

# Enhancing Bayesian parameter estimation by simultaneously adapting to multiple energy-scales with RHIC and LHC data

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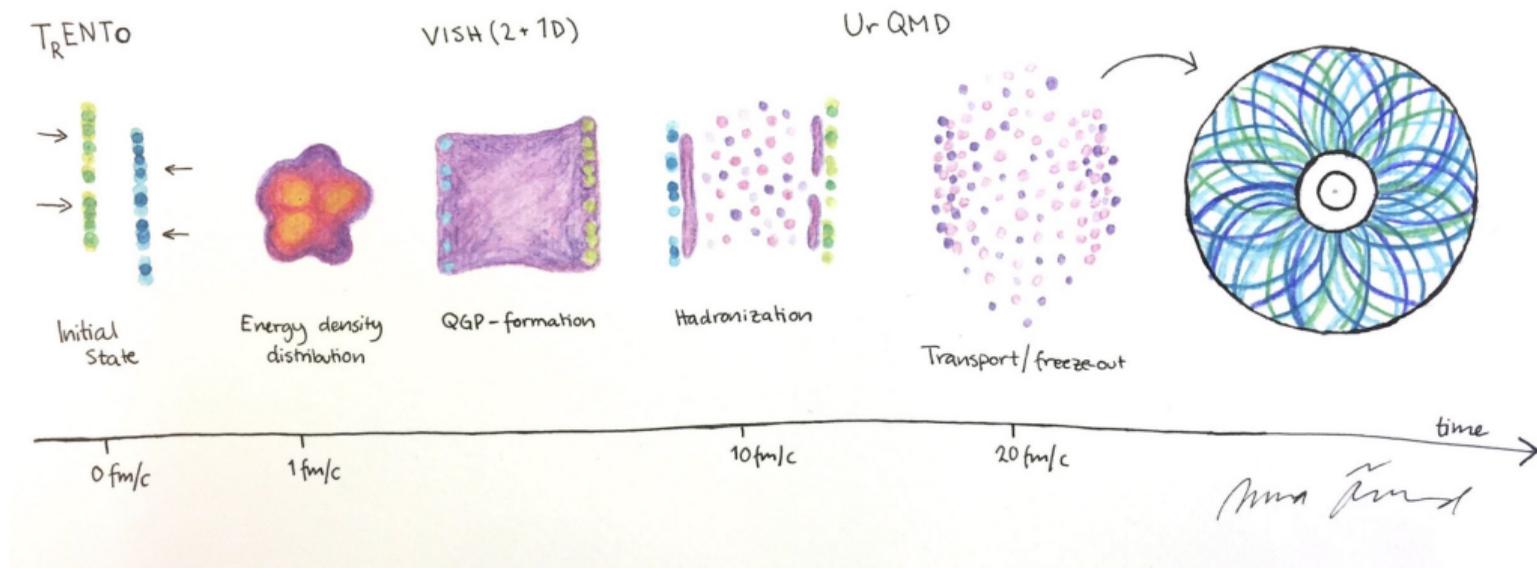
15. January 2025

ATHIC – Berhampur, India

Based on work: arXiv:2411.01932



# THE DIFFERENT STAGES OF HEAVY-ION COLLISIONS

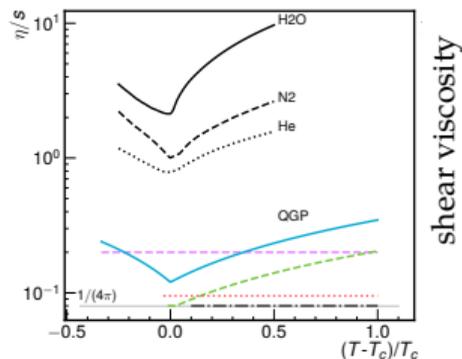


Credits to Anna Önerstad

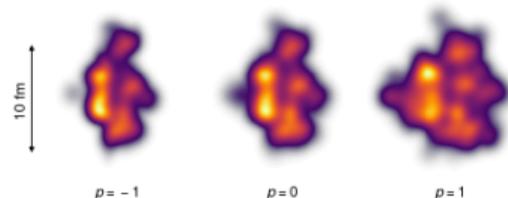
$$T^{\mu\nu} = eu^\mu u^\nu - (P + \Pi)\Delta_{\mu\nu} + \pi^{\mu\nu}, \quad \delta_\mu T^{\mu\nu} = 0$$

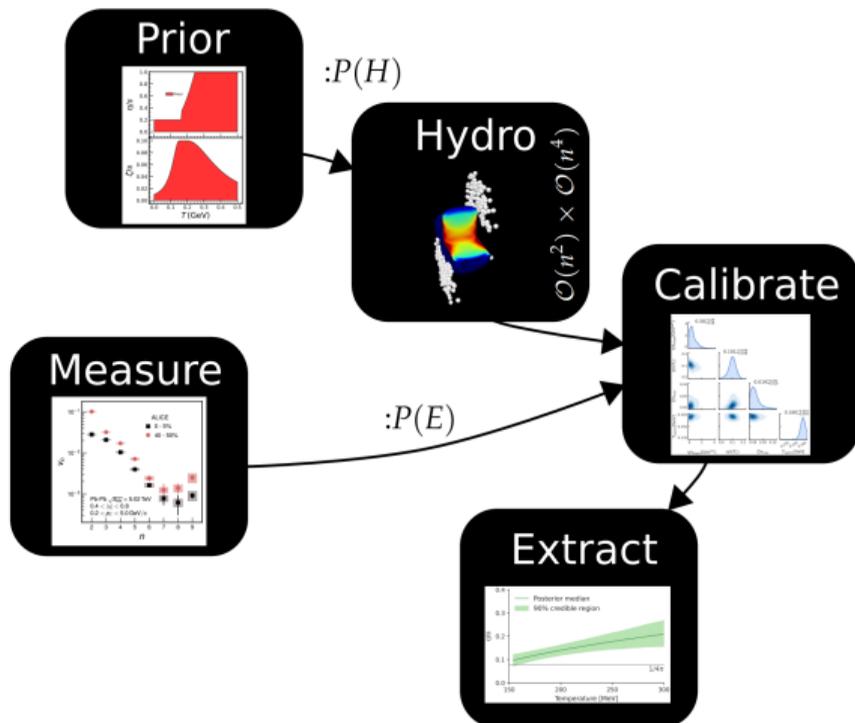
Parameter	Description
$T_c$	Temperature of const. $\eta/s(T)$ , $T < T_c$
$\eta/s(T_c)$	Minimum $\eta/s(T)$
$(\eta/s)_{\text{slope}}$	Slope of $\eta/s(T)$ above $T_c$
$(\eta/s)_{\text{curve}}$	Curvature of $\eta/s(T)$ above $T_c$
$(\zeta/s)_{\text{peak}}$	Temperature of $\zeta/s(T)$ maximum
$(\zeta/s)_{\text{max}}$	Maximum $\zeta/s(T)$
$(\zeta/s)_{\text{width}}$	Width of $\zeta/s(T)$ peak
$T_{\text{switch}}$	Switching / particlization temperature
N(5.02 TeV)	Overall normalization (Pb–Pb 5.02 TeV)
N(2.76 TeV)	Overall normalization (Pb–Pb 2.76 TeV)
N(200 GeV)	Overall normalization (Au–Au 200 GeV)
$p$	Entropy deposition parameter
$w$	Nucleon width
$\sigma_k$	Std. dev. of nucleon multiplicity fluctuations
$d_{\text{min}}^3$	Minimum volume per nucleon
$\tau_{\text{fs}}$	Free-streaming time

Hydrodynamics

Trento p-value, <http://qcd.phy.duke.edu/trento/>

Initial conditions

 $p = -1$  $p = 0$  $p = 1$

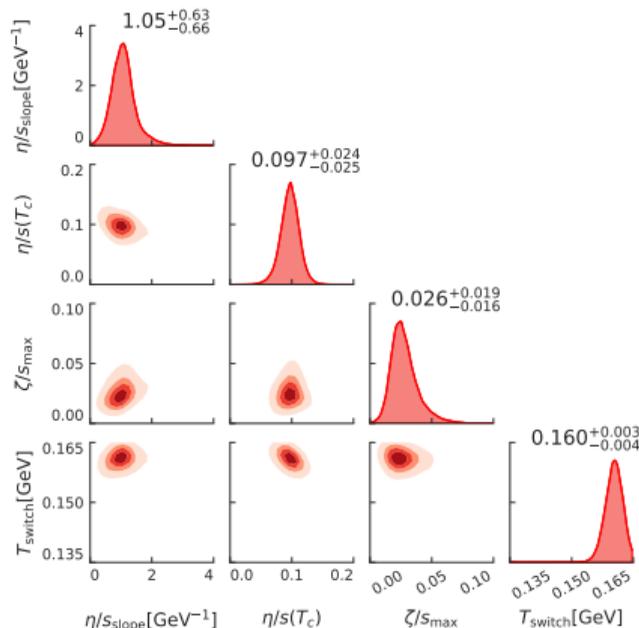
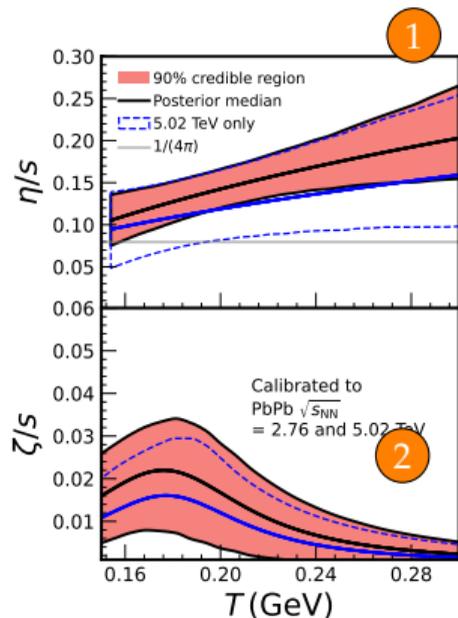


Bayes' theorem:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

$$P(E) = \sum_{i=1}^n P(E|H_i)P(H_i)$$

- Find optimal set of model parameters that best reproduce the experimental data.
- Utilise constraints, such as flow observables, to help narrow down the  $\eta/s(T)$  and such.

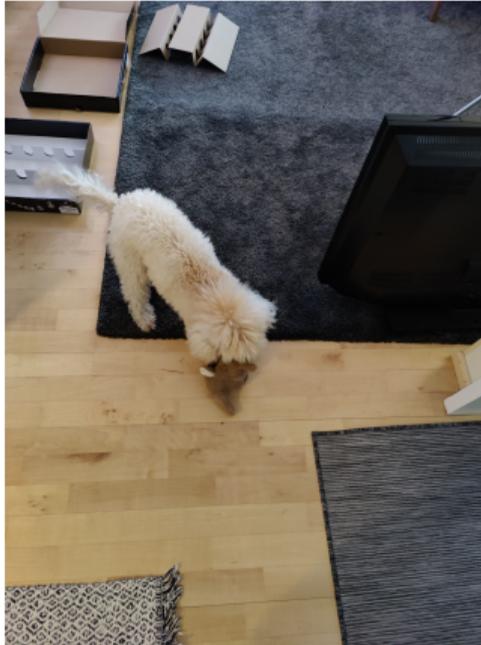


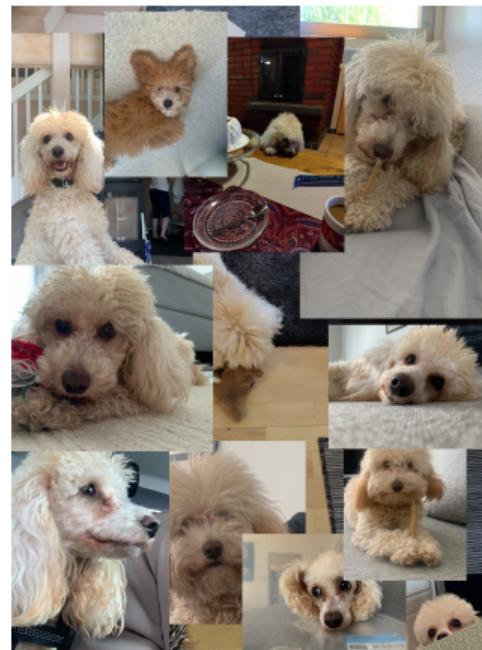
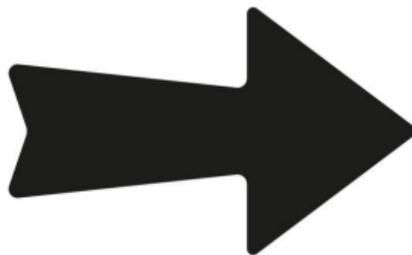
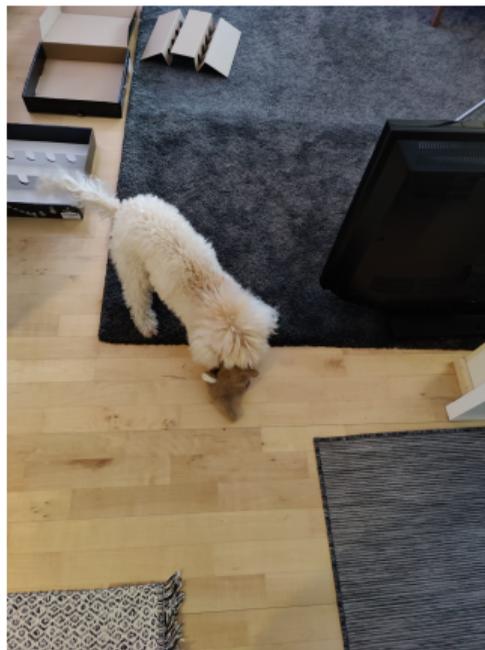
- 1 Significantly improved  $\eta/s(T)$  uncertainty
- 2 Non-zero  $\zeta/s(T)$
- 3 Overall better convergence for parameter components

Together with two collision energies and added observables, the uncertainty has reduced!

J.E. Parkkila, A. Onnerstad, M. Virta, S.F. Taghavi, C. Mordasini, A. Bilandzic, D.J. Kim, Phys. Lett. B 835 (2022) 137485

# HOW TO IMPROVE?

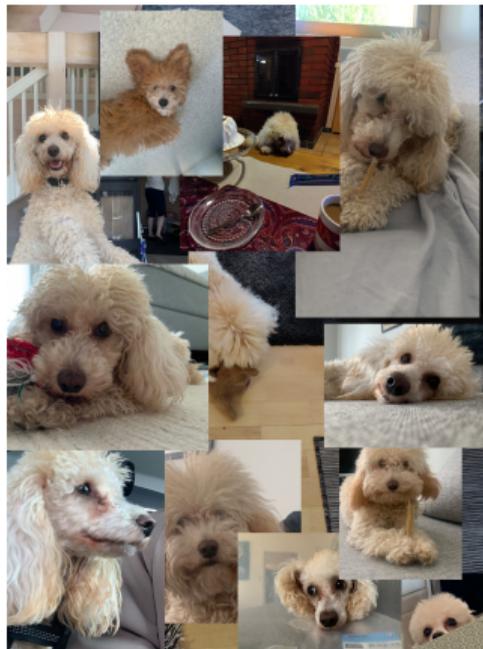




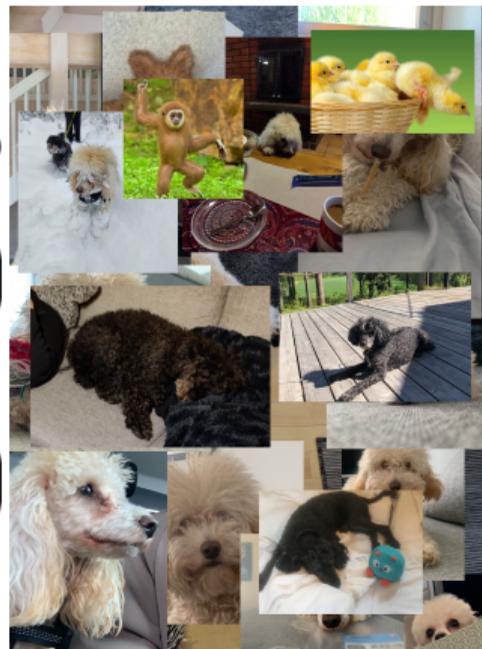
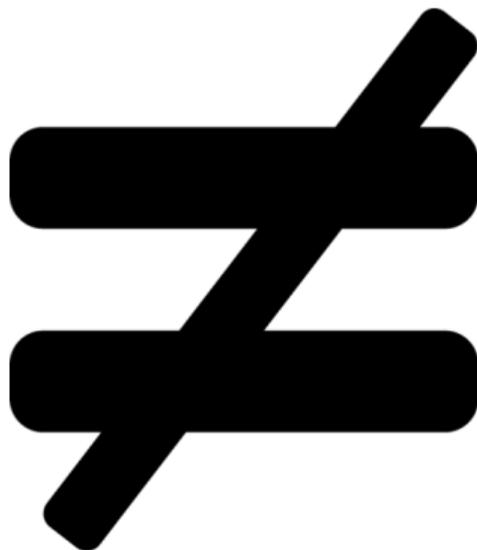
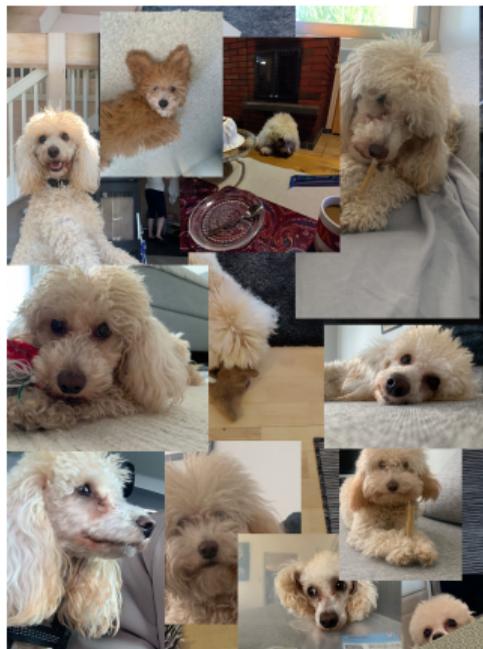
- Add more independent and sensitive data, inclusion of RHIC data
- Use separate centrality binning for each parametrisation

Not quite...

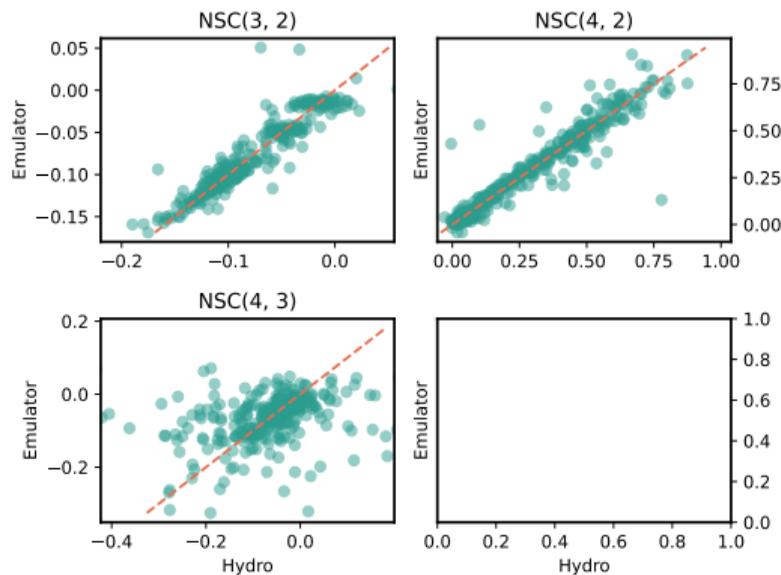
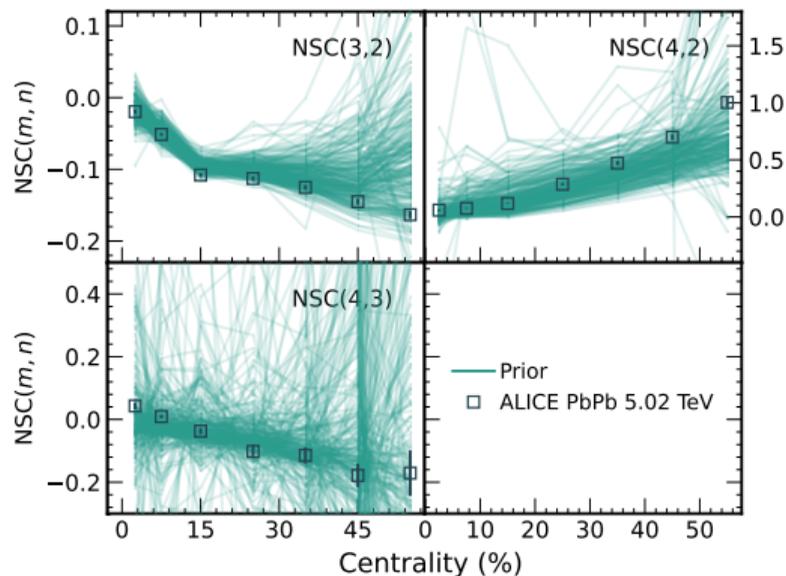
# WHAT ARE WE PUTTING IN?



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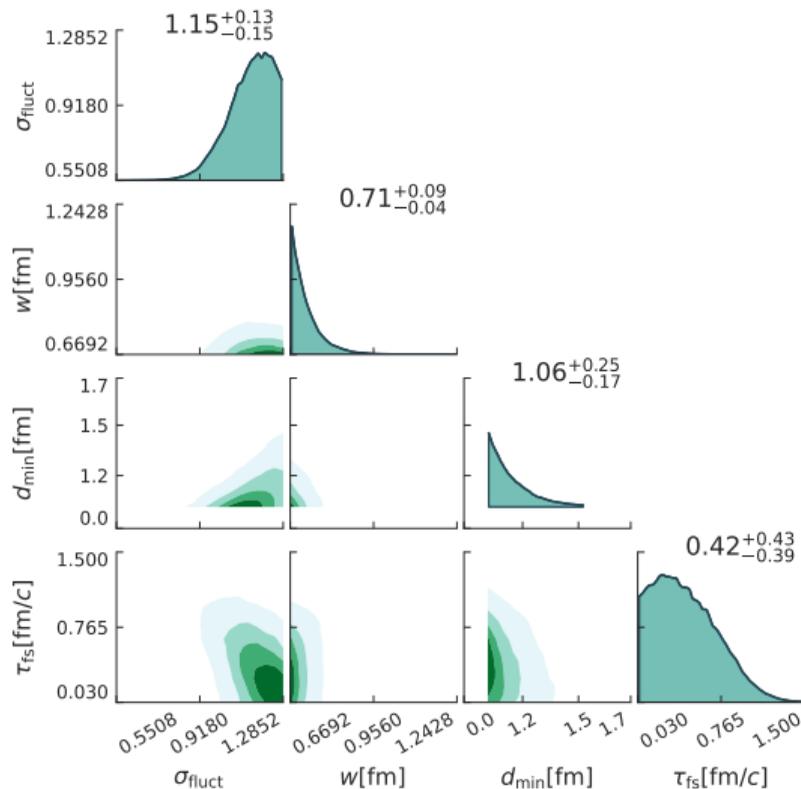
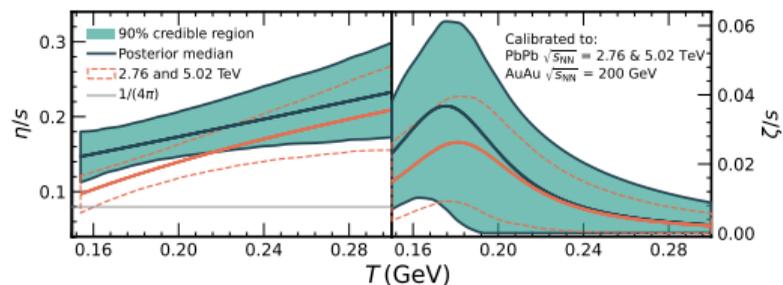
- Including EVERYTHING doesn't necessarily improve results
- Need to separate between birds and dogs



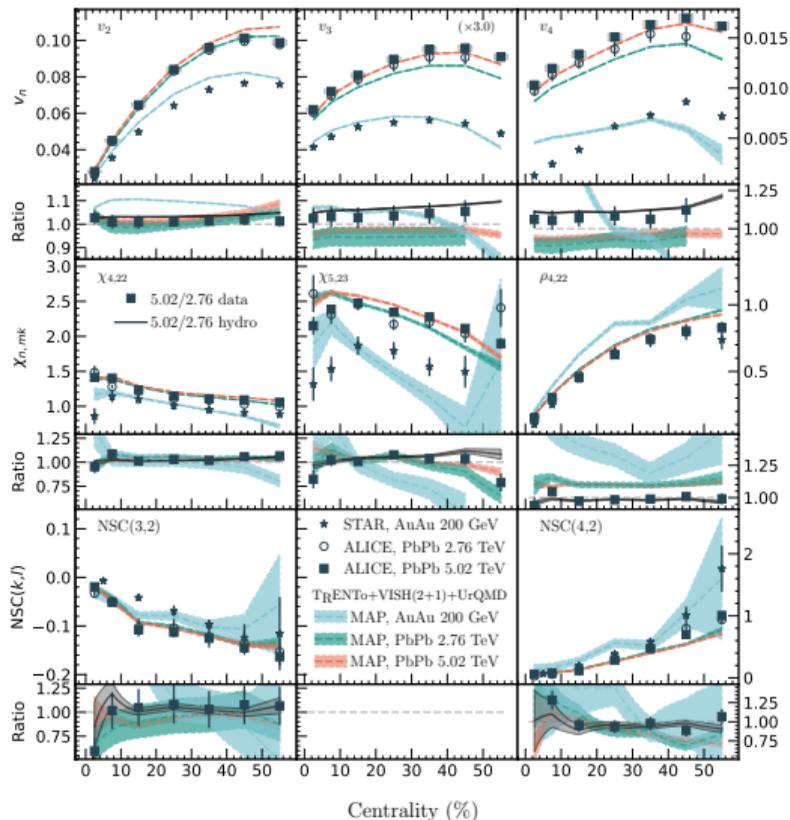
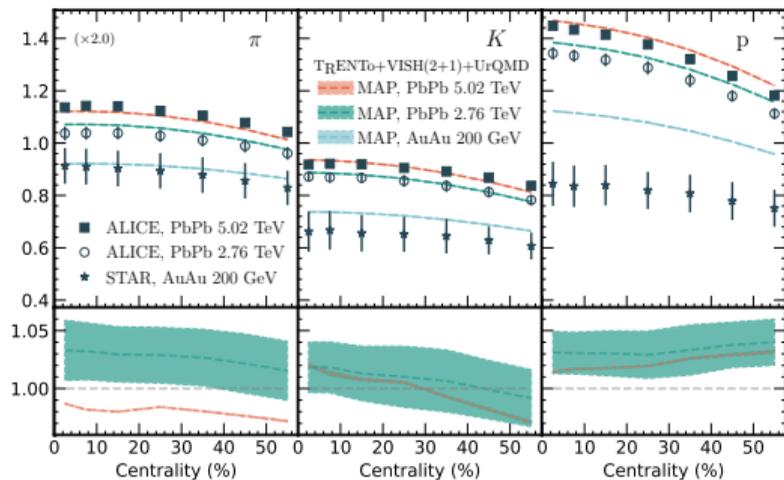
- Prior distributions, how well parameter space covers the data
- Most parametrisations have divergent NSC(4,3)

- Emulator validation, how well the emulator can interpret model results
- NSC(4,3) becomes scattered

# RESULTS



- Posterior distributions are larger due to the separate centrality binning
  - Previous results biased by single centrality definition
  - Some parameters are on the boundaries of the prior range
- ⇒ need to extend the parameter ranges



- Mean transverse momentum within 5% for Pb-Pb
- Improved results for  $v_2$ , while slight overestimation of  $v_3$  and  $v_4$
- Trend captured for  $\chi_{n,mk}$ ,  $\rho_{n,mk}$  and NSC( $n, m$ )

## Conclusions

- Posterior distributions are larger — uncertainty increased
- Inclusion of RHIC data and separate centrality calibration improves MAP results
- Caution is needed with data included in the analysis

## Outlook

- Parameter ranges need to be extended
- Try different initial conditions models
  - EKRT
  - IP+Glasma
- Measurements of independent data
- Improve models to access higher order observables

## Conclusions

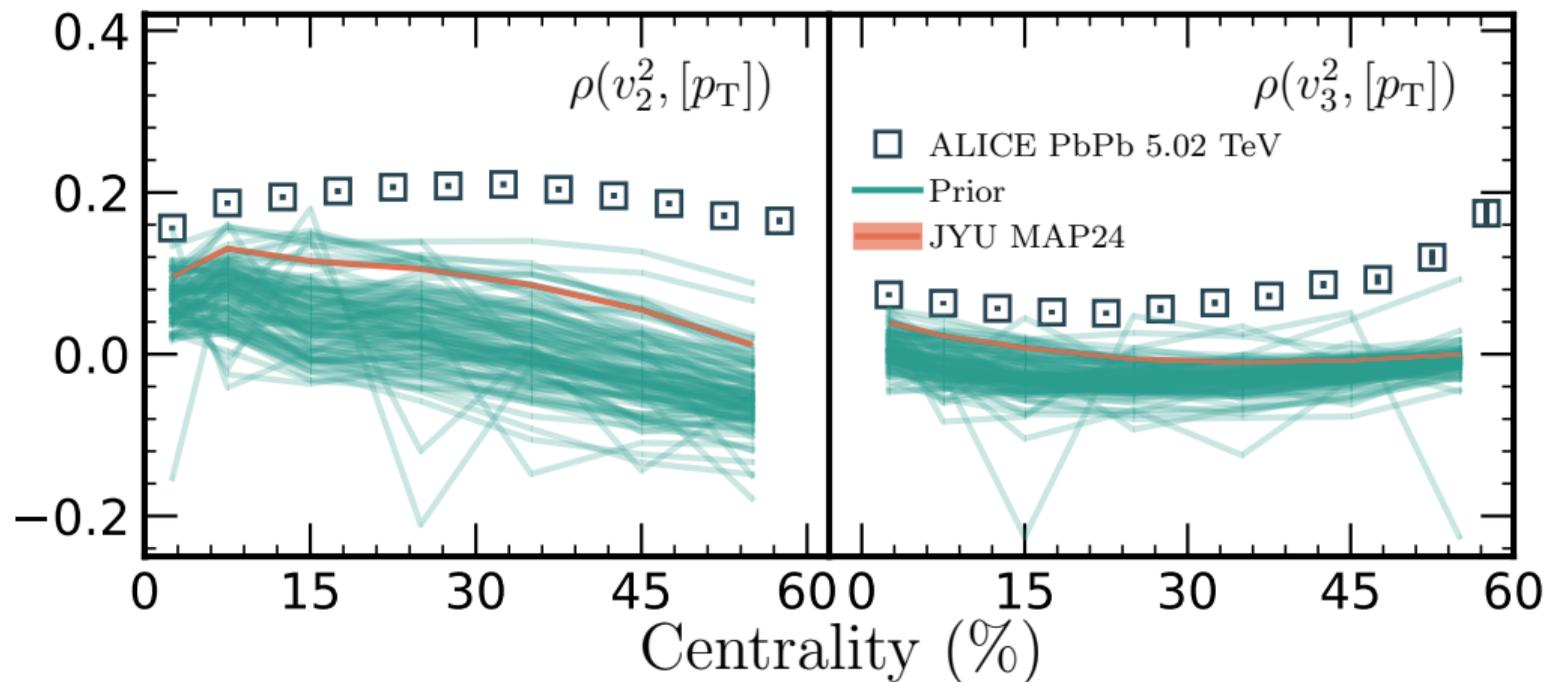
- Posterior distributions are larger — uncertainty increased
- Inclusion of RHIC data → centrality calibration
- Caution is needed in future analysis

Out now: [arxiv:2411.01932!!](https://arxiv.org/abs/2411.01932)

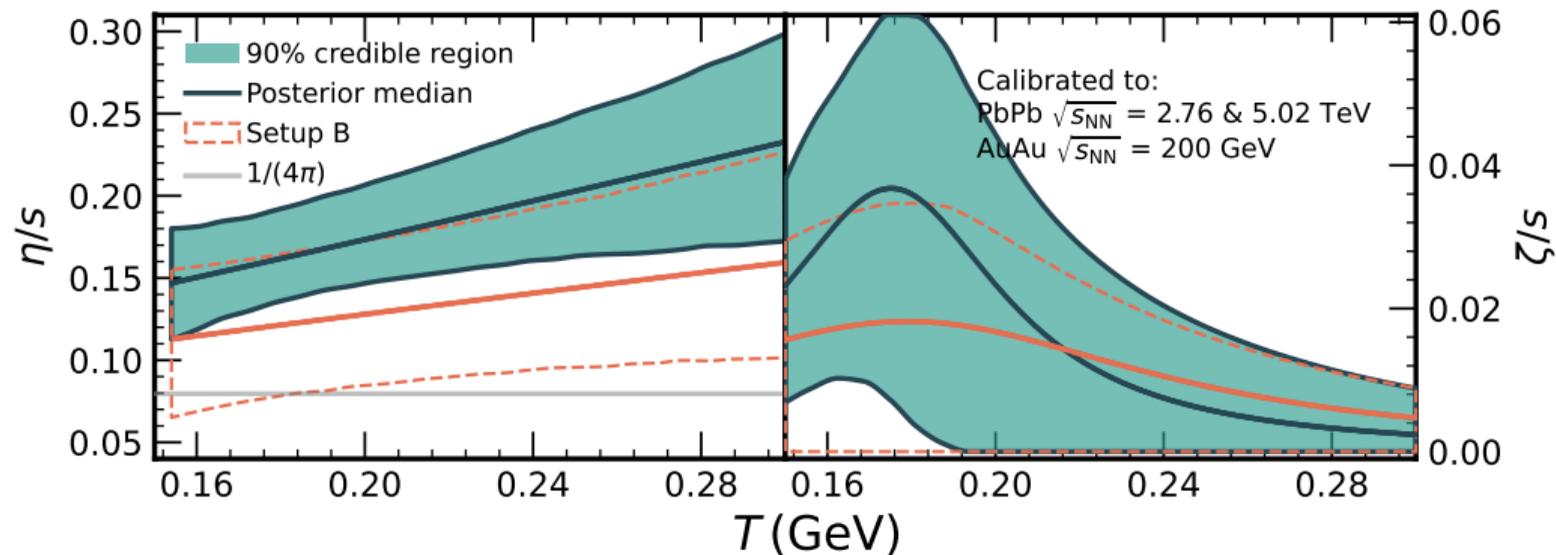
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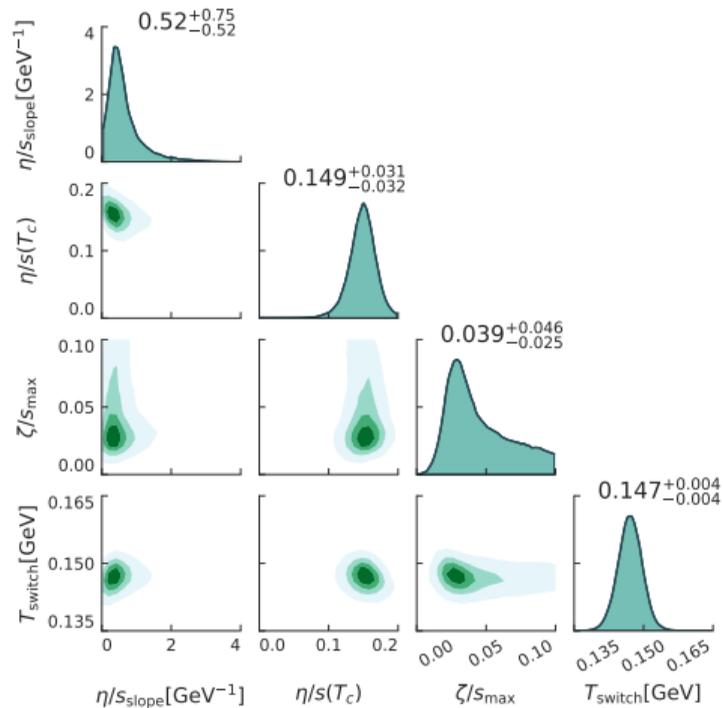
THANK YOU!



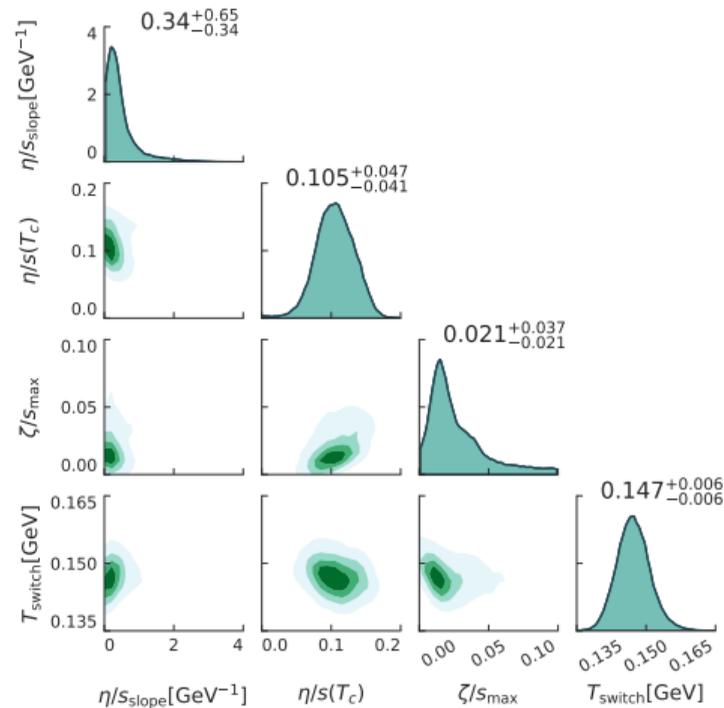
- Data not captured by the prior distribution
- MAP result closer to the data



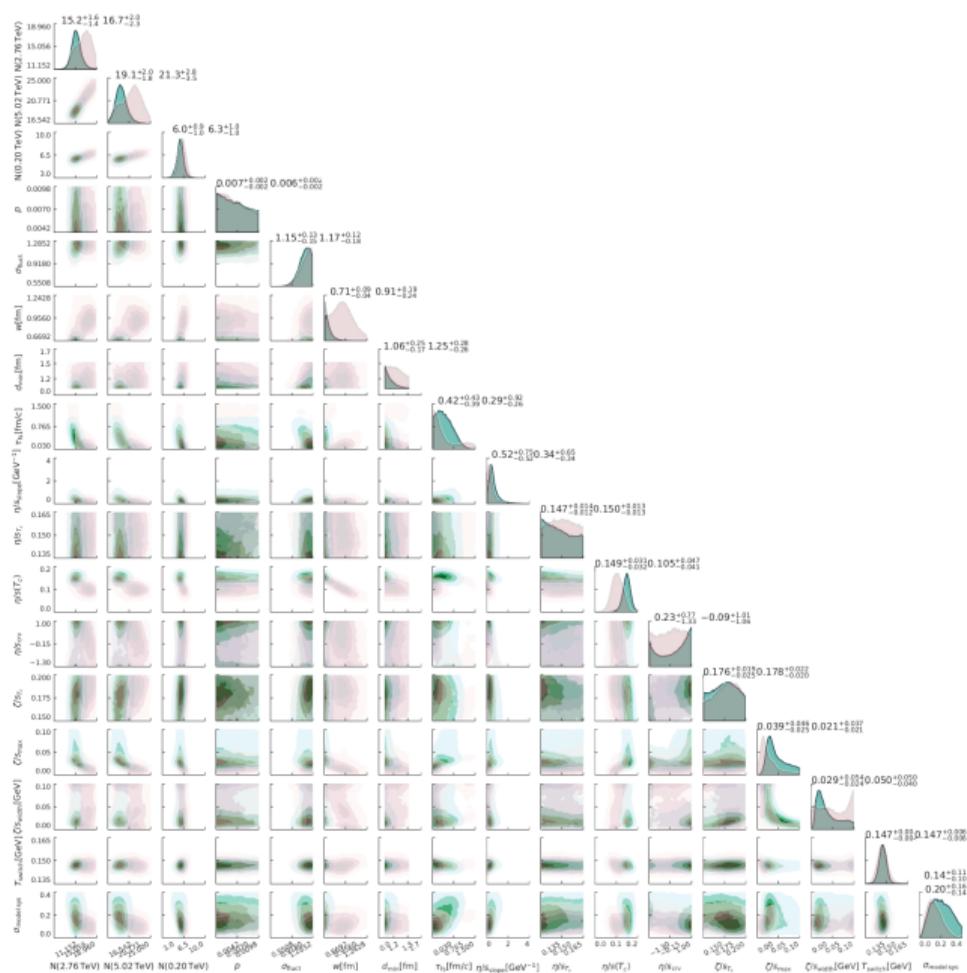
- Inclusion of all observables is marked as *Setup B*
- Including all causes larger uncertainty in  $\eta/s$  and smaller values for  $\zeta/s$

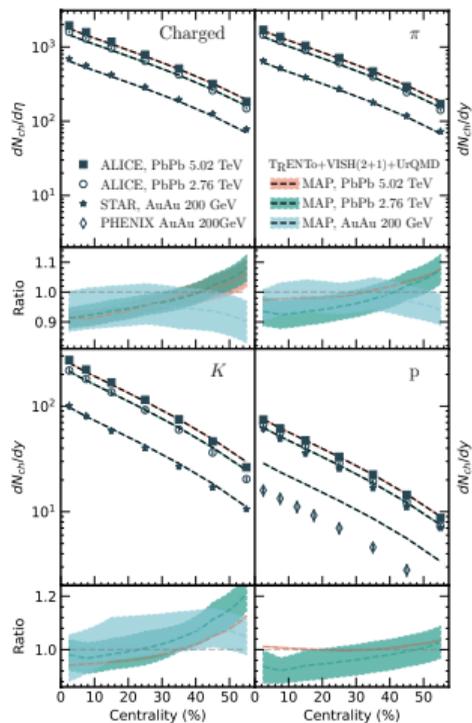


Setup A

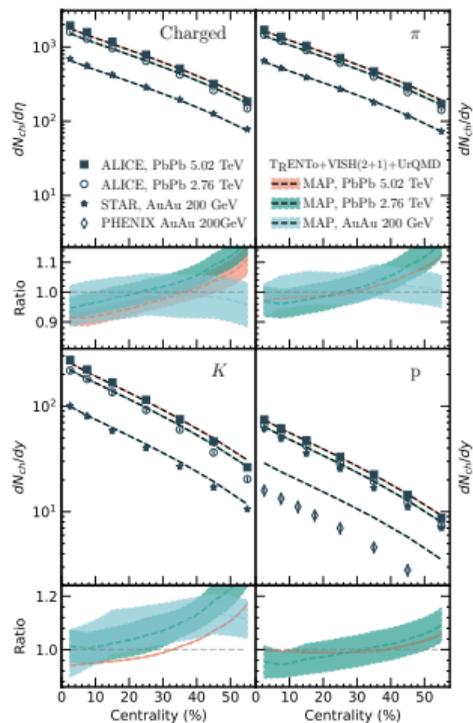


Setup B

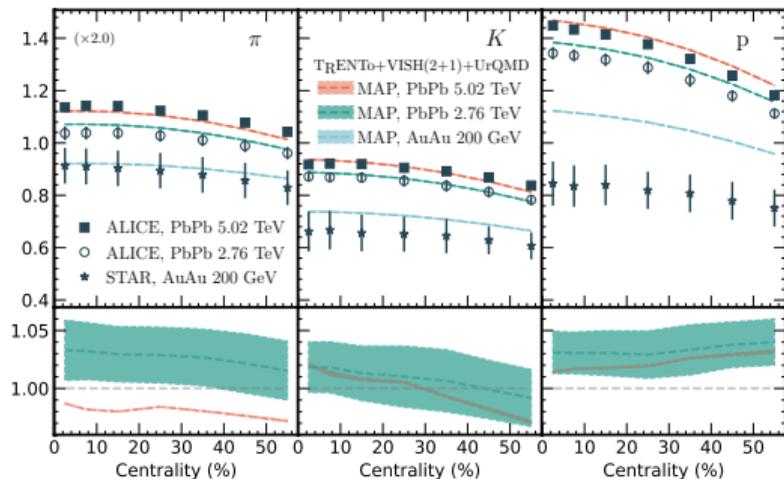




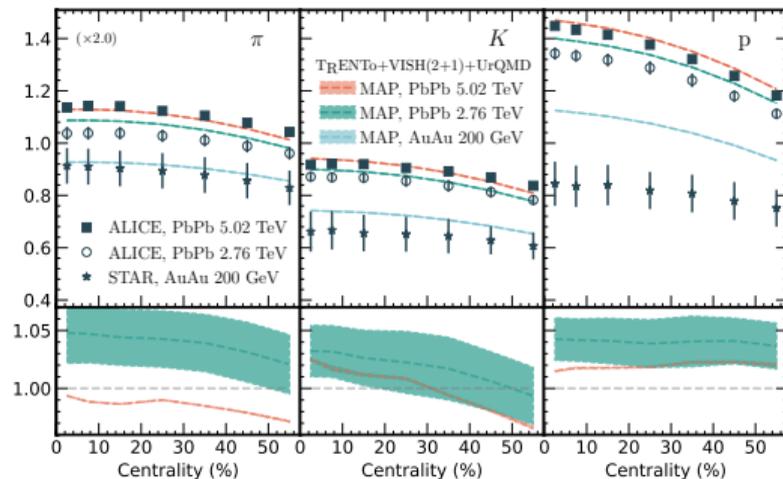
Setup A



Setup B

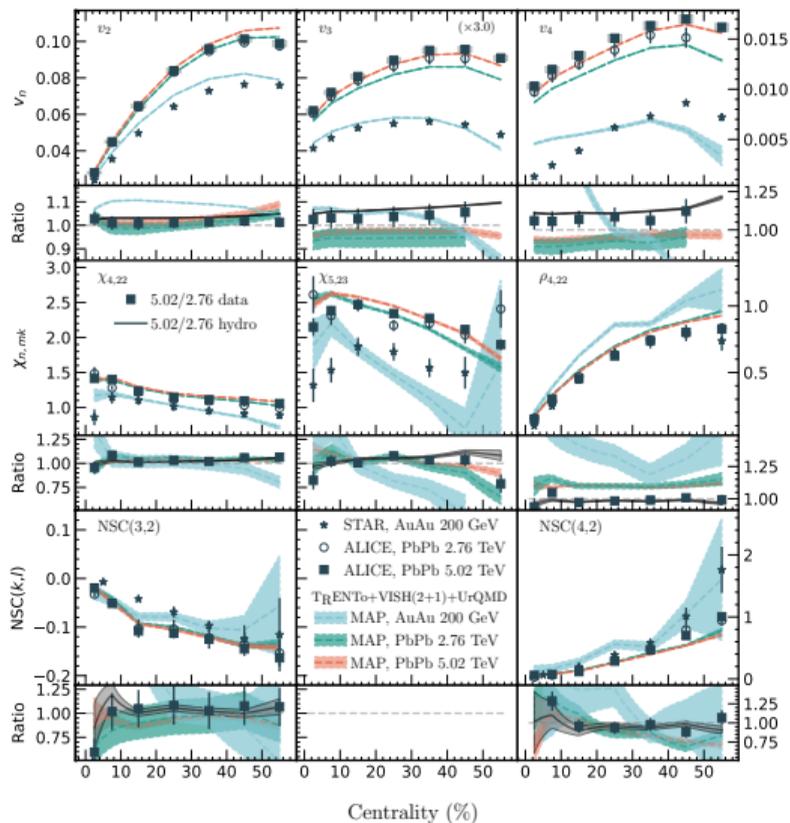


Setup A

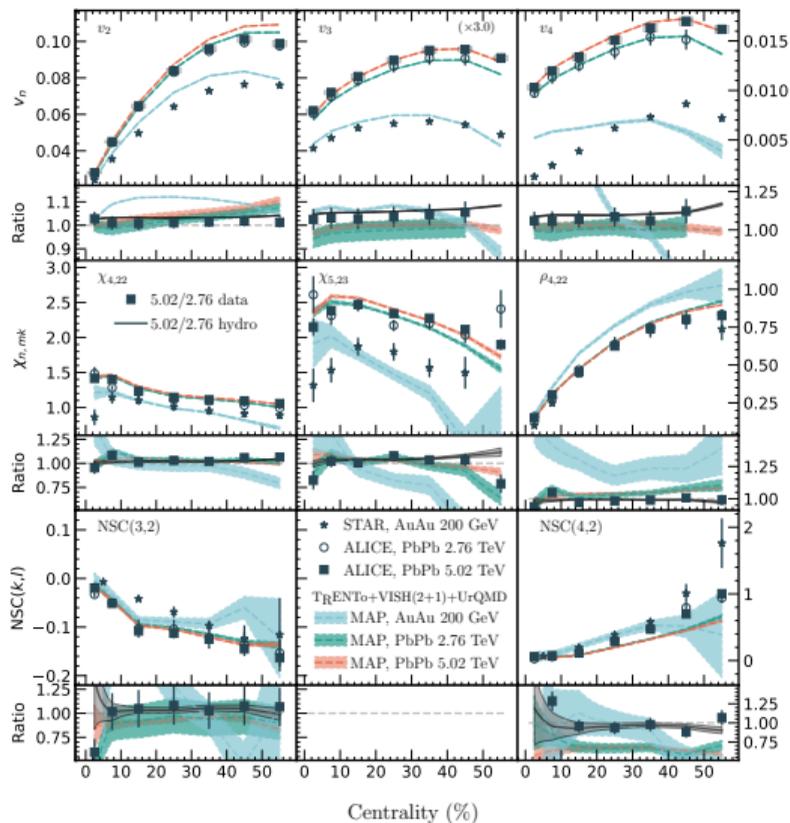


Setup B

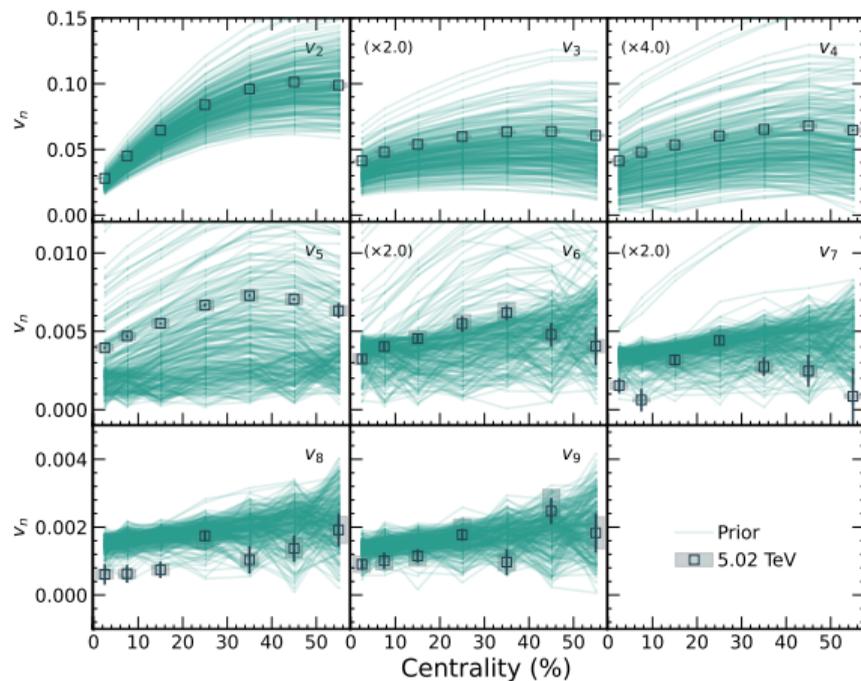
# EFFECT OF INCLUDING ALL OBSERVABLES: FLOW OBSERVABLES



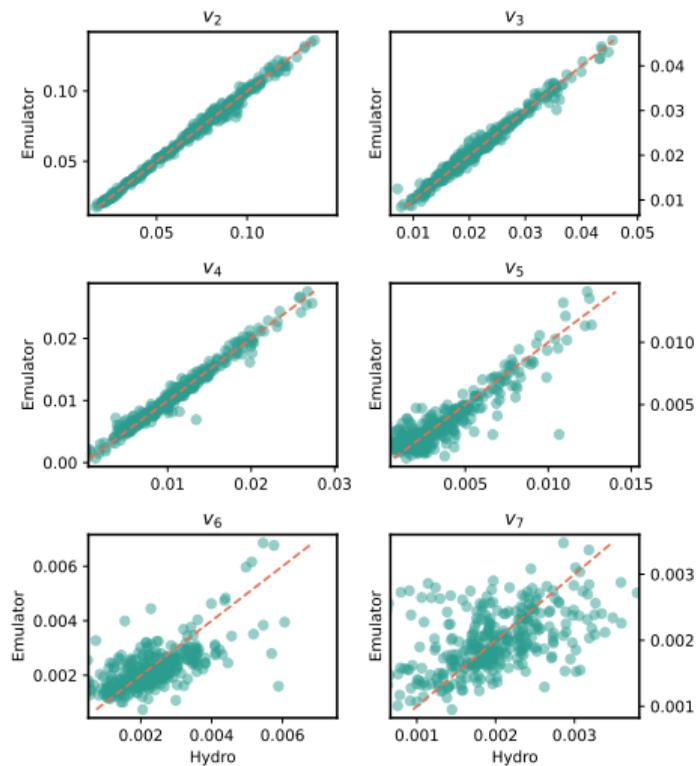
Setup A



Setup B



Prior distribution



Emulator validation

- Together, various flow observables cover the sensitivity for all components of transport properties.

Name	Symbol	Measure	Sensitivity-stochastic approach
Flow coefficients	$v_n$	System expansion and anisotropy of the flow	Average $\langle \eta/s \rangle$ and $\zeta/s(T)$ peak
(Normalized) Symmetric cumulants	(N)SC( $k, l, m$ )	Correlation between magnitudes of flow harmonics	$\eta/s(T)$ temperature dependence
Non-linear flow mode coefficients	$\chi_{n,mk}$	Quantification of the non-linear response	$\eta/s(T)$ at the freeze-out
Symmetry-plane correlations	$\rho_{n,mk}$	Correlations between the directions of flow harmonics	$\eta/s(T)$

Thanks to excellent ALICE papers over years:

- Phys.Rev.Lett. 117 (2016) 182301, Phys.Lett. B773 (2017) 68, Phys.Rev. C 97 (2018) 024906, JHEP05 (2020) 085, Phys.Lett. B818 (2021) 136354, Phys.Rev.Lett. 127 (2021) 092302 - [flow](#)
- Phys.Rev.Lett. 106 (2011) 032301, Phys.Rev.C 88 (2013) 044910, Phys.Lett. B772 (2017) 567-577, Phys.Rev.C 101, 044907 (2020) - [N<sub>ch</sub> and  \$\langle p\_T \rangle\$](#)