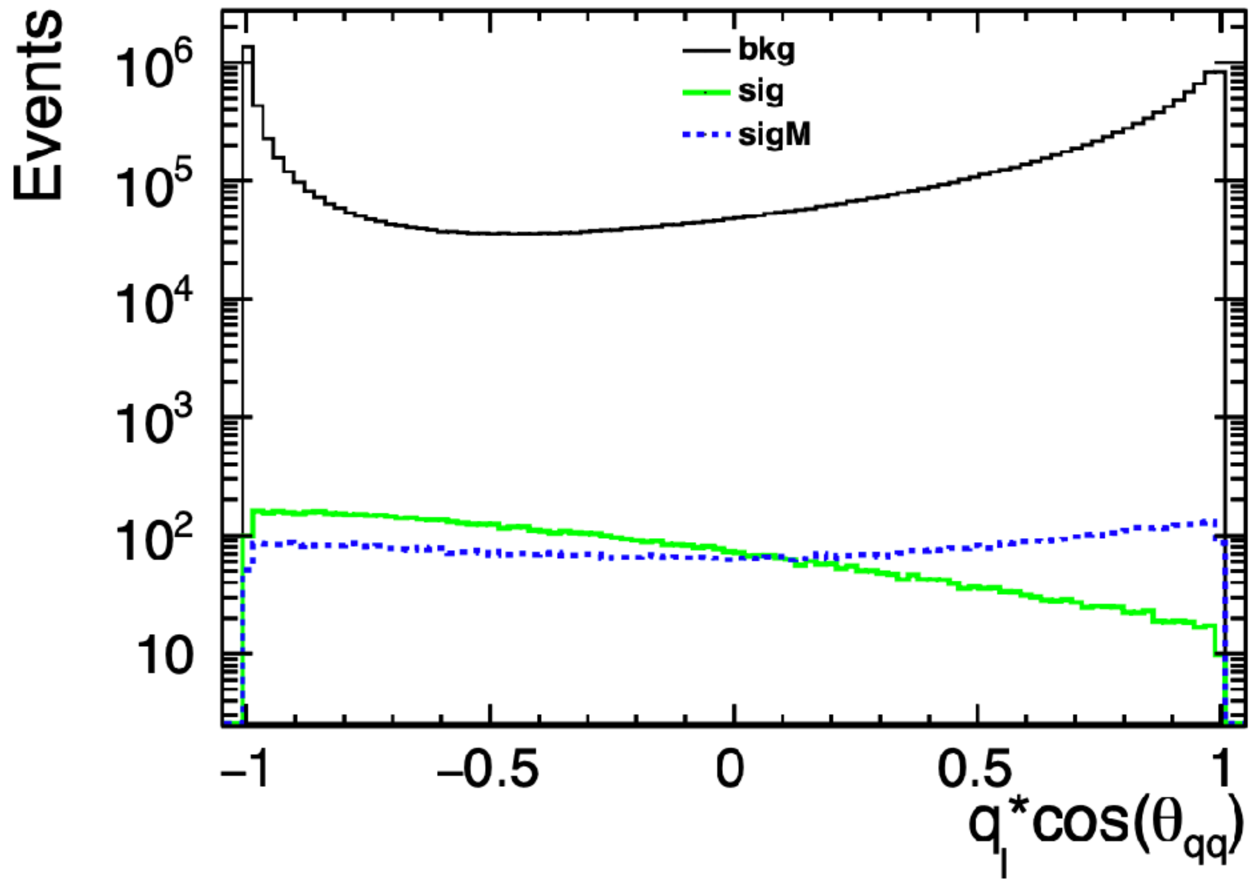




- More sophisticated variable: lepton and dijet angles relative to beam weighted by the lepton charge q_ℓ

ILC 250 GeV, $m_N = 150$ GeV





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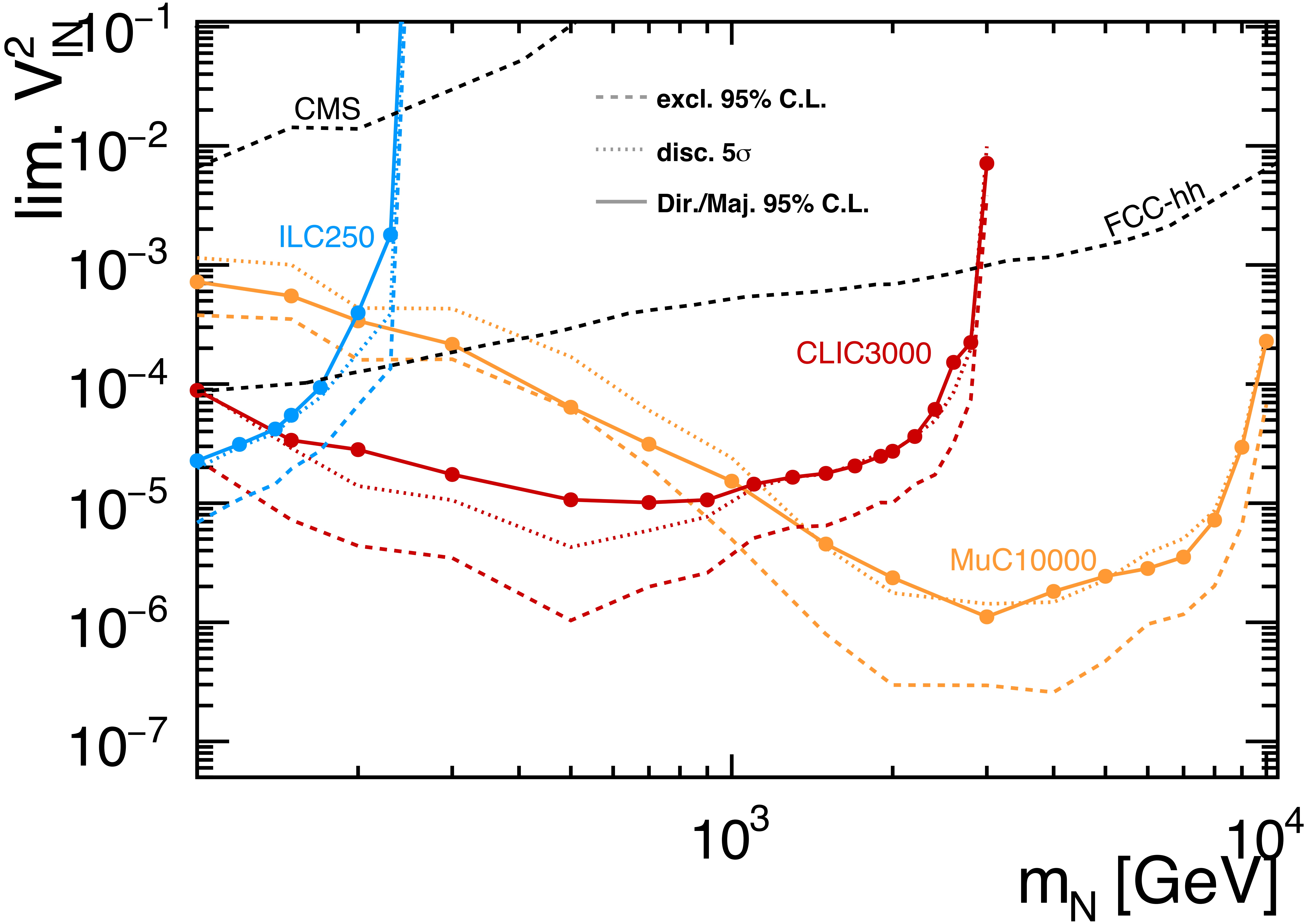

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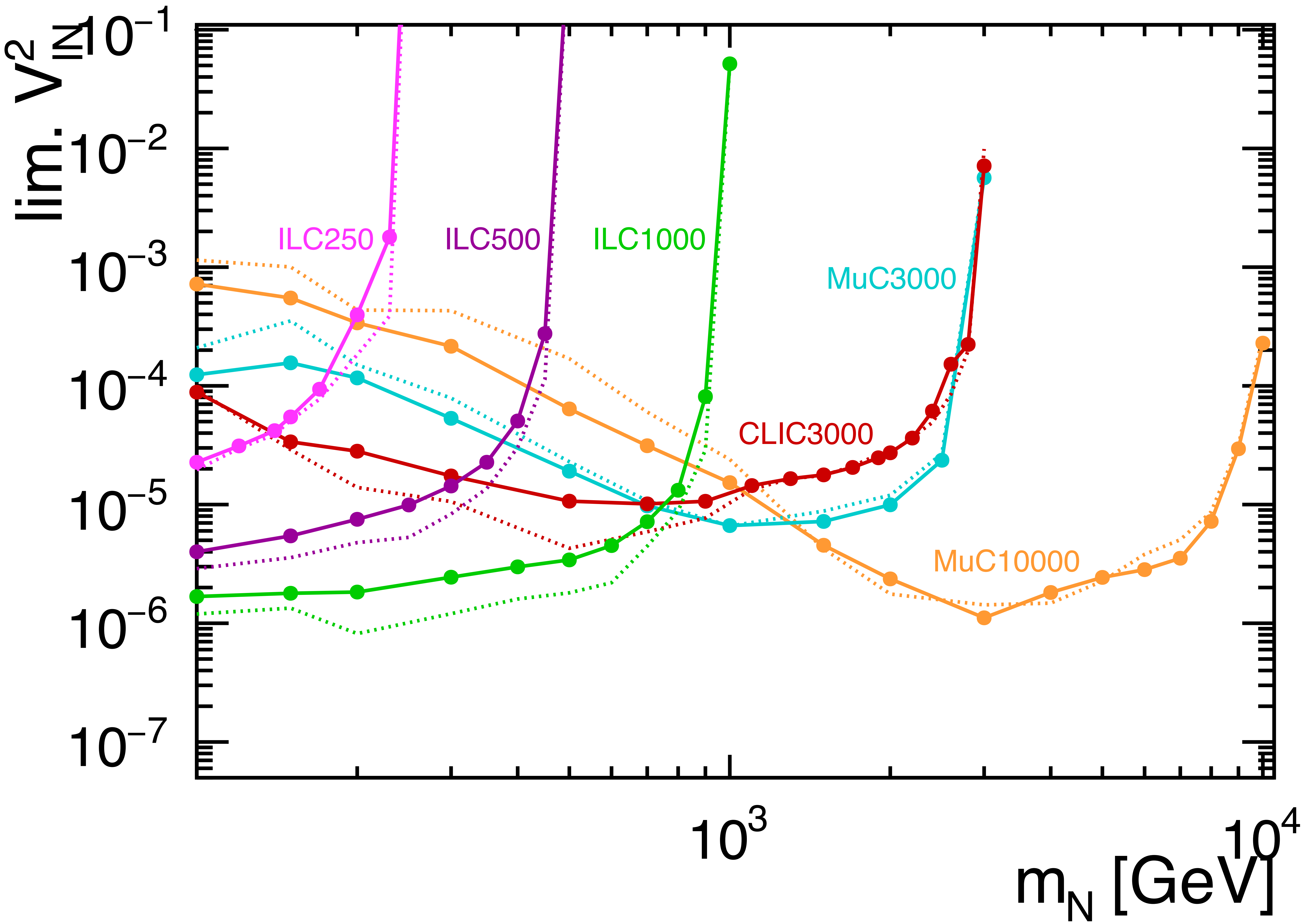
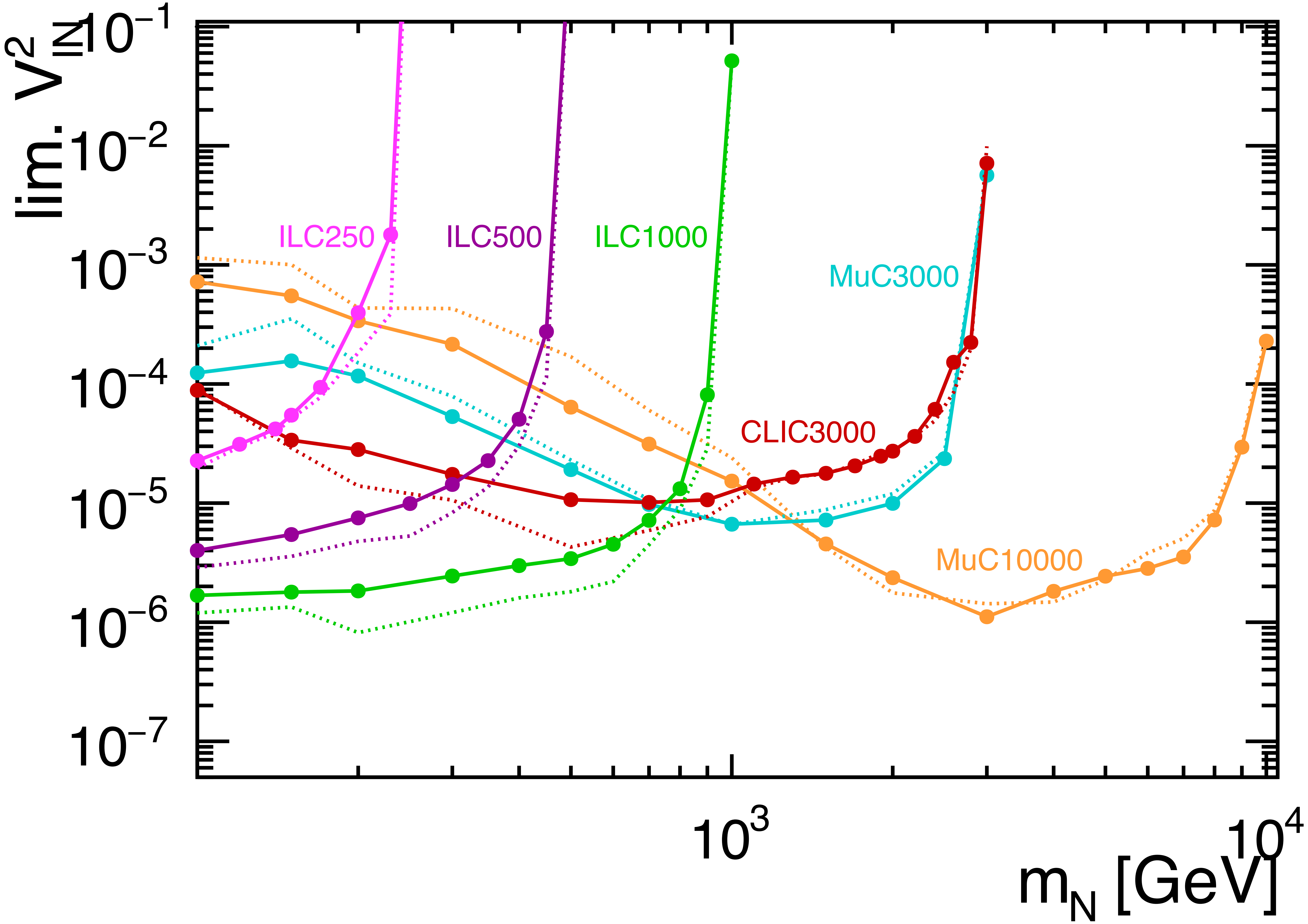


Almost immediately with a discovery a Majorana vs. Dirac discrimnation possible!

Dirac vs. Majorana discrimination



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More difficult, but possible for off-shell case!

Flavor complementarity

- Dominant *t*-channel production (*W* exchange):
- On-shell production
- Off-shell more difficult: need to scan each parameter point

$$\sigma \propto \frac{|V_{\ell_{in}N}|^2 \cdot |V_{\ell_{out}N}|^2}{|V_{eN}|^2 + |V_{\mu N}|^2}$$

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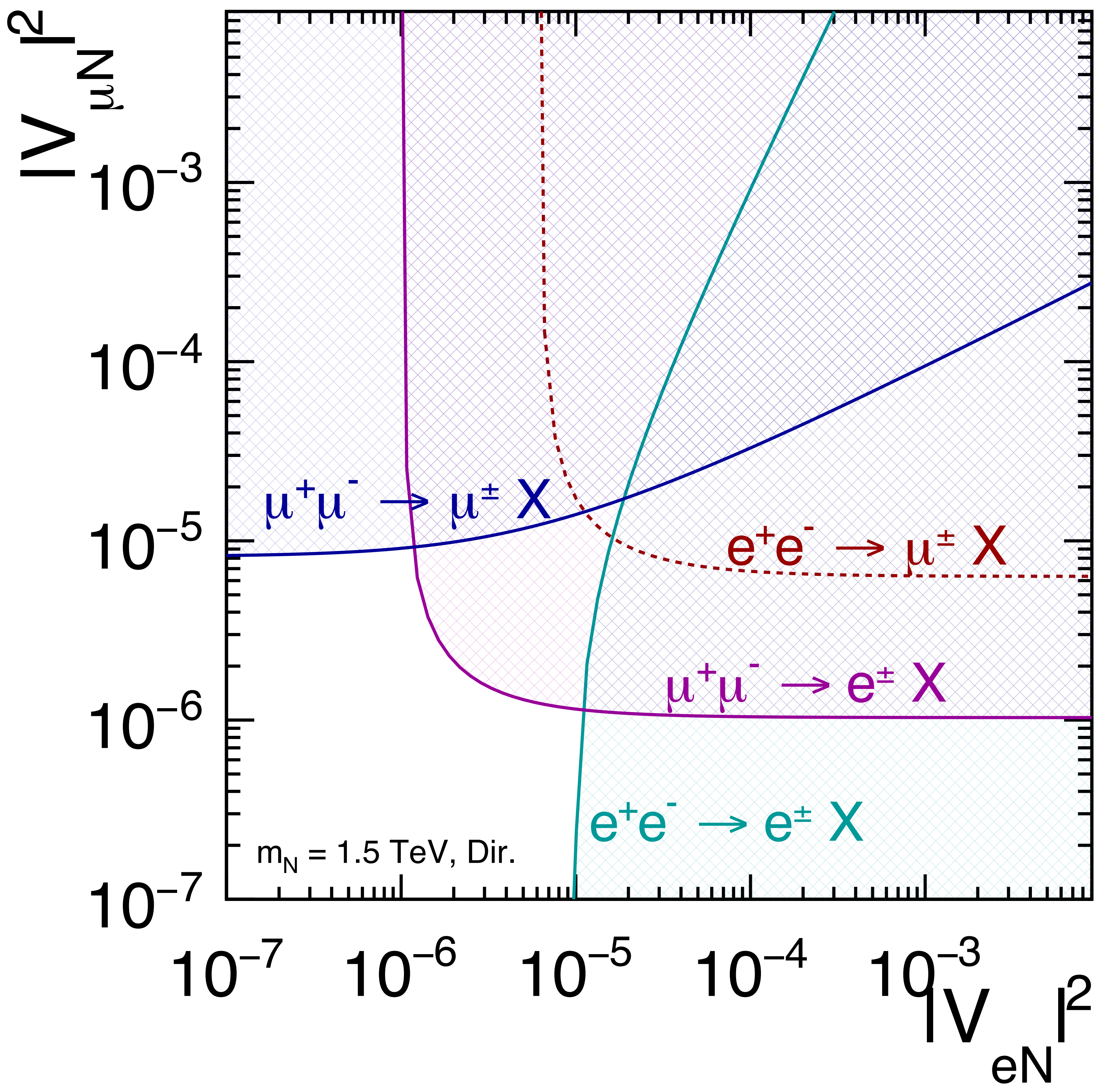

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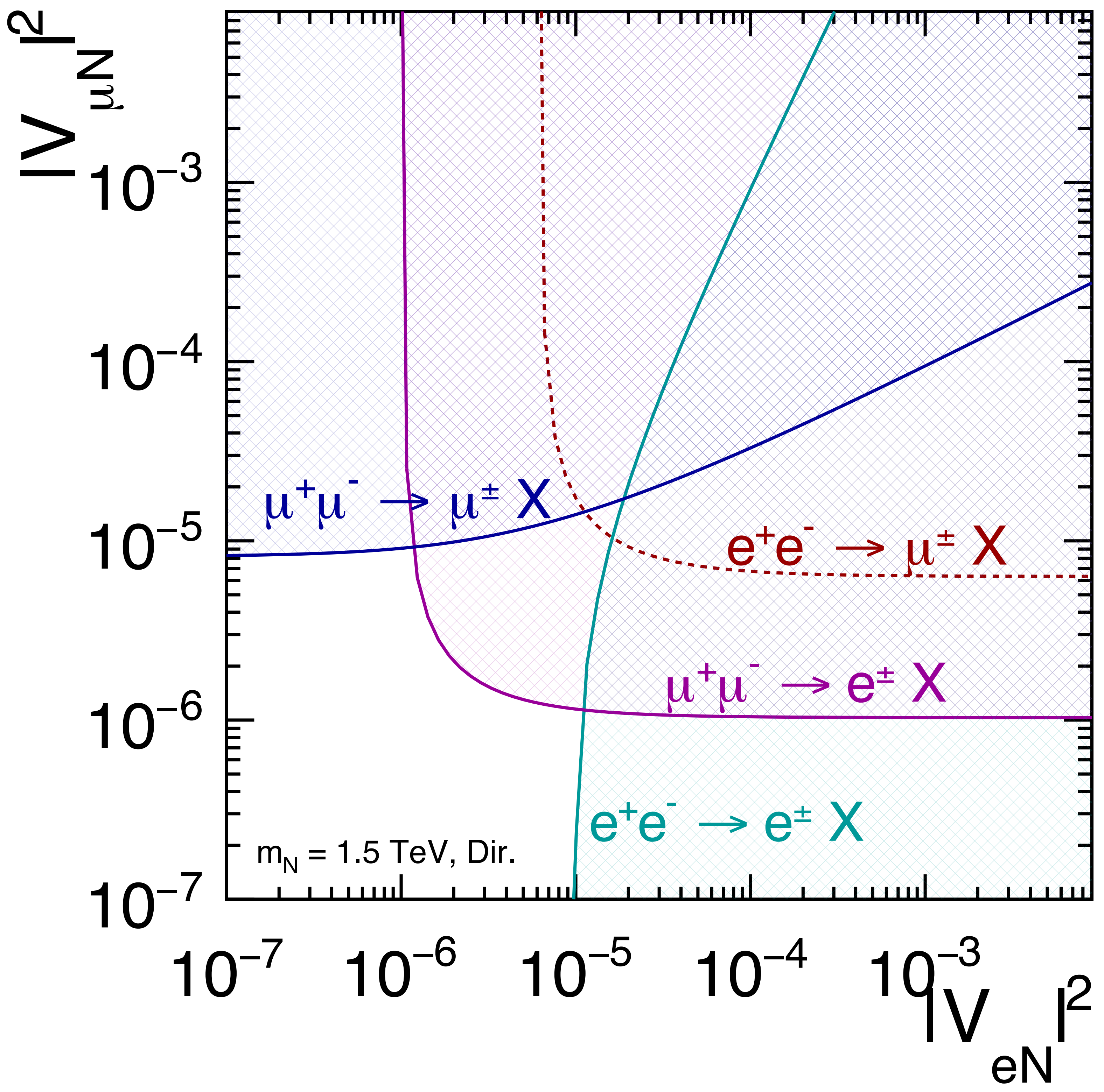

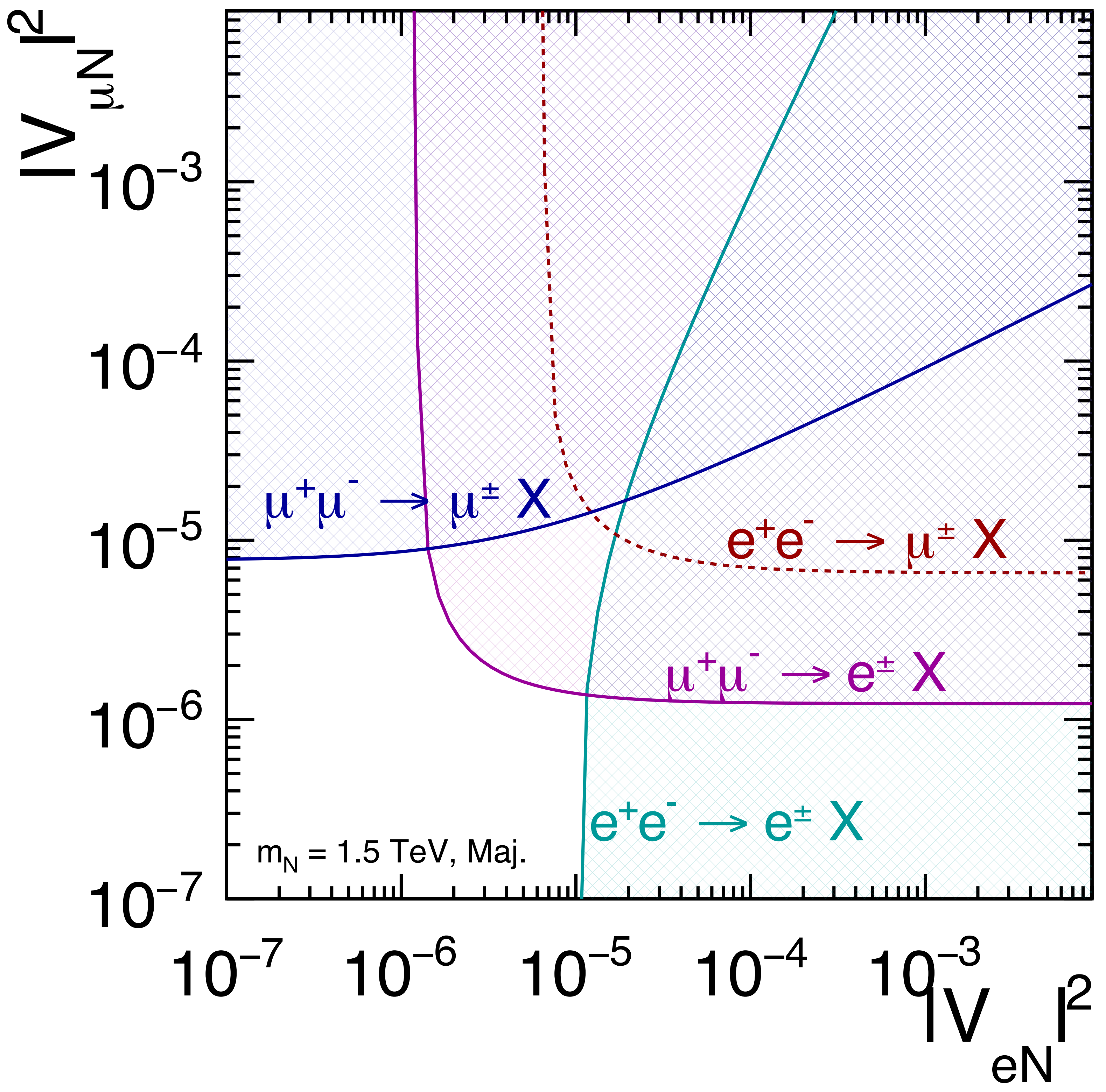

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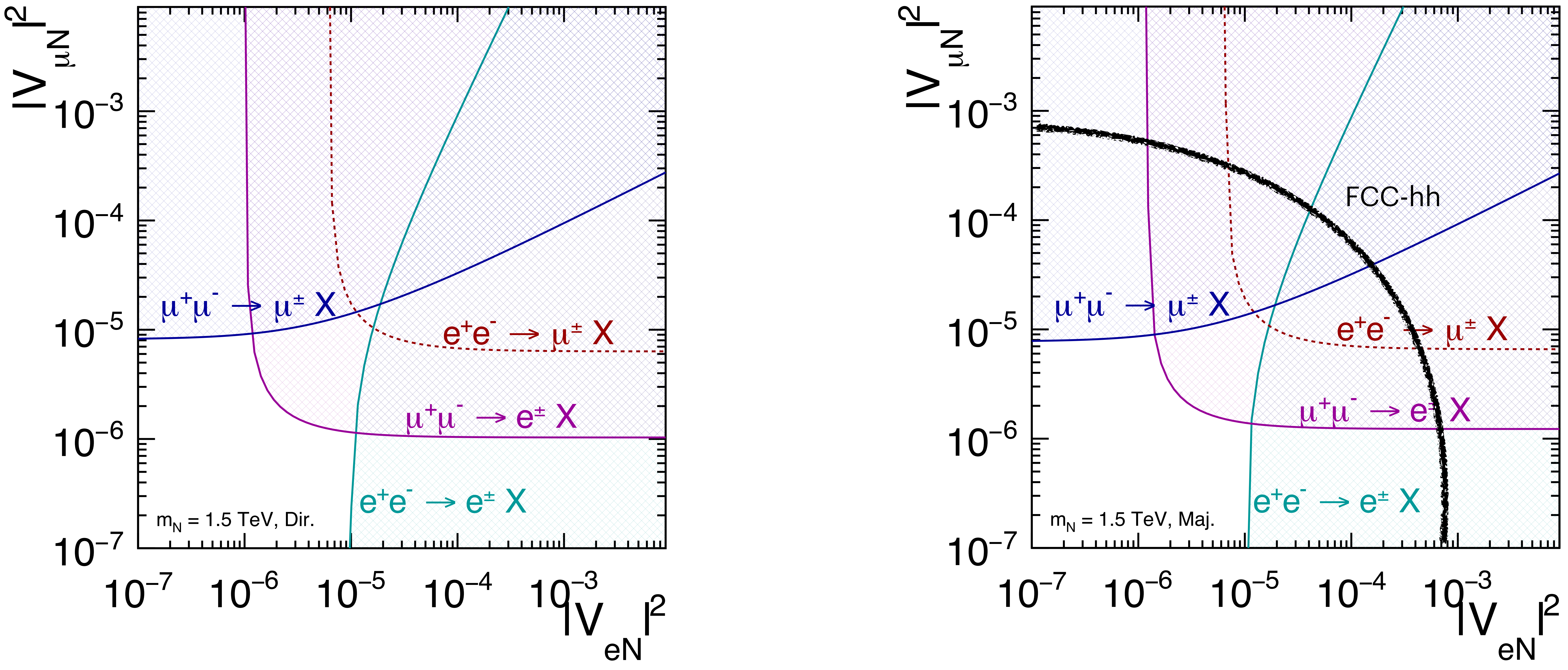
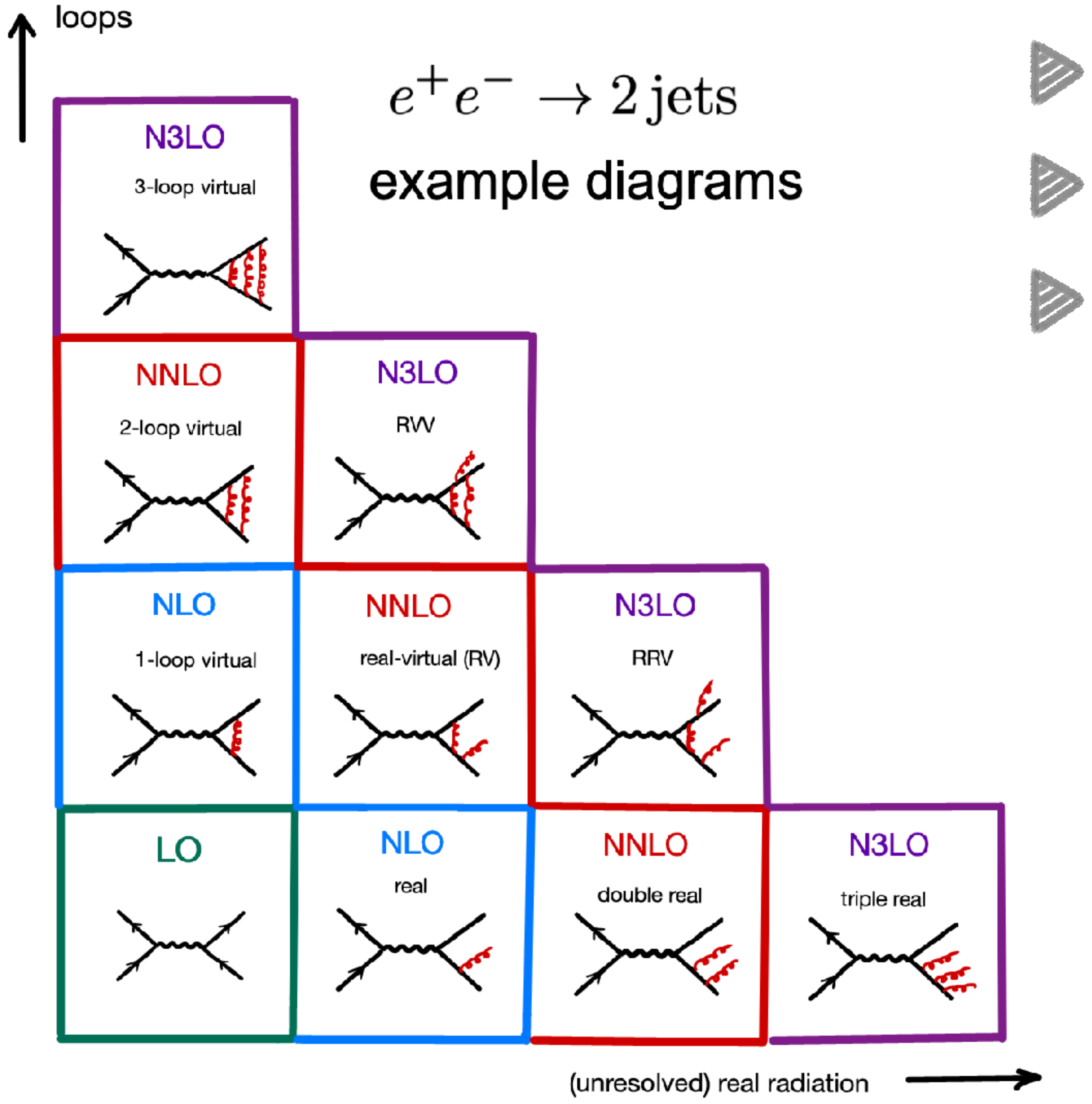


Precision simulation for muon colliders — Loops and Legs

Getty Villa, Pacific Palisades, Etruscan, 525 BC

Precision simulations for muon colliders



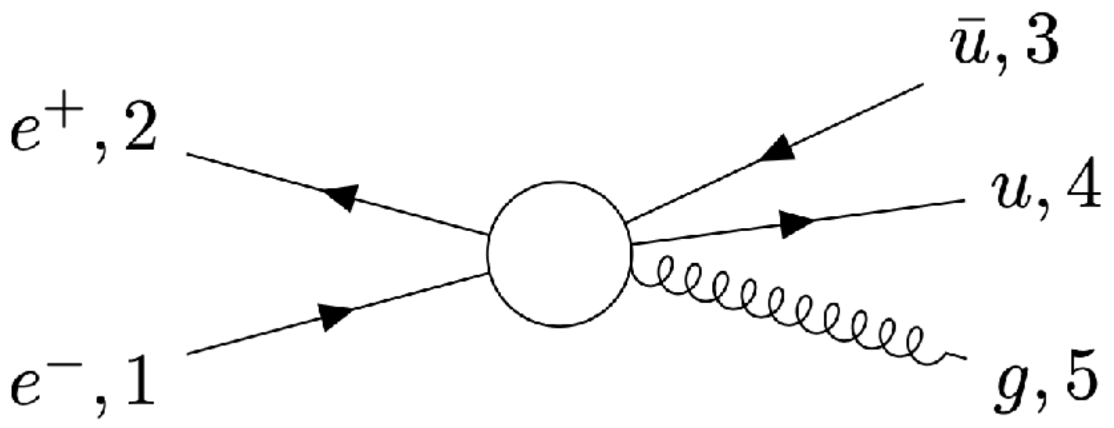
- Automation of NLO corrections in MC generators
- Signal and background samples at full SM QFT interference level:
- $\mu^+\mu^- \to 2f, 3f, 4f, 5f, 6f, [7-10f]$ @ *NLO QCD* $\oplus$ *EW* (arbitrary cuts, fully differential)
  - NLO QCD $\oplus$ EW automated: `Whizard v3.1.0+`

Phd theses: C. Weiss, 2017; Chokoufé, 2017; V. Rothe, 2021, P. Stienemeier, 2022; P. Bredt, 2022

- Subtraction scheme: FKS cf. Frixione/Kunszt/Signer, hep-ph9512328
- Phase-space partition Φ_{n+1} into collinear/soft-singular pairs

$$\sigma_{\text{NLO}} = \int d\Phi_n \mathcal{B} + \int d\Phi_{n+1} \underbrace{[\mathcal{R}(\Phi_{n+1}) - d\sigma_S(\Phi_{n+1})]}_{\text{finite by construction}} + \underbrace{\int d\Phi_n \mathcal{V} + \int d\Phi_n d\sigma_{S,\text{int}}}_{\text{finite by KLN}}$$

$\mathcal{B}$: Born, $\mathcal{R}$: Real emission, $\mathcal{V}$: Virtual



For NLO QCD possible singular regions: $(i,j) \in \{(3,5),(4,5)\}$

NLO QCD — Automation for all colliders

- Automatic differential fixed-order results (histogrammed distributions) in MCs
- Photon isolation, photon recombination, light-, b-, c-jet selection
- Covers also loop-induced processes ("LO", virtual-squared)
- One-loop provider (OLP): `OpenLoops`, `Recola`, `GoSam`
- UFO-based BSM models NLO QCD with `Whizard+GoSam`

P. Bredt, J. Braun, G. Heinrich, M. Höfer: DESY+KIT

arXiv:2104.11141

Process Vector boson (pair) plus jets	WHIZARD $\sigma_{\rm LO}$[fb]	$\sigma_{\rm NLO}$[fb]	K
$pp \to W^\pm$ *	$1.3749(8) \cdot 10^8$	$1.7696(10) \cdot 10^8$	1.29
$pp \to W^\pm j$ *	$2.046(3) \cdot 10^7$	$2.854(5) \cdot 10^7$	1.39
$pp \to W^\pm jj$	$6.856(12) \cdot 10^6$	$7.814(27) \cdot 10^6$	1.14
$pp \to W^\pm jjj$ †	$1.840(5) \cdot 10^6$	$1.978(7) \cdot 10^6$	1.07
$pp \to Z$	$4.2541(3) \cdot 10^7$	$5.4086(16) \cdot 10^7$	1.27
$pp \to Zj$	$7.215(4) \cdot 10^6$	$9.733(10) \cdot 10^6$	1.35
$pp \to Zjj$	$2.364(5) \cdot 10^6$	$2.676(7) \cdot 10^6$	1.13
$pp \to Zjjj$	$6.381(23) \cdot 10^5$	$6.85(3) \cdot 10^5$	1.07
$pp \to W^+W^-(4f)$	$7.352(10) \cdot 10^4$	$10.268(11) \cdot 10^4$	1.40
$pp \to W^+W^-j(4f)$	$2.853(7) \cdot 10^4$	$3.733(7) \cdot 10^4$	1.31
$pp \to W^+W^-jj(4f)$ *	$1.150(5) \cdot 10^4$	$1.372(6) \cdot 10^4$	1.19
$pp \to W^+W^+jj$ *	$1.506(5) \cdot 10^2$	$2.235(7) \cdot 10^2$	1.48
$pp \to W^-W^-jj$	$6.772(24) \cdot 10^1$	$9.982(28) \cdot 10^1$	1.47
$pp \to ZW^\pm$	$2.780(5) \cdot 10^4$	$4.488(4) \cdot 10^4$	1.61
$pp \to ZW^\pm j$	$1.609(4) \cdot 10^4$	$2.0940(28) \cdot 10^4$	1.30
$pp \to ZW^\pm jj$	$8.06(3) \cdot 10^3$	$9.02(4) \cdot 10^3$	1.12
$pp \to ZZ$ *	$1.0969(10) \cdot 10^4$	$1.4183(11) \cdot 10^4$	1.29
$pp \to ZZj$	$3.667(9) \cdot 10^3$	$4.807(8) \cdot 10^3$	1.31
$pp \to ZZjj$ *	$1.356(6) \cdot 10^3$	$1.684(8) \cdot 10^3$	1.24

Process Top quarks plus jets	WHIZARD $\sigma_{\rm LO}$[fb]	$\sigma_{\rm NLO}$[fb]	K
$e^+e^- \to jj$	622.737(8)	639.39(5)	1.03
$e^+e^- \to jjj$	340.6(5)	317.8(5)	0.93
$e^+e^- \to jjjj$	105.0(3)	104.2(4)	0.99
$e^+e^- \to jjjjj$	22.33(5)	24.57(7)	1.10
$e^+e^- \to t\bar{t}$	166.37(12)	174.55(20)	1.05
$e^+e^- \to t\bar{t}j$	48.12(5)	53.41(7)	1.11
$e^+e^- \to t\bar{t}jj$	8.592(19)	10.526(21)	1.23
$e^+e^- \to t\bar{t}jjj$	1.035(4)	1.405(5)	1.36
$e^+e^- \to t\bar{t}t\bar{t}$	$0.6388(8) \cdot 10^{-3}$	$1.1922(11) \cdot 10^{-3}$	1.87
$e^+e^- \to t\bar{t}t\bar{t}j$	$2.673(7) \cdot 10^{-5}$	$5.251(11) \cdot 10^{-5}$	1.96

Top quarks plus boso	$\sigma_{\rm LO}$[fb]	$\sigma_{\rm NLO}$[fb]	K
$e^+e^- \to t\bar{t}H$	2.020(3)	1.912(3)	0.95
$e^+e^- \to t\bar{t}Hj$	$2.536(4) \cdot 10^{-1}$	$2.657(4) \cdot 10^{-1}$	1.05
$e^+e^- \to t\bar{t}Hjj$	$2.646(8) \cdot 10^{-2}$	$3.123(9) \cdot 10^{-2}$	1.18
$e^+e^- \to t\bar{t}Z$	4.638(3)	4.937(3)	1.06
$e^+e^- \to t\bar{t}Zj$	$6.027(9) \cdot 10^{-1}$	$6.921(11) \cdot 10^{-1}$	1.15
$e^+e^- \to t\bar{t}Zjj$	$6.436(21) \cdot 10^{-2}$	$8.241(29) \cdot 10^{-2}$	1.28
$e^+e^- \to t\bar{t}W^\pm jj$	$2.387(8) \cdot 10^{-4}$	$3.716(10) \cdot 10^{-4}$	1.56
$e^+e^- \to t\bar{t}HZ$	$3.623(19) \cdot 10^{-2}$	$3.584(19) \cdot 10^{-2}$	0.99
$e^+e^- \to t\bar{t}ZZ$	$3.788(6) \cdot 10^{-2}$	$4.032(7) \cdot 10^{-2}$	1.06
$e^+e^- \to t\bar{t}HH$	$1.3650(15) \cdot 10^{-2}$	$1.2168(16) \cdot 10^{-2}$	0.89
$e^+e^- \to t\bar{t}W^+W^-$	$1.3672(21) \cdot 10^{-1}$	$1.5385(22) \cdot 10^{-1}$	1.13

NLO QCD — Automation for all colliders

- Automatic differential fixed-order results (histogrammed distributions) in MCs
- Photon isolation, photon recombination, light-, b-, c-jet selection
- Covers also loop-induced processes ("LO", virtual-squared)
- One-loop provider (OLP): `OpenLoops`, `Recola`, `GoSam`
- UFO-based BSM models NLO QCD with `Whizard+GoSam`

P. Bredt, J. Braun, G. Heinrich, M. Höfer: DESY+KIT

arXiv:2104.11141

Process	WHIZARD σ_{LO} [fb]	WHIZARD σ_{NLO} [fb]
$e^+e^- \to jj$	622.737(8)	639.39(5)
$e^+e^- \to jjj$	340.6(5)	317.8(5)
$e^+e^- \to jjjj$	105.0(3)	104.2(4)
$e^+e^- \to jjjjj$	22.33(5)	24.57(7)
$e^+e^- \to jjjjjj$	3.583(17)	4.46(4)

Process Vector boson (pair) plus jets	WHIZARD σ_{LO}[fb]	WHIZARD σ_{NLO}[fb]	K
$pp \to W^\pm$ *	$1.3749(8) \cdot 10^8$	$1.7696(10) \cdot 10^8$	1.29
$pp \to W^\pm j$ *	$2.046(3) \cdot 10^7$	$2.854(5) \cdot 10^7$	1.39
$pp \to W^\pm jj$	$6.856(12) \cdot 10^6$	$7.814(27) \cdot 10^6$	1.14
$pp \to W^\pm jjj$ †	$1.840(5) \cdot 10^6$	$1.978(7) \cdot 10^6$	1.07
$pp \to Z$	$4.2541(3) \cdot 10^7$	$5.4086(16) \cdot 10^7$	1.27
$pp \to Zj$	$7.215(4) \cdot 10^6$	$9.733(10) \cdot 10^6$	1.35
$pp \to Zjj$	$2.364(5) \cdot 10^6$	$2.676(7) \cdot 10^6$	1.13
$pp \to Zjjj$	$6.381(23) \cdot 10^5$	$6.85(3) \cdot 10^5$	1.07
$pp \to W^+W^-(4f)$	$7.352(10) \cdot 10^4$	$10.268(11) \cdot 10^4$	1.40
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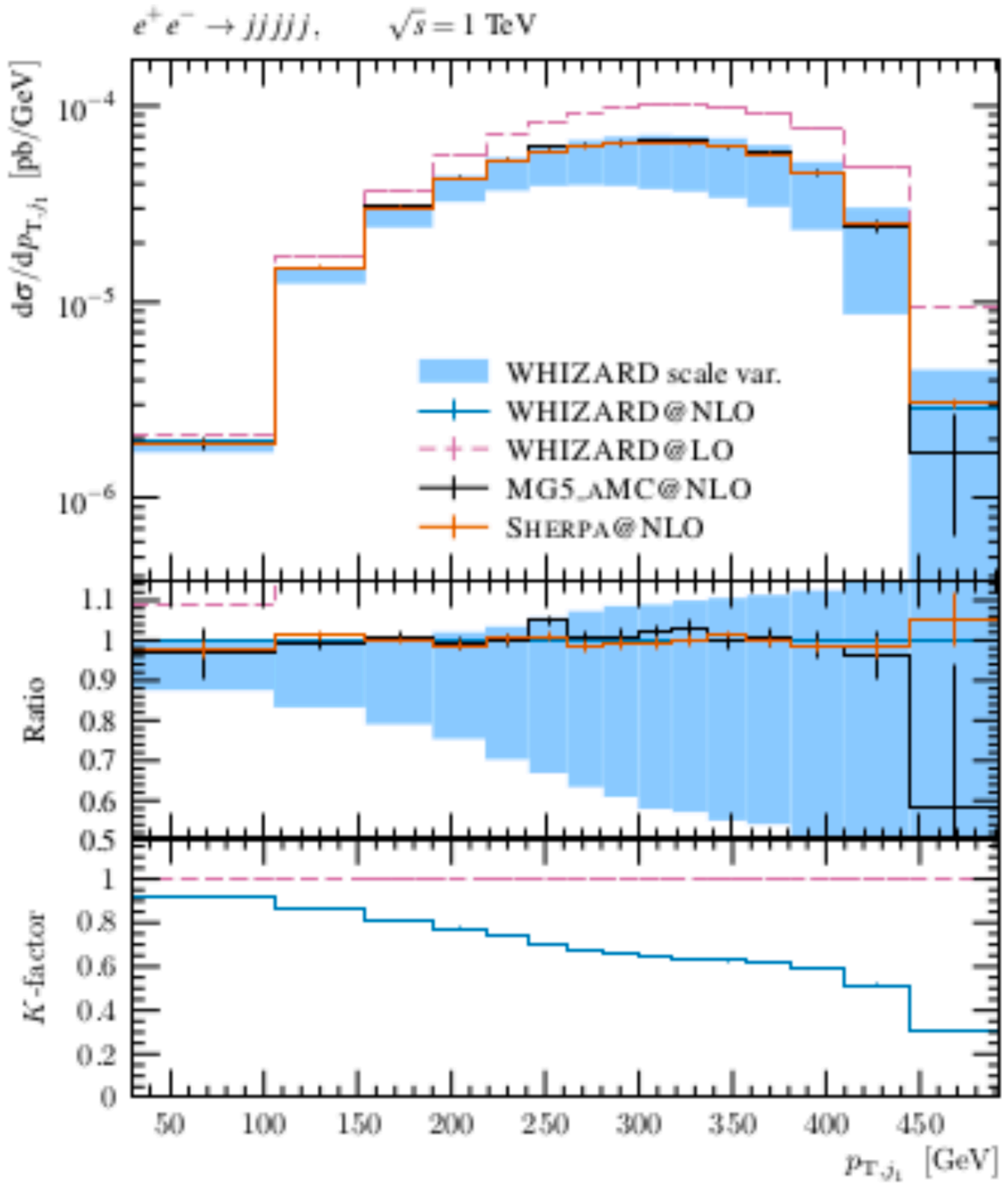
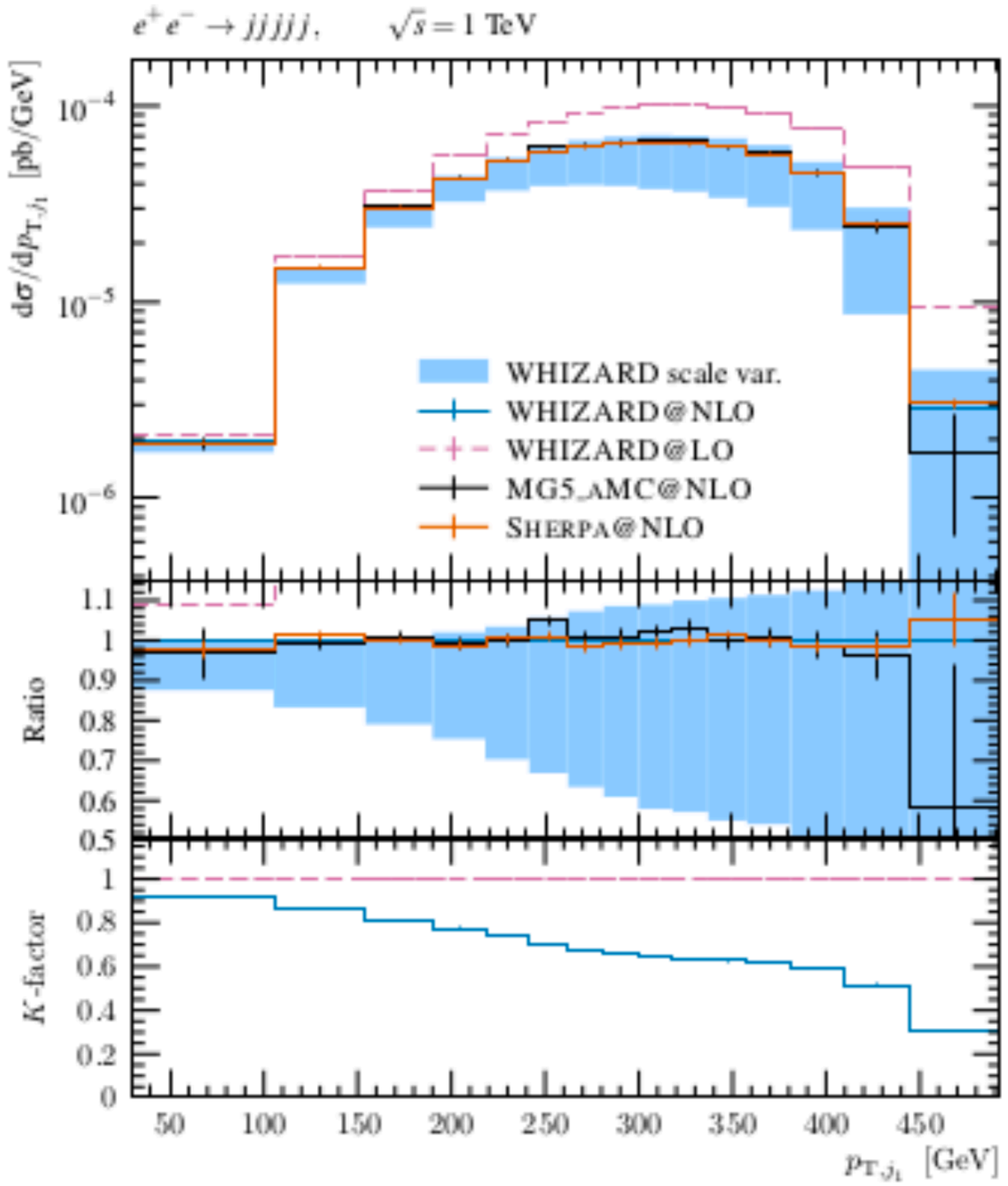
Differential distributions NLO QCD

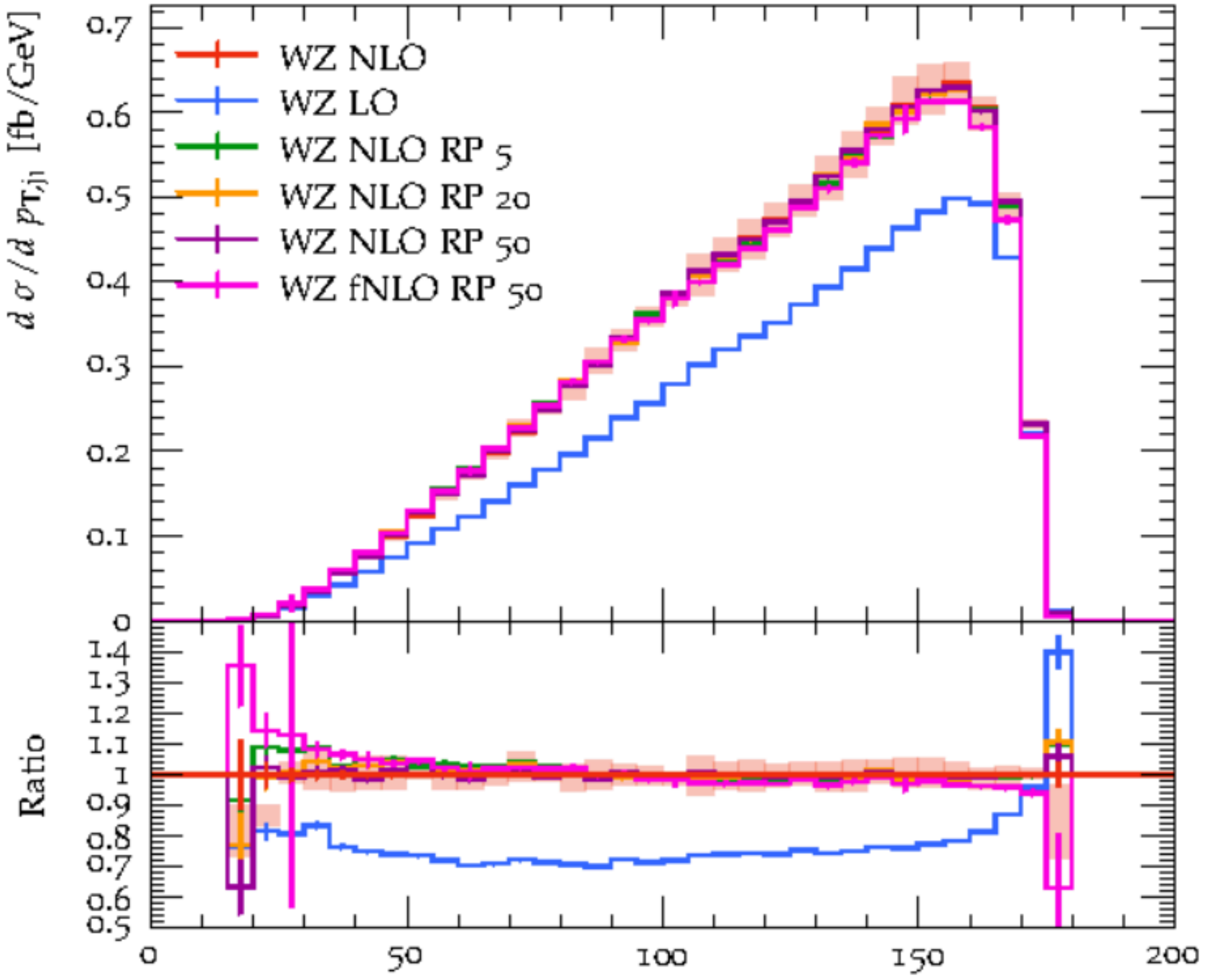


Differential distributions NLO QCD



- Matching between NLO real emission from hard ME and parton shower (PS)
- `Whizard` uses the POWHEG scheme
- Special cases: Massive/massless emitters, back-to-pack kinematics, running α_s
- Real partitioning of phase space into singular and finite regions
- Resonance-aware subtraction: Intermediate resonances handled
- At the moment: NLO QCD; straightforward (?) QED/EW generalization

Differential distributions NLO QCD



ILC 500: $e^+e^- \to t\bar{t}j$

$$\mu_R = {}^{H_T}/_2 \quad \text{with} \quad H_T := \sum_i \sqrt{p_{T,i}^2 + m_i^2}$$

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- `Whizard` uses the POWHEG scheme
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Automation of NLO EW corrections

$\alpha_s \sim 0.1 \qquad \alpha \sim 0.01$

- EW splittings and singular regions
- Careful: EW scheme & renormalization scheme & complex mass scheme & photon definition (off-shell vs. on-shell)
- Photon recombination with fermions for IR-safe observables

EW renorm. schemes & photons entering at Born level

$Q_\gamma^2 \to 0$	$Q_\gamma^2 \sim$ EW scale
on-shell photons no γ splittings	*off-shell* photons $\gamma^* \to f\bar{f}$
$\alpha(0)$	$\alpha\vert_{G_\mu}$, $\alpha(M_Z)$
$\left[\frac{\delta\alpha(0)}{\alpha(0)} + \delta Z_{AA}\right]_{\text{light}} = 0$	$\left[\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} + \delta Z_{AA}\right]_{\text{light}} + \delta Z_{\gamma,\text{PDF}}$ $\to$ finite overall photon factor $\neq 0$

with photon virtuality Q_γ^2

$\to$ α coupling constant, renormalization factors

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$\alpha_s \sim 0.1 \qquad \alpha \sim 0.01$

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process $pp \to$	α^n	MG5_aMC@NLO $\sigma_{\text{NLO}}^{\text{tot}}$ [pb] [1804.10017]	WHIZARD $\sigma_{\text{NLO}}^{\text{tot}}$ [pb]	δ [%]	$\sigma_{\text{LO}}^{\text{sig}}$	$\sigma_{\text{NLO}}^{\text{sig}}$
$e^+\nu_e$	α^2	5200.5(8)	5199.4(4)	–0.73	0.81	1.24
e^+e^-	α^2	749.8(1)	749.8(1)	–0.50	0.082	0.004
$e^+\nu_e\mu^-\bar{\nu}_\mu$	α^4	0.52794(9)	0.52816(9)	+3.69	1.27	1.69
$e^+e^-\mu^+\mu^-$	α^4	0.012083(3)	0.012078(3)	–5.25	0.68	1.26
$He^+\nu_e$	α^3	0.064740(17)	0.064763(6)	–4.04	0.06	1.24
He^+e^-	α^3	0.013699(2)	0.013699(1)	–5.86	0.03	0.32
Hjj	α^3	2.7058(4)	2.7056(6)	–4.23	0.67	0.27
tj	α^2	105.40(1)	105.38(1)	–0.72	0.20	0.74

$$\delta \equiv \frac{\sigma_{\text{NLO}}^{\text{tot}} - \sigma_{\text{LO}}^{\text{tot}}}{\sigma_{\text{LO}}^{\text{tot}}} \qquad \sigma^{\text{sig}} \equiv \frac{|\sigma_{\text{WHIZARD}}^{\text{tot}} - \sigma_{\text{MG5}}^{\text{tot}}|}{\sqrt{\Delta_{\text{err,WHIZARD}}^2 + \Delta_{\text{err,MG5}}^2}}$$

LHC setup (Run II): $\sqrt{s} = 13$ TeV $\quad \mu_R = \mu_F = \frac{1}{2}\sum_i \sqrt{p_{T,i}^2 + m_i^2}$ EW scheme: G_μ CMS

PDF set: LUXqed_plus_PDF4LHC15_nnlo_100 cuts from ref. [1804.10017]

Automation of NLO EW corrections

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- EW splittings and singular regions
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Cross-validation of WHIZARD and MUNICH/MATRIX orig. ref. [Kallweit *et. al.*, 1412.5157]

process $pp \to$	MUNICH(CS) +OpenLoops $\sigma^{\text{tot}}_{\text{NLO}}$ [fb]	WHIZARD +OpenLoops $\sigma^{\text{tot}}_{\text{NLO}}$ [fb]	δ [%]	dev [%]	σ^{sig}
ZZ	$1.05729(1) \cdot 10^4$	$1.05729(11) \cdot 10^4$	-4.20	0.0001	0.01
W^+Z	$1.71505(2) \cdot 10^4$	$1.71507(2) \cdot 10^4$	-0.15	0.001	0.88
W^-Z	$1.08576(1) \cdot 10^4$	$1.08574(1) \cdot 10^4$	$+0.07$	0.001	0.90
W^+W^-	$7.93106(7) \cdot 10^4$	$7.93087(21) \cdot 10^4$	$+4.55$	0.002	0.89
ZH	$6.18523(6) \cdot 10^2$	$6.18533(6) \cdot 10^2$	-5.29	0.002	1.17
W^+H	$7.18070(7) \cdot 10^2$	$7.18072(9) \cdot 10^2$	-2.31	0.0003	0.18
W^-H	$4.59289(4) \cdot 10^2$	$4.59299(5) \cdot 10^2$	-2.15	0.002	1.62
ZZZ	$9.7429(2) \cdot 10^0$	$9.7417(11) \cdot 10^0$	-9.47	0.012	1.01
W^+W^-Z	$1.08288(2) \cdot 10^2$	$1.08293(10) \cdot 10^2$	$+7.67$	0.004	0.45
W^+ZZ	$2.0188(4) \cdot 10^1$	$2.0188(23) \cdot 10^1$	$+1.58$	0.0001	0.01
W^-ZZ	$1.09844(2) \cdot 10^1$	$1.09838(12) \cdot 10^1$	$+3.09$	0.006	0.51
$W^+W^-W^+$	$8.7979(2) \cdot 10^1$	$8.7991(15) \cdot 10^1$	$+6.18$	0.014	0.79
$W^+W^-W^-$	$4.9447(1) \cdot 10^1$	$4.9441(2) \cdot 10^1$	$+7.13$	0.013	2.52
ZZH	$1.91607(2) \cdot 10^0$	$1.91614(18) \cdot 10^0$	-8.78	0.004	0.39
W^+ZH	$2.48068(2) \cdot 10^0$	$2.48095(28) \cdot 10^0$	$+1.64$	0.011	0.96
W^-ZH	$1.34001(1) \cdot 10^0$	$1.34016(15) \cdot 10^0$	$+2.51$	0.011	1.02
W^+W^-H	$9.7012(2) \cdot 10^0$	$9.700(2) \cdot 10^0$	$+9.83$	0.014	0.75
ZHH	$2.39350(2) \cdot 10^{-1}$	$2.39337(32) \cdot 10^{-1}$	-11.06	0.005	0.41
W^+HH	$2.44794(2) \cdot 10^{-1}$	$2.44776(24) \cdot 10^{-1}$	-12.04	0.007	0.74
W^-HH	$1.33525(1) \cdot 10^{-1}$	$1.33471(19) \cdot 10^{-1}$	-11.53	0.041	2.80

LHC setup (Run II), $\delta \equiv (\sigma^{\text{tot}}_{\text{NLO}} - \sigma^{\text{tot}}_{\text{LO}})/\sigma^{\text{tot}}_{\text{LO}}$, $\text{dev} \equiv |\sigma^{\text{tot}}_{\text{WHIZARD}} - \sigma^{\text{tot}}_{\text{MUNICH}}|/\sigma^{\text{tot}}_{\text{WHIZARD}}$

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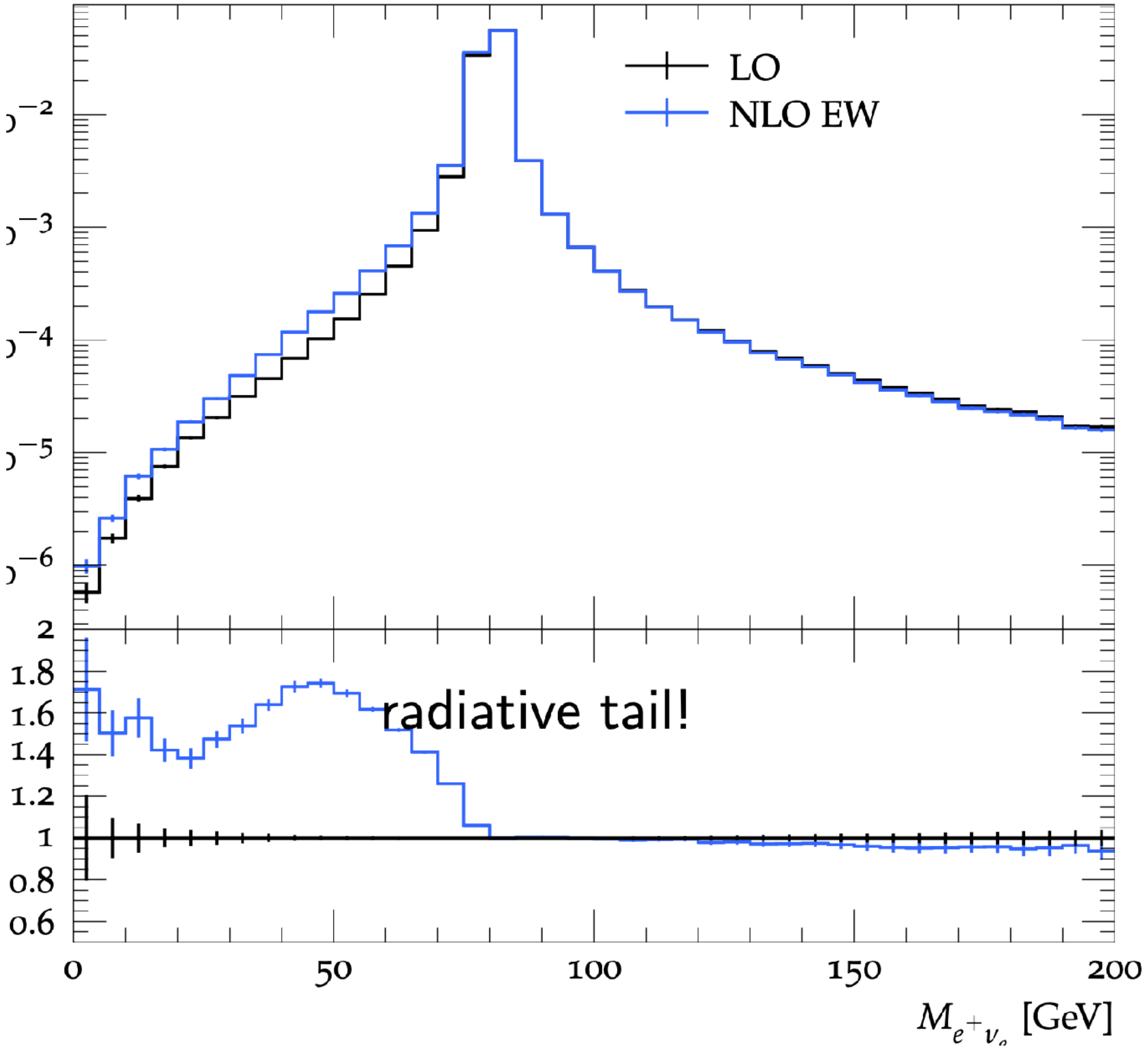
Fixed order differential distributions

for $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$ at NLO EW



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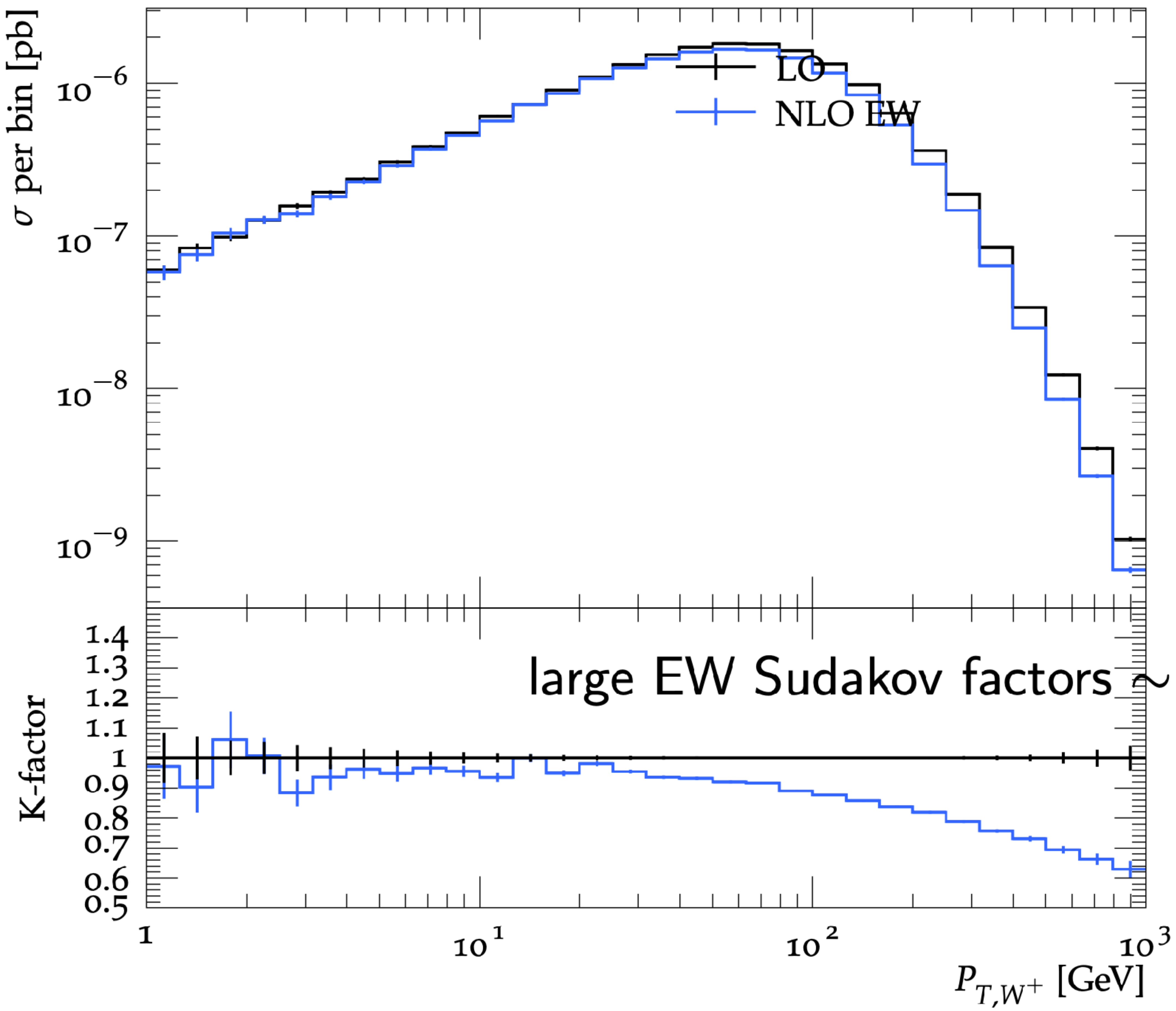
P_{T,W^+} to $pp \to HHW^+$

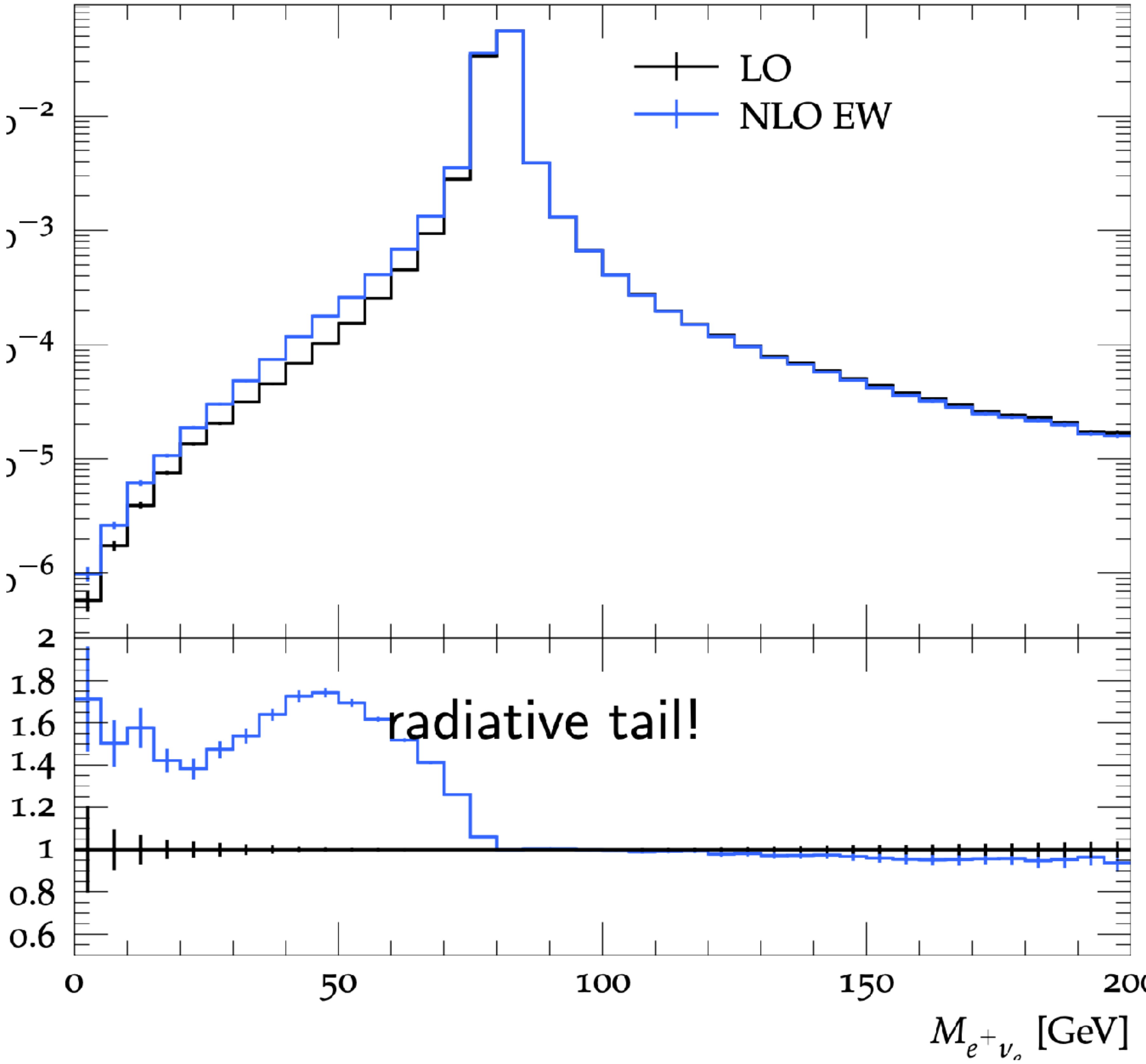


large EW Sudakov factors $\sim \alpha^n \log^{2n} \frac{Q^2}{M_W^2}$!

Fixed order differential distributions for $pp \to HHW^+$ at NLO EW

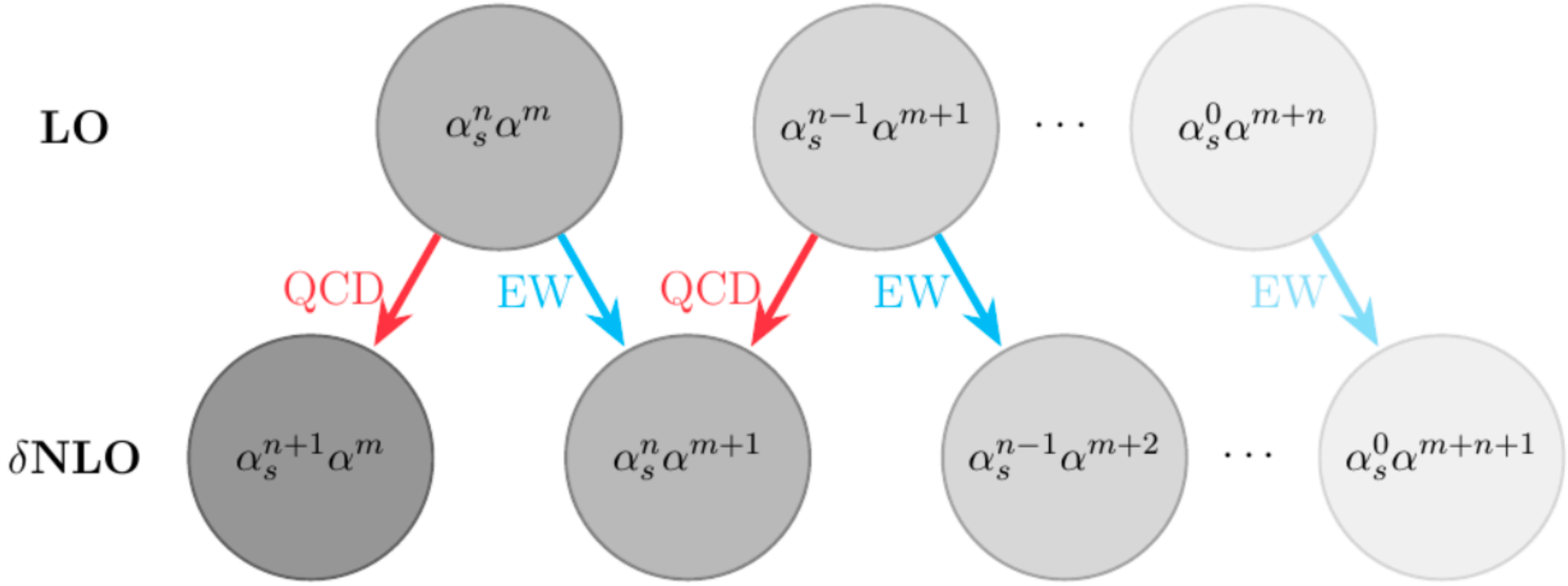
Fixed order differential distributions

for $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$ at NLO EW



Mixed NLO QCD/EW corrections

- Interfering correction type for $\mathcal{O}(\alpha_s^n)$ for $n > 1$:



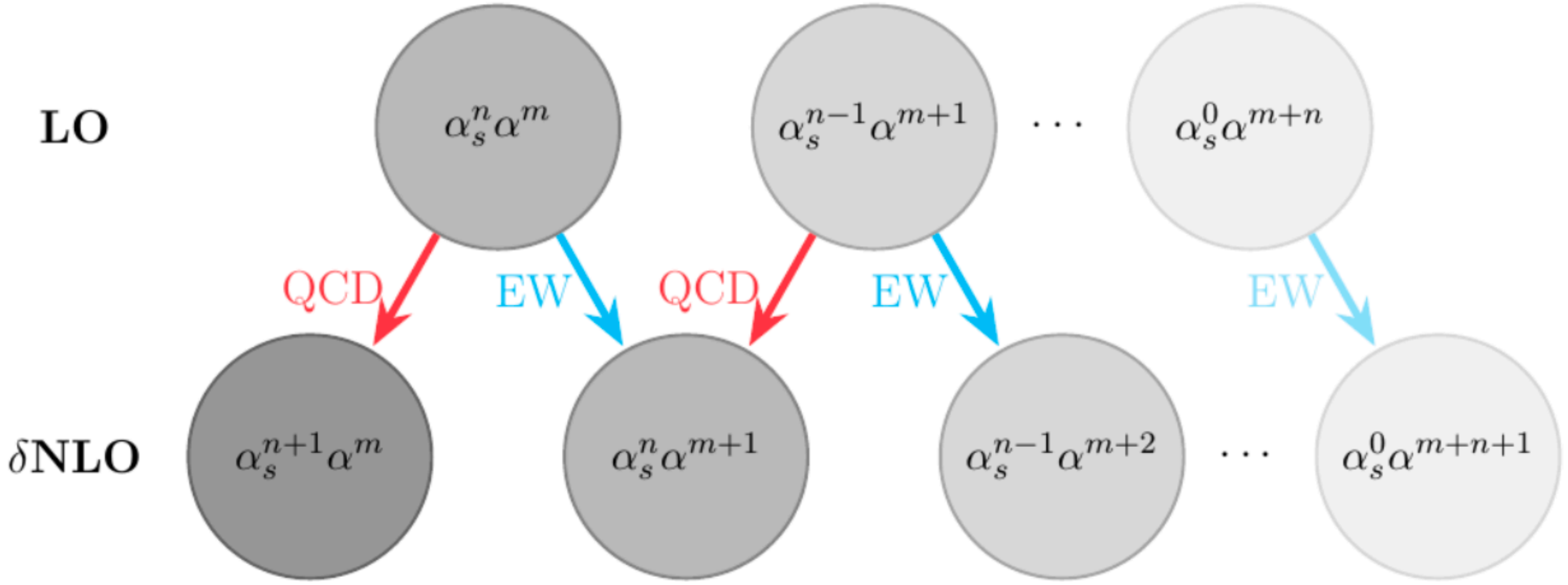
- Treatment of photons & gluons in protons/jets on a democratic basis
- Example: $pp \to Zj$ at $\mathcal{O}(\alpha\alpha_s)$: NLO EW contributions from $pp \to Zg\gamma$ at $\mathcal{O}(\alpha^2\alpha)$ needs mass singularity cancellations from

$[\mathcal{B}(q\bar{q} \to Zg)$ at $\mathcal{O}(\alpha\alpha_s)]$ × [QED splitting]

$[\mathcal{B}(q\bar{q} \to Z\gamma)$ at $\mathcal{O}(\alpha^2)$] × [QCD splitting]

Mixed NLO QCD/EW corrections

- Interfering correction type for $\mathcal{O}(\alpha_s^n)$ for $n > 1$:



- Treatment of photons & gluons in protons/jets on a democratic basis
- Example: $pp \to Zj$ at $\mathcal{O}(\alpha\alpha_s)$: NLO EW contributions from $pp \to Zg\gamma$ at $\mathcal{O}(\alpha^2\alpha)$ needs mass singularity cancellations from

$[\mathscr{B}(q\bar{q} \to Zg)$ at $\mathcal{O}(\alpha\alpha_s)]$ × [QED splitting]

$[\mathscr{B}(q\bar{q} \to Z\gamma)$ at $\mathcal{O}(\alpha^2)$] × [QCD splitting]

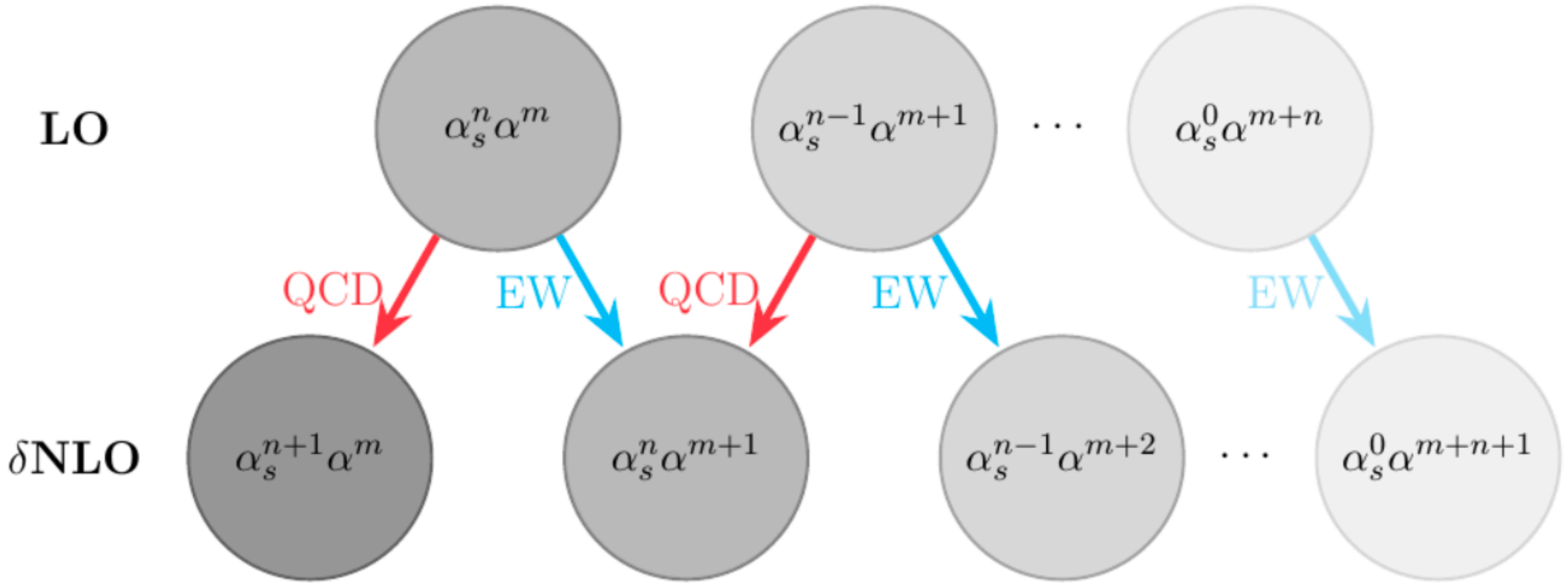
Comparison with `MG5_aMC@NLO` for $pp \to e^+\nu_e j$ and $pp \to e^+e^-j$ at NLO EW

process $pp \to Xj$	$\alpha_s^n \alpha^m$	`MG5_aMC@NLO` $\sigma^{\rm tot}_{\rm LO}$ [pb]	`MG5_aMC@NLO` $\sigma^{\rm tot}_{\rm NLO}$ [pb]	`WHIZARD+OpenLoops` $\sigma^{\rm tot}_{\rm LO}$ [pb]	`WHIZARD+OpenLoops` $\sigma^{\rm tot}_{\rm NLO}$ [pb]	δ [%]	$\sigma^{\rm sig}$ LO/NLO
$e^+\nu_e j$	$\alpha_s\alpha^2$	914.81(6)	904.75(8)	914.74(7)	904.59(7)	−1.11	0.8/1.5
e^+e^-j	$\alpha_s\alpha^2$	150.59(1)	149.09(2)	150.59(1)	149.08(2)	−1.00	0.05/0.4

LHC-setup (Run II), cuts with photon recombination **and** jet clustering

Mixed NLO QCD/EW corrections

- Interfering correction type for $\mathcal{O}(\alpha_s^n)$ for $n > 1$:



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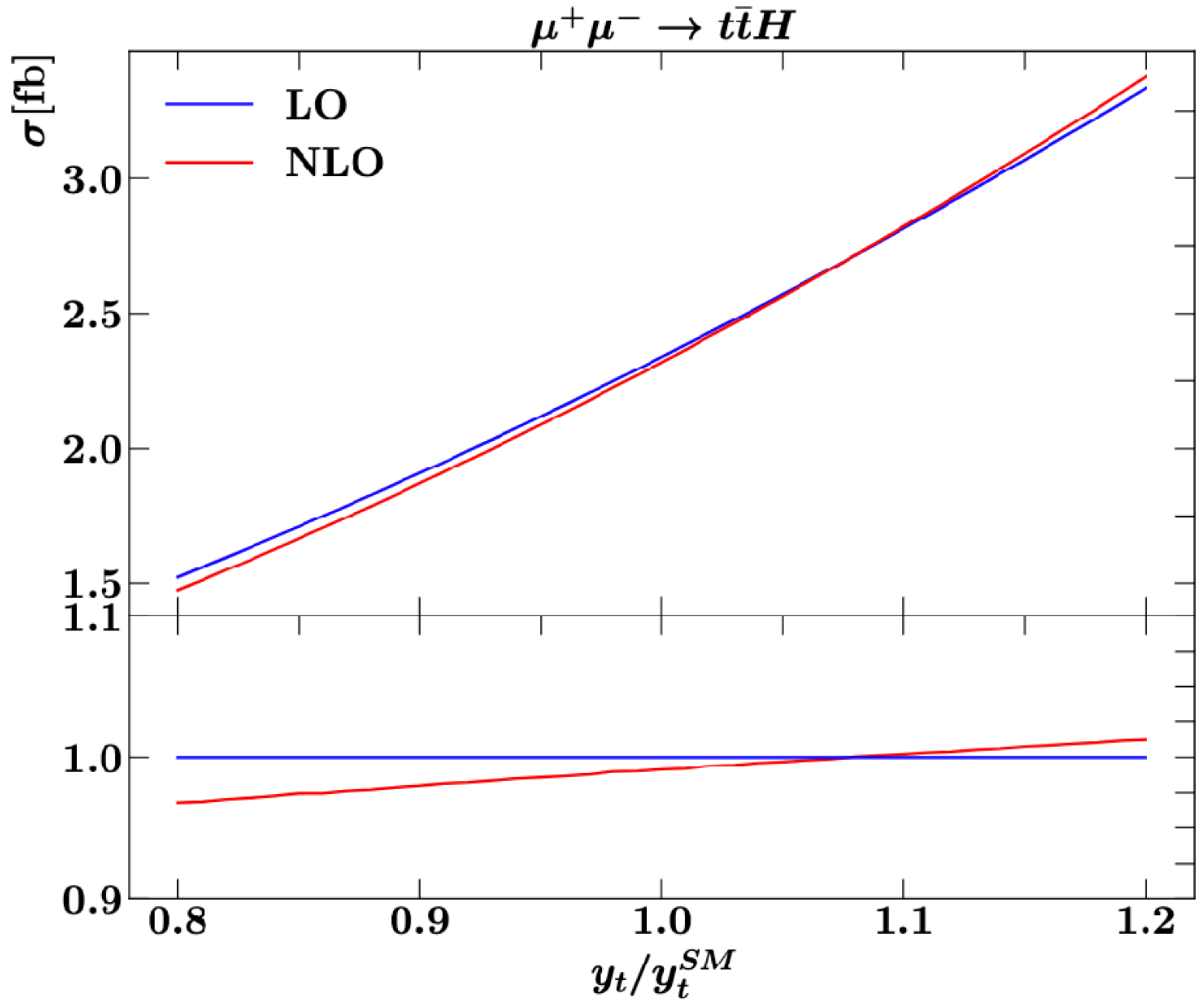
Cross-validation with MUNICH/MATRIX using OpenLoops for $pp \to t\bar{t}$ and $pp \to t\bar{t} + W^\pm/Z/H$ with complete NLO SM corrections, e. g.

$pp \to t\bar{t}W^+$	$\alpha_s^n \alpha^m$	σ^{tot} [fb] MUNICH$_{\text{(CS)}}$	WHIZARD	σ^{sig} / dev MUNICH$_{\text{(CS)}}$-WHIZARD
LO_{21}	$\alpha_s^2\alpha$	$2.411403(1) \cdot 10^2$	$2.4114(1) \cdot 10^2$	0.72 / 0.003%
LO_{12}	$\alpha_s\alpha^2$	0.000	0.000	0.00 / 0.000%
LO_{03}	α^3	$2.31909(1) \cdot 10^0$	$2.3193(1) \cdot 10^0$	1.76 / 0.009%
δNLO_{31}	$\alpha_s^3\alpha$	$1.18993(2) \cdot 10^2$	$1.1905(5) \cdot 10^2$	1.06 / 0.048%
δNLO_{22}	$\alpha_s^2\alpha^2$	$-1.09511(9) \cdot 10^1$	$-1.0947(3) \cdot 10^1$	1.13 / 0.035%
δNLO_{13}	$\alpha_s\alpha^3$	$2.93251(3) \cdot 10^1$	$2.9334(8) \cdot 10^1$	1.14 / 0.030%
δNLO_{04}	α^4	$5.759(3) \cdot 10^{-2}$	$5.756(4) \cdot 10^{-2}$	0.58 / 0.049%

NLO correctios for muon colliders

$\sqrt{s}$ [GeV]	MCSANCee[37] σ^{tot}_{LO} [fb]	σ^{tot}_{NLO} [fb]	WHIZARD+RECOLA σ^{tot}_{LO} [fb]	σ^{tot}_{NLO} [fb]	δ_{EW} [%]	σ^{sig} (LO/NLO)
250	225.59(1)	206.77(1)	225.60(1)	207.0(1)	−8.25	0.4/2.1
500	53.74(1)	62.42(1)	53.74(3)	62.41(2)	+16.14	0.2/0.3
1000	12.05(1)	14.56(1)	12.0549(6)	14.57(1)	+20.84	0.5/0.5



NLO QCD

$\mu^+\mu^- \to t\bar{t}H$

$\sqrt{s}$[GeV]	σ^{LO}[fb]	σ^{NLO}[fb]	K
500	0.272	$0.435^{+3.82\%}_{-3.13\%}$	1.601
800	2.339	$2.319^{+0.01\%}_{-0.09\%}$	0.991
1000	2.008	$1.893^{+0.49\%}_{-0.62\%}$	0.942
1400	1.323	$1.192^{+0.81\%}_{-1.08\%}$	0.900
1000	2.009	$1.894^{+0.45\%}_{-0.65\%}$	0.942
3000	0.406	$0.342^{+1.54\%}_{-1.84\%}$	0.842
6000	0.128	$0.102^{+2.22\%}_{-2.55\%}$	0.794
10000	0.053	$0.040^{+3.01\%}_{-3.11\%}$	0.759
14000	0.030	$0.0221^{+3.33\%}_{-3.13\%}$	0.735

NLO EW

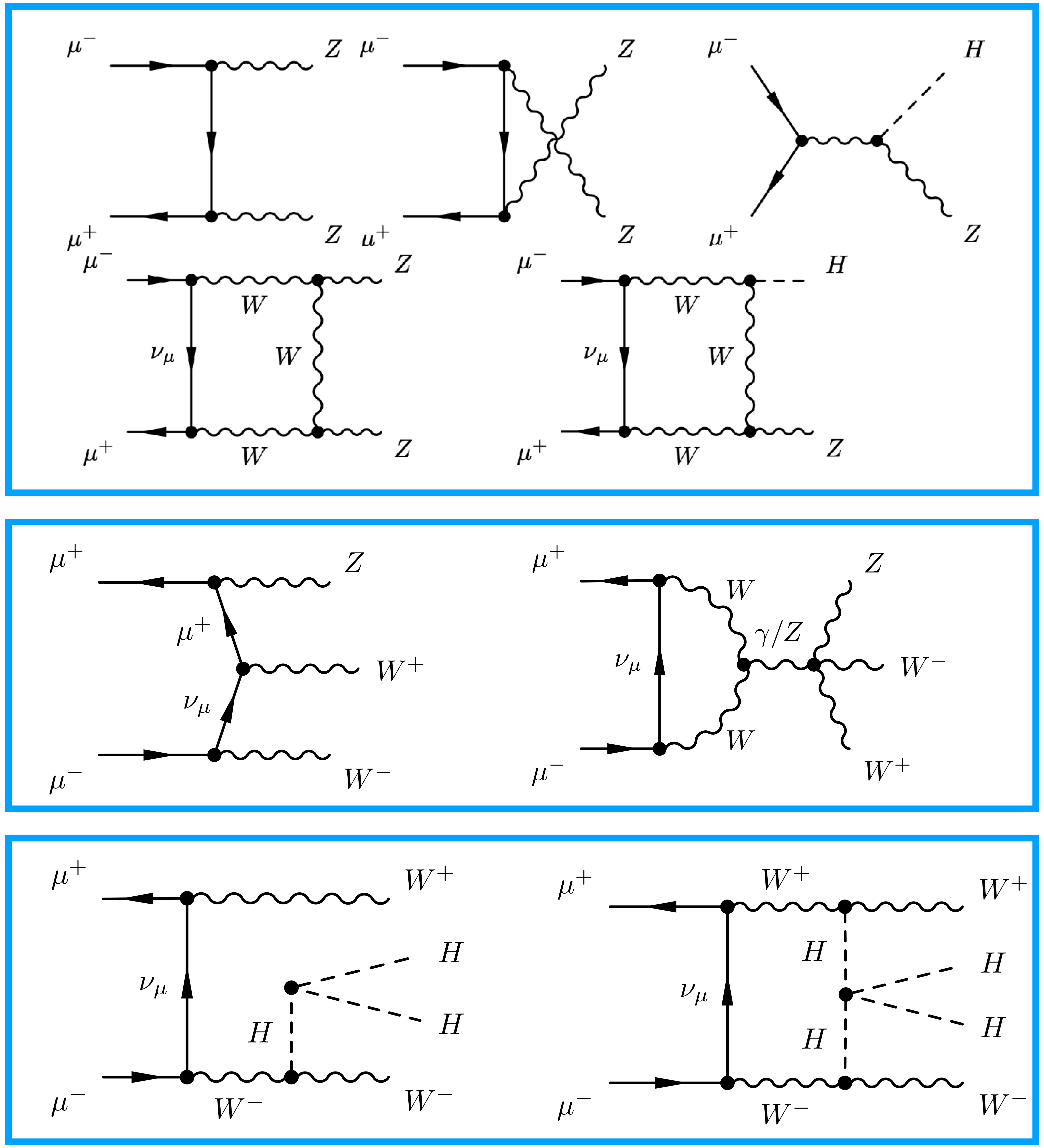
$\mu^+\mu^- \to t\bar{t}H$

$\sqrt{s}$[GeV]	σ^{LO}[fb]	σ^{NLO}[fb]	K
500	0.271	0.091	0.335
800	2.339	1.533	0.655
1000	2.008	1.402	0.698
1400	1.323	0.967	0.731
1000	2.008	1.322	0.658
3000	0.407	0.296	0.728
6000	0.128	0.086	0.669
10000	0.053	0.027	0.516
14000	0.030	0.017	0.579

Francesco Ucci, DESY summer student report, 2022

SM EW Corrections to Multi-Bosons at MuC

arXiv: 2208.09438



$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\rm incl}_{\rm LO}$ [fb]	$\sigma^{\rm incl}_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$4.6591(2)\cdot 10^2$	$4.847(7)\cdot 10^2$	$+4.0(2)$
ZZ	$2.5988(1)\cdot 10^1$	$2.656(2)\cdot 10^1$	$+2.19(6)$
HZ	$1.3719(1)\cdot 10^0$	$1.3512(5)\cdot 10^0$	$-1.51(4)$
HH	$1.60216(7)\cdot 10^{-7}$	$5.66(1)\cdot 10^{-7}$ *	
W^+W^-Z	$3.330(2)\cdot 10^1$	$2.568(8)\cdot 10^1$	$-22.9(2)$
W^+W^-H	$1.1253(5)\cdot 10^0$	$0.895(2)\cdot 10^0$	$-20.5(2)$
ZZZ	$3.598(2)\cdot 10^{-1}$	$2.68(1)\cdot 10^{-1}$	$-25.5(3)$
HZZ	$8.199(4)\cdot 10^{-2}$	$6.60(3)\cdot 10^{-2}$	$-19.6(3)$
HHZ	$3.277(1)\cdot 10^{-2}$	$2.451(5)\cdot 10^{-2}$	$-25.2(1)$
HHH	$2.9699(6)\cdot 10^{-8}$	$0.86(7)\cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1)\cdot 10^0$	$0.993(6)\cdot 10^0$	$-33.1(4)$
W^+W^-ZZ	$1.209(1)\cdot 10^0$	$0.699(7)\cdot 10^0$	$-42.2(6)$
W^+W^-HZ	$8.754(8)\cdot 10^{-2}$	$6.05(4)\cdot 10^{-2}$	$-30.9(5)$
W^+W^-HH	$1.058(1)\cdot 10^{-2}$	$0.655(5)\cdot 10^{-2}$	$-38.1(4)$
$ZZZZ$	$3.114(2)\cdot 10^{-3}$	$1.799(7)\cdot 10^{-3}$	$-42.2(2)$
$HZZZ$	$2.693(2)\cdot 10^{-3}$	$1.766(6)\cdot 10^{-3}$	$-34.4(2)$
$HHZZ$	$9.828(7)\cdot 10^{-4}$	$6.24(2)\cdot 10^{-4}$	$-36.5(2)$
$HHHZ$	$1.568(1)\cdot 10^{-4}$	$1.165(4)\cdot 10^{-4}$	$-25.7(2)$

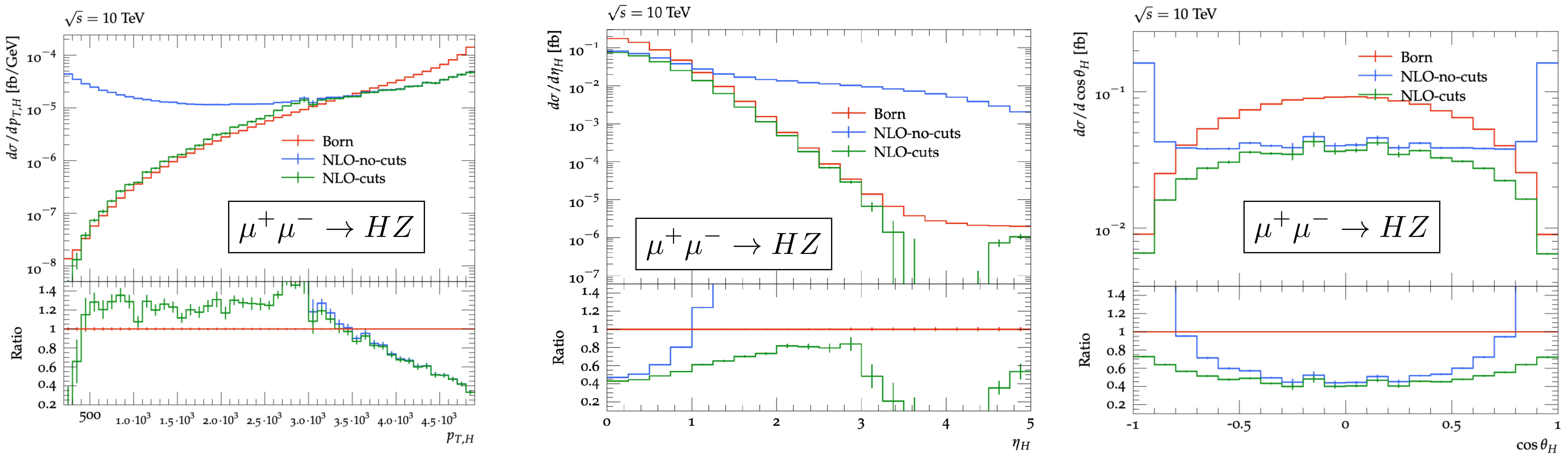
- EW corrections for massive initial state muons
- Massive eikonals need special treatment at high energies

Differential results

arXiv: 2208.09438

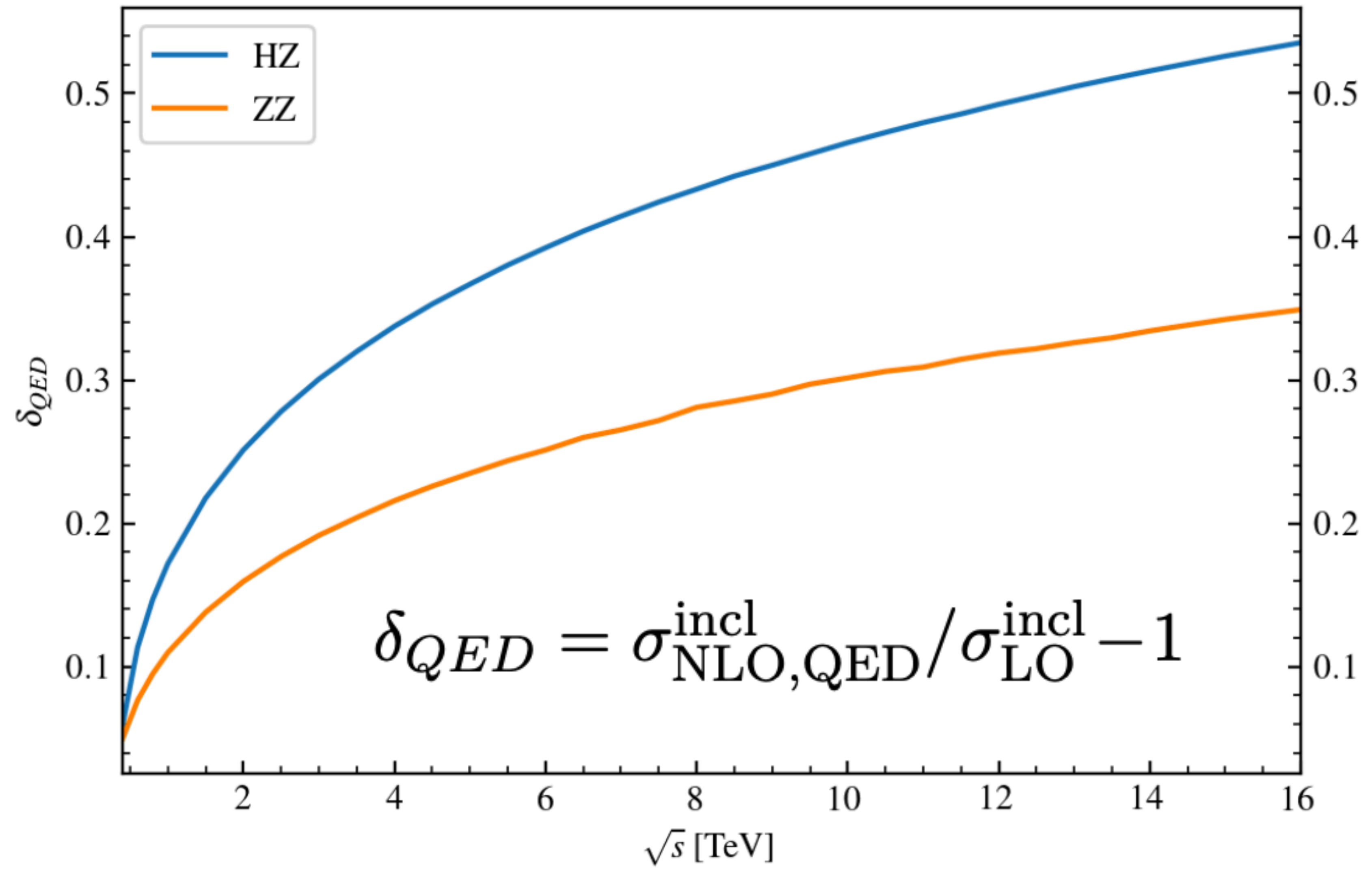
Experimentally motivated photon veto in hard radiation: $E_\gamma < 0.7 \cdot \sqrt{s}/2$

Higgs Transverse Momentum | Higgs rapidity | Higgs scattering angle



More tasks for even more realistic predictions: exclusive events w/ matching to QED/weak showers, resummation, off-shell processes, separate VBF from VBS

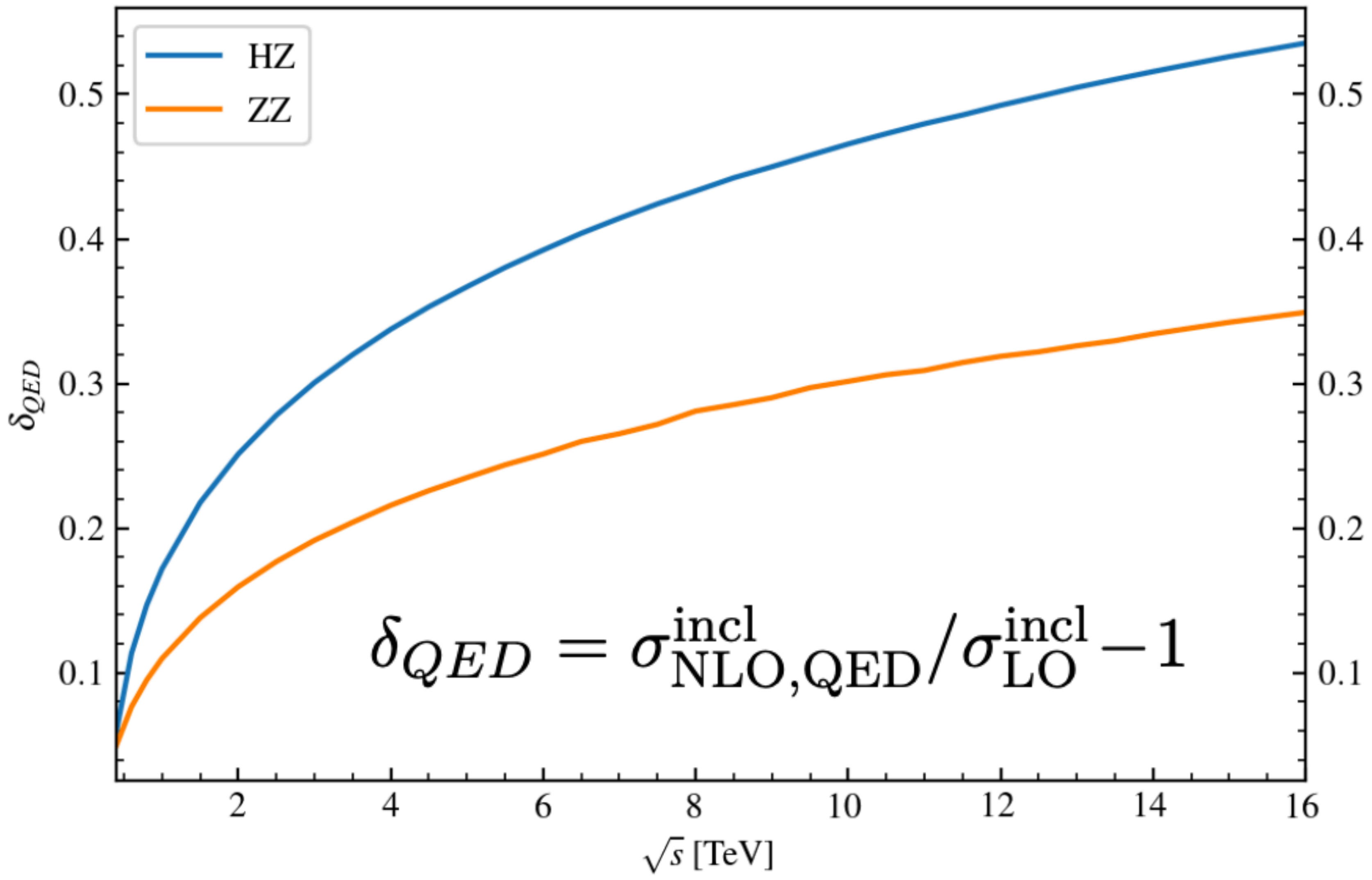
Validation of the QED & Sudakov regime



$\mu^+\mu^- \to X, \sqrt{s} = 10$ TeV	$\sigma^{\rm incl}_{\rm LO}$ [fb]	$\sigma^{\rm incl}_{\rm LO+ISR}$ [fb]	$\delta_{\rm ISR}$ [%]
W^+W^-	$5.8820(2) \cdot 10^1$	$7.295(7) \cdot 10^1$	$+24.0(1)$
ZZ	$3.2730(4) \cdot 10^0$	$4.119(4) \cdot 10^0$	$+25.8(1)$
HZ	$1.22929(8) \cdot 10^{-1}$	$1.8278(5) \cdot 10^{-1}$	$+48.69(4)$
W^+W^-Z	$9.609(5) \cdot 10^0$	$10.367(8) \cdot 10^0$	$+7.9(1)$
W^+W^-H	$2.1263(9) \cdot 10^{-1}$	$2.410(2) \cdot 10^{-1}$	$+13.3(1)$
ZZZ	$8.565(4) \cdot 10^{-2}$	$9.431(7) \cdot 10^{-2}$	$+10.1(1)$
HZZ	$1.4631(6) \cdot 10^{-2}$	$1.677(1) \cdot 10^{-2}$	$+14.62(8)$
HHZ	$6.083(2) \cdot 10^{-3}$	$6.916(3) \cdot 10^{-3}$	$+13.68(6)$

arXiv: 2208.09438

Validation of the QED & Sudakov regime



$$L(s, M_W^2) = \frac{\alpha}{4\pi}\log^2\frac{s}{M_W^2} \overset{10\text{ TeV}}{\sim} 6\%$$

$$l(s, M_W^2) = \frac{\alpha}{4\pi}\log\frac{s}{M_W^2} \overset{10\text{ TeV}}{\sim} 0.6\%$$

- EW corrections at high energies dominated by EW double & single Sudakov logs
- Relevant in kinematic region of Sudakov limit $r_{kl} = (p_k + p_l)^2 \sim s \gg M_W^2$
- IR quasi-divergencies of virtual corrections not cancelled by real EW radiation
- Both initial and final states no EW “color” singlets

$\mu^+\mu^- \to X, \sqrt{s} = 10$ TeV	$\sigma^{\text{incl}}_{\text{LO}}$ [fb]	$\sigma^{\text{incl}}_{\text{LO+ISR}}$ [fb]	δ_{ISR} [%]
W^+W^-	$5.8820(2)\cdot 10^{1}$	$7.295(7)\cdot 10^{1}$	+24.0(1)
ZZ	$3.2730(4)\cdot 10^{0}$	$4.119(4)\cdot 10^{0}$	+25.8(1)
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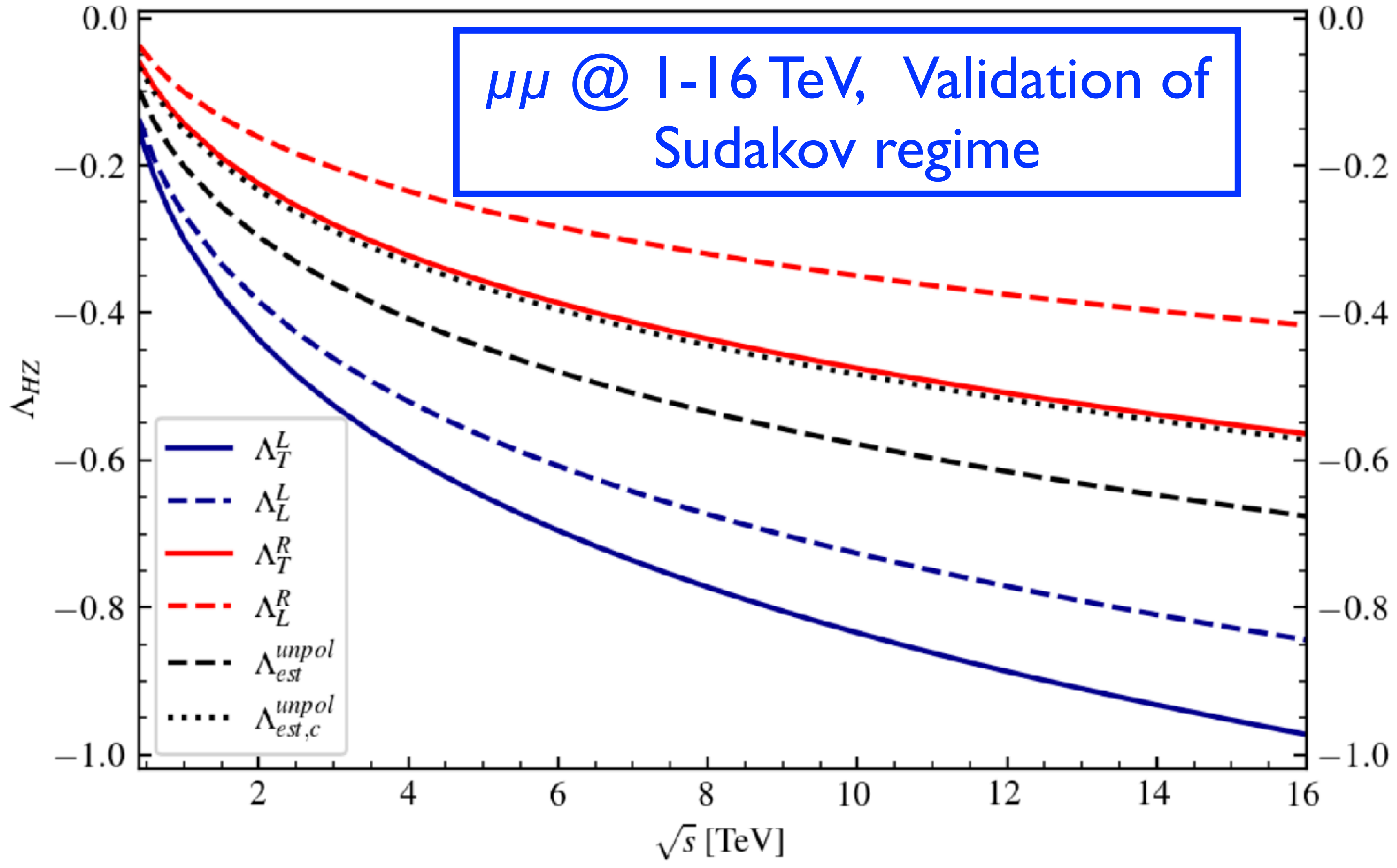
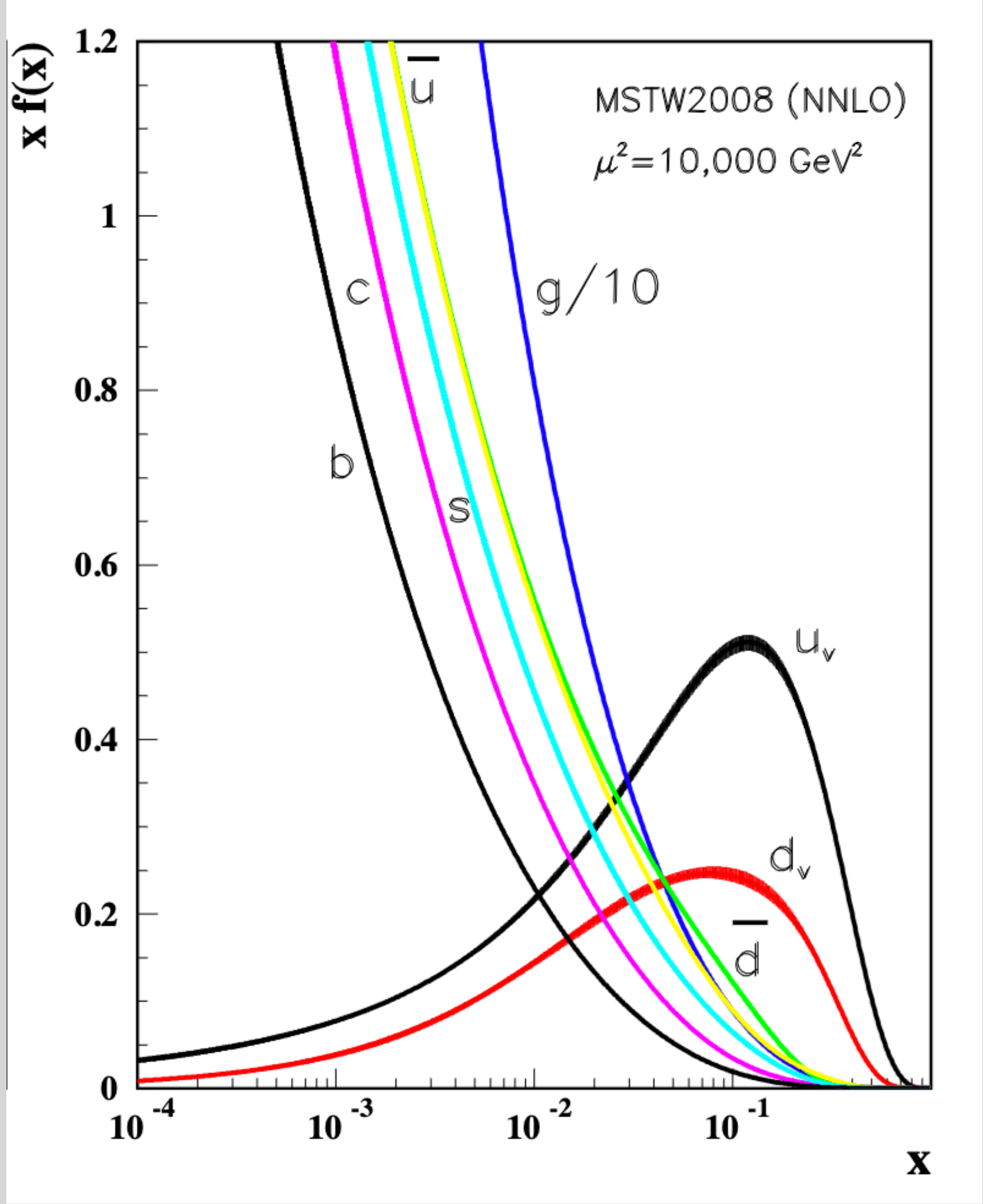
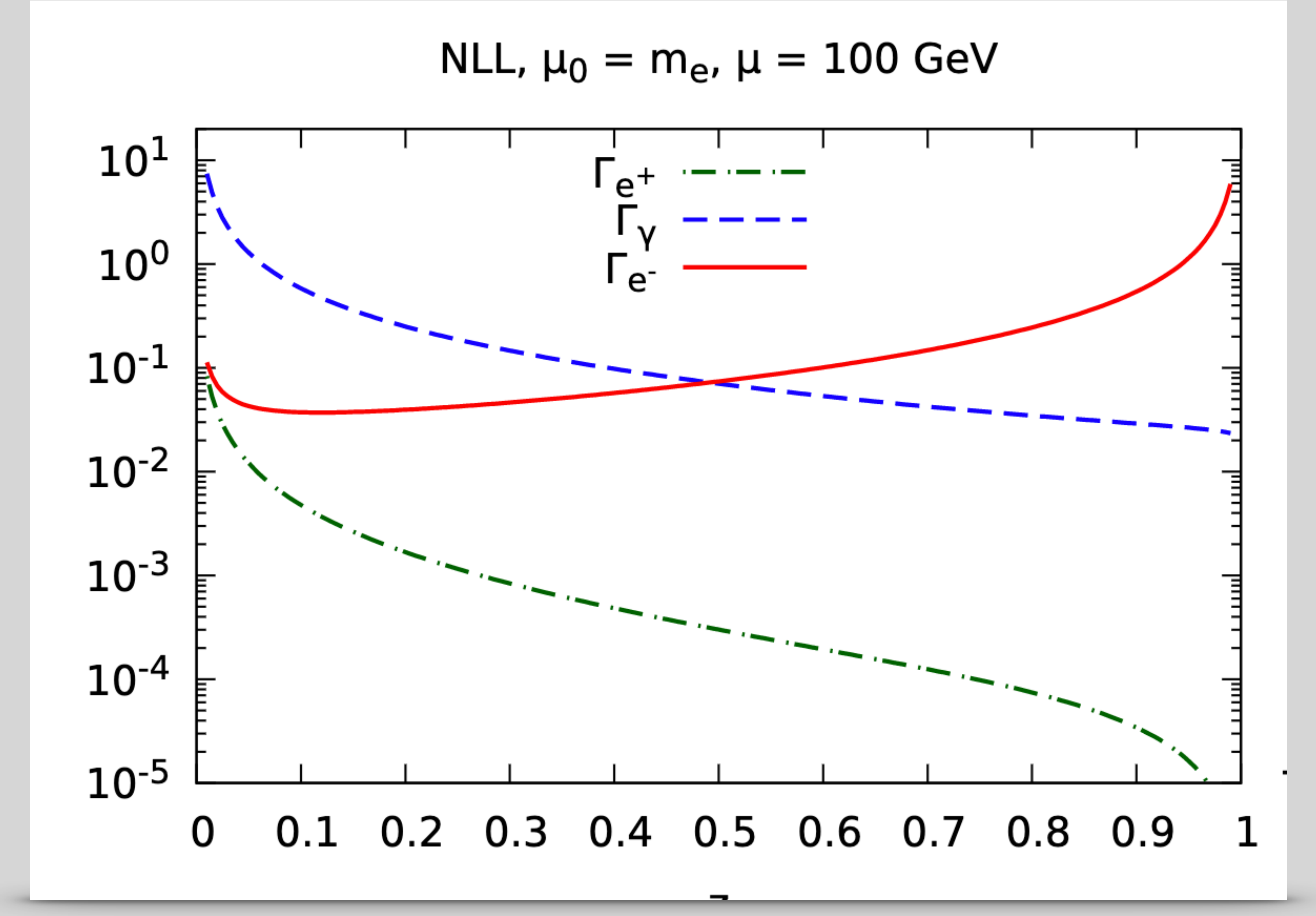


arXiv: 2208.09438

Initial State Radiation — Lepton PDFs





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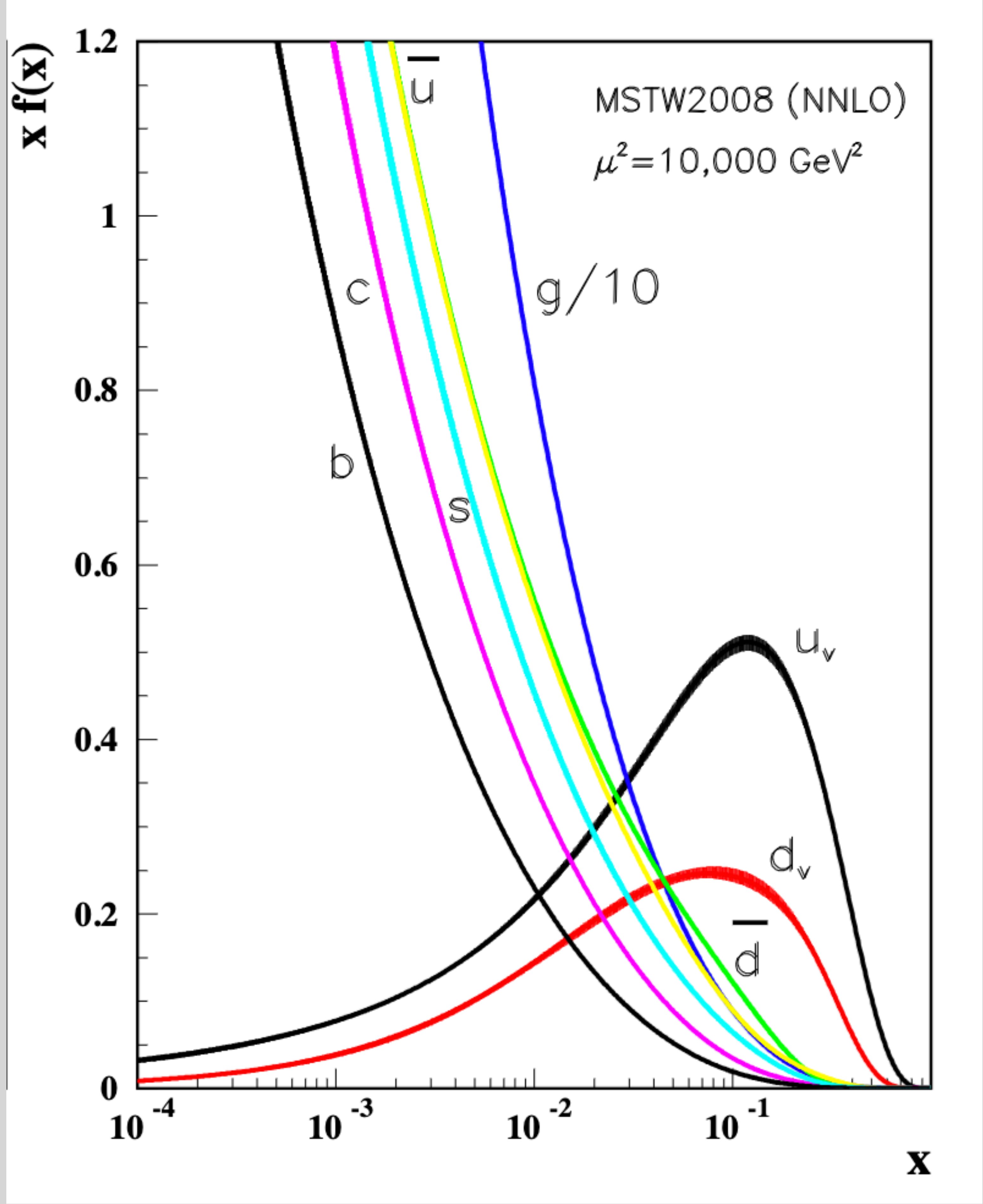
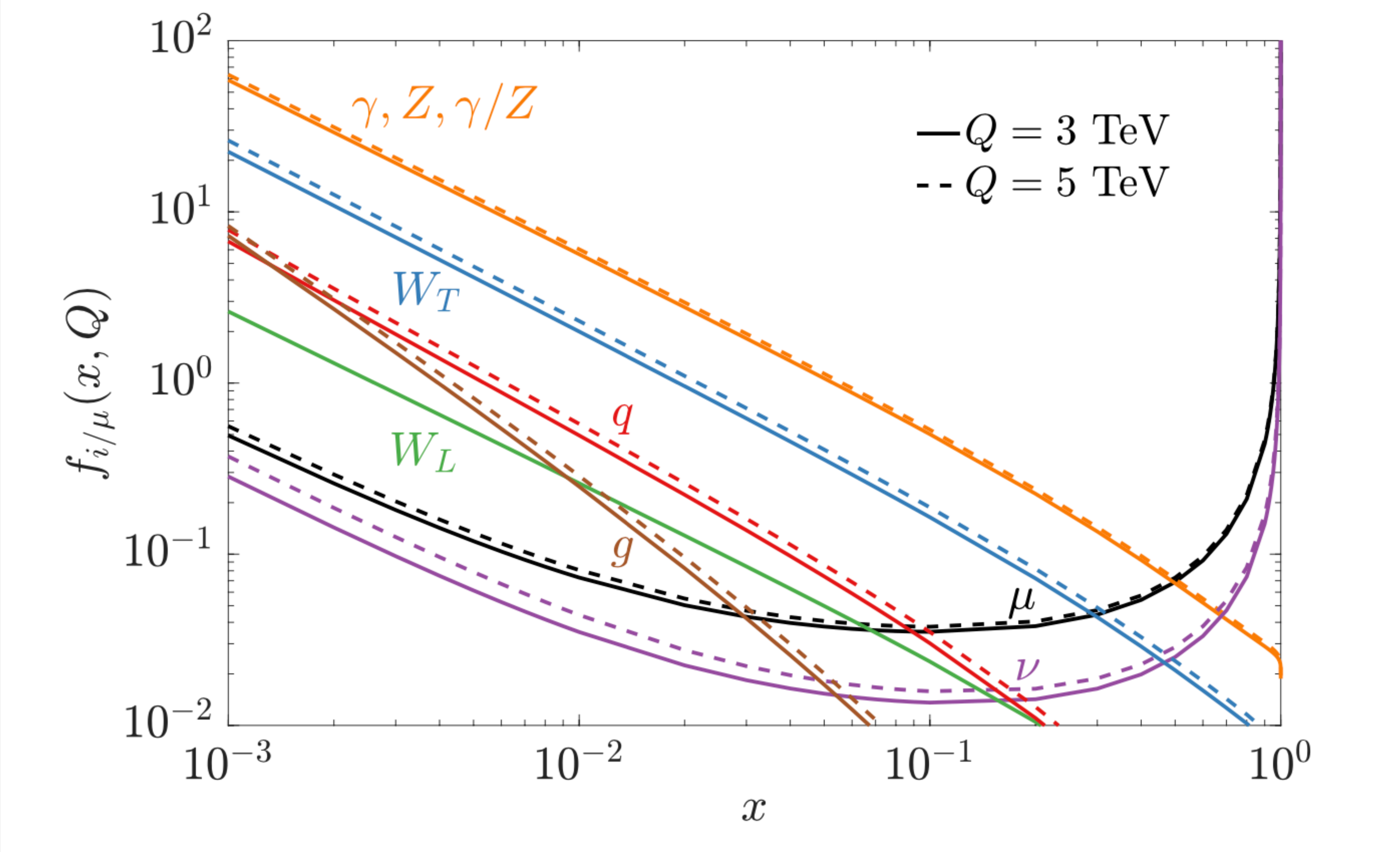




Conclusions & Outlook

- Muon colliders offer a gigantic physics potential: combination of energy & precision

- Three paradigm search channels presented:
- Anomalous muon-Higgs coupling: sign determination and precision up to 1%
- Heavy Neutrinos: mass reach in off-shell production to several 10s of TeV, Majorana/Dirac disc.
- Heavy Z′ (neutral currents): reach up to 70 TeV, with hadronic observables ca. 100 TeV

- Theoretical modelling very challenging, but very interesting:
- Important (but still in infancy) work in QED + EW parton showers with matching
- Regime of EW PDFs, EW parton showers and EW fragmentation: deep in Sudakov regime
- Matching and merging, definition of exclusive vs. inclusive events very complicated

IMCC Annual Meeting DESY 2025

Just confirmed this week: IMCC annual workshop 2025 at DESY!!! Very likely: May 12-16, 2025

Local Organizing team: Federico Meloni (chair), Jenny List, Priscila Pani, Jürgen Reuter ~ 200—250 participants

International UON Collider Collaboration

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

BACKUP

EFT modelling of SM deviations

$$F_U(H) = 1 + \sum_{n\geq 1} f_{U,n} \left(\frac{H}{v}\right)^n$$

Non-linear representation (HEFT)

Scalar H NGB $U = e^{i\phi^a \tau_a / v}$ $\phi^a \tau_a = \sqrt{2} \begin{pmatrix} \frac{\phi^0}{\sqrt{2}} & \phi^+ \\ \phi^- & -\frac{\phi^0}{\sqrt{2}} \end{pmatrix}$

Linear representation ([truncated] SMEFT)

H doublet $\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi^+ \\ v + H + i\phi^0 \end{pmatrix}$

Generalized (μ) Yukawa sector

$$\mathcal{L}_{UH} = \frac{v^2}{4} \mathrm{tr}[D_\mu U^\dagger D^\mu U] F_U(H) + \frac{1}{2} \partial_\mu H \partial^\mu H - V(H)$$
$$- \frac{v}{2\sqrt{2}} \left[\sum_{n\geq 0} y_n \left(\frac{H}{v}\right)^n (\bar{\nu}_L, \bar{\mu}_L) U (1 - \tau_3) \begin{pmatrix} \nu_R \\ \mu_R \end{pmatrix} + \text{h.c.} \right]$$

$$\mathcal{L}_\varphi = \left[-\bar{\mu}_L y_\mu \varphi \mu_R + \sum_{n=1}^{N} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} (\varphi^\dagger \varphi)^n \bar{\mu}_L \varphi \mu_R + \text{h.c.} \right]$$

Parameterization of μ mass and Yukawa modifier

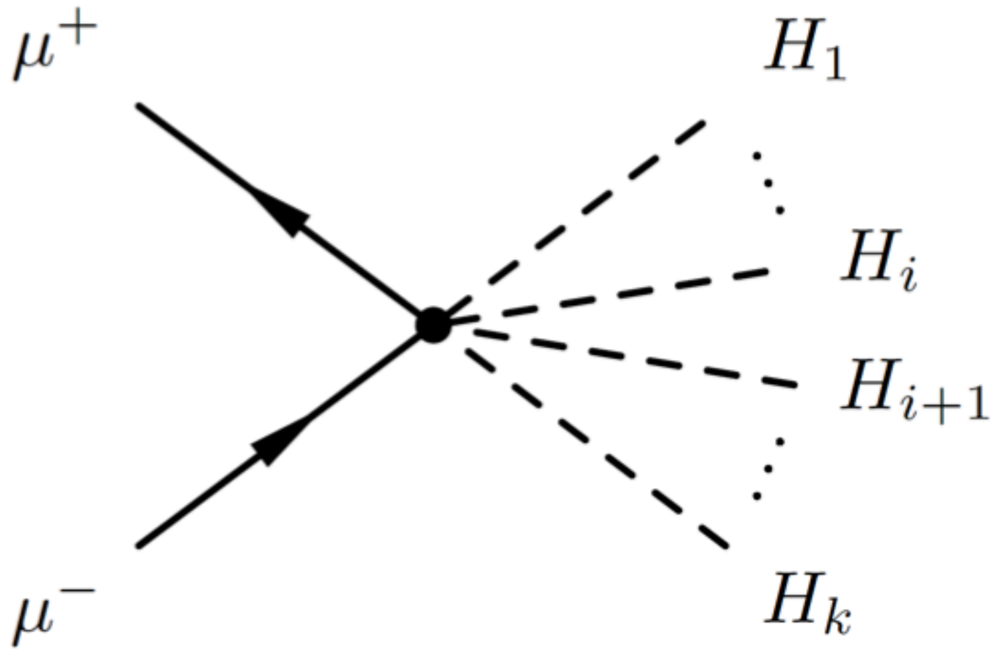
$$m_\mu = \frac{v}{\sqrt{2}} y_0 \qquad \kappa = \frac{v}{\sqrt{2} m_\mu} y_1$$

$$m_\mu = \frac{v}{\sqrt{2}} \left[y_\mu - \sum_{n=1}^{N} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \frac{v^{2n}}{2^n} \right]$$

$$\kappa = 1 - \frac{v}{\sqrt{2} m_\mu} \sum_{n=1}^{N} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \frac{n v^{2n}}{2^{n-1}}$$

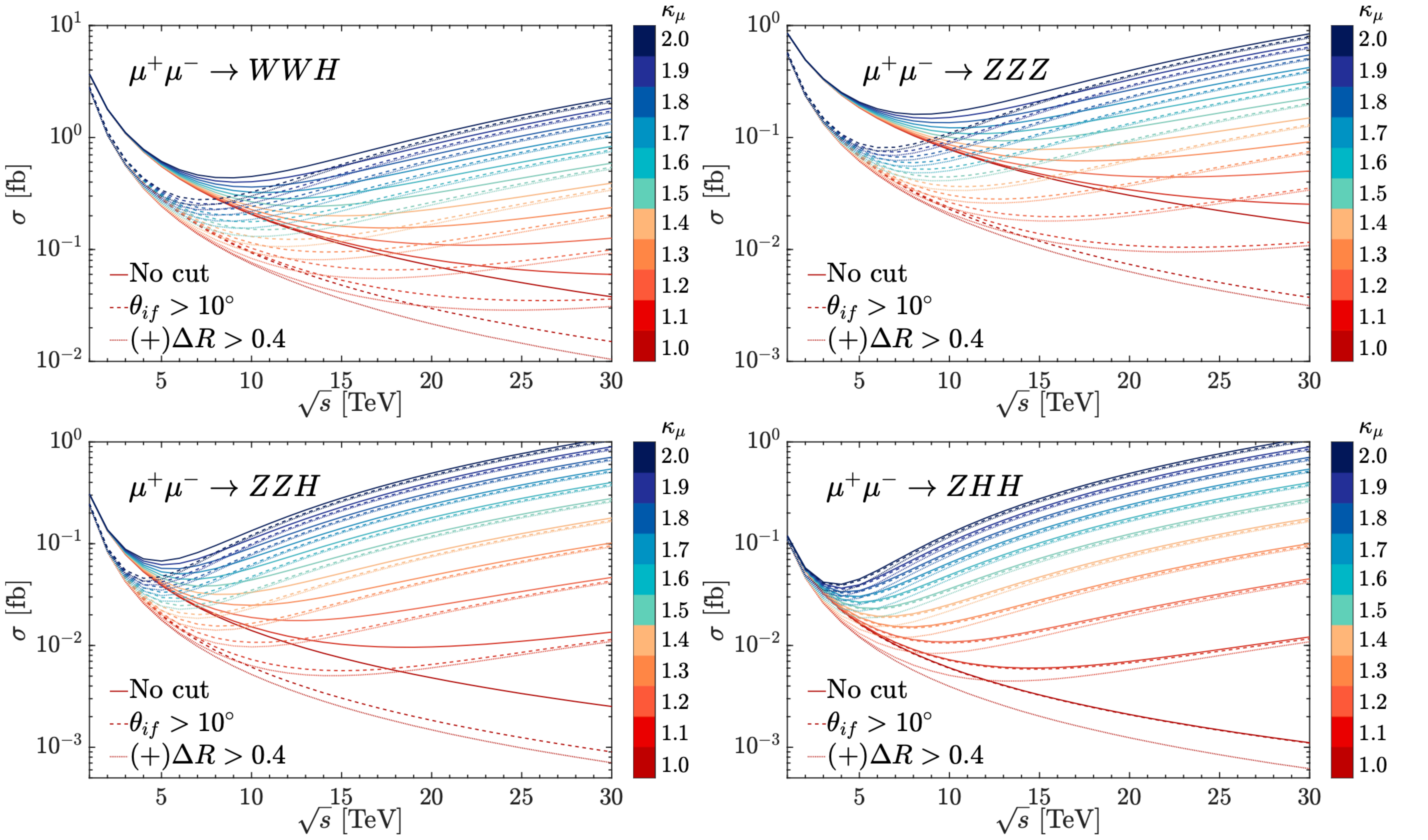
Extreme case: vanishing μ Yukawa: no pure Higgs final states at tree-level !

$$-i \frac{k!}{\sqrt{2}} \left[Y_\ell \delta_{k,1} - \sum_{n=n_k}^{M-1} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \binom{2n+1}{k} \frac{v^{2n+1-k}}{2^n} \right] = 0 \quad =$$



Benchmark scenario: "matched" case

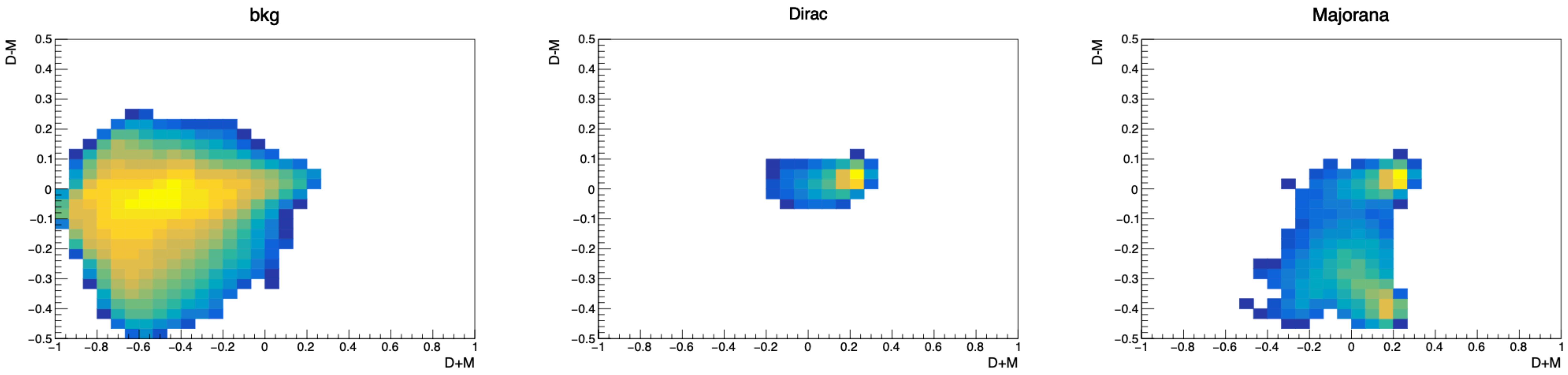
Variations of cross sections with κ



BDT framework for model discrimination

- 2 independent BDT trainins: Dirac vs. (α_{BDT} · Majorana + Bkgd.) & Majorana vs. (α_{BDT} · Dirac + Bkgd.)
- χ^2-like statistics: $T' = \sum_{bins} \frac{[(B+D)-(B+M)]^2}{\frac{1}{2}[(B+D)+(B+M)]} + \#\,\text{DOF} = \sum_{bins} \frac{(D-M)^2}{B+\frac{D+M}{2}} + \#\,\text{DOF}$ $\qquad T' \longrightarrow T'(\alpha_{lim}) = \sum_{bins} \frac{\alpha_{lim}^2 (D-M)^2}{B + \alpha_{lim}\cdot\frac{D+M}{2}}$
- Statistical test: $T \geq \chi^2_{crit}(\text{DOF}) \implies$ signal hypotheses distinguishable
- 2D histograms: $\text{BDT}_D + \text{BDT}_M$, $\text{BDT}_D - \text{BDT}_M$
- Technical procedure:

1. Train BDT for different values α_{BDT}
2. For each α_{BDT}: calculate 95% CL limit
   α_{lim} such that $T(\alpha_{lim}) = \chi^2_{crit}(\text{DOF})$
3. Select the best limit: $\alpha_{min} = \min\left\{\alpha_{lim}\right\}$
4. Set final limit as $V^{\lim}_{\ell N} = \alpha_{min} \cdot V^{ref}_{\ell N}$

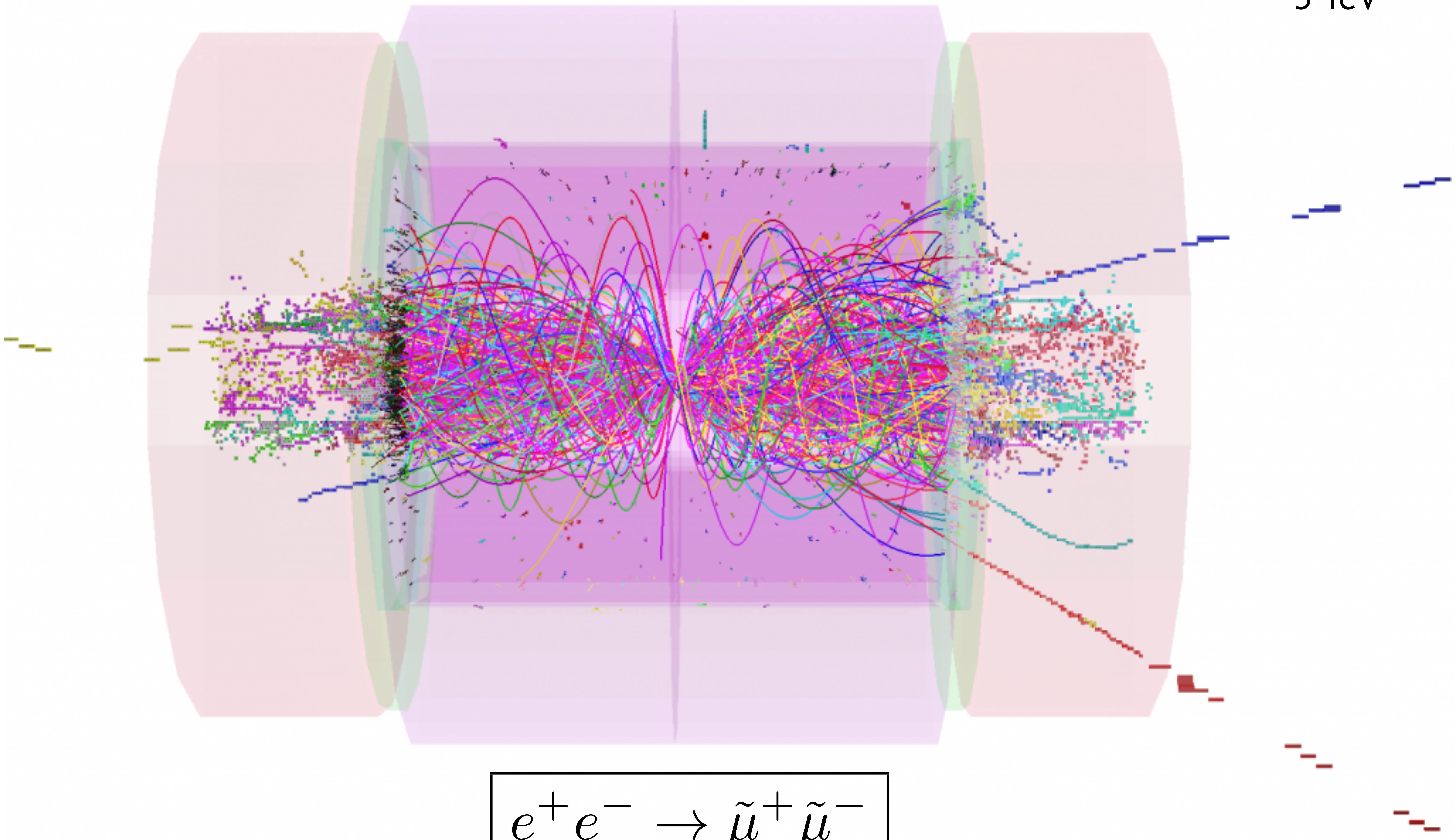


The importance of MC event generators

Why are event generators important? Because all our forward simulation chain depends on them!

Why are event generators non-trivial? Because they contain *all* our knowledge of particle physics!

3 TeV



$$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-$$

Courtesy to Philipp Roloff

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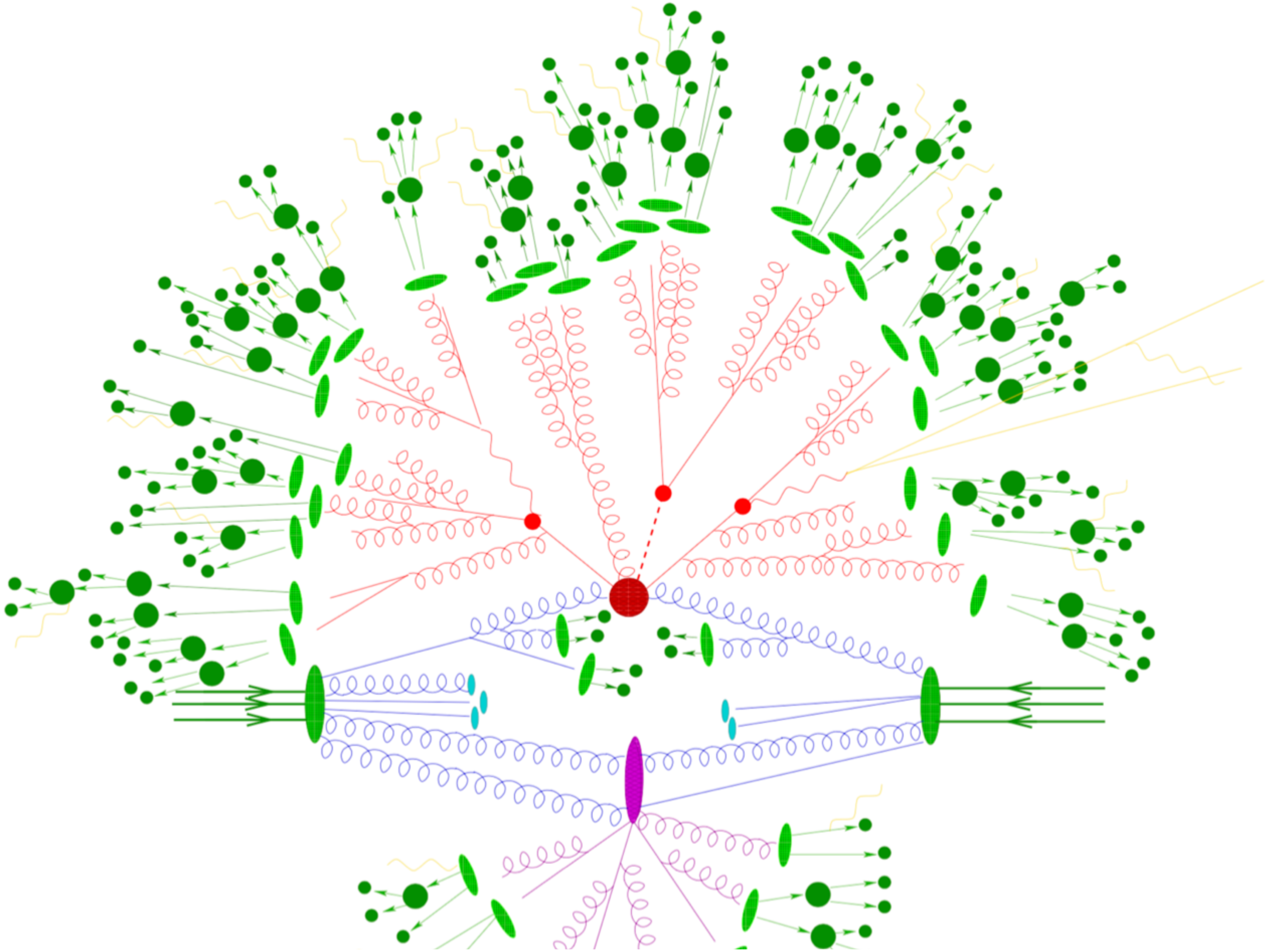

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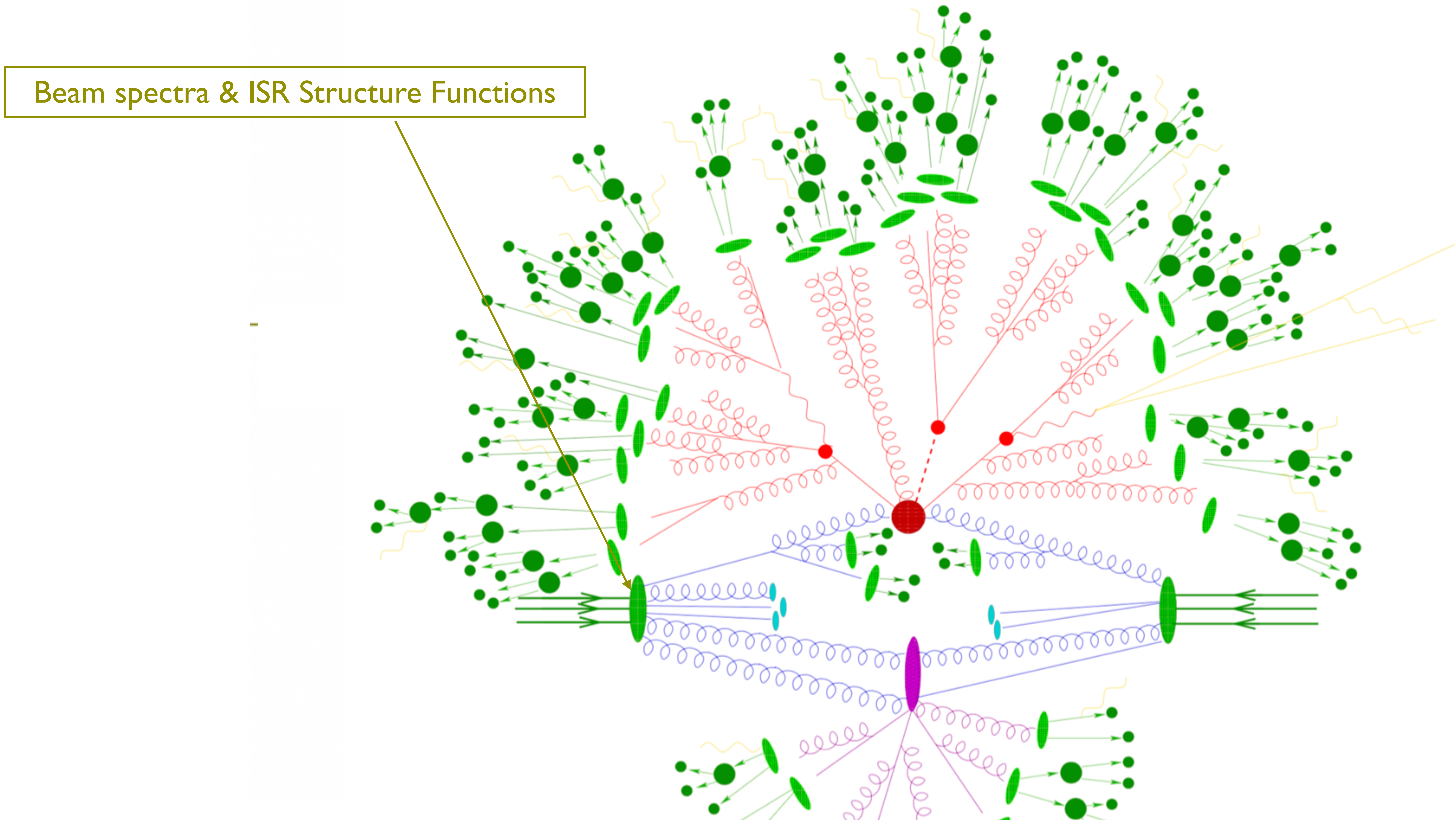

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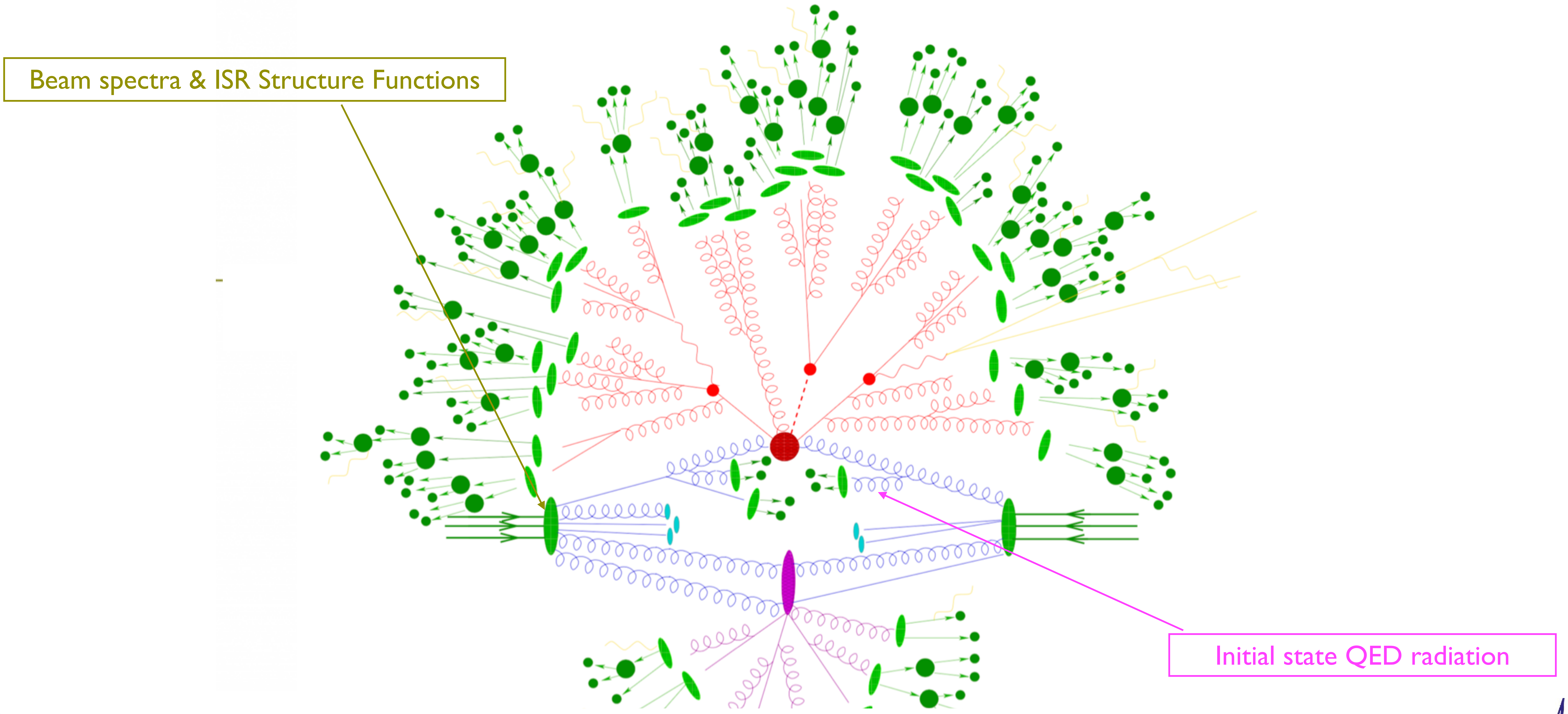

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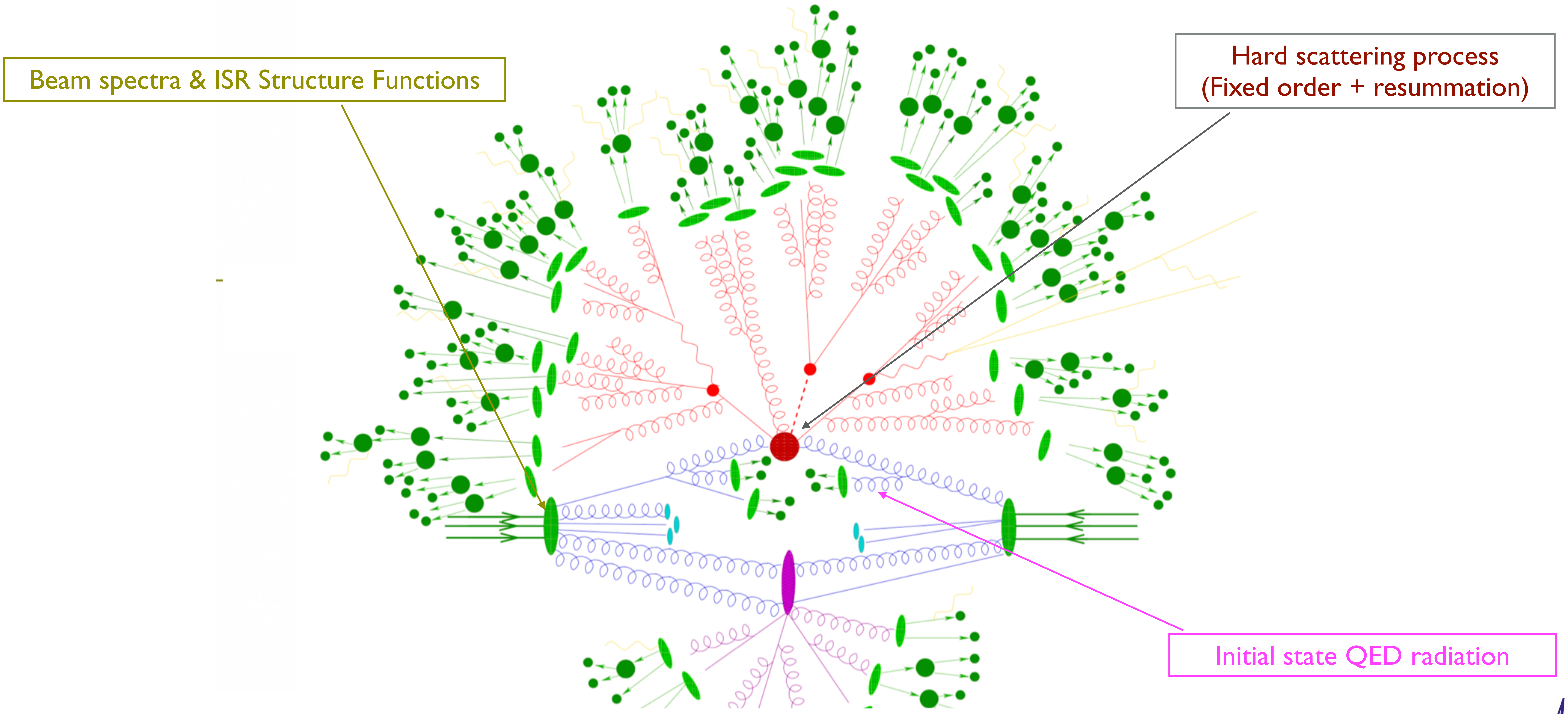

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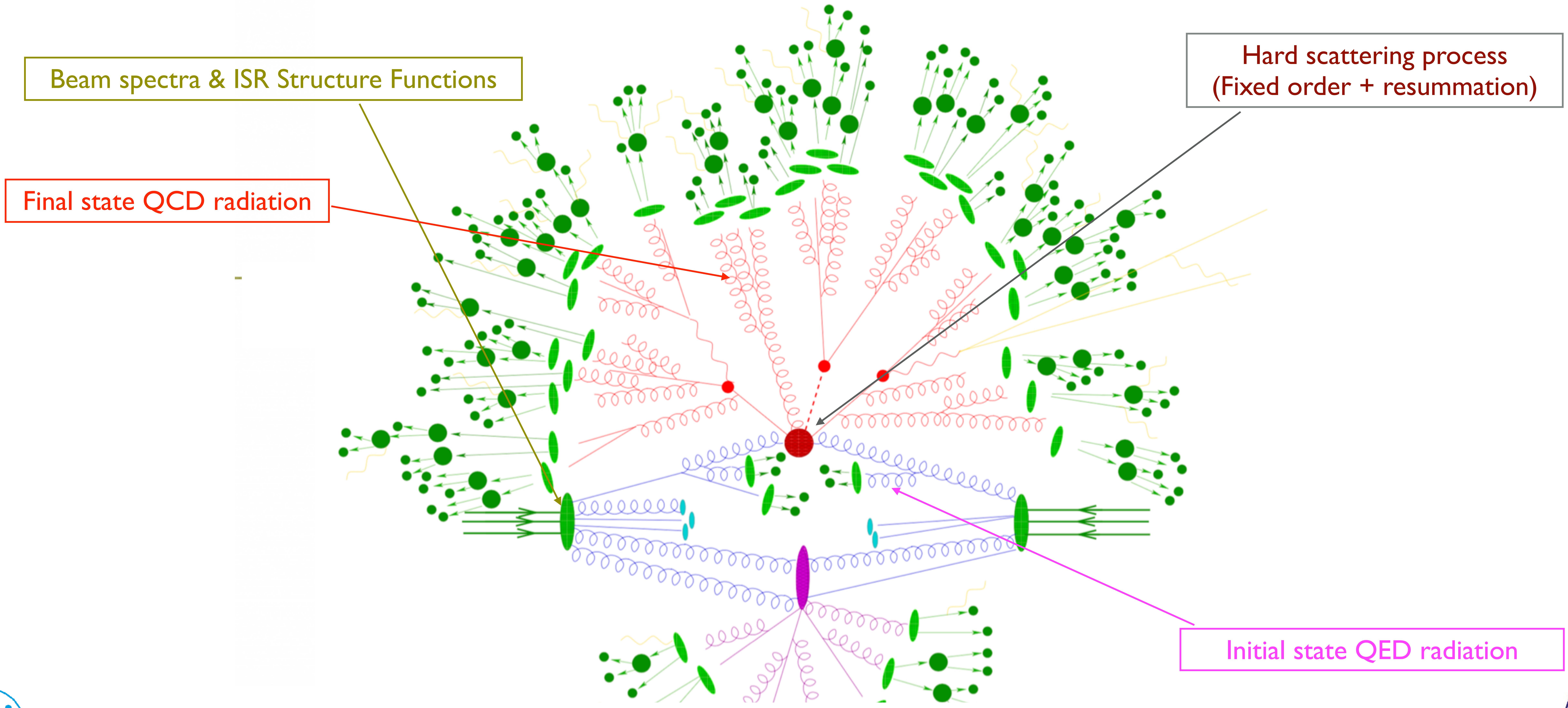

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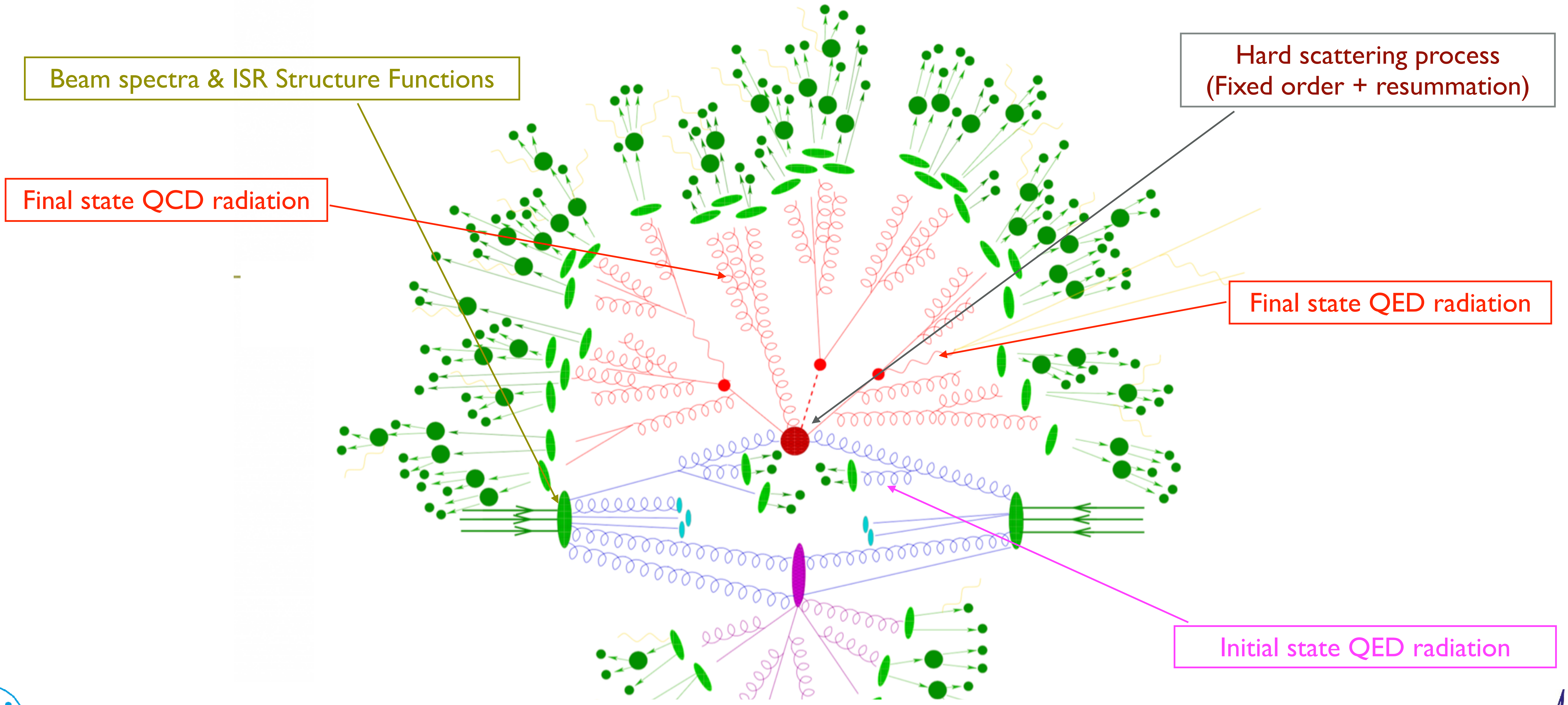

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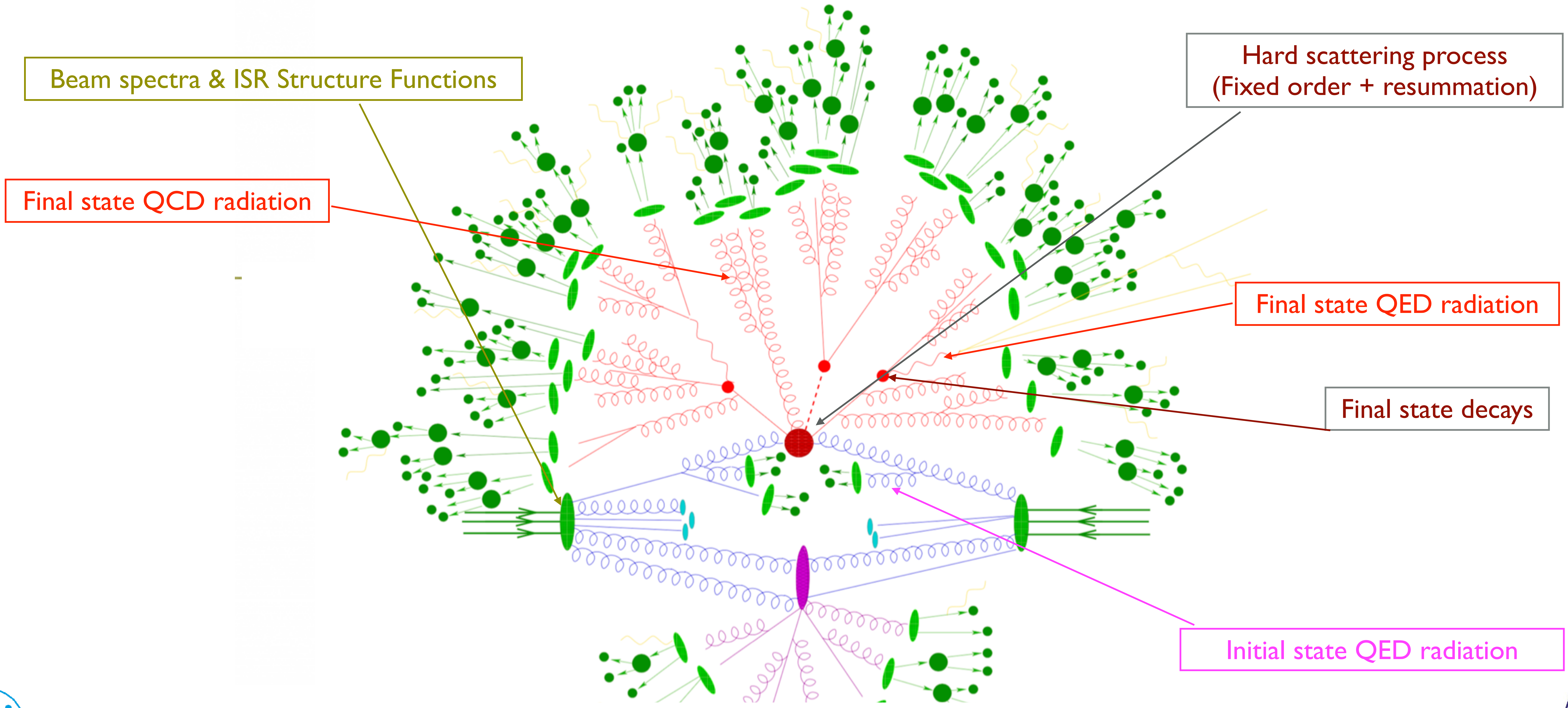

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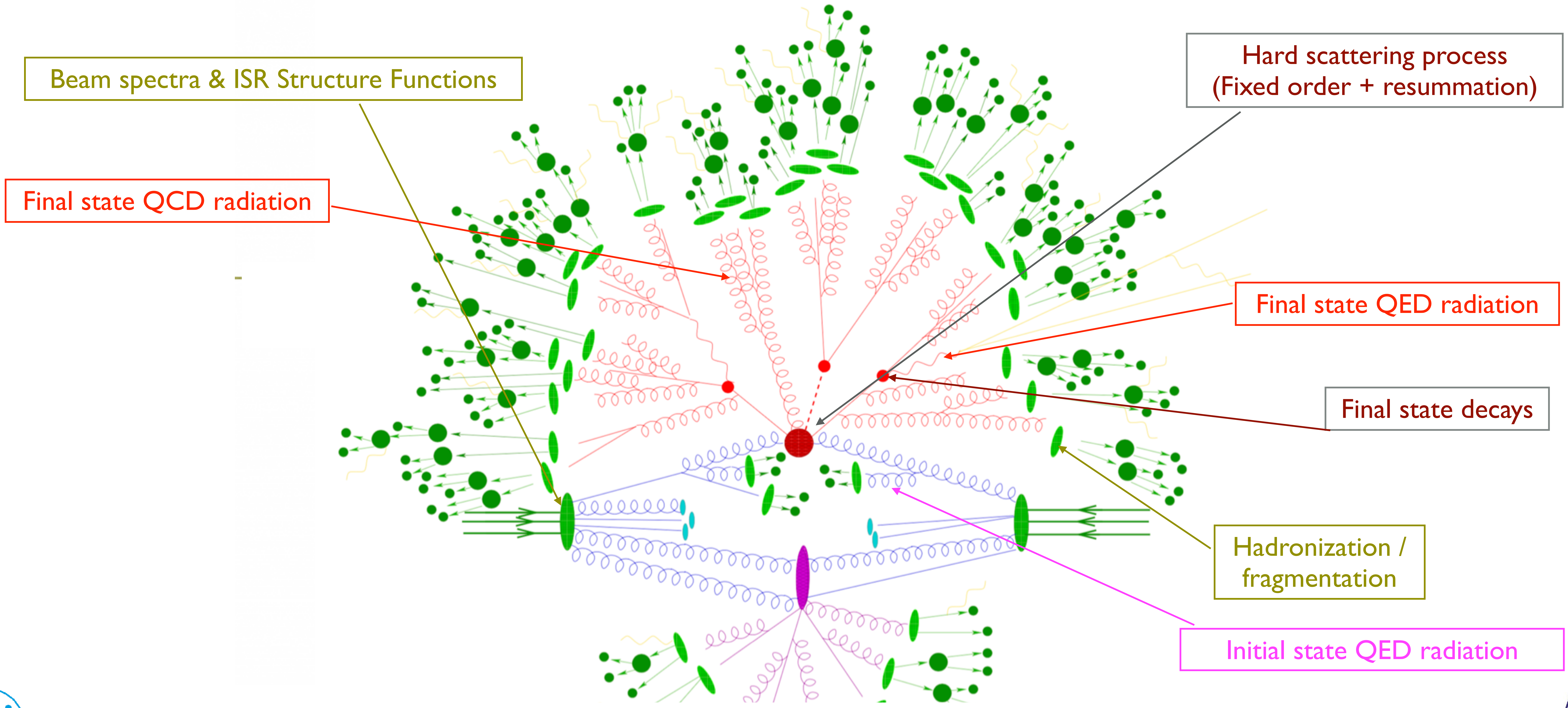

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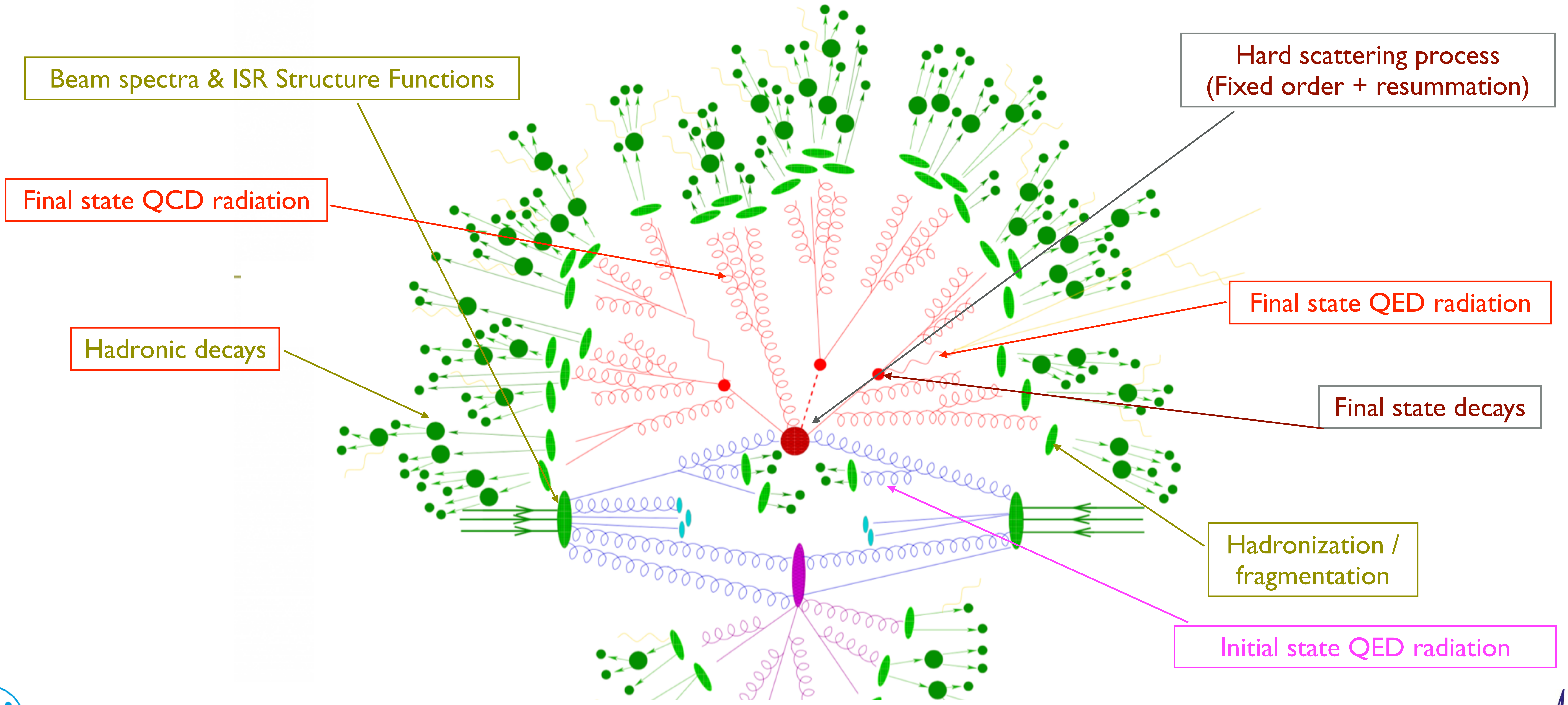

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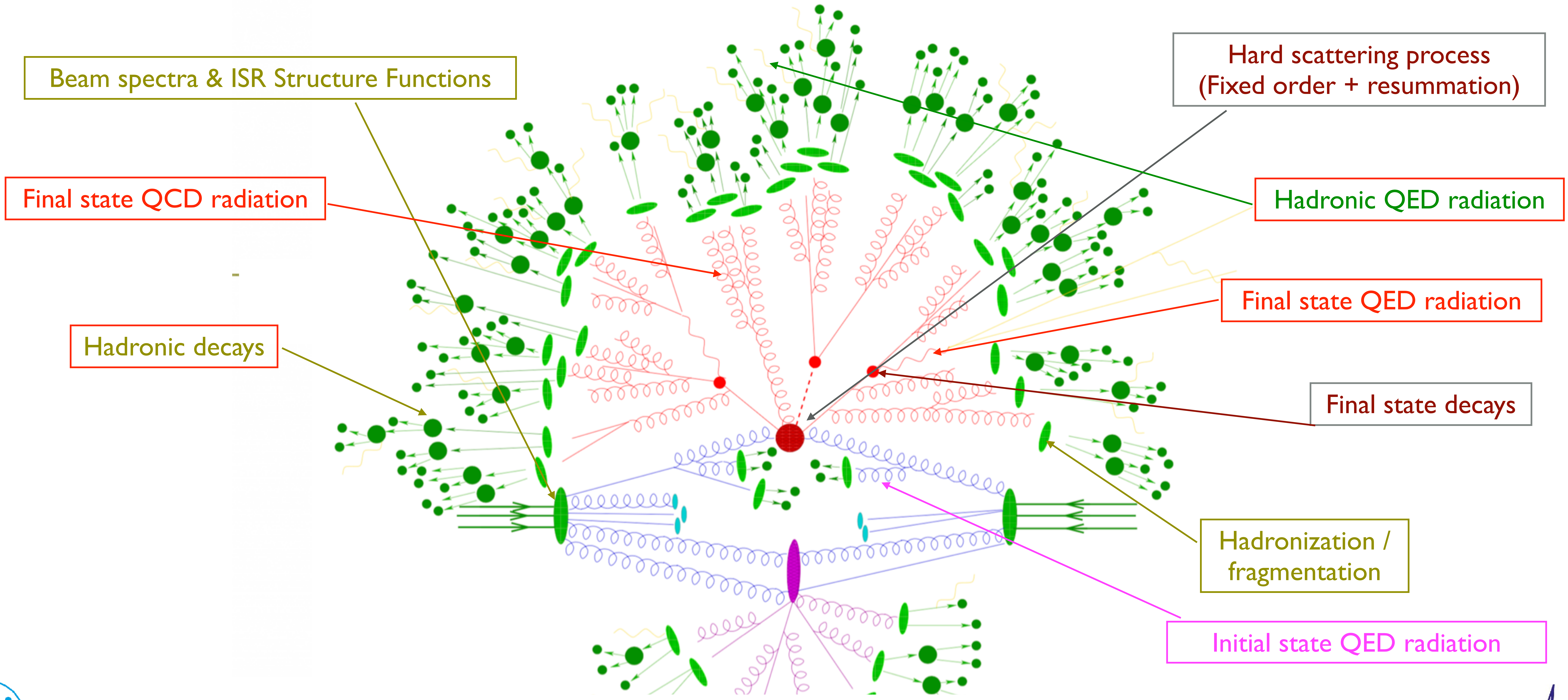

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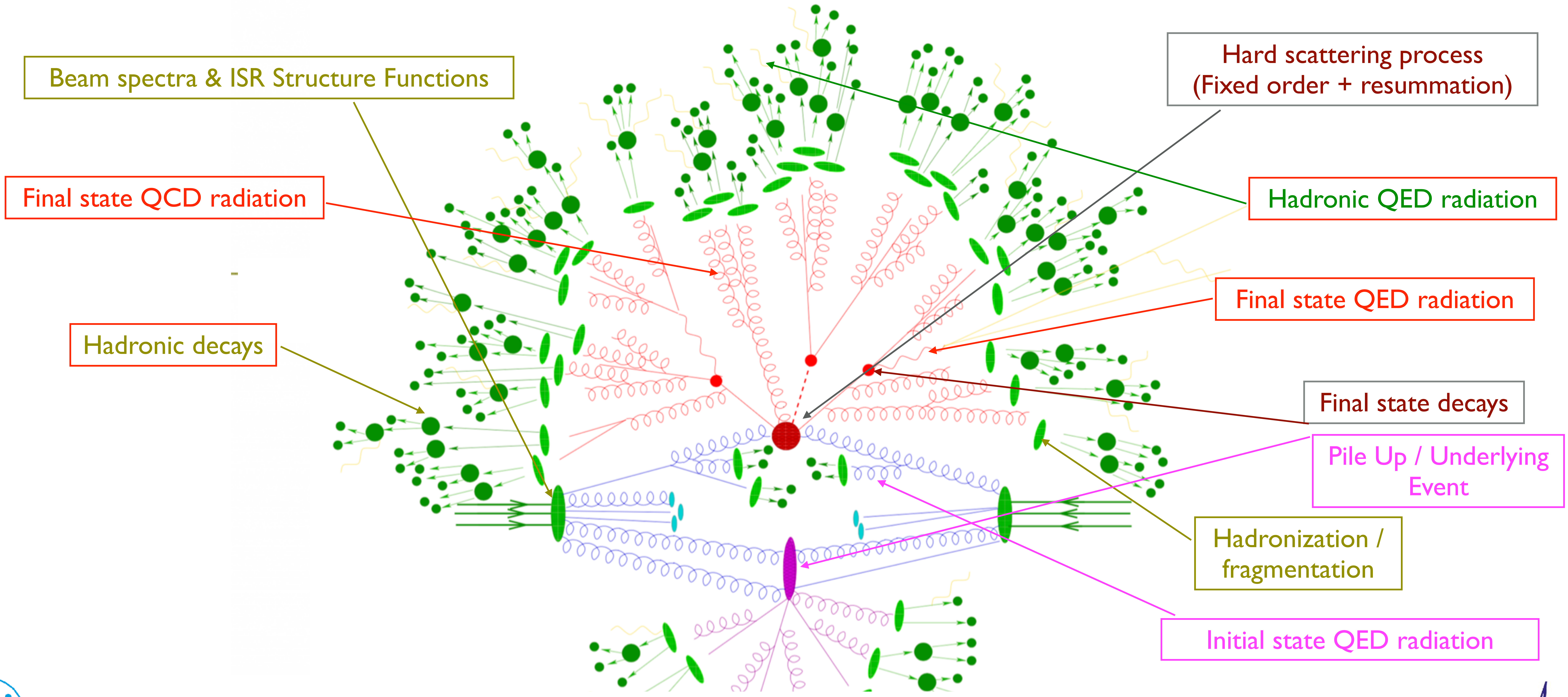

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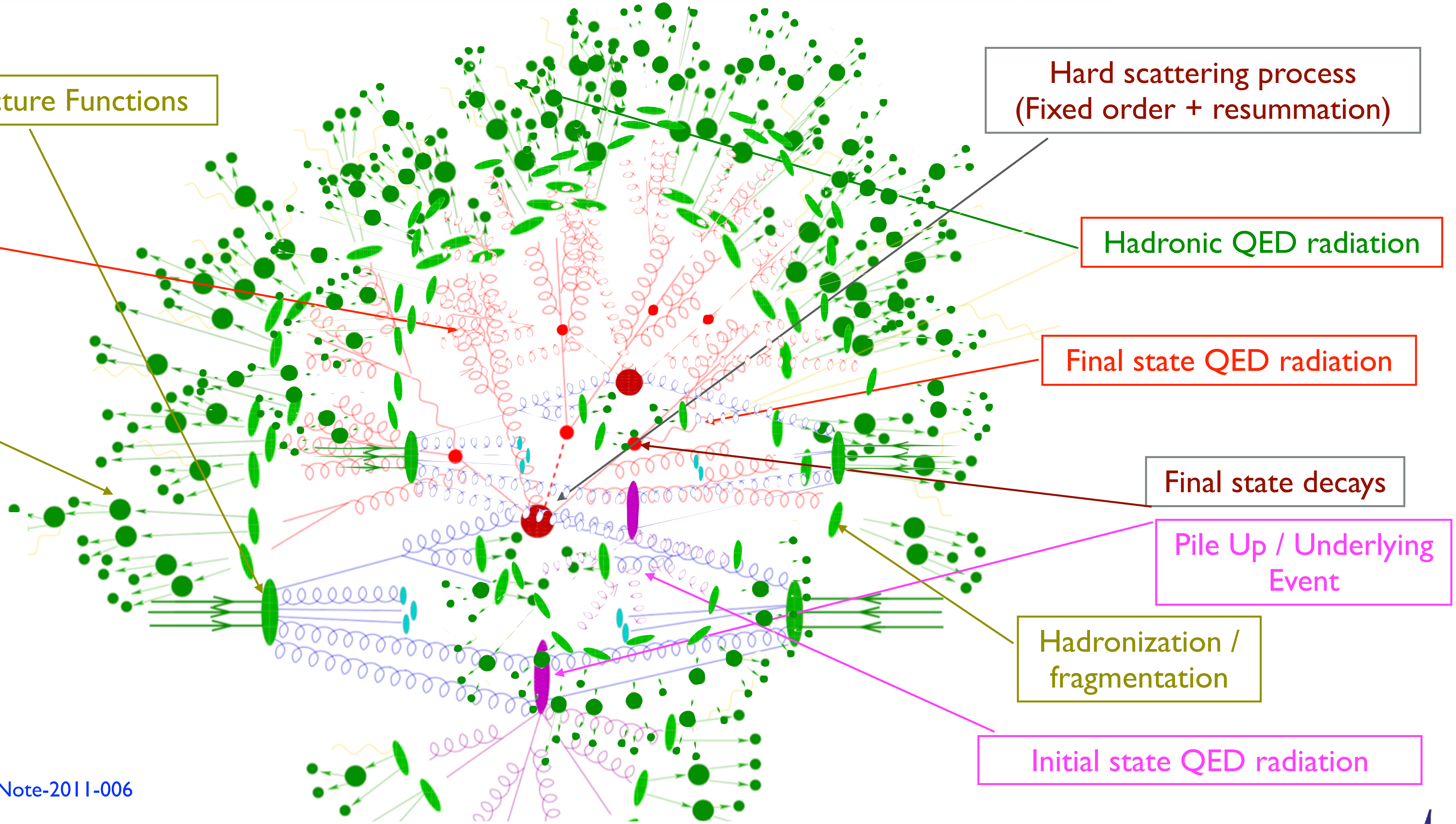
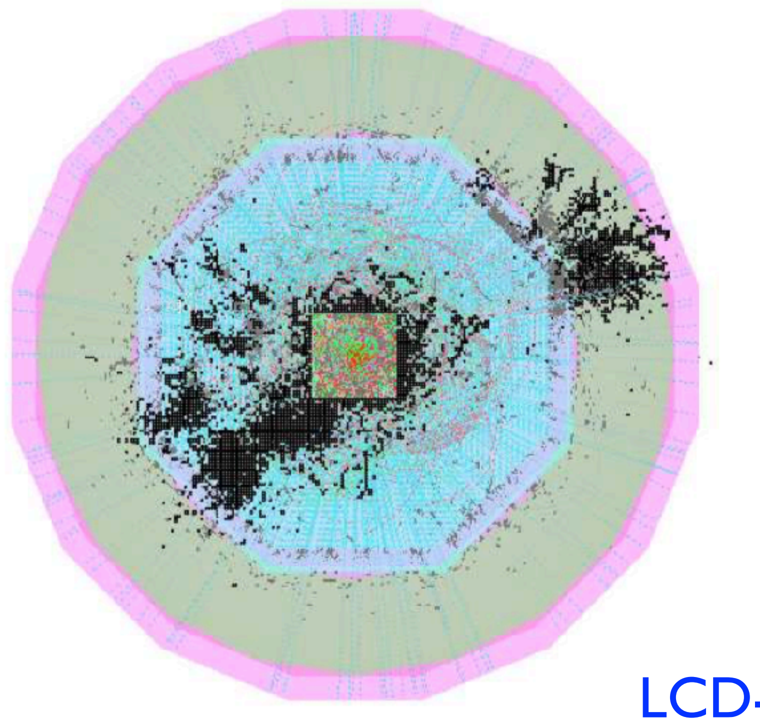
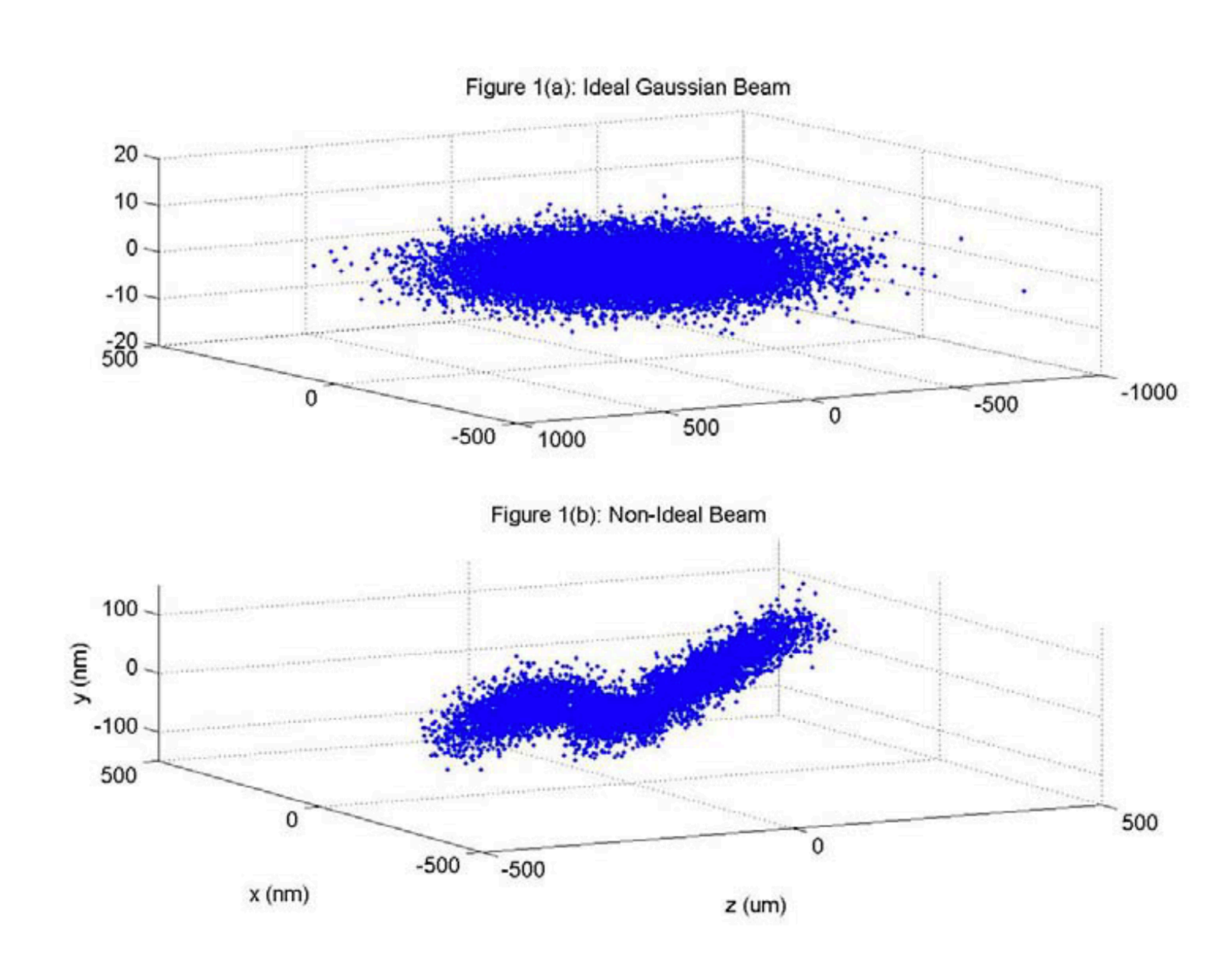




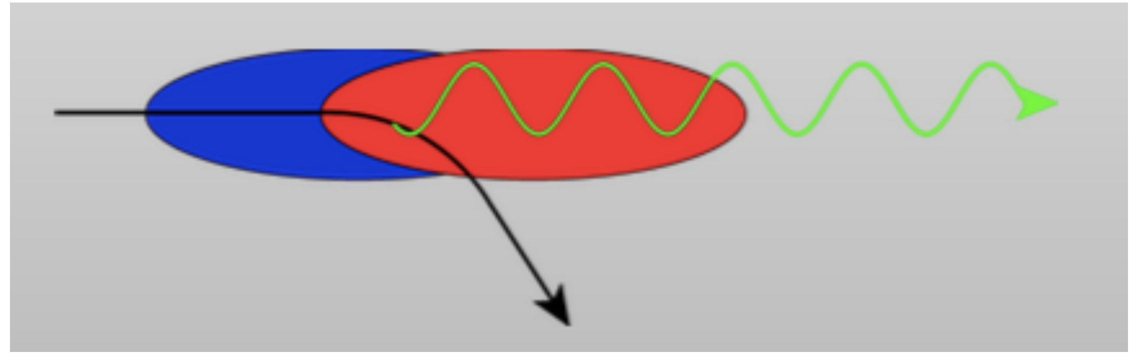
LCD-Note-2011-006

Beam simulations



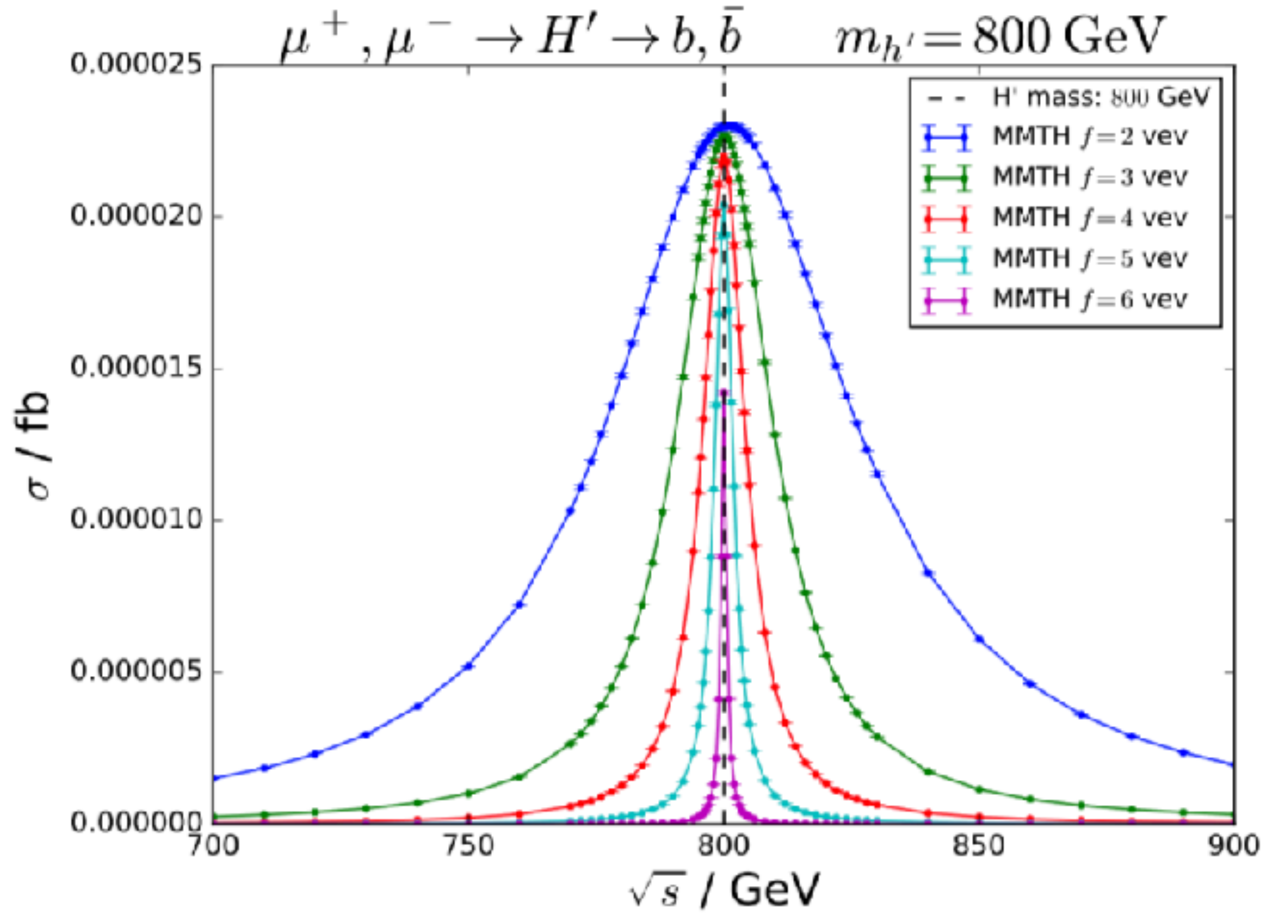
Beam simulations

- Micro-scale bunches create beam structure/-strahlung
- Mostly Gaussian shape for circular machines, but not fully
- Machine simulation with tools like `GuineaPig(++)`, `CAIN`
- Has to be folded into realistic MC simulations



$$L \approx \frac{N}{4\pi\sigma_x\sigma_y}\,\frac{\eta P_{AC}}{E_{CM}}$$

1. Gaussian shape with specific spreads — Avail.: ✔
2. Parameterized (delta peak ⊕ power law) — Avail.: (✔)
3. Generator for 2D histogrammed fit — Avail.: [✔]



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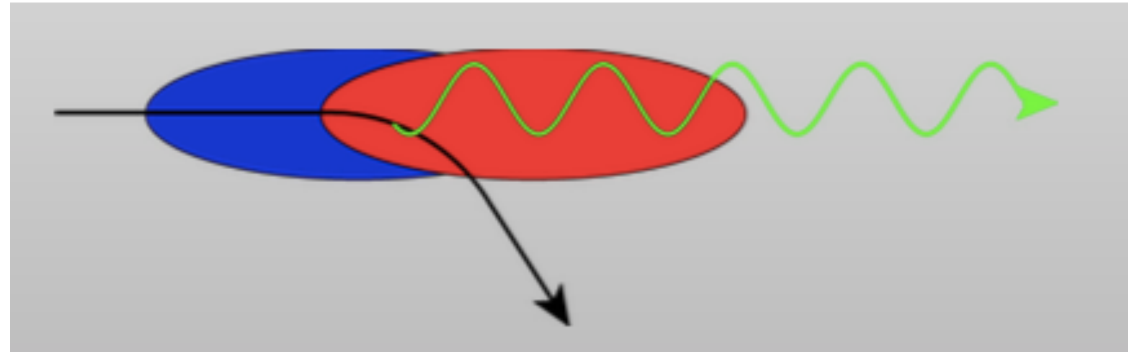

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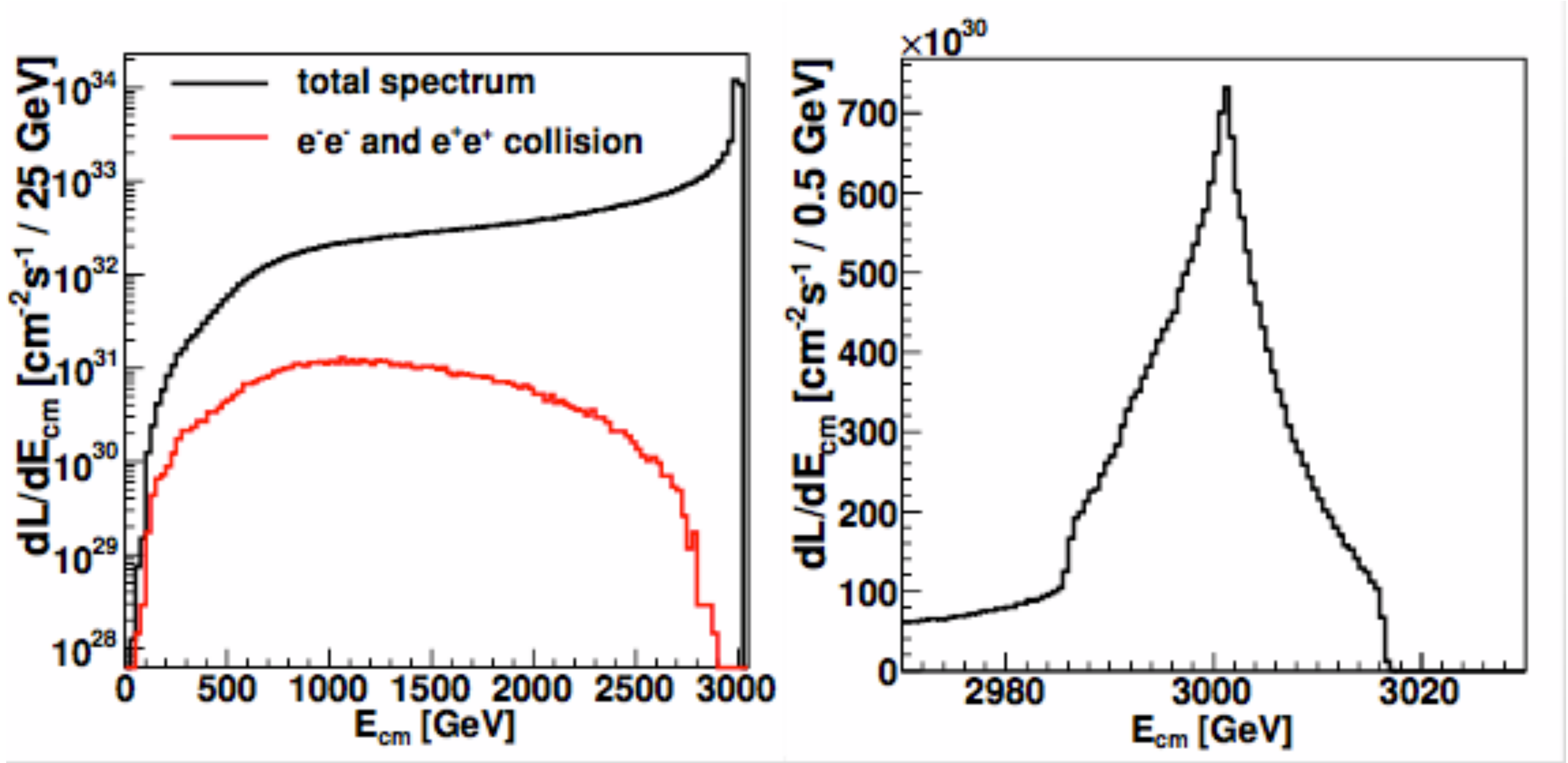

Dalena/Esbjerg/Schulte [LCWS 2011]

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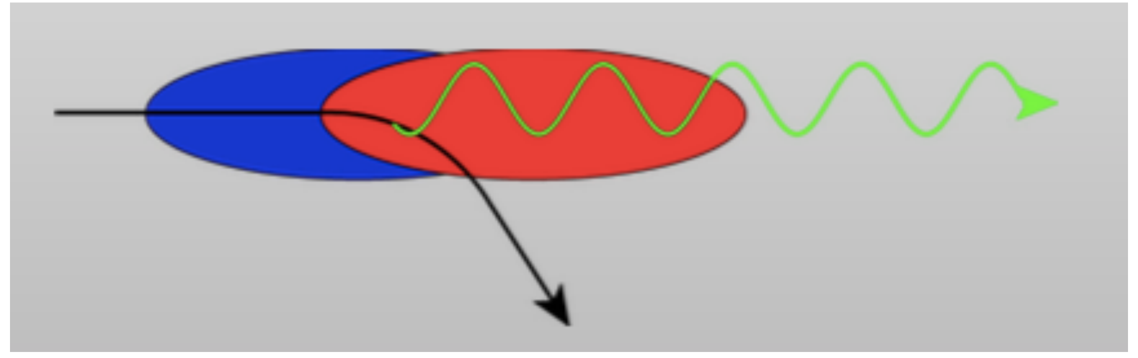

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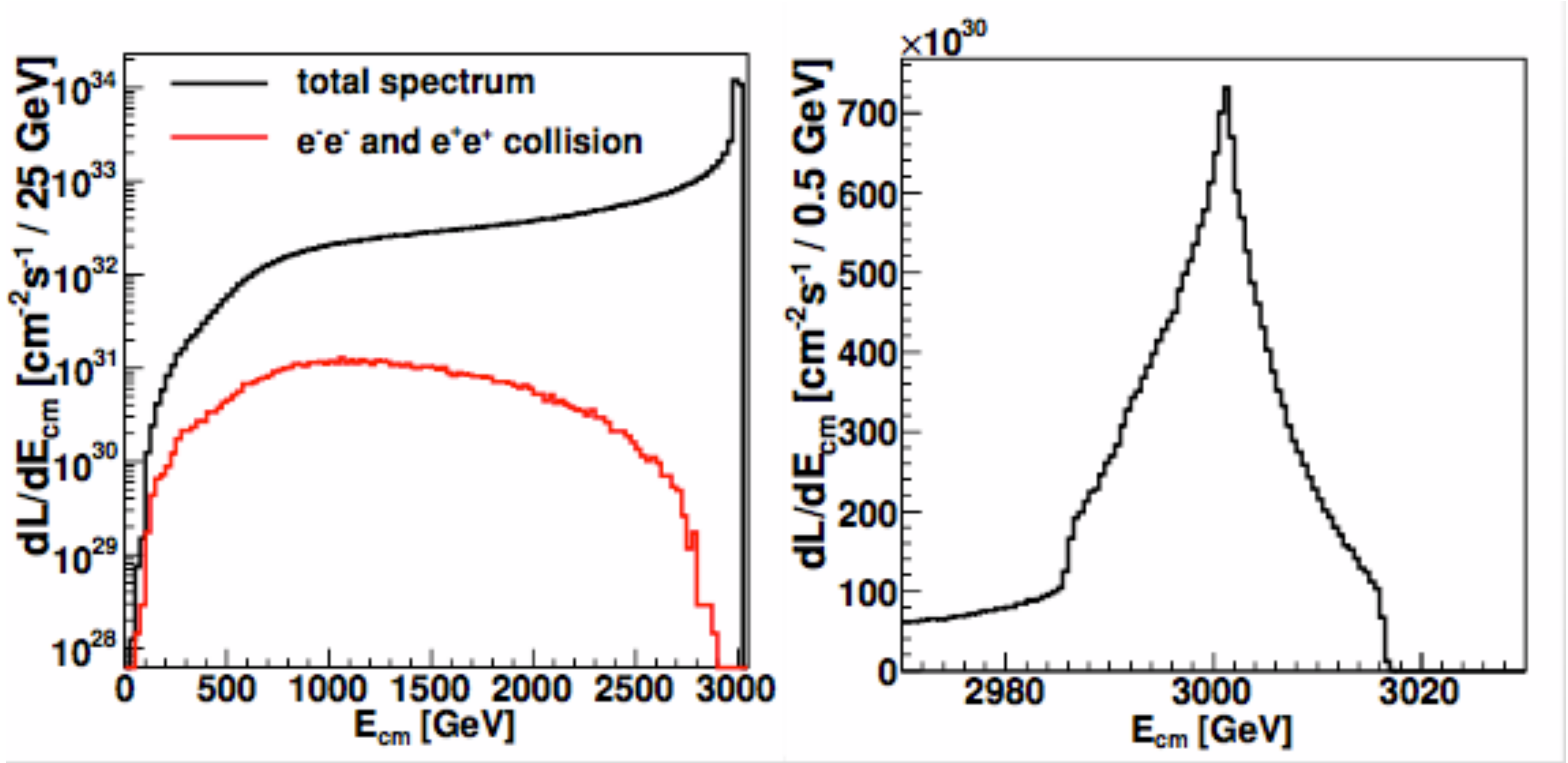


Dalena/Esbjerg/Schulte [LCWS 2011]

$$D_{B_1B_2}(x_1,x_2) \neq D_{B_1}(x_1)\cdot D_{B_2}(x_2)$$
$$D_{B_1B_2}(x_1,x_2) \neq x_1^{\alpha_1}(1-x_1)^{\beta_1}x_2^{\alpha_2}(1-x_2)^{\beta_2}$$

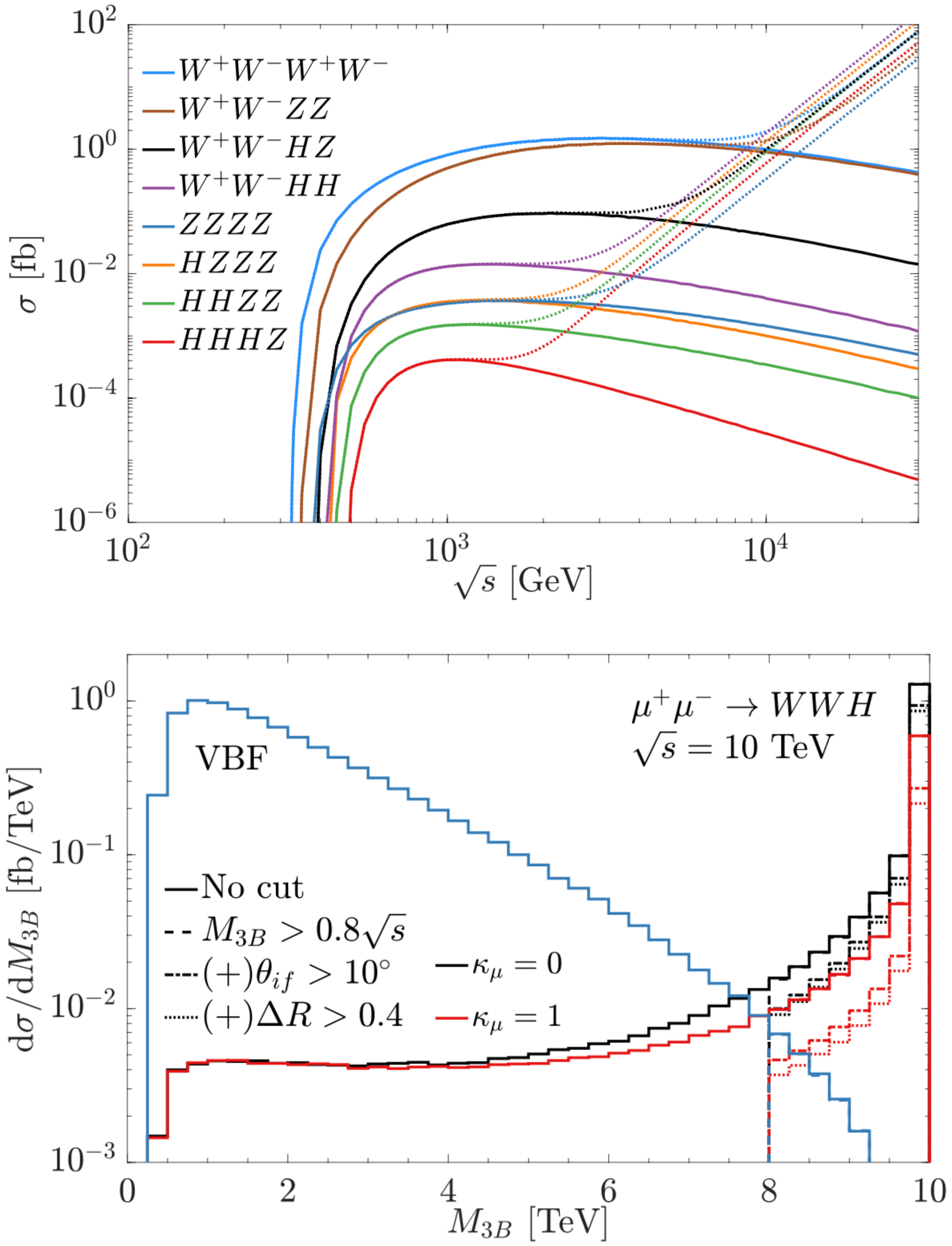
- Pro (1.): Easy implementation, covers main features
- Con (1.): Gaussian approximative, exceeds nominal collider energy
- Pro (2.): Relatively easy implementation
- Con (2.): Delta peak behaves badly in MC, beams maybe not factorizable/simple power law
- Pro (3.): most exact simulation, generator mode avoids artifacts in tails
- Con (3.): only available (yet) in dedicated tools like `LumiLinker` and `CIRCE2`

BSM Modelling in Simulation

BSM Models: UFO magic

- BSM models available from Lagrangian level tools (LanHEP, SARAH, FeynRules)
- Transferred to MC generator via UFO format: v1 1108.2040 v2:2304.09883
- Allows for all Lagrangian-based BSM models

- [x] Spin 0, 1/2, 1, 3/2, 2 supported (some 3/2, 2 features missing in some MC)
- [x] Majorana fermions and fermion-number violating vertices
- [x] 5-, 6-, 7-, 8-, ... point vertices (optimization for code generation pending)
- [x] Arbitrary Lorentz structures in vertices
- [x] Keeping track of the order of insertions
- [x] Customized propators
- [x] Exotic colored objects (sextets, decuplets, epsilon structures)
- [x] (S)LHA-style input files from spectrum generators to MC generators (scans!)
- [x] Automated calculations of widths (UFO side vs. MC generator side)
- [ ] Long-lived particles, displaced vertices, oscillations in decays (not all MCs yet)
- [x] Lots of bug reports and constructive feedback from many different users
- [ ] LO fully supported, NLO (QCD) available on UFO side, but not all MCs

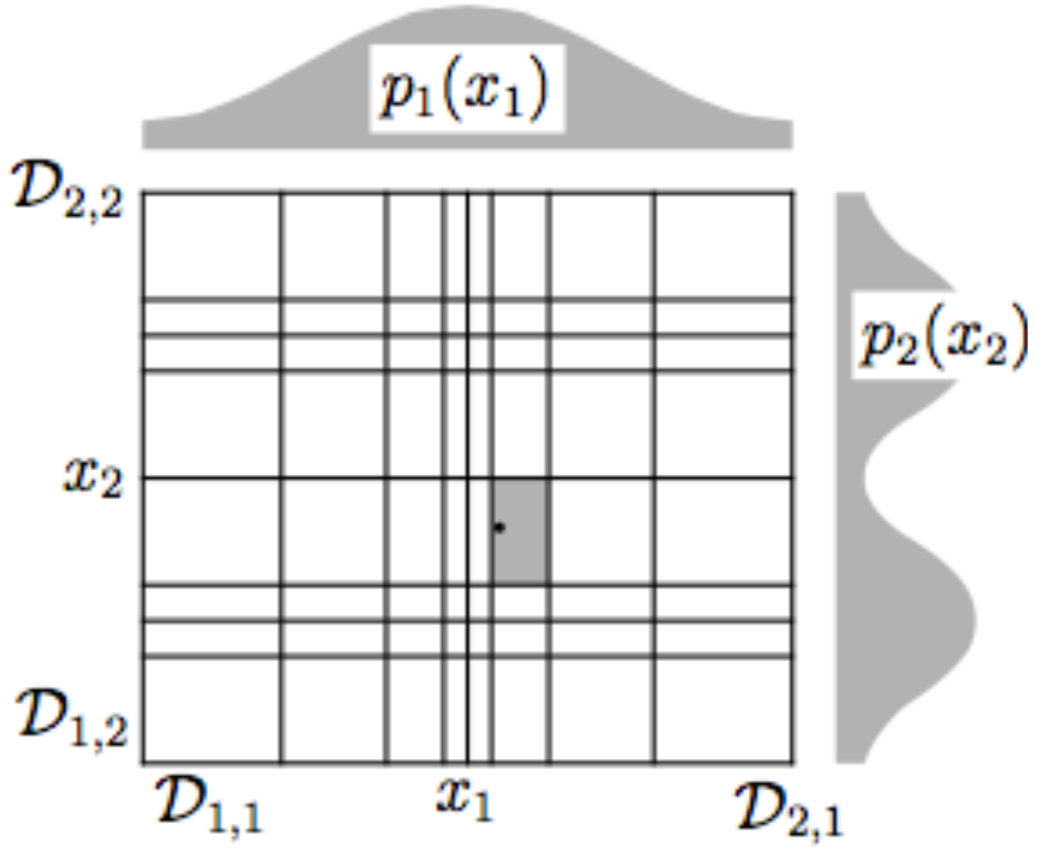


MuC example for SMEFT/HEFT UFO, from: T. Han et al. arXiv:2108.05362

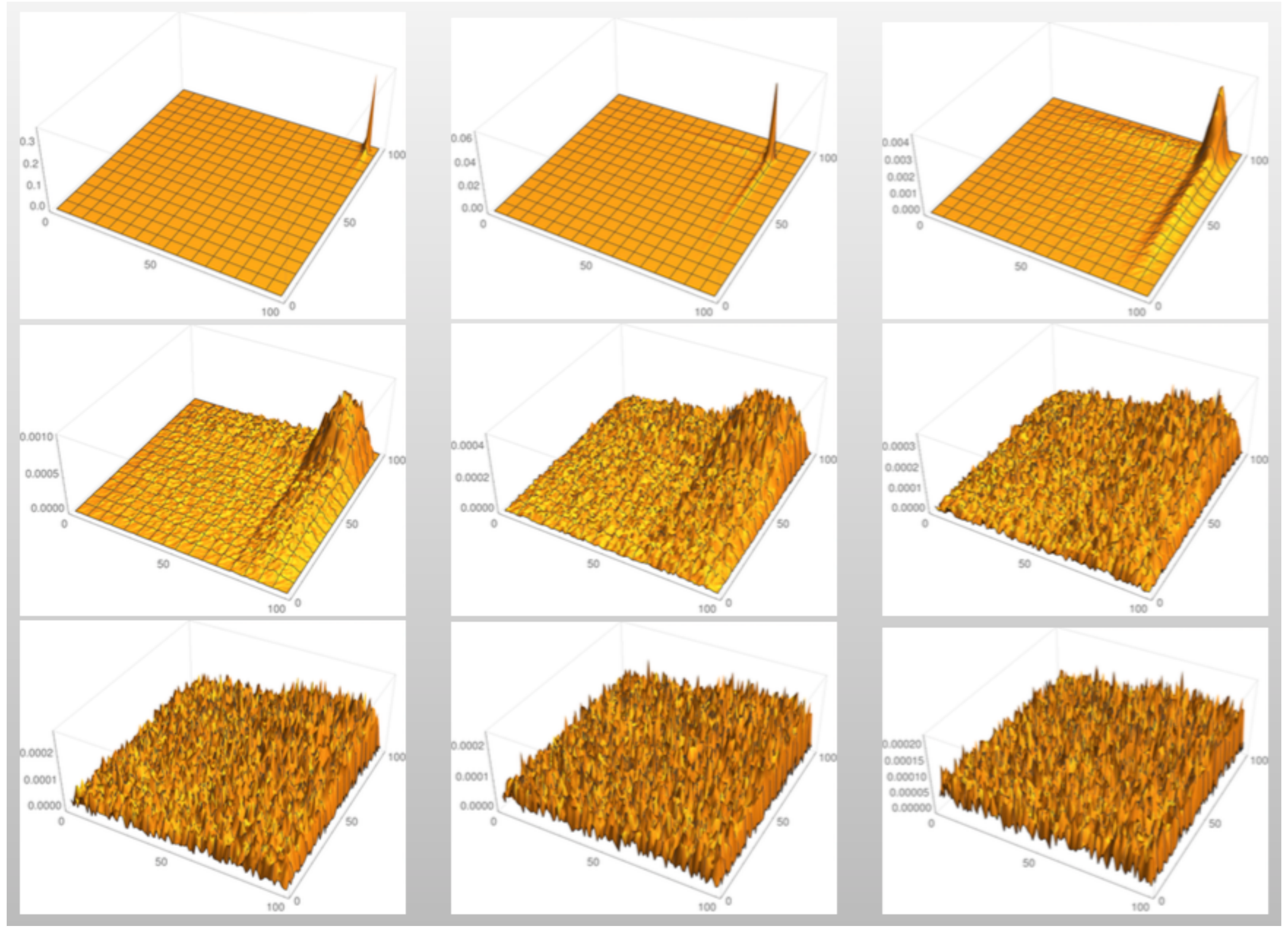
Beam simulations (technial details)

CIRCE2 algorithm T. Ohl, 1996, 2005

↪ Talk by Thorsten Ohl 06/2023: https://indico.cern.ch/event/1266492/



- Adapt 2D factorized variable width histogram to steep part of distribution
- Smooth correlated fluctuations with moderate Gaussian filter [suppresses artifacts from limited GuineaPig statistics
- Smooth continuum/boundary bins separately [avoid artificial beam energy spread]



(171,306 GuineaPig events in 10,000 bins)

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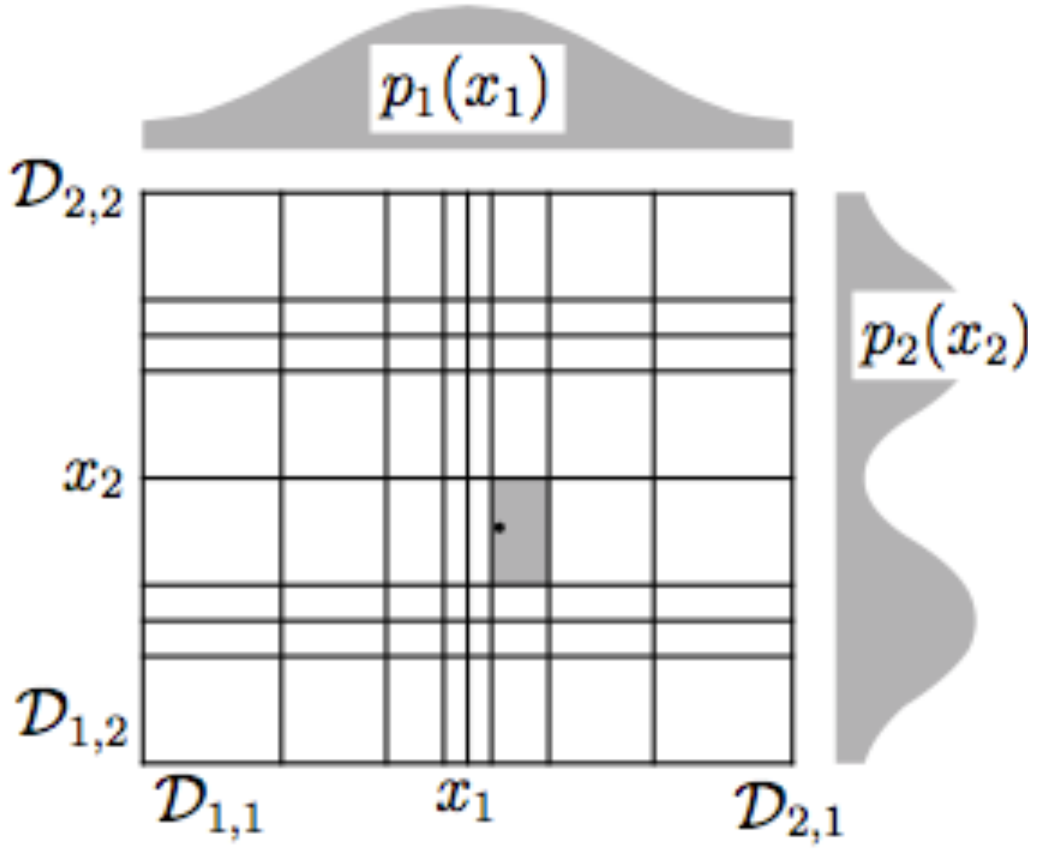

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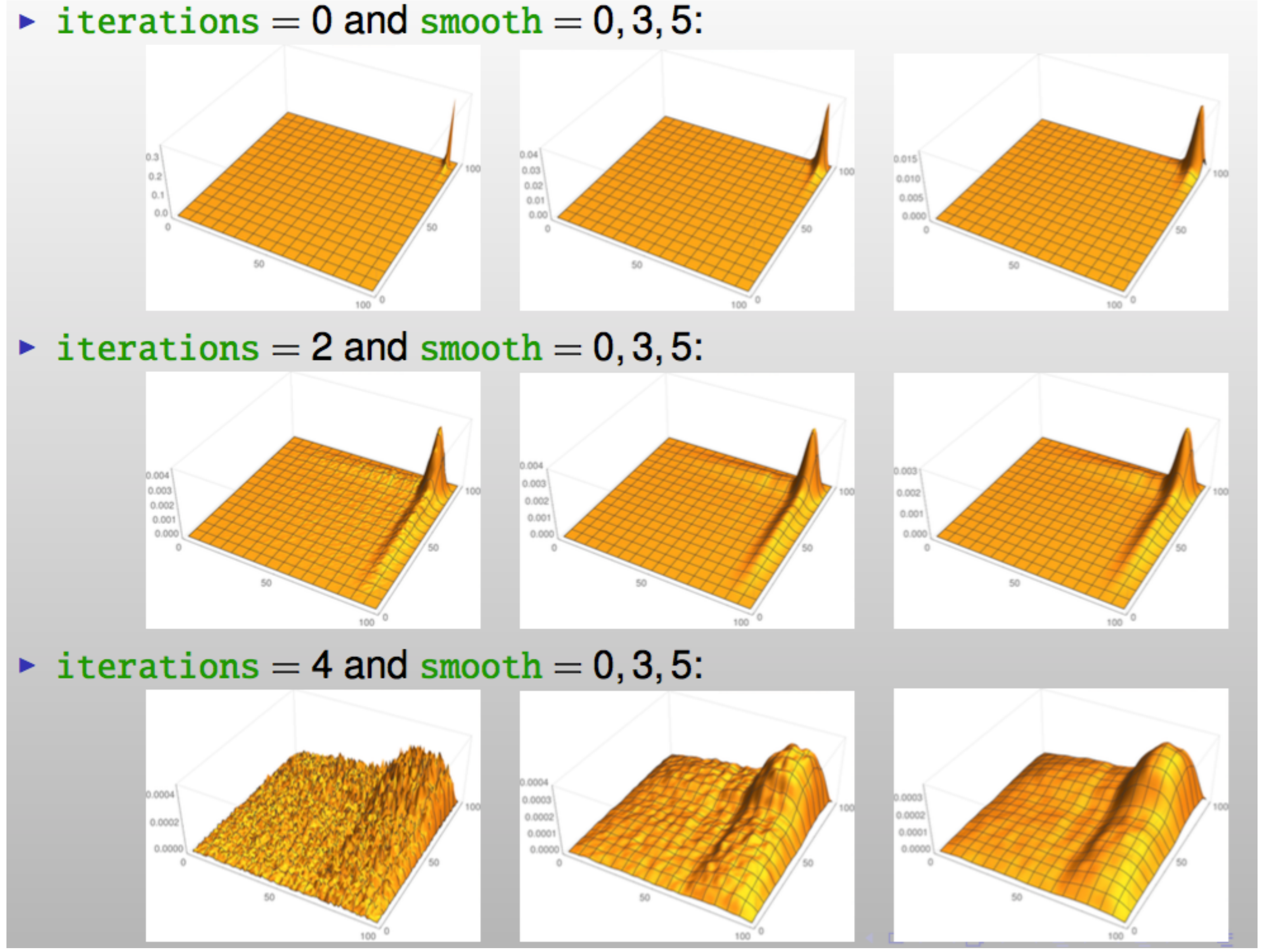

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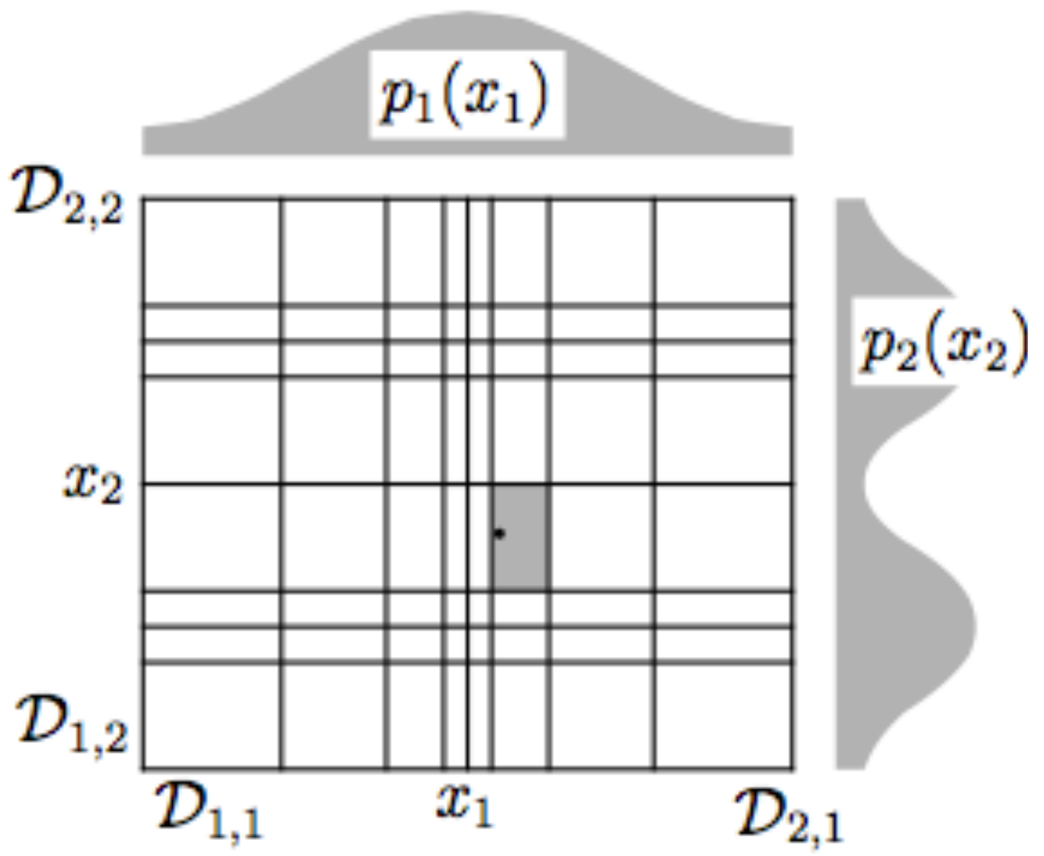

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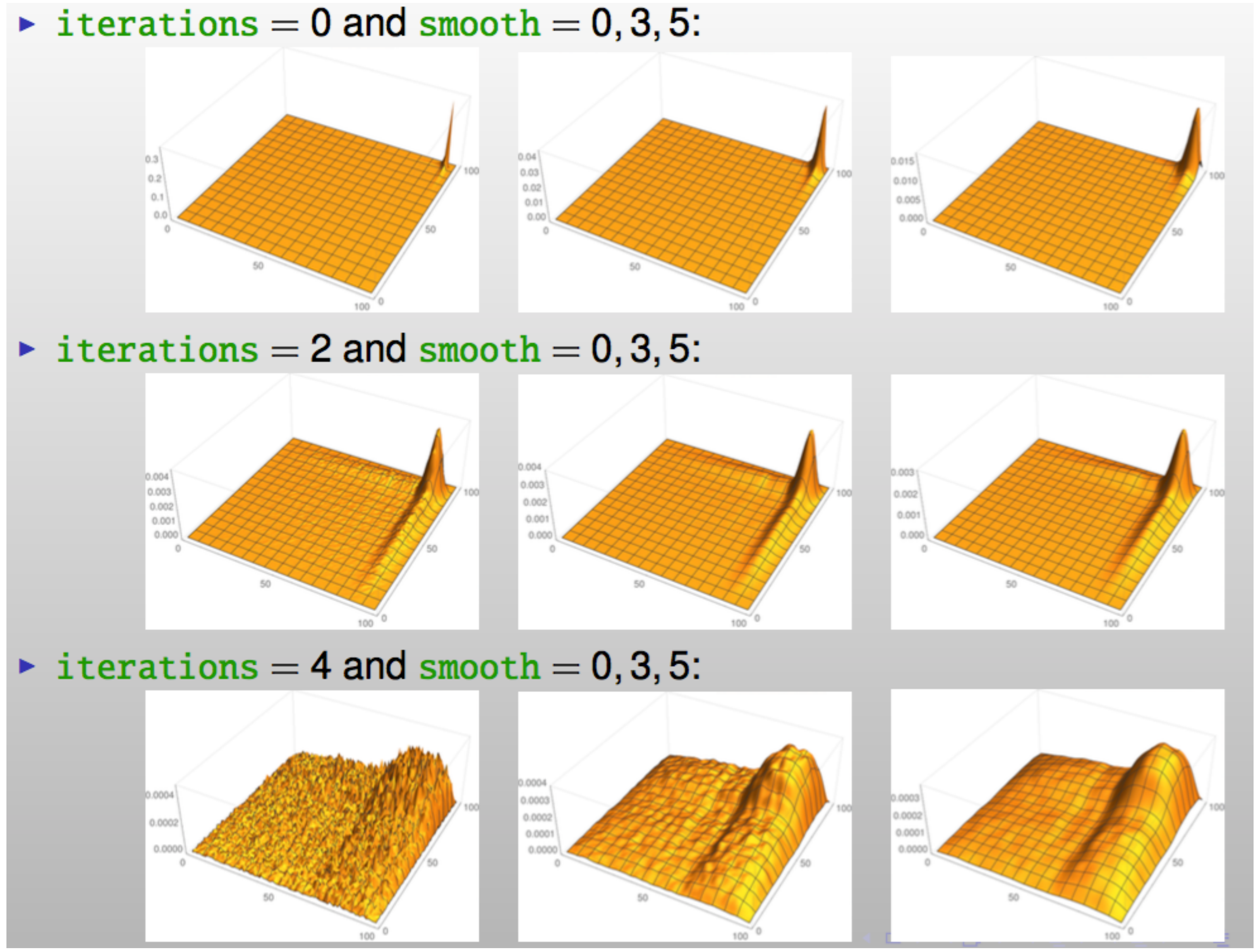


1. Run Guinea-Pig++ with

```
do_lumi=7;num_lumi=100000000;num_lumi_eg=100000000;num_lumi_gg=100000000;
```

to produce `lumi.[eg][eg].out` with (E_1, E_2) pairs.

[Large event numbers, as Guinea-Pig++ will produce only a small fraction!]

2. Run circe2_tool.opt with steering file

```
{ file="ilc500/beams.circe"                          # to be loaded by WHIZARD
  { design="ILC" roots=500 bins=100 scale=250 # E in [0,1]
    { pid/1=electron pid/2=positron pol=0     # unpolarized e-/e+
      events="ilc500/lumi.ee.out" columns=2   # <= Guinea-Pig
      lumi = 1564.763360                      # <= Guinea-Pig
      iterations = 10                         # adapting bins
      smooth = 5 [0,1) [0,1)                  # Gaussian filter 5 bins
      smooth = 5 [1] [0,1) smooth = 5 [0,1) [1] } } }
```

to produce correlated beam description

3. Run WHIZARD with SINDARIN input:

```
beams = e1, E1 => circe2
$circe2_file = "ilc500.circe"
$circe2_design = "ILC"
?circe_polarized = false
```

3 simulation options

1. Unpolarized simulation with unpol. spectra
2. Pol. simulation: unpol. spectra + pol. beams
3. Polarized spectrum with helicity luminosities

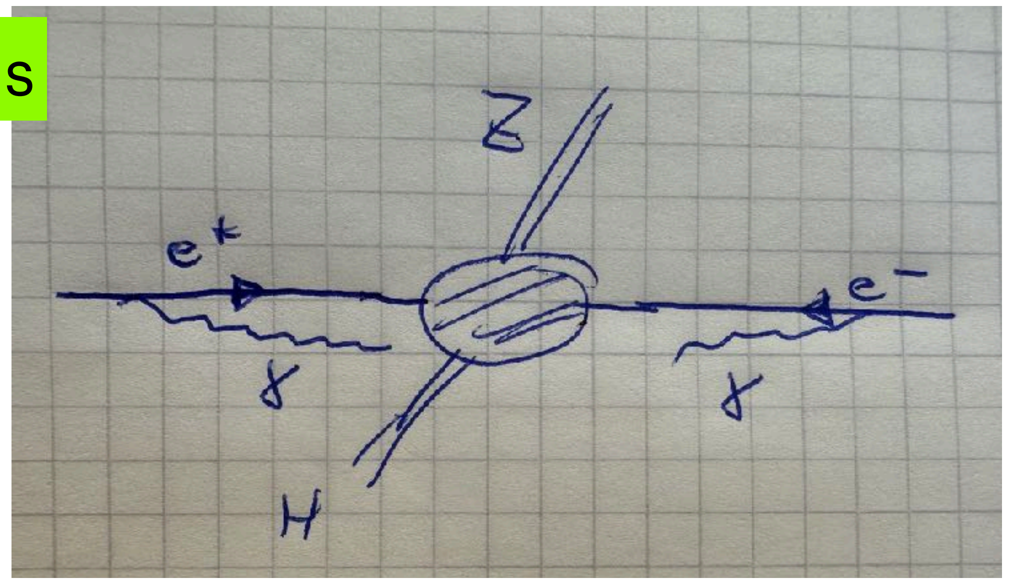
Precision simulations for muon colliders

- What is different to MC event generators for the LHC?
- What is different to MC event generators for (high-energy) electron-positron colliders?
- Where do we stand and what is still needed?

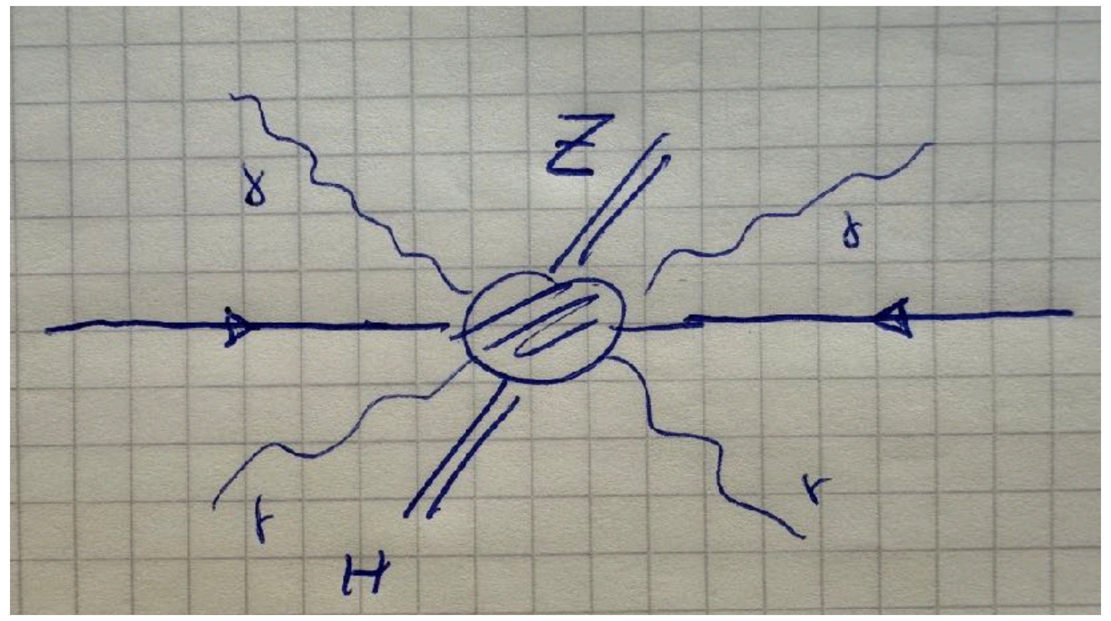
1. Beam simulation: mostly Gaussian beam spread (0.01%, very clean)
2. Initial-state structure: PDFs, collinear vs. soft resummation, cross section predictions ...
3. Hard process (SM): NLO SM automation , NNLO automation (?), QED/EW dominated; EW Sudakov regime
4. Exclusive processes (QED/QCD/EW): photons, interleaved showers, EW fragmentation (?)

Collinear logarithms

$$L = \log \frac{Q^2}{m^2}$$



$$\sigma = \alpha^b \sum_{n=0}^{\infty} \alpha^n \sum_{i=0}^{n} \sum_{j=0}^{n} \varsigma_{n,i,j} L^i \ell^j$$



Soft logarithms

$$\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}$$

QED PDFs — QED Initial State Resummation

Collinear logarithms

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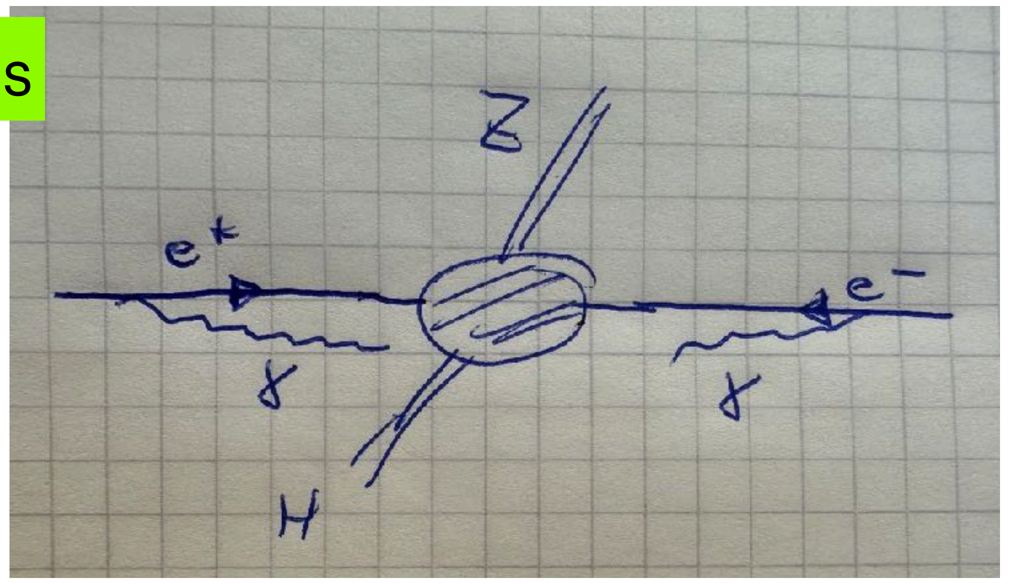

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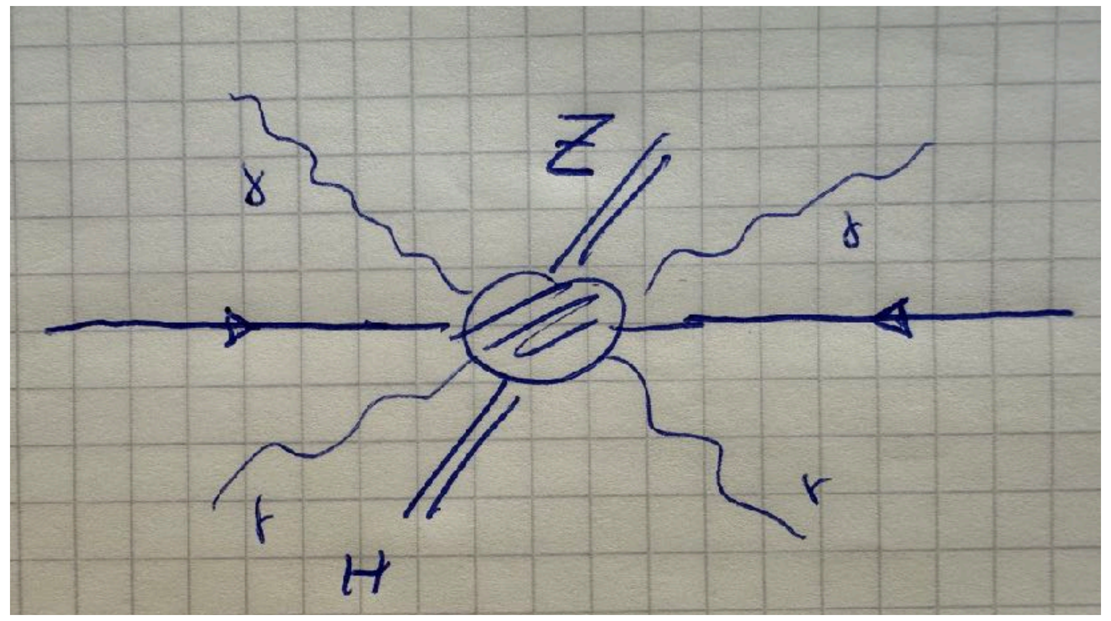


Soft logarithms

$\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}$

- Different factorization schemes: focus on collinear logs, $\log \frac{Q^2}{m_\mu^2}$, vs. soft logs, $\log \frac{Q^2}{\overline{E_\gamma^2}}$, cf. 2203.12557
- YFS (Yennie-Frautschi-Suura), cf. e.g. 2203.10948 $\quad d\sigma = \sum_{n_\gamma}^{\infty} \frac{\exp[Y_{res.}]}{n_\gamma!} \prod_{j=1}^{n_\gamma} \left[d\text{LIPS}_j^\gamma S_{res.}(k_j) \right] [\sigma_0 + \text{corrections}]$
  - Universal soft exponentiation factor, provides n_γ exclusive resolved photons with (almost) exact kinematics
  - Implemented in "Krakow" MCs (`BHLUMI/BHWIDE`, `KORAL(W/Z)`, `KKMC-ee`, `YFS(WW/ZZ)`, also: `Sherpa`, w.i.p.: `Whizard`
- Collinear factorization: universal lepton QED PDFs, LL: $(\alpha L)^k$, NLL: $\alpha(\alpha L)^{k-1}$

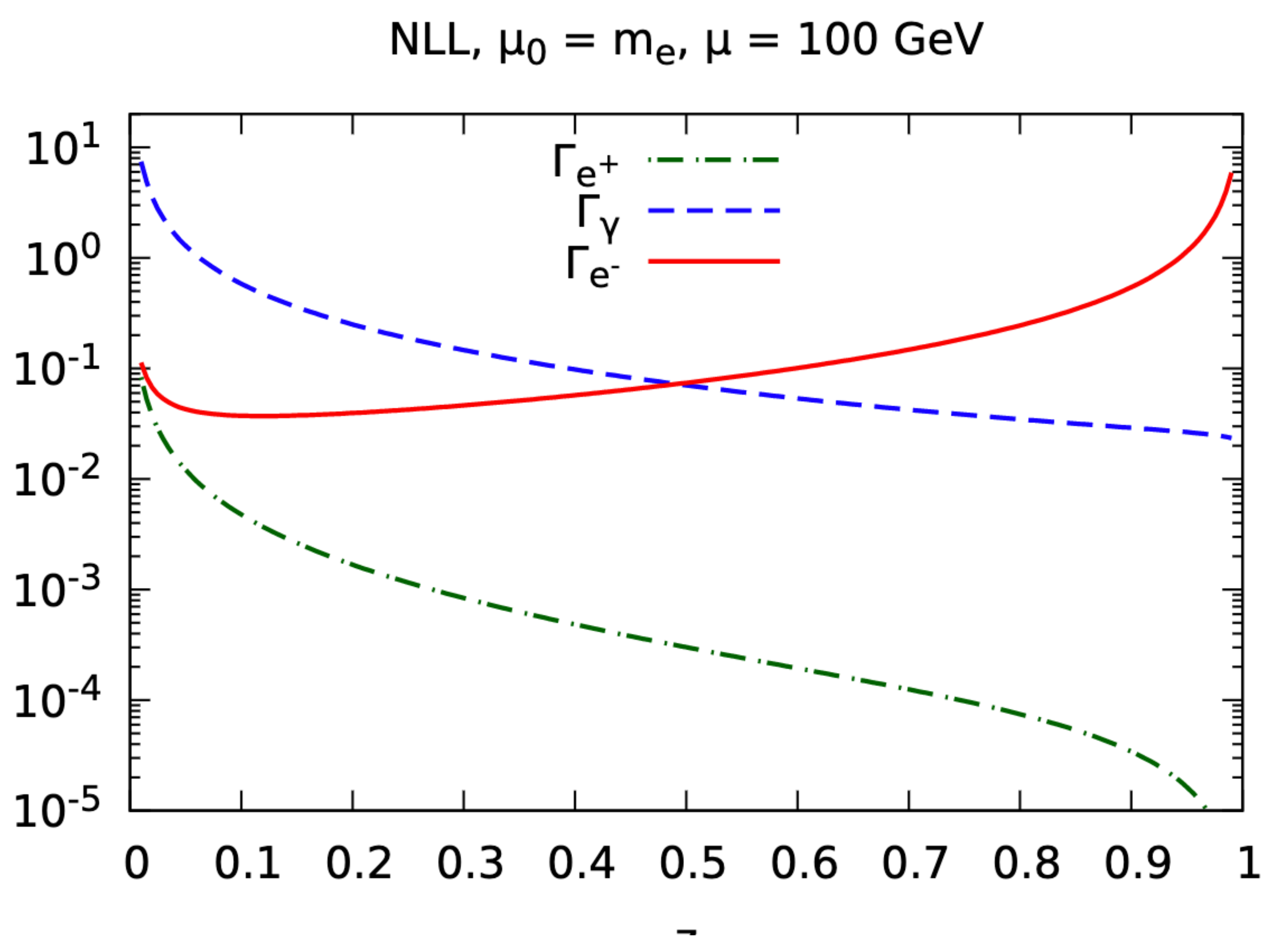
$$d\sigma_{kl}(p_k, p_l) = \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_- \, \Gamma_{i/k}(z_+, \mu^2, m^2) \, \Gamma_{j/l}(z_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) + \mathcal{O}\left(\left(\frac{m^2}{s} \right)^p \right)$$

$$\mathbb{P}_S = \begin{pmatrix} P_{\Sigma\Sigma} & P_{\Sigma\gamma} \\ P_{\gamma\Sigma} & P_{\gamma\gamma} \end{pmatrix} ,$$

$$P_{NS} = P_{e^\pm e^\pm} - P_{e^\pm e^\mp} \equiv P_{ee}^V - P_{e\bar{e}}^V .$$

QED PDFs — Collinear Factorization

- Collinear resummation LO/LL Gribov/Lipatov, 1972; Kuraev/Fadin, 1985; Skrzypek/Jadach, 1992; Cacciari/Deandrea/Montagna/Nicrosini, 1992
- NLO QED PDFs, collinear evolution @ NLL Frixione, 1909.0388; Bertone/Cacciari/Frixione/Stagnitto, 1911.12040 + 2207.03265
- Inclusive in all initial-state photons
- Gives most precise normalization of total cross section
- Integrable power-like singularity $1/(1-z)$ for $z \to 1$
- Numerical stability differs in different QED renormalization schemes, DIS vs. $\overline{\text{MS}}$
- *Also:* fast interpolation (CTEQ-like) grids available
- Implementations available in `MG5` and `Whizard`
- Different levels of precision possible: NLL+NLO, LL+NLO, LL+NLO, LL+LO
- Different names in literature: electron structure functions, ISR structure functions
- "Photon PDF" (a.k.a. EPA, Weizsäcker-Williams) Γ_γ, peaked at small *z*
- Very well known from ILC/CLIC simulations: "virtual photon"-induced processes
- At very high energies lepton colliders become $\gamma\gamma$ colliders (like LHC is *gg*)



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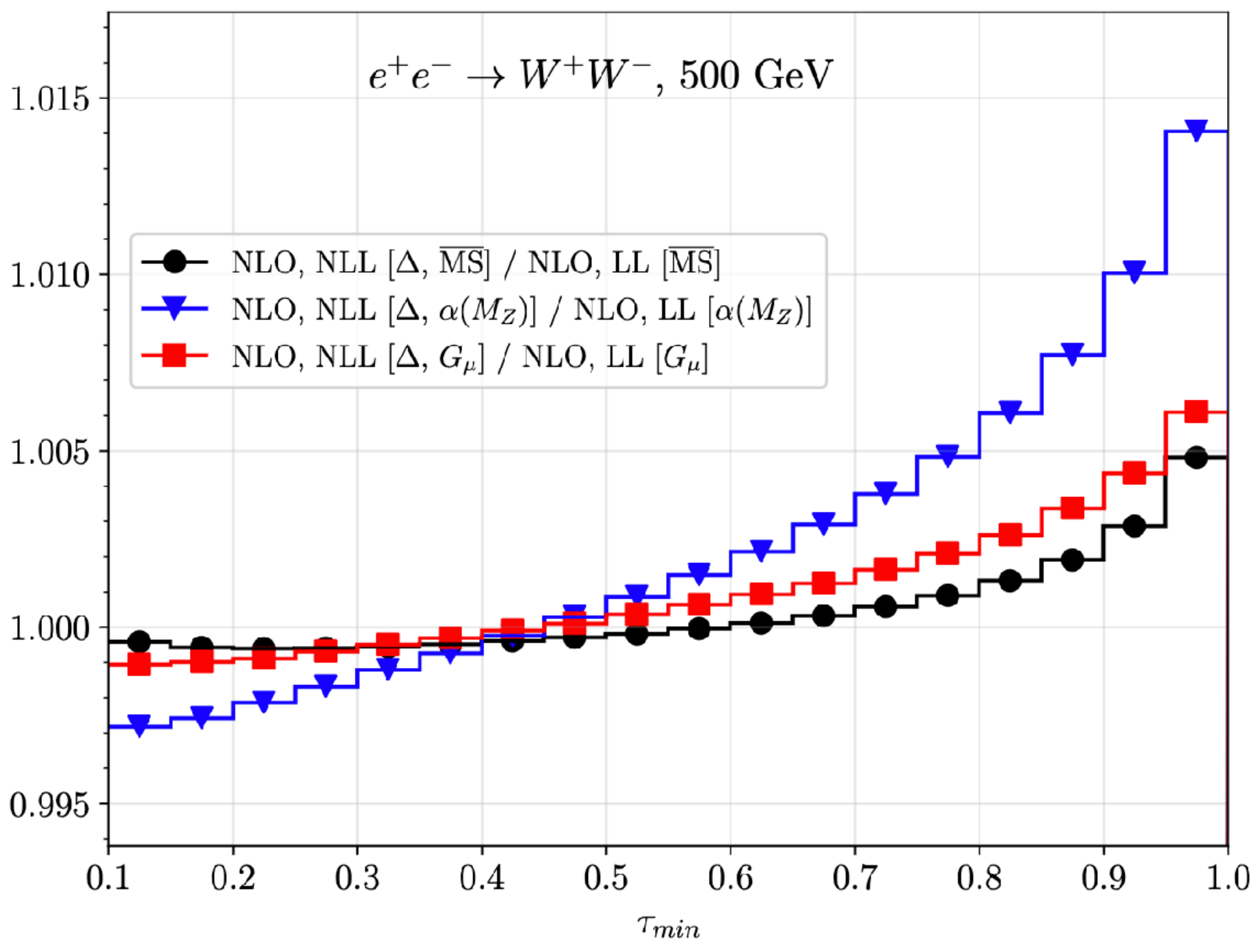

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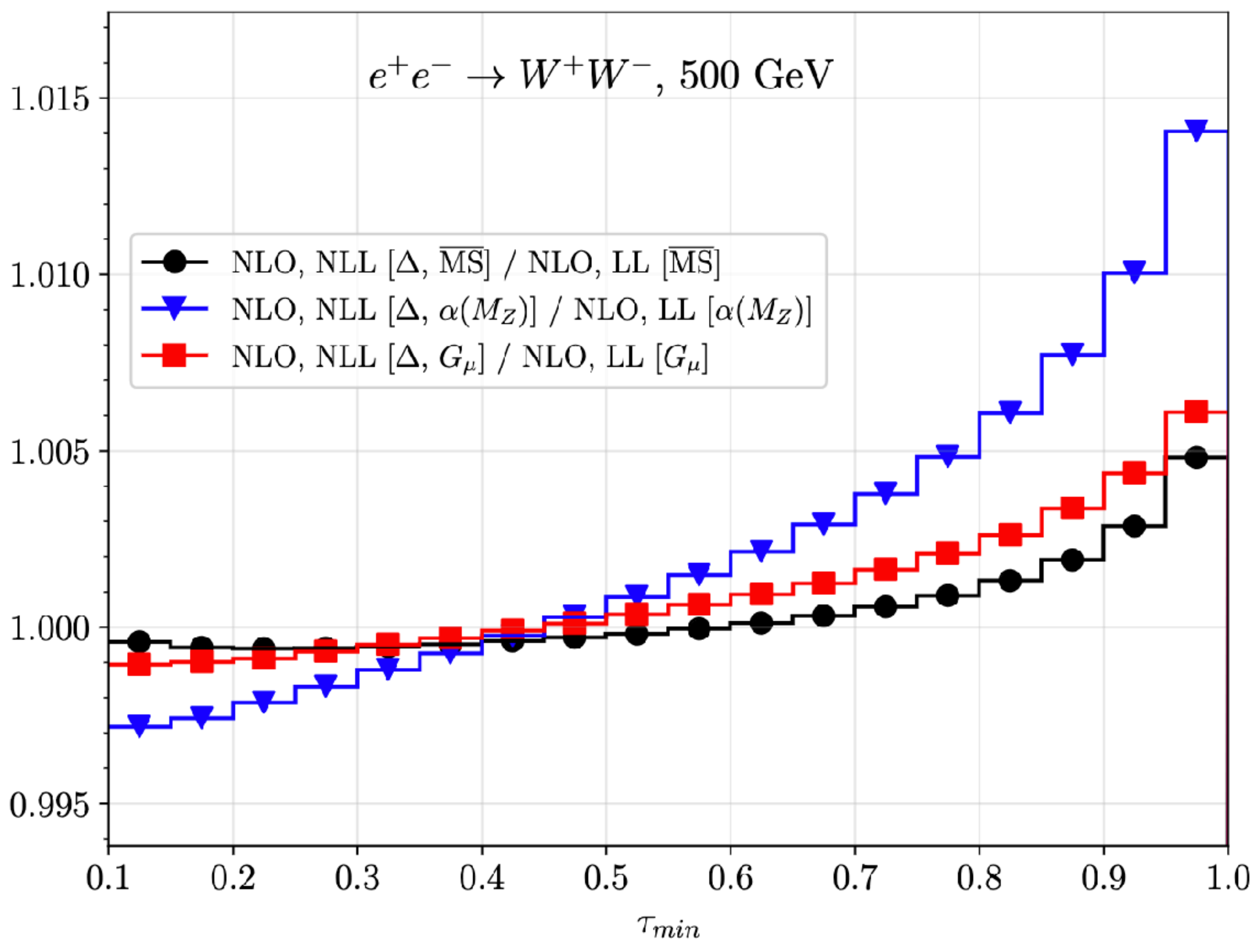

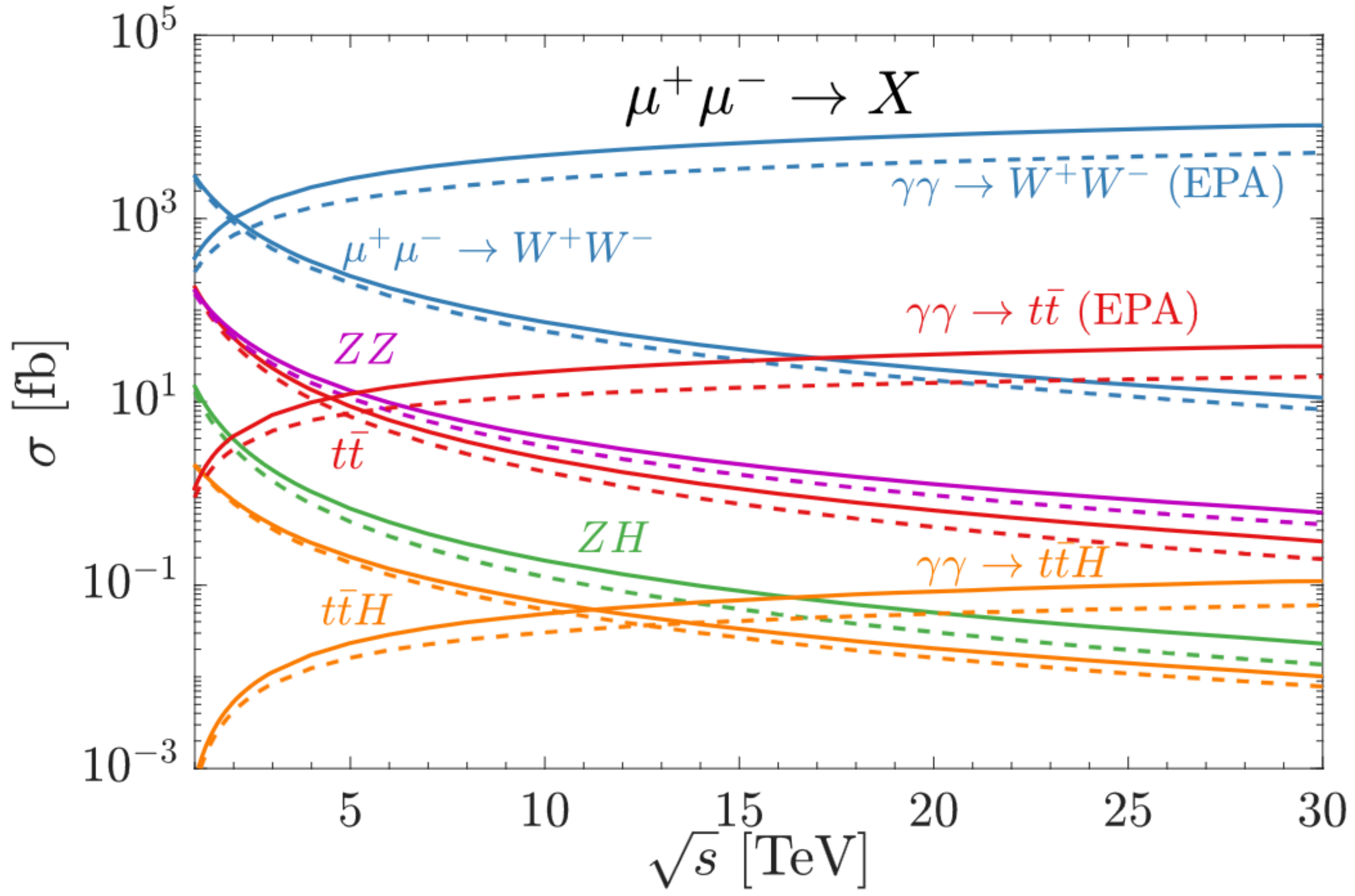

Han/Ma/Xie, 2007.14300

EW PDFs — EW Collinear Factorization

- Collinear factorization not in QED, but in full SM

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- Ancient name (from SSC times!): EWA (“Effective *W* approximation)
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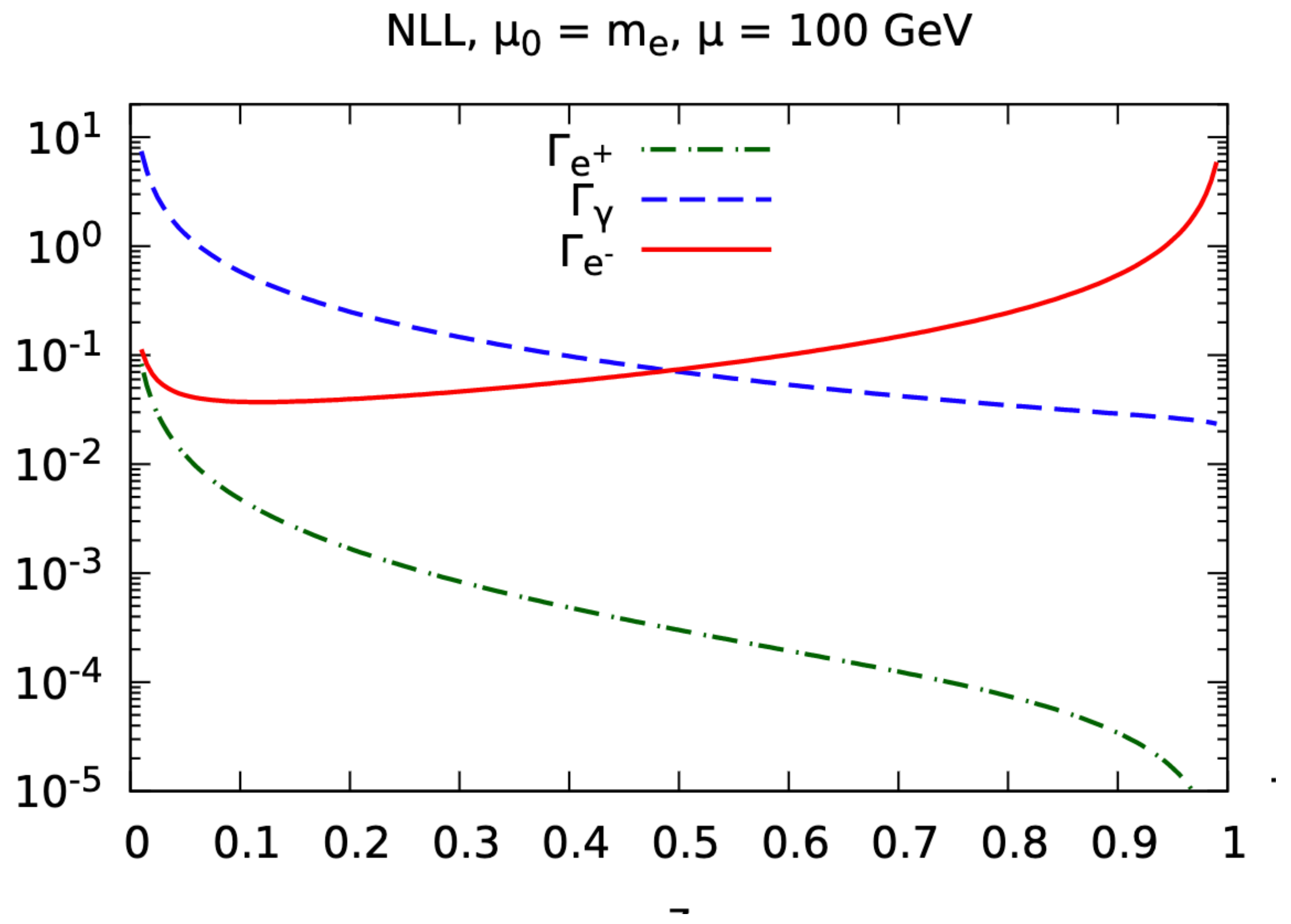

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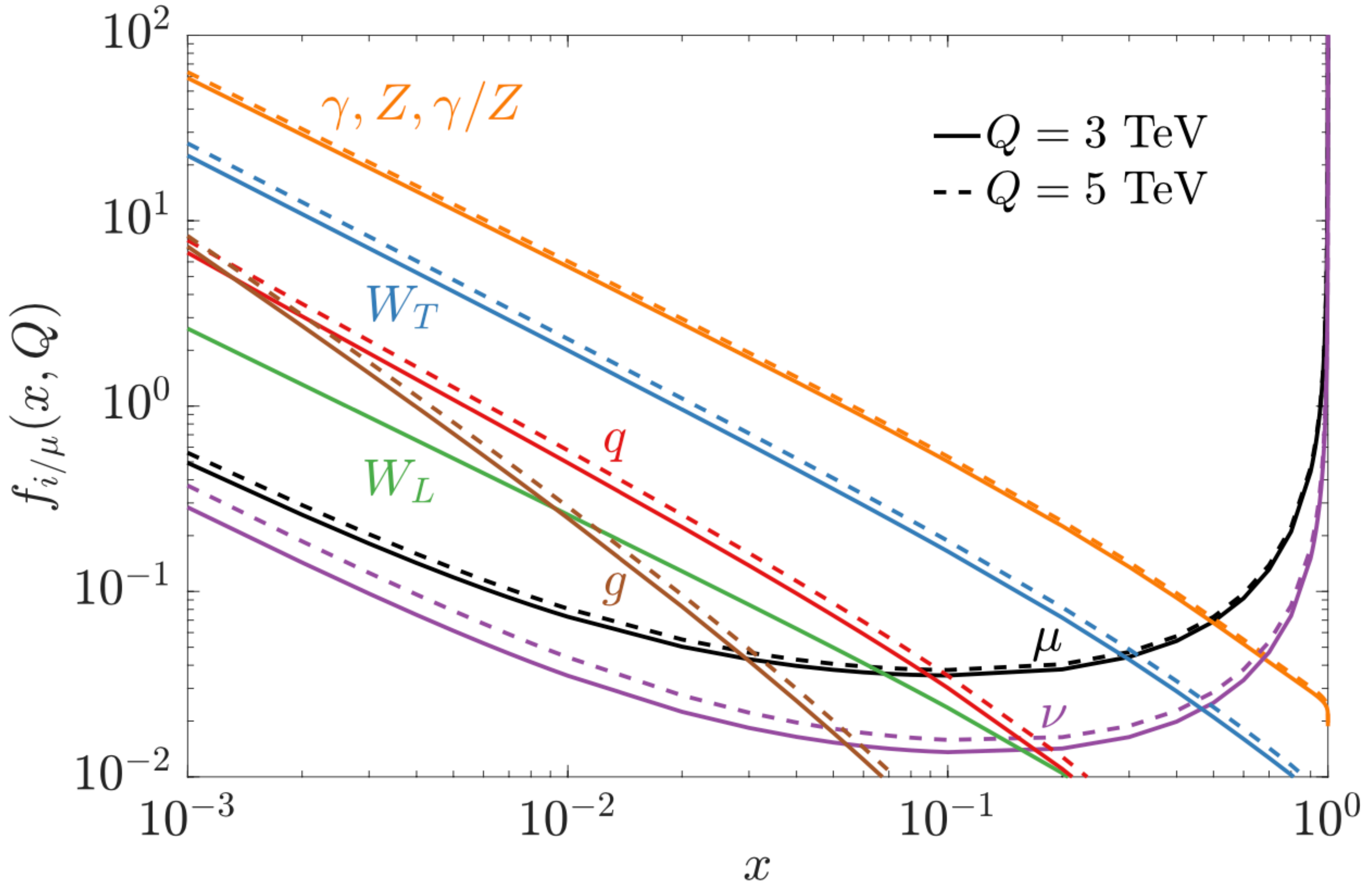

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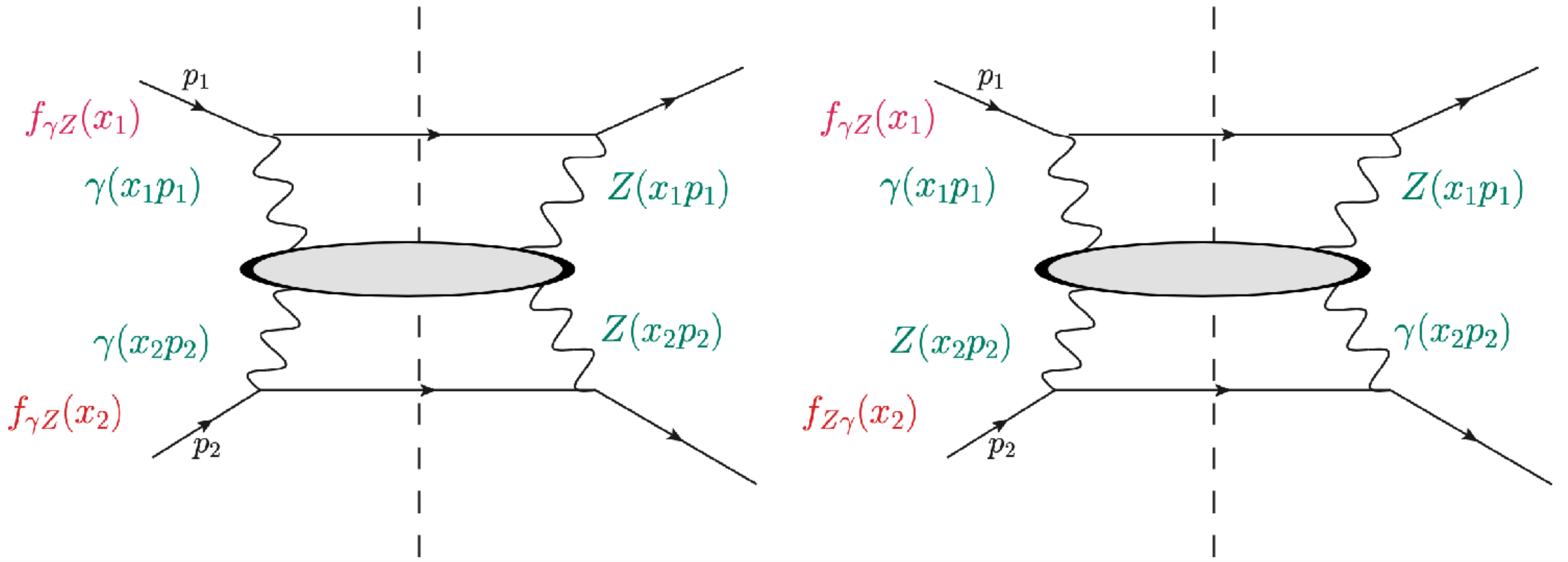
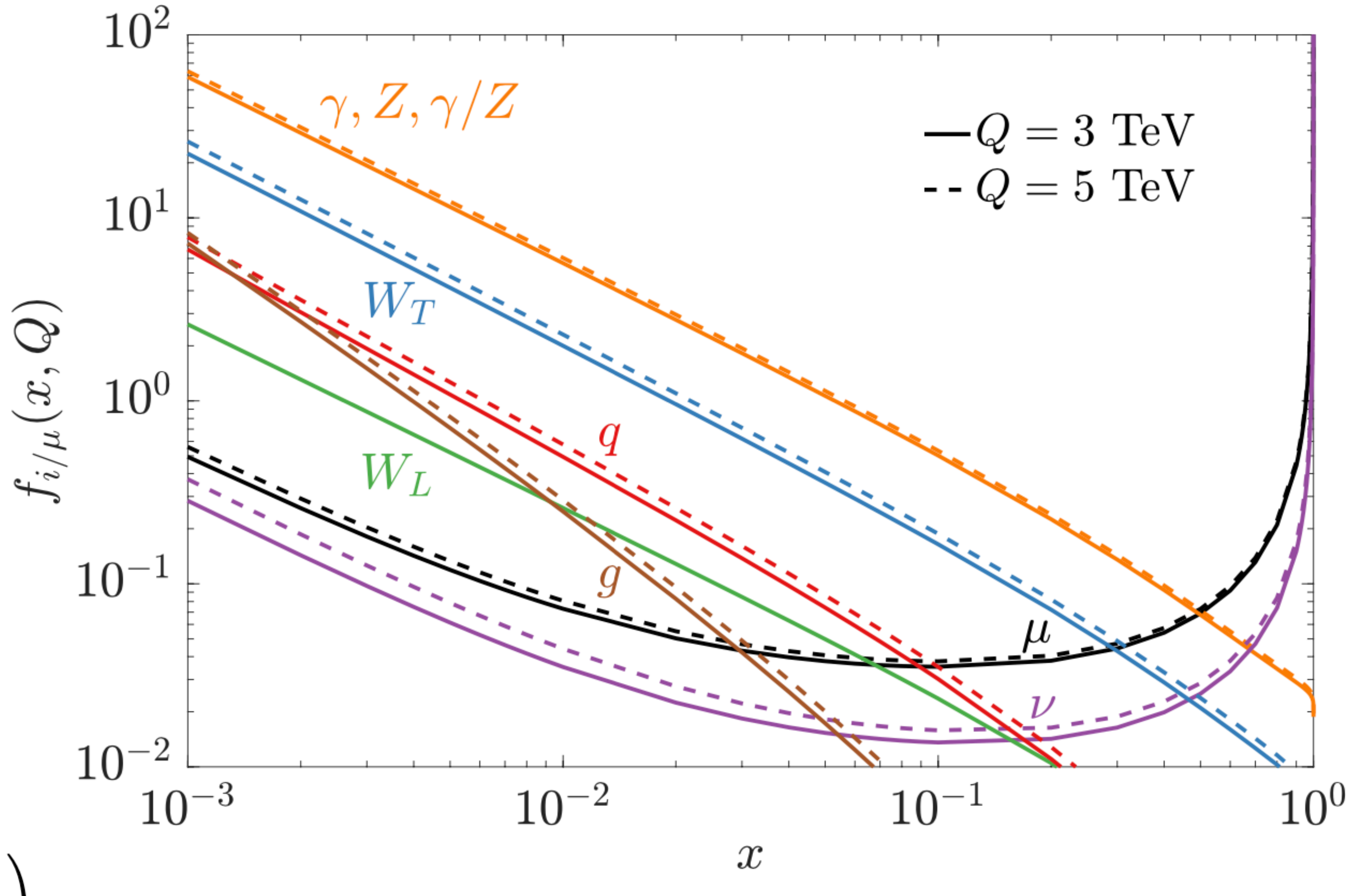


$$\begin{array}{c} \\ \gamma \\ Z \\ \gamma/Z \end{array} \begin{array}{ccc} \gamma & Z & \gamma/Z \\ \hat\sigma(\gamma,\gamma) & \hat\sigma(\gamma,Z) & \hat\sigma(\gamma,\gamma/Z) \\ \hat\sigma(Z,\gamma) & \hat\sigma(Z,Z) & \hat\sigma(Z,\gamma/Z) \\ \hat\sigma(\gamma/Z,\gamma) & \hat\sigma(\gamma/Z,Z) & \hat\sigma(\gamma/Z,\gamma/Z) \end{array}$$



- $\gamma\gamma$ part (quasi-) identical to collinear QED lepton PDFs
- Factorization has coherent interference $\gamma\gamma/\gamma Z/ZZ$
- Trivial on the PDF infrastructure side, complication for ME generation
- Work in progress in `MG5` and `Whizard`
- Has to be accompanied by EW fragmentation functions (event selection!)

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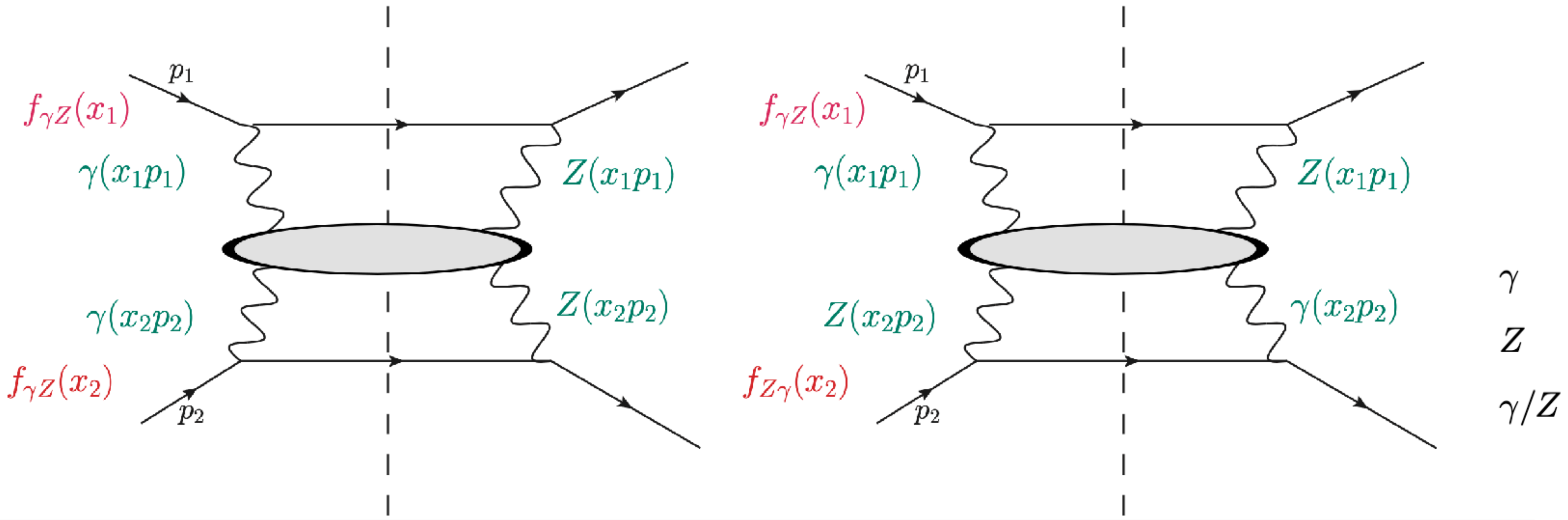
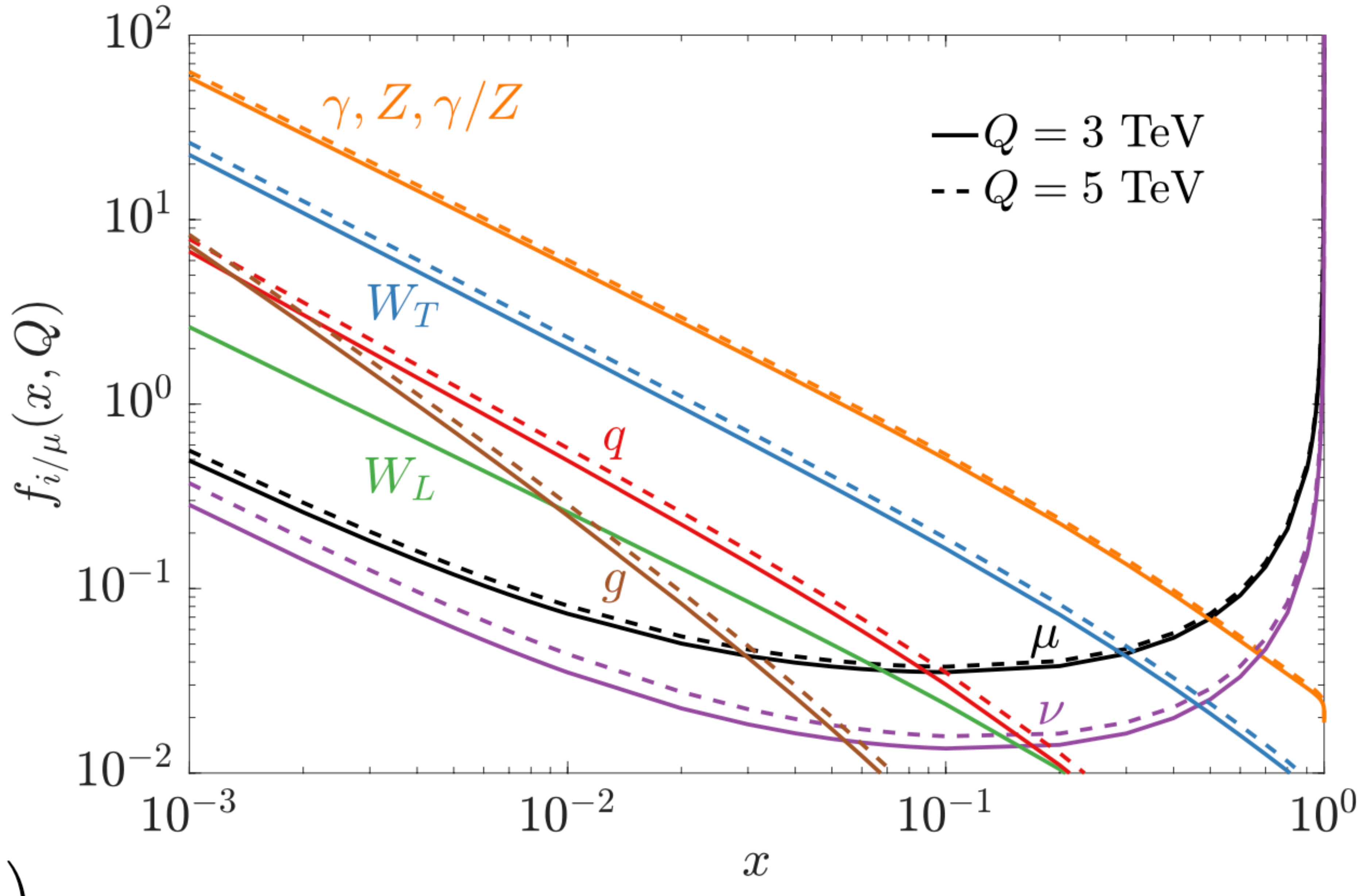
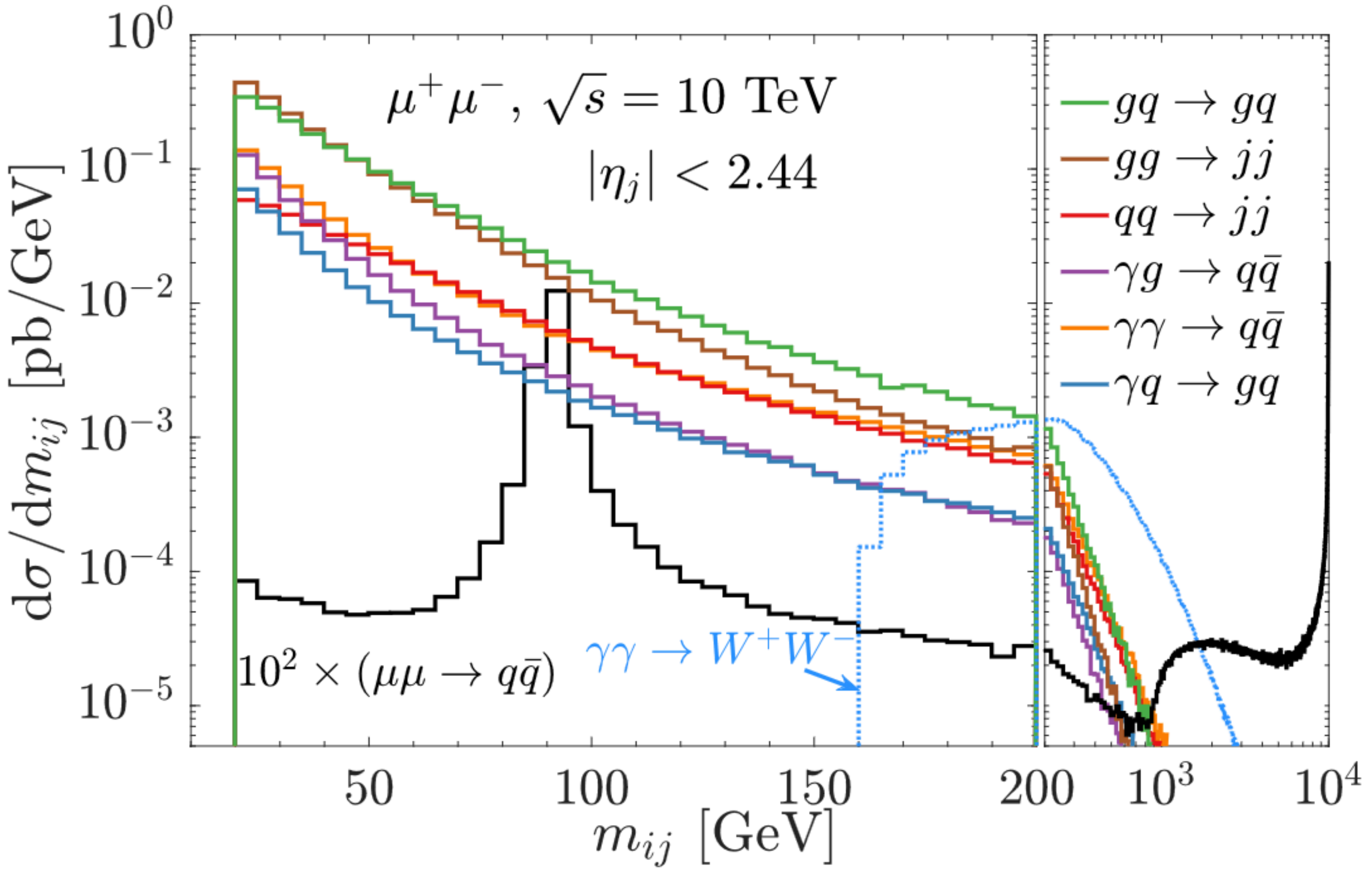
- Fully inclusive in collinear/forward/beam direction
- *Also:* fast interpolation (CTEQ-like) grids available



$$\begin{array}{c} \\ \gamma \\ Z \\ \gamma/Z \end{array} \begin{array}{ccc} \gamma & Z & \gamma/Z \\ \left(\begin{array}{c} \hat\sigma(\gamma,\gamma) \\ \hat\sigma(Z,\gamma) \\ \hat\sigma(\gamma/Z,\gamma) \end{array}\right. & \begin{array}{c} \hat\sigma(\gamma,Z) \\ \hat\sigma(Z,Z) \\ \hat\sigma(\gamma/Z,Z) \end{array} & \left.\begin{array}{c} \hat\sigma(\gamma,\gamma/Z) \\ \hat\sigma(Z,\gamma/Z) \\ \hat\sigma(\gamma/Z,\gamma/Z) \end{array}\right) \end{array}$$





- $\gamma\gamma$ part (quasi-) identical to collinear QED lepton PDFs
- Factorization has coherent interference $\gamma\gamma/\gamma Z/ZZ$
- Trivial on the PDF infrastructure side, complication for ME generation
- Work in progress in `MG5` and `Whizard`
- Has to be accompanied by EW fragmentation functions (event selection!)

(Resonance) Matching to shower / hadronization

- Problem: $\mu^+\mu^- \to jjjj$ not dominated by highest α_s power, but by resonances $\mu^+\mu^- \to WW/ZZ \to (jj)(jj)$

  Solution: proper merging w/ resonant subprocesses by resonance histories
- MC generators allow to pass resonance history to SMC

```
?resonance_history = true
resonance_on_shell_limit = 4
resonance_on_shell_turnoff = 1
resonance_background_factor = 1e-10
```