Induced electric field due to thermoelectric effects in evolving quark-gluon plasma formed in heavy-ion collisions

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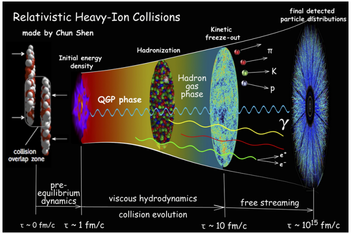
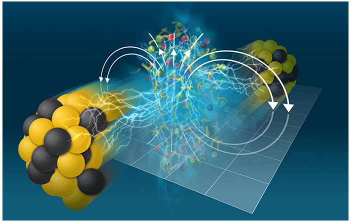
Dynamics of heavy-ion collisions and origin of magnetic field

Heavy-ion collisions aim to form and characterize macroscopic deconfined state of quarks and gluons in local thermal equilibrium. .





Evolution of Little Bang

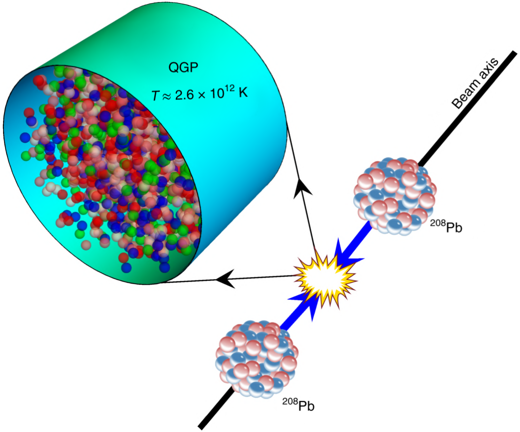
- Collision of two Lorentz contracted nuclei
- Pre-equilibrium phase
- Quark-gluon plasma phase
- Hadronic phase
- Freeze out (Chemical and Kinetic)
- Free streaming

Origin of magnetic field

- Spectator protons in off-central collisions
- Strongest magnetic field ($\approx 10^{18}$ Gauss) produced around this direction of motion - A right-hand thumb rule

Motivation

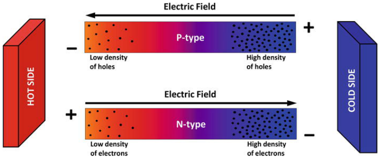
How does the produced time-varying magnetic field affect the thermoelectric transport properties of the evolving QGP medium?

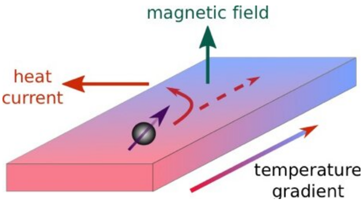


- Expansion of the medium leads to the cooling
- Evolving medium has to follow a certain cooling rate (Bjorken flow, Gubser flow)
- This cooling rate is affected by external magnetic field
- The radial expansion of medium also affect the cooling rate
- Hence, it affects the transport properties of the medium

Thermoelectric transport properties

The system driven out of local thermal equilibrium exhibit the transport properties such as thermal and electrical conductivity.





- The gradients of temperature in a system leads to the transportation of heat. The ratio of heat current to the temperature gradient is known as thermal conductivity
- A medium composite of charged particles exhibit thermoelectric behavior in the presence of temperature gradients
- If charged particles conduct this transportation of heat, the presence of external magnetic field affects it

Formalism - Solving Boltzmann transport equation

- Total single particle distribution function $f_i = f_i^0 + \delta f_i$
- To find the expression of δf_i, we solve the Boltzmann transport equation (BTE) with the help of relaxation time approximation (RTA)

$$\frac{\partial f_i}{\partial \tau} + \frac{\vec{k}_i}{\omega_i} \cdot \frac{\partial f_i}{\partial \vec{x}_i} + q_i \left(\vec{E} + \frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \vec{k}_i} = -\frac{\delta f_i}{\tau_R^i} \qquad (1)$$

$\tau_R^i \rightarrow$ relaxation time, $\omega_i \rightarrow$ energy and $k_i \rightarrow$ momentum of the ith particle

- Ansatz $\rightarrow$ $\delta f_i = (\vec{k}_i.\vec{\Omega}) \frac{\partial f_i^0}{\partial \omega_i}$
- $\vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_3 \vec{\nabla} T + \alpha_4 \vec{\nabla} \dot{T} + \alpha_5 (\vec{\nabla} T \times \vec{B}) + \alpha_6 (\vec{\nabla} T \times \dot{\vec{B}}) + \alpha_7 (\vec{\nabla} \dot{T} \times \vec{B}) + \alpha_8 (\vec{E} \times \vec{B}) + \alpha_9 (\vec{E} \times \dot{\vec{B}}) + \alpha_{10} (\dot{\vec{E}} \times \vec{B})$

Formalism - Thermoelectricity (In the absence of magnetic field)

- In kinetic theory, electric current density for such a system can be expressed as $\vec{j} = \sum_i q_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i}{\omega_i} \delta f_i$ Here, $q_i \rightarrow$ the electric charge, and $g_i \rightarrow$ the degeneracy of the i^{th} species particles
- A general form of unknown vector $\vec{\Omega}$ can be assumed as $\vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \vec{\nabla} T + \alpha_3 \vec{\nabla} \dot{T}$
- Using the expressions of δf_i , we can express electric current as $\vec{j} = \sum_i \frac{q_i g_i}{3} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} v_i^2 \tau_R^i \Big[- q_i \vec{E} + \frac{(\omega_i - \mathrm{b}_i h)}{T} \Big\{ \vec{\nabla} T - \tau_R^i \vec{\nabla} \dot{T} \Big\} \Big] \frac{\partial f_i^0}{\partial \omega_i}$
- For a open circuit system i.e. $\vec{j} = 0$, we get $\vec{E} \alpha \vec{\nabla} T$
- The constant of proportionality is Seebeck coefficient S i.e. $\vec{E} = S \vec{\nabla} T$

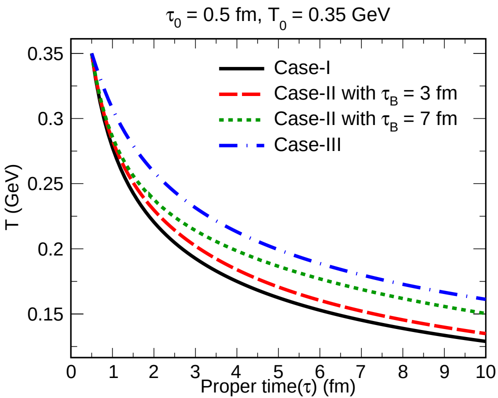
Formalism - Modification of the net current in magnetic field

$$
\begin{aligned}
j^x = &\sum_i \frac{q_i g_i}{3} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} v_i^2 \frac{\tau_R^i q_i}{(1+\chi_i+\chi_i^2)(1+\chi_i)} \Bigg\{ \Bigg(\frac{(1+\chi_i^2)+\chi_i(2+\chi_i)}{(1+\chi_i^2)} \Bigg) E^x \\
&\pm \chi \Bigg(\frac{(1+\chi_i^2)(1+\chi_i)+\chi_i(2+\chi_i)}{(1+\chi_i^2)} \Bigg) (E^z - E^y) \Bigg\} \Bigg(-\frac{\partial f_i^0}{\partial \omega_i} \Bigg) \\
&+ \sum_i \frac{q_i g_i}{3} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} v_i^2 \frac{\tau_R^i(\omega_i - \mathrm{b}_i h)}{T(1+\chi_i+\chi_i^2)(1+\chi_i)} \Bigg\{ (1+\chi_i) \frac{\partial T}{\partial x} - \tau_R^i \frac{(1+\chi_i-\chi_i^2)}{(1+\chi_i^2)} \frac{\partial \dot{T}}{\partial x} \\
&\pm \chi_i(1+\chi_i) \Bigg(\frac{\partial T}{\partial z} - \frac{\partial T}{\partial y} \Bigg) \mp \tau_R^i \frac{\chi_i(2+\chi_i)}{(1+\chi_i^2)} \Bigg(\frac{\partial \dot{T}}{\partial z} - \frac{\partial \dot{T}}{\partial y} \Bigg) \Bigg\} \frac{\partial f_i^0}{\partial \omega_i} \, .
\end{aligned} \tag{2}
$$

Where, b_i is the baryon quantum number of i^{th} species. h is enthalpy per baryon number, and $\chi_i = \frac{\tau_R^i}{\tau_B} = \frac{\tau_R^i}{\tau_E}$. The components j^y and j^z can be obtained from the above equation by changing x, y, and z in cyclic order.

Cooling rates: Bjorken flow

Bjorken flow is theoretical model used to describe the dynamics of quark-gluon plasma (QGP), a state of matter created in high-energy heavy-ion collisions. The medium expands uniformly in the longitudinal direction (along the beam axis).



[1]

- Case-I (Ideal hydrodynamics)

$$T = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}}.$$

- Case-II (Ideal magneto-hydrodynamics)

$$T = \left[T_0^4\left(\frac{\tau_0}{\tau}\right)^{\frac{4}{3}} + \frac{4\alpha}{(2\beta\tau)^{\frac{4}{3}}}\left\{\Gamma(4/3, 2\beta\tau) - \Gamma(4/3, 2\beta\tau_0)\right\} - \frac{2\alpha}{(2\beta\tau)^{\frac{7}{3}}}\left\{\Gamma(7/3, 2\beta\tau) - \Gamma(7/3, 2\beta\tau_0)\right\}\right]^{\frac{1}{4}}.$$

- Case-III (First-order hydrodynamic)

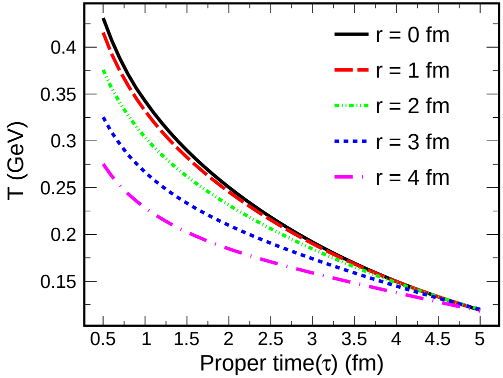
$$T = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}}\left\{1 + \frac{b}{6a}\frac{1}{\tau_0 T_0}\left(1 - \left(\frac{\tau_0}{\tau}\right)^{\frac{2}{3}}\right)\right\},$$

with $a = \left(16 + \frac{21}{2}N_f\right)\frac{\pi^2}{90}$ and
$b = (1 + 1.70N_f)\frac{0.342}{(1+N_f/6)\alpha_s^2 \ln(\alpha_s^{-1})}$.

[1] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D 109, no.1, 014018 (2024)

Cooling rates: Gubser flow

Gubser flow introduces radial expansion in the transverse directions while maintaining boost invariance along the longitudinal direction.



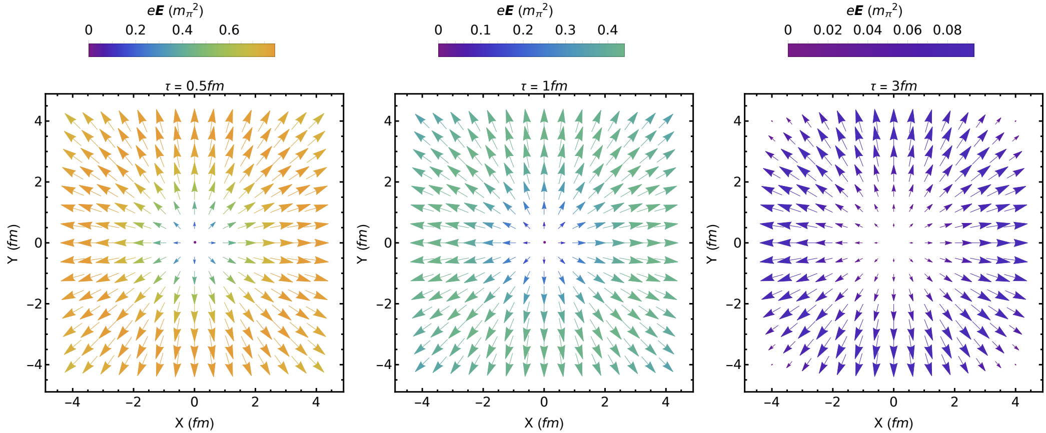
[2]

$$T = \frac{1}{\tau f_*^{1/4}} \left[\frac{T_0}{(1+g^2)^{1/3}} + \frac{\mathrm{H}_0 g}{\sqrt{1+g^2}} \left\{ 1 - (1+g^2)^{1/6} {}_2F_1\left(\frac{1}{2}, \frac{1}{6}; \frac{3}{2}; -g^2\right) \right\} \right] \quad (3)$$

Where T_0 is an integration constant and ${}_2F_1$ denotes a hypergeometric function with $g = \frac{1-q^2\tau^2+q^2r^2}{2q\tau}$. For numerical estimations, we used semi-realistic numbers for a central gold-gold collision at $\sqrt{s_{\mathrm{NN}}} = 200\,\mathrm{GeV}$ are $\hat{T}_0 = 5.55$ and $\mathrm{H}_0 = 0.33$ if we choose $1/q = 4.3$ fm.

[2] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D 109, no.1, 014018 (2024)

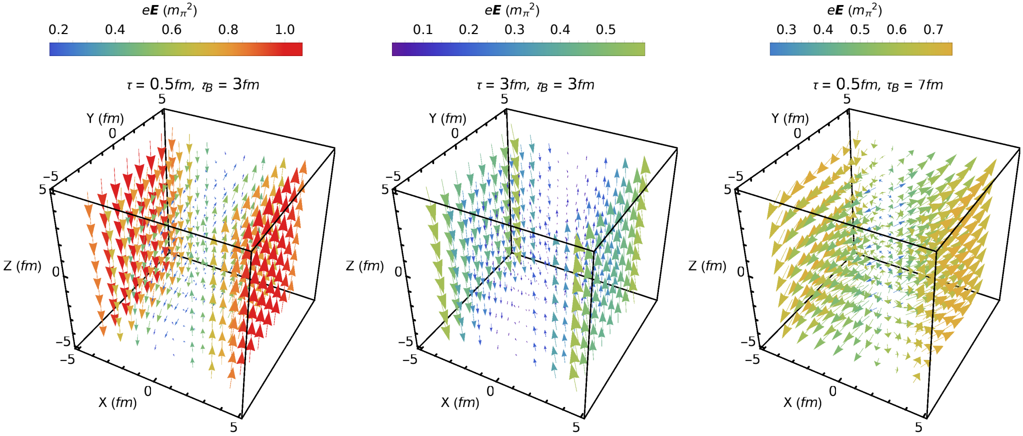
Results: Induced electric field vector in absence of magnetic field



Spatial profile of the induced electric field at different times (τ) in the QGP created in head-on collisions. [3]

[3] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D 110, no.11, 114051 (2024)

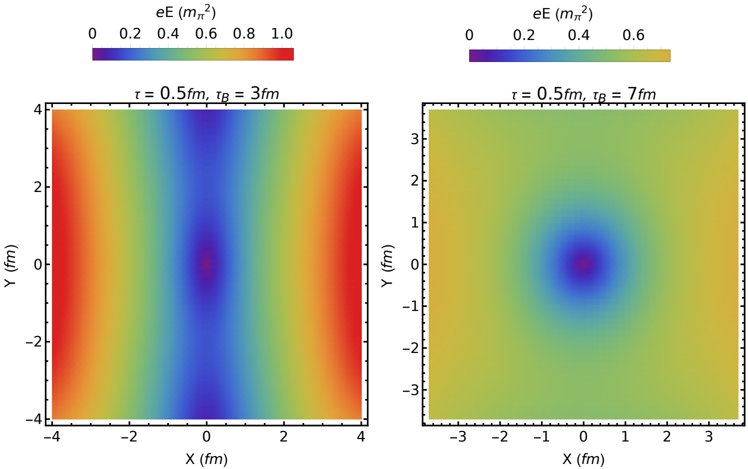
Results: Induced electric field vector in the presence of magnetic field



Spatial profile of the induced electric field at different times (τ) in the QGP created in peripheral collisions. [4]

[4] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D 110, no.11, 114051 (2024)

Results: Magnitude of induced electric field



Magnitude of the induced electric field in the transverse plane with $eB_0 = 5\ m_\pi^2$.[5]

[5] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D 110, no.11, 114051 (2024)

Summary and conclusion

- The presence of magnetic field creates Hall-like transport coefficients in medium
- Bjorken and Gubser flow pictures are used to study the cooling effects on the medium
- Cooling rates also get affected in the presence of magnetic field
- Induced electric field get asymmetrical in the presence of external magnetic field
- The thermoelectric coefficients (Seebeck and Nernst) are crucial to estimate the induced electric field in the medium
- The maximum electric field produced in head-on and peripheral collisions is $\sim 0.7\ m_\pi^2$ *and* $1.0\ m_\pi^2$, respectively

References

K. Singh, J. Dey and R. Sahoo, Phys. Rev. D **109**, no.1, 014018 (2024)

K. Singh, J. Dey and R. Sahoo, Phys. Rev. D **110**, no.11, 114051 (2024)


THANK YOU