Exploring the flow harmonic correlations via multi-particle Symmetric and Asymmetric Cumulants in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

Kaiser Shafi

with Prabhupada Dixit, Sandeep Chatterjee, and Md. Nasim
Indian Institute of Science Education and Research, Berhampur
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Outline

Initial anisotropy in coordinate space

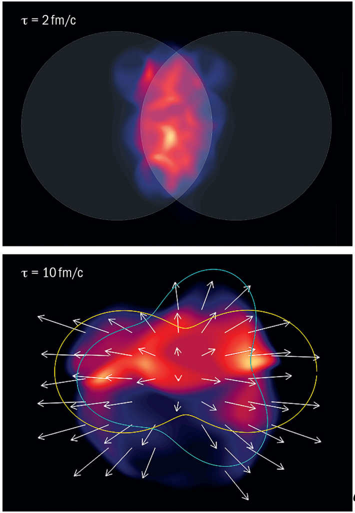
⇓

Thermalized medium

⇓

Final anisotropy in momentum space

Anisotropic flow is a sensitive probe both of initial conditions in heavy-ion collisions, and of QGPs transport properties (e.g. of its shear viscosity)



[a]

[a] https://cerncourier.com/a/going-with-the-flow

Anisotropic Flow

Fourier series to describe anisotropic emission of particles in the plane transverse to the beam direction after every heavy-ion collision. [1]

$$E\frac{d^3N}{dp^3} = \frac{d^2N}{2\pi p_t dp_t dy}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos\left[n\left(\phi - \psi_n\right)\right]\right]$$

- v_n : flow amplitudes
- ψ_n : symmetry planes

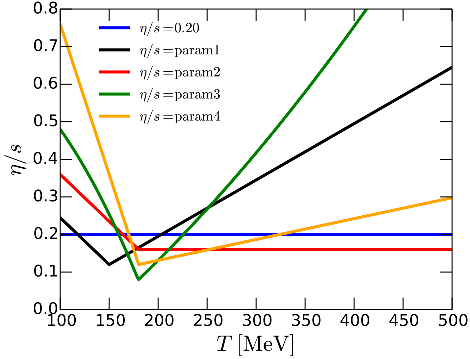
Anisotropic flow is quantified with v_n and ψ_n.

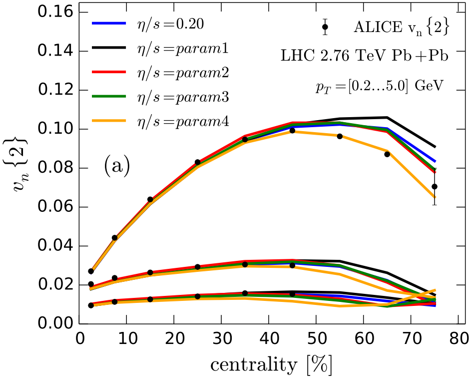
- v_1 : directed flow
- v_2 : elliptic flow
- v_3 : triangular flow
- v_4 : quadrangular flow, etc.

[1] Poskanzer, Voloshin, Phys.Rev.C 58 (1998) 1671-1678

Traditional flow observables

- v_n are insensitivite to temperature dependence of η/s [2]



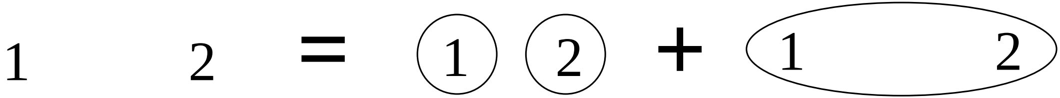


[2] H. Niemi, K. J. Eskola, R. Paatelainen, Phys. Rev. C 93, 024907 (2016)

Multi-particle correlations

Two-particle correlations

$$f(\mathbf{p}_1,\mathbf{p}_2) = f(\mathbf{p}_1)f(\mathbf{p}_2) + f_c(\mathbf{p}_1,\mathbf{p}_2)$$



Decomposition of the two-particle distribution into uncorrelated and correlated components. [3]

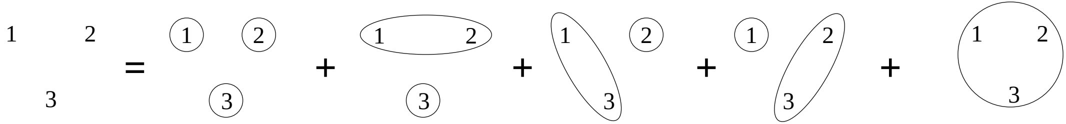
[3] Borghini, Dinh, Ollitrault, PhysRevC.63.054906

Multi-particle correlations

Three-particle correlations

$$f(\mathbf{p}_1,\mathbf{p}_2,,\mathbf{p}_3) = f(\mathbf{p}_1)f(\mathbf{p}_2)f(\mathbf{p}_3) + f_c(\mathbf{p}_1,\mathbf{p}_2)f(\mathbf{p}_3) + f_c(\mathbf{p}_1,\mathbf{p}_3)f(\mathbf{p}_2)$$

$$+f_c(\mathbf{p}_2,\mathbf{p}_3)f(\mathbf{p}_1) + f_c(\mathbf{p}_1,\mathbf{p}_2),\mathbf{p}_3)$$

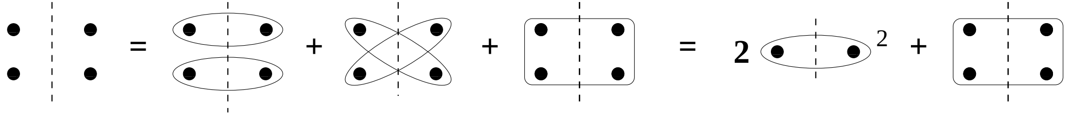


Decomposition of the three-particle distribution. [4]

[4] Borghini, Dinh, Ollitrault, PhysRevC.63.054906

Multi-particle correlations

Four-particle correlations



Decomposition of four-particle distribution. [5]

$$
\begin{aligned}
c_n\{4\} &\equiv \langle\langle e^{in(\varphi_1+\varphi_2-\varphi_3-\varphi_4)}\rangle\rangle \\
&- \langle\langle e^{in(\varphi_1-\varphi_3)}\rangle\rangle\langle\langle e^{in(\varphi_2-\varphi_4)}\rangle\rangle \\
&- \langle\langle e^{in(\varphi_1-\varphi_4)}\rangle\rangle\langle\langle e^{in(\varphi_2-\varphi_3)}\rangle\rangle .
\end{aligned}
$$

$$c_n\{4\} \equiv \langle v_n^4\rangle - 2\langle v_n^2\rangle^2$$

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

[5] Borghini, Dinh, Ollitrault, PhysRevC.63.054906

Cumulants definitions

The Symmetric Cumulants, defined below, can probe the genuine correlations between different flow harmonics [6]

$$
\begin{aligned}
SC(m,n) &\equiv \langle v_m^2 v_n^2\rangle_c = \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle \\
&= \langle\langle e^{i(m\varphi_1+n\varphi_2-m\varphi_3-n\varphi_4)}\rangle\rangle - \langle\langle e^{i(m\varphi_1-n\varphi_3)}\rangle\rangle\langle\langle e^{i(m\varphi_2-n\varphi_4)}\rangle\rangle
\end{aligned}
$$

The Asymmetric Cumulants, which are more generalized, probe the genuine correlations between the different moments of different flow harmonics: [7]

$$
\begin{aligned}
\mathrm{AC}_{2,1}(m,n) &\equiv \langle (v_m^2)^2 v_n^2\rangle_c \equiv \langle v_m^4 v_n^2\rangle_c, \\
&= \langle v_m^4 v_n^2\rangle - \langle v_m^4\rangle\langle v_n^2\rangle - 2\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle + 2\langle v_m^2\rangle^2\langle v_n^2\rangle, \\
\mathrm{AC}_{3,1}(m,n) &\equiv \langle (v_m^2)^3 v_n^2\rangle_c \equiv \langle v_m^6 v_n^2\rangle_c
\end{aligned}
$$

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[6] Bilandzic, Christensen, Gulbrandsen, Hansen, Zhou, Phys.Rev.C 89 (2014) 6, 064904

[7] Bilandzic, Lesch, Mordasini, Taghavi, Phys.Rev.C 105 (2022) 2, 024912

Normalized Cumulants

To eliminate the effect of the magnitudes of v_m and v_n on the cumulants, we define the normalized cumulants. This enables us to compare data and model calculations in a quantitative way and compare the fluctuations of the initial and final states.
The normalized symmetric and asymmetric cumulants are defined as: [8]

$$\mathrm{NSC}(m,n) = \frac{\mathrm{SC}(m,n)}{\langle v_m^2\rangle\langle v_n^2\rangle}$$

$$\mathrm{NAC}_{2,1}(m,n) = \frac{\mathrm{AC}_{2,1}(m,n)}{\langle v_m^2\rangle^2\langle v_n^2\rangle}$$

[8] Taghavi, Eur.Phys.J.C 81 (2021) 7, 652

The Bootstrap method: The calculation of sampling variance

Let $\hat{O}$ be the observable on which we intend to find the standard error.

- Given a parent sample of size n, we construct B independent bootstrap samples $X_1^*, X_2^*, \ldots, X_B^*$, each with n data points randomly drawn with replacement.
- We evaluate the observable for each bootstrap sample.

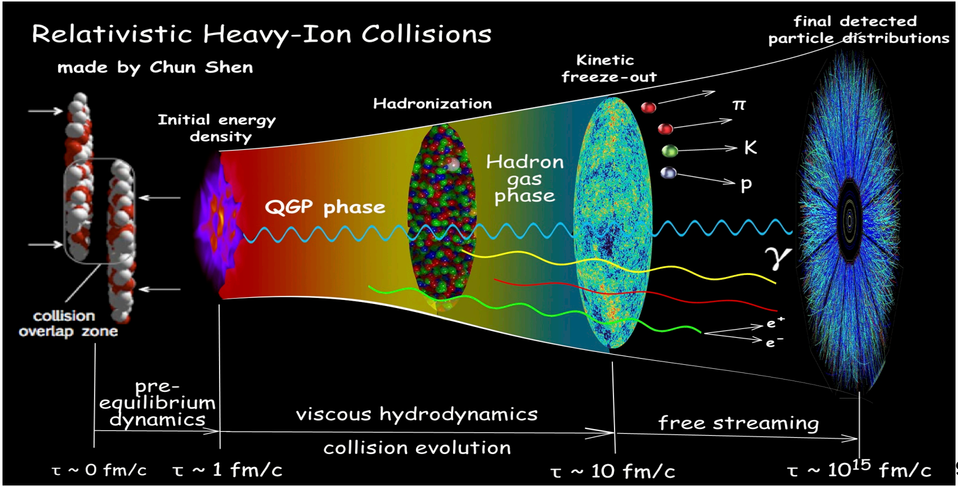
$$\hat{O}_b^* = \hat{O}(X_b^*), \qquad b = 1, 2, ..., B.$$

- The sampling variance of the observable can then be calculated as follows:

$$Var(\hat{O}) = \frac{1}{B-1}\sum_{b=1}^{B}(\hat{O}_b^* - \bar{\hat{O}}),$$

where, $\bar{\hat{O}} = \frac{1}{B}\sum_{b=1}^{B} \hat{O}_b^*$.

Framework

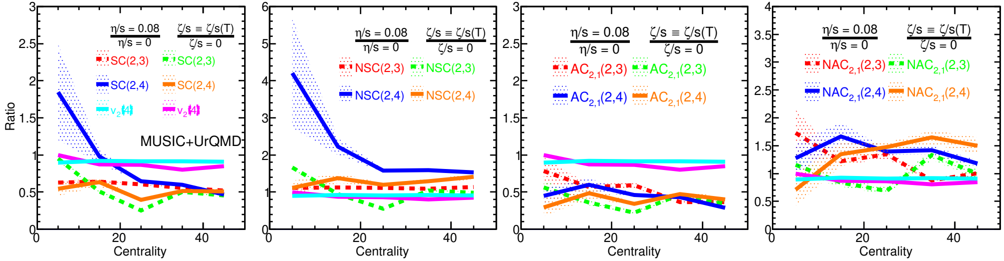


[9]

MCGM → MUISC → UrQMD

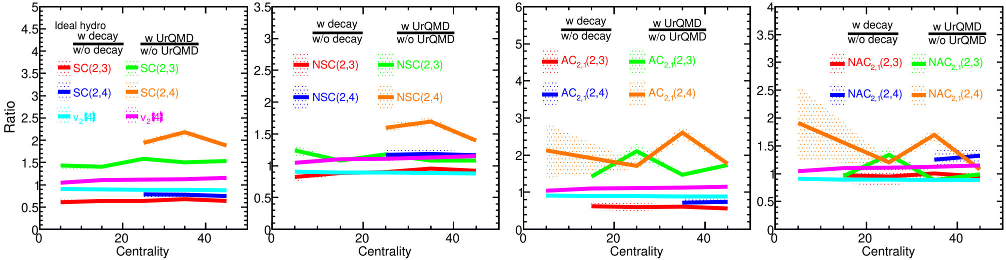
[9] Chun Shen: https://chunshen1987.github.io/

Sensitivity to hydrodynamic transport coefficients



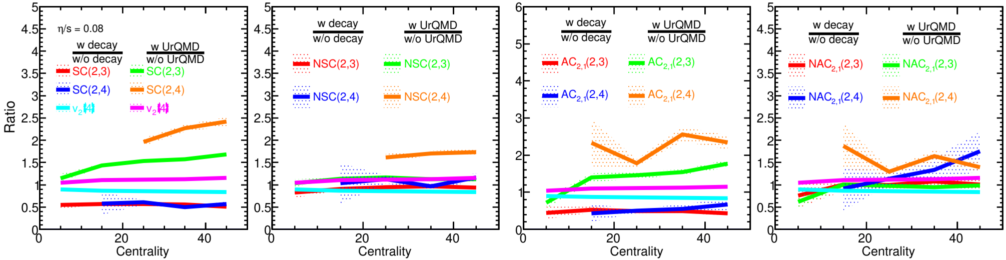
- The large sensitivity of symmetric cumulants primarily arises from the characteristics of anisotropic flow, as the symmetric cumulants involve higher powers of flow coefficients.
- NSC(2,3) and $NAC_{2,1}(2,3)$ are insensitive to the hydro model parameters. Reliable for constraining the initial state of the system's evolution.
- NSC(2,4) and $NAC_{2,1}(2,4)$ show considerable sensitivity to these parameters, allowing them to effectively constrain hydrodynamic models.

Sensitivity to hadronic interactions



- The large sensitivity of symmetric cumulants primarily arises from the characteristics of anisotropic flow, as the symmetric cumulants involve higher powers of flow coefficients.
- NSC(2,3) and $NAC_{2,1}(2,3)$ are insensitive to late-stage hadronic interactions. Reliable for constraining the initial state of the systems evolution.
- NSC(2,4) and $NAC_{2,1}(2,4)$ show considerable sensitivity to these stages, allowing them to effectively constrain transport models.

Sensitivity to hadronic interactions



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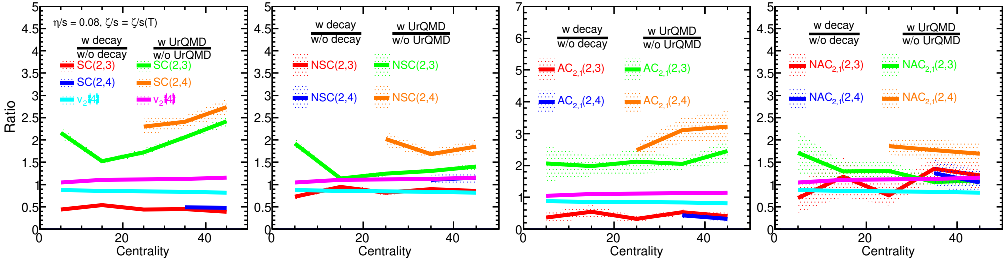

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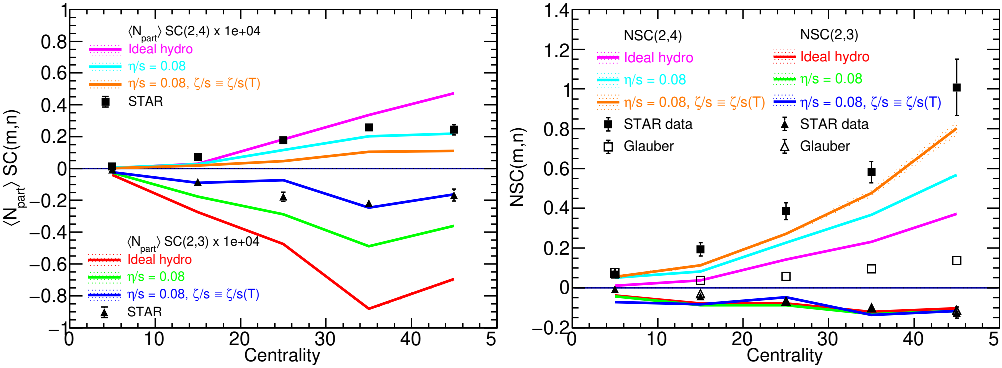
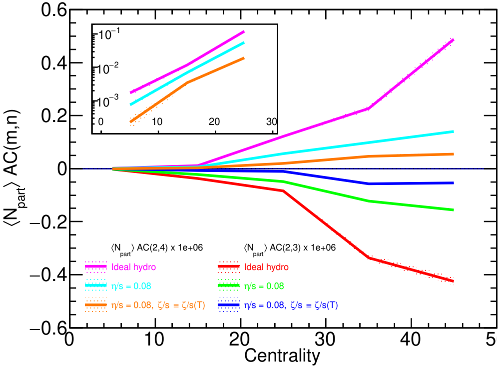
Centrality dependence of $SC(m,n)$

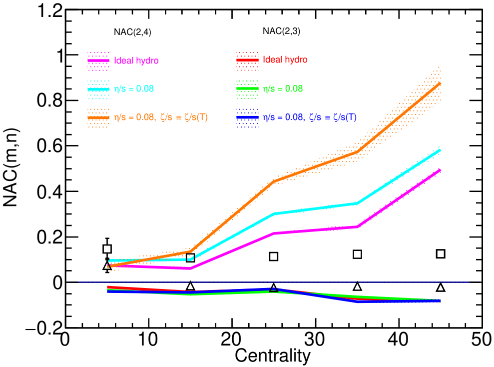


- (Normalized) Symmetric cumulant vs centrality from four-particle correlations compared with STAR data [10] in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. SC(2,3) is consistently negative while as SC(2,4) is consistently positive.
- Note that we have shown the Glauber results for NSC(2,4) to highlight that in non-central collisions v_4 is mainly driven by ε_2^2 and not ε_4.

[10] Adam et al., Phys.Lett.B 783 (2018) 459-465

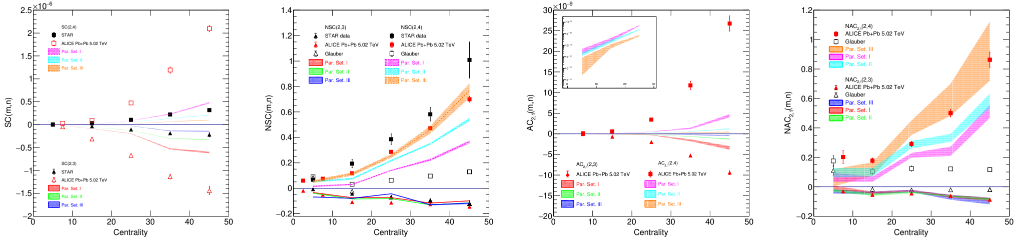
Centrality dependence of $AC_{2,1}(m, n)$





- (Normalized) Asymmetric cumulant vs centrality from six-particle correlations in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. $AC_{2,1}(2,3)$ is consistently negative while as $AC_{2,1}(2,4)$ is consistently positive.
- Note that we have shown the Glauber results for $NAC_{2,1}(2,4)$ to highlight that in non-central collisions v_4 is mainly driven by ε_2^2 and not ε_4.

$\sqrt{s_{NN}}$ dependence



- The values of SC(m,n) and AC$_{2,1}(m,n)$ at ALICE [11] [12] are larger-by factors ranging from 3 to 8-compared to those at STAR [13].
- Important to note the similarities in the magnitude and centrality dependence of the normalized cumulants. Minimal energy dependence for the normalized cumulants.

[11] Acharya et. al., Phys.Rev.C 108 (2023) 5, 055203
[12] Acharya et al., Phys.Lett.B 818 (2021) 136354
[13] Adam et al., Phys.Lett.B 783 (2018) 459-465

Summary

- v_n are insensitivite to temperature dependence of η/s.
- NSC(2, 3) and $\text{NAC}_{2,1}(2,3)$ are to both the hydro model parameters (shear and bulk viscosity) and late-stage hadronic interactions. Reliable for constraining the initial state of the systems evolution.
- NSC(2, 4) and $\text{NAC}_{2,1}(2,4)$ show considerable sensitivity to these stages, allowing them to effectively constrain hydrodynamic and transport models.
- The values of SC(m,n) and $\text{AC}_{2,1}(m,n)$ at ALICE are larger-by factors ranging from 3 to 8-compared to those at STAR.
- Important to note the similarities in the magnitude and centrality dependence of the normalized cumulants. Minimal energy dependence for the normalized cumulants.

Thank you for your attention!

Kaiser Shafi

kaisers@iiserbpr.ac.in