Thermal diffusion properties of a rotating QGP medium

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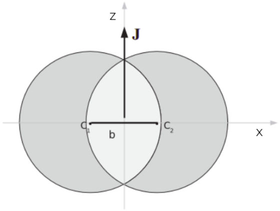
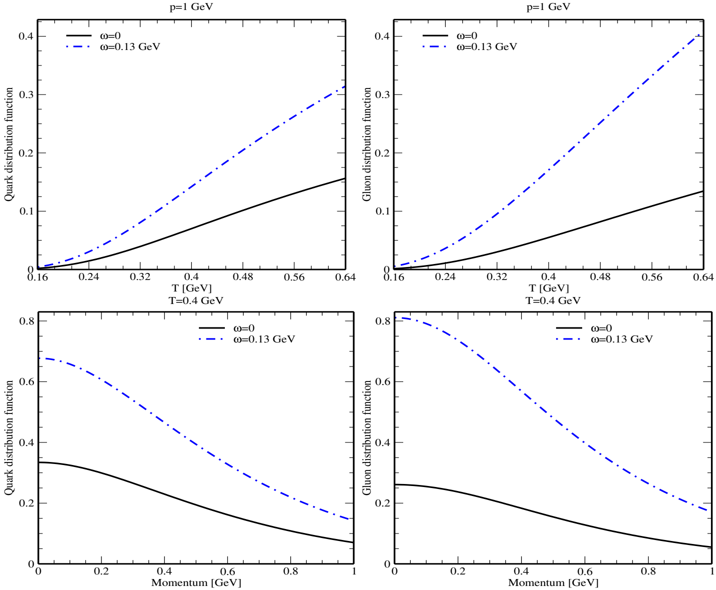
Introduction

- In the noncentral heavy ion collisions, a large angular momentum of the order $J \propto b\sqrt{s_{NN}}$ is produced.
- The interaction zone could possess a significant fraction of the angular momentum varying between $10^3\hbar$ and $10^5\hbar$ with angular velocity in the range 0.01 GeV - 0.1 GeV or even more, which further gets transferred to the produced fireball.
- The polarization of Λ hyperons in heavy ion collisions reveals that the produced fireball has a strong vorticity.
- The fireball or QGP may sustain the large angular momentum for a longer time due to the conservation of the total angular momentum.
- The rapid rotation influences both quark and gluon degrees of freedom and thus, various properties of the QGP medium could be altered.



- Emergence of rotation modifies the quark, antiquark and gluon distribution functions respectively as

$$
\begin{aligned}
f_q &= \frac{1}{e^{\beta(u_\mu p^\mu - \mu)}+1}\frac{\sinh(\beta\omega)}{\sinh(\beta\omega/2)} = f_0^q \chi_q(\beta\omega), \\
\bar{f}_q &= \frac{1}{e^{\beta(u_\mu p^\mu + \mu)}+1}\frac{\sinh(\beta\omega)}{\sinh(\beta\omega/2)} = f_0^{\bar{q}} \chi_{\bar{q}}(\beta\omega), \\
f_g &= \frac{1}{e^{\beta u_\mu p^\mu}-1}\frac{\sinh(3\beta\omega/2)}{\sinh(\beta\omega/2)} = f_0^g \chi_g(\beta\omega).
\end{aligned}
$$


p=1 GeV
ω=0
ω=0.13 GeV
Quark distribution function
T [GeV]
p=1 GeV
ω=0
ω=0.13 GeV
Gluon distribution function
T [GeV]
T=0.4 GeV
ω=0
ω=0.13 GeV
Quark distribution function
Momentum [GeV]
T=0.4 GeV
ω=0
ω=0.13 GeV
Gluon distribution function
Momentum [GeV]

Thermal conductivity of a rotating QGP medium

- The spatial component of heat flow due to the action of external disturbance:
$Q^i = \sum_f g_f \int \frac{d^3\mathrm{p}}{(2\pi)^3} \frac{p^i}{\omega_f} \left[(\omega_f - h_f)\delta f_q + (\omega_f - \bar{h}_f)\delta \bar{f}_q\right]$.
- Navier-Stokes equation: $Q^i = -\kappa\delta^{ij}\left[\partial_j T - \frac{T}{\varepsilon+P}\partial_j P\right]$.
- Relativistic Boltzmann transport (RBT) equation in the novel relaxation time approximation (RTA):

$$p^\mu \frac{\partial f_q'}{\partial x^\mu} + q_f F^{\rho\sigma} p_\sigma \frac{\partial f_q'}{\partial p^\rho} = -\frac{p_\nu u^\nu}{\tau_f}\left[\delta f_q - \frac{\langle(\omega_f/\tau_f)\,\delta f_q\rangle_0}{\langle\omega_f/\tau_f\rangle_0} + \frac{P_1^{(0)}\left\langle(\omega_f/\tau_f)\,P_1^{(0)}\delta f_q\right\rangle_0}{\left\langle(\omega_f/\tau_f)\,P_1^{(0)}P_1^{(0)}\right\rangle_0} + \frac{p^{\langle\mu\rangle}\left\langle(\omega_f/\tau_f)\,p_{\langle\mu\rangle}\delta f_q\right\rangle_0}{(1/3)\left\langle(\omega_f/\tau_f)\,p_{\langle\mu\rangle}p^{\langle\mu\rangle}\right\rangle_0}\right],$$

where τ_f represents the novel relaxation time and $f_q' = \delta f_q + f_q$.

$$P_1^{(0)} = 1 - \frac{\langle\omega_f/\tau_f\rangle_0\,\omega_f}{\left\langle\omega_f^2/\tau_f\right\rangle_0},$$

$$p^{\langle\mu\rangle} = (g^{\mu\nu} - u^\mu u^\nu)\,p_\nu .$$

Momentum integral of A (A can be different terms appearing in RBT equation) relative to the local equilibrium distribution function is defined as

$$\langle A\rangle_0 = \int \frac{d^3\mathrm{p}}{(2\pi)^3\omega_f} A f_q \,.$$

- The novel relaxation time for quarks (antiquarks):

$$\tau_{f(\bar{f})} = (\beta\omega_f)^{\xi}\, t_{f(\bar{f})},$$

where ξ is an arbitrary constant and $t_{f(\bar{f})}$ is momentum-independent, whose form is given by

$$t_{f(\bar{f})} = \frac{1}{5.1\, T\alpha_s^2 \log\left(1/\alpha_s\right)\left[1 + 0.12(2N_f + 1)\right]}\ .$$

- In QCD kinetic theories, $\xi = \frac{1}{2}$.
- Expanding the gradient of the particle distribution function in terms of the gradients of the flow velocity and the temperature in the novel relaxation time approximation, the RBT equation can be rewritten as

$$-p^{\mu} f_0^q \left(1 - f_0^q\right) \chi_q(\beta\omega) \left[\left(u_{\alpha} p^{\alpha}\right) \partial_{\mu}\beta + \beta\partial_{\mu}\left(u_{\alpha} p^{\alpha}\right) - \partial_{\mu}\left(\beta\mu\right)\right] + p^{\mu} f_0^q \partial_{\mu}\left(\chi_q(\beta\omega)\right) = -\frac{p_{\nu} u^{\nu}}{\tau_f}\left[\delta f_q - \frac{\left\langle\left(\omega_f/\tau_f\right)\delta f_q\right\rangle_0}{\left\langle\omega_f/\tau_f\right\rangle_0} + \frac{P_1^{(0)}\left\langle\left(\omega_f/\tau_f\right) P_1^{(0)}\delta f_q\right\rangle_0}{\left\langle\left(\omega_f/\tau_f\right) P_1^{(0)} P_1^{(0)}\right\rangle_0} + \frac{p^{\langle\mu\rangle}\left\langle\left(\omega_f/\tau_f\right) p_{\langle\mu\rangle}\delta f_q\right\rangle_0}{(1/3)\left\langle\left(\omega_f/\tau_f\right) p_{\langle\mu\rangle} p^{\langle\mu\rangle}\right\rangle_0}\right]\ .$$

- For quarks:

$$\begin{aligned}\delta f_q \quad = \quad & -\frac{\beta\tau_f f_0^q\left(1 - f_0^q\right)}{J}\chi_q(\beta\omega)\left[\frac{\left(\omega_f - h_f\right) p^j}{T\omega_f}\left(\partial_j T - \frac{T}{nh_f}\partial_j P\right) + p_0\frac{DT}{T} - \frac{p^{\mu}p^{\alpha}}{p_0}\nabla_{\mu}u_{\alpha} + TD\left(\frac{\mu}{T}\right)\right] \\ & -\frac{\beta\tau_f f_0^q}{J}\sinh\left(\frac{\beta\omega}{2}\right)\left[TD\left(\frac{\omega}{T}\right) + \frac{Tp^{\alpha}}{p_0}\nabla_{\alpha}\left(\frac{\omega}{T}\right)\right].\end{aligned}$$

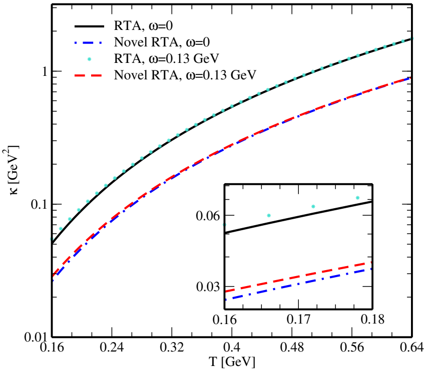
- $$J = \frac{\left(1 - \frac{\omega_f \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^q}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi-1}} f_0^q}\right) \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^q \left(1 - \frac{\omega_f \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^q}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi-1}} f_0^q}\right)}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^q \left(1 - \frac{\omega_f \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^q}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi-1}} f_0^q}\right)^2} + \frac{3\mathrm{p} \int d\mathrm{p}\, \frac{\mathrm{p}^3}{\omega_f^{\xi}} f_0^q}{\int d\mathrm{p}\, \frac{\mathrm{p}^4}{\omega_f^{\xi}} f_0^q}.$$
- For antiquarks:

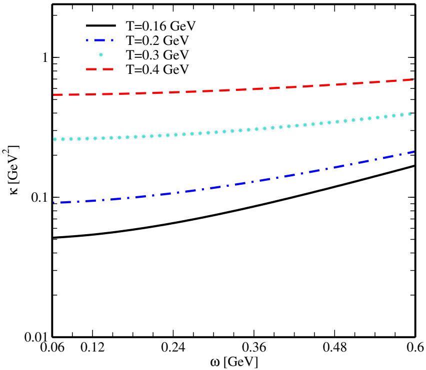
$$\begin{aligned} \delta \bar{f}_q \quad = \quad & -\frac{\beta \tau_{\bar{f}} f_0^{\bar{q}} \left(1 - f_0^{\bar{q}}\right)}{\bar{J}} \chi_{\bar{q}}(\beta\omega) \left[\frac{(\omega_f - \bar{h}_f)\, p^j}{T\omega_f} \left(\partial_j T - \frac{T}{n\bar{h}_f} \partial_j P\right) + p_0 \frac{DT}{T} - \frac{p^\mu p^\alpha}{p_0} \nabla_\mu u_\alpha - TD\left(\frac{\mu}{T}\right) \right] \\ & -\frac{\beta \tau_{\bar{f}} f_0^{\bar{q}}}{\bar{J}} \sinh\left(\frac{\beta\omega}{2}\right) \left[TD\left(\frac{\omega}{T}\right) + \frac{Tp^\alpha}{p_0} \nabla_\alpha \left(\frac{\omega}{T}\right) \right]. \end{aligned}$$

- $$\bar{J} = \frac{\left(1 - \frac{\omega_f \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^{\bar{q}}}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi-1}} f_0^{\bar{q}}}\right) \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^{\bar{q}} \left(1 - \frac{\omega_f \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^{\bar{q}}}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi-1}} f_0^{\bar{q}}}\right)}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^{\bar{q}} \left(1 - \frac{\omega_f \int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi}} f_0^{\bar{q}}}{\int d\mathrm{p}\, \frac{\mathrm{p}^2}{\omega_f^{\xi-1}} f_0^{\bar{q}}}\right)^2} + \frac{3\mathrm{p} \int d\mathrm{p}\, \frac{\mathrm{p}^3}{\omega_f^{\xi}} f_0^{\bar{q}}}{\int d\mathrm{p}\, \frac{\mathrm{p}^4}{\omega_f^{\xi}} f_0^{\bar{q}}}.$$

- The thermal conductivity for a rotating QGP medium:

$$\kappa = \frac{\beta^{2+\xi}}{6\pi^2}\sum_f g_f \int d\mathrm{p}\, \frac{\mathrm{p}^4}{\omega_f^{2-\xi}} \left[\frac{t_f}{J}(\omega_f - h_f)^2 f_0^q\left(1-f_0^q\right)\chi_q(\beta\omega) + \frac{t_{\bar{f}}}{\bar{J}}(\omega_f - \bar{h}_f)^2 f_0^{\bar{q}}\left(1-f_0^{\bar{q}}\right)\chi_{\bar{q}}(\beta\omega)\right].$$





Thermal diffusion of a rotating QGP medium

- The thermal diffusion constant (D^T) is associated with the rate of heat transfer in a system. It appears in the diffusion equation for heat as

$$\frac{\partial T}{\partial t} = D^T \nabla^2 T,$$

which is carried out at constant pressure.

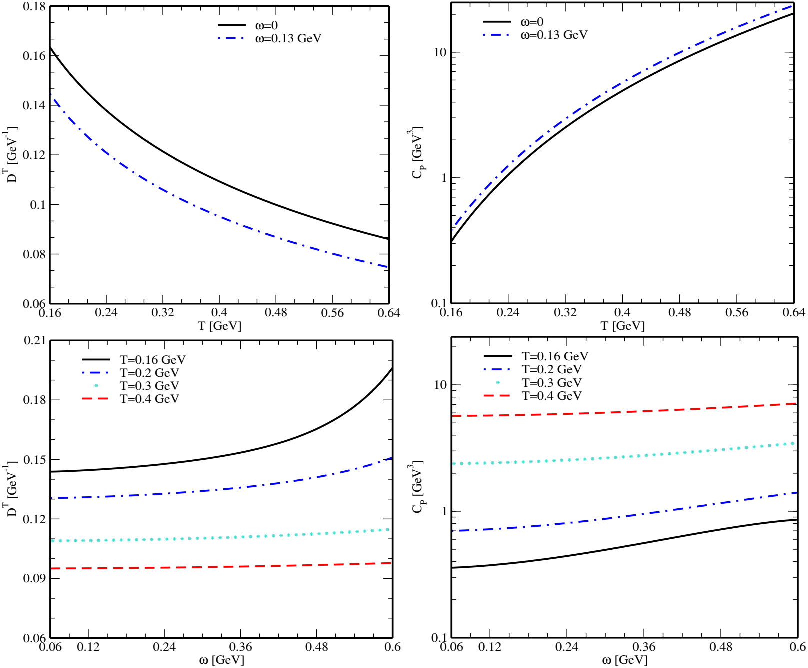
- The thermal diffusion constant in the first order relativistic viscous hydrodynamics:

$$D^T = \frac{\kappa}{C_P},$$

where the specific heat at constant pressure is given by

$$\begin{aligned} C_P \quad = \quad & \frac{\beta^2}{6\pi^2} \sum_f g_f \int d\mathrm{p} \left(3\mathrm{p}^2\omega_f + \frac{\mathrm{p}^4}{\omega_f}\right) \left[\left\{(\omega_f - \mu)\, f_0^q \left(1 - f_0^q\right) + (\omega_f + \mu)\, f_0^{\bar{q}} \left(1 - f_0^{\bar{q}}\right)\right\}\right. \\ & \left. \times \frac{\sinh(\beta\omega)}{\sinh(\beta\omega/2)} - \left(f_0^q + f_0^{\bar{q}}\right) \omega \sinh(\beta\omega/2)\right] \\ & + \frac{\beta^2}{6\pi^2} g_g \int d\mathrm{p} \left(3\mathrm{p}^2\omega_g + \frac{\mathrm{p}^4}{\omega_g}\right) \left[\omega_g f_0^g \left(1 + f_0^g\right) \frac{\sinh(3\beta\omega/2)}{\sinh(\beta\omega/2)} - 2 f_0^g \omega \sinh(\beta\omega)\right]. \end{aligned}$$

- Both κ and C_P are useful in discerning the influence of rotation on the thermal diffusion constant of the QGP medium.


ω=0
ω=0.13 GeV
D^T [GeV^{-1}]
T [GeV]
C_P [GeV^3]
T=0.16 GeV
T=0.2 GeV
T=0.3 GeV
T=0.4 GeV
ω [GeV]

Conclusions

- We calculated the thermal conductivity for a rotating medium by solving the relativistic Boltzmann transport equation in the novel relaxation time approximation within the kinetic theory approach.
- The introduction of rotation enhances the conduction of heat in the medium.
- Observation on the specific heat at constant pressure reveals that the change in enthalpy with the temperature becomes larger as the rotation becomes more rapid.
- The thermal diffusion constant gets decreased with the emergence of angular velocity, and this indicates a slower heat transfer in a rotating medium as compared to that in a nonrotating medium.

Thank you.