Shear and Bulk viscosity in semi-QGP region

for pure glue theory

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for pure glue theory

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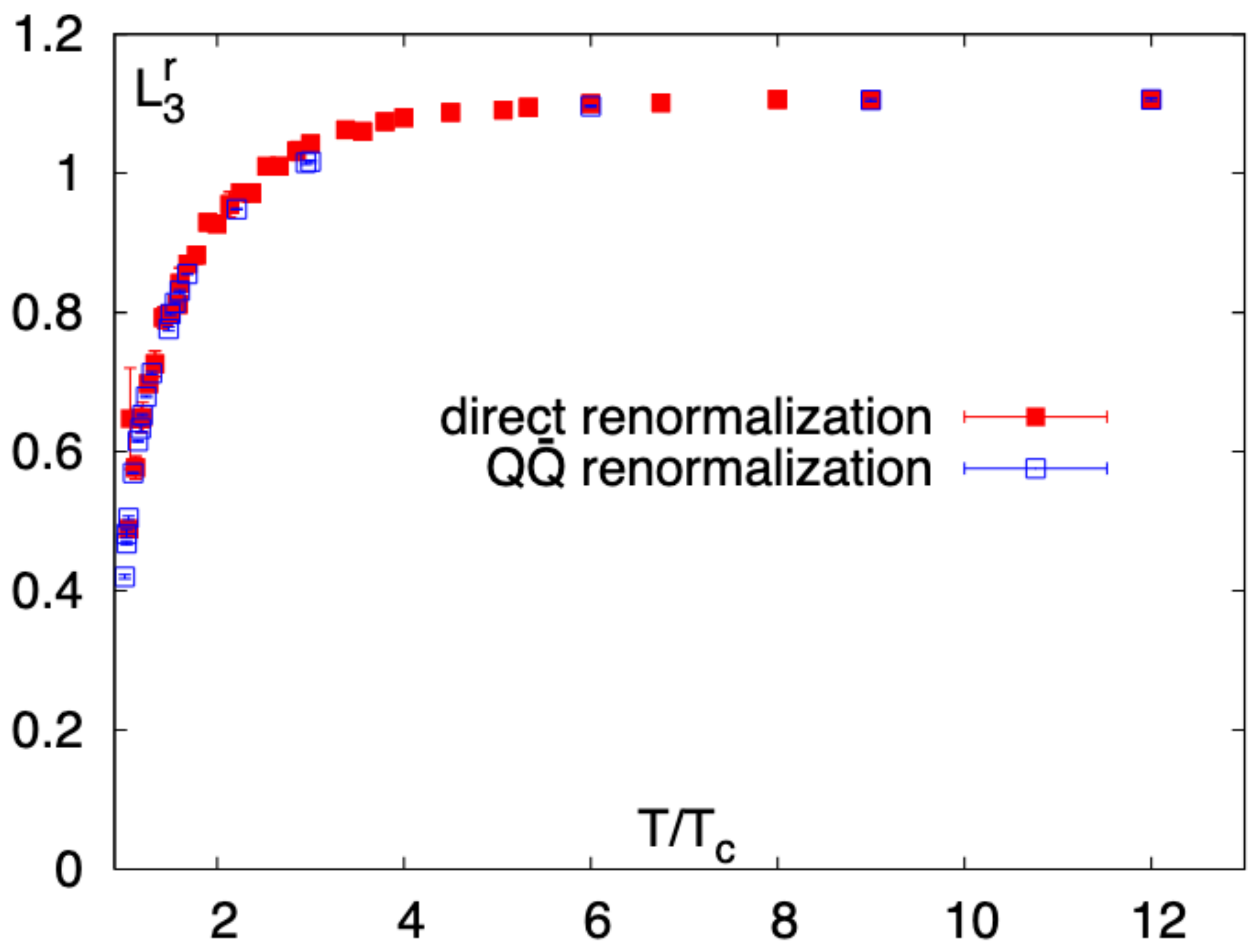
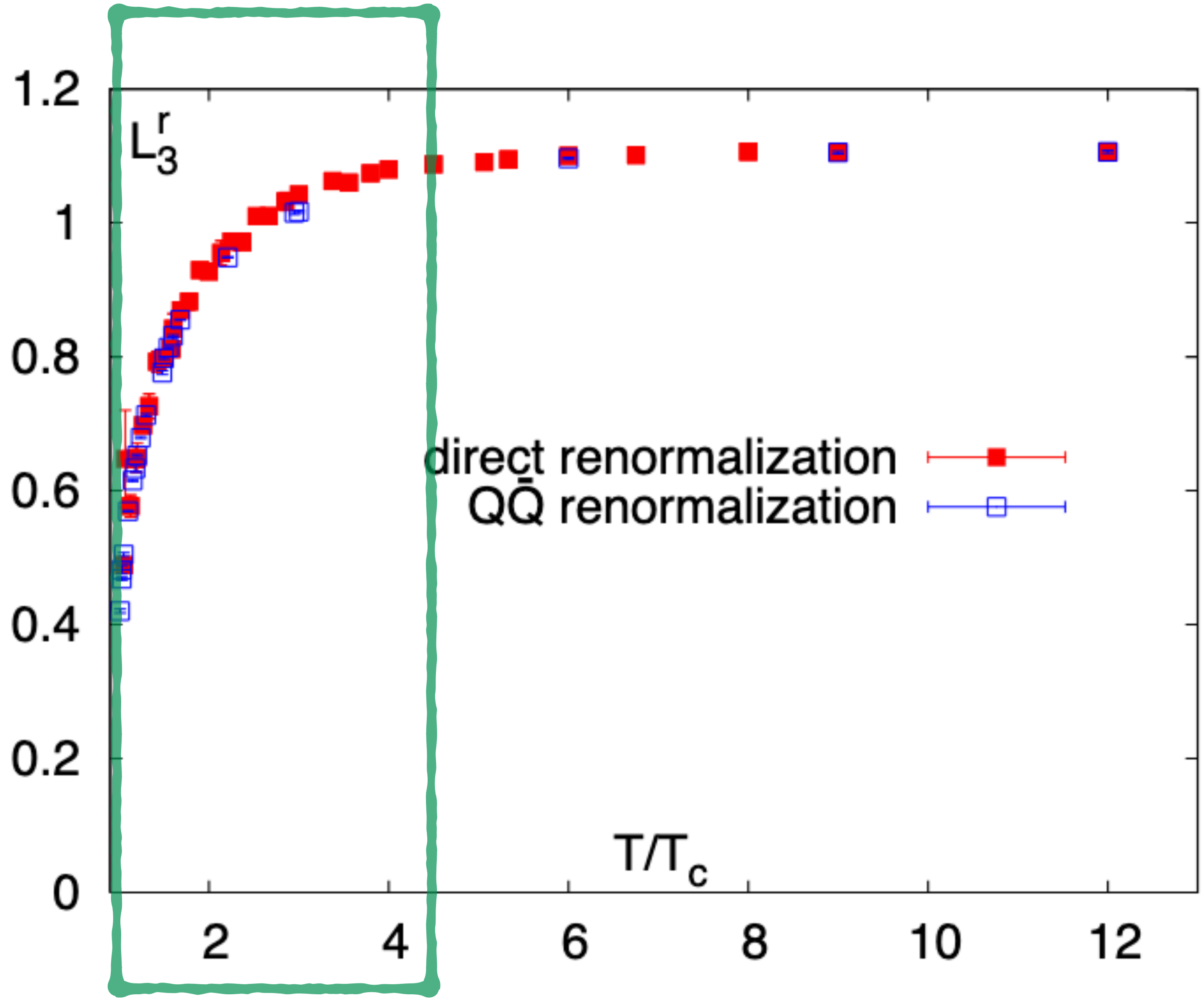
Yoshimasa

Semi-QGP region



SU(3) Polyakov loop.[Ref: Gupta, Hubner and Kaczmarek, 0711.2251]

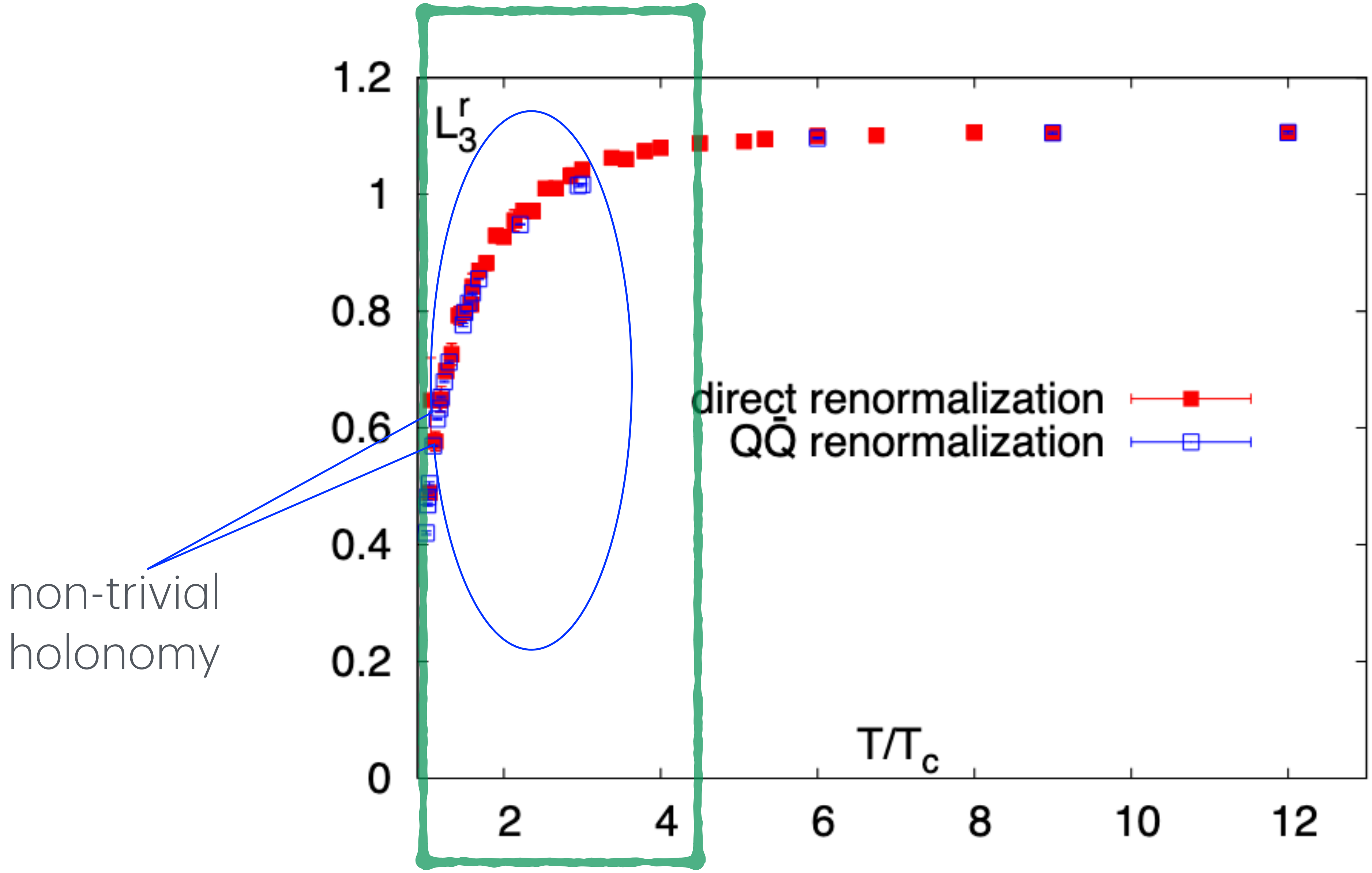
Semi-QGP region



SU(3) Polyakov loop.[Ref: Gupta, Hubner and Kaczmarek, 0711.2251]

1. Range: T_c to $4T_c$
2. $0.4 < \ell_1 < 1$ for $SU(3)$ gauge theory, without quarks. $\left(\ell_1 = \mathrm{Tr}L(\mathbf{x})\right)$

Semi-QGP region



SU(3) Polyakov loop.[Ref: Gupta, Hubner and Kaczmarek, 0711.2251]

To explain thermodynamics of this region of non-trivial holonomy, we need to add some potential due to non-zero holonomy or **Holonomous potential** to the free energy in the Semi-QGP region.

(our) Matrix Model

We can obtain the Free energy with non-trivial Holonomy in the semi-QGP region in our matrix model.

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- We add a background field to the gauge field. $A_{bg,\mu} = \frac{\mathbf{Q}}{g}\delta_{0,\mu} = \frac{2\pi T}{g}\mathbf{q}\,\delta_{0,\mu}$

$$L(\mathbf{x}) = \exp\left(\frac{2\pi i}{N_c}\mathbf{q}\right)$$

where $(\mathbf{q})_{ab} = q^a \delta_{ab}$ has $(N_c - 1)$ independent element, satisfying $\sum_{a=1}^{N_c} q^a = 0$

Ref: Dumitru, Guo, Hidaka, Altes, Pisarski; 1205:0137

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where $(\mathbf{q})_{ab} = q^a \delta_{ab}$ has $(N_c - 1)$ independent element, satisfying $\sum_{a=1}^{N_c} q^a = 0$

Hence the total field, $A_\mu = A_{bg,\mu} + B_\mu$

(B_μ is quantum gauge field, that is considered in the perturbative study.)

Ref: Dumitru, Guo, Hidaka, Altes, Pisarski; 1205:0137

(our) Matrix Model

Free energy computation to Leading order:

$$V_{pert} \sim \sum_{a,b=1}^{N_c} \mathscr{P}^{ab,ab} T^4 \left(|q^{ab}|^2 (1 - |q^{ab}|)^2 - \frac{1}{30} \right)$$

$$\mathscr{P}^{ab,cd} = \delta^{ac}\delta^{bd} - \frac{1}{N_c}\delta^{ab}\delta^{cd} \qquad q^{ab} = \mathrm{mod}(q^a - q^b, 1)$$

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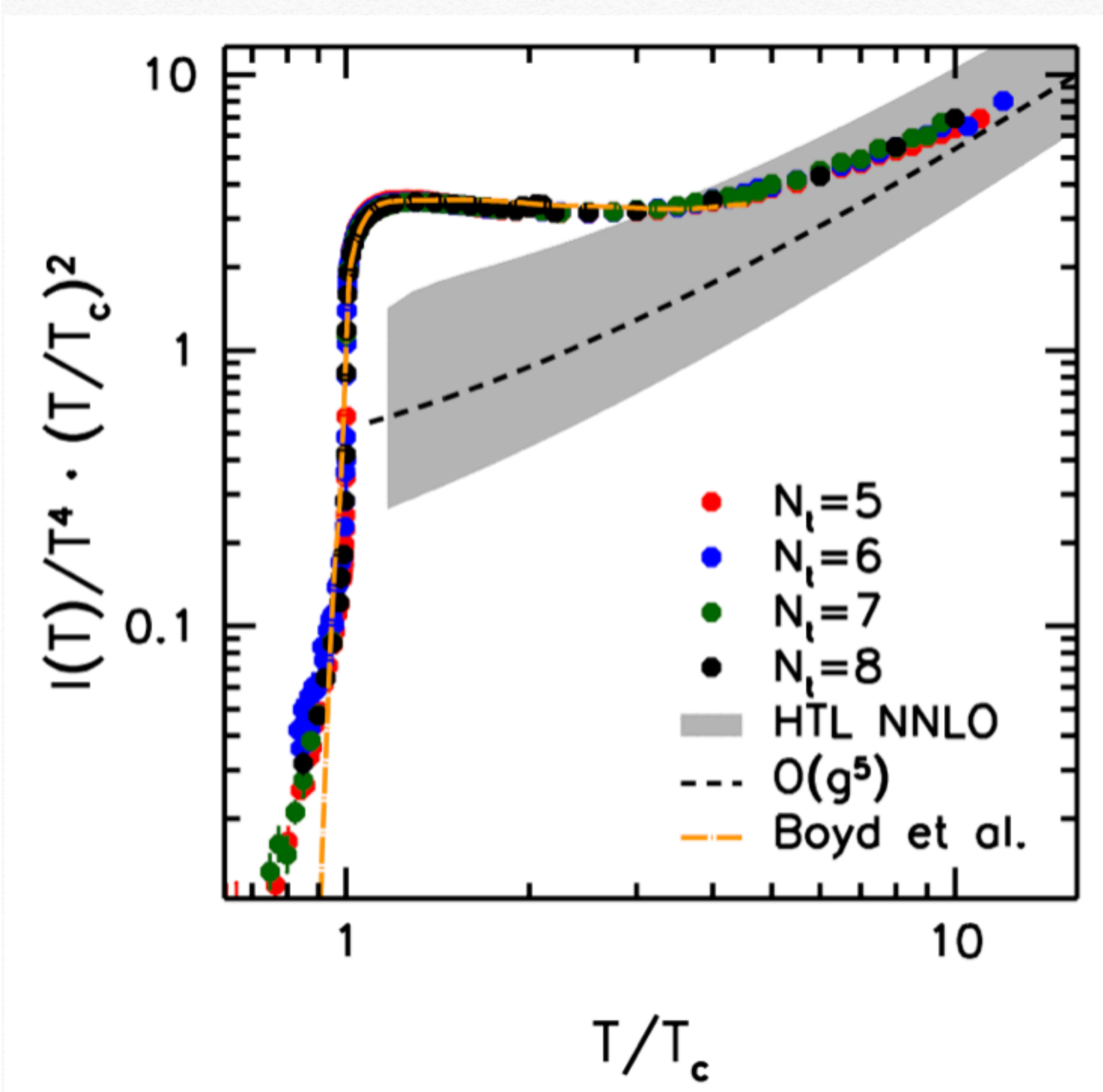
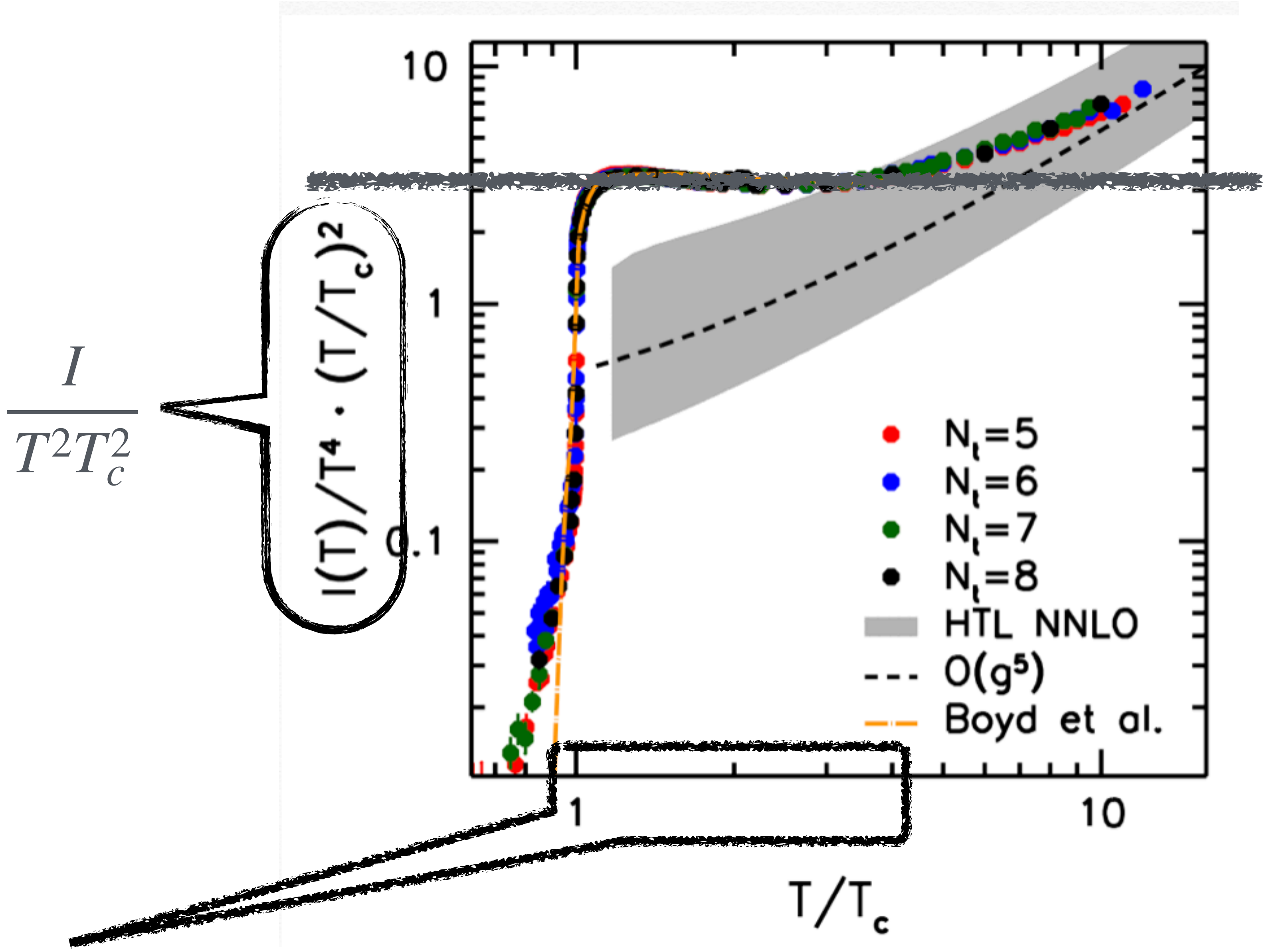
This is not enough!

(our) Matrix Model



Interaction Measure. [Ref: Borsanyi, Endrodi, Fodor, Katz, Szabo, 1204.6184]

(our) Matrix Model



$1.2\,T_c \to 4\,T_c$

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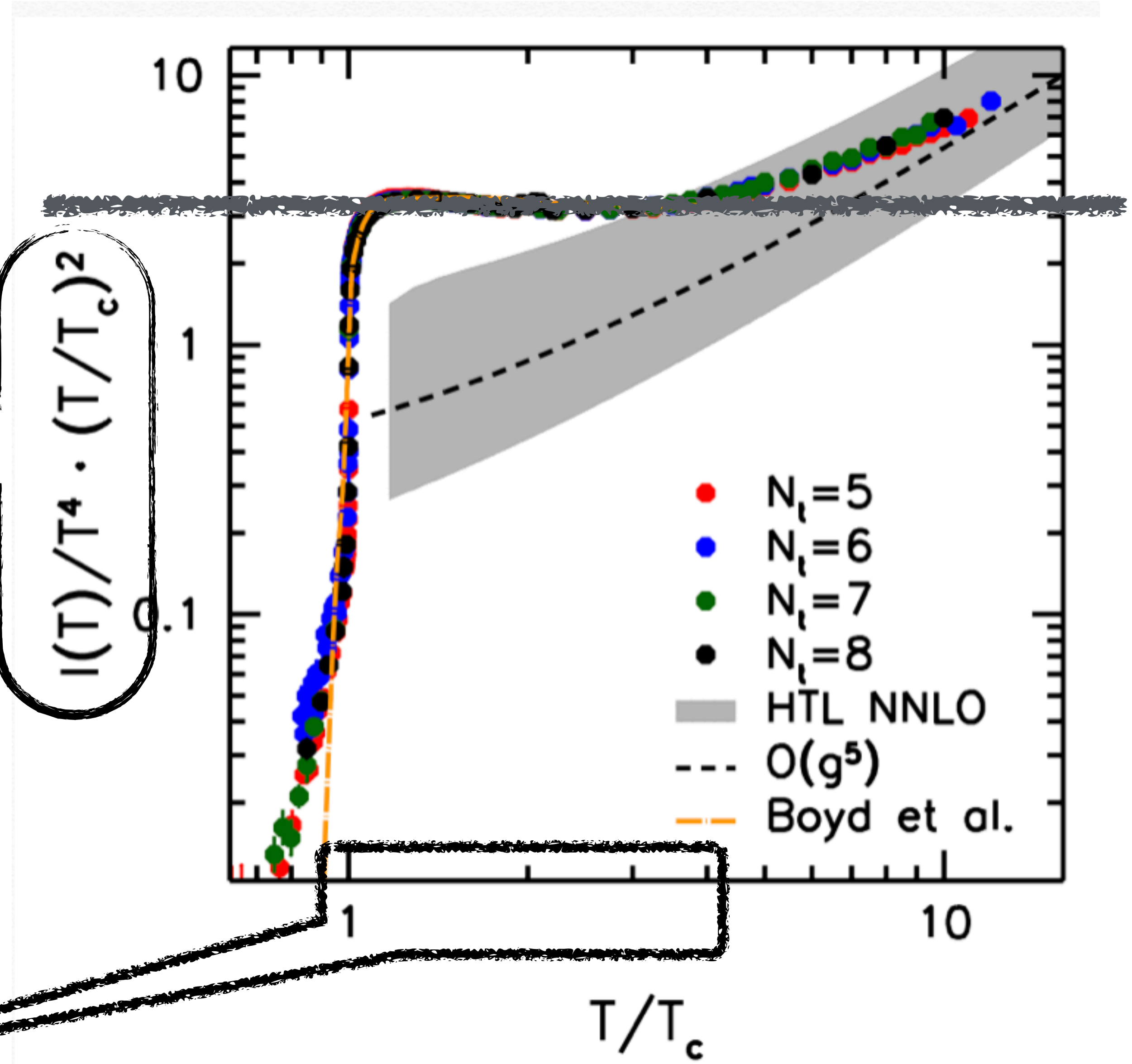
(our) Matrix Model



$\frac{I}{T^2 T_c^2}$

$1.2\,T_c \to 4\,T_c$

Interaction Measure. [Ref: Borsanyi, Endrodi, Fodor, Katz, Szabo, 1204.6184]

In $1.2\,T_c \to 4\,T_c$, leading correction to pressure in pure gauge theory, behaves $\sim T^2$. Not exact, but approximately.

(our) Matrix Model

$$V_{pert} \sim \sum_{a,b=1}^{N_c} \mathscr{P}^{ab,ab} T^4 \left(|q^{ab}|^2 (1 - |q^{ab}|)^2 - \frac{1}{30} \right)$$

In addition to the above Free energy, we need a non-perturbative term to the free energy that behaves $\sim T^2$.

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So, we need such a term, $V_{non-pert} \sim \sum_{a,b} \mathscr{P}^{ab,ab} \, C T^2 \left(-q^{ab}(1 - q^{ab}) + \frac{1}{6} \right)$

(our) Matrix Model- Teen Field (ও)

- Pisarski and Hidaka suggested that, we can consider a dynamical field that can generate non-perturbative term in the Free energy. [Hidaka, Pisarski, arXiv: 2009.03903]
- It has to be Two dimensional in nature, that is, its momentum is suppressed in two spatial directions to give $\sim T^2$ contribution in the free energy.
- It can be represented in adjoint representation and it is a ghost field.

We named it "Teen" field (notation for later use: ও)

(our) Matrix Model- Teen Field (ত)

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ত (Teen) field propagator in momentum space,

$$\Delta_{ত}^{ab}(p_0^{ab}, \mathbf{p}) = \frac{1}{(p_0^{ab})^2 + p_{||}^2 + p_{\perp}^2}$$

where $p_{\perp} \leq T_c$ and $T_c \ll gT \ll T$.

Kinetic theory and Transport Coefficient

We use Kinetic theory in the semi-QGP region, that is when $q^{ab} \neq 0$ for gluons and teens.

$$\mathcal{S}_{\text{ব}}\, 2P^{\mu,a_1b_1}\partial_\mu f_{\text{ব},a_1b_1}(\mathbf{x},\mathbf{p},t) = -\mathcal{C}_{\text{ব},a_1b_1}[f_{\text{দ}}]$$

ব= gluon, teen and $\mathcal{S}_{\text{gluon}} = 1,\ \mathcal{S}_{\text{teen}} = -1$

Energy-Momentum Tensor: $T^{\mu\nu}_{\text{g}}(\boldsymbol{r},t) = 2\int_{\text{g}} d\Gamma_{ab}\, P^{\mu,ab}P^{\nu,ab}\, f_{\text{g},ab}(\boldsymbol{r},\boldsymbol{p},t)$

$$T^{\mu\nu}_{\text{৩}} = 2\int_{\text{৩}} d\Gamma_{ab}\, P^{\mu,ab}P^{\nu,ab}\, \mathcal{S}_{\text{৩}}\, f_{\text{৩},ab}\,.$$

$$f^0_{\text{g},ab} = f^0_{ab}(E_p) \equiv \frac{1}{e^{(E_p - i(Q^a - Q^b))/T} - 1}, \quad f^0_{\text{৩},ab}(E_p) = \pm f^0_{\text{g},ab}(E_p)$$

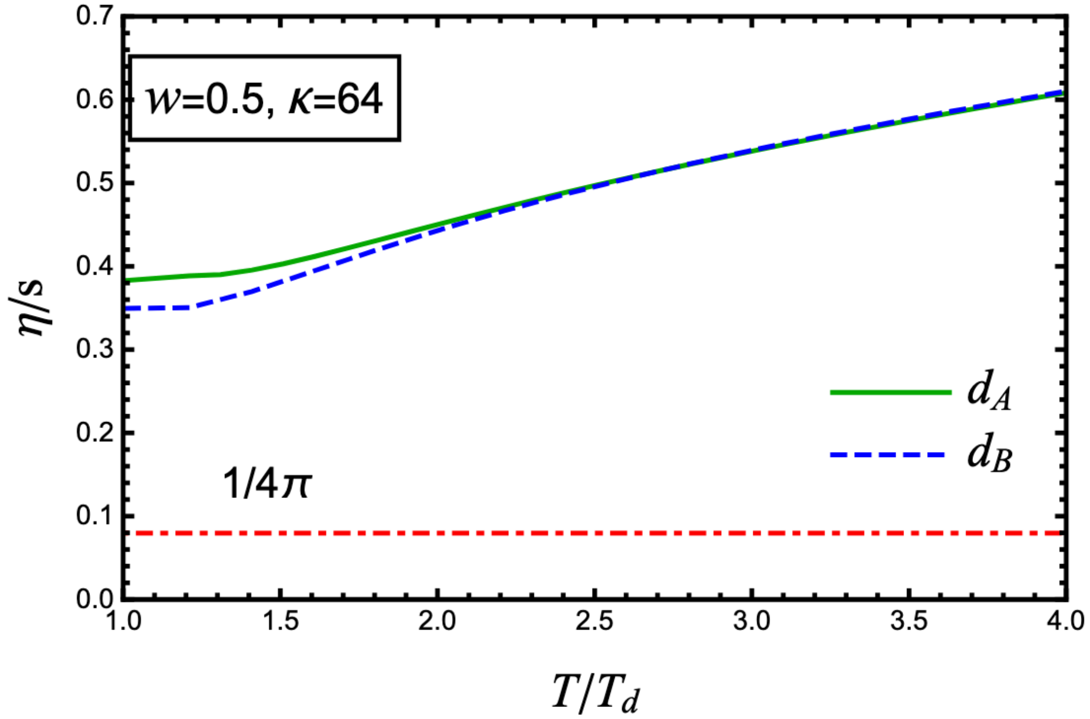
Since, $T^{\mu\nu} = \mathscr{E}u^\mu u^\nu + \mathscr{P}\Delta^{\mu\nu} + \Pi^{\mu\nu}$

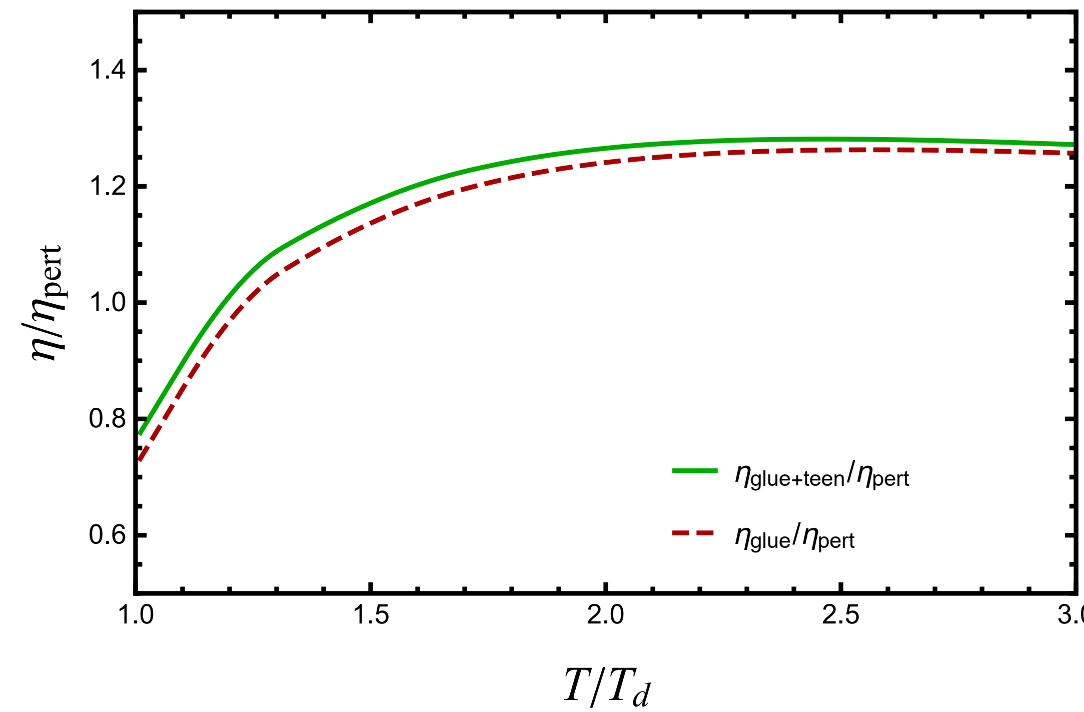
$$\Pi^{\mu\nu} = \eta\,\sigma^{\mu\nu} + \zeta\,\Delta^{\mu\nu}\Delta^{\alpha\beta}\partial_\alpha u_\beta + \cdots$$

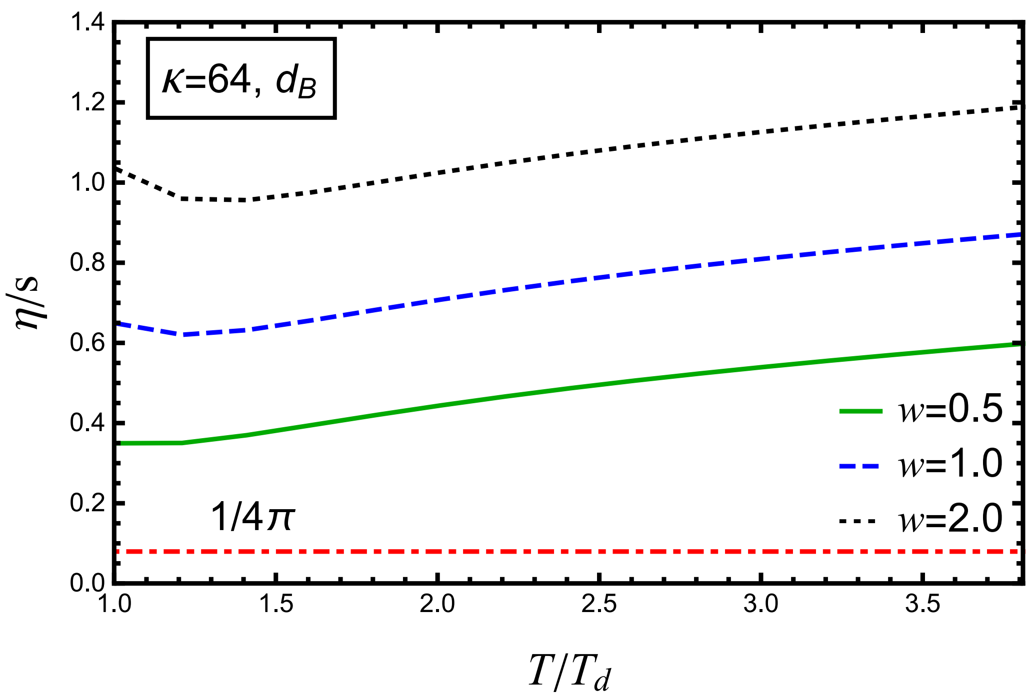
Result- Shear Viscosity

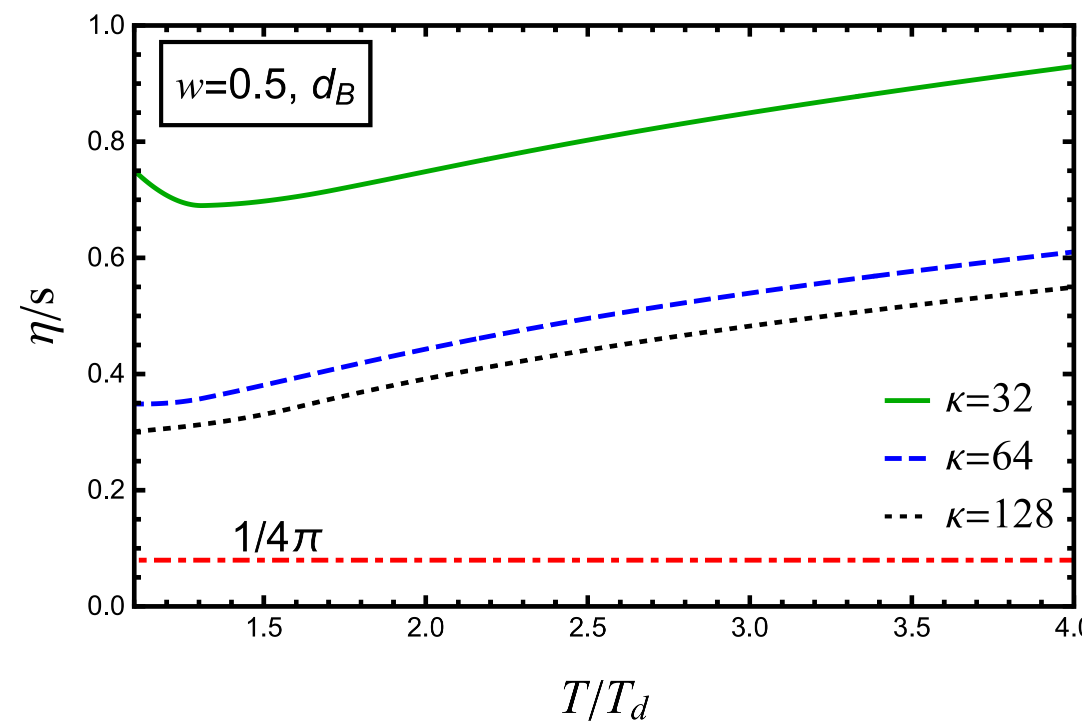
MD, R Ghosh, N Haque, Y Hidaka, R Pisarski
(in preparation)

$$\eta = \frac{T^3}{g^4 \log(\kappa/g^2 N_c)} F(T/T_d)$$









Result- Shear Viscosity

MD, R Ghosh, N Haque, Y Hidaka, R Pisarski
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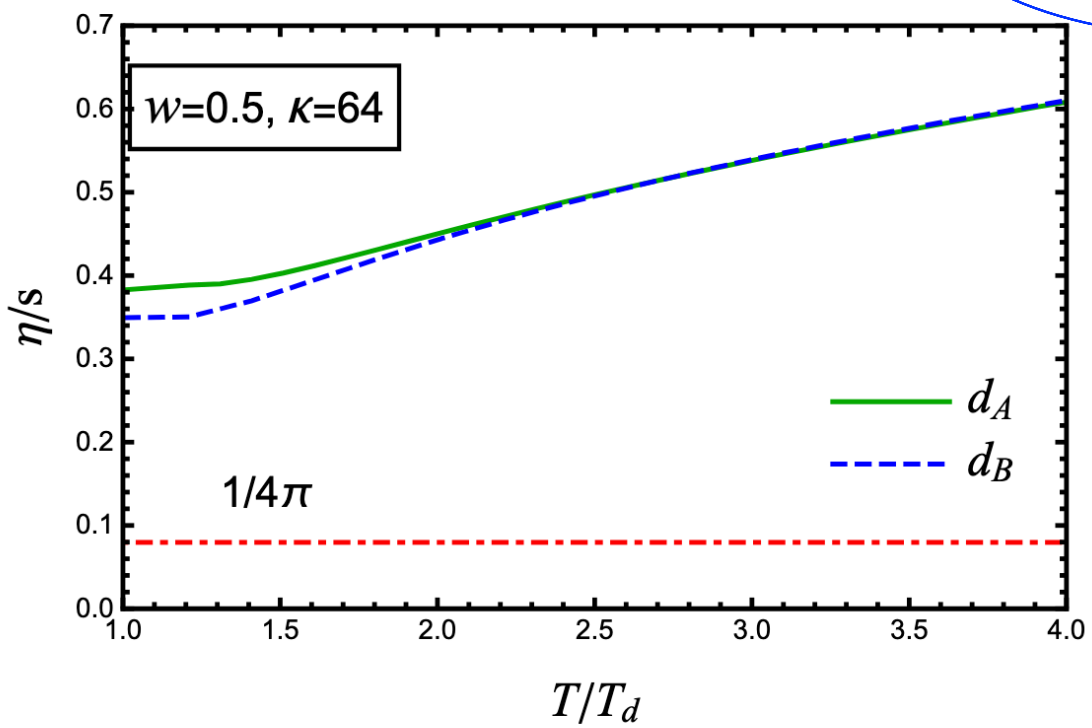
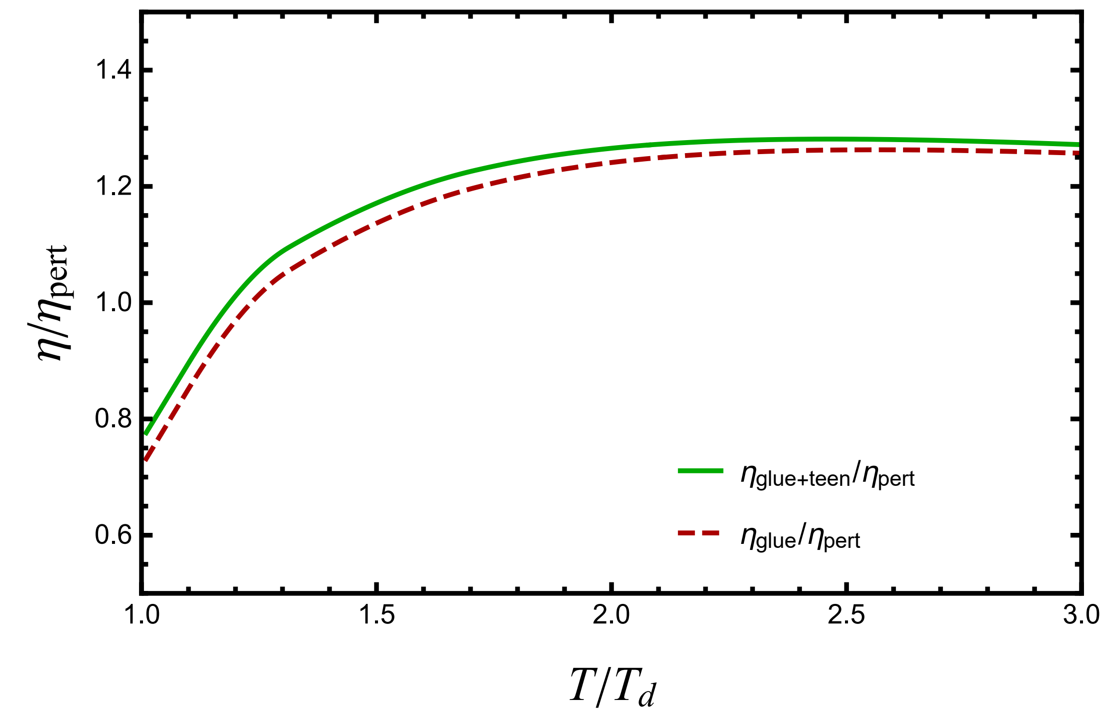
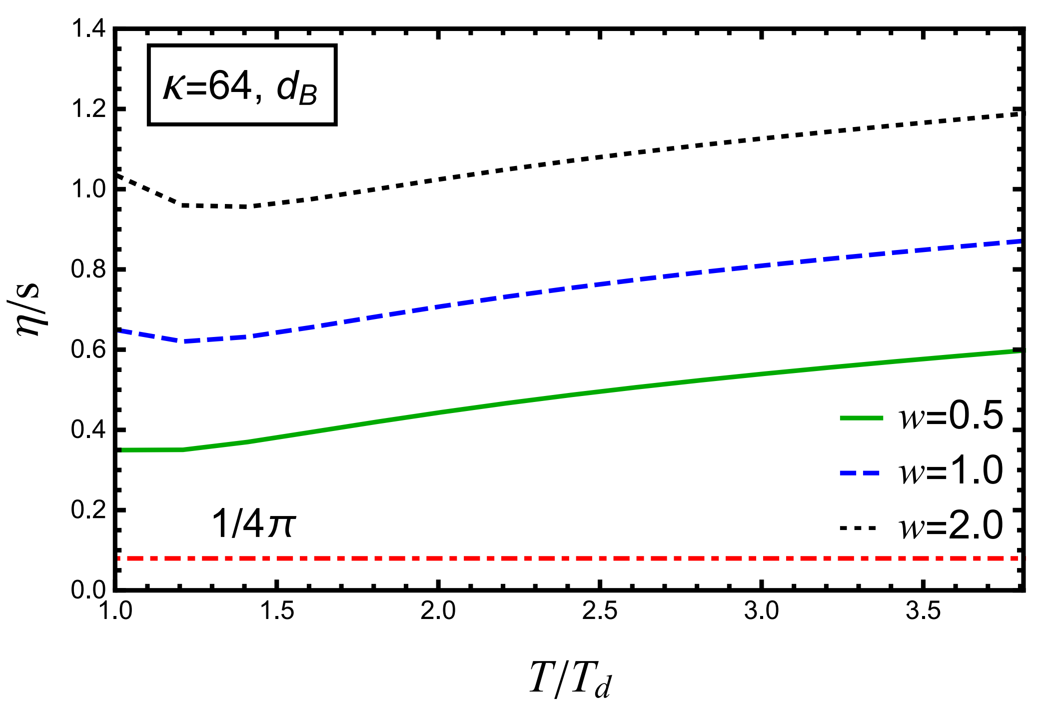
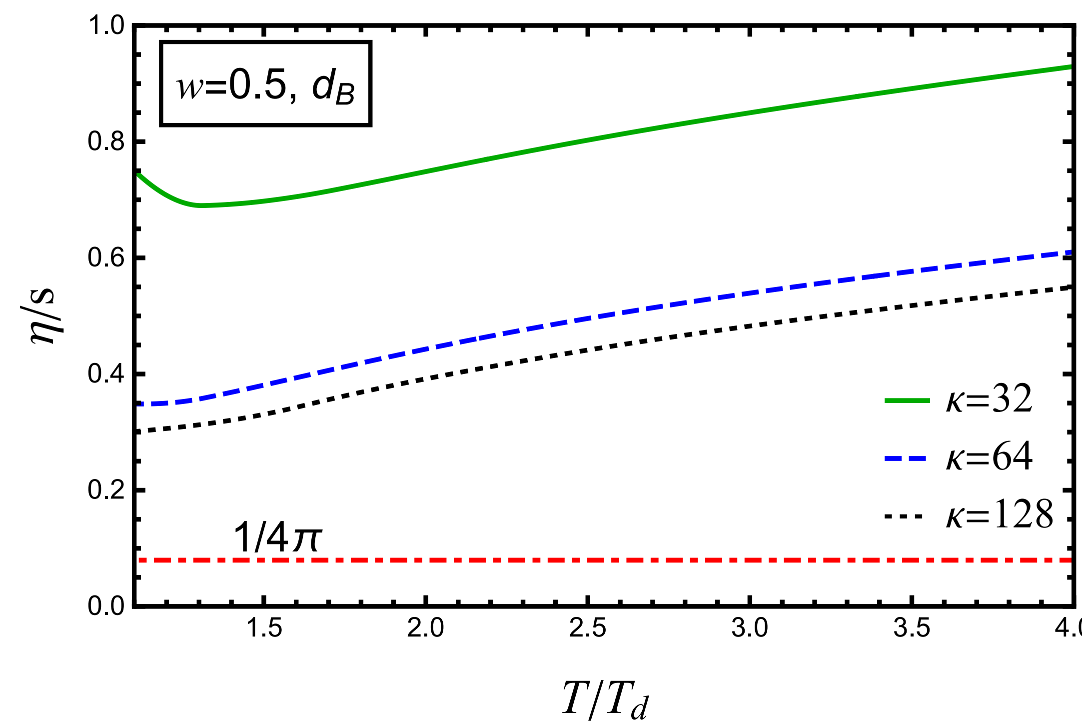
Appears in perturbative computation

$$\eta = \frac{T^3}{g^4 \log(\kappa/g^2 N_c)} F(T/T_d)$$





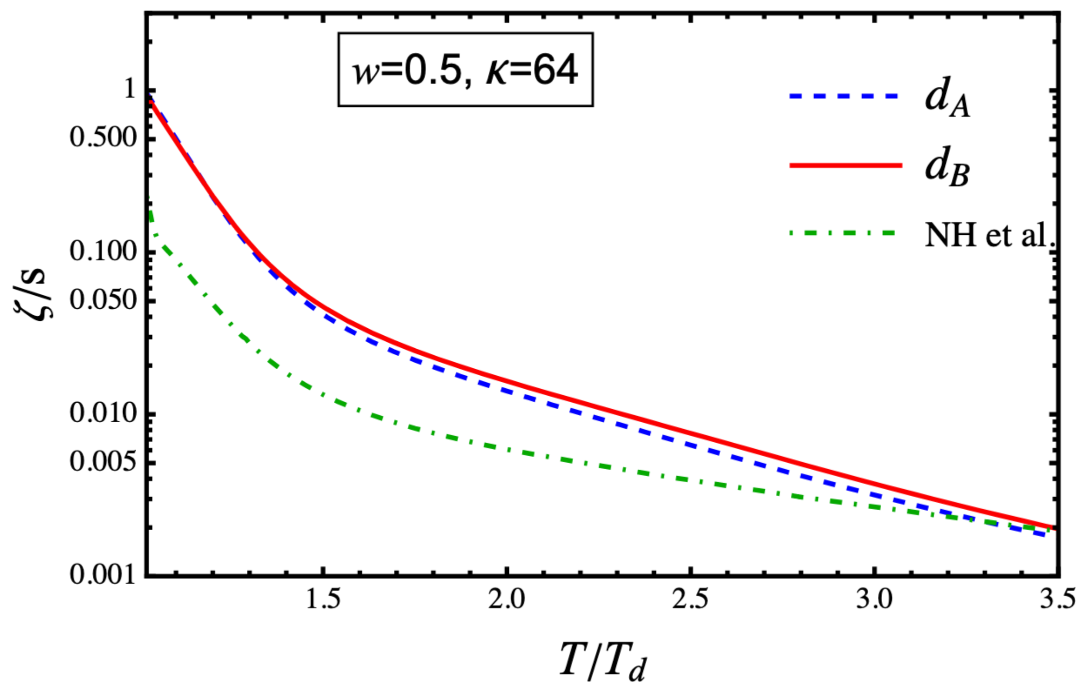


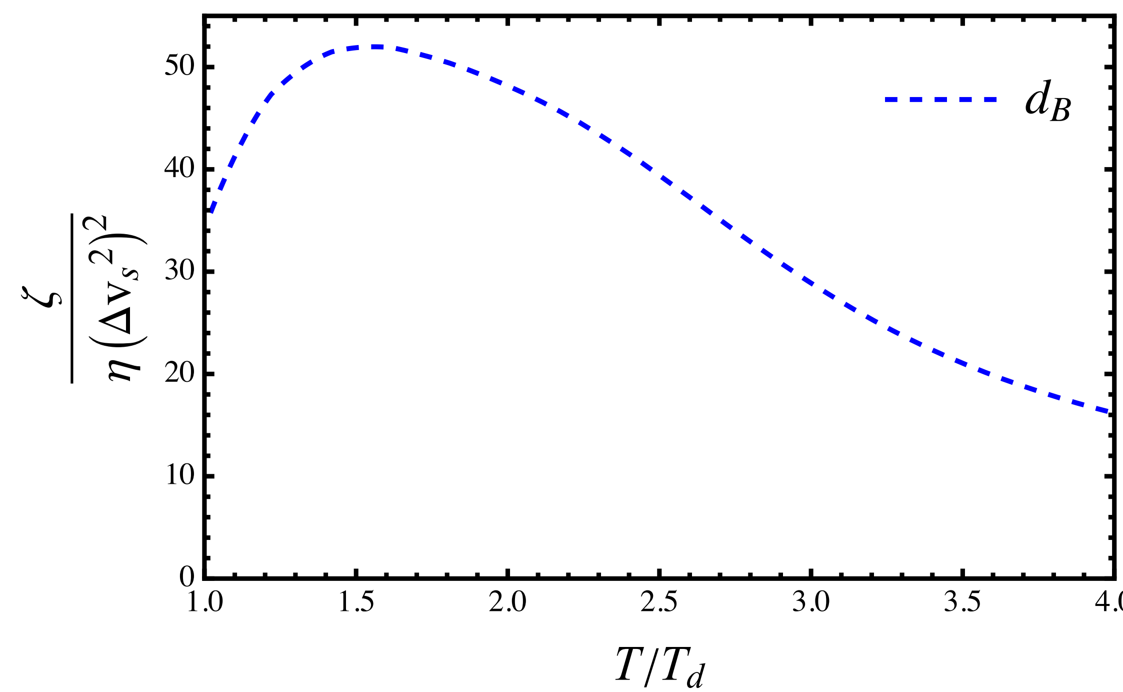


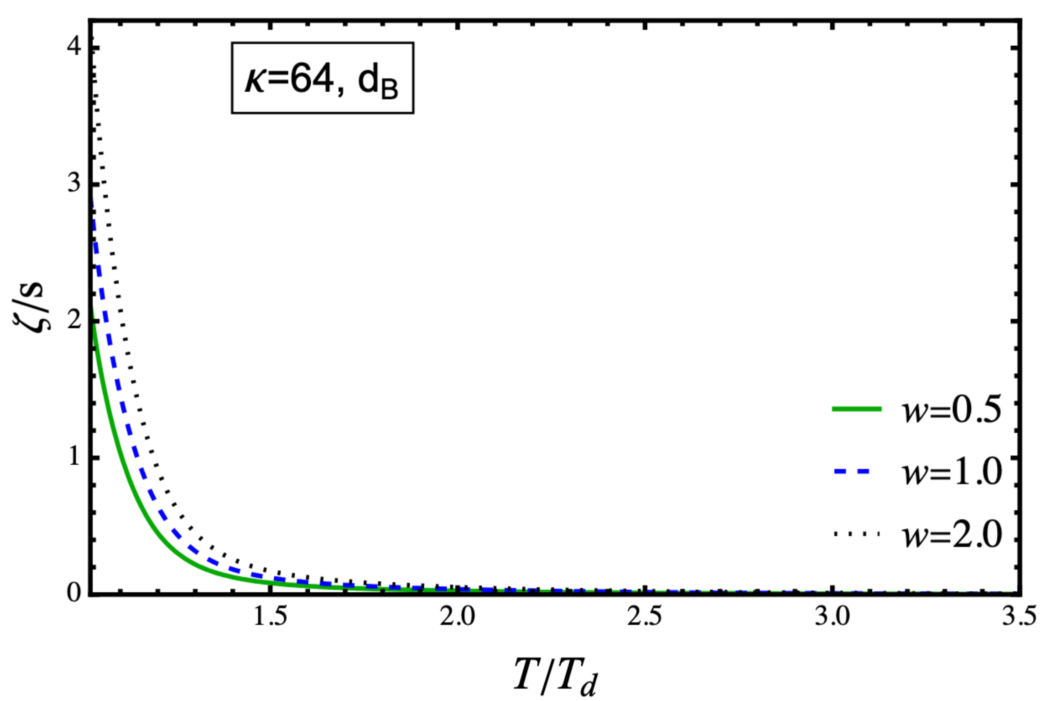
Result- Bulk Viscosity

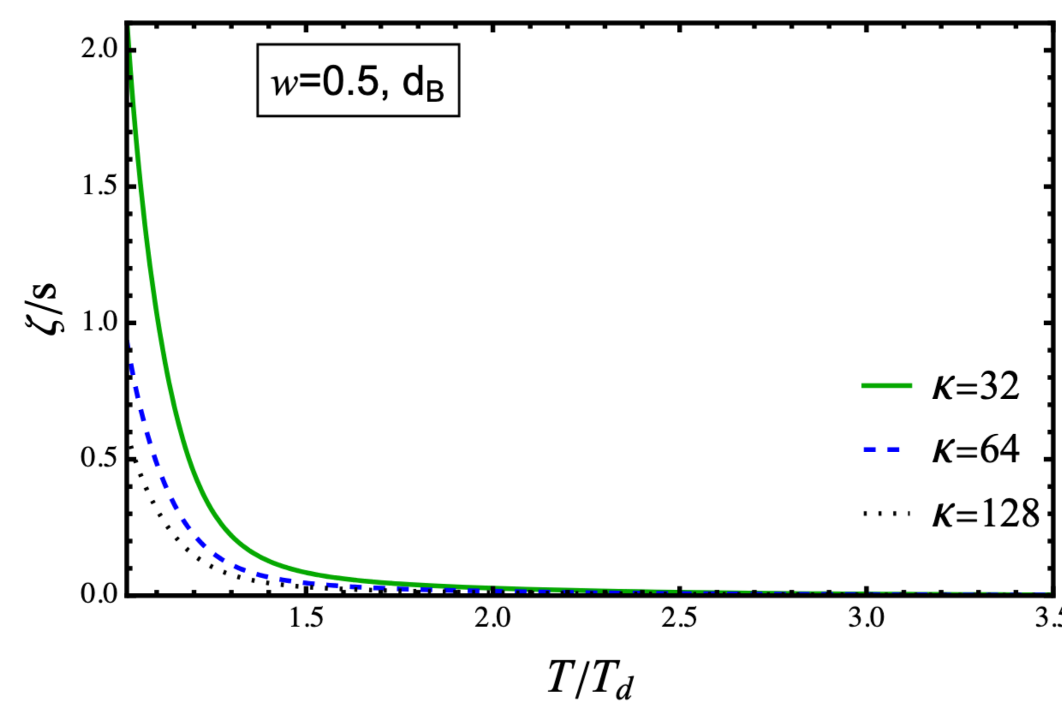
MD, R Ghosh, N Haque, Y Hidaka, R Pisarski
(in preparation)

$$\zeta = \frac{T^3(\Delta v_s^2)^2}{g^4 \log(\kappa/g^2 N_c)} G(T/T_d)$$









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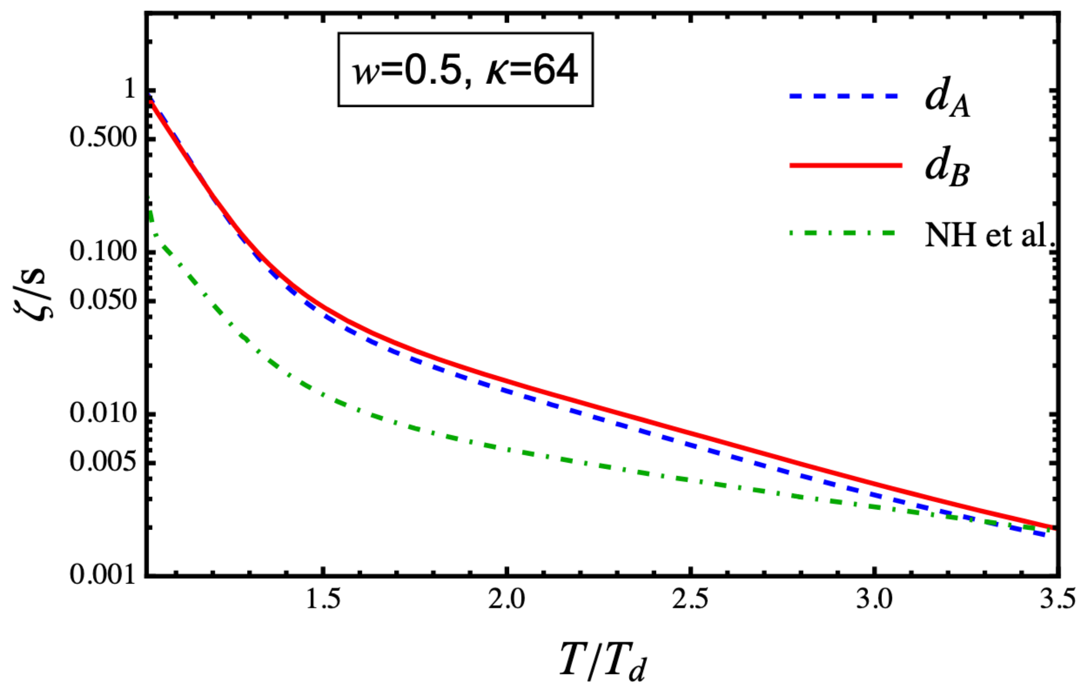
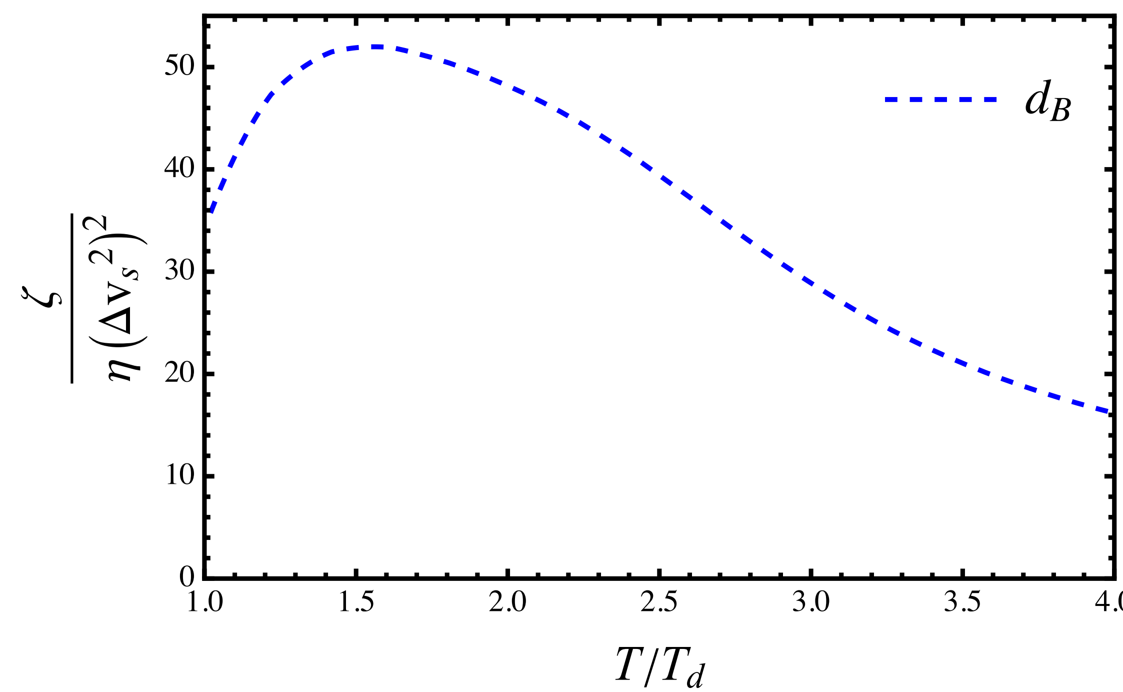
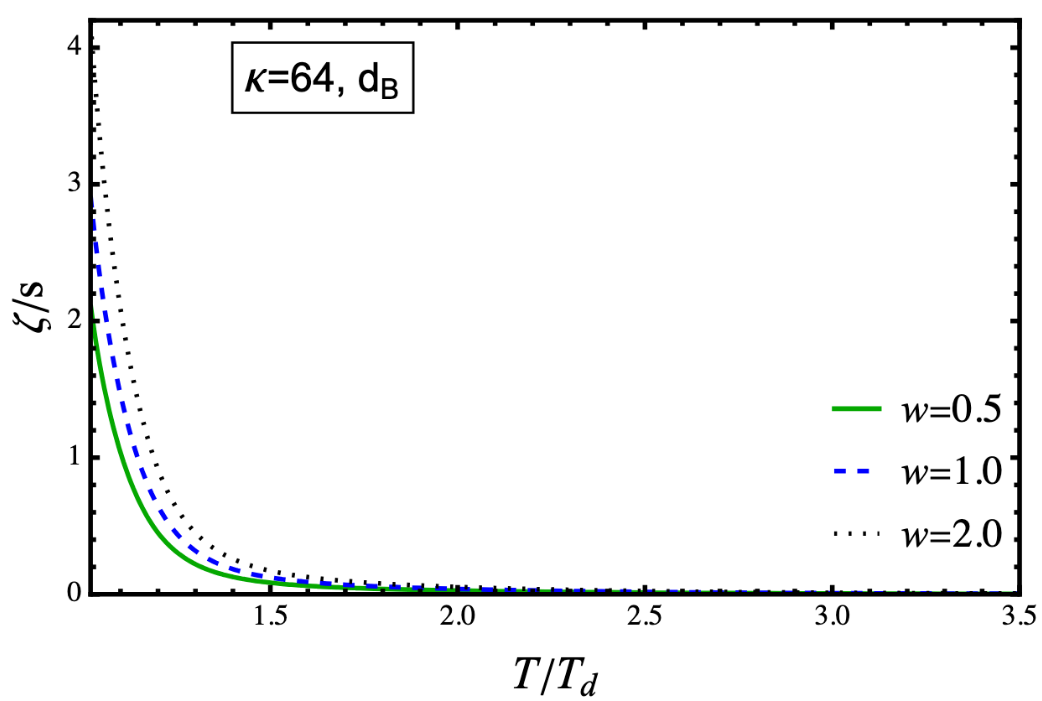
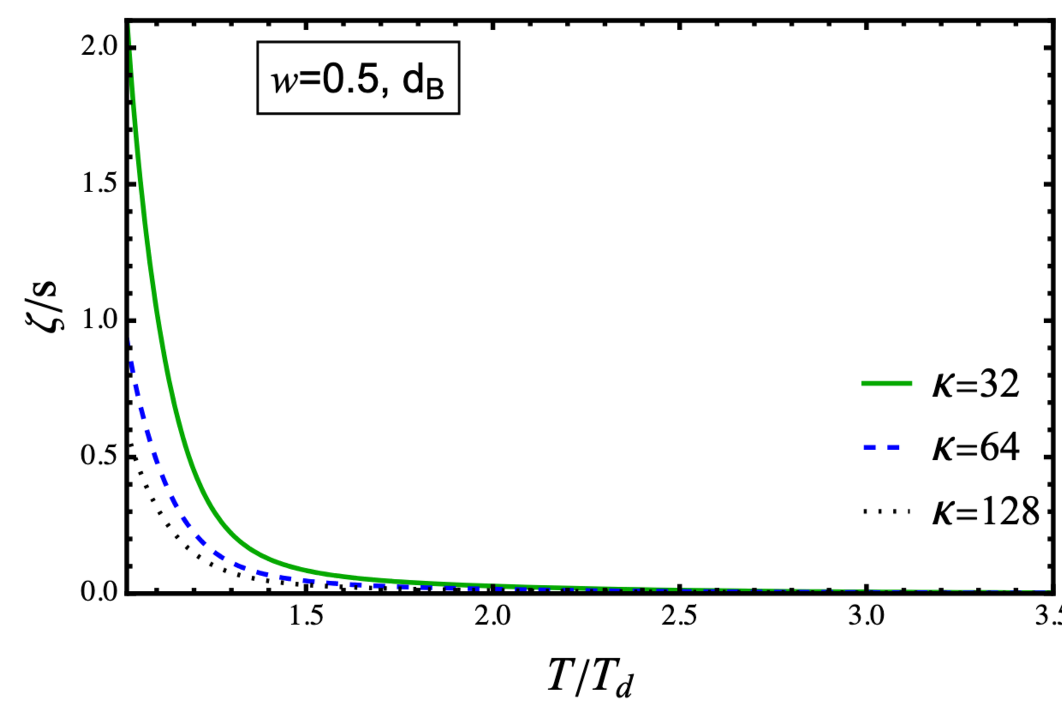
$$\zeta = \frac{T^3(\Delta v_s^2)^2}{g^4 \log(\kappa/g^2 N_c)} G(T/T_d)$$

Does not appear in perturbative study









Conclusion (so far):

- Teen field can explain the $\sim T^2$ behaviour of pressure near the transition region.

- η/s is still larger than the AdS/CFT bound even at transition temperature T_c. With Quarks, Polyakov loop value is smaller at T_{χ}, so we η/s will be smaller. We want to compare our result with recent lattice data of η/s from Altenkort et. al. (2211.08230).

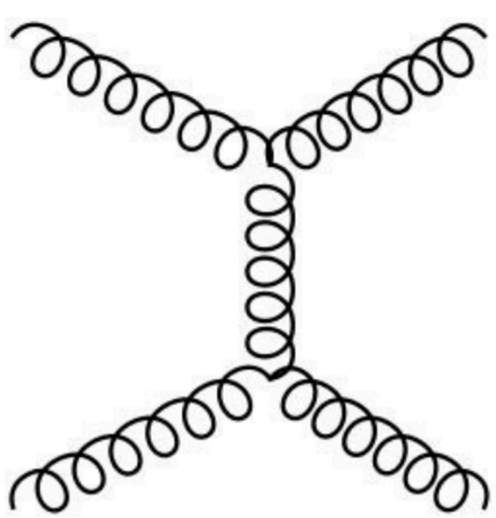
Thank You for your attention

Back-up Slide

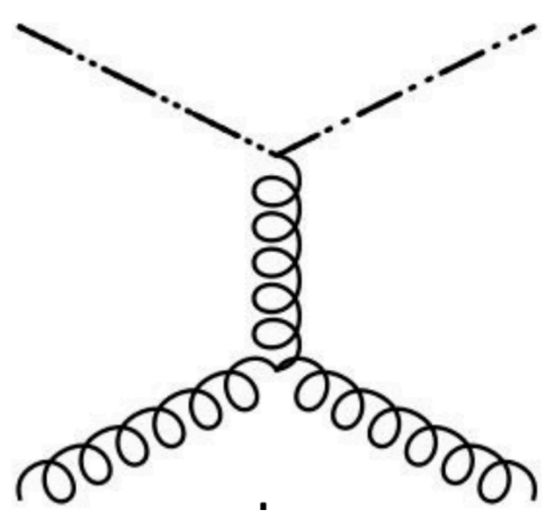
Collision Kernel:

$$\mathcal{C}_{\text{ব},a_1b_1}[f_{\text{দ}}] = \frac{1}{2}\sum_{\text{দ},\text{ব}',\text{দ}'} \mathcal{S}_{\text{ব}}\mathcal{S}_{\text{দ}} \int_{\text{দ}} d\Gamma_{a_2b_2} \int_{\text{ব}'} d\Gamma_{a_3b_3} \int_{\text{দ}'} d\Gamma_{a_4b_4}\, (2\pi)^4\delta^4(P_1+P_2-P_3-P_4)|\mathcal{M}_{\text{বদ}\to\text{ব}'\text{দ}'}|^2$$
$$\times\Big[f_{\text{ব},a_1b_1} f_{\text{দ},a_2b_2}(1+f_{\text{ব}',a_3b_3})(1+f_{\text{দ}',a_4b_4}) - f_{\text{ব}',a_3b_3} f_{\text{দ}',a_4b_4}(1+f_{\text{ব},a_1b_1})(1+f_{\text{দ},a_2b_2})\Big]\,.$$

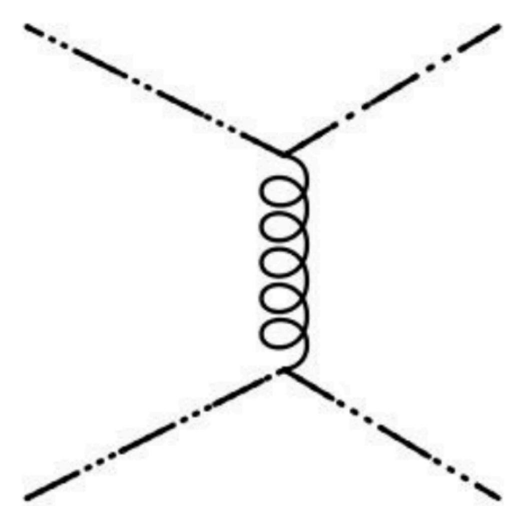
Contributing diagrams (at tree level):



gluon-gluon



gluon-teen



teen-teen

Integration Measure:

For Integration over gluon momenta:

$$\int_{\mathrm{g}} d\Gamma_{ab} = \sum_{s} \sum_{a,b=1}^{N_c} \mathscr{P}^{ab,ba} \int \frac{d^3p}{(2\pi)^3 2E_p}$$

For Integration over teen(ს) momenta:

$$\int_{2\mathrm{D}} d\Gamma_{ab} = \sum_{s} \sum_{a,b=1}^{N_c} \mathscr{P}^{ab,ba} \int \frac{d^3p}{(2\pi)^3 2E_p} = \sum_{s} \sum_{a,b=1}^{N_c} \mathscr{P}^{ab,ba} \int_0^{T_d} d^2p_\perp \int \frac{dp_\| d\Omega}{(2\pi)^3 2p_\|}$$

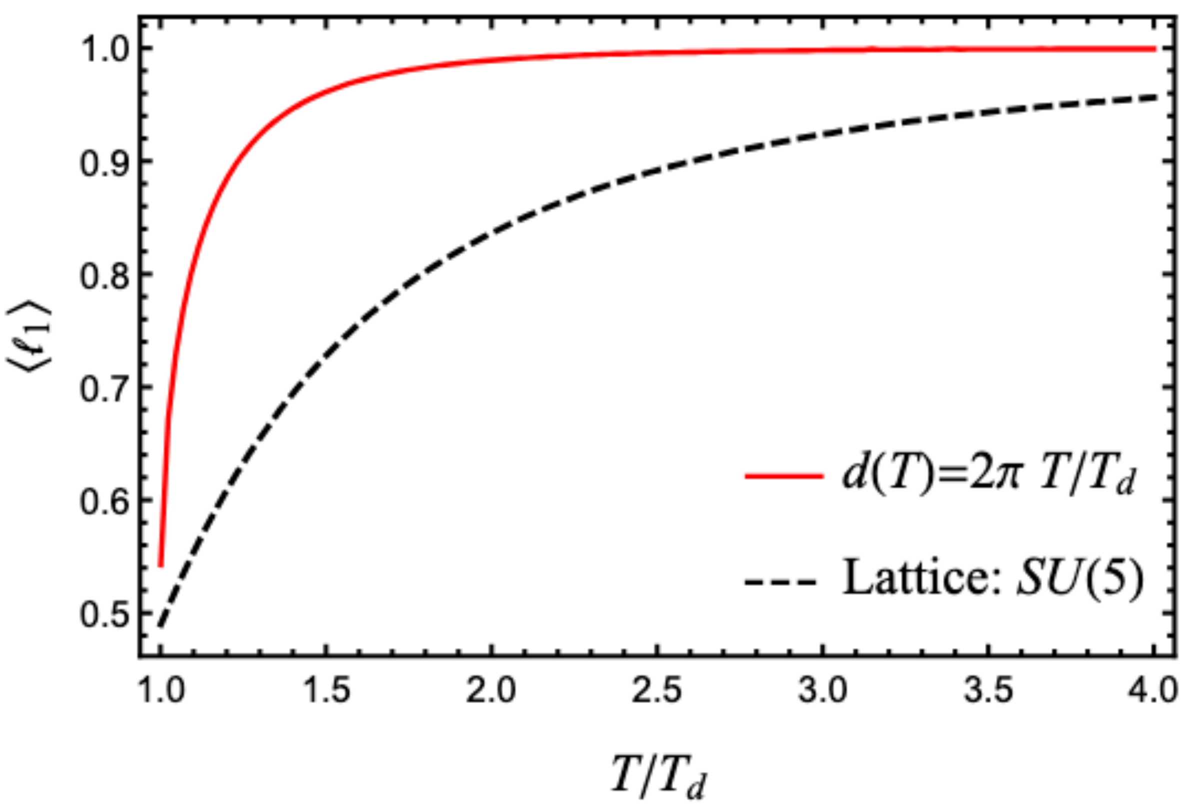
$$= \sum_{s} \sum_{a,b=1}^{N_c} \mathscr{P}^{ab,ba} \frac{T_d^2}{2} \int \frac{dp_\| d\Omega}{(2\pi)^3 2p_\|}$$

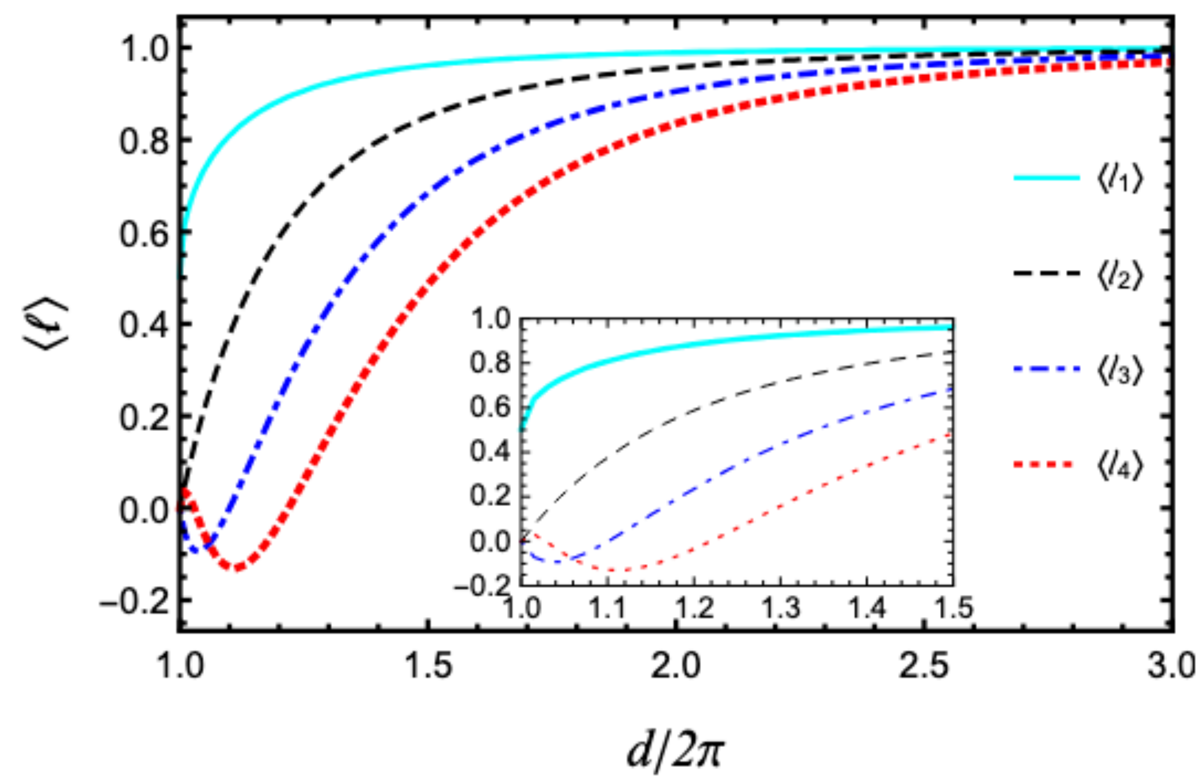
Back-up Slide

Loops!!

For the simplest ansatz, $d(T) = 2\pi T/T_d$, Polyakov loop at kth order,

$$\ell_k = \int_{-q_0}^{+q_0} dq\, \rho(q) \cos(2\pi k q)$$





Back-up Slide

Loops!!

We tried another few ansatzs, but two best choice were,

$$d_A(T) = 1.08\,t + 5.2032$$
$$d_B(T) = \frac{0.26}{t^3} + 1.105\,t + 4.9182\ ,\ t = \frac{T}{T_d}$$