Viscous effects of a hot QGP medium in time dependent magnetic field

ATHIC 2025

Gowthama K K, IIT Bombay
(Debarshi Dey, Sadhana Dash, Basanta Kumar Nandi)

Introduction- Shear and bulk viscosity and importance of time dependent EM fields

Shear viscosity results

Phenomenologically significant quantities

The energy-momentum tensor of the QGP fluid can be expressed as

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 p^0} p^\mu p^\nu f(x,p),$$

The spatial components of the dissipative part of the energy momentum tensor can be related to the shear and bulk viscous coefficients in the following manner,

$$\Delta T^{ij} = -\eta W^{ij} - \zeta \delta^{ij} \partial_l u^l, \tag{1}$$

and the distribution function, $f(x,p) = f_0(x,p) + \delta f(x,p)$

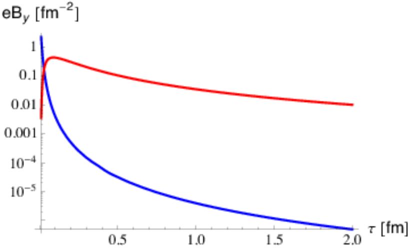

Figure: Evolution of magnetic field in vacuum (blue) and in medium (red).

Heavy-ion collision experiments at RHIC and LHC have shown the presence of magnetic fields. The form of the decay of the magnetic field can be represented as,

$$eB(t) = eB_0 Exp[-\frac{t}{\tau_B}]$$

In the presence of external EM fields, the away from equilibrium distribution function is, $\delta f(t) \propto E(t), B(t)$

Writing the Boltzmann equation out into its components, we have

$$\frac{p^\mu}{p^0}\frac{\partial f_0}{\partial x^\mu} + \frac{\partial(\delta f(t))}{\partial t} + qE(t)\cdot\frac{\partial f_0}{\partial p} + q(v\times B(t))\frac{\partial(\delta f(t))}{\partial p} = -\frac{\delta f(t)}{\tau_R}$$

using the following *Ansatz* for δf

$$\delta f(t) = -\tau_R qE(t)\cdot\frac{\partial f_0}{\partial p} - \Gamma(t)\cdot\frac{\partial f_0}{\partial p},$$

To obtain the shear viscosity, we compare with Eq.1, and read off the coefficient of $-W^{ij}$. The expression reads

$$\eta(t) = -\Bigg[\sum_f g_f\left(\frac{2}{15}\right)\int\frac{d^3p}{(2\pi)^3}\frac{p^4}{2\epsilon_f^2}\left(\frac{\partial f^0}{\partial\epsilon_f}\right)$$
$$(e^{\int(-1/\tau_R - iqB(t)/\epsilon)dt}\int dt\, e^{-\int(-1/\tau_R - iqB(t)/\epsilon)dt} +$$
$$e^{\int(-1/\tau_R + iqB(t)/\epsilon)dt}\int dt\, e^{-\int(-1/\tau_R + iqB(t)/\epsilon)dt}) + G.C\Bigg]$$

Consider the magnetic field, $eB(t) = eB_0 Exp[-\frac{t}{\tau_B}]$, in the limiting case we can see that $\lim_{\tau_B \to \infty} eB(t) = eB_0$. Then the factor,

$$\lim_{\tau_B \to \infty} e^{\int(-1/\tau_R - iqB(t)/\epsilon)dt} \int dt\, e^{-\int(-1/\tau - iqB(t)/\epsilon)dt} = \frac{e^{(1/\tau_R - i\omega)t} e^{-(1/\tau_R - i\omega)t}}{(1/\tau_R - i\omega)}$$

$$= \frac{1}{(1/\tau_R - i\omega)}.$$

here, $\omega = qB_0/\epsilon$. Combining the two terms,

$$\frac{1}{(1/\tau_R - i\omega)} + \frac{1}{(1/\tau_R + i\omega)} = \frac{2\tau_R}{(1 + \omega^2 \tau_R^2)},$$

and hence we reduce to the equation in literature for the constant field case,

$$\eta = -\Bigg[\sum_f g_f \left(\frac{2}{15}\right) \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{2\epsilon_f^2} \left(\frac{\partial f^0}{\partial \epsilon_f}\right) \left[\frac{2\tau_R}{(1 + \omega^2 \tau_R^2)}\right] + G.C\Bigg]$$

Shear viscosity Vs Temperature

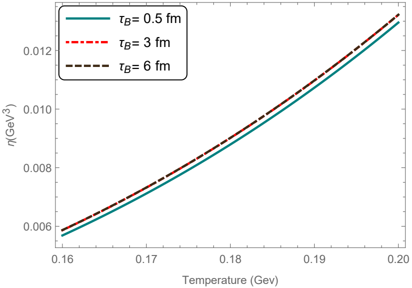

Figure: Temperature dependence of shear viscosity and its dependency on the decay rate of the magnetic field, τ_B at $t = 5\ fm$, $qB_0 = 0.01\ GeV^2$ and $\mu = 0.1\ GeV$.

Shear viscosity Vs Time

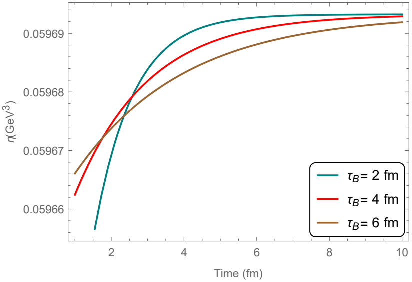

Figure: Time dependence of shear viscosity and its dependency on the decay rate of the magnetic field, τ_B at $T = 0.2\ GeV$, $qB_0 = 0.01\ GeV^2$ and $\mu = 0.1\ GeV$.

Shear viscosity as a function of qB_0 and τ_B

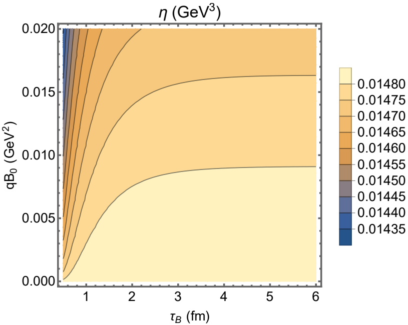

Figure: Decay rate, τ_B and amplitude qB_0 dependency of the shear viscosity at $T = 0.2\ GeV$, $t = 5\ fm$ and $\mu = 0.1\ GeV$.

The Shear viscosity to entropy density ratio

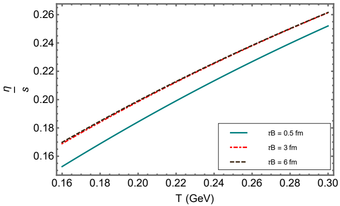

Figure: Shear viscosity to entropy density as a function of temperature and its dependency on the decay rate of the magnetic field, τ_B at $t = 5$ fm, $qB_0 = 0.01\ GeV^2$ and $\mu = 0.1\ GeV$.

Thermalisation time

The thermalisation time, τ_{th} can be modelled as, $\tau_{th} = 4\pi\eta/Ts$,

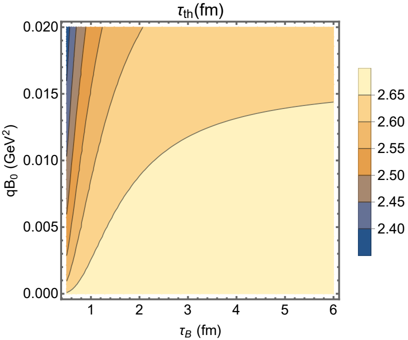

Figure: Thermalistaion time, τ_{th} dependency on the decay rate, τ_B, and amplitude, qB_0, at $T = 0.2\ GeV$, $t = 5\ fm$ and $\mu = 0.1\ GeV$.