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Kubo formula for a dissipative spin hydrodynamic framework: spin chemical potential as the leading order term in the hydrodynamic gradient expansion

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Journal References: 2410.04141 [nucl-th]

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PIONEERING EDUCATION PARADIGMS

Observation of the spin polarization of hadrons and the ``*Spin-sign problem*"

STAR collaboration, Nature 548, 62-65 (2017); Becattini, F., et. al. PRC 95, 054902 (2017)

⟹ New challenges in the modeling of the ``*spin dynamics*" in an evolving QCD medium.

Z.-T. Liang, et. al., PRL 94 (2005) 102301; PLB 629 (2005) 20–26 J.-H. Gao, et al., PRC 77 (2008) 044902; S.-W. Chen, et. al. , Front. Phys. China 4 (2009) 509–516 B. Betz, M. Gyulassy, G. Torrieri, PRC 76 (2007) 044901, F.Becattini, et. al. Ann.Phys.323,2452(2008)

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``Spin hydrodynamics frameworks"

N. Weickgenannt, et. al., Phys.Rev.D 100 (2019) 5, 056018; Phys.Rev.Lett. 127 (2021) 5, 052301 ; Phys.Rev.D 104 (2021) 1, 016022; W. Florkowski, et.al., Prog.Part.Nucl.Phys. 108 (2019) 103709; Phys.Rev.C 98 (2018) 4, 044906; S. Bhadury, et.al., Phys.Lett.B 814 (2021) 136096 ; Phys.Rev.D 103 (2021) 1, 014030; K. Hattori, et.al. PLB 795 (2019) 100-106; K. Fukushima, S. Pu, Phys.Lett.B 817 (2021) 136346, Golam Sarwar et. al. Phys.Rev.D 107 (2023) 5, 054031, R. Biswas, et.al. Phys.Rev.D 107 (2023) 9, 094022......

Macroscopic conservation equations: $\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J^\mu = 0, \quad \partial_\lambda J^{\lambda\mu\nu} = 0,$

Conservation of the total angular momentum ⟹ Evolution of six component anti-symmetric tensor: **Spin chemical potential**

Generalized thermodynamic relations:

$$\varepsilon + P = Ts + \mu n + \omega_{\alpha\beta} S^{\alpha\beta},$$
$$d\varepsilon = Tds + \mu dn + \omega_{\alpha\beta} dS^{\alpha\beta},$$
$$dP = sdT + nd\mu + S^{\alpha\beta} d\omega_{\alpha\beta}.$$

$$T, \mu, u^\mu \to \mathcal{O}(1)$$
$$\omega^{\mu\nu} \to \mathcal{O}(1)$$

D. She, et.al., 2105.04060

K. Hattori, et. al. PLB 795 (2019) 100-106; A. Daher et. al. PRD 107 (2023) 5, 054043.

Entropy current analysis: $\mathcal{S}^{\mu} = T^{\mu\nu}\beta_{\nu} + P\beta^{\mu} - \alpha J^{\mu} - \beta\omega_{\alpha\beta}S^{\mu\alpha\beta}$

K. Hattori, et.al. PLB 795 (2019) 100-106; K. Fukushima, S. Pu, Phys.Lett.B 817 (2021) 136346

IRRep of dissipative currents:

$$T^{\{\alpha\beta\}}_{(1)} = h^{\alpha}u^{\beta} + h^{\beta}u^{\alpha} + \pi^{\alpha\beta} + \Pi\Delta^{\alpha\beta},$$

$$S^{\mu\alpha\beta}_{(1)} = 2u^{[\alpha}\Delta^{\mu\beta]}\Phi + 2u^{[\alpha}\tau^{\mu\beta]}_{(s)} + 2u^{[\alpha}\tau^{\mu\beta]}_{(a)} + \Theta^{\mu\alpha\beta}$$

R. Biswas, et.al. Phys.Rev.D 108 (2023) 1, 014024
D. She, et.al., 2105.04060

Entropy production in dissipative systems:

Entropy production

$$\partial_{\mu}\mathcal{S}^{\mu} \geq 0$$

$$\Pi = \zeta\theta,$$ → Bulk viscous term

$$h^{\mu} = -\kappa_{11}\frac{S^{\alpha\beta}}{\varepsilon + P}\nabla^{\mu}\Omega_{\alpha\beta} - \kappa_{12}\nabla^{\mu}\alpha,$$

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu},$$ → Shear viscous term

$$\mathcal{J}^{\mu} = \widetilde{\kappa}_{11}\nabla^{\mu}\alpha + \widetilde{\kappa}_{12}\frac{S^{\alpha\beta}}{\varepsilon + P}\nabla^{\mu}\Omega_{\alpha\beta},$$

$$\Phi = -2\chi_1 u^{\alpha}\nabla^{\beta}(\beta\omega_{\alpha\beta}),$$

$$\tau^{\mu\beta}_{(s)} = -2\chi_2\Delta^{\mu\beta,\gamma\rho}\nabla_{\gamma}(\beta\omega_{\alpha\rho})u^{\alpha},$$

$$\tau^{\mu\beta}_{(a)} = -2\chi_3\Delta^{[\mu\beta][\gamma\rho]}\nabla_{\gamma}(\beta\omega_{\alpha\rho})u^{\alpha},$$

$$\Theta^{\mu\alpha\beta} = \chi_4\Delta^{\delta\alpha}\Delta^{\rho\beta}\Delta^{\gamma\mu}\nabla_{\gamma}(\beta\omega_{\delta\rho}).$$

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$$\mathcal{J}^\mu = \widetilde{\kappa}_{11}\nabla^\mu\alpha + \widetilde{\kappa}_{12}\frac{S^{\alpha\beta}}{\varepsilon + P}\nabla^\mu\Omega_{\alpha\beta},$$

Jin Hu Phys. Rev. C 107, 024915

→ Cross diffusion like terms

$$\kappa_{11} \geq 0, \quad \widetilde{\kappa}_{11} \geq 0$$
$$\kappa_{12}^2 - \kappa_{11}\widetilde{\kappa}_{11} \leq 0$$

$$\Phi = -2\chi_1 u^\alpha\nabla^\beta(\beta\omega_{\alpha\beta}),$$

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→ Spin transport

Kubo relations: $S_{(1)}^{\mu\alpha\beta} = \left\langle \widehat{S}_{(1)}^{\mu\alpha\beta} \right\rangle = -\int d^3x' \int_{-\infty}^{t} dt'\ e^{\epsilon(t'-t)} \left(\widehat{S}_{(1)}^{\mu\alpha\beta}(\vec{x},t), \widehat{S}_{(1)}^{\rho\gamma\delta}(\vec{x}',t') \right)_l \nabla_\rho(\beta\omega_{\gamma\delta})$

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We need to have the tensor decomposition of the dissipative currents $S_{(1)}^{\mu\alpha\beta} = \Sigma^{\mu\alpha\beta\eta\gamma\delta}\nabla_\eta(\beta\omega_{\gamma\delta})$

$$\Sigma^{\mu\alpha\beta\rho\gamma\delta} = -\int d^3x' \int_{-\infty}^{t} dt'\ e^{\epsilon(t'-t)} \left(\widehat{S}_{(1)}^{\mu\alpha\beta}(\vec{x},t), \widehat{S}_{(1)}^{\rho\gamma\delta}(\vec{x}',t') \right)_l$$

X.-G. Huang, et. al., Annals Phys. 326 (2011) 3075–3094; D. N. Zubarev, Nonequilibrium statistical thermodynamics; A. Hosoya, et.al., Annals Phys. 154 (1984) 229; J. Hu, Phys. Rev. D 103 no. 11, (2021) 116015.

- Orthogonality
- Symmetry property
- Onsager relation

⇒ Generic decomposition of the spin tensor $\widehat{S}_{(1)}^{\mu\alpha\beta} = \widehat{\Xi}^{\mu\alpha\beta} + \widehat{\mathcal{V}}^{\mu[\alpha}u^{\beta]}$

Spin tensor decompositions:

$$S_{(1)}^{\mu\alpha\beta} = 2u^{[\alpha}\Delta^{\mu\beta]}\Phi + 2u^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}$$

$$S_{(1)}^{\mu\alpha\beta} = -(\Sigma_1+\Sigma_3)\Delta^{\mu[\alpha}\Phi_\perp^{\beta]} + (\Sigma_2+\Sigma_3)\varepsilon^{\mu\alpha\beta}\varphi + \Sigma_3\Phi^{\mu\alpha\beta} + \Lambda_1\Delta^{\mu[\alpha}\Phi_{||}^{\beta]} + \Lambda_2\Gamma_s^{\mu[\alpha}u^{\beta]} + \Lambda_3\Gamma_a^{\mu[\alpha}u^{\beta]}$$

Spin transport:

$$\chi_1 = \int d^3x' \int_{-\infty}^{t} dt'\ e^{\epsilon(t'-t)} \left(\widehat{\Phi}(\vec{x},t), \widehat{\Phi}(\vec{x}',t') \right)_l = -\frac{1}{\beta}\frac{\partial}{\partial\omega}\mathrm{Im}\, G^R_{\widehat{\Phi},\widehat{\Phi}}(\mathbf{0},\omega)\Big|_{\omega\to 0},$$

X.-G. Huang, et. al., Annals Phys. 326 (2011) 3075–3094

$$\Sigma_1+\Sigma_3 \le 0,\ \Sigma_2+\Sigma_3 \le 0,\ \Sigma_3 \ge 0,$$
$$\Lambda_1 \le 0,\ \Lambda_2 \le 0,\ \Lambda_3 \le 0.$$

Kubo relations: $S_{(1)}^{\mu\alpha\beta} = \left\langle \widehat{S}_{(1)}^{\mu\alpha\beta} \right\rangle = -\int d^3x' \int_{-\infty}^{t} dt' \; e^{\epsilon(t'-t)} \left(\widehat{S}_{(1)}^{\mu\alpha\beta}(\vec{x},t), \widehat{S}_{(1)}^{\rho\gamma\delta}(\vec{x}',t') \right)_l \nabla_\rho (\beta\omega_{\gamma\delta})$

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$$= -\frac{1}{\beta}\frac{\partial}{\partial\omega} \mathrm{Im}\, G^R_{\widehat{\Phi},\widehat{\Phi}}(\mathbf{0},\omega)\Big|_{\omega\to 0},$$

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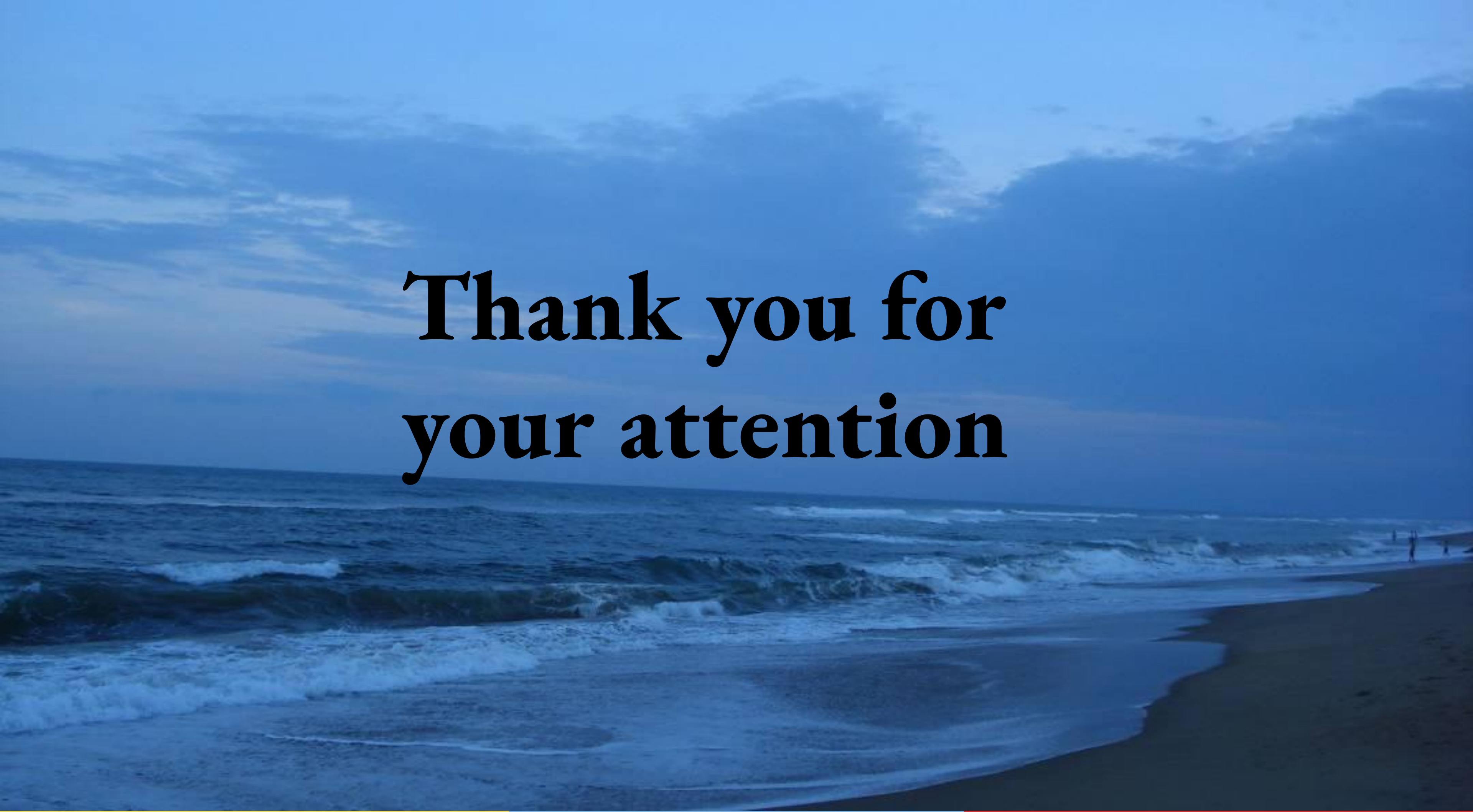
Equivalence:

$$\Lambda_1 = -4\chi_1 \,, \Lambda_2 = -2\chi_2$$
$$\Lambda_3 = -2\chi_3 \,, \chi_4 = \frac{5}{9}\Sigma_3 - \frac{3}{9}\Sigma_1 - \frac{1}{9}\Sigma_2$$



Thank you for your attention