# Spin-Polarization in Neutron Stars and investigating its impact on Gravitational wave signatures

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# Introduction

- **Spin-polarized baryon in neutron stars.**

$\Downarrow$

- **CDM3Y interaction $\rightarrow$ Hartree-Fock framework.**

$\Downarrow$

- **Study of $P$-$\epsilon$ relation and magnetization.**

$\Downarrow$

- **Investigate possible connection with gravitational waves.**

*Hi Tan et al., 2020*

*Khoa, Oretzen, 1996*

# Hartree-Fock approach to the spin polarized neutron-matter

- **Neutron-proton asymmetry :** $\boxed{\delta = \frac{(n_n - n_p)}{n_b}}$

- **Spin polarization parameter :** $\boxed{\Delta_{n,p} = \frac{(n\uparrow_{n,p} - n\downarrow_{n,p})}{n_{n,p}}}$

$$E = E_{\text{kin}} + \frac{1}{2} \sum_{k\sigma\tau, k'\sigma'\tau'} \Big[ \langle k\sigma\tau, k'\sigma'\tau' | v_D | k\sigma\tau, k'\sigma'\tau' \rangle + \langle k\sigma\tau, k'\sigma'\tau' | v_{\text{EX}} | k'\sigma\tau, k\sigma'\tau' \rangle \Big]$$

*Vidana, Polls (2002)*

# Direct and exchange Potentials :

- **Direct and exchange potentials :**

$$v^{D(\mathrm{EX})}(n_b, r) = F_{00}(n_b) v_{00}^{D(\mathrm{EX})}(r) + F_{10}(n_b) v_{10}^{D(\mathrm{EX})}(r)(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma})$$
$$+ F_{01}(n_b) v_{01}^{D(\mathrm{EX})}(r)(\boldsymbol{\tau} \cdot \boldsymbol{\tau}) + F_{11}(n_b) v_{11}^{D(\mathrm{EX})}(r)(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma})(\boldsymbol{\tau} \cdot \boldsymbol{\tau})$$

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- **M-3Y model :**

$$\boxed{v_{\mathrm{st}}^{D(\mathrm{EX})}(r) = \sum_{\nu=1}^{3} Y_{\mathrm{st}}^{D(\mathrm{EX})}(\nu) \frac{\exp(-R_\nu r)}{R_\nu r}}$$

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- **Expression for Energy Density :**

$$E = \frac{3}{10} \sum_{\sigma\tau} \frac{\hbar^2 k_{F\sigma\tau}^2}{m_\tau} n_{\sigma\tau} + F_{00}(n_b) E_{00} + F_{10}(n_b) E_{10}$$
$$+F_{01}(n_b) E_{01} + F_{11}(n_b) E_{11}$$

*D.T.Khoa et al.(2016)*

*Vidana, Polls (2002)*

# Energy density using CDM3Y interaction

- **Spin-Isospin independent Energy :**

$$E_{00} = \frac{1}{2} n_b^2 J_{00}^D + \int A_{00}^2 v_{00}^{EX}(r)\, d^3 r$$

- **Spin dependent-Isospin independent Energy :**

$$E_{10} = \frac{1}{2} n_b^2 J_{10}^D \left( \frac{\Delta_n}{2}(1+\delta) + \frac{\Delta_p}{2}(1-\delta) \right)^2 + \int A_{10}^2 v_{10}^{EX}(r)\, d^3 r$$

- **Spin independent-Isospin dependent Energy :**

$$E_{01} = \frac{1}{2} n_b^2 J_{01}^D \delta^2 + \int A_{01}^2 v_{01}^{EX}(r)\, d^3 r$$

- **Spin-Isospin dependent Energy :**

$$E_{11} = \frac{1}{2} n_b^2 J_{11}^D \left( \frac{\Delta_n}{2}(1+\delta) - \frac{\Delta_p}{2}(1-\delta) \right)^2 + \int A_{11}^2 v_{11}^{EX}(r)\, d^3 r$$

# Energy density using CDM3Y interaction

- $CDM3Y_8$ **interaction realistic density dependencies** $F_{st(n_b)}$

$$\boxed{F_{st}(n_b) = C_{st}\left[1 + \alpha_{st}\exp(-\beta_{st} n_b) + \gamma_{st} n_b\right]}$$

*N.H.Tan et al. (2020)*

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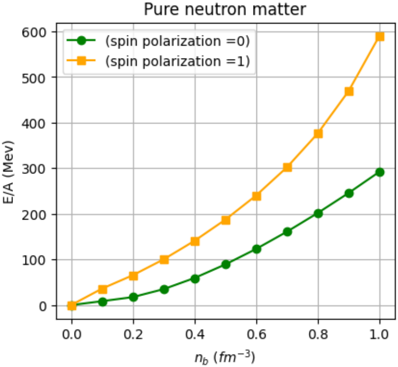
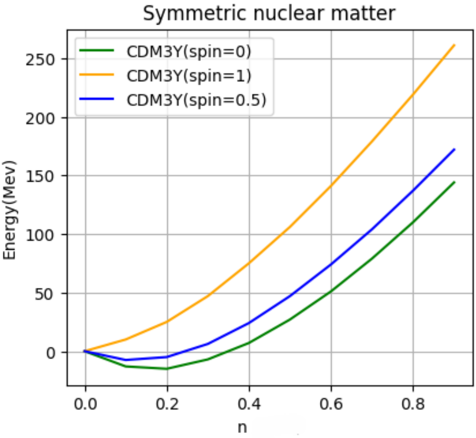
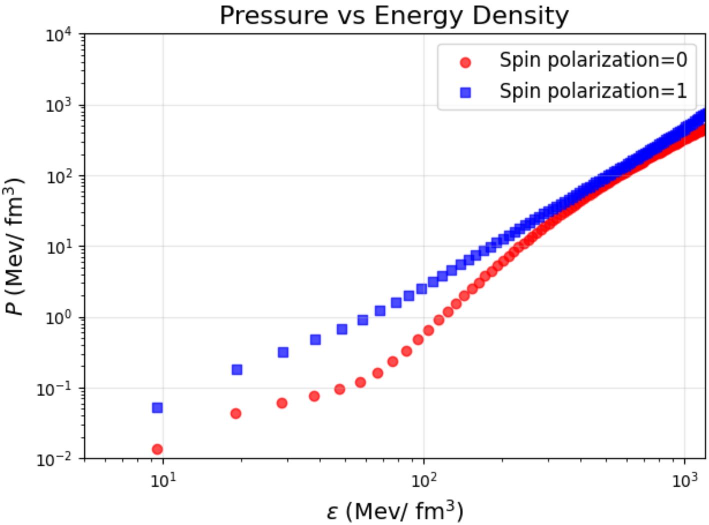


# Energy vs spin polarization parameter

## Symmetric nuclear matter for different Spin polarization :



# Pressure vs Energy density



# Magnetic field strength and spin polarization

- **Magnetization :**

$$\boxed{M_{n_b,\Delta} = \mu\rho\Delta}$$

- **Spin polarization due to magnetic field:**

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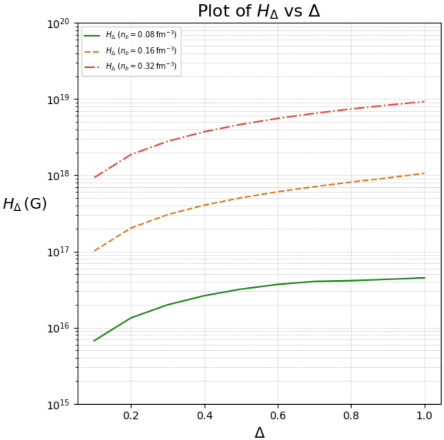

# EOS dependence of gravitational waves

**Gravitational waves characteristic strain from a rotating isolated neutron star (NS) :**

$$h_c = f h_0 \sqrt{\frac{dt}{d\omega}} = \frac{32\pi^2 G I_0 \epsilon}{D c^4} \sqrt{\frac{\pi I_0}{N_{\text{tot}}}}$$

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- **Spin down rate via Gravitational waves :**

$$\dot{\omega}(t) = -\frac{32GI_0}{5c^5}\left[\epsilon_{mag} + \epsilon_{sta}(t) + \epsilon_{acc}(t)\right]^2\omega^5$$

*Huang, Lu, Rice (2020)*

*N.H.Tan et al. (2020)*

## Magnetically induced deformation :

$$\boxed{\epsilon_{mag} = 6.626 \times 10^{-6} B_{15}^2 M_{\odot 1.4}^2 R_6^4 \left(1 - \frac{0.385}{\Lambda}\right)}$$

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## Accretion induced ellipticity :

$$\epsilon_{acc}(t) = 2 M_{acc} R^2 I_0^{-1}$$

*Giliberti, Cambiotti (2022)*

# Acknowledgments

# Thank You!

For your attention and participation.

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