Using Bayesian inference of RHIC Beam Energy Scan data to explore the properties of the Quark-Gluon Plasma at finite baryon density

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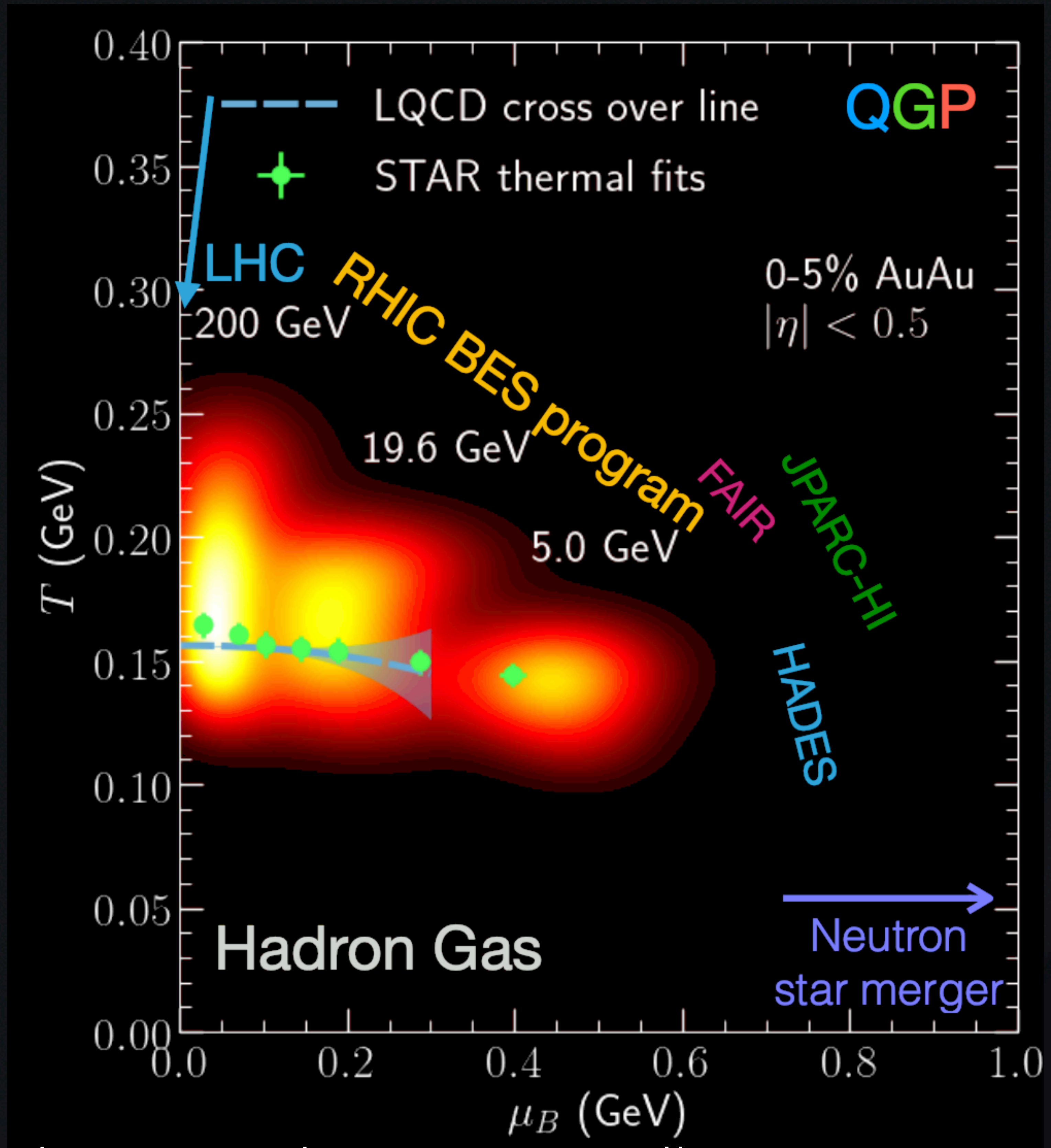
Phys.Rev.C 110 (2024) 4, 044904
Phys.Rev.C 110 (2024) 5, 054905

10th Asian Triangle Heavy-Ion Conference - ATHIC 2025

Motivation: Probing the Nuclear Phase Diagram in Finite Baryon Density Region

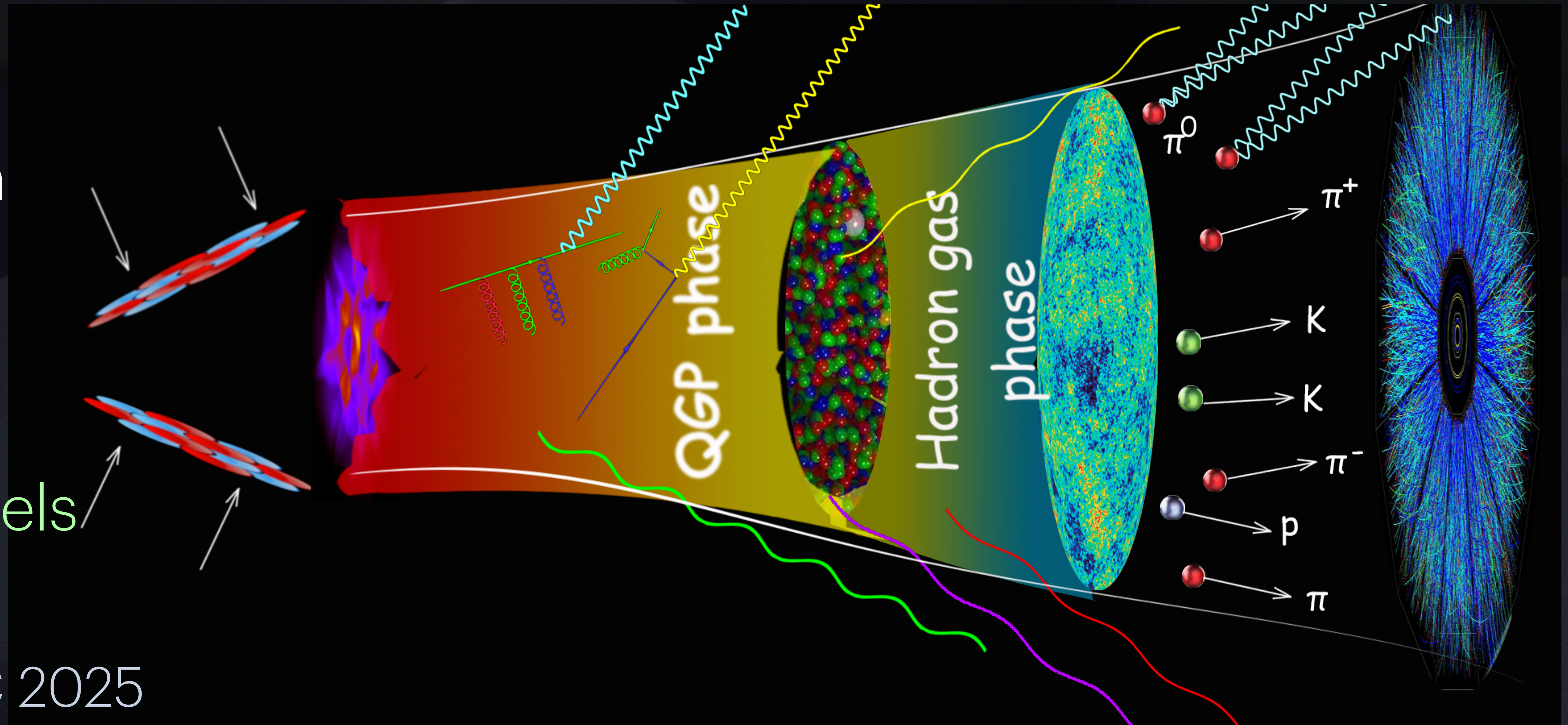


We want to explore:

- Search for a critical point and 1st order phase transition
- Properties of many body QCD such as QGP transport properties at finite baryon density i.e $(\eta/s)(T, \{\mu_q\})$, $(\zeta/s)(T, \{\mu_q\})$.
- Solve fundamental questions of QCD like chiral symmetry breaking and confinement.

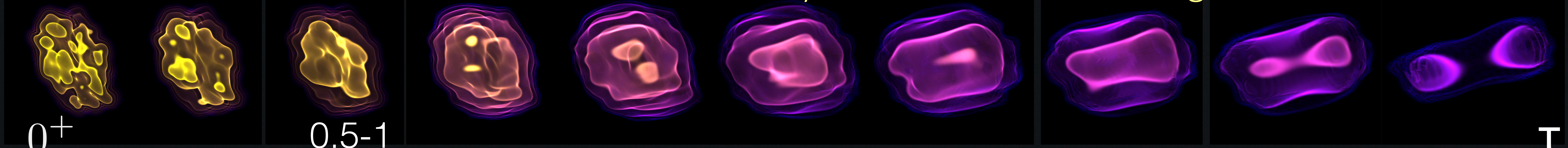
High energy heavy ion collisions are complex processes with multiple stages. So first principle calculations are not always possible.

Comparison between exp data and phenomenological models are the way to go!



Our Model : iEBE-MUSIC

Basic Scheme of Heavy Ion Collision Modeling



Initial State +
Pre-equilibrium dynamics

Hydrodynamics

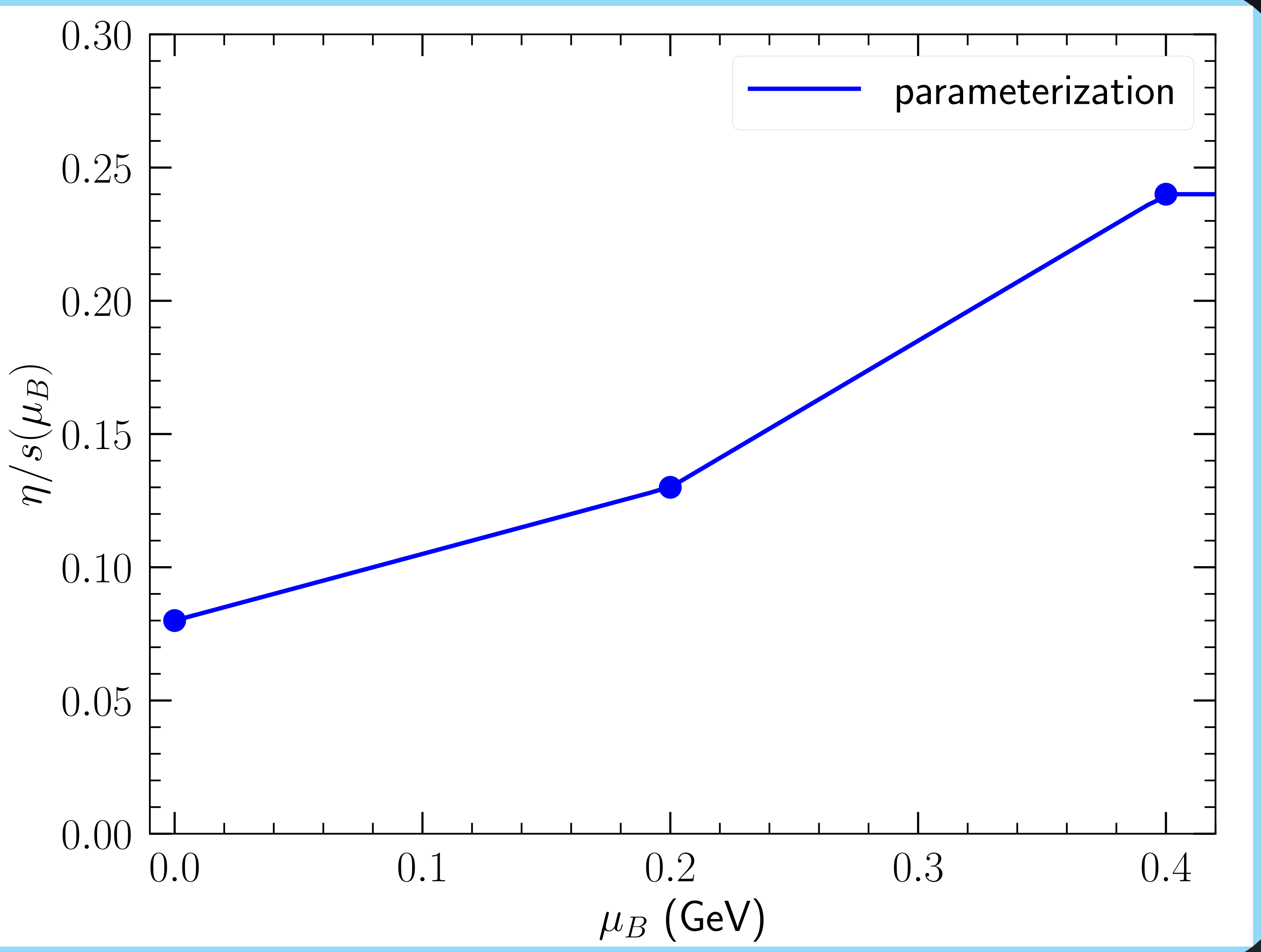
Hadronic Transport

3D MC Glauber Model

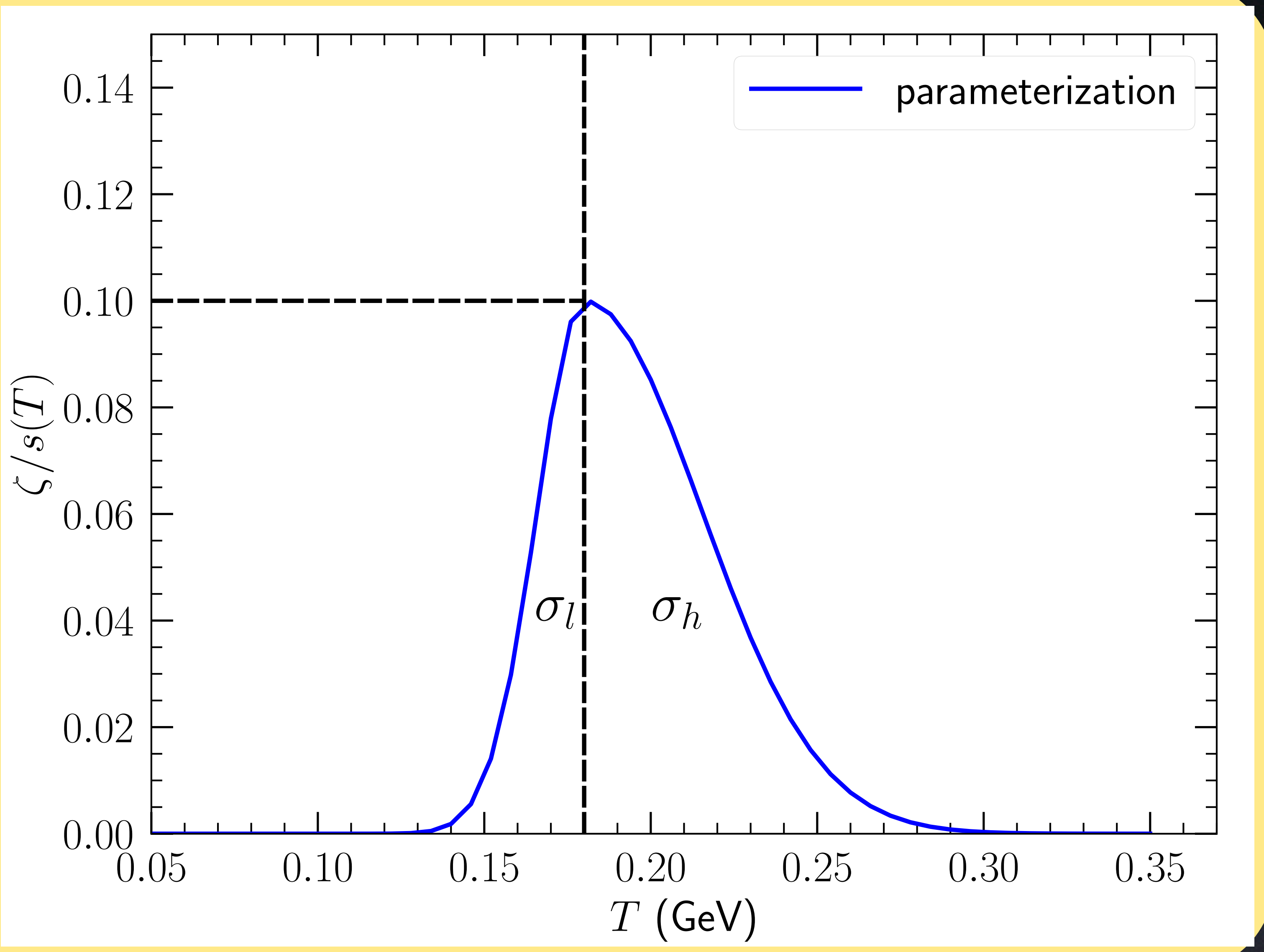
MUSIC

iSS + UrQMD

$\eta/s(\mu_B)$ has a piece-wise parameterization



$\zeta/s(T)$ is parameterized with an two-piece asymmetric Gaussian



Bayesian Analysis: A Way of Systematic Comparison between Models and Experimental Data

An Inverse Problem:

Bayes Theorem $P(\theta|Data) = \frac{P(Data|\theta)P(\theta)}{P(Data)}$

θ : Model Parameters

TABLE I. The 20 model parameters and their prior ranges.

Parameter	Prior	Parameter	Prior
B_G (GeV^{-2})	[1, 25]	$\alpha_{\text{string tilt}}$	[0, 1]
$\alpha_{\text{shadowing}}$	[0, 1]	α_{preFlow}	[0, 2]
$y_{\text{loss},2}$	[0, 2]	η_0	[0.001, 0.3]
$y_{\text{loss},4}$	[1, 3]	η_2	[0.001, 0.3]
$y_{\text{loss},6}$	[1, 4]	η_4	[0.001, 0.3]
$\sigma_{y_{\text{loss}}}$	[0.1, 0.8]	ζ_{max}	[0, 0.2]
α_{Rem}	[0, 1]	$T_{\zeta,0}$ (GeV)	[0.15, 0.25]
λ_B	[0, 1]	$\sigma_{\zeta,+}$ (GeV)	[0.01, 0.15]
σ_x^{string} (fm)	[0.1, 0.8]	$\sigma_{\zeta,-}$ (GeV)	[0.005, 0.1]
$\sigma_\eta^{\text{string}}$	[0.1, 1]	e_{sw} (GeV/fm^3)	[0.15, 0.5]

Likelihood $P(Data|\theta)$ →

← Posterior $P(\theta|Data)$

$\sqrt{s_{\text{NN}}}$ [GeV]	STAR	PHOBOS
200	$dN/dy(\pi^+,\pi^-,K^+,K^-,p,\bar{p})$ $\langle p_T\rangle(\pi^+,\pi^-,K^+,K^-,p,\bar{p})$ $v_2^{\text{ch}}\{2\}$, $v_3^{\text{ch}}\{2\}$ $\langle\delta p_T\rangle$	$dN_{\text{ch}}/d\eta$ $v_2^{\text{ch}}(\eta)$
19.6	$dN/dy(\pi^+,\pi^-,K^+,K^-,p,\bar{p})$ $\langle p_T\rangle(\pi^+,\pi^-,K^+,K^-,p,\bar{p})$ $v_2^{\text{ch}}\{2\}$, $v_3^{\text{ch}}\{2\}$ $\langle\delta p_T\rangle$	$dN_{\text{ch}}/d\eta$
7.7	$dN/dy(\pi^+,\pi^-,K^+,K^-,p,\bar{p})$ $\langle p_T\rangle(\pi^+,\pi^-,K^+,K^-,p,\bar{p})$ $v_2^{\text{ch}}\{2\}$, $v_3^{\text{ch}}\{2\}$ $\langle\delta p_T\rangle$	

Posterior distributions provide statistically robust constraints on the model parameters. But the likelihood must be calculated at millions of parameter points, so we need emulators!

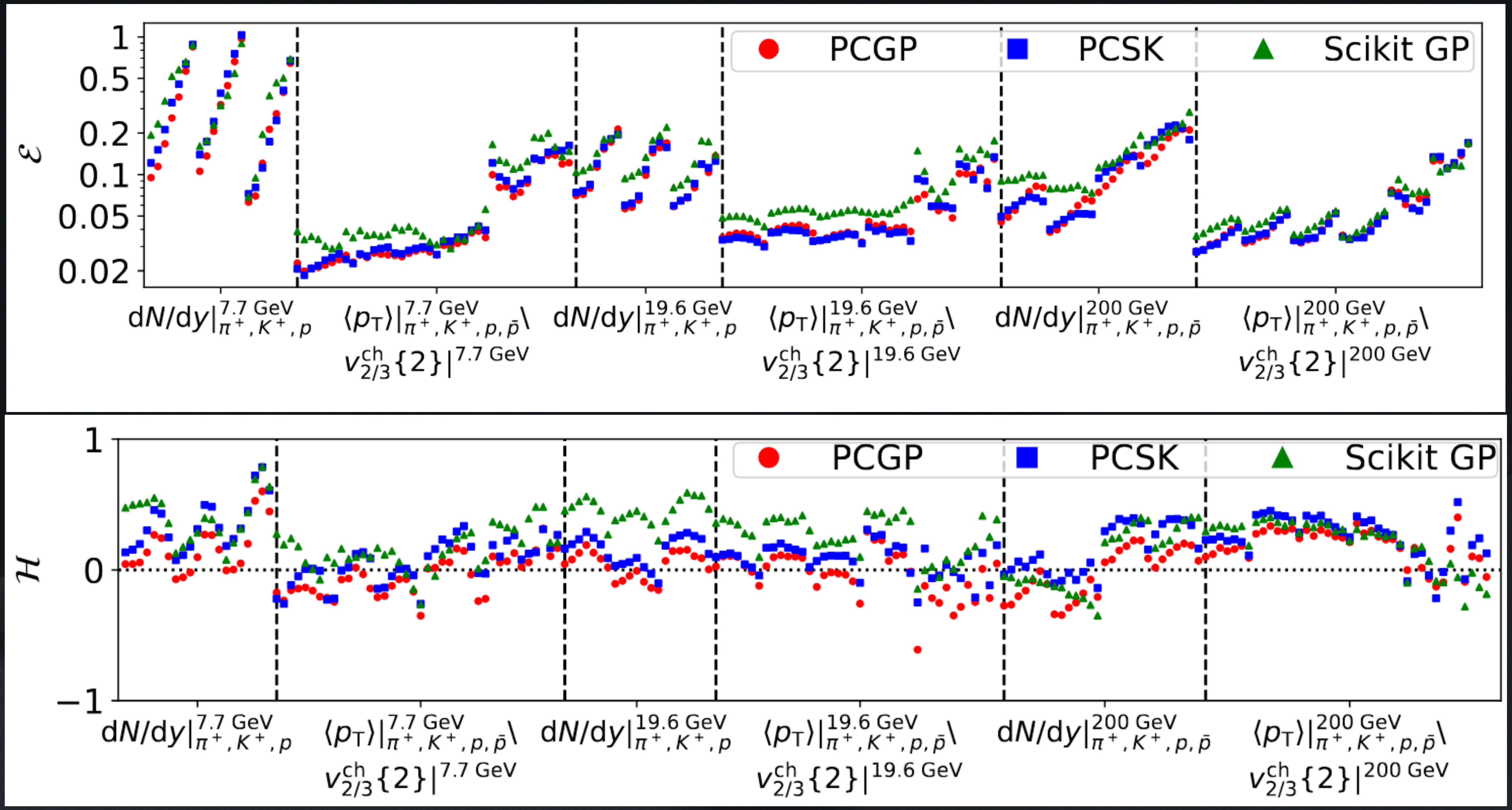
Quantifying Emulator Performance

$$\mathcal{E} \equiv \sqrt{\left\langle \left(\frac{\text{prediction} - \text{truth}}{\text{truth}} \right)^2 \right\rangle}$$

(RMS Error is ideally zero)

$$\mathscr{H} \equiv \ln\left(\sqrt{\left\langle \left(\frac{\text{prediction} - \text{truth}}{\text{prediction uncertainty}} \right)^2 \right\rangle} \right)$$

$\mathscr{H} = 0$: Best
$\mathscr{H} > 0$: Predicted uncertainty is too small
$\mathscr{H} < 0$: Predicted uncertainty is too large



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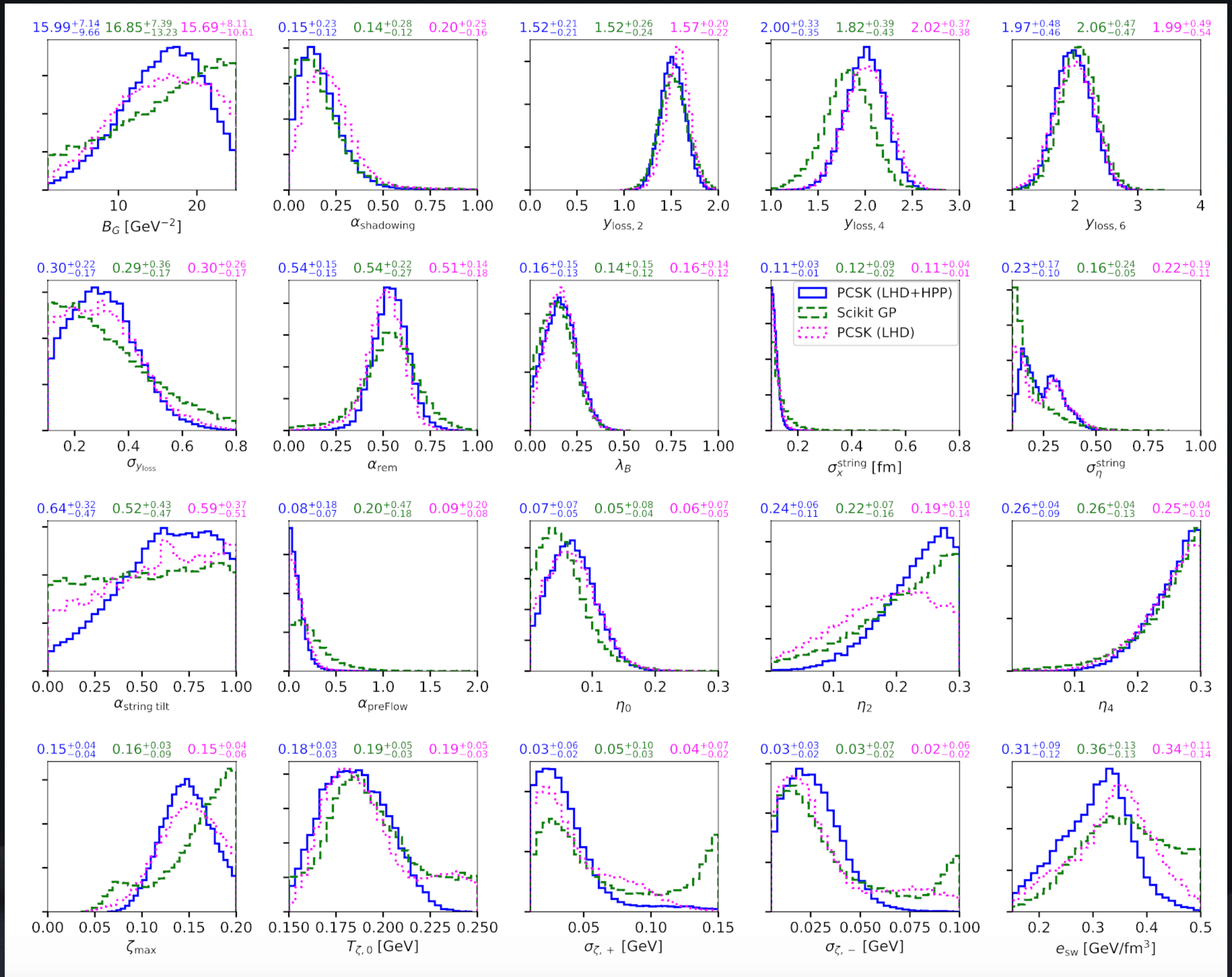
Out of all the 3 emulators, PCSK gives the least error and the most reliable estimate of uncertainty

Posterior from **Different Emulators**



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KL Divergence

$$D_{\rm KL}(p||q) = \int d\boldsymbol{\theta}\, p(\boldsymbol{\theta}) \ln\left(\frac{p(\boldsymbol{\theta})}{q(\boldsymbol{\theta})}\right)$$

GP Model	$D_{\rm KL}$
PCSK (LHD + HPP)	24.6
Scikit GP	20.9
PCSK (LHD only)	22.4

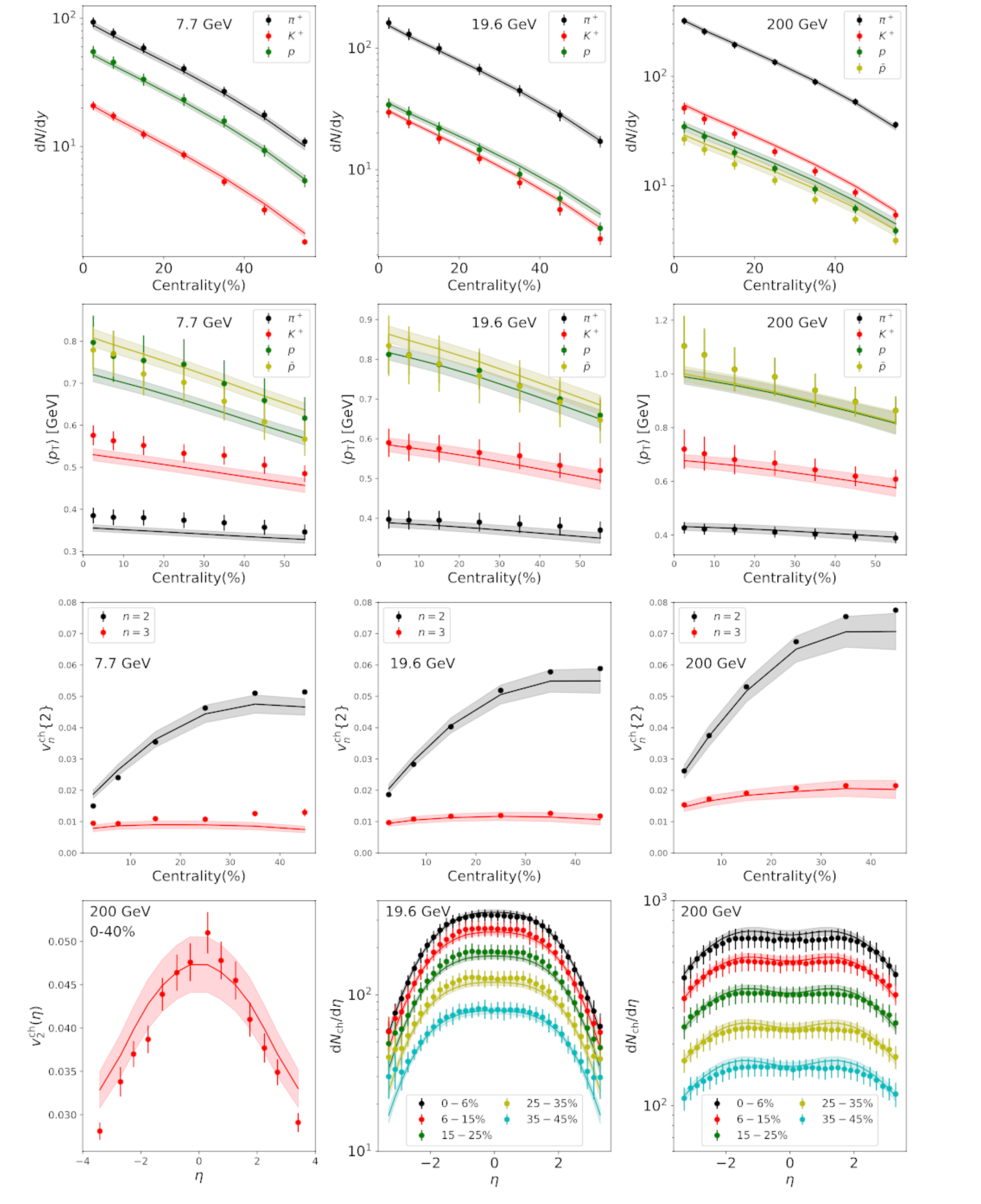
Bayes Factor

$$\mathcal{B}_{A/B} \equiv \frac{\mathcal{P}(A|\mathbf{y}_{\rm exp})}{\mathcal{P}(B|\mathbf{y}_{\rm exp})} = \frac{\mathcal{P}(\mathbf{y}_{\rm exp}|A)\mathcal{P}(A)}{\mathcal{P}(\mathbf{y}_{\rm exp}|B)\mathcal{P}(B)}$$

Model A	Model B	$\ln(\mathcal{B}_{A/B})$
PCSK (LHD + HPP)	Scikit GP	6.94 ± 0.03
PCSK (LHD + HPP)	PCSK (LHD only)	-1.41 ± 0.03

Both metrics show PCSK gives the best results for Posterior distribution!

Predicting Observables from Posterior Distribution which are/are not Part of Calibration



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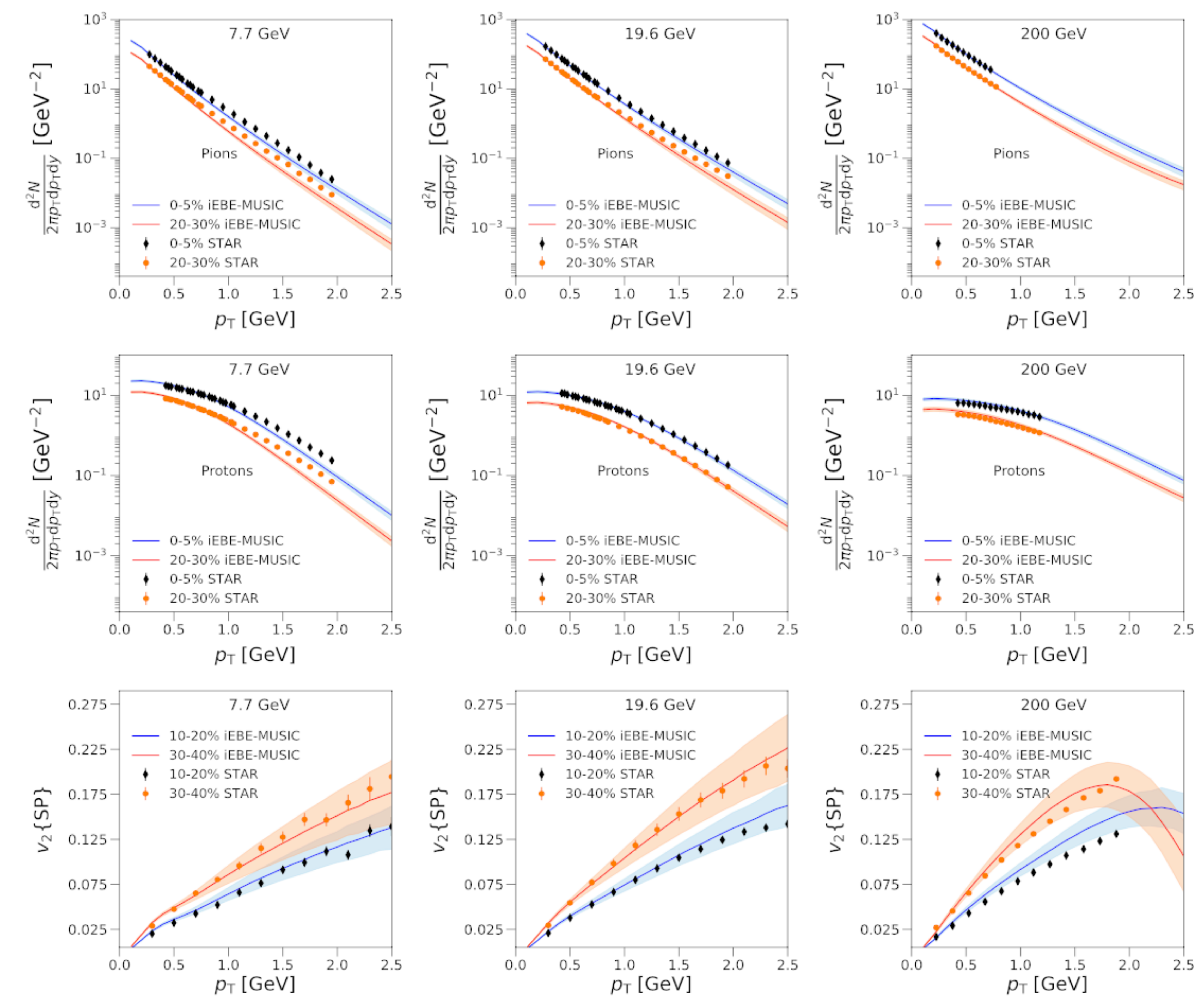
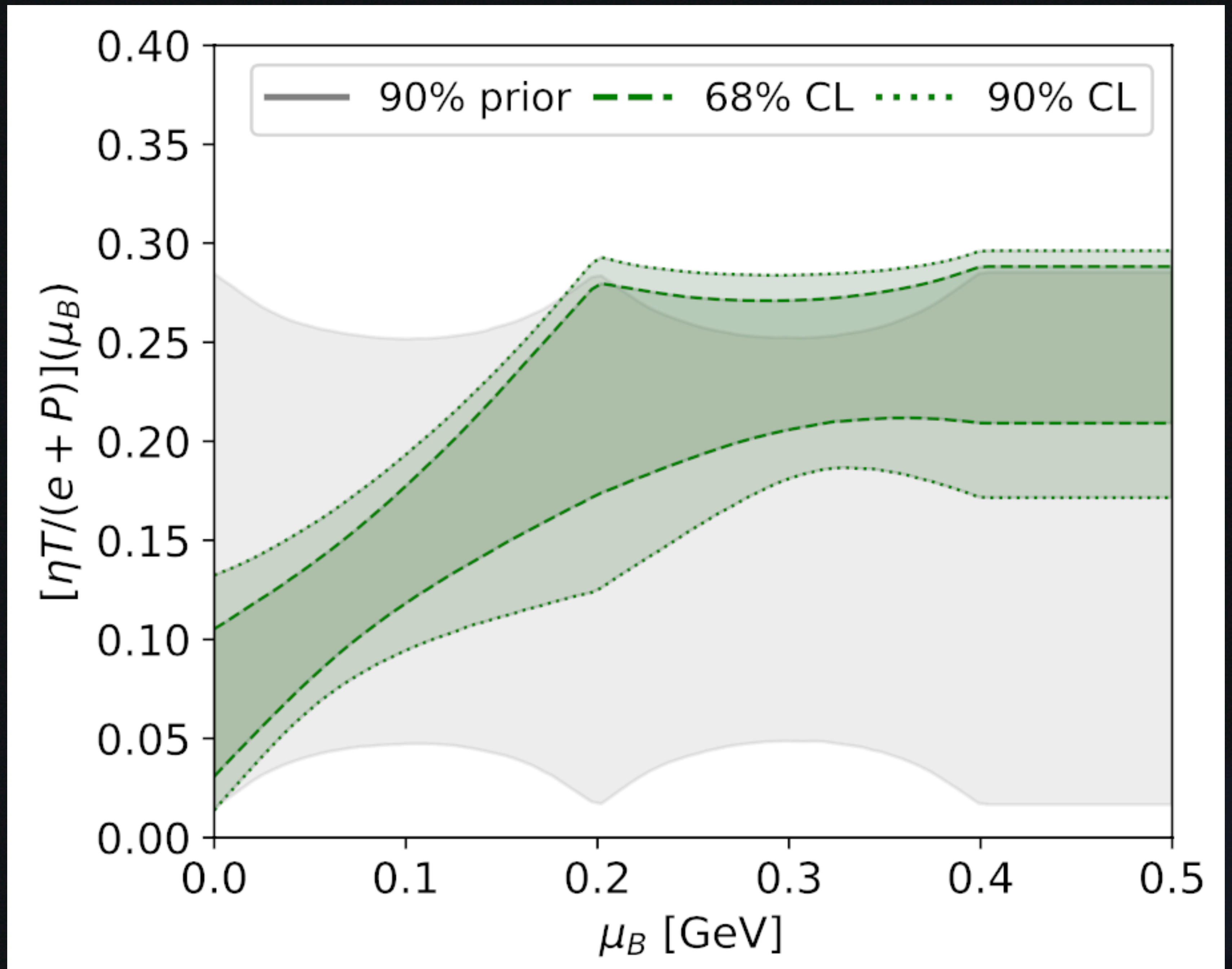


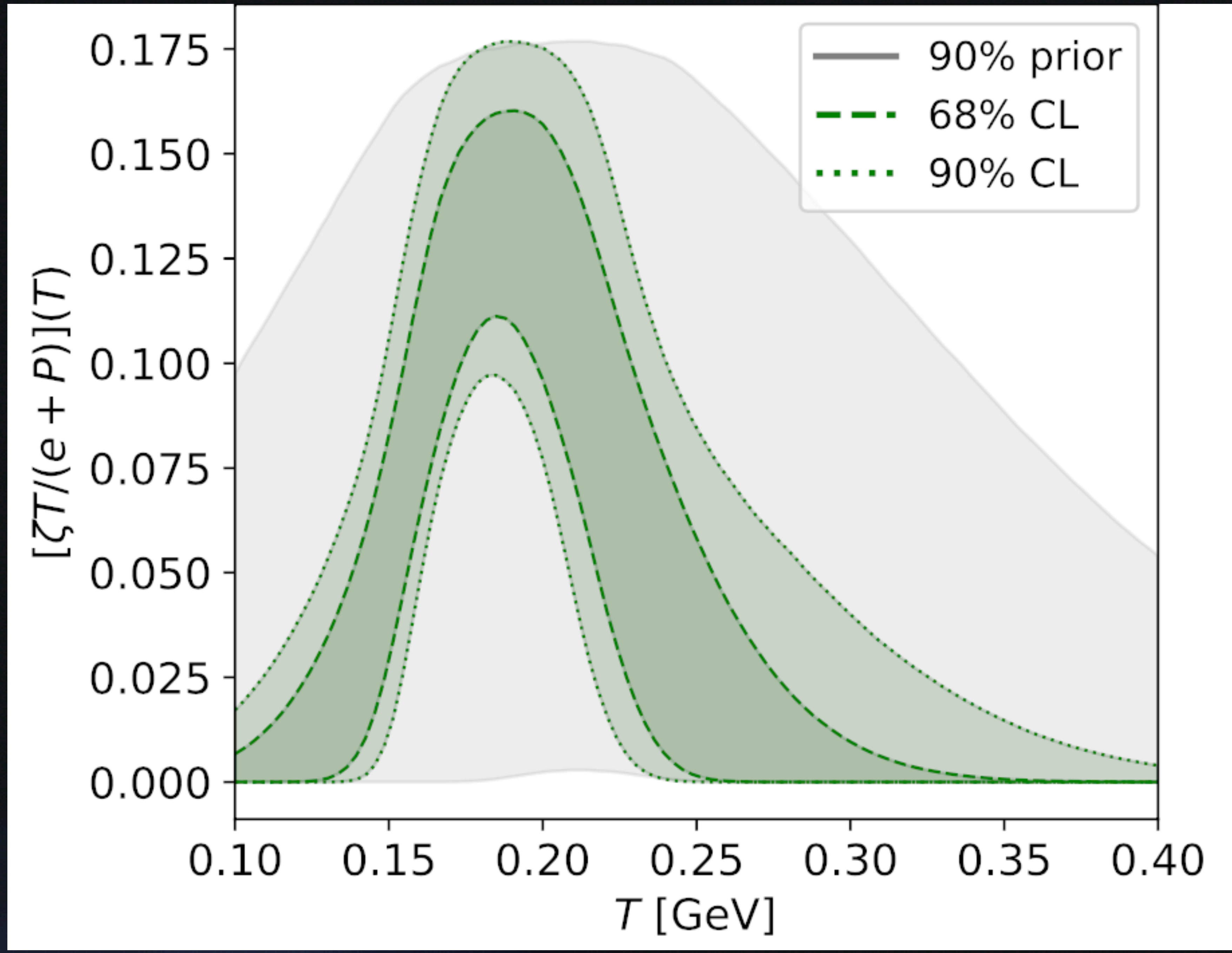
Our analysis makes reasonable predictions for both!

Constraints on QGP Viscosity

Shear Viscosity $[\eta T/(e+P)](\mu_B)$



Bulk Viscosity $[\zeta T/(e+P)](T)$



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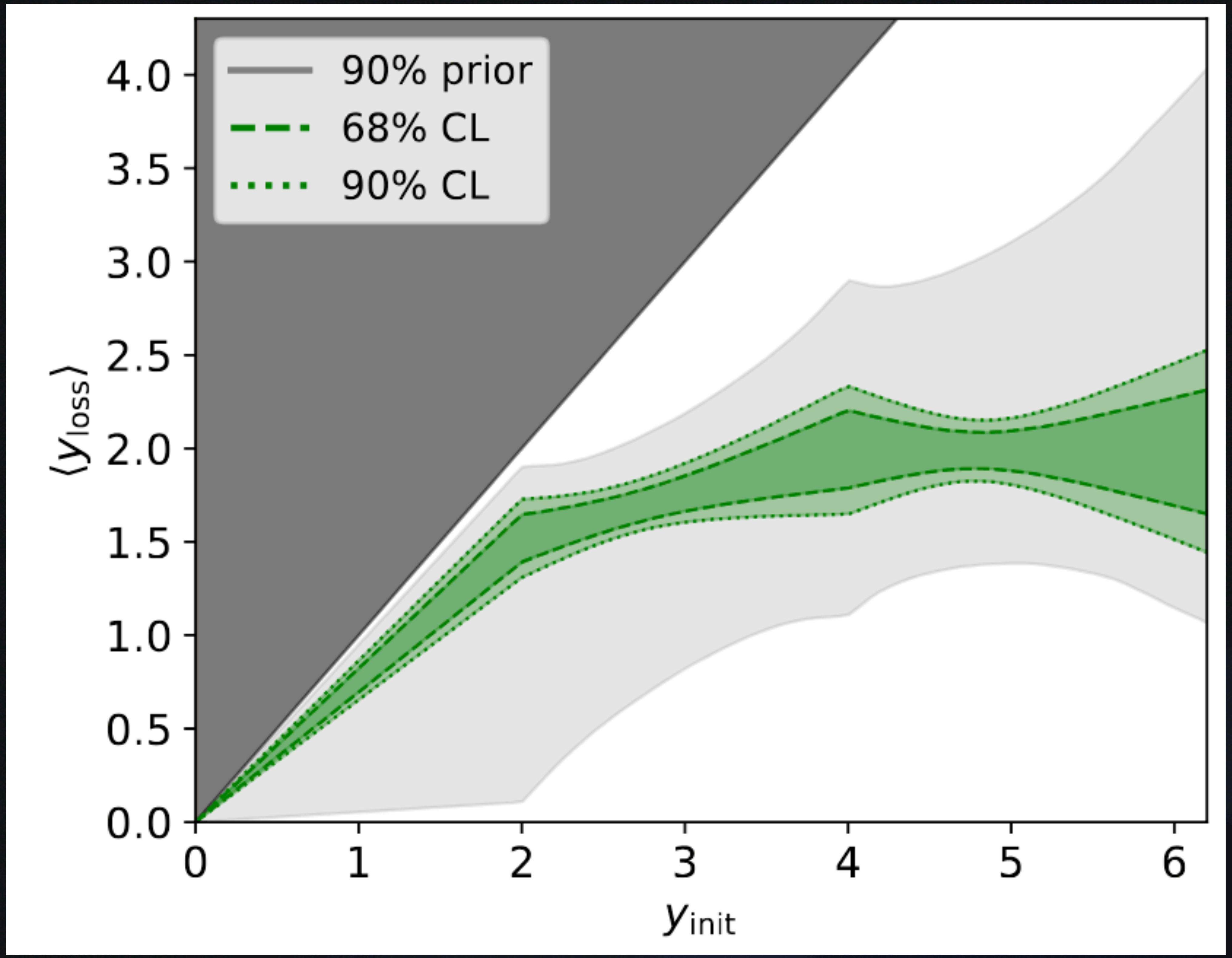
- The Shear viscosity shows a significant increase in the interval [0, 0.2 GeV].
- The bulk viscosity reaches a peak $\sim$ 0.12 at around 200 MeV.

Constraints on initial stopping



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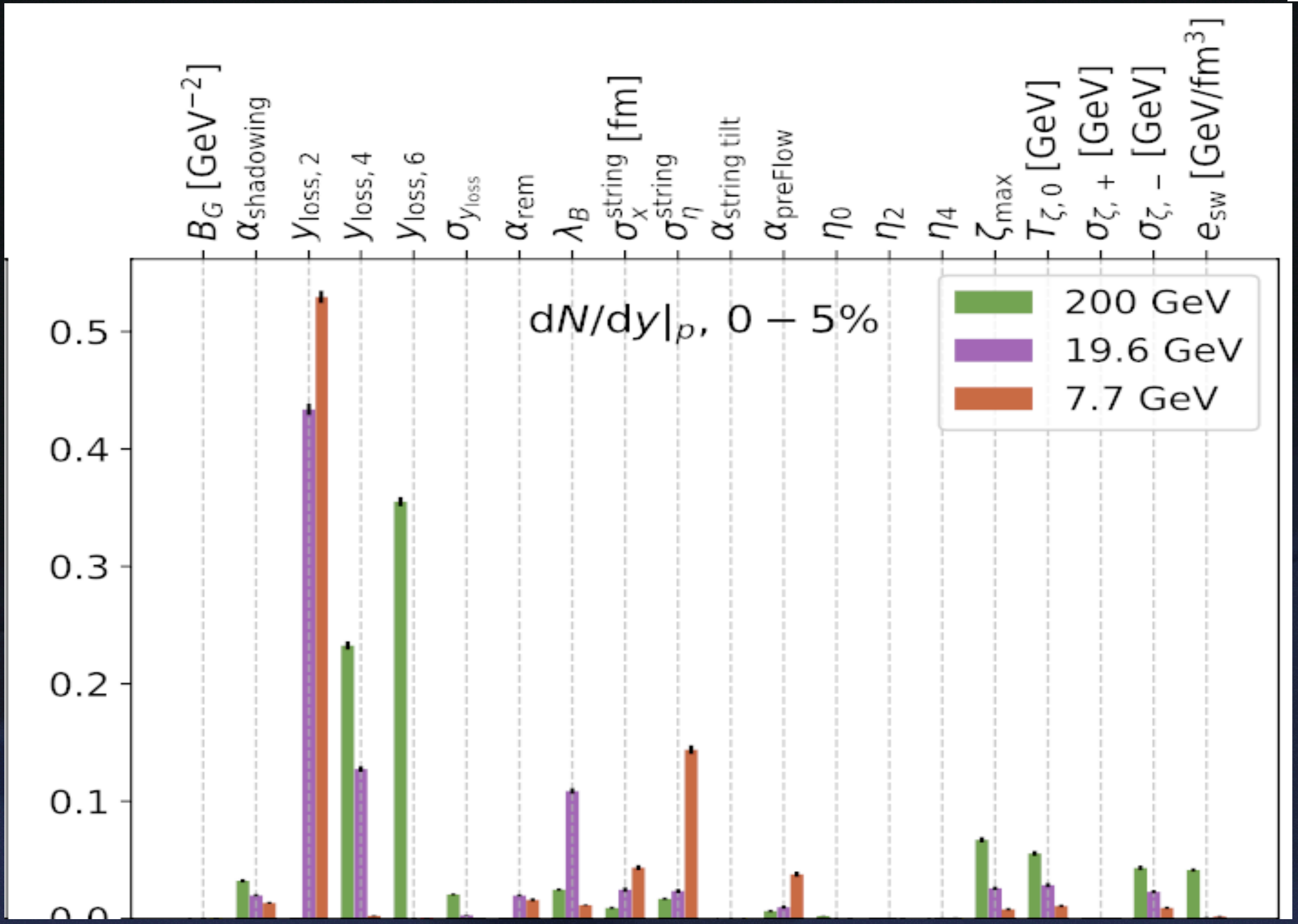
y_{init} is the initial rapidity and $\langle y_{loss} \rangle$ is the average rapidity loss per parton pair. We see about 2 units of rapidity loss at the top energy. The constraints obtained from this work is consistent with the BRAHMS Collaboration and previous studies.

Sensitivity Analysis: What Observables Help to Constrain Which Parameters?

Sobol Indices quantifies global sensitivity of each observable to the model parameters

$$S_i^{\mathbb{O}} \equiv \frac{\mathrm{Var}_{\theta_i}\left(\mathbb{E}_{\boldsymbol{\theta}_{-i}}(\mathbb{O}(\boldsymbol{\theta})|\theta_i)\right)}{\mathrm{Var}_{\boldsymbol{\theta}}(\mathbb{O}(\boldsymbol{\theta}))}, \quad i = 1, \ldots, m.$$



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Particle yields can strongly constrain rapidity loss parameters as seen from our results as well.

Summary

- We have performed a systematic Bayesian analysis with RHIC BES data using a (3+1)D framework.
- We got robust constraints on shear and bulk viscosity as a function of μ_B and T respectively.
- We performed global and local sensitivity analysis to quantify how much each observable can constrain each parameter. The observables present in our analysis showed sensitivity to viscosity and rapidity loss parameters.
- Our next step is to expand our data set, explore energy dependence of various parameters using Bayesian model selection method.

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Sensitivity Analysis (Contd.)

To understand sensitivity in the relevant parameter region we define,

$$\mathcal{R}_{ai} \equiv \left\langle \frac{\partial y_a}{\partial \theta_i} \right\rangle_{\text{post}} = \sum_j \langle \delta y_a \delta \theta_j \rangle_{\text{post}} \langle \delta \theta_j \delta \theta_i \rangle^{-1}_{\text{post}}.$$

Green is positive correlation and pink is negative correlation

- Looking at how the bulk viscosity parameters change with particle yield: bulk viscosity has a positive correlation with particle yield (entropy production).
- It has a negative correlation with anisotropic flow (viscous damping) and mean p_T (bulk viscosity resists expansion) .