Deciphering accretion-driven starquakes in recycled millisecond pulsars using gravitational waves

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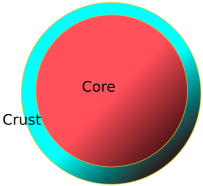
Motivation

- Our goal is to understand how the continuous gravitational wave (GW) signals generated by misaligned NS changes under the influence of starquakes.
- Accretion of the star in such a system can lead to spin-up process. This spin-up process upon reaching a certain breaking frequency can lead to starquakes.
- In this work, we have showed how the pre-existing GW signal from such a star can change due to starquakes.



Strain angle and breaking frequency

- The strain tensor can be defined as:

$$u_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

- The strain angle α is defined as the difference between the local maximum and local minimum between the eigenvalues.
- We used the Tresca criterion (Gao et al. 2014), which gives:

$$\alpha = \frac{\sigma_{max}}{2}$$

- We redefined the strain angle as (Giliberti & Cambiotti 2022):

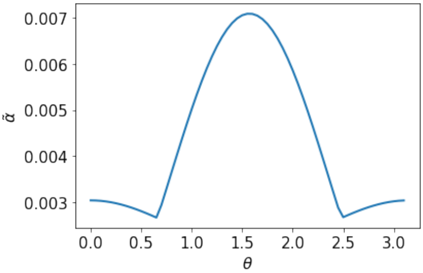
$$\alpha = \tilde{\alpha}\nu^2$$

Where $\tilde{\alpha}$ is a term that depends on the structure of the NS.

$$\nu_b = \sqrt{\frac{\sigma_{max}}{2\tilde{\alpha}_{max}}}$$

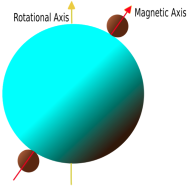
Strain angle and breaking frequency

- The strain angle α against the azimuthal angle is shown:



- Here we see that the value of the maximum strain angle is near the equator of the star.
- In our calculations of breaking frequency, we take the value of σ_{max} to be 0.04 as given by (Baiko D. A. & Chugunov A. I. 2018). This gives the value of the breaking frequency to be $\nu_b = 580$ Hz.

Moment of Inertia Calculations For a Spherical Star



- The moment of inertia takes the form:

$$I' = \mathrm{diag}\left(\frac{2MR^2}{5} + 2\delta m R^2, \frac{2MR^2}{5} + 2\delta m R^2, \frac{2MR^2}{5} + 2\frac{2}{5}\delta m a^2\right)$$

$$\mathcal{I}_{ij} = -I_{ij} + \frac{1}{3} I^k_k \delta_{ij}$$

$$P = \begin{pmatrix} \cos\Omega t & -\sin\Omega t & 0 \\ \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\chi & \sin\chi \\ 0 & -\sin\chi & \cos\chi \end{pmatrix}$$

Moment of Inertia Calculations For a Spherical Star

- The mass quadrupole moment in this frame can hence be given as $\mathcal{I}^{dist} = P \times \mathcal{I} \times P^t$. P^t is the transpose of the P-matrix.
- The leading term in the gravitational radiation field can be given as :

$$h_{ij}^{TT} = \frac{2G}{c^4}\frac{1}{r}\left[P_i^k P_j^l - \frac{1}{2}P_{ij}P^{kl}\right]\ddot{\mathcal{I}}_{kl}$$

Here P_{ij} represents the transverse projection operator and is given as $P_{ij} = \delta_{ij} - n_i n_j$.

$$h_+ = h_0 \sin\chi\left[\frac{1}{2}\cos\chi \sin i \cos i \cos\Omega t - \sin\chi \frac{1+\cos^2 i}{2}\cos 2\Omega t\right]$$

$$h_- = h_0 \sin\chi\left[\frac{1}{2}\cos\chi \sin i \sin\Omega t - \sin\chi \cos i \sin 2\Omega t\right]$$

$$h_0 := -\frac{6G}{c^4}\mathcal{I}_{zz}^{dist}\frac{\Omega^2}{r}$$

$$\mathcal{I}_{zz}^{dist} = \frac{4}{3}\delta m R^2$$

Change in ellipticity due to starquake

- The amount of ellipticity of our configuration can be given as:

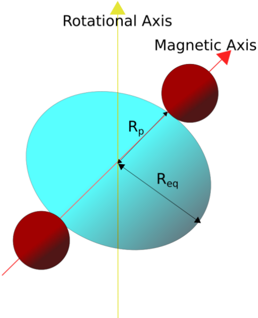
$$\epsilon_i = \left| \frac{I_{zz} - I_{xx}}{I_0} \right|, \epsilon_i = \frac{2\delta m R^2}{I_0}$$

- For an accreted mass of about $\delta m = 10^{-4}\mathrm{M}_\odot$ we get the value as: $\epsilon_i = 3.4246 \times 10^{-4}$ (Cyg-X2, mass $= 1.46 M_\odot$, radius $= 12.3$ km).

$$\epsilon_{max} = \frac{R^3}{3 I_0 G} \left[\Phi^F(R) - \Phi^E(R) \right]$$ (Giliberti et al. 2022)

Φ^F and Φ^E denotes the total perturbed gravitational potential in spherical harmonics for a star with fluid configuration and a star with elastic crust configuration respectively.

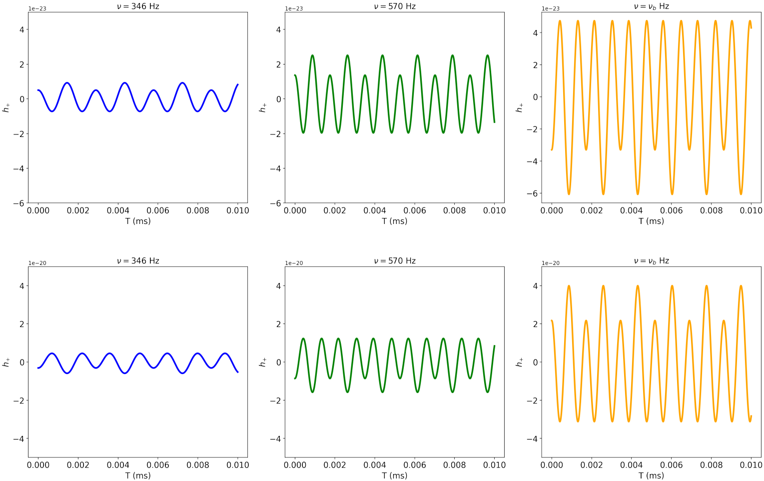
Moment of Inertia Calculations For Deformed Star



$$I' = \mathrm{diag}\left(\frac{M(R_{eq}^2 + R_p^2)}{5} + 2\delta m R_p^2, \frac{M(R_{eq}^2 + R_p^2)}{5} + 2\delta m R_p^2, \frac{2MR_{eq}^2}{5} + 2\frac{2}{5}\delta m a^2\right)$$

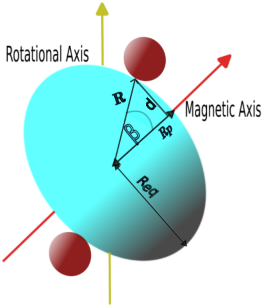
$$\mathcal{I}_{zz}^{dist} = \frac{1}{15}\left(20\,\delta m\,R_{eq}^2 + M\left(R_p^2 - 3R_{eq}^2\right)\cos\chi + M\left(R_{eq}^2 + R_p^2\right)\right)$$

Summary of Results



Chatterjee S., Nath K. K., Mallick R., 2024, MNRAS, 534, 97

Effects of the position of magnetic mountain



$$\mathcal{I}_{zz}^{dist} = \frac{2}{15}\left[M(R_{eq}^2 - R_p^2) - 10\delta m(R(\beta)^2 - d^2)\right]$$

In this case, $h_0 \propto \mathcal{I}_{zz}^{dist}$, which implies that the GW amplitude for *model II* depends on the position of the mountain as:

$$h_0 \propto \cos^2 \beta$$

Effect of the mass of mountains

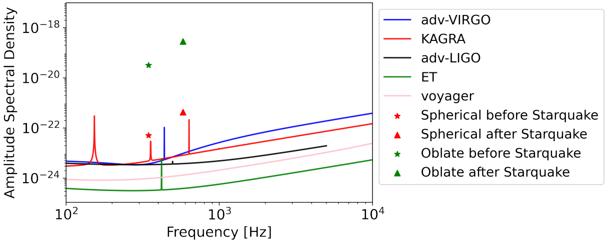
	model I	**model II**	
$\delta m\ [M_\odot]$	—h_0—	R_p/R_{eq}	—h_0—
10^{-4}	3.0328×10^{-23}	0.80	1.91×10^{-20}
		10^{-2}	6.41×10^{-23}
		10^{-4}	3.312×10^{-23}
10^{-6}	3.0328×10^{-25}	0.80	1.913×10^{-20}
		10^{-2}	9.7161×10^{-23}
		10^{-4}	6.47×10^{-25}
10^{-8}	3.0328×10^{-27}	0.80	1.9131×10^{-20}
		10^{-2}	9.749×10^{-23}
		10^{-4}	9.779×10^{-25}

Prospects of detection of starquakes from the GW signal

The power spectral density as defined by Takami et al. 2015:

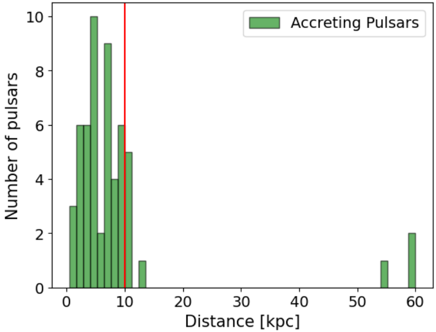
$$\tilde{h}(f) = \sqrt{\frac{|\tilde{h_+}(f)|^2 + |\tilde{h_\times}(f)|^2}{2}}$$

$$\tilde{h}_{+,\times}(f) = \int h_{+,\times}(t) \exp^{-i2\pi f t} \, dt \quad \text{, where } f \geq 0$$



The points show the $2\tilde{h}\sqrt{f}$ for the two cases before and after the starquake.

Prospects of detection of starquakes from the GW signal



- Next-generation detectors are expected to detect continuous GW signals from nearby pulsars.
- Therefore, it would be interesting to check how likely we are to observe a star with an accreting pulsar within a distance of 10 kpc.

Thank You