Energy evolution of QGP under two loop correction at finite chemical potential

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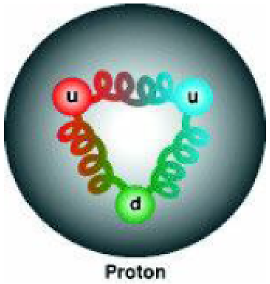
Outlines

- Introduction
- Quark Gluon Plasma
- QGP fireball in mean field potential
- Two loop correction at finite chemical potential
- Result and conclusion

What is Quark-Gluon Plasma?

At room temperature, quarks and gluons are always confined inside colorless objects (hadrons):

protons, neutrons, pions, …..

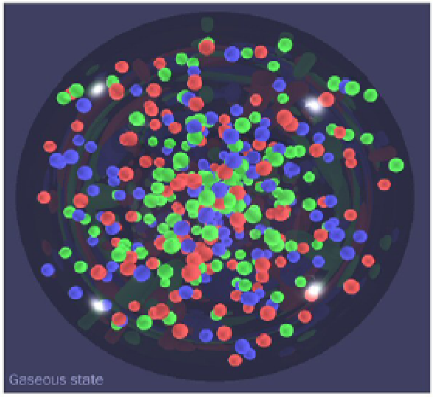


Very high temperature (asymptotic freedom):

→ Interactions become weak

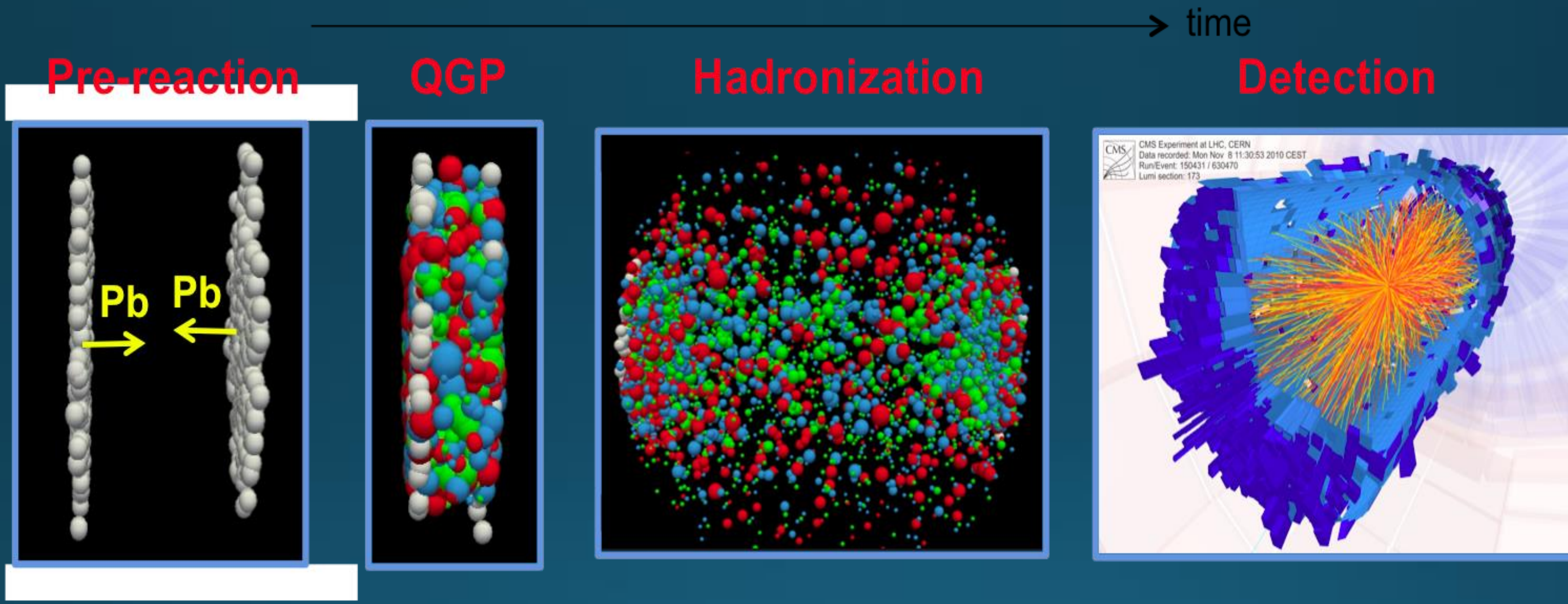
→ quarks and gluons deconfined

→ Quark-gluon plasma (QGP)



Infinitely high temperature:
QGP may behave like an ideal gas.

Experimental Behaviour of QGP



Partition Function and Free Energy at chemical potential

The general Partition function of the system defined as[1]:

$$Z(T,\mu,V) = Tr\{\exp[-\beta(\widehat{H} - \mu\widehat{N})]\}$$

$$F_i = T ln Z(T,\mu,V)$$

$$\beta = {}^{1}/_{T}$$

where, μ and $\widehat{N}$ are the chemical potential and quark number.

QGP Fireball in mean field potential

Free energy is given by [2]

$$F_i = \mp T g_i \int dk \rho_i(k) \ln\left(1 \pm e^{-\left(\frac{\sqrt{m_i^2+k^2}}{T}\right)}\right)$$

Where, $\rho_i(k)$ is density of states of particular particle, i (quarks, gluons, pions etc.) being the number of states with momentum between k and k+dk in spherically symmetric situation. This density of state is calculated through Thomas-Fermi model

- sign corresponds to fermions
+ sign corresponds to bosons
T corresponds to temperature

$g_i =$ Color degeneracy, 6 for quarks and 8 for gluons

The Thomas Fermi model develops the density of states like the following

$$\int \rho_{q,g}(k)dk = \frac{\left(-V_{conf}(k)^3\right)\nu}{3\pi^2}$$

$$\rho_{q,g}(k) = \left(\frac{\nu}{\pi^2}\right)\left[-V_{Conf}(k)^2\left(\frac{dV_{Conf}(k)}{dk}\right)\right]_{q,g}$$

Where

ν = volume occupied by QGP

k = relativistic four momentum in natural units

V_{conf} = confining potential for quarks , gluons

Interface Free Energy, which is used to replace MIT Bag is [4]

$$F_{interface} = \frac{1}{4}\gamma R^2 T^3$$

$$\gamma = \sqrt{2} \times \sqrt{\left(\frac{1}{\gamma_g}\right)^2 + \left(\frac{1}{\gamma_q}\right)^2}$$

Where,

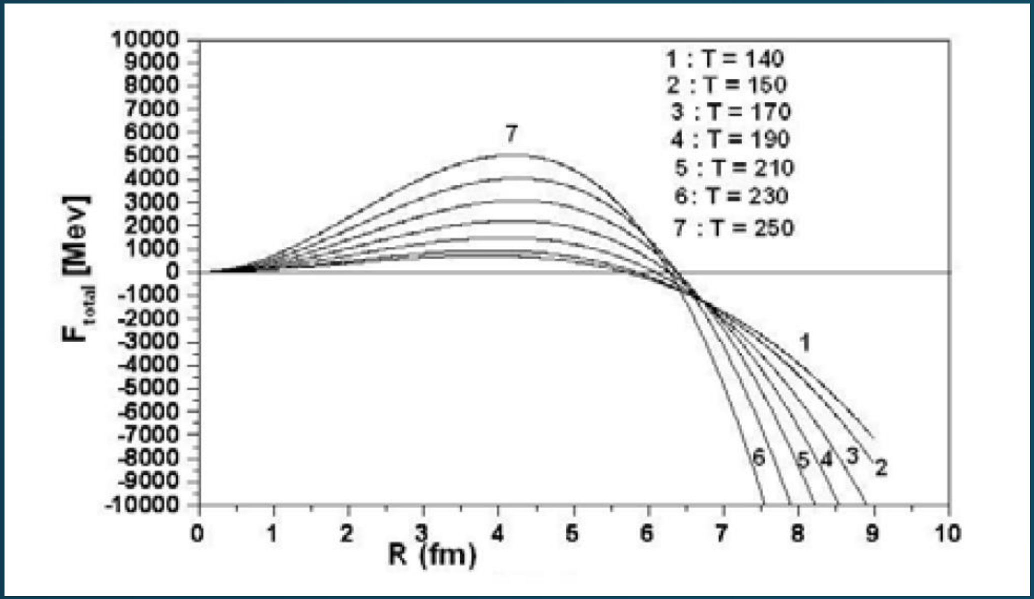
γ = modification introduced to take care of quark and gluons respectively

R = radius of droplet

γ_q, γ_g are the quark and gluons flow parameters

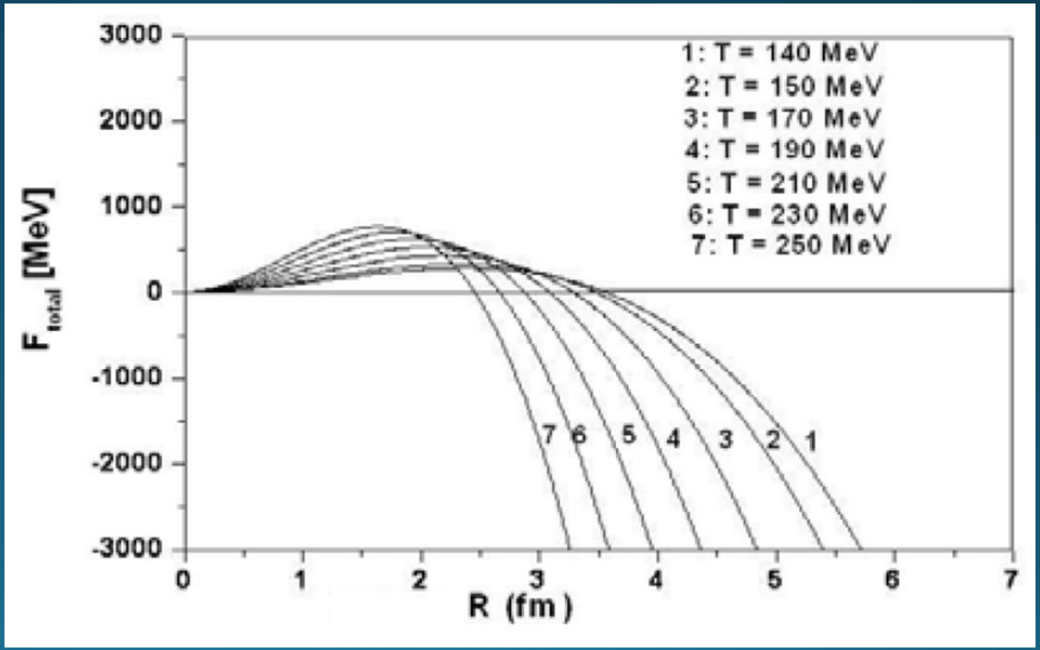
Total free energy of QGP through this model is

$$F_{total} = \sum F_{q=u,d,s} + F_{interface} + F_{mesons} + F_{gluons}$$



$F_{total}\ at\ \gamma_g = 6\gamma_q, \qquad \gamma_q = 1/6$
for various temperatures

$F_{total}\ at\ \gamma_g = 8\gamma_q, \qquad \gamma_q = 1/6$
for various temperatures



Density of state in phase space with two loop correction [3]

$$V_{conf}(k) = \frac{8\pi}{k}\gamma_{g,q}\alpha_s(k,\mu)\left(T^2 + \frac{\mu^2}{\pi^2}\right)\left[1 + \frac{\alpha_s(k,\mu)}{4\pi}\alpha_1 + \left(\frac{\alpha_s(k,\mu)}{4\pi}\right)^2\alpha_2\right] - \frac{m_0^2}{2k}$$

Where,

For μ *=300 MeV*

$$\gamma_g = 1/6$$

$$\gamma_q = (8-10)\gamma_g$$

For $\mu = 0$

$$\gamma_q = 1/14$$

$$\gamma_g = (44-56)\gamma_q$$

$$\alpha_s(k,\mu) = \frac{4\pi}{(33-2n_f)\ln\left(1+\frac{k^2+\mu^2}{\Lambda^2}\right)}$$

Λ = QCD Parameter having taken as 1.5 GeV

$a_1 = 2.5833 - 0.2778n_1$

$\alpha_2 = 28.5433 - 4.1471n_l + 0.0722n_l^2$

ν = Volume occupied by QGP

n_1 = Number of light quark elements

Density of state in phase space with two loop correction at chemical potential

$$m^2(T,\mu) = 2\gamma^2 g^2(k)\left(T^2 + \frac{\mu^2}{\pi^2}\right)(1 + g^2(k)a_1)$$

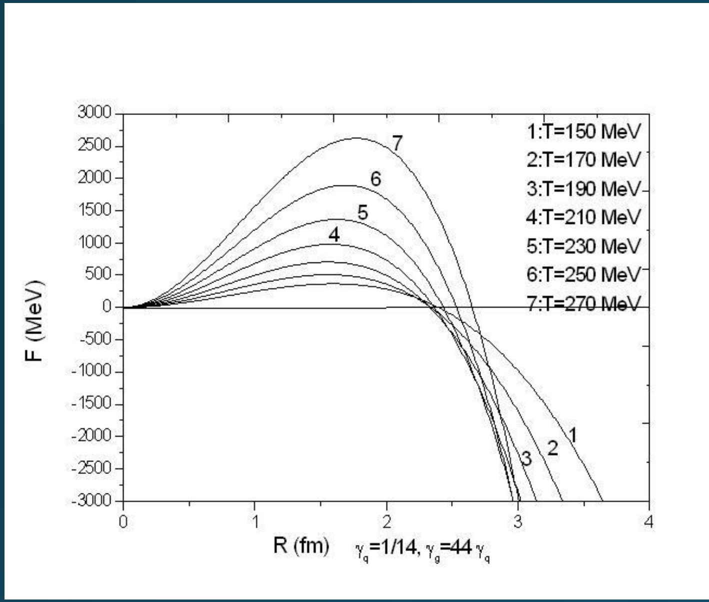
$$\rho_{q,q}(k,\mu) = \left(\frac{\nu}{\pi^2}\right)\left[\frac{\gamma^3(T^2+\mu^2/\pi^2)}{2}\right]^3 g^6(k)A$$

$$\rho_{q,q}(k,\mu = 0) = \left(\frac{\nu}{\pi^2}\right)\left[\frac{\gamma^3(T^2)}{2}\right]^3 g^6(k)A$$

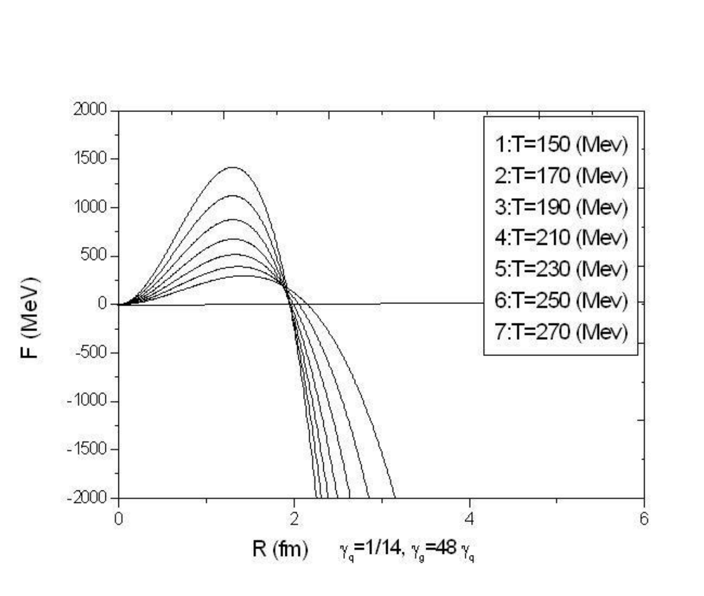
$$A = \left\{1 + \frac{(\alpha_s(k,\mu)\alpha_1)}{\pi} + \frac{((\alpha_s(k,\mu))^2\alpha_2}{\pi^2}\right\}^2 \left[\frac{1 + \frac{(\alpha_s(k,\mu)\alpha_1)}{\pi}}{k^4} + \frac{\frac{((\alpha_s(k,\mu))^2\alpha_2}{\pi^2}}{k^4} + \frac{1 + \frac{(2\alpha_s(k,\mu)\alpha_1)}{\pi} + \frac{((3\alpha_s(k,\mu))^2\alpha_2}{\pi^2}}{k^2(k^2+\Lambda^2)\ln\left(1 + \frac{k^2+\mu^2}{\Lambda^2}\right)}\right]$$

$$\alpha_s(k,\mu) = \frac{4\pi}{29\ln\left(1 + \frac{k^2+\mu^2}{\pi^2}\right)}$$

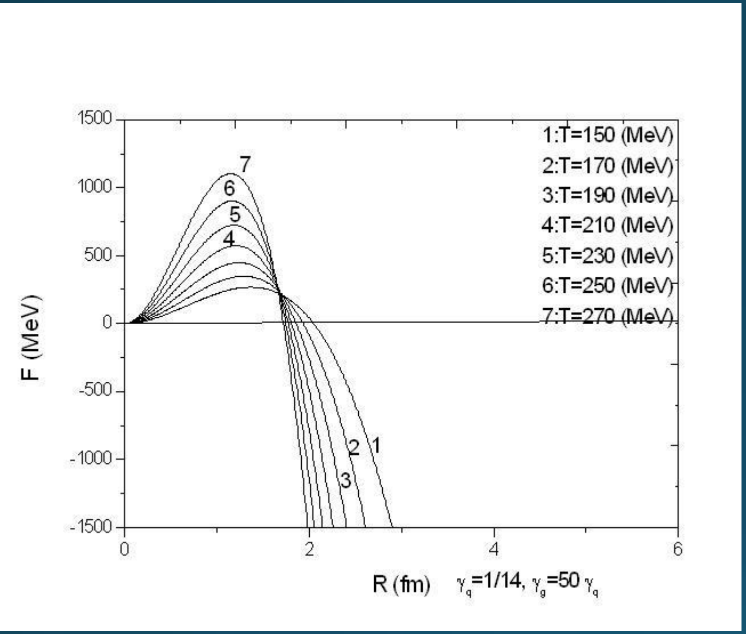
$$\alpha_s(k) = \frac{4\pi}{29\ln\left(1 + \frac{k^2}{\pi^2}\right)}$$

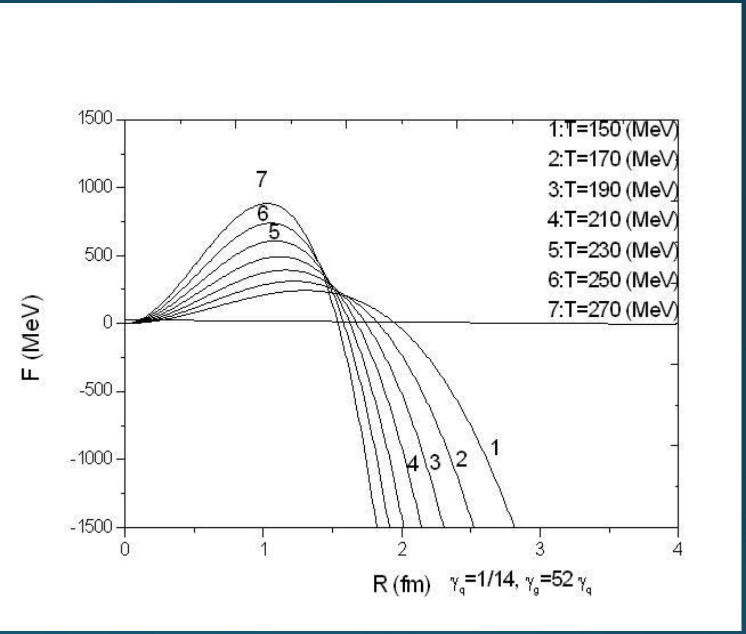


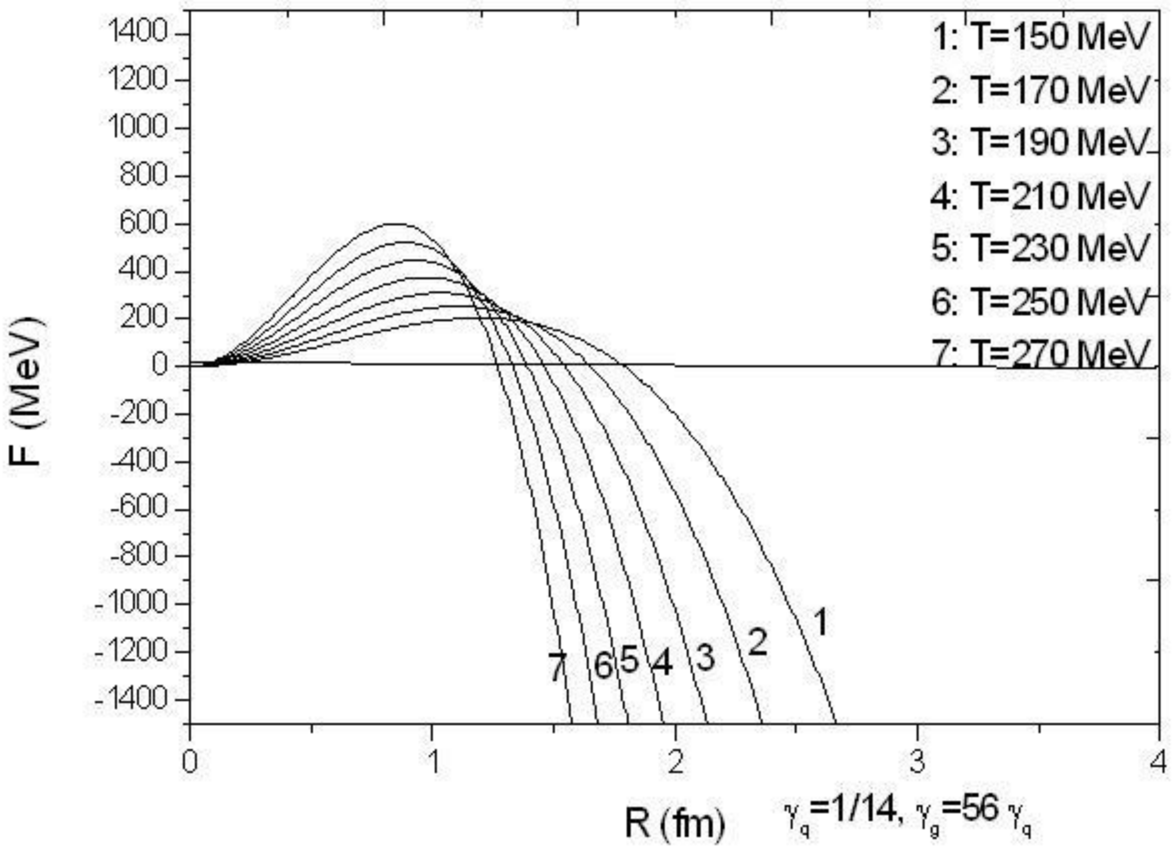
3000
2500
2000
1500
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
F (MeV)
0
1
2
3
4
R (fm)
γ_q=1/14, γ_g=44 γ_q
1:T=150 MeV
2:T=170 MeV
3:T=190 MeV
4:T=210 MeV
5:T=230 MeV
6:T=250 MeV
7:T=270 MeV
1
2
3
4
5
6
7



2000
1500
1000
500
0
-500
-1000
-1500
-2000
F (MeV)
0
2
4
6
R (fm)
γ_q=1/14, γ_g=48 γ_q
1:T=150 (Mev)
2:T=170 (Mev)
3:T=190 (Mev)
4:T=210 (Mev)
5:T=230 (Mev)
6:T=250 (Mev)
7:T=270 (Mev)


1500
1000
500
0
-500
-1000
-1500
F (MeV)
0
2
4
6
R (fm)
γq=1/14, γg=50 γq
1:T=150 (MeV)
2:T=170 (MeV)
3:T=190 (MeV)
4:T=210 (MeV)
5:T=230 (MeV)
6:T=250 (MeV)
7:T=270 (MeV)


1500
1000
500
0
-500
-1000
-1500
F (MeV)
0
1
2
3
4
R (fm)
γq=1/14, γg=52 γq
1:T=150 (MeV)
2:T=170 (MeV)
3:T=190 (MeV)
4:T=210 (MeV)
5:T=230 (MeV)
6:T=250 (MeV)
7:T=270 (MeV)


1400
1200
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1200
-1400
F (MeV)
0
1
2
3
4
R (fm)
γ_q=1/14, γ_g=56 γ_q
1: T=150 MeV
2: T=170 MeV
3: T=190 MeV
4: T=210 MeV
5: T=230 MeV
6: T=250 MeV
7: T=270 MeV
1
2
3
4
5
6
7

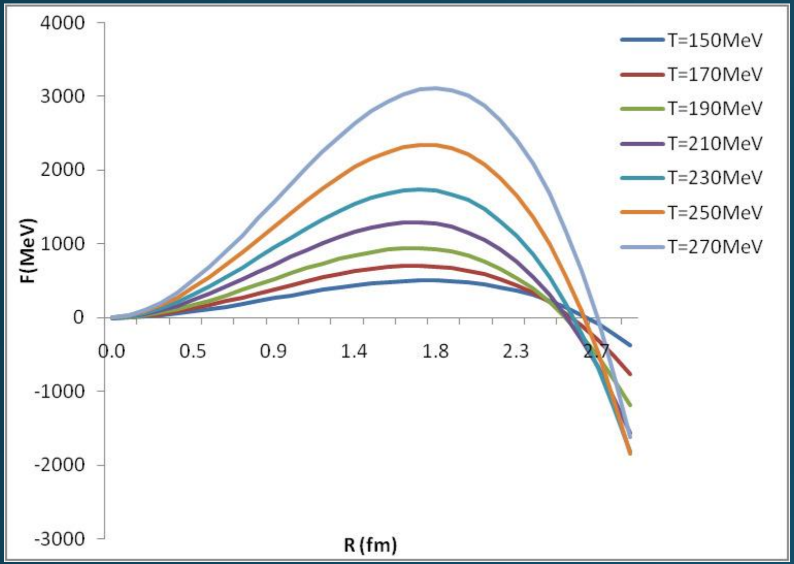


Fig: Free energy vs Radius variation at $\gamma_g = \frac{1}{6}$ and $\gamma_q = 52\gamma_g$

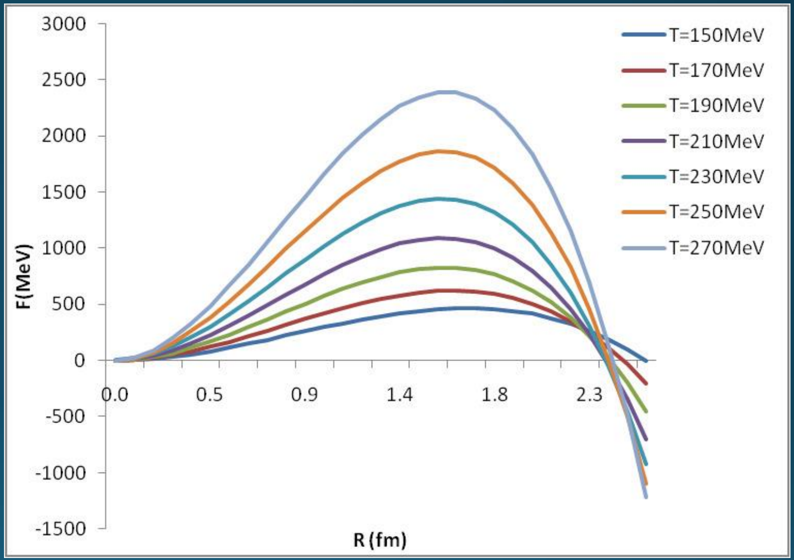


Fig: Free energy vs Radius variation at $\gamma_g = \frac{1}{6}$ and $\gamma_q = 54\gamma_g$

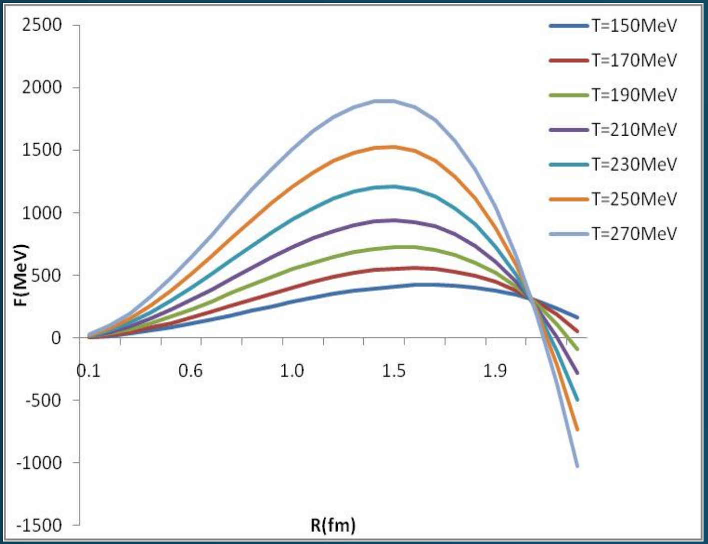


Fig: Free energy vs Radius variation at $\gamma_g = \frac{1}{6}$ and $\gamma_q = 56\gamma_g$

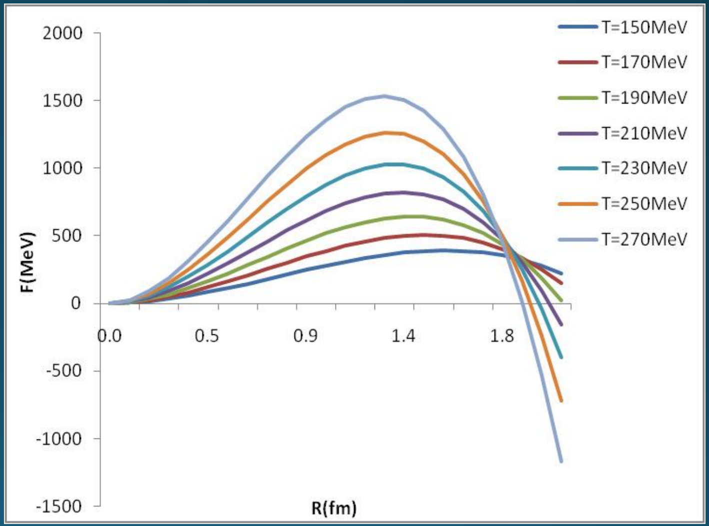


Fig: Free energy vs Radius variation at $\gamma_g = \frac{1}{6}$ and $\gamma_q = 58\gamma_g$

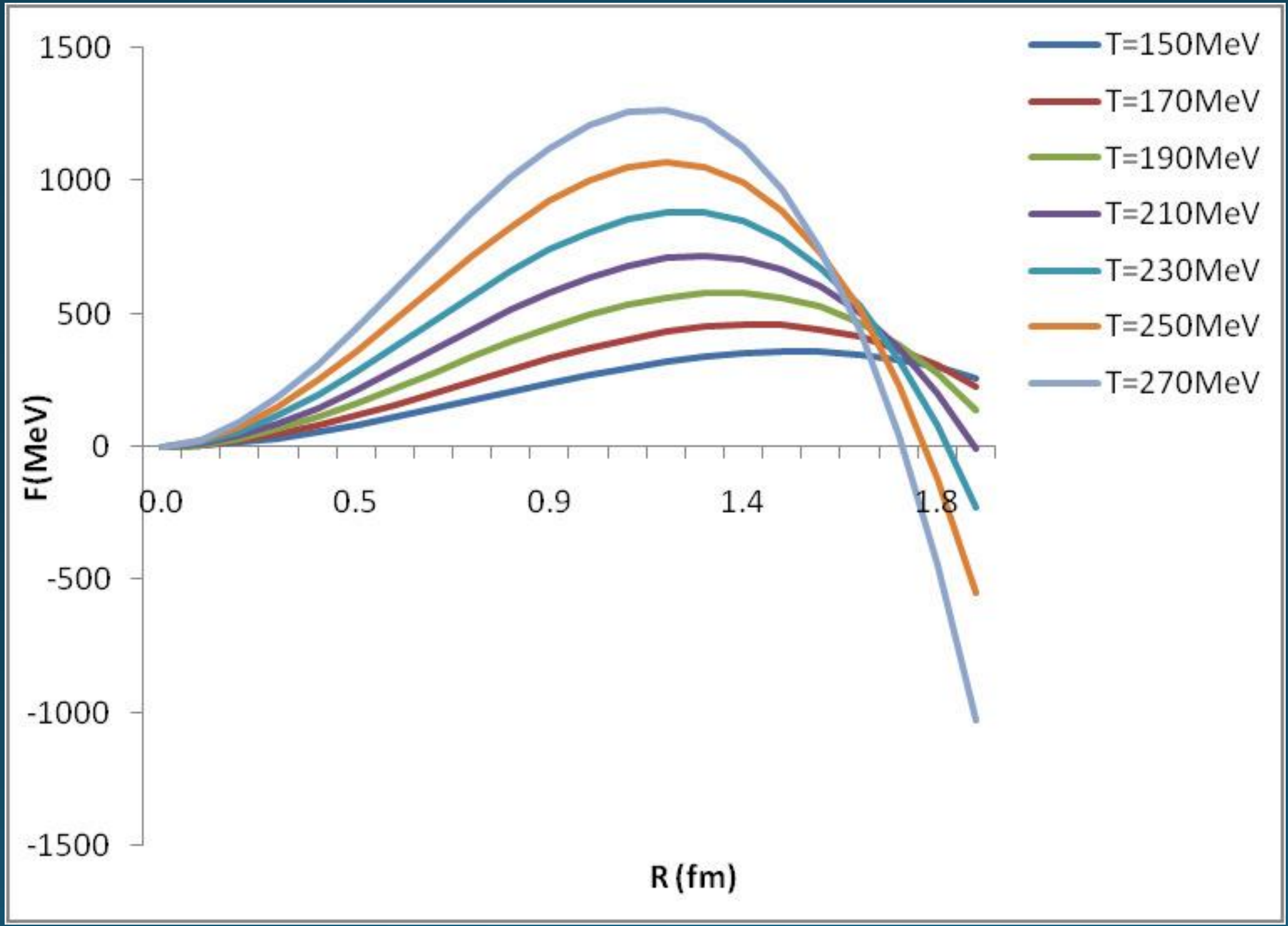
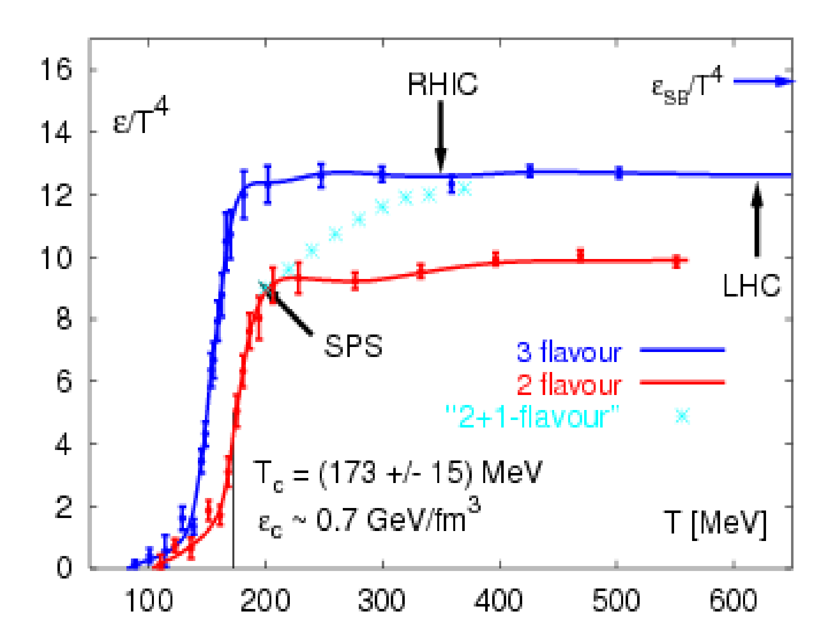
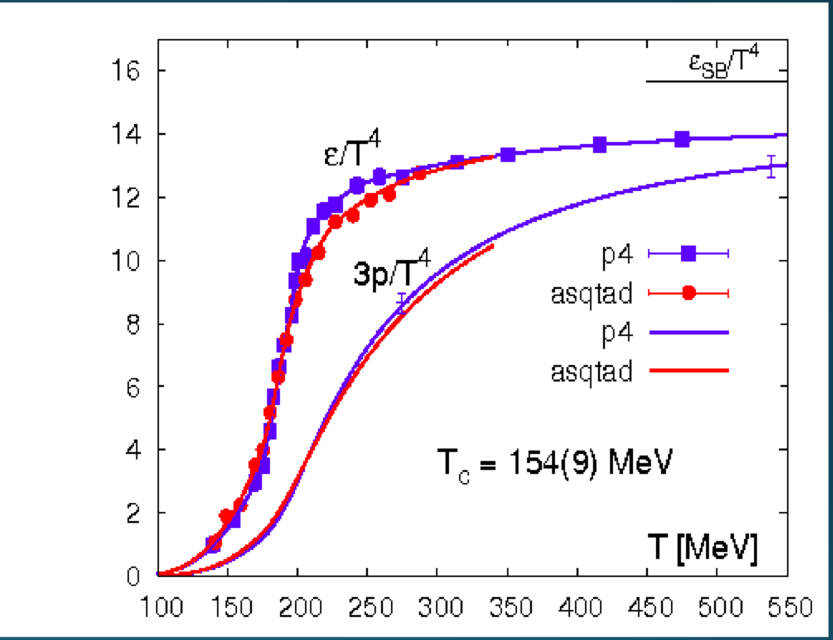


Fig: Free energy vs Radius variation at $\gamma_g = \frac{1}{6}$ and $\gamma_q = 60\gamma_g$

Equation of State





To look all these equations of state of QGP, We need to set up models and depending on the model, it can predict the QCD phase structure. There are many phenomenological models, we also create a very simple model to see all these physical parameters through one loop correction.

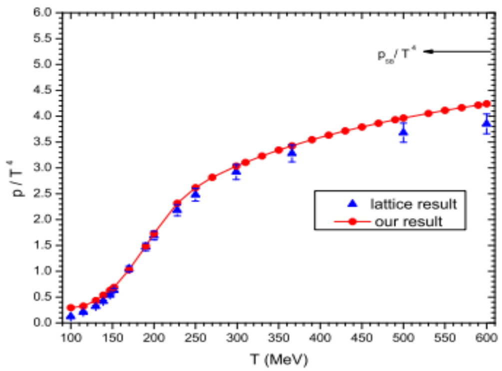


FIG. 4: Pressure vs. T at $\gamma_q = 1/14$, $\gamma_g = 68\gamma_q$.

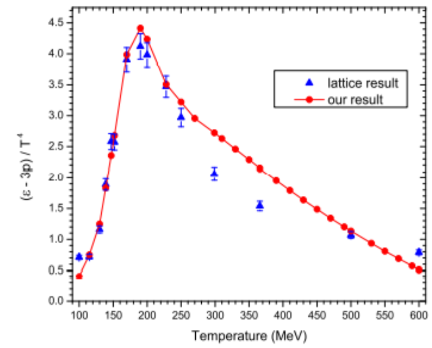
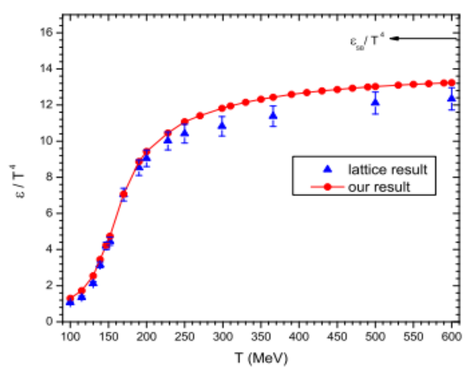


FIG. 6: Interaction measurement vs. T at $\gamma_q = 1/14$, $\gamma_g = 68\gamma_q$.



Energy density vs. T at $\gamma_q = 1/14$, $\gamma_g = 68\gamma_q$.

Conclusion

QGP Fireball under mean field potential and one loop at finite chemical potential

- The model predict a weakly first order transition at temperature range (160+ 5) MeV or (160-5) MeV
- It is also consistent with expectations of QGP- hadron phase transition. Due to presence of one loop correction in mean field potential, The following parameters are observed.
- The stability of droplets increases.
- Their size decreases in comparison with result with uncorrected potential.
- Energy evolution with effect of one loop correction in potential shows a higher transition temperature in range of T= 180 to 250 MeV.
- The transition of temperature is effected by dynamical flow parameter used in potential.
- It results in decreasing observable QGP droplets of stable radius 2.5-4.5 fm.

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Thank you