Chiral cross-over in Hadron Resonance Gas model

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Pseudo-critical line

- For 2+1 flavor with non-zero quark mass, the chiral crossover occurs at $T_{pc} = 156.5(1.5)$ MeV. [HotQCD 2018]
- For small μ_B/T_{pc}, the line can be extended as,

$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_{pc}(0)}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_{pc}(0)}\right)^4 .$$

- LQCD estimates curvature coefficients $\kappa_2 = 0.012(4)$ [HotQCD 2018] and 0.0153(18) [BMW 2020], $\kappa_4 = 0$ (within variance).
- $0.015 < \kappa_2 < 0.02$ for the freeze-out line. [HotQCD 2015]

Freeze-out and HRG

- Hadron resonance gas model (HRG) suitably describes the experimental yield and estimates the freeze-out line. [A. Andronic et.al 2007, 2009, 2017, S.Bhattacharyya, DB et.al 2019]
- It will be interesting to provide estimations of the pseudo-critical line from the HRG.
- How far the pseudo-critical and freeze-out line match in HRG?

Cross-over line in HRG around $\mu_B = 0$

- The order parameter is

$$\left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right] = \frac{\partial P}{\partial m_l}$$

- Earlier studies with HRG found a higher $T_{pc}(0) \sim 170$ MeV.
  [J. Jankowski et al. 2013, A. N Tawfik, N. Magdy 2015]
- We need precise determination of the hadronic σ terms, while evaluating $\partial P/\partial m_l$

$$\frac{\partial P}{\partial m_l} = -\sum_{\alpha} \frac{g_\alpha}{2\pi^2} \int_0^\infty dp\ p^2\ n_\alpha\ (E_\alpha) \frac{1}{2E_\alpha} \frac{\partial M_\alpha^2}{\partial m_l}.$$

Renormalized chiral condensate

- A renormalized chiral condensate is defined in LQCD as,

$$-m_s \left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right] = -m_s \frac{\partial P}{\partial m_l}$$

- A natural choice for dimensionless condensate [HotQCD 2012],

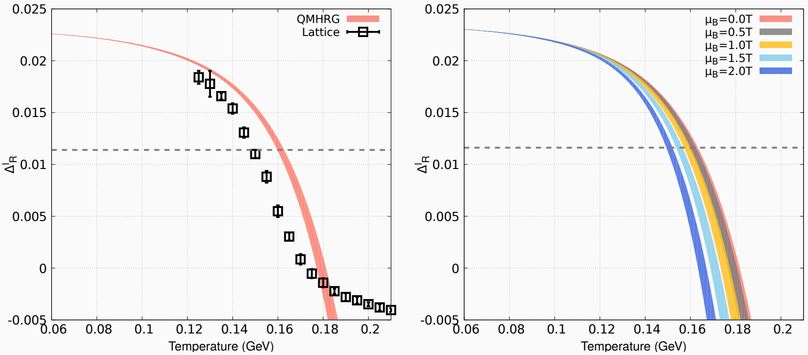
$$\Delta_R^l = d + m_s r_1^4 \left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right]$$

- Using low energy constant of $SU(2)$ χ_{PT}, $\Sigma^{1/3} = 272(5)$ MeV, $m_s = 92.2(1.0)$ MeV, and $r_1 = 0.3106$ fm, one gets $d = 0.022791$. [FLAG 2022],

Improved σ terms:

- Quark mass variation of pseudoscalar mesons have been included following χPT.
- We have done extensive compilation of the LQCD results to find $\frac{\partial M_\alpha}{\partial m_l}$ at a constant m_s, set at the physical value. [DB, PP, SS 2022]
- For the first time, σ terms for η, $\rho(770)$, $K^*(892)$, and η' have been calculated from LQCD data.
  [RQCD Bali et al. 2016, D. Guo et al. 2016, RQCD Bali et al. 2021] .
- Precise σ terms for all baryons and resonances have been included [PM. Copeland et al. 2023] .

$\Delta_R^l = d + m_s r_1^4 \left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right]$



- In lattice Δ_R^l goes to half of its low-temperature value at T_{pc}.
- We use this fact to estimate T_{pc} from HRG model calculations.

Pseudo-critical temperature:

- This improved calculation gives $T_{pc} = 161.2 \pm 1.7$ MeV at $\mu_B = 0$. [DB, S.Sharma, P.Petreczky 2022]
- $\kappa_2 = 0.0203(7)$ and $\kappa_4 = -3(2) \times 10^{-4}$.
- Results are in agreement with LQCD estimations of $\kappa_2 = 0.016(6)$ [HotQCD 2018] and, $\kappa_4 = 0.001(7)$.

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- How to extend this line at higher μ_B?

Mean-filed repulsive HRG

- The pressure for an interacting ensemble of (anti-) baryons is [P. Huovinen P. Petreczky 2018]

$$P_{int}^{B\{\bar{B}\}} = T \sum_{i \in B\{\bar{B}\}} \int g_i \frac{d^3p}{(2\pi)^3} \ln\left[1 + e^{-\beta(E_i - \mu_{eff})}\right] + \frac{K}{2} n_{B\{\bar{B}\}}^2$$

- The effective chemical potential, $\mu_{eff} = B_i \mu_B - K n_{B\{\bar{B}\}}$.
- The number densities can be solved self-consistently from:

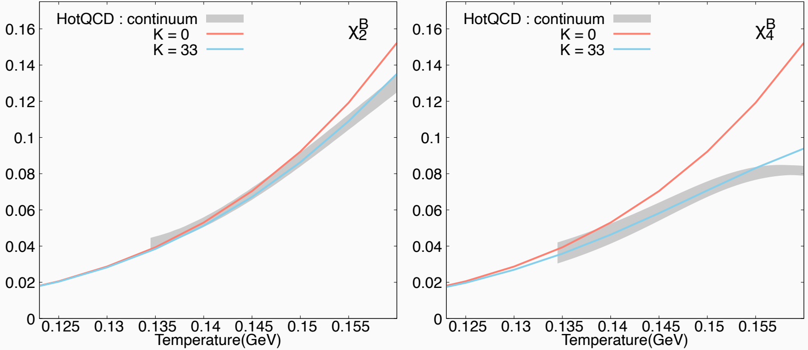
$$n_{B\{\bar{B}\}} = \sum_{i \in B\{\bar{B}\}} \int g_i \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(E_i - \mu_{eff})} + 1}$$

- The total pressure is the sum of the interacting ensemble of the (anti)baryons and the non-interacting ensemble of mesons.

Fitting the parameter K

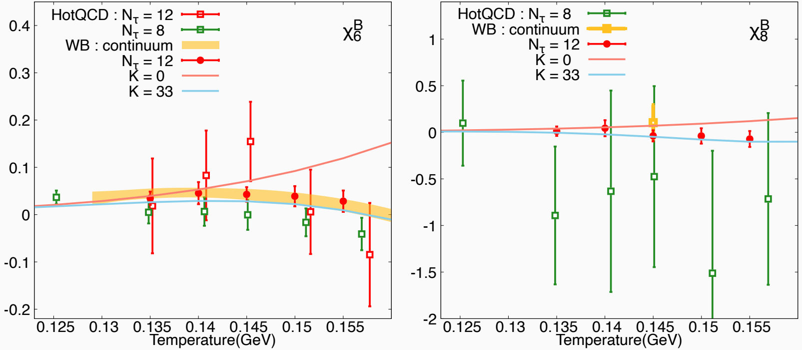
- The mean-field coefficient K can be estimated by fitting the baryon susceptibilities.

$$\chi_B^n = \frac{\partial^n \left[P(\mu_B/T)/T^4 \right]}{\partial (\mu_B/T)^n}$$



HotQCD 2020

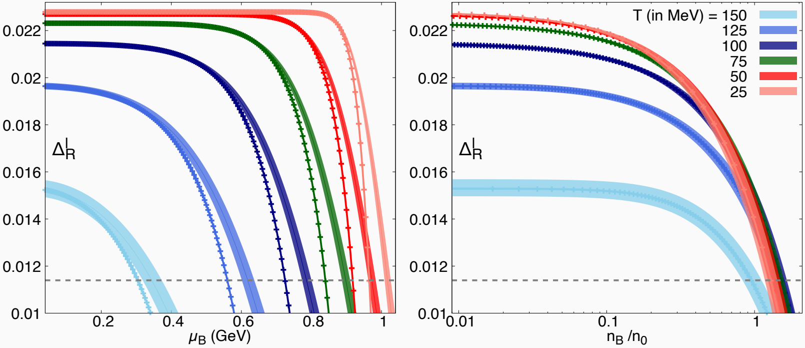
Fitting the parameter K



HotQCD 2020, WB 2018, 2023

Chiral condensate variation with μ_B

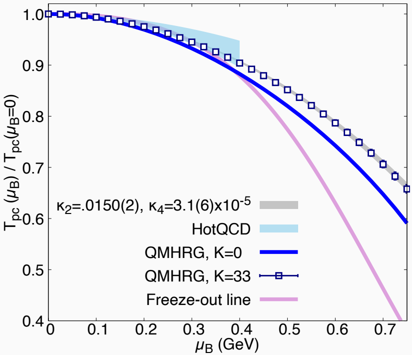
- The repulsion effectively decreases the chiral condensate.
- This effect is prominent at low temperatures.



Pseudo-critical line

- Lattice QCD results disfavor $\mu_B^{CEP} < 400$ MeV [WB 2020, HotQCD 2019].
- This conclusion allows us to extend the cross-over line estimation at high μ_B. [DB, S. Sharma, P.Petreczky 2024]

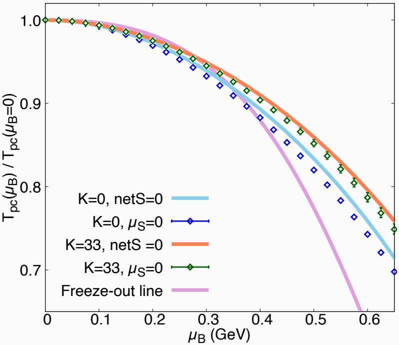
- $\kappa_2 = 0.0150(2)$ and $\kappa_4 = 3.1(6) \times 10^{-5}$.
- The estimation of κ_4 is distinctly different from zero given the estimated errors.



Effect of strangeness neutrality

- $n_S = 0$ restricts the density of strange particles, giving higher values of T_{pc}.
- In agreement with estimations from NJL model
  [MSA, DB et al. 2024]

- Freeze-out line deviates from the pseudo-critical line around $\mu_B = 400$ MeV.
- Indicates a longer lifetime of hadronic phase at lower collision energies.



Summary and outlook

- We have improved the chiral description within the HRG model with precise estimations of σ terms.
- With suitable value of K the pseudo-critical line has been extended at higher μ_B.
- κ_2 and κ_4 betters and matches with LQCD for $K \neq 0$.
- Freeze-out might occur at much later time at higher μ_B.
- Strangeness neutrality increases $T_{pc} \rightarrow$ lower value of κ_2.
- Chiral mean-field model would provide insight into the curvature coefficients and interplay with strangeness. Talk by MS Ali

Collaborators:

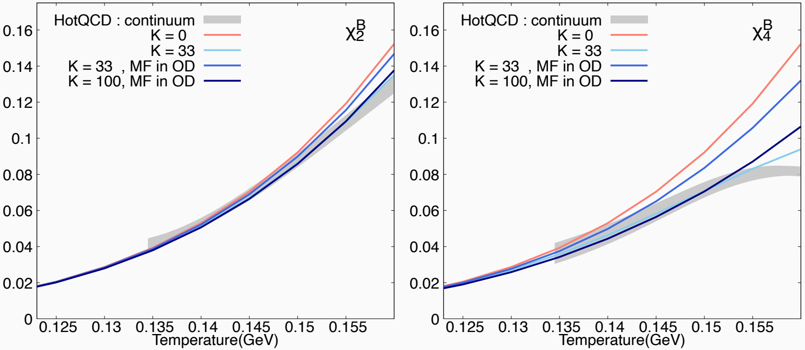
Peter Petreczky, Sayantan Sharma

References:

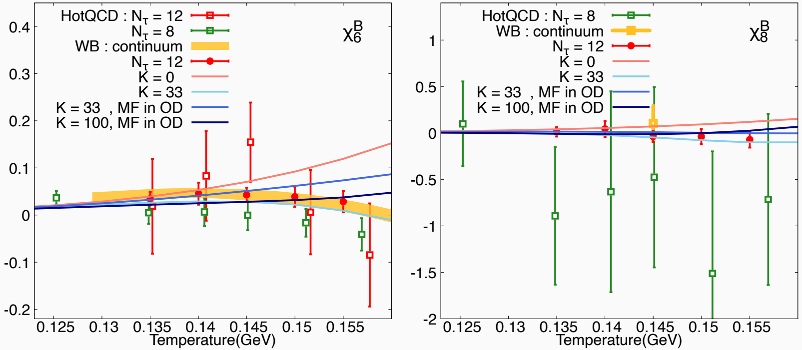
Phys.Rev.C 106, 045203 and Phys.Rev.C 109, 055206

BACKUP

Quantification of the mean-field parameter K



Quantification of the mean-field parameter K



Pseudoscalar ground states

- From $SU(2)$ $\chi_{\rm PT}$,

$$M_\pi^2 = M^2 \left[1 - \frac{1}{2}\zeta\ \bar{l}_3 + \mathcal{O}(\zeta^2)\right] \quad , \quad \zeta = \frac{M^2}{16\pi^2 F_\pi^2}$$

- Kaon properties are predicted well from 2+1 $\chi_{\rm PT}$
  [RBC 2014, Durr 2015]
  $M_K^2 = B_K(m_s)m_s\left[1 + \frac{\lambda_1(m_s)+\lambda_2(m_s)}{F^2}M^2\right]$
  $M^2 = 2Bm_l,\ B = \Sigma/F^2$

- From LQCD the pion mass is consistent with LO result $M_\pi^2 \approx 2Bm_l$. [RQCD Bali et al. 2016] .

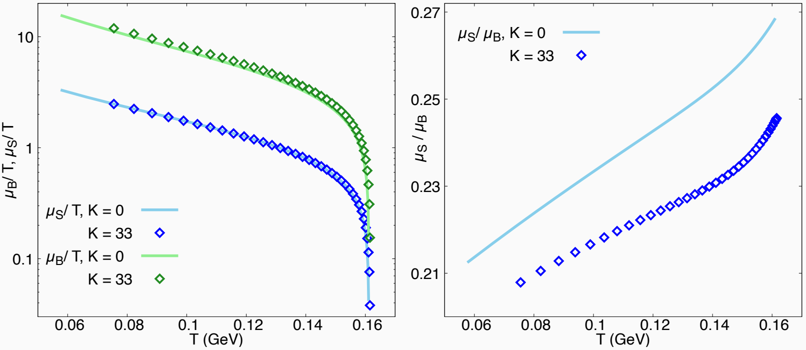
Sigma terms for Heavier hadrons

$$\sigma_\alpha = m_l \frac{\partial M_\alpha}{\partial m_l}\Big|_{m_l = m_l^{phys}} = m_l \langle \alpha | \bar{u}u + \bar{d}d | \alpha \rangle = M_\pi^2 \frac{\partial M_\alpha}{\partial M_\pi^2}\Big|_{M_\pi = M_\pi^{phys}}.$$

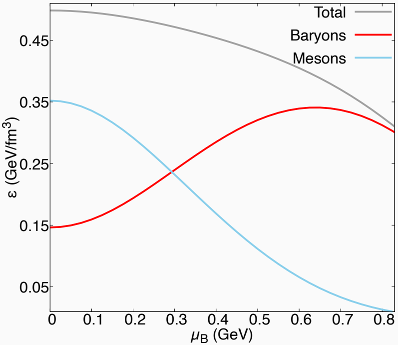
N	Λ	Σ	Ξ
44(3)(3)	31(1)(2)	25(1)(1)	15(1)(1)
Δ	Σ^*	Ξ^*	Ω^-
29(9)(3)	18(6)(2)	10(3)(2)	5(1)(1)

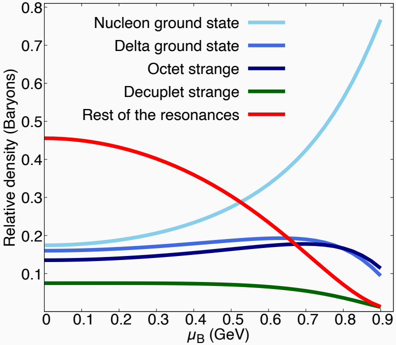
The sigma terms of ground state baryons have been only recently calculated with precision. [Copeland et al. 2021] .

Strangeness chemical potential for neutrality case



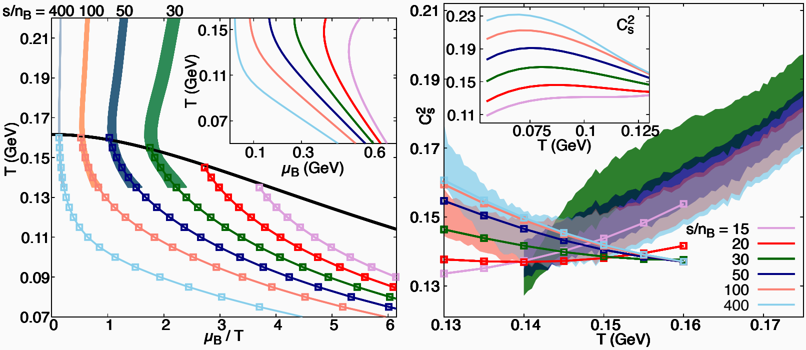
Contribution of different sectors along the phase line





An alternate equation of state

The agreement at lower μ_B/T can be utilized to evaluate the isentropic trajectories and speed of sound at higher values.



Lattice results from HotQCD 2023.