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Quantum Hall effect for quarks in chiral effective model

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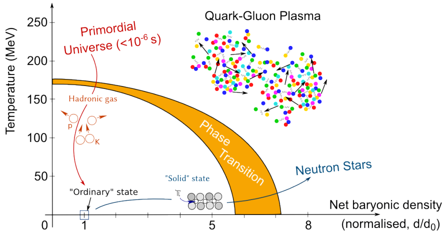
Introduction

In the context of the QCD phase diagram there are two extreme scenarios,

- **Hot and dense phases** – the quark-gluon plasma that existed in the early universe just after the Big Bang – at high temperatures and nearly vanishing net baryon densities. Probed at ultra-relativistic heavy-ion collision experiments at the LHC and RHIC.
- **Cold nuclear matter** – systems like the atomic nuclei and neutron stars – that correspond to conditions of low temperatures and high density Upcoming facilities of Compressed Baryonic Matter (CBM) experiment in FAIR and Nuclotron-based Ion Collider fAcility (NICA) at JINR.



QCD phase diagram. [1]

CBM will probe the highest baryonic densities ever obtained in laboratory environments

In this work, we employ the chiral effective model to obtain the constituent quark mass in this non-pQCD regime and determine the electrical conductivity at $T = 0$ in the kinetic theory framework.

[1] A. Maire, PhD thesis, Strasbourg U., 2011.

Constituent quark mass from chiral effective model

We use the chiral effective model to obtain the expressions for the constituent quark masses. We start with a Lagrangian density, [2]

$$L = L_{kin} + L_{BX} + L_{SSB} + L_{scale-break} + L_{ESB}, \tag{1}$$

where, $$L_{BX} = -\sum_{i=n,p} \bar{\psi}_i m_i^* \psi_i, \qquad m_i^* = -g_{\sigma i}\sigma - g_{\zeta i}\zeta - g_{\delta i}\delta,$$

$$L_{SSB} = -\frac{1}{2}k_0\chi^2(\sigma^2+\zeta^2+\delta^2) + k_1(\sigma^2+\zeta^2+\delta^2)^2 + k_2\left(\frac{\sigma^4}{2}+\frac{\delta^4}{2}+3\sigma^2\delta^2+\zeta^4\right) + k_3\chi(\sigma^2-\delta^2)\zeta - k_4\chi^4,$$

$$L_{scale-break} = -\frac{1}{4}\chi^4 \ln\frac{\chi^4}{\chi_0^4} + \frac{d}{3}\chi^4 \ln\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\left(\frac{\chi}{\chi_0}\right)^3\right),$$

$$L_{ESB} = -\left(\frac{\chi}{\chi_0}\right)^2 Tr\left[\text{diag}\left(\frac{1}{2}m_\pi^2 f_\pi(\sigma+\delta),\ \frac{1}{2}m_\pi^2 f_\pi(\sigma-\delta),\ \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right)\right].$$

Here, m_i^* is the effective mass of the baryon field ψ_i. Since we consider nuclear matter, $i = p, n$ stand for proton and neutron, respectively. σ, δ and ζ are the scalar fields and χ is the dilaton field.

[2] A. Mishra *et al.*, *Phys. Rev. C* **69**, 024903, arXiv: `nucl-th/0308064` (2004).

Constituent quark mass from chiral effective model

The equations of motion for the fields σ, ζ, δ and χ, obtained from the Lagrangian density in Eq. (1), are [3]

$$k_0\chi^2\sigma - 4k_1\sigma(\sigma^2+\zeta^2+\delta^2) - 2k_2(\sigma^3+3\sigma\delta^2) - 2k_3\chi\sigma\zeta - \frac{d}{3}\chi^4\left(\frac{2\sigma}{\sigma^2-\delta^2}\right) + \left(\frac{\chi}{\chi_0}\right)^2 m_\pi^2 f_\pi - \sum_i g_{\sigma i}\rho_i^s = 0,$$

$$k_0\chi^2\zeta - 4k_1\zeta(\sigma^2+\zeta^2+\delta^2) - 4k_2\zeta^3 - k_3\chi(\sigma^2-\delta^2) - \frac{d}{3}\frac{\chi^4}{\zeta} + \left(\frac{\chi}{\chi_0}\right)^2\left[\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right] - \sum_i g_{\zeta i}\rho_i^s = 0,$$

$$k_0\chi^2\delta - 4k_1\delta(\sigma^2+\zeta^2+\delta^2) - 2k_2\delta(\delta^2+3\sigma^2) + 2k_3\chi\delta\zeta + \frac{2}{3}d\chi^4\left(\frac{\delta}{\sigma^2-\delta^2}\right) - \sum_i g_{\delta i}\rho_i^s = 0,$$

$$k_0\chi(\sigma^2+\zeta^2+\delta^2) - k_3\zeta(\sigma^2-\delta^2) + \chi^3\left[1+4\ln\frac{\chi}{\chi_0}\right]$$
$$+ (4k_4-d)\chi^3 - \frac{4}{3}d\chi^3\ln\left[\left(\frac{(\sigma^2-\delta^2)\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right] + \left(\frac{2\chi}{\chi_0^2}\right)\left[m_\pi^2 f_\pi\sigma + \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right] = 0,$$

where $\rho_i^s \equiv \langle\bar{\psi}_i\psi_i\rangle$ is the scalar density of the i^{th} baryon. Solving the coupled equations, one can obtain the values of σ, ζ, δ and χ as a function of baryon density ρ_i.

[3] P. Papazoglou *et al.*, *Phys. Rev. C* **59**, 411–427, arXiv: nucl-th/9806087 (1999).

Constituent quark mass from chiral effective model

Comparing with the explicit symmetry breaking term in the QCD Lagrangian and using δ=0 for simplicity, we get

$$m_q \langle \bar{q}q \rangle = \frac{1}{2} m_\pi^2 f_\pi \sigma, \tag{2}$$

where $m_q = (m_u + m_d)/2$ is the current quark mass and $\langle \bar{q}q \rangle$ is the quark condensate.

By normalizing the density-dependent quark condensate, we express the constituent u-quark mass as [4],

$$M_q(\rho_B) = \left(\frac{\langle \bar{q}q \rangle (\rho_B)}{\langle \bar{q}q \rangle (\rho_B = 0)} \right)^{1/3} M_q(\rho_B = 0), \tag{3}$$

where $\langle \bar{q}q \rangle (\rho_B = 0) = (263.5 \text{ MeV})^3$ and $M_q(\rho_B = 0) = 313$ MeV are the vacuum expectation values.

Assuming ρ_B as $\rho_q = 3\rho_B$, the Fermi momentum of the quark matter is related to number density as $p_F = (6\pi^2 \rho_q / g)^{1/3}$, where $g = g_s \times g_c = 2 \times 3 = 6$. This gives the expression for quark chemical potential,

$$\mu_q = \sqrt{(3\pi^2 \rho_B)^{2/3} + M_q^2(\rho_B)}, \tag{4}$$

[4] D. R. J. Marattukalam *et al.*, arXiv: 2410.22890 (nucl-th) (Oct. 2024).

Classical expressions of electrical conductivity

In the kinetic theory framework [5,6], at $T \to 0$ the conductivity tensor has the form,

$$\sigma^{ij} = gQ^2 \int \frac{d^3\vec{p}}{(2\pi)^3}\delta(\mu_q - E)\frac{\tau_c}{1+(\tau_c/\tau_B)^2}\frac{p^i p^k}{E^2}\left[\delta^{jk} + \left(\frac{\tau_c}{\tau_B}\right)^2 b^j b^k + \frac{\tau_c}{\tau_B}\epsilon^{kjm} b^m\right]. \quad (5)$$

Here, τ_c is the thermal relaxation time and $\tau_B = E/QB$ is the magnetic relaxation time.

For a relatively weak magnetic field $\vec{B} = B\hat{k}$,

$$\sigma^{||}_{CM} = gQ^2 \int \frac{d^3p}{(2\pi)^3}\tau_c\delta(\mu_q - E)\frac{p^2}{3E^2} = \frac{Q^2}{\pi^2}\tau_c\frac{(\mu_q^2 - M^2)^{3/2}}{\mu_q}, \quad (6)$$

$$\sigma^{\perp}_{CM} = gQ^2 \int \frac{d^3p}{(2\pi)^3}\frac{\tau_c}{1+(\tau_c/\tau_B)^2}\delta(\mu_q - E)\frac{p^2}{3E^2} = \frac{Q^2}{\pi^2}\frac{\tau_c}{1+(\tau_c/\tau_B)^2}\frac{(\mu_q^2 - M^2)^{3/2}}{\mu_q}, \quad (7)$$

$$\sigma^{\times}_{CM} = gQ^2 \int \frac{d^3p}{(2\pi)^3}\delta(\mu_q - E)\frac{\tau_c(\tau_c/\tau_B)}{1+(\tau_c/\tau_B)^2}\frac{p^2}{3E^2} = \frac{Q^2}{\pi^2}\frac{\tau_c(\tau_c/\tau_B)}{1+(\tau_c/\tau_B)^2}\frac{(\mu_q^2 - M^2)^{3/2}}{\mu_q}, \quad (8)$$

[5] S. Ghosh *et al.*, *Phys. Rev. D* **102**, 114015, arXiv: 1911.10005 (hep-ph) (2020).

[6] J. Dey *et al.*, *Nucl. Phys. A* **1034**, 122654, arXiv: 2103.15364 (hep-ph) (2023).

Quantum expressions of electrical conductivity

The conductivity tensor σ^{ij} in Eq. (5) can be re-expressed by modifying the phase space integrals to incorporate the quantization of the quark momenta in the xy-plane, *i.e.*,

$$g_s \int \frac{d^3\vec{p}}{(2\pi)^3} \rightarrow \frac{QB}{(2\pi)^2} \sum_{l=0}^{\infty} \alpha_l \int d\phi \int \frac{dp^z}{2\pi}. \tag{9}$$

For a strong magnetic field $\vec{B} = B\hat{k}$,

$$\sigma_{QM}^{||} = 3Q^2 \sum_{l=0}^{\infty} \alpha_l \frac{QB}{(2\pi)^2} \int_{-\infty}^{\infty} \delta(\mu_q - E_l)\tau_c \frac{(p^z)^2}{E_l^2} dp^z = 6Q^2 \sum_{l=0}^{l_{max}} \alpha_l \frac{QB}{(2\pi)^2} \tau_c \frac{\sqrt{\mu_q^2 - M_l^2}}{\mu_q}, \tag{10}$$

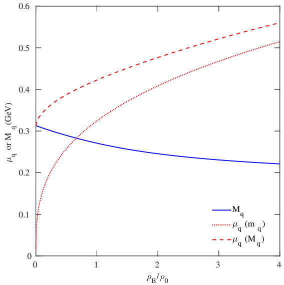
$$\sigma_{QM}^{\perp} = 3Q^2 \sum_{l=0}^{\infty} l\alpha_l \frac{(QB)^2}{(2\pi)^2} \int_{-\infty}^{\infty} \delta(\mu_q - E_l) \frac{\tau_c}{1 + (\tau_c/\tau_B)^2} \frac{dp^z}{E_l^2} = 6Q^2 \sum_{l=0}^{l_{max}} l\alpha_l \frac{Q^2B^2}{(2\pi)^2} \frac{\tau_c}{1 + (\tau_c/\tau_B)^2} \frac{1}{\mu_q \sqrt{\mu_q^2 - M_l^2}}, \tag{11}$$

$$\sigma_{QM}^{\times} = 3Q^2 \sum_{l=0}^{\infty} l\alpha_l \frac{(QB)^2}{(2\pi)^2} \int_{-\infty}^{\infty} \delta(\mu_q - E_l) \frac{\tau_c(\tau_c/\tau_B)}{1 + (\tau_c/\tau_B)^2} \frac{dp_r^z}{E_l^2} = 6Q^2 \sum_{l=0}^{l_{max}} l\alpha_l \frac{Q^2B^2}{(2\pi)^2} \frac{\tau_c(\tau_c/\tau_B)}{1 + (\tau_c/\tau_B)^2} \frac{1}{\mu_q \sqrt{\mu_q^2 - M_l^2}}, \tag{12}$$

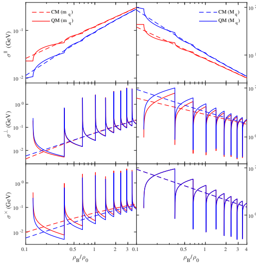
where $M_l \equiv \sqrt{M^2 + 2lQB}$ and $E_l \equiv \sqrt{(p^z)^2 + M_l^2}$. A physical solution exists only when $\mu > M_l$, which puts a constraint on the maximum number of Landau levels included, $l_{max} = \text{Integer}\left[\frac{\mu_q^2 - M^2}{2QB}\right]$.

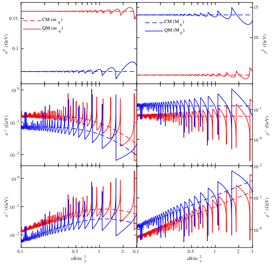
Results

- A quantitative comparison of the conductivity components obtained from the relaxation time approximated (RTA) BTE in the limit $T \to 0$ at finite baryon density ρ_B.
- A simplified system consisting of a single quark flavour - the u quark.
- We compare the conductivity components in two different mass scenarios:
    - current quark mass $m_q = (m_u + m_d)/2 = 4.63$ MeV
    - density-dependent in-medium constituent quark mass $M_q(\rho_B)$ obtained using the chiral effective model.



Results





Magnetic fields create anisotropy. For a magnetic field along the z-axis,

- parallel conductivity ($\sigma^{||} = \sigma^{zz}$)
- perpendicular conductivity ($\sigma^{\perp} = \sigma^{xx} = \sigma^{yy}$)
- Hall conductivity ($\sigma^{\times} = \sigma^{xy} = -\sigma^{yx}$)

We observe Shubnikov-de Haas (SdH) type oscillations in the conductivity and resistivity components as a function of baryon density and magnetic fields. Quantization of Hall conductivity and resistivity are also observed.

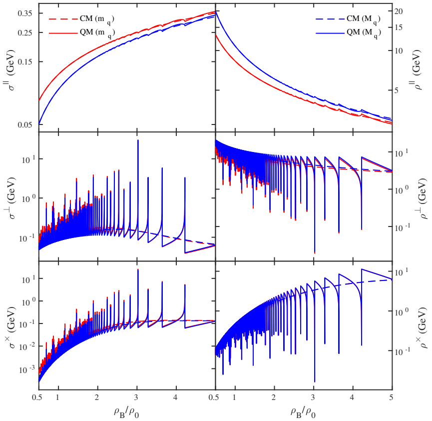
Results

As we move from the surface to the core of a neutron star, the density and the magnetic field increases gradually.

$$B(\rho_B/\rho_0) = B_{surf} + B_0[1 - \exp(-\beta(\rho_B/\rho_0)^{\gamma})]^7$$

β=0.01, γ=3, $B_{surf} = 10^{14}$ G, $B_0 = 5 \times 10^{18}$ G.

The seemingly opposite effects of baryon density and magnetic field with regard to the quantization of electrical conductivity and resistivity can give rise to a rather unique conductivity/resistivity profile in neutron stars.

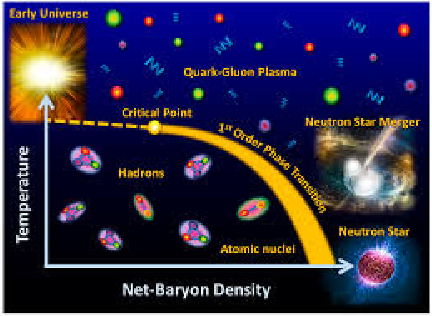


7 D. Bandyopadhyay *et al.*, *Phys. Rev. Lett.* **79**, 2176–2179, arXiv: astro-ph/9703066 (1997).

Summary

Motivated by the nearly degenerate matter inside neutron stars and the reaction zone of upcoming CBM/NICA experiments, we have systematically generated the results of the classical and quantum expressions of conductivity and resistivity components along density and magnetic field axes using straightforward current quark mass and constituent quark mass obtained from the chiral effective model.

- SdH oscillations and QHE phenomena – an oscillatory or quantized behaviour along the density and magnetic field axes due to the Landau quantization.
- The effects of quantization are pronounced in the low density and high magnetic field region – the quantum domain.
- Possibility of observing SdH-type oscillations and QHE in neutron stars – using a density-dependent magnetic field profile.



Thank you!