Axion effects on the nonradial oscillations of neutron stars

THIC 2025

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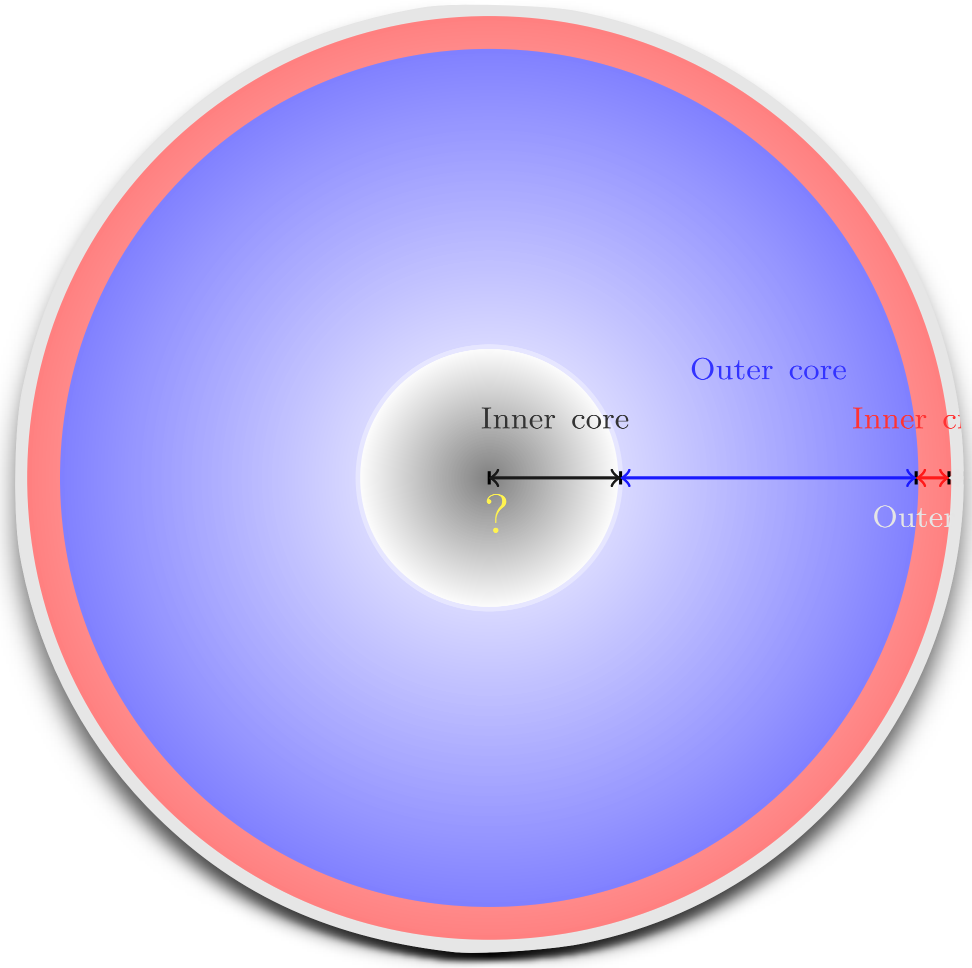
arXiv: 2411.17828

(DK, Hiranmaya Mishra)

- Compact stars, (NS/HS)
  - NSs are the exciting natural astrophysical laboratories to study the behaviour of matter at extreme densities.
  - Macroscopic properties of such NS like mass, radius, moment of inertia, tidal deformability depend on the equation of state of matter of NS.

$$\frac{dp}{dr} = -(\epsilon + p)\frac{m + 4\pi r^3 p}{r(r-2m)}, \qquad \frac{dm}{dr} = 4\pi r^2 \epsilon$$

The boundary conditions: $m(0) = p(R) = 0, \quad p(r=0) = p_0,$



- **Non-radial oscillations can unveil the NS matter:**

$$Q' - \frac{1}{c_s^2}\left[\omega^2 r^2 e^{\Lambda - 2\Phi} Z + \Phi' Q\right] + l(l+1)e^{\Lambda} Z = 0$$

$$Z' - 2\Phi' Z + e^{\Lambda}\frac{Q}{r^2} - \frac{\omega_{BV}^2 e^{-2\Phi}}{\Phi'\left(1 - \frac{2m}{r}\right)}\left(Z + \Phi' e^{-\Lambda + 2\Phi}\frac{Q}{\omega^2 r^2}\right) = 0$$

The boundary conditions:

$$Q(r) = Cr^{l+1}, \quad Z(r) = -Cr^l/l \qquad \omega^2 r^2 e^{\Lambda - 2\Phi} Z + \Phi' Q\Big|_{r=R} = 0$$

Introduction

- Some recent studies have shown that quark-matter core can appear in massive NS, and the presence of a first-order phase transition from hadronic to quark matter can imprint signatures in binary NS merger observations.

- The binary NS merger events have recently emerged as a new tool for probing the beyond standard-model (BSM) particles, such as axions.

  Axions are the hypothetical elementory particles introduced to solve the strong CP problem.

- To address the strong CP problem, physicists Peccei and Quinn proposed the "Peccei-Quinn mechanism" in 1977.

- This mechanism introduces a new global symmetry called PQ symmetry. When it is spontaneously broken, it gives rise to a new pseudo-Nambu-Goldstone boson, the axion.

- Here, we are taken the Nambu--Jona-Lasino (NJL) mode to study low energies.

$$\mathscr{L} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - \hat{m})q + G_s \sum_{A=0}^{8} \left[(\bar{q}\lambda^A q)^2 + (\bar{q}i\gamma_5\lambda^A q)^2\right]$$
$$-K\left[e^{i\theta}\mathrm{det}\{\bar{q}(1+\gamma^5)q\} + e^{-i\theta}\mathrm{det}\{\bar{q}(1-\gamma^5)q\}\right] - G_v\left[(\bar{q}\gamma^{\mu}q)^2 + (\bar{q}\gamma^{\mu}\gamma^5 q)^2\right].$$

Equation of state (NJL model)

- Thermodynamic potential

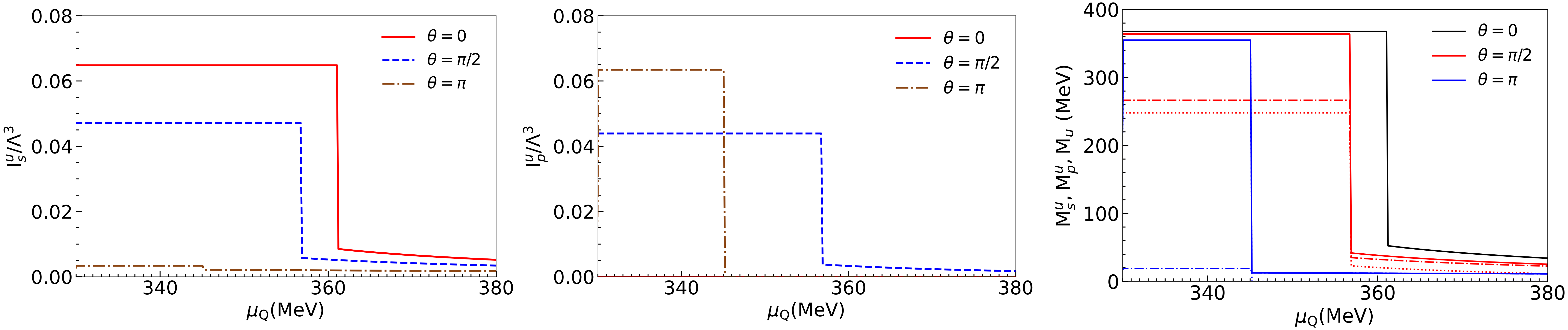
$$\Omega(I_s^i, I_p^i, \theta, T, \mu) = \Omega_{\bar{q}q} + \sum_{i=u,d,s} 2G_s(I_s^{i2} + I_p^{i2}) + 4K(\cos\theta I_s^u I_s^d I_s^s + \sin\theta I_p^u I_p^d I_p^s)$$
$$-4K\Big(\cos\theta(I_s^u I_p^d I_p^s + I_s^d I_p^u I_p^s + I_s^s I_p^d I_p^u) + \sin\theta(I_p^u I_s^d I_s^s + I_p^d I_s^u I_s^s + I_p^s I_s^u I_s^d)\Big)$$

where, scalar condensate $I_s^i = \langle \bar{q}^i q^i \rangle$

and pseudo-scalar condensate $I_p^i = \langle \bar{q}^i i\gamma^5 q^i \rangle$

$$p = -\Omega(I_s, I_p, \theta, T, \mu)$$

$$\epsilon = \sum_i \mu_i n_i - p$$



Equation of state (RMF model)

Walecka's mean field model

- Relativistic nucleons interact through exchange of mesons
  - scalar meson exchange ⇒ *Attraction*
  - vector meson exchange ⇒ *Repulsion*
- Adjust the couplings and the masses so that B.E. per nucleon at saturation densities is reproduce.
- More terms/parameters are introduced to describe other properties of nuclear matter.

Nuclear matter (RMF model) EOS

Annals Phys 83 (1974) pp 491-529

$$\mathscr{L} = \sum_i \bar{\Psi}_i \left(i\gamma_\mu \partial^\mu - m_i + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \vec{I}_i \vec{\rho}^\mu \right) \Psi_i + \mathscr{L}_{\text{mes}}$$

$$\mathscr{L}_{\text{mes}} = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu$$

$$+ \frac{\kappa}{3!}(g_{\sigma\text{N}}\sigma)^3 + \frac{\lambda}{4!}(g_{\sigma\text{N}}\sigma)^4 - \frac{\xi}{4!}(g_{\omega\text{N}}^2 \omega_\mu \omega^\mu)^2 - \Lambda'(g_{\omega\text{N}}^2 \omega_\mu \omega^\mu)(g_{\rho\text{N}}^2 \rho_\mu \rho^\mu)$$

$$\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$$

$$\vec{R}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$$

- At the mean field level i.e. $\langle \sigma \rangle = \sigma_0$, $\langle \omega_\mu \rangle = \omega_0 \delta_{\mu 0}$ and $\langle \rho_\mu^a \rangle = \delta_{\mu 0}\delta_3^a \rho_3^0$

$$m_i^* = m_i - g_\sigma \sigma_0 \quad \text{and} \quad \mu_i^* = \mu_i - g_\omega \omega_0 - g_\rho I_{3i} \rho_3^0$$

- Equation of state ($T \neq 0$):

$$\epsilon = -\gamma \sum_i \int \frac{d^3\mathbf{k}}{(2\pi)^3} E_i^* \left(1 - f_-^i - f_+^i\right) \quad \Rightarrow \quad \frac{1}{\pi^2} \sum_{i=n,p,e} H(m^*/k_F^i)$$

$$+\frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{3}\kappa\sigma_0^3 + \frac{1}{4}\lambda\sigma_0^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_0^{3\,2} + \frac{\xi}{8}\left(g_{\omega\mathrm{N}}\omega\right)^4 + 3\Lambda'\left(g_{\omega\mathrm{N}}g_{\rho\mathrm{N}}\omega_0\rho_0^3\right)^2$$

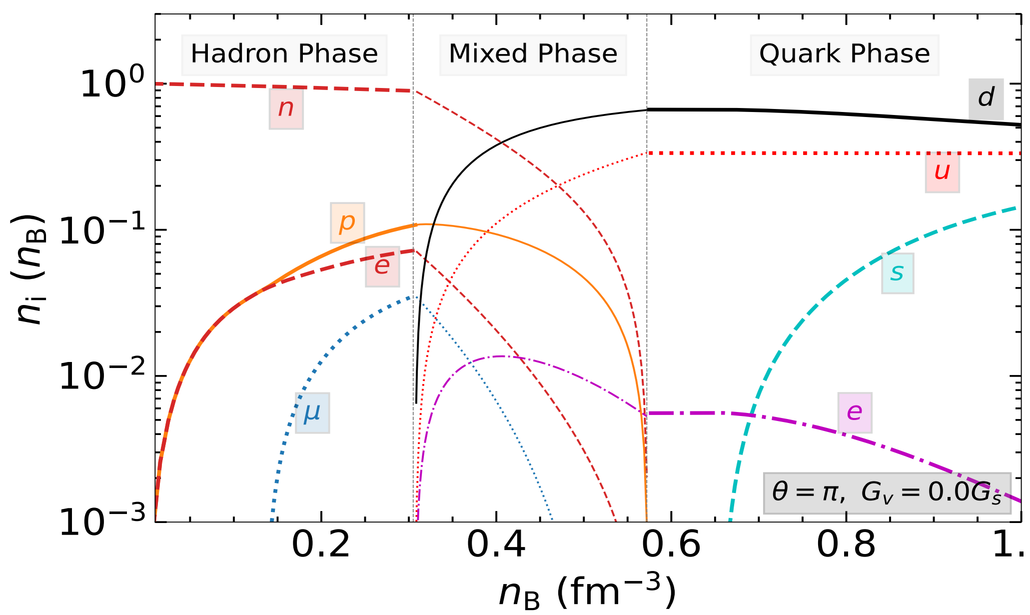
where,

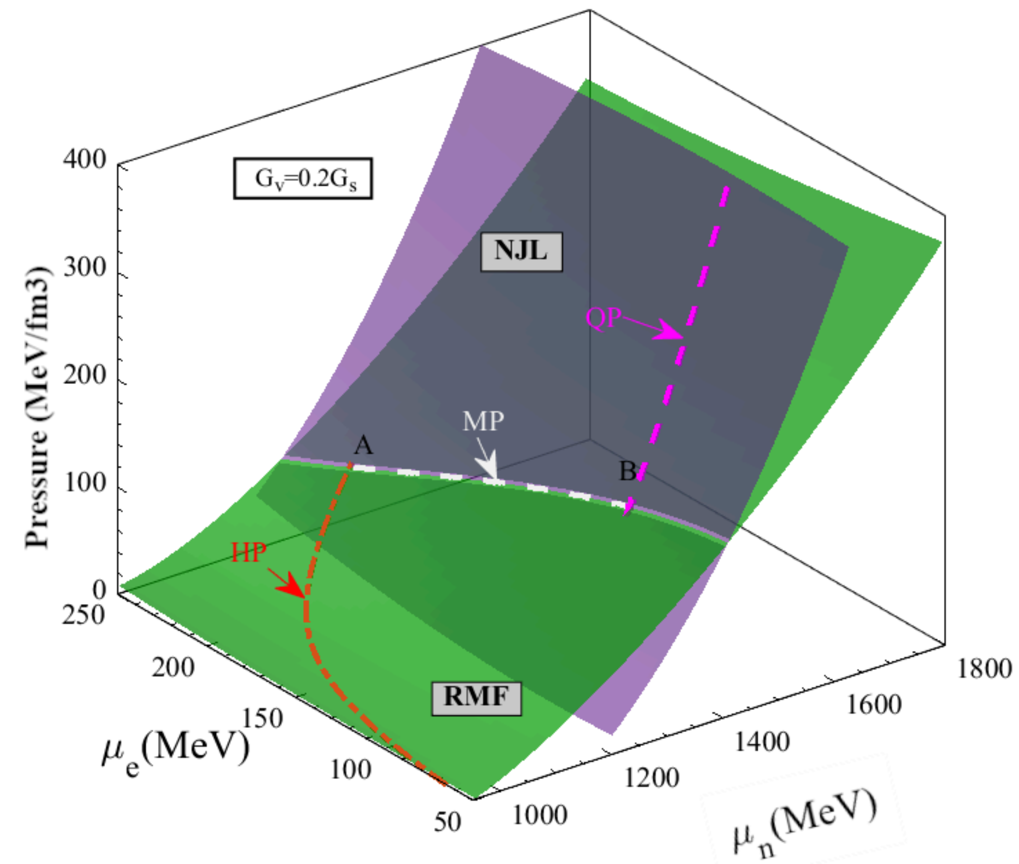
$$H(x) = \frac{1}{8}\left(\sqrt{1+x^2}(2+x^2) - x^4 \ln\left(\frac{x+\sqrt{1+x^2}}{x}\right)\right)$$

$$p = \sum_{i=n,p,e} \mu_i n_i - \epsilon$$

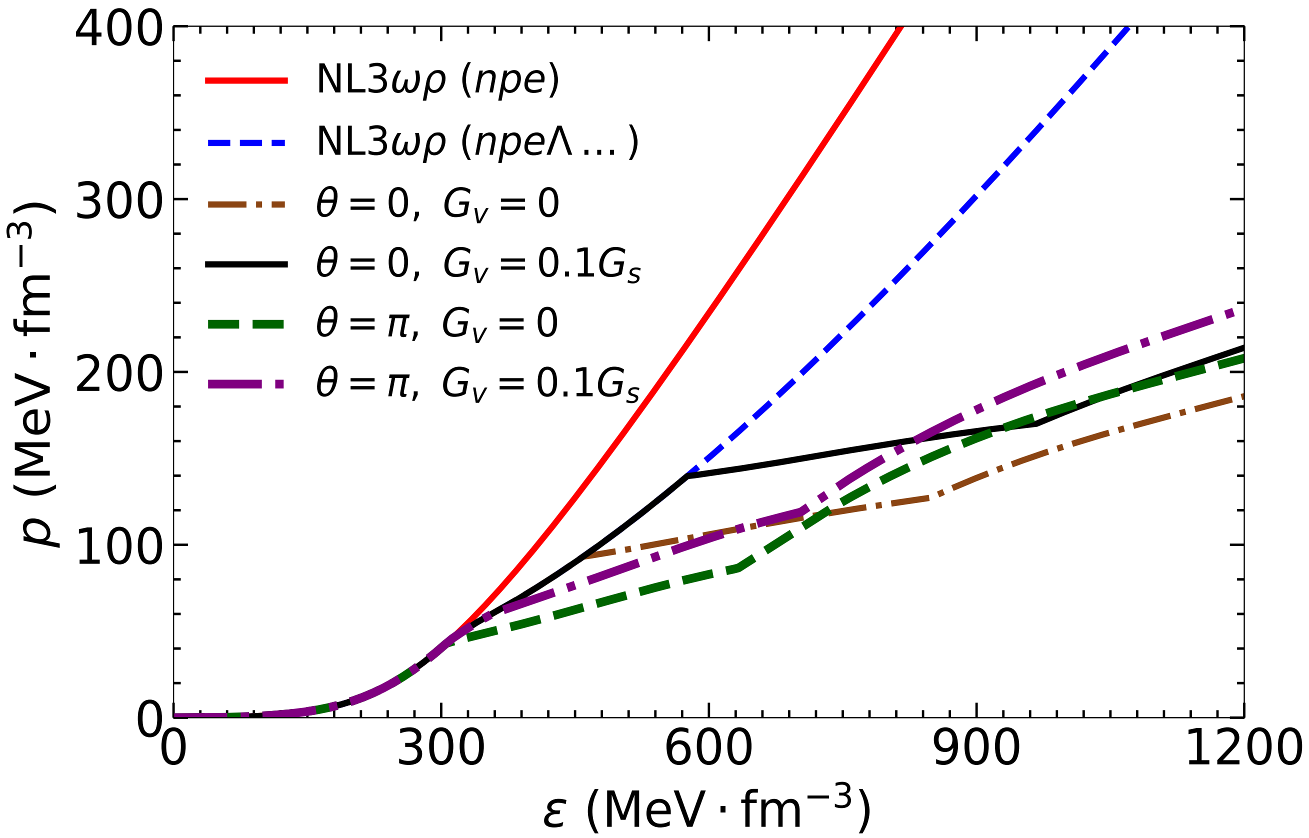
- Gibbs construction for the hadron-quark phase transition

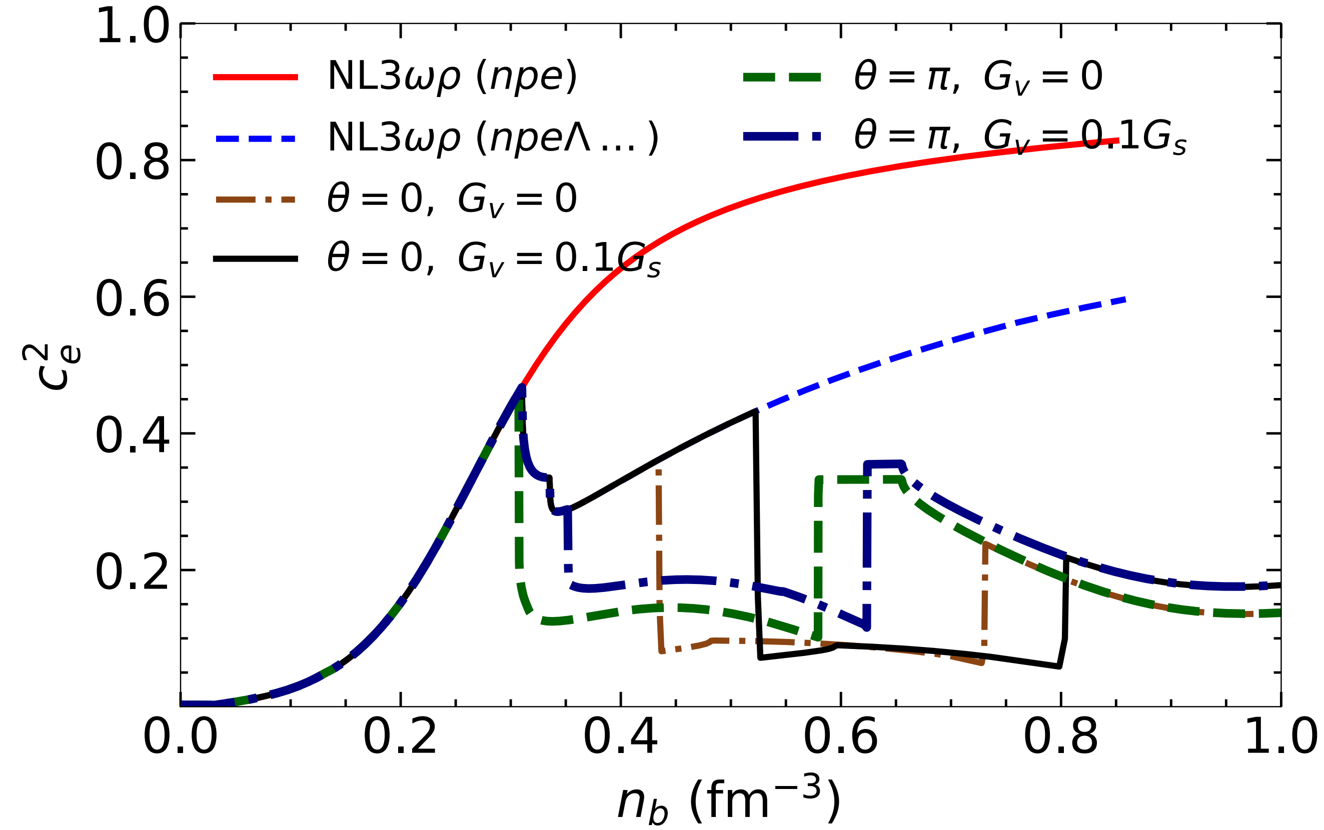
$$p_{\mathrm{HP}}(\mu_{\mathrm{B}}^c, \mu_{\mathrm{E}}^c) = p_{\mathrm{QP}}(\mu_{\mathrm{B}}^c, \mu_{\mathrm{E}}^c) = p_{\mathrm{MP}}(\mu_{\mathrm{B}}^c, \mu_{\mathrm{E}}^c),$$

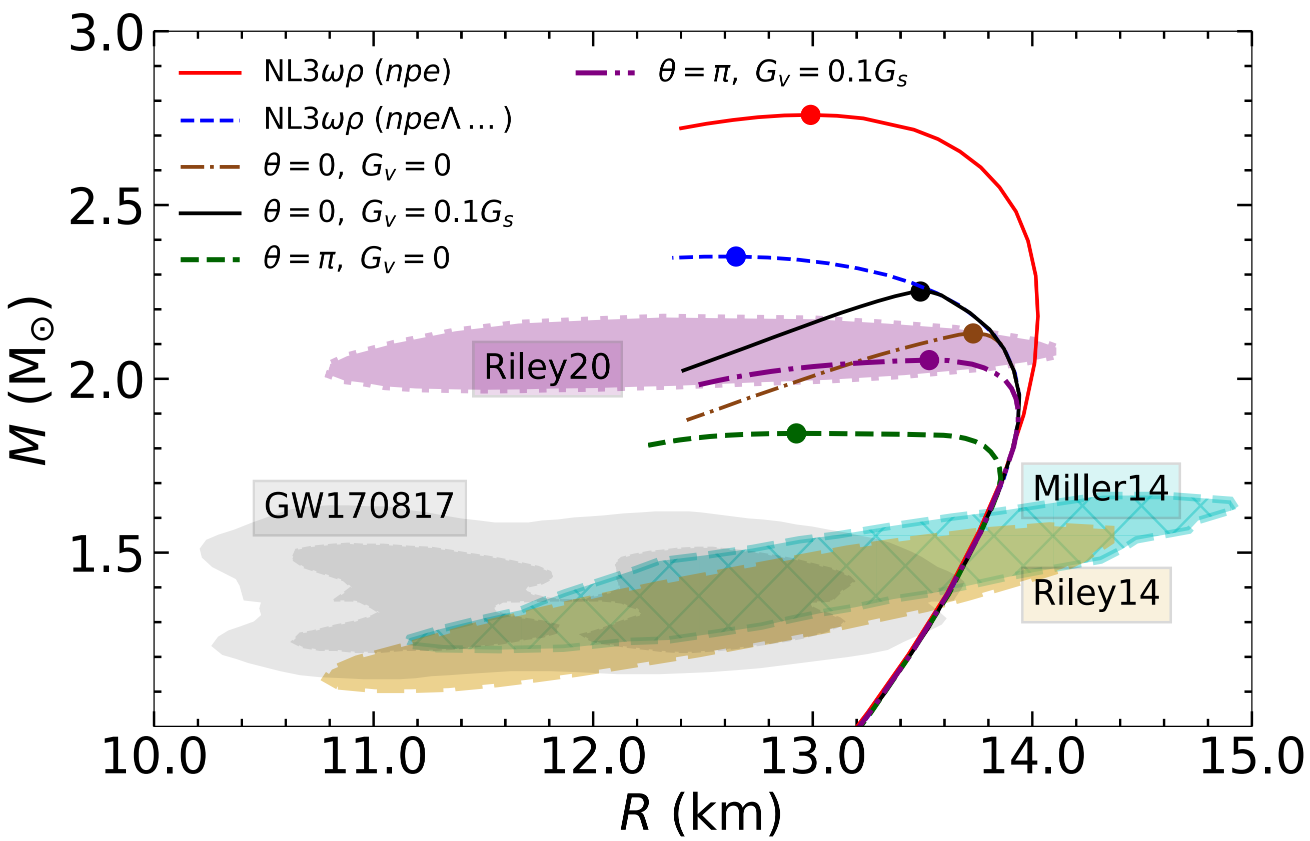


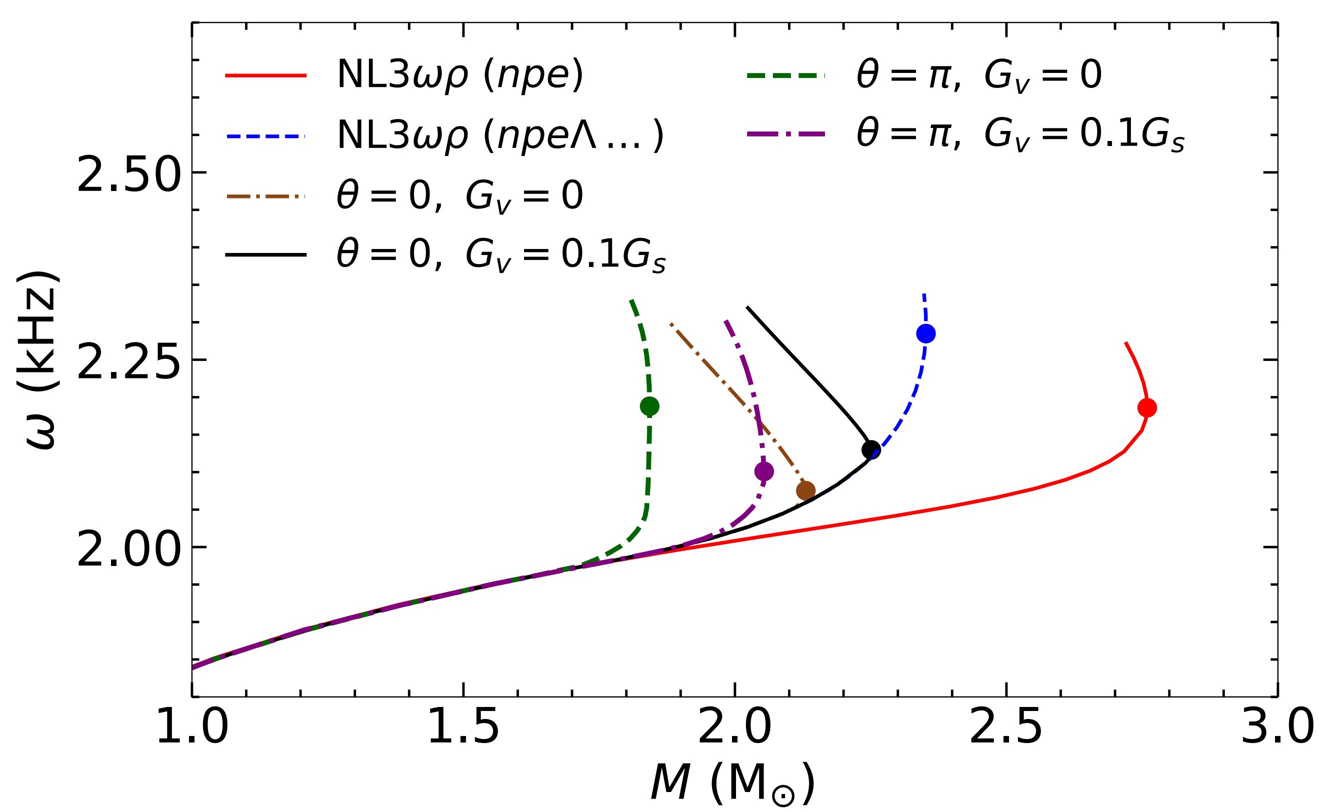


Equation of state (Hybrid matter)









Summary and Conclusions

- NSs are the exciting natural astrophysical laboratories to study the behaviour of matter at extreme densities.
- We discussed that the *f* mode non-radial oscillation frequencies are more sensitive to the low density part of equation of state.
- The presence of quark matter in the core of the neutron stars enhances the *f*-mode oscillation frequencies.
- The presence of axion enhances further the *f*-mode oscillation frequencies.
- In the future detectors like advanced LIGO/Virgo, Einstine telescope etc may give some light on the presence or absence of axions/dark matter in the core of neutron stars, if they are able to detect *f* mode of NS.

Thank you

Equation of state (NJL model)

- Thermodynamic potential and effective mass