UNIVERSITÄT BIELEFELD

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Quarkonia Spectral Function at Finite Momenta

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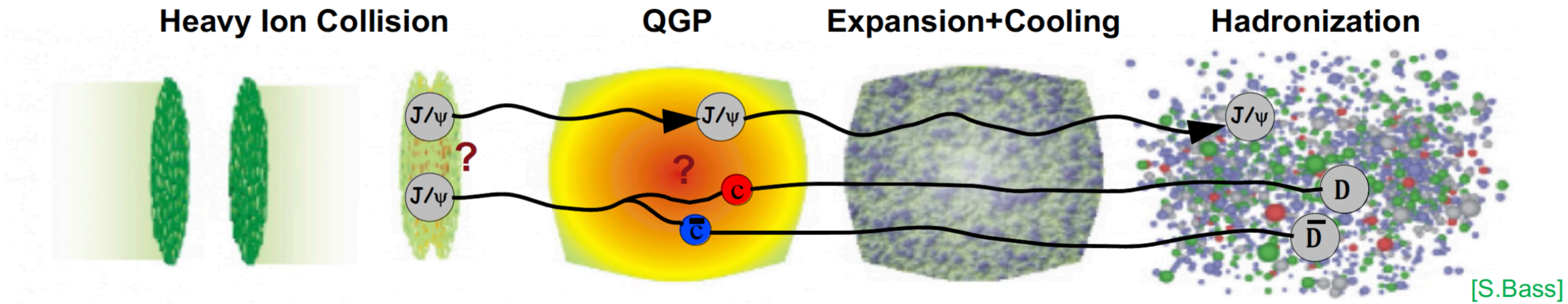
Motivation



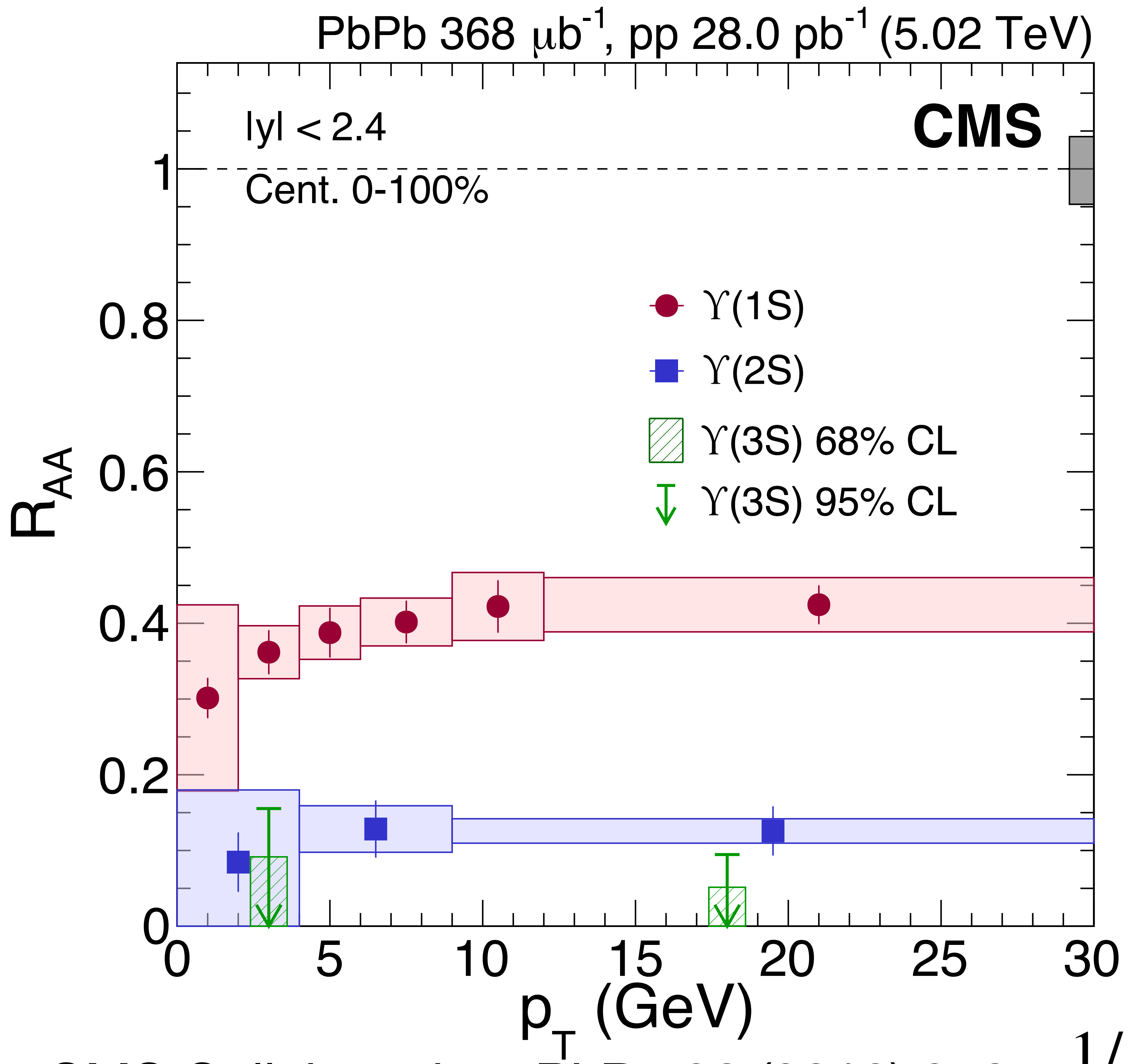
QGP cause suppression of Quarkonia (bound states of heavy $\bar{q}q$) due to colour screening.

J/ψ suppression: T. Matsui and H. Satz, Phys. Lett. B 178, 416 (1986)

It have rich structure of separated energy scales that makes it an ideal probe of confinement and deconfinement.

The Quarkonia in-medium properties of bound states are encoded in the spectral function.

We aim to study Quarkonia at finite momenta that is helpful in interpreting the experimental results.



CMS Collaboration, PLB 790 (2019) 270

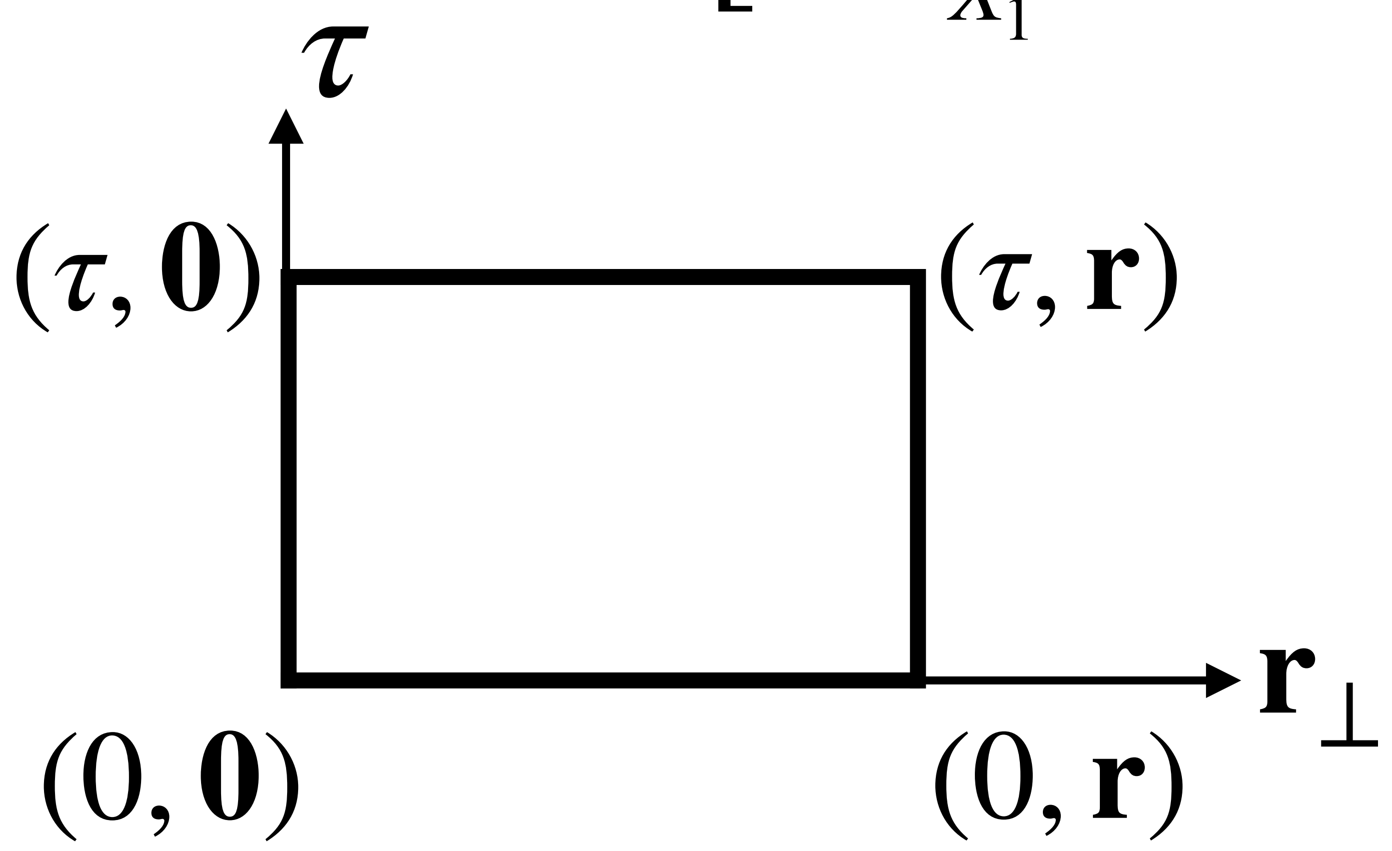
Wilson-line Correlators

The expectation of Wilson loop is given as

$$C_E(\tau, r) \equiv \frac{1}{N_c} \mathrm{Tr} \left\langle W\big[(0,\mathbf{0});\ (\tau,\mathbf{0});\ (\tau,\mathbf{r});\ (0,\mathbf{r});\ (0,\mathbf{0})\big] \right\rangle_T$$

M. Laine *et at.*, JHEP03 (2007) 054

$$W[X_1, X_2] = \mathscr{P} \exp\left[i\, g \int_{X_1}^{X_2} A_\mu dX_\mu\right]$$



We need analytical continuation for this euclidean time quantity.

The potential can be extracted by following relation

$$i\partial_t C_>(t, r, r') \equiv V_>(t, r)\ C_>(t, r, r')$$

Spectral function at zero-momenta

In the presence of the interaction

M. Laine *et at.*, JHEP03 (2007) 054

$$\left\{ i\partial_t - \left[2M - \frac{\nabla^2_{\vec{r}}}{M} + V_T(r) \right] \right\} C_>(t; \vec{r}, \vec{r}') = 0 \qquad C_>(0, \vec{r}, \vec{r}') = 6\,\delta^3(\vec{r} - \vec{r}')$$

where V_T is defined in static limit

$$V_T(r) = i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t} = V_{re}(r) - iV_{im}(r)$$

The spectral function is related to the Fourier transform of $C_>(\vec{r}, \vec{r}', t)$

$$\rho(\omega) \propto \lim_{\vec{r}\to 0} \lim_{\vec{r}'\to 0} \int_{-\infty}^{\infty} d\omega\, C_>(\vec{r}, \vec{r}', t)\, \exp(i\omega t)$$

Which is related to lattice correlators in following fashion

$$C(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh\left[\omega(\tau - \frac{1}{2T})\right]}{\sinh\left[\frac{\omega}{2T}\right]}$$

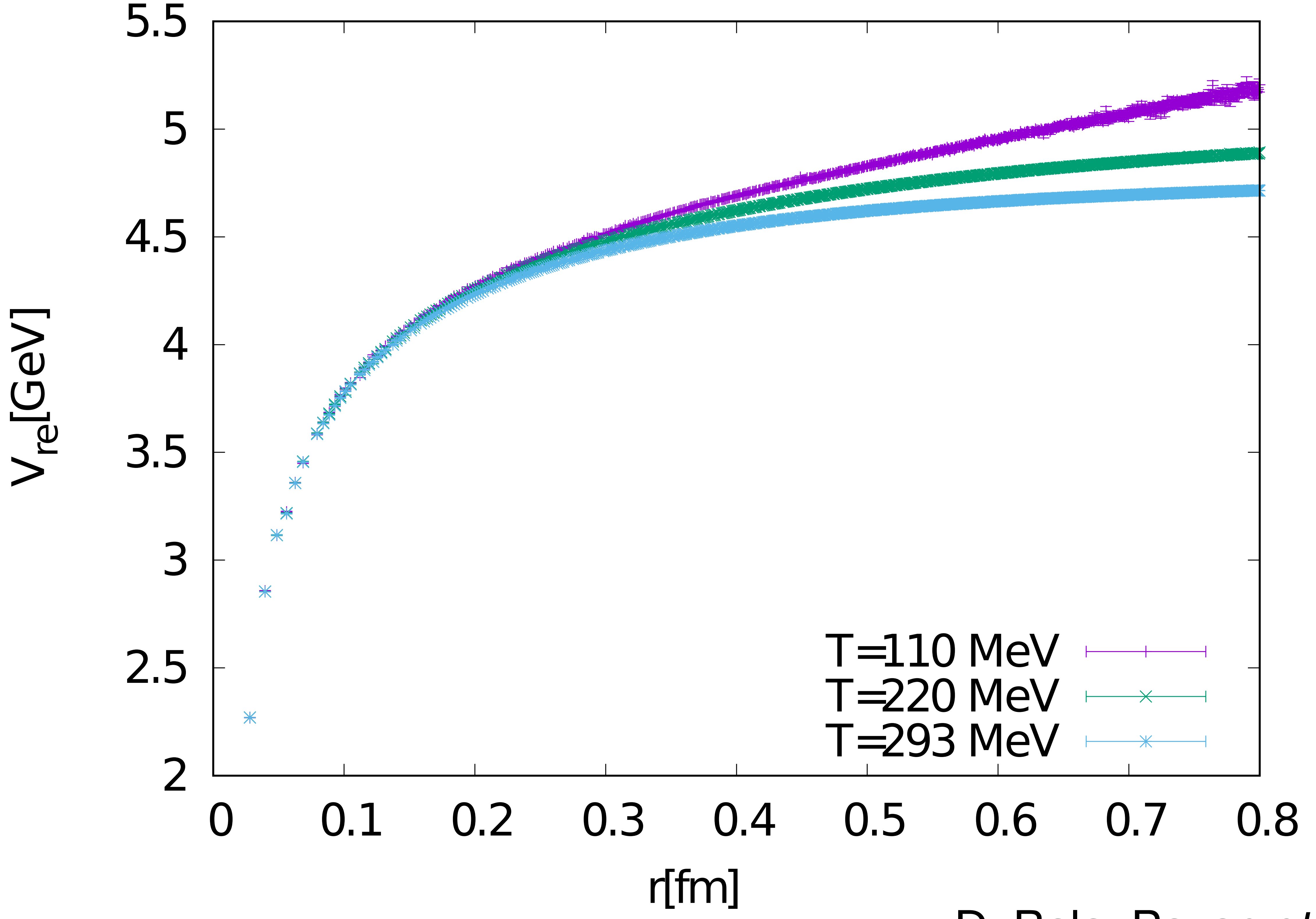
Potential at zero-momenta

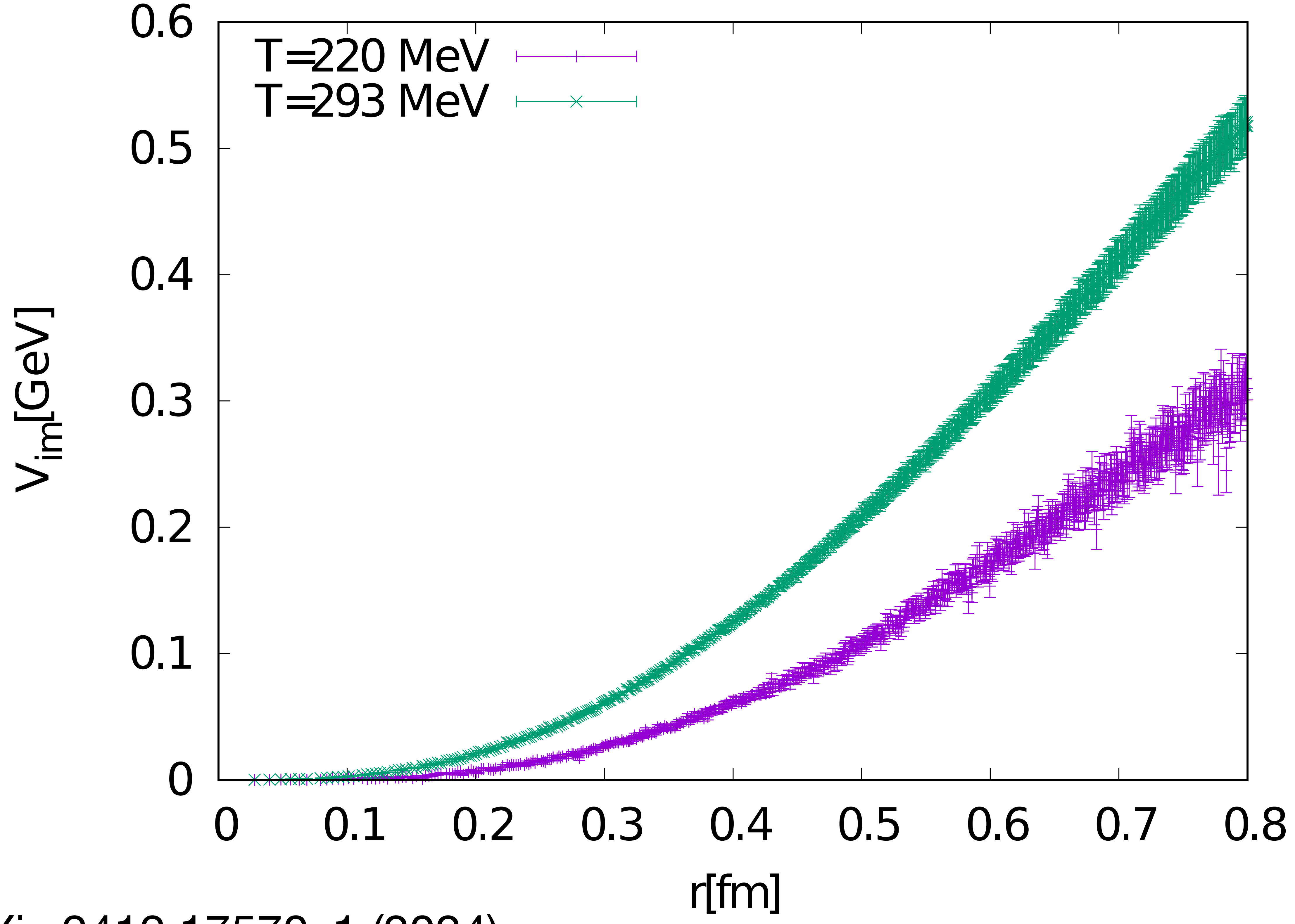
We have used following parametrisation to extract the thermal static potential

$$W(r,\tau) = A\exp\left(-V_{re}(r)\tau + \frac{V_{im}(r)}{\pi T}\log(\pi\tau T) + \ldots \right)$$

D. Bala and S. Datta, Phys. Rev. D. 101 (2020)

The imaginary part increases with both temperature and distance



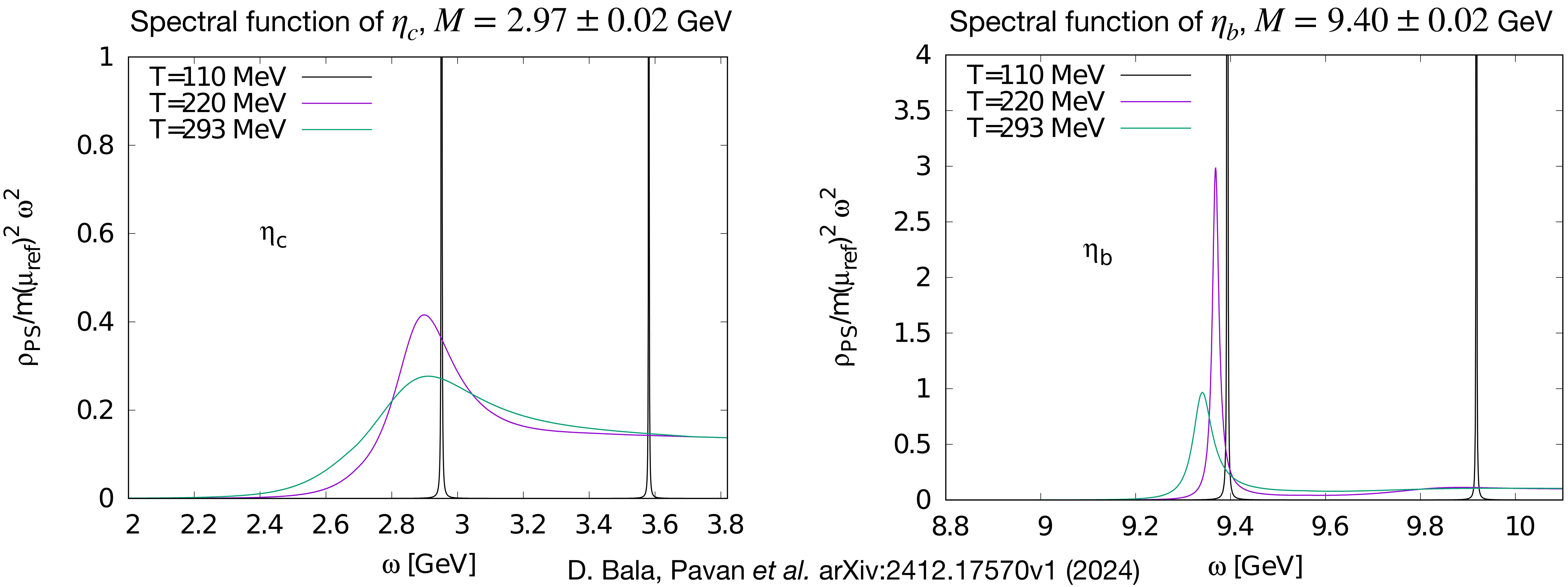


D. Bala, Pavan *et al.* arXiv:2412.17570v1 (2024)

Spectral Function at zero-momenta

Matched spectral function with vacuum part

Spectral function is not Gaussian around the peak

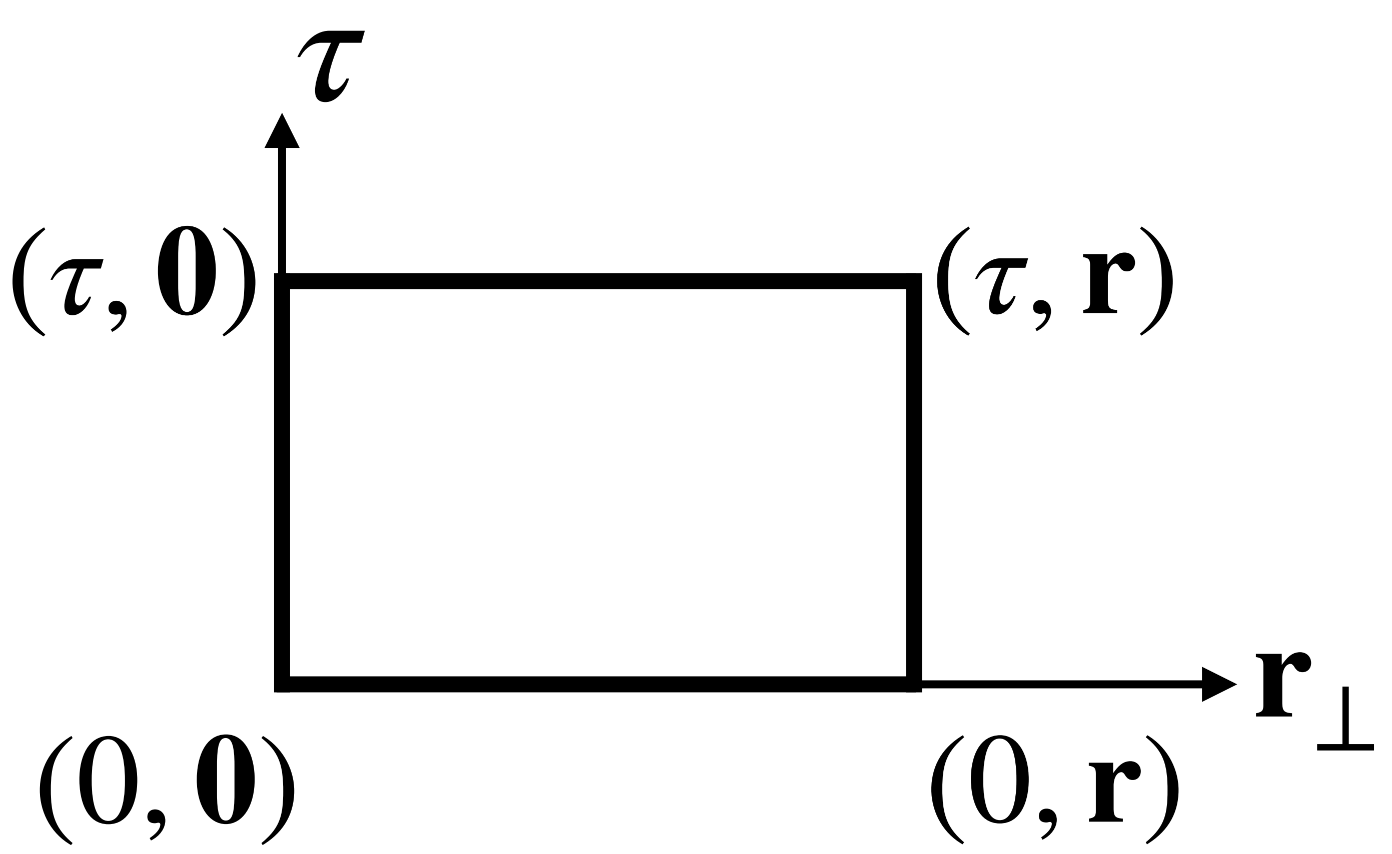


D. Bala, Pavan *et al.* arXiv:2412.17570v1 (2024)

Wilson loop at finite-momenta

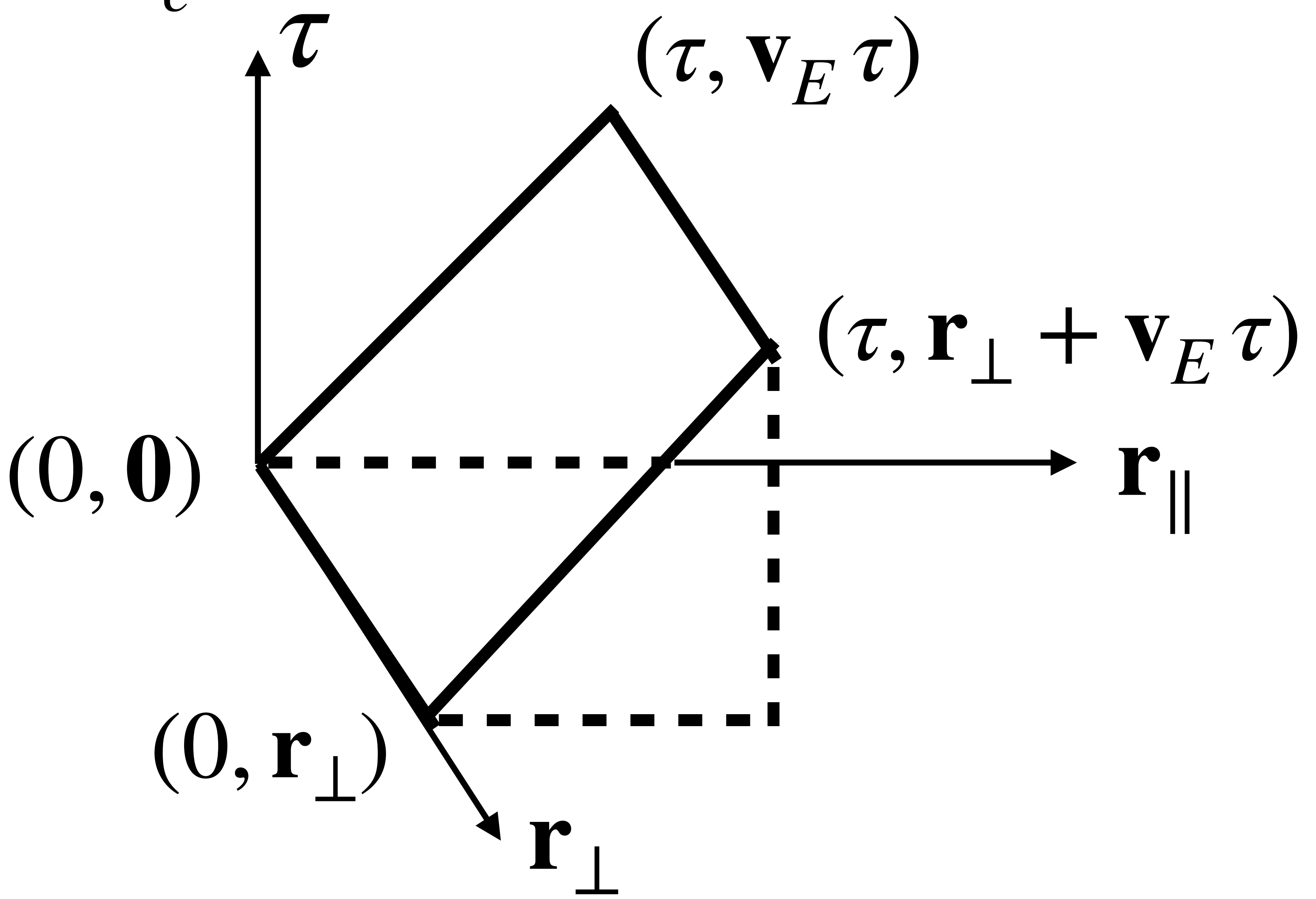
At zero-momenta the expectation of Wilson loop is given as

$$C_E(\tau, v_E, r_\perp) \equiv \frac{1}{N_c}\mathrm{Tr}\left\langle W\Big[(0,\mathbf{0});\ (\tau,\mathbf{0});\ (\tau,\mathbf{r});\ (0,\mathbf{r});\ (0,\mathbf{0})\Big]\right\rangle_T$$



Wilson loop expectation at finite momenta (tilted Wilson loop)

$$C_E(\tau, v_E, r_\perp) \equiv \frac{1}{N_c}\mathrm{Tr}\left\langle W\Big[(0,\mathbf{0});\ (\tau,\mathbf{v}_E\,\tau);\ (\tau,\mathbf{r}_\perp+\mathbf{v}_E\,\tau);\ (0,\mathbf{r}_\perp);\ (0,\mathbf{0})\Big]\right\rangle_T$$



$$W[X_1, X_2] = \mathscr{P}\exp\left[i\,g\int_{X_1}^{X_2} A_\mu dX_\mu\right]$$

Spectral function at finite-momenta

Finite momentum modifies the Schrödinger equation in following fashion

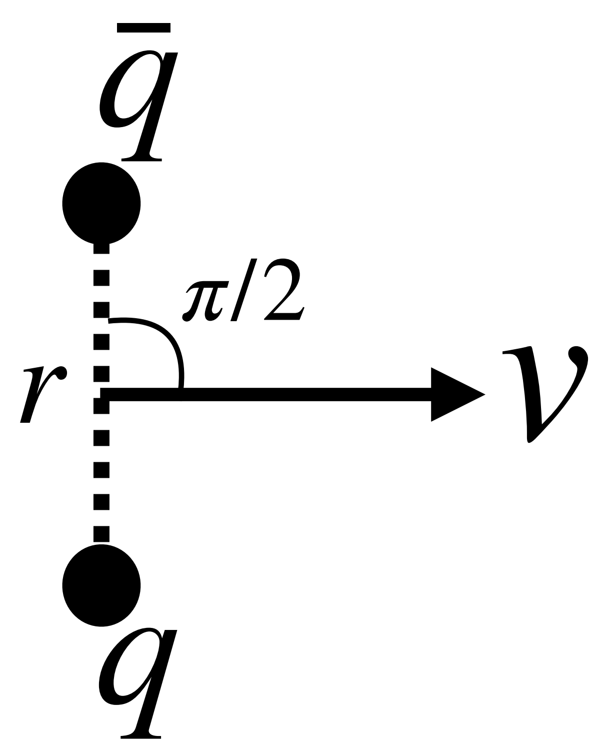
$$\left\{ i\,\partial_t - \left[2M\sqrt{1+\frac{\lambda^2}{4}} - \frac{1}{\left(1+\frac{\lambda^2}{4}\right)^{\frac{3}{2}}}\,\frac{\nabla^2}{M} + V_T(r_\perp, r_\parallel, v) \right] \right\} C_>(t,\vec{q},\vec{r}) = 0$$

$$C_>(0,\vec{q},\vec{r}) = -\frac{24\,N_c}{4+\lambda^2}\,\delta^3(\vec{r}) \qquad\qquad v_q = \frac{q}{\sqrt{q^2+4M^2}}$$

λ parameterises the momentum and given as $\lambda = q/M$

We follow the same procedure as zero momentum after onwards.

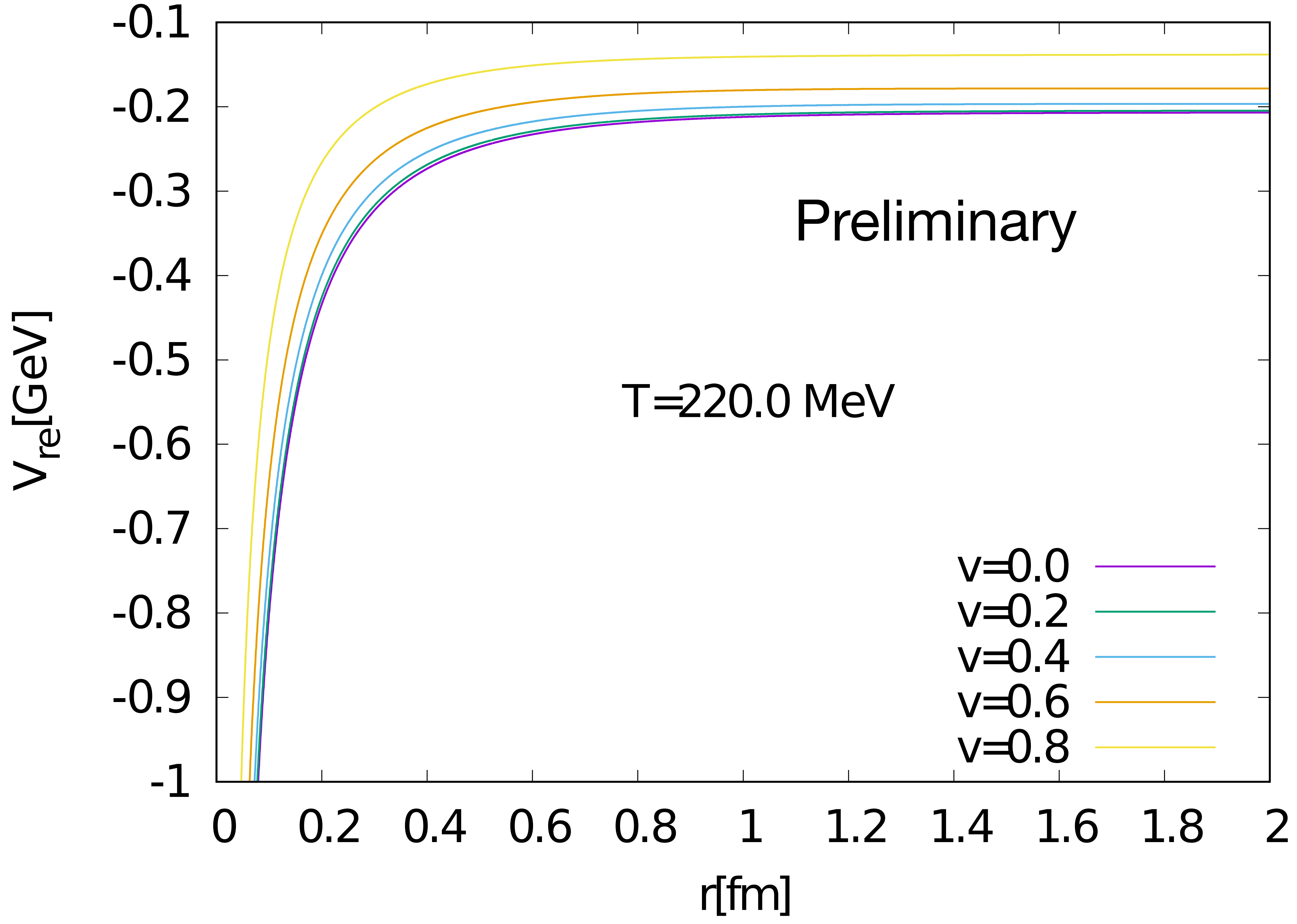
Potential at finite-momenta

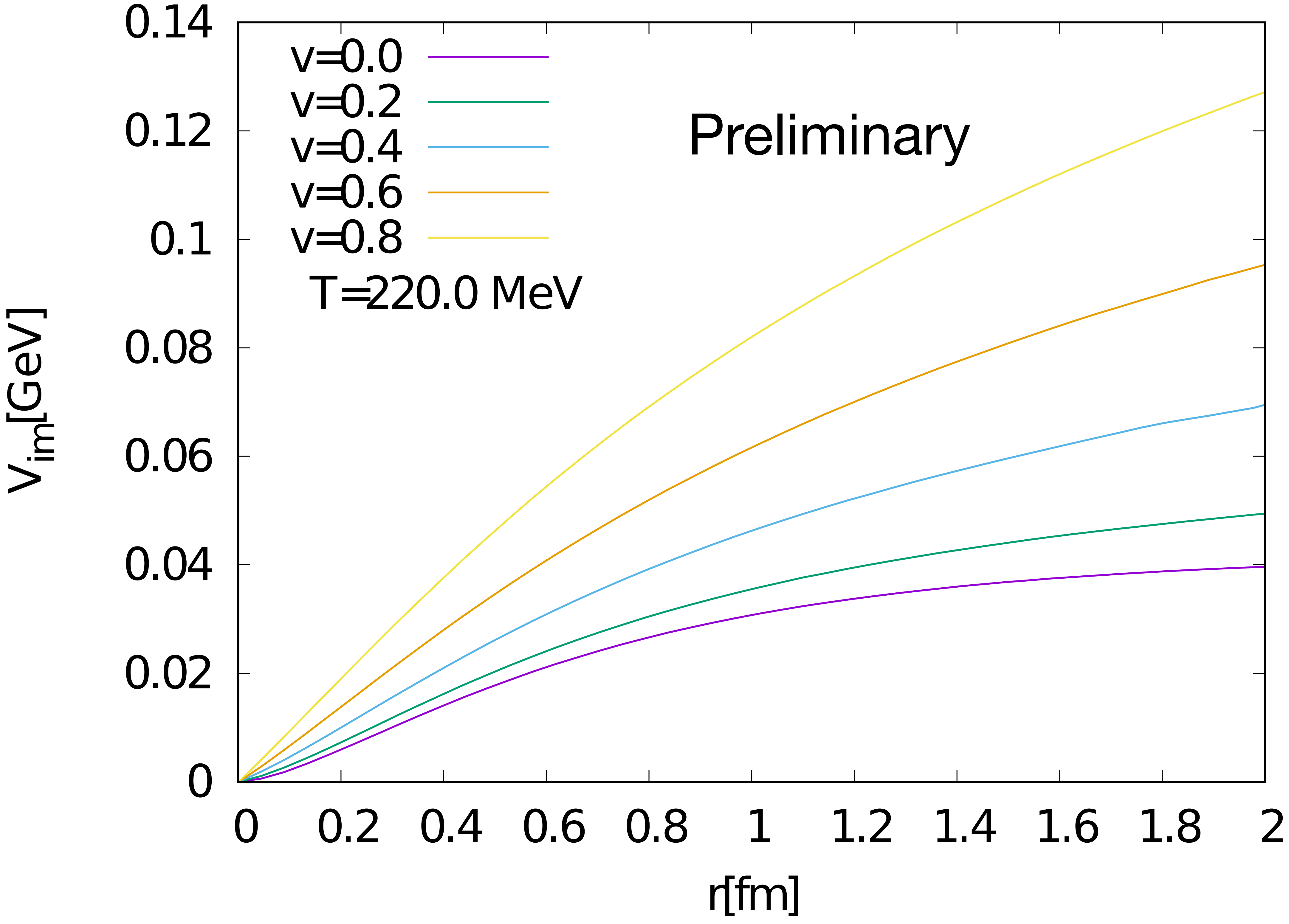


We have plotted this potential considering only the contributions from the transverse component.

This is perturbative potential calculated up to order g^2 in coupling.

The real part of potential gets contribution from transverse part of spectral function.



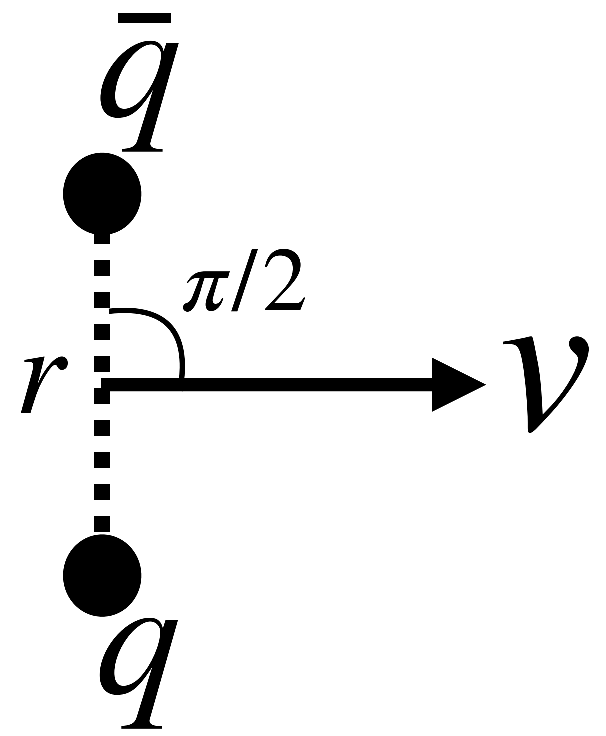


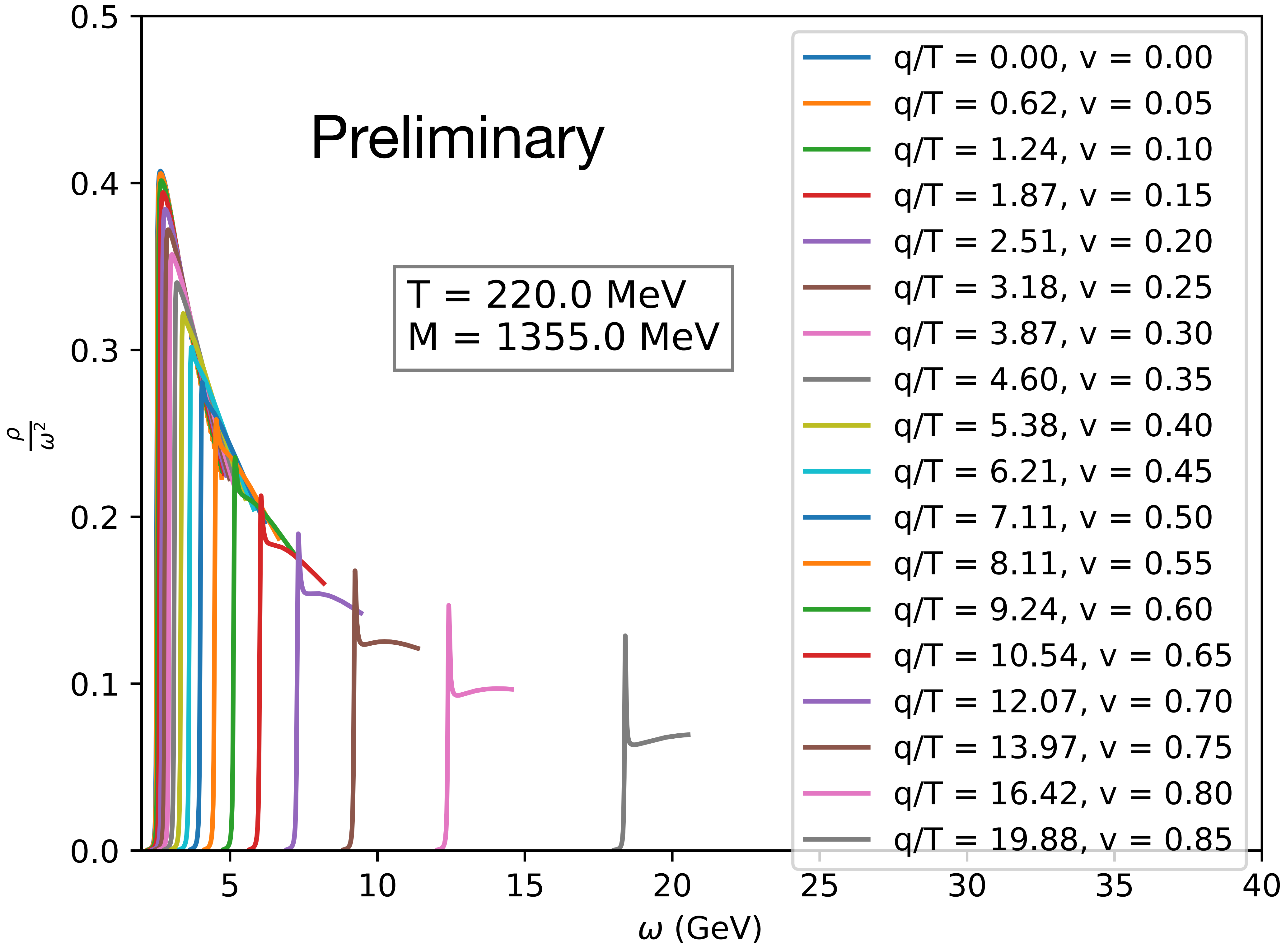
Spectral function at finite-momenta

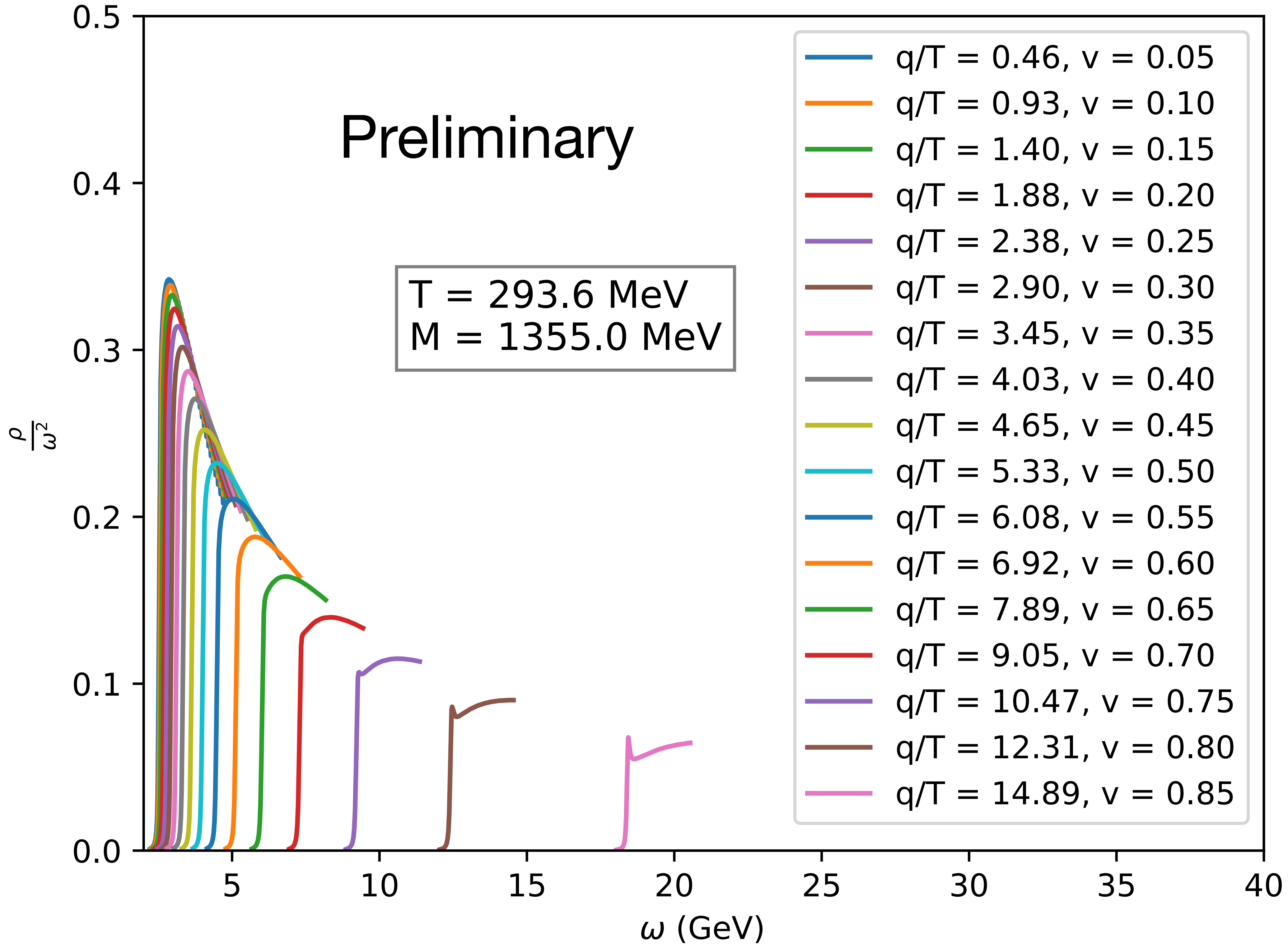
The spectral function in intermediate region $\omega \sim \sqrt{4M^2 + q^2}$

We have considered only the transverse motion to simplify the solution of the Schrödinger equation.

The peak of spectral function decreases with increasing momenta.







Conclusion and Outlook

We solved the Schrödinger equation to extract the spectral function in intermediate region and matched with perturbative UV part for zero momenta.

In finite momenta, the real part of potential gets contribution from transverse part of spectral function that is not case at zero momenta.

We observed that both real and imaginary contribution of potential increases with increasing momenta.

We are working on extracting potential and spectral function considering both longitudinal and transverse motions.

We also want to extend the finite momenta perturbative analysis on lattice by calculating the Wilson-line correlators.