Using Multiparticle Cumulants to Characterize the Initial State in XeXe at 5.44 TeV and PbPb at 5.36 TeV in CMS

15th January, 2025

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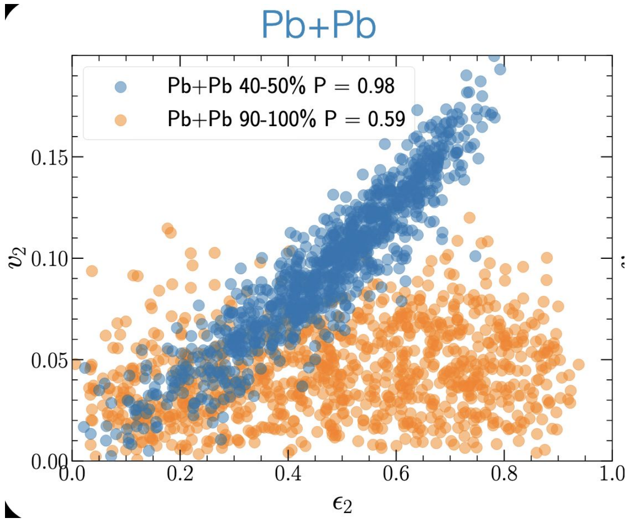
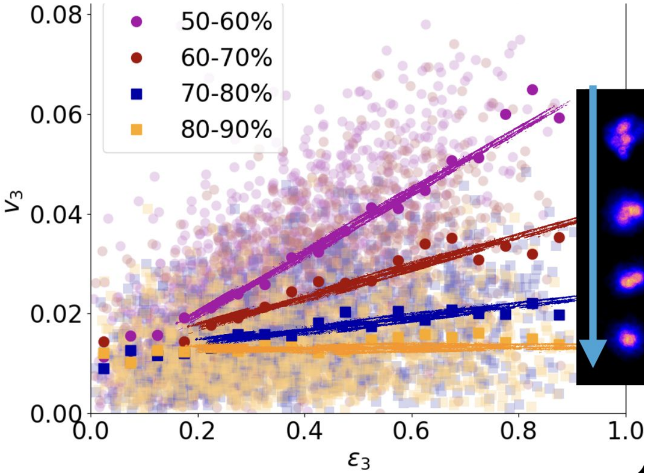
Probing the Initial State through Anisotropic Flow

- Naive expectation :
  - $v_n \propto \varepsilon_n$ for n = 2,3 [Ref1, Ref2]
- Reality :

Ref.





- Non-linear response (v_2, v_3) from semi-peripheral collisions
- Higher order flow harmonics (n > 3) :
  - Non-linear response observed right from semi-central collisions [Ref]

Studying Initial-state vs Final-state Effects

- **Higher-order moments of $\langle v_n^k \rangle$** (n = 2, 3, 4; k = 2 [Ref], 4, 6) : **probe increasing non-linearity** [Ref1, Ref2, Ref3]
  - Initial-state vs final-state effects that go beyond naive expectations from single flow harmonic calculations

- Advantages -

  - **Study effects of different initial-state model conditions**
  - **Provide tighter constraints on nuclear deformation parameters**
  - **Instigate more stringent future Bayesian analyses to constrain QGP properties like η/s or $\zeta/s(T)$**

Observables

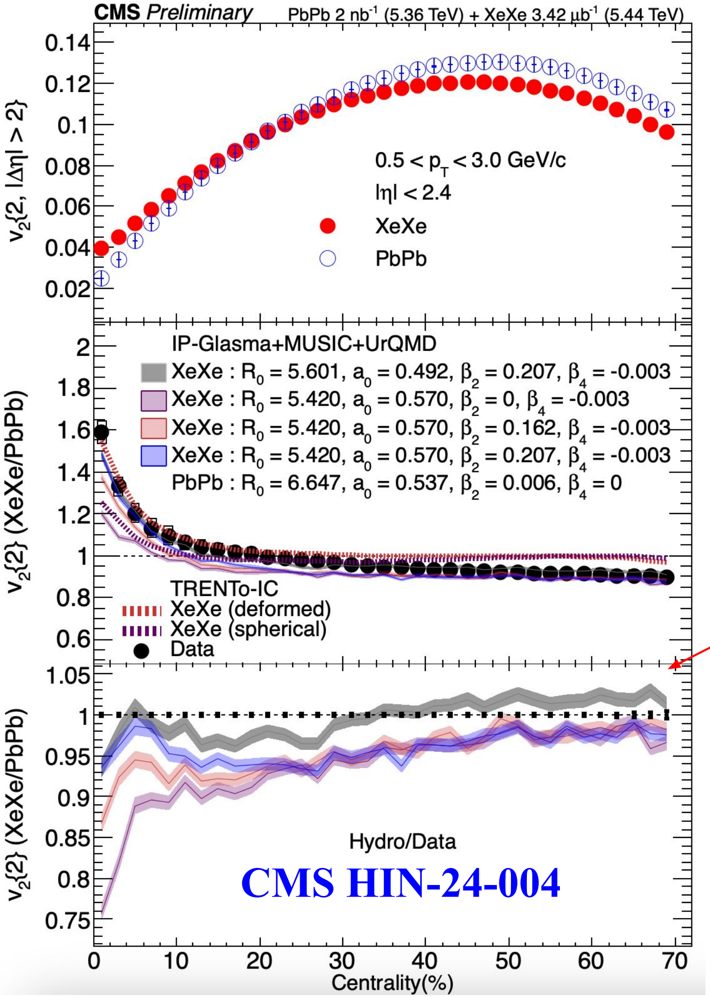
$$\frac{dN}{d\varphi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos n(\varphi - \Psi_n)$$

Requires a lot of statistics as we go to higher and higher orders - thanks to the CMS detector's large acceptance and pseudorapidity range, $|\eta| < 2.4$

- **Mixed Harmonic Cumulants (MHC)** [Ref.]
    - **Genuine** correlations of **higher-order moments of >= 2 different flow harmonics**
    - 6- and 8-particle cumulants : **insensitive to non-flow effects [Ref.]**

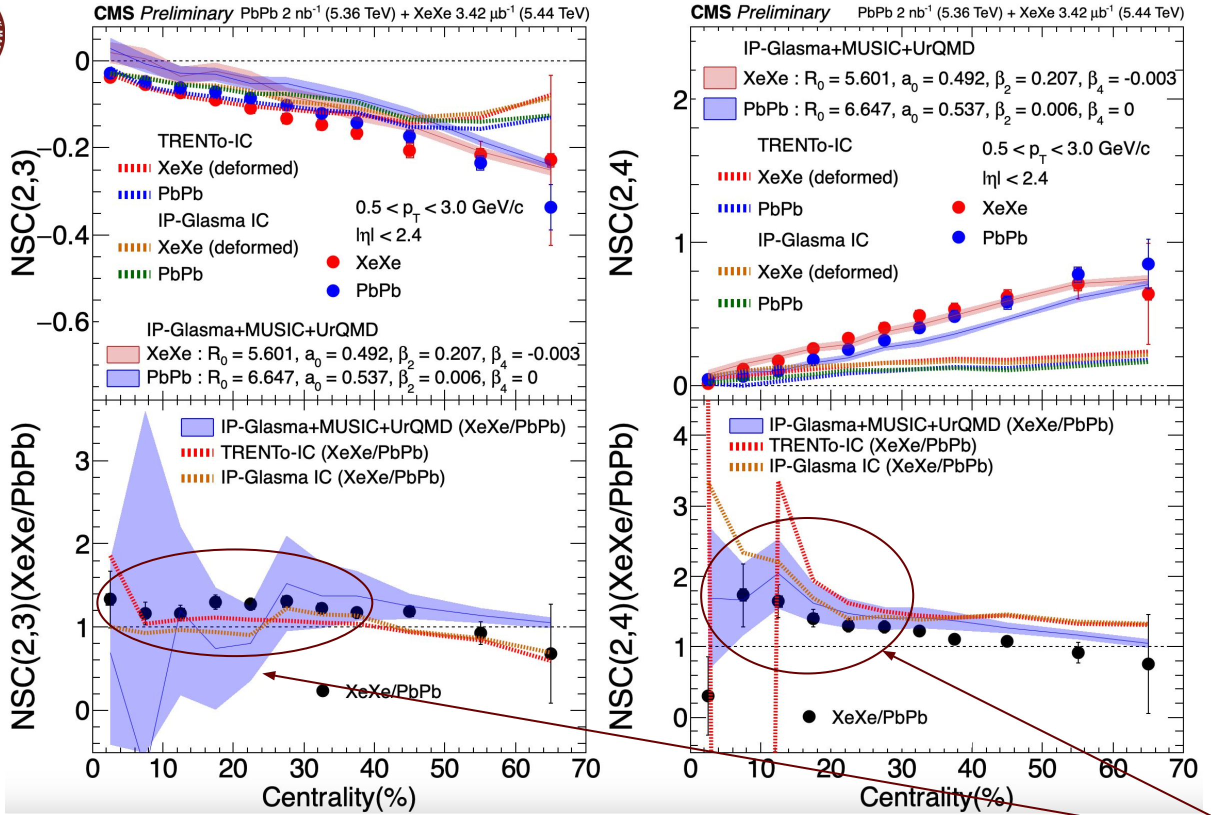
Single flow harmonic correlations	Correlations between two flow harmonics	Correlations between three flow harmonics	Correlation between higher-order moments of two flow harmonics
1. $v_n\{2, \lvert\Delta\eta\rvert > 2\}$ 2. $v_n\{4\}/v_n\{2\}$ $(n = 2, 3)$	1. NSC(k,l) (k, l = 2, 3, 4)	1. SC(k,l,m) 2. NSC(k,l,m) (k, l, m = 2, 3, 4, 5, 6)	1. $\mathrm{nMHC}(v_2^p, v_3^q)$ 2. $\mathrm{nMHC}(v_2^p, v_4^q)$ (p, q = 2, 4, 6)

$v_2\{2, |\Delta\eta| > 2\}$ vs Centrality



- $v_2\{2\}$(XeXe) > $v_2\{2\}$(PbPb) till ~20% centrality
  - More elliptic flow fluctuations
  - Sensitivity to Xe nuclear deformation [Ref]
- **Hydrodynamic predictions** by IP-Glasma+MUSIC+UrQMD for $v_2\{2\}$(XeXe/PbPb): closest match with parameter set
  R_0 = 5.601 fm, a_0 = 0.492 fm, β_2 = 0.207, β_4 = -0.003
  - Hydro/data : Maximum difference of 5% in the most central region

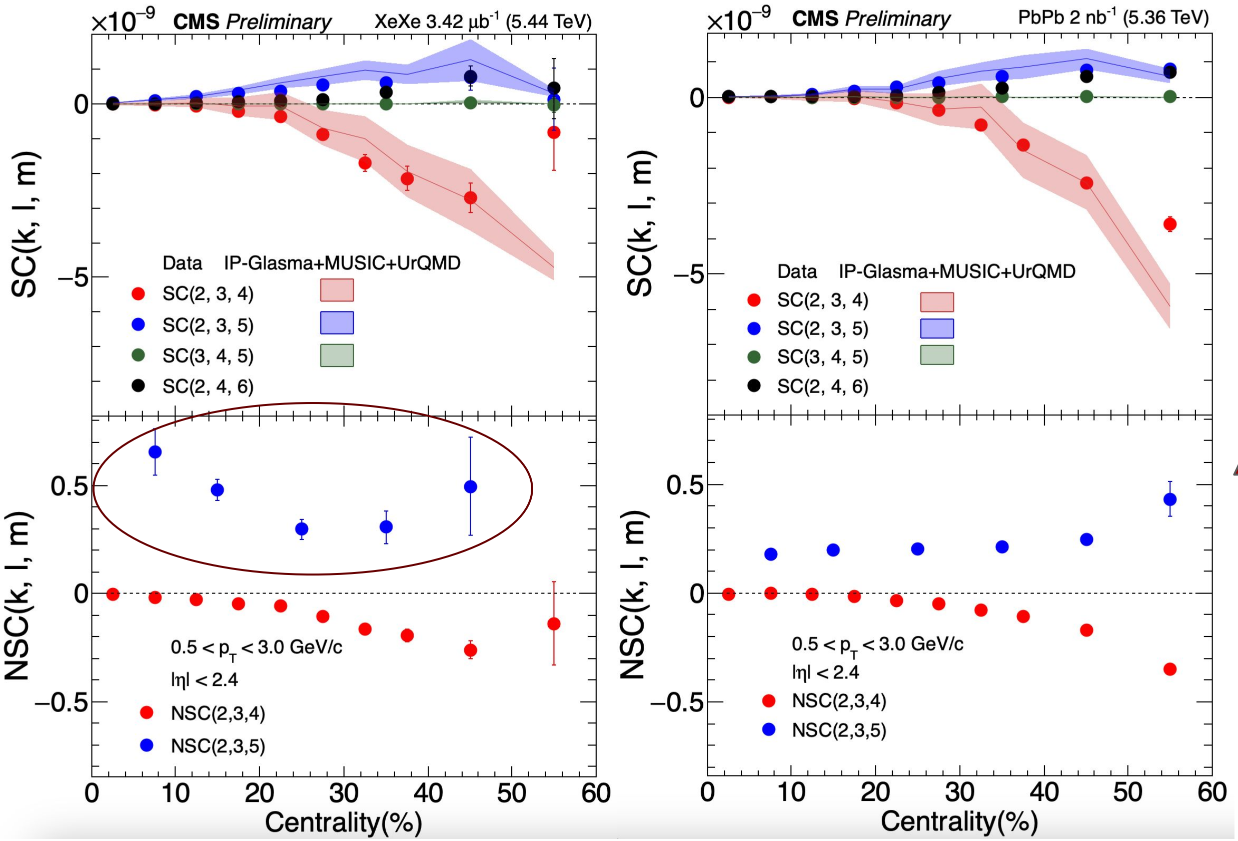
NSC(m,n) vs centrality



CMS HIN-24-004

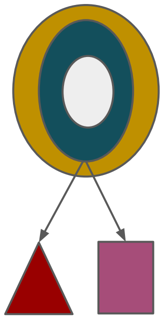
- Study correlation between **second-order moments of v_2 and v_3 or v_4**
- Trend is recreated by hydrodynamics, little quantitative discrepancy
- Considerable difference seen between both IS models
  - Sensitive to initial state NSC(2,3)
  - Sensitive to IS + QGP properties : NSC(2,4)

SC(k,l,m) and NSC(k,l,m) vs centrality



CMS HIN-24-004

- **NSC(2,3,4) ≠ 0** :
  - Fluctuations in **magnitude of persistent ellipsoidal shape** (also indicated by non-zero NSC(2,4)) + **shape of ellipsoid itself** [Ref.]
  - Greater for XeXe till 50% centrality - **more fluctuation in initial ellipsoidal shape for Xe**

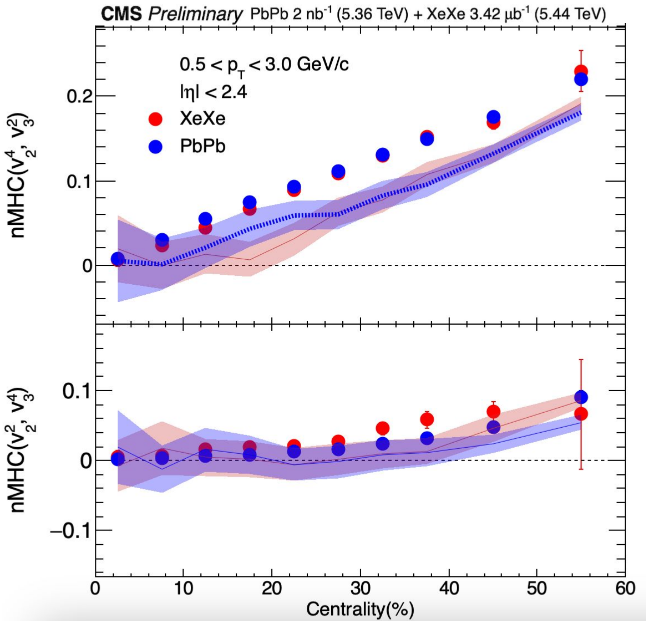


- **NSC(2,3,5) ≠ 0** :
  - **v_5** : non-linear contribution from both v_2 and v_3
  - More inconsistent trend observed between SC(2,3,5) and NSC(2,3,5) in XeXe - more **nonlinear hydrodynamic response**

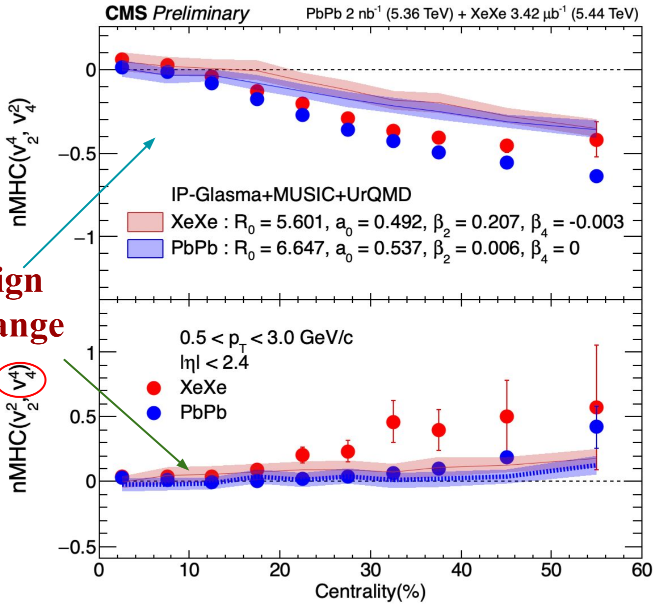
6-particle nMHCs vs centrality

$|nMHC(v_2^k, v_3^l)| < |nMHC(v_2^k, v_4^l)|$

CMS HIN-24-004



Sign change



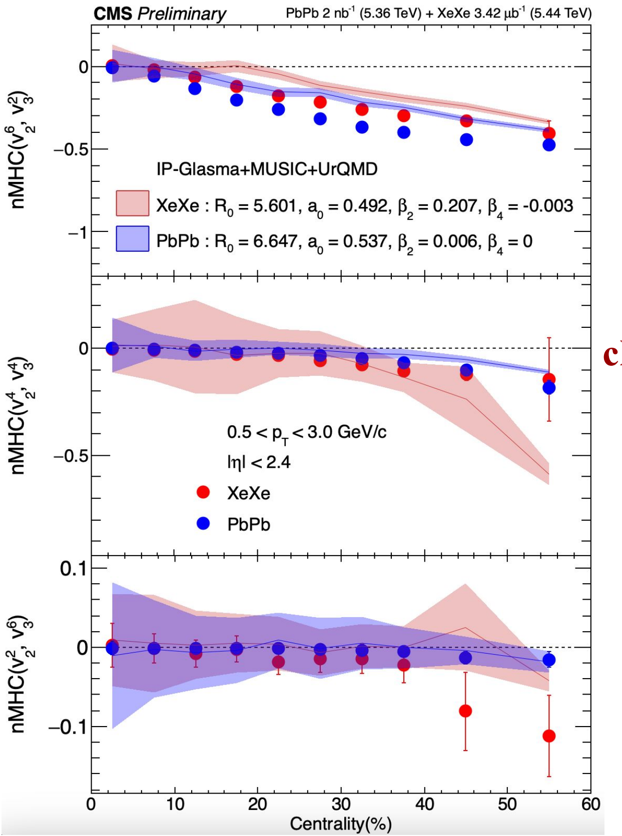
- $nMHC(v_2^k, v_3^l)$ - **same sign** for all 6-particle nMHCs
- $nMHC(v_2^k, v_4^l)$ - **change sign** depending on **lower (v_4^2) or higher-order moment of (v_4^4) of v_4** - **POSSIBLE CONTRIBUTION OF NON-LINEAR RESPONSE**

$$v_2 \approx a\varepsilon_2 , v_3 \approx b\varepsilon_3$$
$$v_4 \approx c\varepsilon_4 + d(\varepsilon_2)^2$$

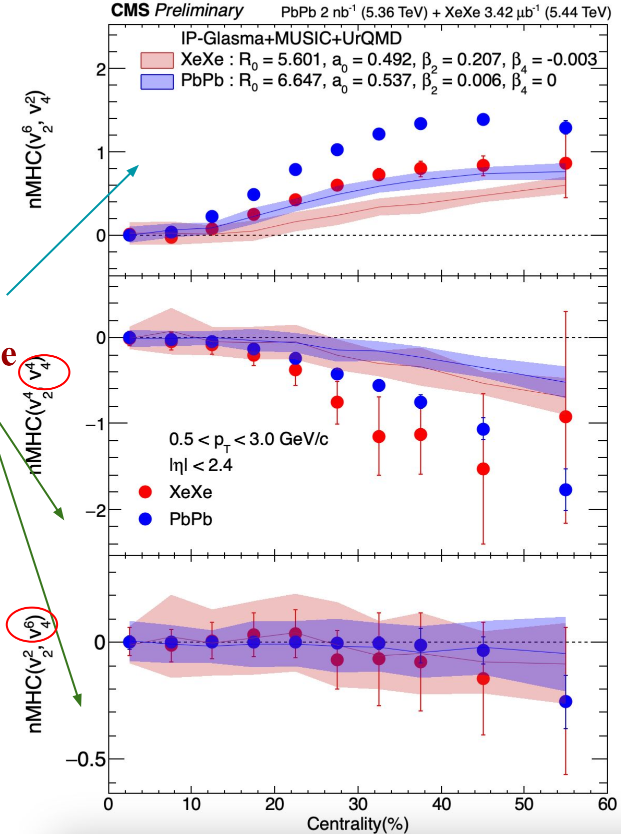
8-particle nMHCs vs centrality

CMS HIN-24-004

$|\mathrm{nMHC}(v_2^k, v_3^l)| < |\mathrm{nMHC}(v_2^k, v_4^l)|$



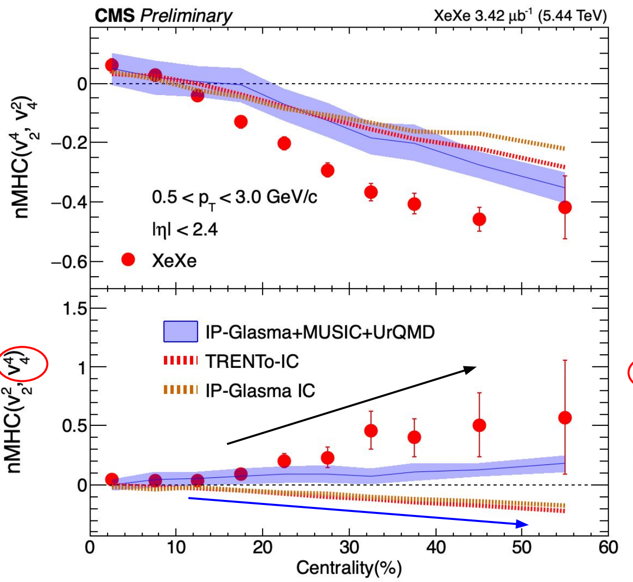
Sign change

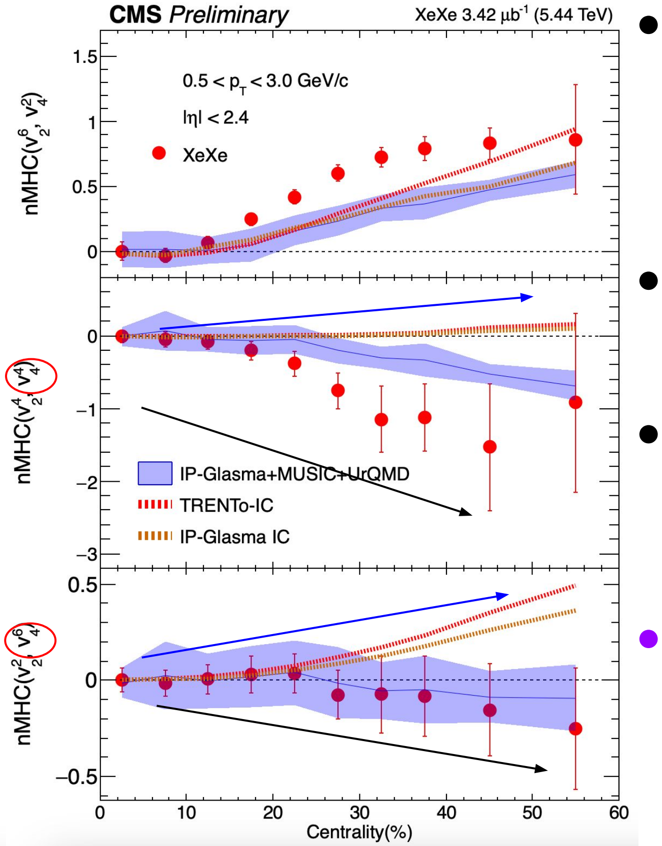


- $\mathrm{nMHC}(v_2^k, v_3^l)$ - **same sign** for all 8-particle nMHCs
- $\mathrm{nMHC}(v_2^k, v_4^l)$ - **change sign** depending on **lower** (v_4^2) **or higher-order moment of** (v_4^4/v_4^6) **of** v_4 - **POSSIBLE CONTRIBUTION OF NON-LINEAR RESPONSE**

$$v_2 \approx a\varepsilon_2\ ,\ v_3 \approx b\varepsilon_3$$
$$v_4 \approx c\varepsilon_4 + d(\varepsilon_2)^2$$

Initial vs final-state predictions for nMHC(v_2^k, v_4^l)





- Difference between TRENTo-IC vs IP-Glasma IC :
  - Sensitivity of these observables to **initial-state conditions**
- **Higher-order moments of v_4^k** (k=4,6) : **completely different results from IC models**
- **Increasing non-linearity of v_4 with increase in k**, moving from central to peripheral regions
- **Goes way beyond the naive expectations of "v_2 and v_4 being positively correlated", as suggested by just NSC(2,4)**

CMS HIN-24-004

$v_2 \approx a\varepsilon_2$, $v_3 \approx b\varepsilon_3$

$v_4 \approx c\varepsilon_4 + d(\varepsilon_2)^2$

Summary and Outlook

- First systematic study of 2-, 4-, 6- and 8-particle mixed harmonic cumulants in XeXe and PbPb collisions at 5.44 TeV and 5.36 TeV respectively

- Best match of XeXe using final-state IP-Glasma+MUSIC+UrQMD hydrodynamic prediction with parameter set :
  - **(R_0 = 5.601 fm, a_0 = 0.492 fm, β_2 = 0.207, β_4 = -0.003)**

- Study of higher-order mixed harmonic cumulants and three-harmonic symmetric cumulants :
  - Further **probe non-linearity** of flow harmonics impossible to be probed through single-harmonic flow studies
  - Harmonics involving higher-order moments of **v_4 and v_5** - non-linear effects ; different predictions as compared to initial-state models

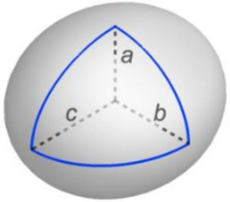
- More precise study :
  - Further constrain nuclear deformation
  - Compare between different systems to study (possible?) increasing effect of non-linearity in smaller system - **looking forward to the OO collisions this year!**

Thank you for your kind attention!

BACKUP

Why study XeXe Collisions?

β_2=0.2, γ=30°



$a \neq b \neq c$

J. Jia, Phys.Rev.C 105 (2022) 4, 044905

- ^{129}Xe : predicted to have **deformed**(1) and **triaxial** structure($r_1 \neq r_2 \neq r_3$)(2)

Nucleon density : **Woods-Saxon profile**

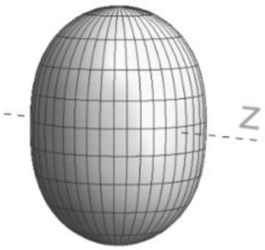
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{[r-R(\theta,\phi)]/a_0}},$$

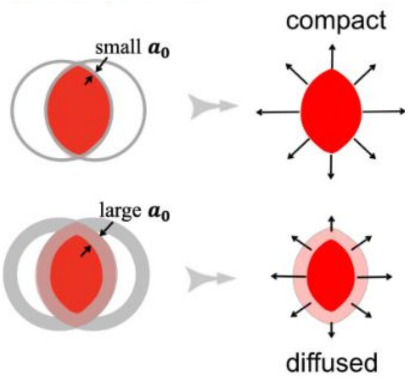
$$R(\theta,\phi) = R_0(1+\beta_2[\cos\gamma Y_{2,0}+\sin\gamma Y_{2,2}]+\beta_3\sum_{m=-3}^{3}\alpha_{3,m}Y_{3,m}+\beta_4\sum_{m=-4}^{4}\alpha_{4,m}Y_{4,m})$$

PRC 102 024901 (2020)

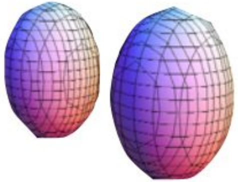
R_0 : mean nuclear radius
β_2 : quadrupole deformation parameter
a_0 : neutron skin depth
γ : triaxiality parameter

$1+\beta_2 Y_{2,0}(\theta,\phi)$

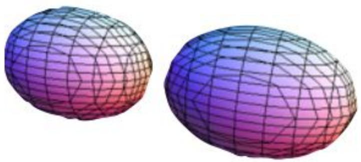




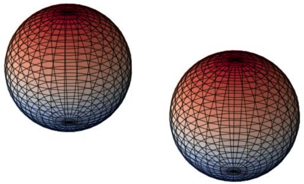
PRC 102 024901 (2020), PRL 128 (2022) 8, 082301



Xe+Xe, side-side

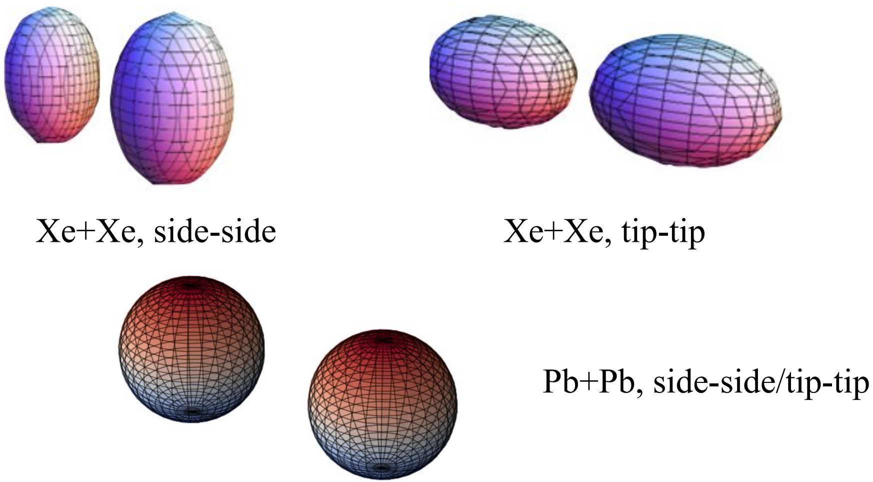


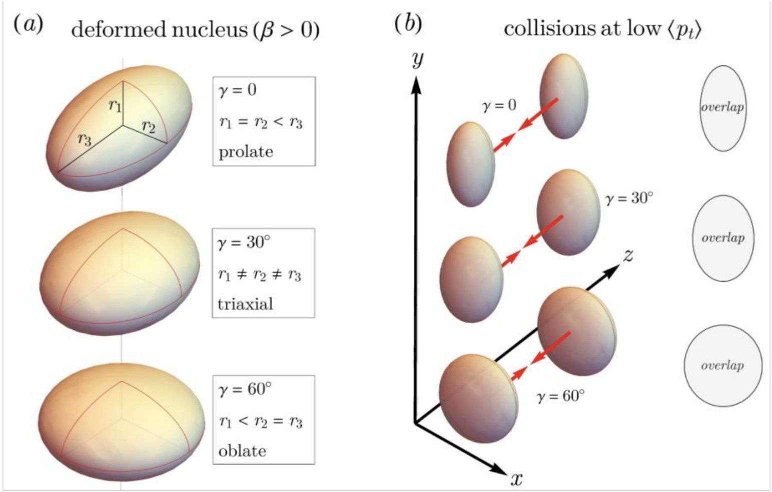
Xe+Xe, tip-tip



Pb+Pb, side-side/tip-tip

Why study XeXe collisions? (continued)





PRL 128 (2022) 8, 082301

- **Flow fluctuations** may arise due to **variation in orientation of colliding nuclei** + **Stronger** fluctuations in **smaller system**

- Comparison between XeXe and PbPb :
  - **Nuclear deformation effect**
  - **System size effect**

Size and geometry of collision matter

Change in size and initial collision geometry : change in :

- Nuclear overlap
- Number of nucleonic interactions
- Number of particles produced

Analysis Techniques

- **2-particle correlation method** -
  - $v_n\{2, |\Delta\eta| > 2\}$ vs centrality
- **Multi-particle cumulant method** -
  - All 4, 6 and 8-particle cumulants vs centrality
  - Used **subevents** (eta gap) for further **non-flow removal**

Requires a lot of statistics as we go to higher and higher orders - thanks to the CMS detector's large acceptance and pseudorapidity range, $|\eta| < 2.4$

Datasets and Selections for XeXe 2017

- XeXe, $\sqrt{s_{NN}}$ **= 5.44 TeV**
- HIMinimumBias/XeXeRun2017-13Dec2017-v1/AOD

/HIMinimumBias{1-20}/XeXeRun2017-13Dec2017-v1/AOD

(Total ~18 million events)

Event Selections

- $|V_z| < 15$ cm
- primaryVertexFilter
- beamScrapingFilter
- hfCoincFilter3
- hiCentrality
- centralityBin
- HLT_HIL1MinimumBiasHF_OR_SinglePixelTrack_part*
- $\rho <= 0.2$

Track Selections

- generalTracks
- highPurity
- $|d_z / \sigma_z| < 3.0$
- $|d_{xy} / \sigma_{xy}| < 3.0$
- $|\sigma_{pT} / p_T| < 0.1$

- $\mathbf{0.5 < p_T}$ **(GeV/c) < 3.0**
- $\mathbf{-2.4 < \eta < 2.4}$

Datasets and Selections for PbPb 2023

- PbPb, $\sqrt{s_{NN}}$ **= 5.36 TeV**
- /HIPhysicsRawPrime0/HIRun2023A-PromptReco-v2/MINIAOD (Total ~300 million events)

Event Selections

- $|V_z| < 15$ cm
- phfCoincFilter2Th4
- primaryVertexFilter
- clusterCompatibilityFilter
- hiCentrality
- centralityBin
- HLT_HIMinimumBiasHF1AND*

Track Selections

- packedPFCandidates
- highPurity
- $|d_z / \sigma_z| < 3.0$
- $|d_{xy} / \sigma_{xy}| < 3.0$

- **$0.5 < p_T$ (GeV/c) < 3.0**
- **$-2.4 < \eta < 2.4$**

Systematic Uncertainties

XeXe 5.44 TeV

- Track selections -
  (i) nominal : $|d_z / \sigma_z| < 3.0$, $|d_{xy} / \sigma_{xy}| < 3.0$, $|\sigma_{pT} / p_T| < 0.1$
  (ii) tight : $|d_z / \sigma_z| < 2.0$, $|d_{xy} / \sigma_{xy}| < 2.0$, $|\sigma_{pT} / p_T| < 0.05$
  (iii) loose : $|d_z / \sigma_z| < 5.0$, $|d_{xy} / \sigma_{xy}| < 5.0$, $|\sigma_{pT} / p_T| < 0.1$

- Vertex cuts -
  (i) nominal : $-15 < v_z < 15$ cm
  (ii) narrow : $-3 < v_z < 3$ cm
  (iii) wide : $-15 < v_z < -3$ cm and $3 < v_z < 15$ cm

- Centrality calibration -
  (i) nominal : eff+contam = 95%
  (ii) systematics 1 : eff+contam = 92%
  (iii) systematics 1 : eff+contam = 98%

- Systematic uncertainty from MC closure test

Summary of Systematic Uncertainties

XeXe 5.44 TeV

Source	**Percentage of uncertainty**
Track selection	2-6%
Vertex cut	2-6%
Centrality calibration	3-6%
Monte-carlo closure	1-9%
Total	**4-13%**

Systematic Uncertainties

PbPb 5.36 TeV

- Track selections -

(i) nominal : $|d_z / \sigma_z| < 3.0$, $|d_{xy} / \sigma_{xy}| < 3.0$
(ii) tight : $|d_z / \sigma_z| < 2.0$, $|d_{xy} / \sigma_{xy}| < 2.0$
(iii) loose : $|d_z / \sigma_z| < 5.0$, $|d_{xy} / \sigma_{xy}| < 5.0$

- Vertex cuts -
  (i) nominal : $-15 < v_z < 15$ cm
  (ii) narrow : $-3 < v_z < 3$ cm
  (iii) wide : $-15 < v_z < -3$ cm and $3 < v_z < 15$ cm

- Centrality calibration -
  (i) nominal
  (ii) systematics HF up
  (iii) systematics HF down

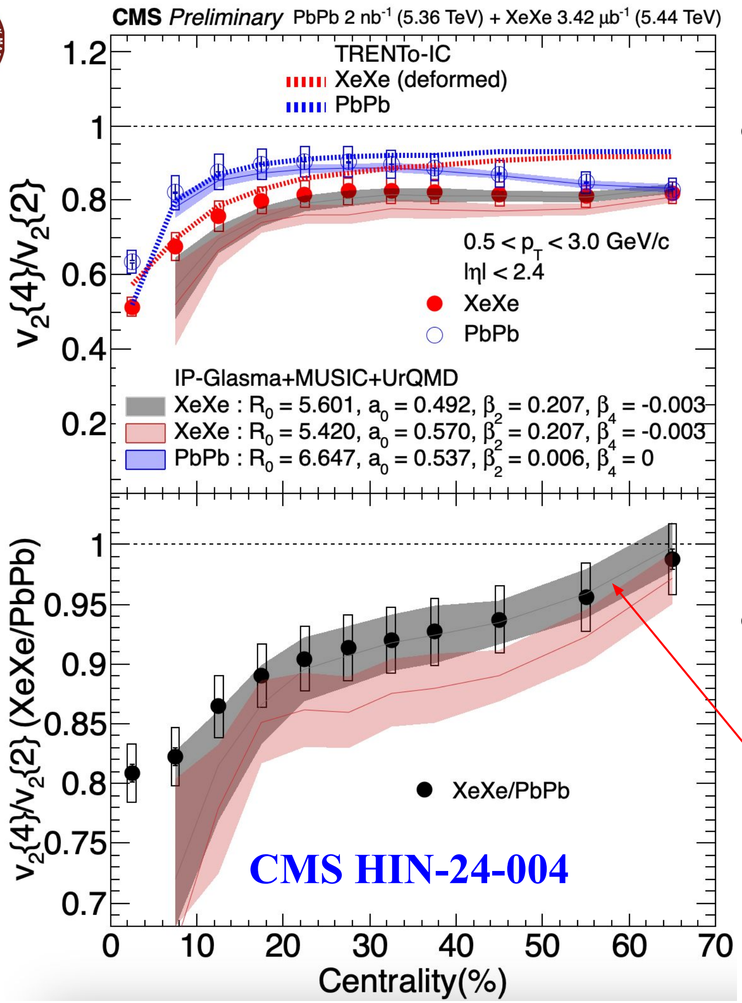
- Systematic uncertainty from MC closure test

Summary of Systematic Uncertainties

PbPb 5.36 TeV

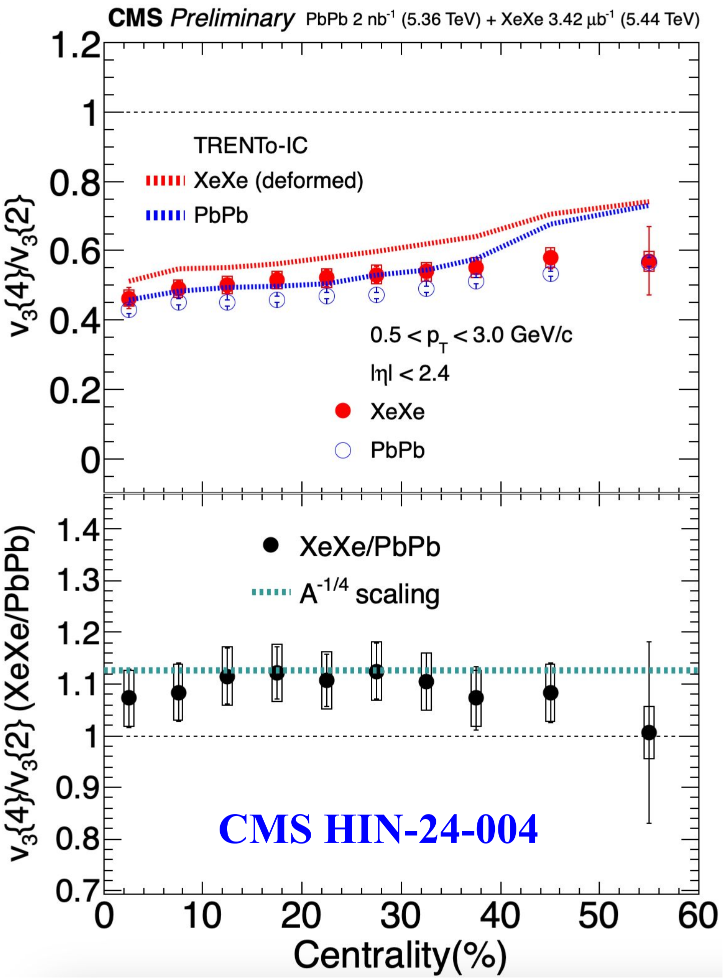
Source	Percentage of uncertainty
Track selection	1-5%
Vertex cut	1-5%
Centrality calibration	1-3%
Monte-carlo closure	1-5%
Total	**2-9%**

$v_2\{4\}/v_2\{2\}$ vs Centrality



- $v_2\{4\}/v_2\{2\}$ **:** relative fluctuations of v_n ; = 1 if v_n is the same for all events, smaller than 1 otherwise[Ref]
  - Lesser values for XeXe : greater flow fluctuations
  - Largest deviation in most central region
  - IS pred greater from central to peripheral - more deviation for XeXe : **greater non-linear hydro response**
- $v_2\{4\}/v_2\{2\}$ known to be sensitive to neutron skin (a_0) [Ref]. **Hydrodynamic predictions** by IP-Glasma+MUSIC+UrQMD :
  - Keeping same $\beta_2 = 0.207$, compared with two different a_0
  - Individual comparison + ratio (XeXe/PbPb) : closer match with ($a_0 = 0.492, \beta_2 = 0.207$)

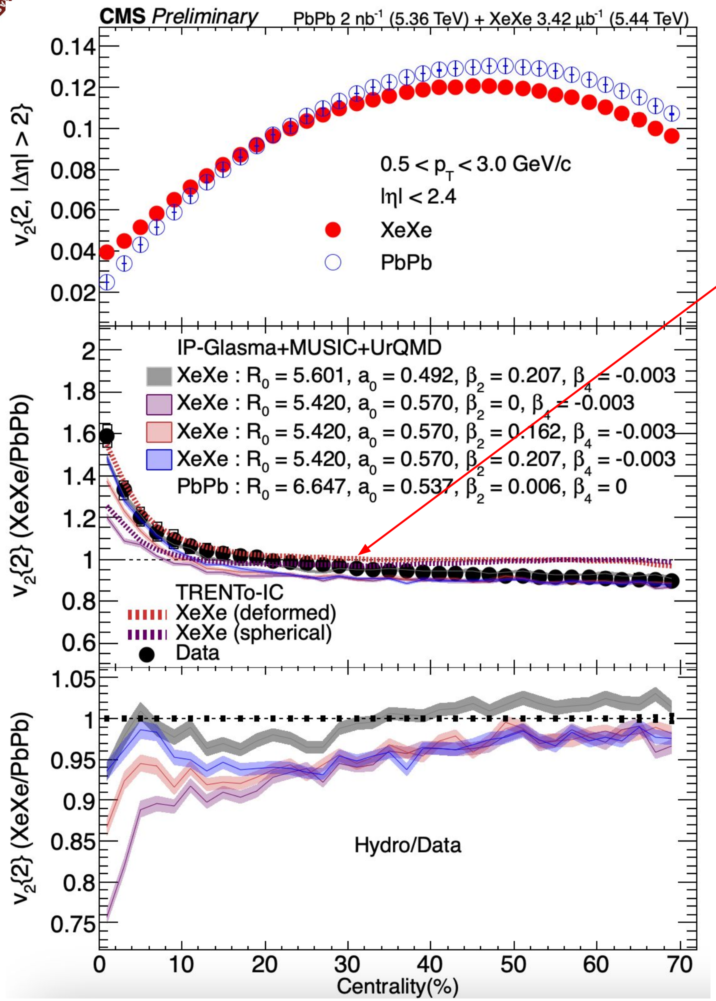
$v_3\{4\}/v_3\{2\}$ vs centrality



- Much flatter than $v_2\{4\}/v_2\{2\}$
- Order reverses -
  $v_3\{4\}/v_3\{2\}$(XeXe) > $v_3\{4\}/v_3\{2\}$(PbPb)
- $v_3\{4\}/v_3\{2\}$: expected to be proportional to $A^{-1/4}$ [Ref1 , Ref2]
  - **$v_3\{4\}/v_3\{2\}$(XeXe/PbPb)** should be ≅ $(129/208)^{-1/4}$ ≅ **1.1268560**
  - **Very good agreement within error bars**
- **Initial-state prediction** by TRENTo-IC:
  $\varepsilon_3\{4\}/\varepsilon_3\{2\}$ follows same order as $v_3\{4\}/v_3\{2\}$
  - Greater than the corresponding v_3 ratios
  - **nonlinear hydrodynamic response of v_n to ε_n**

For paper

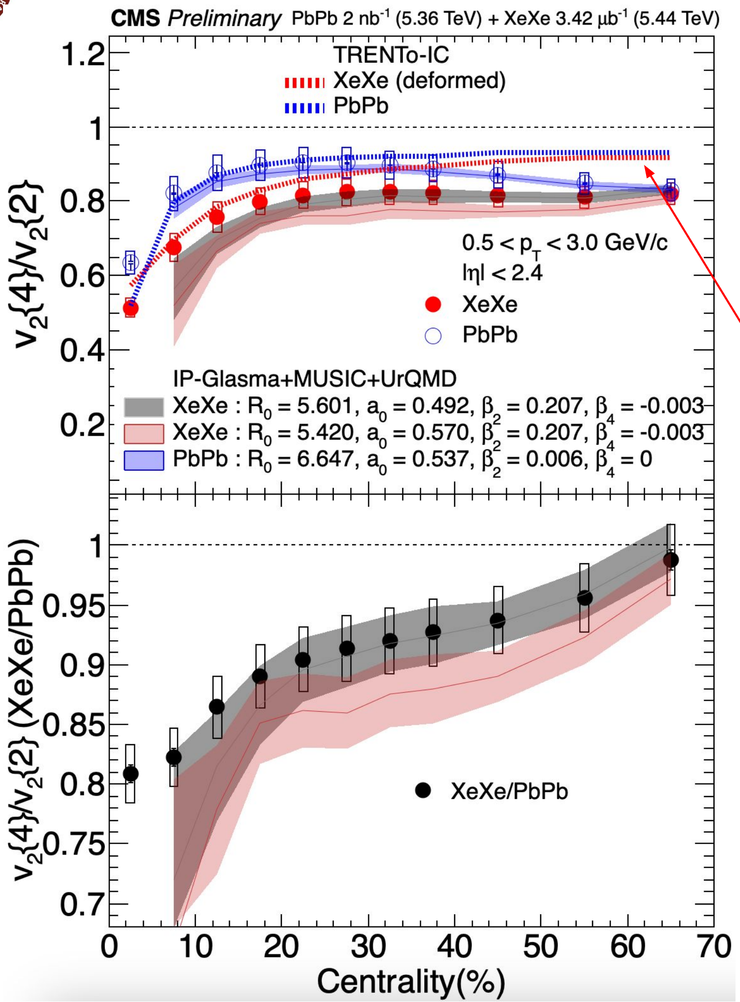
$v_2\{2, |\Delta\eta| > 2\}$ vs centrality



- **Initial-state prediction** by TRENTo-IC: very good match with set **$R_0 = 5.601$ fm, $a_0 = 0.492$ fm, $\beta_2 = 0.207$, $\beta_4 = -0.003$** till ~20% centrality
  - Spherical Xe : ~30% off in the central region - continues decreasing till 20% centrality, but still deviating
  - Matches up after 20% centrality with deformed Xe - same effect of deformation as seen in data and hydro
  - Hydro and data ratio **$v_2\{2\}$(XeXe/PbPb) < 1 after 20% centrality** but ~1 for initial-state model throughout - likely due to effect of greater viscous damping in smaller system (XeXe) [Ref]

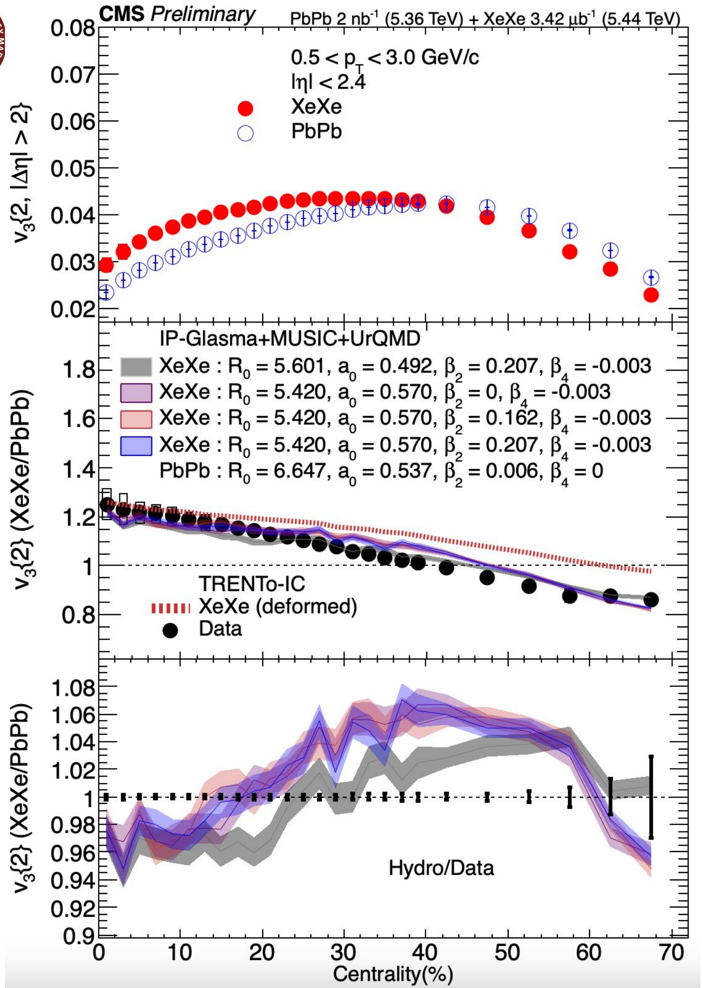
For paper

$v_2\{4\}/v_2\{2\}$ vs centrality



- **Initial-state prediction** by TRENTo-IC: $\varepsilon_2\{4\}/\varepsilon_2\{2\}$
  - Greater than the corresponding v_2 ratios
  - Possibly due to **nonlinear hydrodynamic response of v_n to ε_n**, increasing towards peripheral collisions [Ref 1, Ref 2]
  - Note : **larger difference** between **$\varepsilon_2\{4\}/\varepsilon_2\{2\}$ and $v_2\{4\}/v_2\{2\}$in case of XeXe** - **larger nonlinear hydrodyamic response for XeXe**

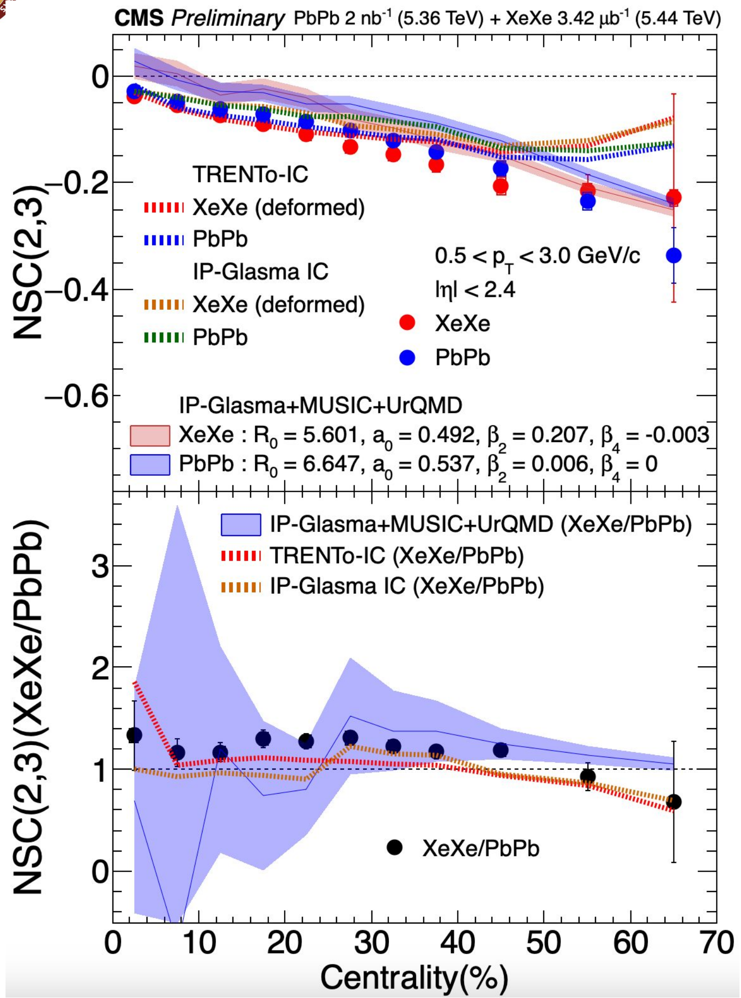
$v_3\{2, |\Delta\eta| > 2\}$ vs centrality



- $v_3\{2\}$(XeXe) > $v_3\{2\}$(PbPb) till 40% centrality
  - More triangular flow fluctuations
  - Sensitivity to initial density fluctuations - greater for smaller system [Ref]
- **Hydrodynamic predictions** by IP-Glasma+MUSIC+UrQMD :
  - Seems to be closest to (**a_0 = 0.492, β_2 = 0.207**) again
  - Hydro/data : not very sensitive to different sets
- **Initial-state prediction** by TRENTo-IC :
  - **$\varepsilon_3\{2\}$(XeXe/PbPb) above 1** for all centralities
  - $v_3\{2\}$(XeXe/PbPb) < 1 above 40% centrality - again, likely due to viscous damping [Ref]

For paper

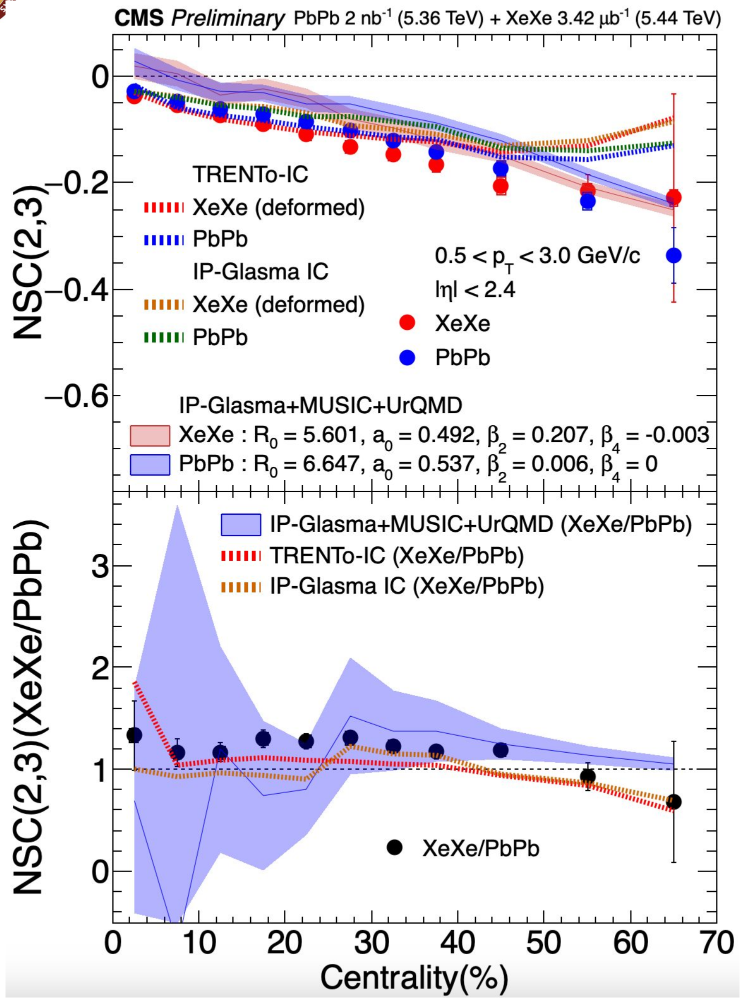
NSC(2,3) vs centrality



- Studies correlation between **second-order moments of v_2 and v_3**
- Similar trend as predicted by hydrodynamics
  - **v_2^2 and v_3^2 increasingly anti-correlated** from central to peripheral collisions
  - **NSC(2,3) (XeXe) > NSC(2,3) (PbPb)** till 50% centrality - higher degree of anti-correlation between v_2^2 and v_3^2
- **Hydrodynamic predictions** by IP-Glasma+MUSIC+UrQMD :
  - Slightly underestimate data for individual NSC(2,3) but qualitatively agree
  - Compatible with data within error bands for all centralities for XeXe/PbPb ratio

For paper

NSC(2,3) vs centrality

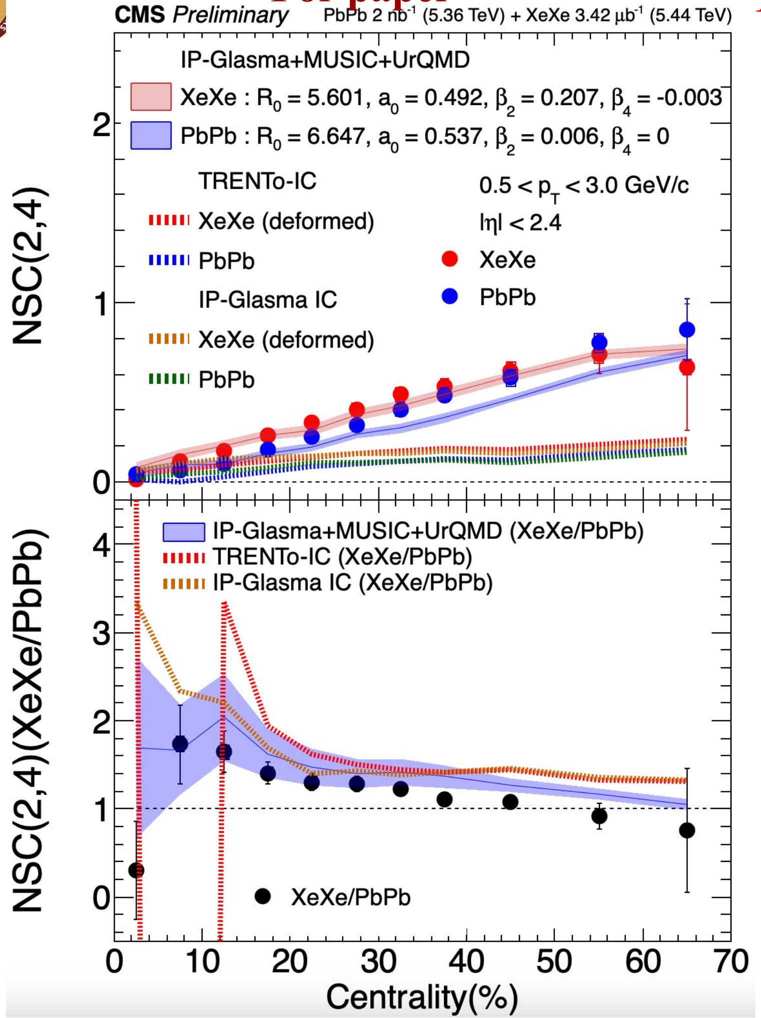


- NSC(2,3) : **sensitive to initial-state correlations** between ε_2^2 and ε_3^2 [Ref]
  - Both IS models qualitatively reproduce the trend
  - Increasing deviation towards peripheral region - **increasing non-linear hydrodynamic response**

- **NSC(2,3) (XeXe/PbPb) :**
  - **Considerable difference between TRENTo-IC and IP-Glasma IC till ~45% centrality!**
  - Sensitivity to initial-state correlations

NSC(2,4) vs centrality

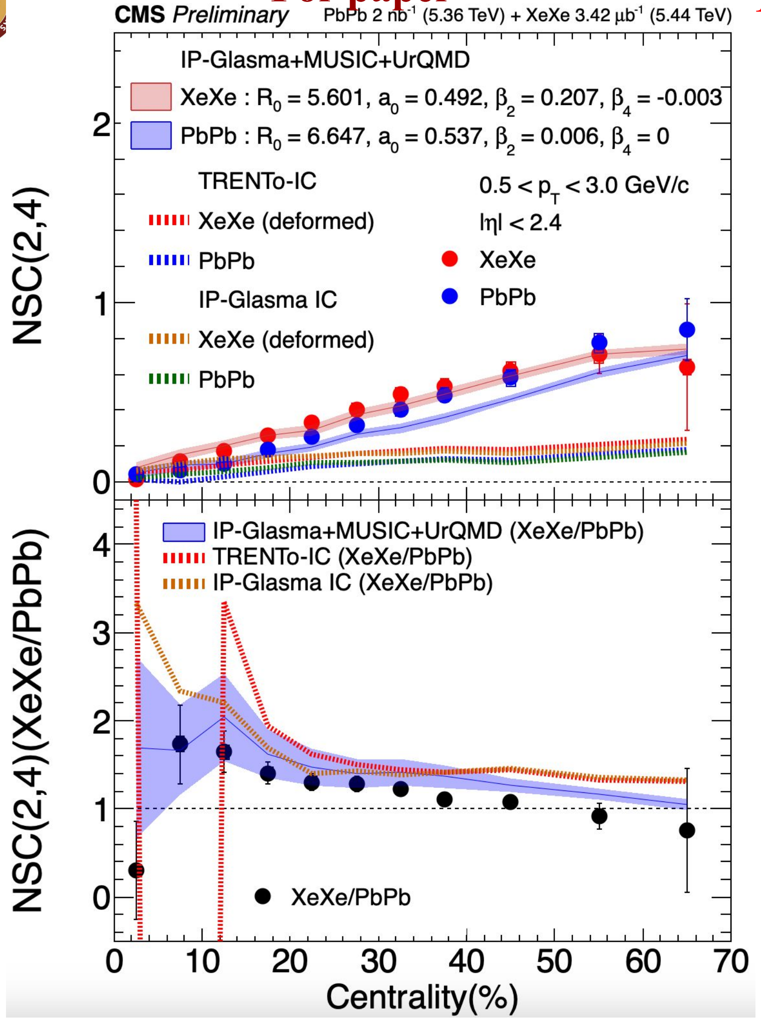
For paper



- Studies correlation between **second-order moments of v_2 and v_4**
- Similar trend as predicted by hydrodynamics
  - **v_2^2 and v_4^2 increasingly correlated** from central to peripheral collisions
  - **NSC(2,4) (XeXe) > NSC(2,4) (PbPb)** for most centralities - higher degree of anti-correlation between v_2^2 and v_4^2
- **Hydrodynamic predictions** by IP-Glasma+MUSIC+UrQMD :
  - Pretty good agreement with XeXe, slightly underestimate for PbPb
  - Compatible with data within error bands for XeXe/PbPb ratio till ~35% centrality, slight discrepancy after that - likely due to discrepancy in PbPb prediction

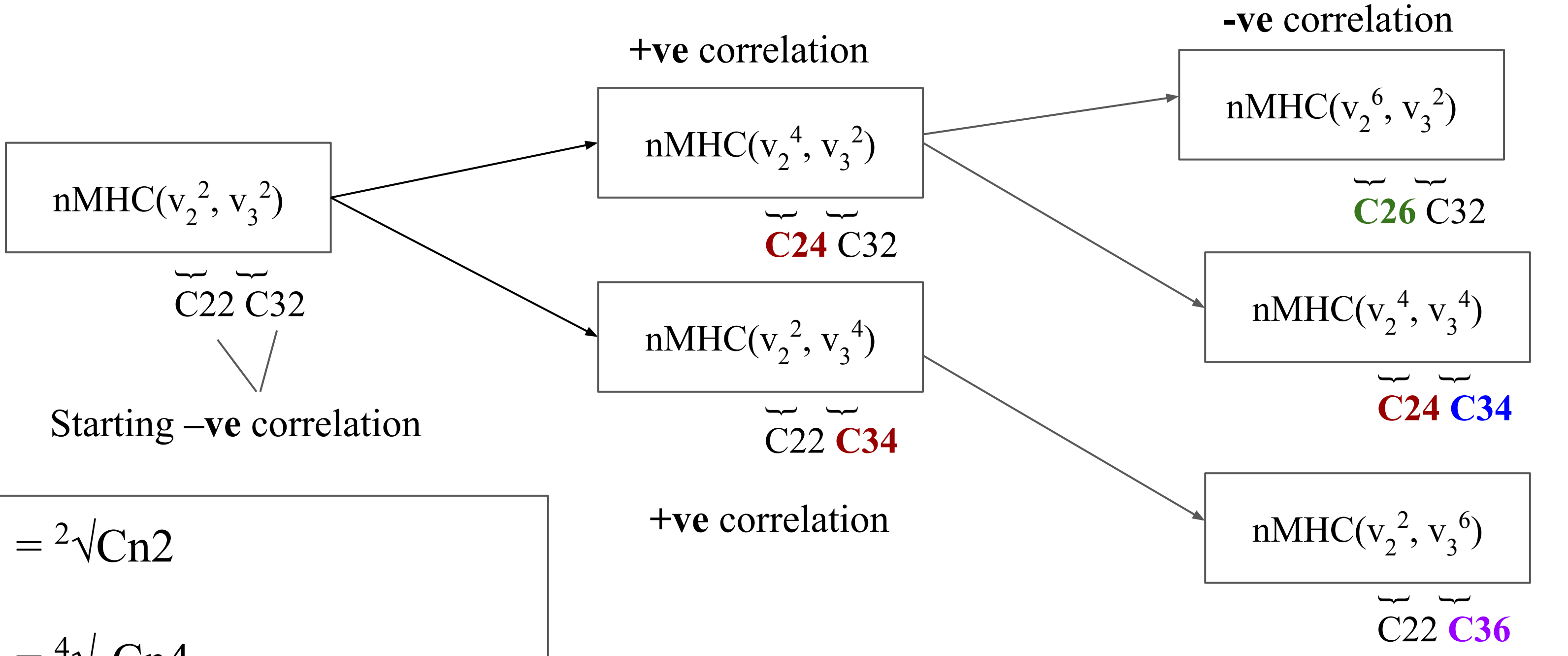
NSC(2,4) vs centrality

For paper



- NSC(2,4) : **sensitive to both initial-state correlations** and QGP transport properties [Ref]
  - $v_4 = a\varepsilon_4 + b(\varepsilon_2^2)$ right from the most central region
  - IS predictions from both TRENTo-IC and IP-Glasma IC increasingly underestimate the data from central to peripheral collisions due to increasing non-linear response contribution to v_4 [Ref]
- **NSC(2,4) (XeXe/PbPb) :**
  - Drastically different for the two IS models till ~40% centrality

A Possible Explanation …



$v_n\{2\} = \sqrt[2]{Cn2}$

$v_n\{4\} = \sqrt[4]{-Cn4}$

$v_n\{6\} = \sqrt[6]{(Cn6/4)}$

$v_n\{8\} = \sqrt[8]{-(Cn8/33)}$

$$\mathbf{v_2 \approx a\varepsilon_2 \,,\, v_3 \approx b\varepsilon_3}$$

A Possible Explanation …

$nMHC(v_2^2, v_4^2)$

C22 C42

Starting **+ve** correlation

-ve correlation

$nMHC(v_2^4, v_4^2)$

C24 C42

$nMHC(v_2^2, v_4^4)$

C22 **C44**

STILL +ve correlation : POSSIBLE NON-LINEAR EFFECT OF v_4

+ve correlation

$nMHC(v_2^6, v_4^2)$

C26 C42

$nMHC(v_2^4, v_4^4)$

C24 **C44**

STILL +ve correlation

$nMHC(v_2^2, v_4^6)$

C22 **C46**

STILL +ve correlation : POSSIBLE NON-LINEAR EFFECT OF v_4

$$v_n\{2\} = \sqrt[2]{Cn2}$$

$$v_n\{4\} = \sqrt[4]{-Cn4}$$

$$v_n\{6\} = \sqrt[6]{(Cn6/4)}$$

$$v_n\{8\} = \sqrt[8]{-(Cn8/33)}$$

$$\mathbf{v_4 = c\varepsilon_4 + d(\varepsilon_2)^2}$$

Initial vs final-state predictions for nMHC(v_2^k, v_3^l) and nMHC(v_2^k, v_4^l)

- **Symmetry observed for nMHC(v_2^k, v_3^l) breaks in nMHC(v_2^k, v_4^l) for higher-order moments of v_4**
- **Strong probe of non-linearity, not visible through lower-order or multiparticle cumulants of only one flow harmonic**

N-particle cumulant	nMHC(v_2^k, v_3^l)	Sign of correlation		nMHC(v_2^k, v_4^l)	Sign of correlation	
		Initial-state model	Final-state model		Initial-state model	Final-state model
n = 4	k = 2, l = 2	-	-	k = 2, l = 2	+	+
n = 6	k = 4, l = 2	+	+	k = 4, l = 2	-	-
n = 6	k = 2, l = 4	+	+	k = 2, l = 4	-	+
n = 8	k = 6, l = 2	-	-	k = 6, l = 2	+	+
n = 8	k = 4, l = 4	-	-	k = 4, l = 4	+	-
n = 8	k = 2, l = 6	-	-	k = 2, l = 6	+	-

Quantitative Discrepancies between Data and IP-Glasma+MUSIC+UrQMD Hydrodynamic Predictions

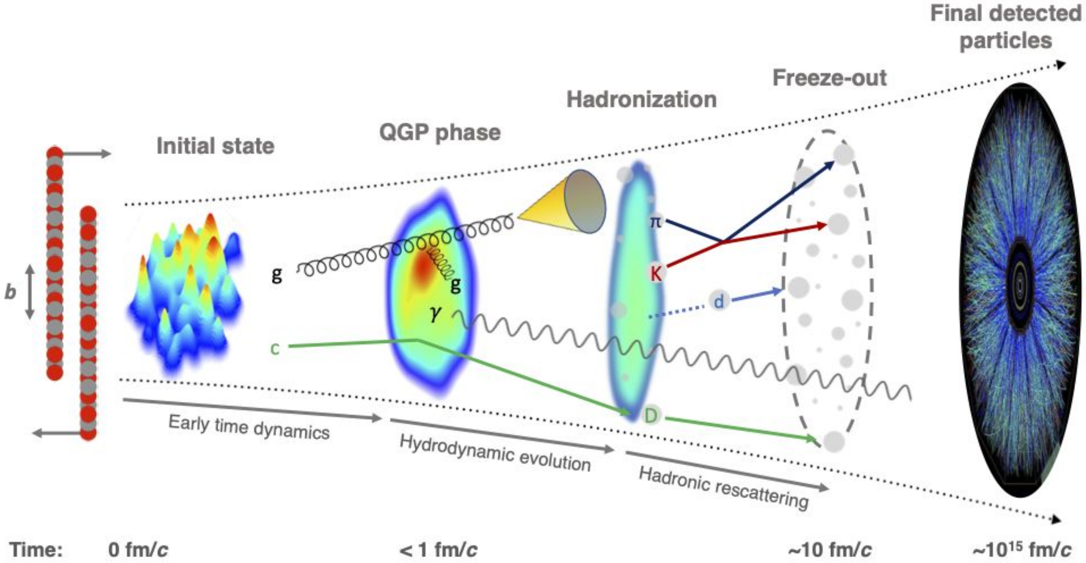
Some possible factors (personal opinion) :

- Overestimation/underestimation of flow harmonics for different p_T cuts in case of integrated flow (v_n vs centrality) [Ref.], which have been used to extract the **QGP transport coefficients**, η/s and ζ/s(T)
  - Current analysis : value of $\eta/s = 0.12$ has been taken in the IP-Glasma+MUSIC+UrQMD framework [Ref.]
- Octupole deformation parameter : $\boldsymbol{\beta_3}$ and triaxiality parameter : $\boldsymbol{\gamma}$ taken to be zero for this model
  - Xe nucleus shown to have potential γ-soft deformation associated with the second-order shape phase transition along the Xe isotope chain [Ref.]

Possible solutions :

- Fine-tuning of deformation parameters through more refined Bayesian analysis
- Fine-tuning of QGP transport coefficients and/or freezeout criteria

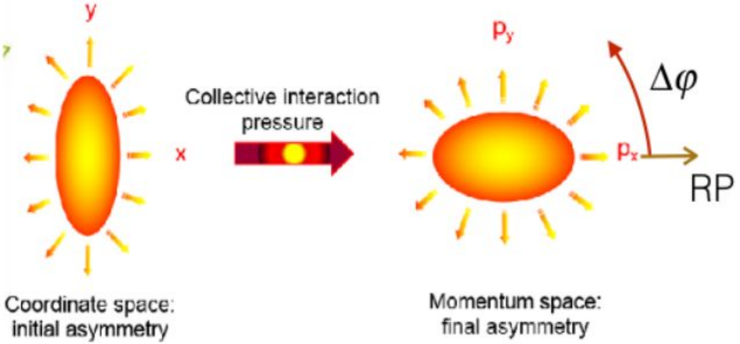
From Anisotropic Flow to Nuclear Structure



https://arxiv.org/pdf/2303.17254

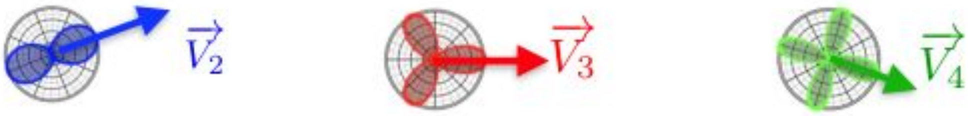
Nuclear structure ↔ Initial conditions ← **Probe** — Anisotropic flow of particles

Quantifying Anisotropic Flow



$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T\, dp_T\, dy}\left(1+\sum_{n=1}^{\infty} 2 v_n \cos\left(n\left(\varphi-\Psi_n\right)\right)\right)$$

$$v_n = \langle \cos\left(n\left(\varphi-\Psi_n\right)\right)\rangle$$

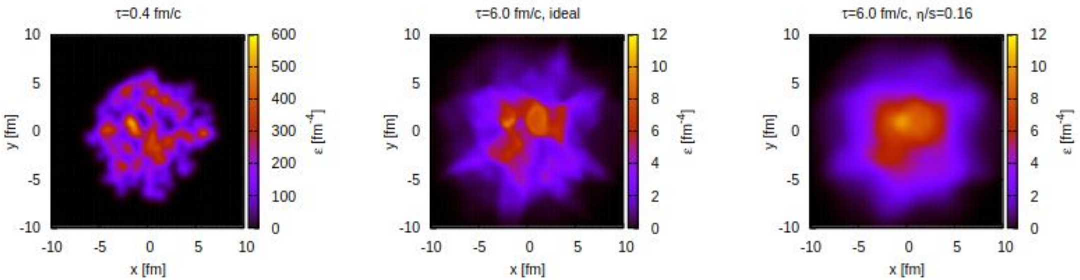


Fourier series[1] :
v_n quantify momentum anisotropy
ε_n quantify spatial anisotropy
v_1 : directed, v_2 : elliptic, v_3 : triangular flow, v_4 : quadrangular flow,...

[1]S. Voloshin and Y. Zhang, Z. Phys. C 70 (1996)

What Anisotropic Flow Measures

B. Schenke et al., PRL 106 (2011) 042301



Initial condition Ideal hydrodynamics Viscous hydrodynamics

v_n is sensitive to :

- Initial state fluctuations of participating nucleons,
- Equation of state (EoS),
- Flow fluctuations,
- Transport properties : shear/bulk viscosity

Toolset for Flow Measurement

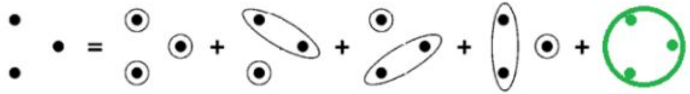
- 2-particle cumulants
- Scalar Product (SP) Method (not done here)
- Symmetric Cumulants
- Multi-particle cumulants
  - ❖ Generic framework[(1)]

[1]A. Bilandzic et al., Phys. Rev. C 89, 064904 (2014)

Multiparticle Cumulants

- k-particle cumulant (k > 1)
  - **collective nature of flow**
  - [1]Largely suppresses lower order non-flow correlations

- stochastic event-by-event fluctuations of v_n harmonics[2]



R. Kubo, J. Phys. Soc. Jpn. 17 (1962)

- generalizing to more than two flow harmonics - not trivial - generic framework[1]

[1]A. Bilandzic et al., Phys. Rev. C 89, 064904 (2014)

Mixed Harmonic Cumulants(MHC)

- recently introduced higher-order flow cumulants
- quantify **genuine** correlations of **higher-order moments of >= 2 different flow harmonics**

- k^{th} moment of v_n : $<v_n^k>$

Example[1] -

$$< v_2^4 > = << \cos(2\phi_1 + 2\phi_2 - 2\phi_3 - 2\phi_4) >>$$

- Correlation of k^{th} moment of v_n with l^{th} moment of v_m: $<v_n^k v_m^l>$

Example -

$$< v_2^4 v_3^2 > = << \cos(2\phi_1 + 2\phi_2 + 3\phi_3 - 2\phi_4 - 2\phi_5 - 3\phi_6) >>$$

Examples of higher order MHC[2]

(4 particles)

$$SC(n,m) = \langle v_n^2 v_m^2 \rangle_{\mathrm{ev,N^4}} - \langle v_n^2 \rangle_{\mathrm{ev,N^2}} \langle v_m^2 \rangle_{\mathrm{ev,N^2}}$$

(6 particles)

$$MHC(v_2^2, v_3^4) = \langle v_2^2 v_3^4 \rangle_6 - 4\langle v_2^2 v_3^2 \rangle_4 \langle v_3^2 \rangle_2 - \langle v_2^2 \rangle_2 \langle v_3^4 \rangle_4 + 4\langle v_2^2 \rangle_2 \langle v_3^2 \rangle_2^2$$

(8 particles)

$$MHC(v_2^6, v_3^2) = \langle v_2^6 v_3^2 \rangle_8 - 9\langle v_2^4 v_3^2 \rangle_6 \langle v_2^2 \rangle_2 - \langle v_2^6 \rangle_6 \langle v_3^2 \rangle_2 - 9\langle v_2^4 \rangle_4 \langle v_2^2 v_3^2 \rangle_4 - 36\langle v_2^2 \rangle_2^3 \langle v_3^2 \rangle_2 + 18\langle v_2^2 \rangle_2 \langle v_3^2 \rangle_2 \langle v_2^4 \rangle_4 + 36\langle v_2^2 \rangle_2^2 \langle v_2^2 v_3^2 \rangle_4,$$

[1] S. Acharya et al., Phys. Lett. B 818, 136354 (2021)
[2] H. Hirvonen et al., Phys. Rev. C 106, 044913 (2022)

Some more MHC formulae - 6 particle cumulants

$$\begin{aligned} MHC(v_2^4, v_3^2) \;=\;& \langle\langle\cos(2\varphi_1 + 2\varphi_2 + 3\varphi_3 - 2\varphi_4 - 2\varphi_5 - 3\varphi_6)\rangle\rangle \\ & -4\,\langle\langle\cos(2\varphi_1 + 3\varphi_2 - 2\varphi_3 - 3\varphi_4)\rangle\rangle\,\langle\langle\cos(2\varphi_1 - 2\varphi_2)\rangle\rangle \\ & -\langle\langle\cos(2\varphi_1 + 2\varphi_2 - 2\varphi_3 - 2\varphi_4)\rangle\rangle\,\langle\langle\cos(3\varphi_1 - 3\varphi_2)\rangle\rangle \\ & +4\,\langle\langle\cos(2\varphi_1 - 2\varphi_2)\rangle\rangle^2\,\langle\langle\cos(3\varphi_1 - 3\varphi_2)\rangle\rangle\,. \end{aligned}$$

$$MHC(v_2^4, v_3^2) = \langle v_2^4 v_3^2\rangle - 4\,\langle v_2^2 v_3^2\rangle\,\langle v_2^2\rangle - \langle v_2^4\rangle\,\langle v_3^2\rangle + 4\,\langle v_2^2\rangle^2\,\langle v_3^2\rangle$$

$$MHC(v_2^2, v_3^4) \;=\; \langle v_2^2 v_3^4\rangle - 4\,\langle v_2^2 v_3^2\rangle\,\langle v_3^2\rangle - \langle v_3^4\rangle\,\langle v_2^2\rangle + 4\,\langle v_2^2\rangle\,\langle v_3^2\rangle^2$$

Some more MHC formulae - 8 particle cumulants

$$
\begin{aligned}
MHC(v_2^6, v_3^2) \;=\;& \left\langle v_2^6 v_3^2\right\rangle - 9\left\langle v_2^4 v_3^2\right\rangle\left\langle v_2^2\right\rangle - \left\langle v_2^6\right\rangle\left\langle v_3^2\right\rangle - 9\left\langle v_2^4\right\rangle\left\langle v_2^2 v_3^2\right\rangle - 36\left\langle v_2^2\right\rangle^3\left\langle v_3^2\right\rangle \\
& + 18\left\langle v_2^2\right\rangle\left\langle v_3^2\right\rangle\left\langle v_2^4\right\rangle + 36\left\langle v_2^2\right\rangle^2\left\langle v_2^2 v_3^2\right\rangle,
\end{aligned}
$$

$$
\begin{aligned}
MHC(v_2^4, v_3^4) \;=\;& \left\langle v_2^4 v_3^4\right\rangle - 4\left\langle v_2^4 v_3^2\right\rangle\left\langle v_3^2\right\rangle - 4\left\langle v_2^2 v_3^4\right\rangle\left\langle v_2^2\right\rangle - \left\langle v_2^4\right\rangle\left\langle v_3^4\right\rangle - 8\left\langle v_2^2 v_3^2\right\rangle^2 \\
& - 24\left\langle v_2^2\right\rangle^2\left\langle v_3^2\right\rangle^2 + 4\left\langle v_2^2\right\rangle^2\left\langle v_3^4\right\rangle + 4\left\langle v_2^4\right\rangle\left\langle v_3^2\right\rangle^2 + 32\left\langle v_2^2\right\rangle\left\langle v_3^2\right\rangle\left\langle v_2^2 v_3^2\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
MHC(v_2^2, v_3^6) \;=\;& \left\langle v_2^2 v_3^6\right\rangle - 9\left\langle v_2^2 v_3^4\right\rangle\left\langle v_3^2\right\rangle - \left\langle v_3^6\right\rangle\left\langle v_2^2\right\rangle - 9\left\langle v_3^4\right\rangle\left\langle v_2^2 v_3^2\right\rangle - 36\left\langle v_2^2\right\rangle\left\langle v_3^2\right\rangle \\
& + 18\left\langle v_2^2\right\rangle\left\langle v_3^2\right\rangle\left\langle v_3^4\right\rangle + 36\left\langle v_3^2\right\rangle^2\left\langle v_2^2 v_3^2\right\rangle.
\end{aligned}
$$

4-particle Symmetric Cumulants(SC)

$$\langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle \equiv SC(m,n)$$

- Studies correlation between **second-order moments** of **two** different harmonics

- Hydrodynamics predicts :

  - ❖ Positive correlation between v_2 and v_4 (even-even)
  - ❖ Anti-correlation between v_2 and v_3 (even-odd)
  - ❖ Anti-correlation between v_3 and v_4 (odd-even)

4-particle Normalized Symmetric Cumulants(NSC)

$$\mathrm{NSC}(m,n) \equiv \frac{\mathrm{SC}(m,n)}{\langle v_m^2 \rangle \langle v_n^2 \rangle}$$

- Removes the dependence on magnitude of flow harmonics
- Investigate the intrinsic correlation between v_n coefficients + model vs data
- Compare across different collision systems - removes dependence on p_T range
- It was found that NSC(3, 2), which studies the correlations between v_2^2 and v_3^2, is very sensitive to the initial conditions and can be used as a good tool to probe initial state ε_2^2 and ε_3^2 correlations.
- NSC(4, 2) and also NSC involving higher order flow coefficients, are sensitive to both initial conditions and the QGP properties.

To avoid short-range, non-flow correlations : $|\Delta\eta| > 2.0$ has been introduced in denominator

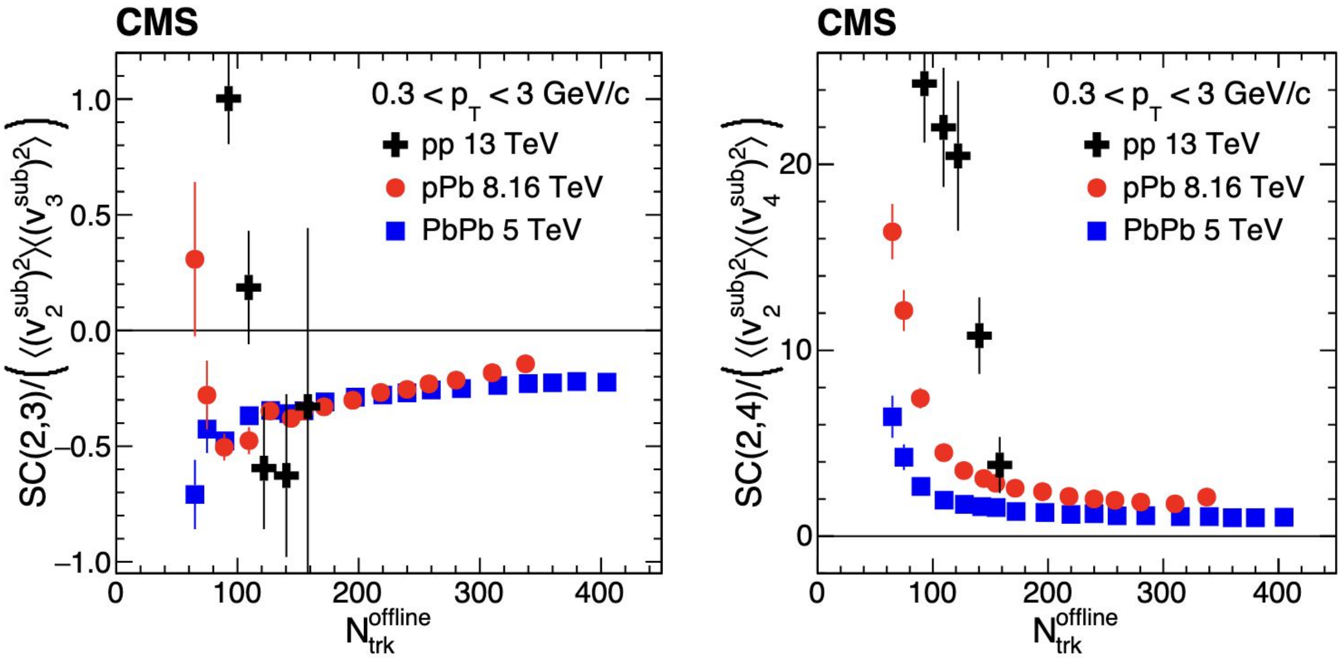


Figure 3: The SCs for the second and third coefficients (left) and the second and fourth coefficients (right) normalized by $\langle (v_2^{\text{sub}})^2 \rangle \langle (v_3^{\text{sub}})^2 \rangle$ and $\langle (v_2^{\text{sub}})^2 \rangle \langle (v_4^{\text{sub}})^2 \rangle$ from two-particle correlations. The results are shown as a function of $N_{\text{trk}}^{\text{offline}}$ in 13 TeV pp, 8.16 TeV pPb, and 5.02 TeV PbPb collisions.

CMS HIN-16-022

Mixed Harmonic Cumulants (MHC)

- Higher order cumulants with n = 4, 6, 8,... particles

- To investigate genuine multi-particle correlations involving more than two different flow coefficients and to study the relationship between **higher moments** of different flow coefficients in heavy-ion collisions at the LHC

- MHC is expected to be insensitive to non-flow effects - genuine multi-particle correlations

- Can be compared to various models to predict initial state conditions and transport properties.

Eg : $\boxed{\mathrm{MHC}(v_m^2, v_n^2) = \mathrm{SC}(m,n)}$

Studied for the first time in XeXe collisions and, for some, also in PbPb collisions

Normalized Mixed Harmonic Cumulants (nMHC)

2 flow coefficients :

$$nMHC(v_m^k, v_n^l) = \frac{MHC(v_m^k, v_n^l)}{\langle v_m^k \rangle \langle v_n^l \rangle}$$

3 flow coefficients :

$$nMHC(v_m^k, v_n^l, v_p^q) = \frac{MHC(v_m^k, v_n^l, v_p^q)}{\langle v_m^k \rangle \langle v_n^l \rangle \langle v_p^q \rangle}$$

- Remove dependence on magnitude of flow harmonics, similar to NSC
- Better observable to compare across different collision systems

Why study MHC?

- [1]Flow fluctuations : manifestation of **initial state fluctuations** in ultra-relativistic collisions
  - ❖ positions of nucleons
  - ❖ quark and gluon fields
  - ❖ event-by-event fluctuations for collisions with same impact parameter

- Each **moment** of individual flow amplitude, $\langle v_n^k \rangle$ ($k > 1$) : independent information of **event-by-event fluctuations**

- More detailed information of QGP properties, e.g. $\eta/s = f(T)$, **cannot be constrained with the measurements of individual flow amplitudes** - insensitive to flow fluctuations

- distribution of **final-state anisotropies** - $P(v_m, v_n, \ldots, \Psi_m, \Psi_n, \ldots)$ is sensitive to :
  - ε_n
  - event-by-event fluctuations
  - correlations between different orders of anisotropy coefficients
  - early state dynamics and transport properties of the QGP.

- Not straightforward to measure in experiments

[1]A. Bilandzic et al., Phys. Rev. C 89, 064904 (2014)
[2]ALICE Collaboration, Phys. Lett. B 818 (2021) 136354

Why study MHC?

Non-flow[1] of k particles : $\delta_k \sim \frac{1}{M^{k-1}}$ → Decreases with increase in number of particles

(M = multiplicity)

↓

[2]**MHC is expected to be insensitive to non-flow effects**

https://arxiv.org/abs/nucl-th/0105040v2

Higher order MHC :

- **Quantify fluctuations of magnitude in initial state geometry**
- **Explain fluctuations in initial shape itself (sensitive to spatial anisotropy ε_n)**
- **Tight constraint on initial state conditions + QGP properties**
- **Distinguish between various models of QGP evolution in hydrodynamic and transport models**
- **Minutely examine initial-state vs final-state effects that go beyond naive expectations from single flow harmonic calculations**

Parameter Sets for Model Comparisons

Nucleus	R_0 (fm)	a_0 (fm)	β_2	β_4
^{208}Pb	6.647	0.537	0.006	0
^{129}Xe (1)	5.601	0.492	0.207	-0.003
^{129}Xe (2) (spherical)	5.420	0.570	0	-0.003
^{129}Xe (3)	5.420	0.570	0.162	-0.003
^{129}Xe (4)	5.420	0.570	0.207	-0.003

IP-Glasma+MUSIC +UrQMD and IP-Glasma IC

Used for comparisons where all sets are compatible within error bands

Nucleus	R_0 (fm)	a_0 (fm)	β_2	β_4
^{208}Pb	6.647	0.537	0.006	0
^{129}Xe(deformed)	5.601	0.492	0.207	-0.003
^{129}Xe(spherical)	5.420	0.570	0	-0.003

TRENTo-IC

Nonlinear response of higher-order harmonics

$v_n \propto \varepsilon_n$ (n = 2, 3) :

$NSC(m,n) = (\langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle\langle v_n^2 \rangle)/(\langle v_m^2 \rangle\langle v_n^2 \rangle)$

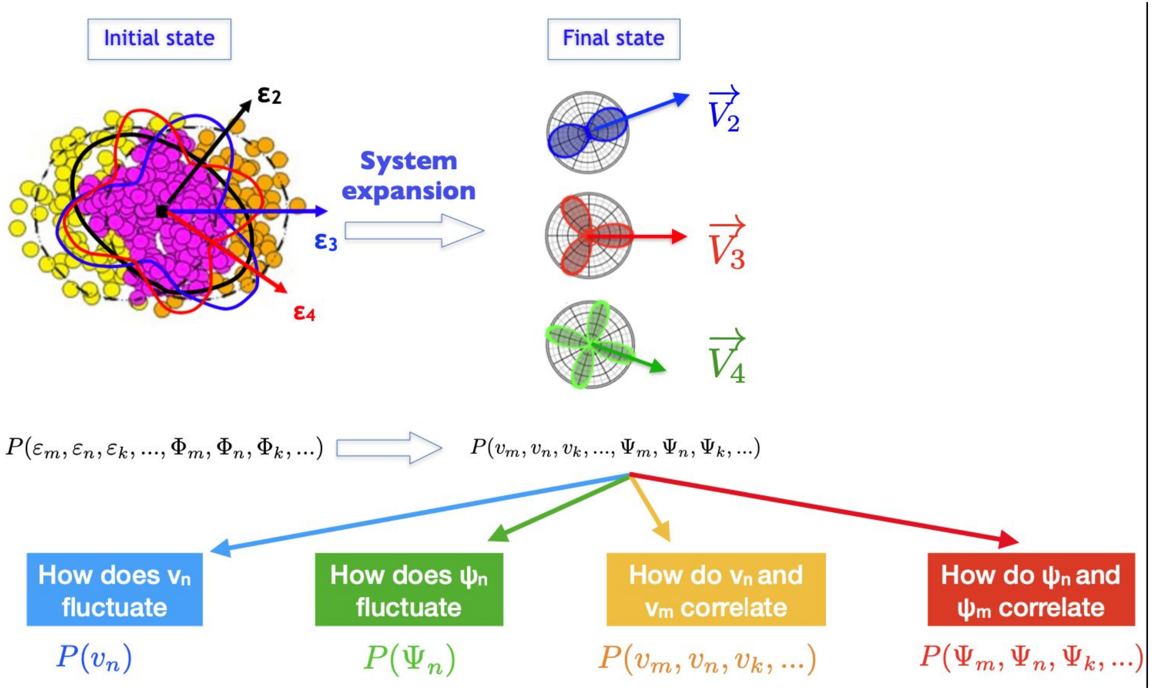
$\cong (\langle \varepsilon_m^2 \varepsilon_n^2 \rangle - \langle \varepsilon_m^2 \rangle\langle \varepsilon_n^2 \rangle)/(\langle \varepsilon_m^2 \rangle\langle \varepsilon_n^2 \rangle) = NSC(m,n)$

But …

$v_4 \cong a\varepsilon_4 + b(\varepsilon_2)^2$

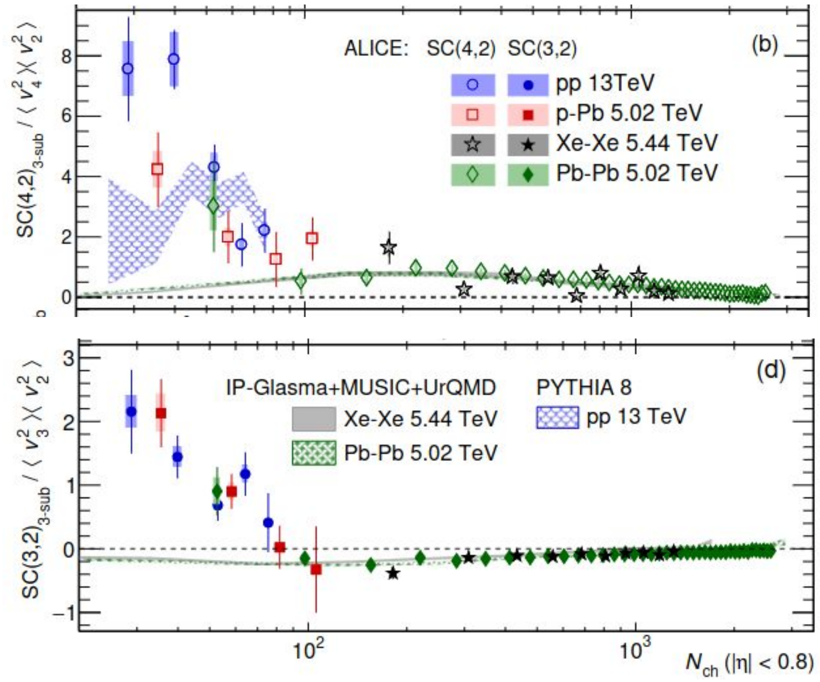
$v_5 \cong c\varepsilon_5 + d(\varepsilon_2\varepsilon_3)$ …

Nonlinear contributions from higher-order flow harmonics might cause discrepancy between SC(m,n) and NSC(m,n) during normalization[Ref.].



Courtesy : You Zhou

What has been published already in XeXe?



ALICE, Phys. Rev. Lett. 123 (2019) 142301

NSC(m,n)

Observables in the denominator are obtained from the $v_2\{2, |\Delta\eta| > 1.4\}$ and $v_n\{2, |\Delta\eta| > 1.0\}$ for higher harmonics.

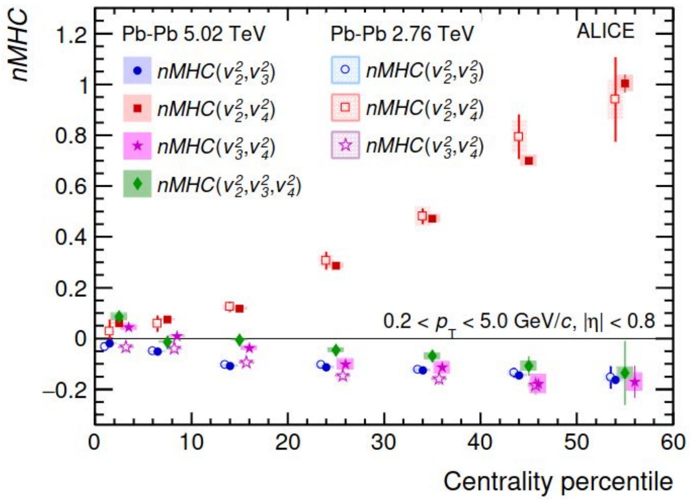
What can we do better ?

$v_n\{2, |\Delta\eta| > 2.0\}$ (n = 2,3,4,..)

- **More non-flow subtraction**

Thanks to the CMS detector's wide pseudorapidity range of $|\eta| < 2.4$

What has been published for nMHC in PbPb?

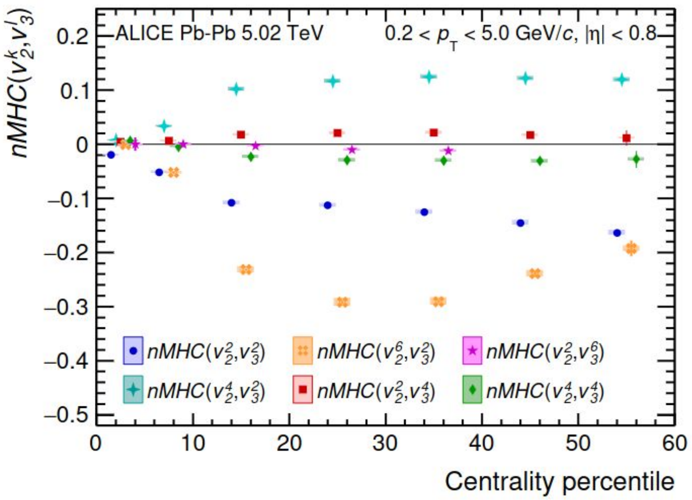


ALICE Collaboration, Phys. Lett. B 818 (2021) 136354

Observables in the denominator have $v_n\{2, |\Delta\eta| > 0.8\}$

What we can do better : $v_n\{2, |\Delta\eta| > 2.0\}$

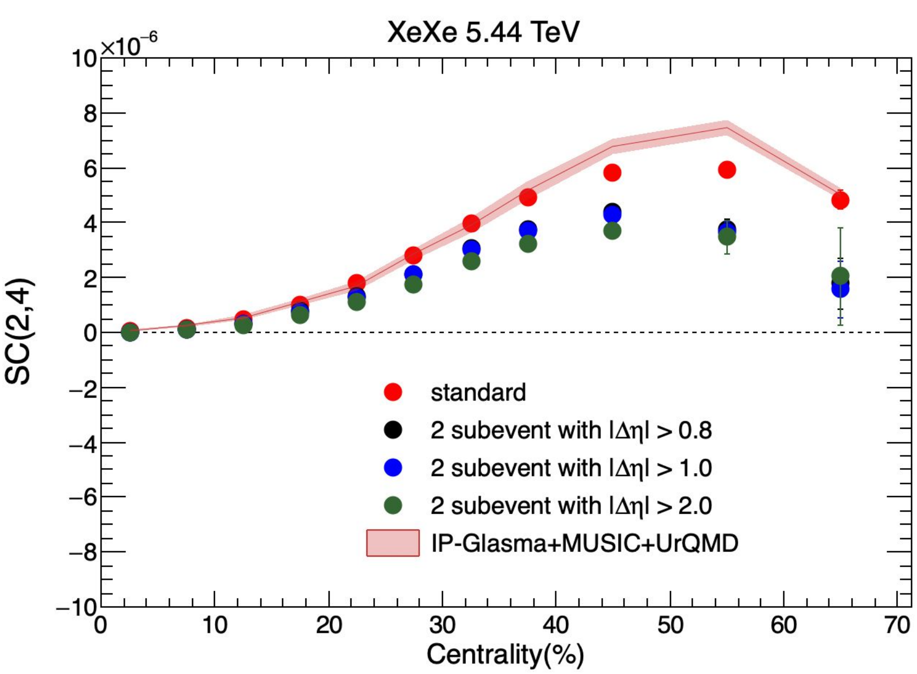
- ❖ **Better non-flow subtraction**

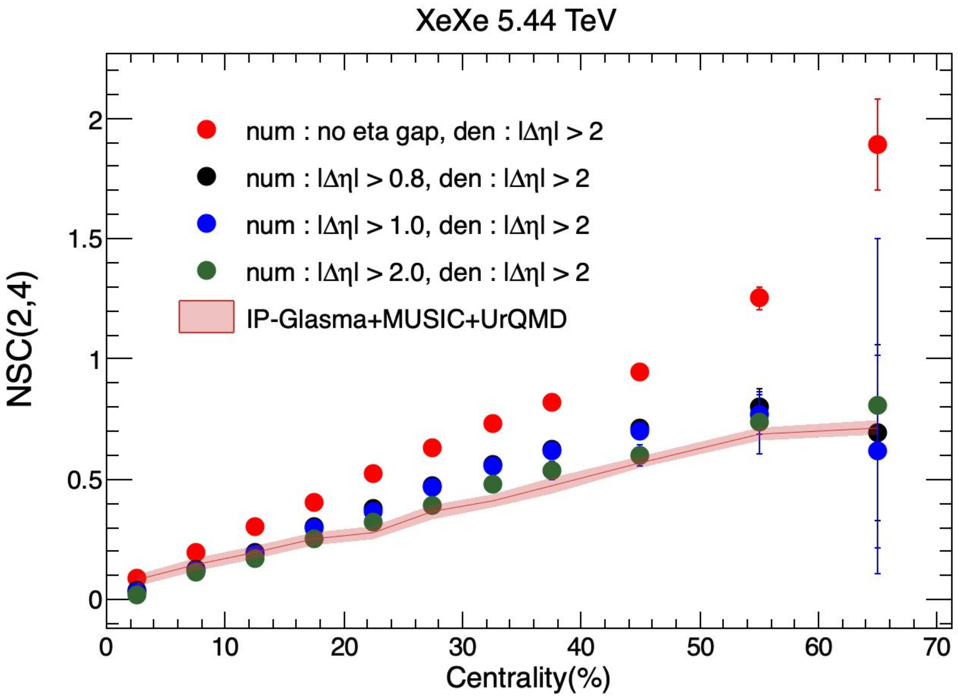


- nMHC(v_2^k, v_3^l) in central and semi-central collisions - direct constraint on the initial correlation between $\langle\varepsilon_2^k\rangle$ and $\langle\varepsilon_3^l\rangle$.
- potential nonlinearity of v_2 and v_3 - more pronounced in peripheral collisions - dynamical evolution of the created QGP

First time measurements : nMHC(v_2^k, v_3^l) in XeXe - what are the effects on v_2 and v_3 in XeXe collisions?
Also checked nMHC(v_2^k, v_4^l) in XeXe and PbPb - initial-state correlations + dynamical system evolution
What effect does nuclear deformation play on the correlation between (ε_2^k,ε_3^l) and (ε_2^k,ε_4^l) ?

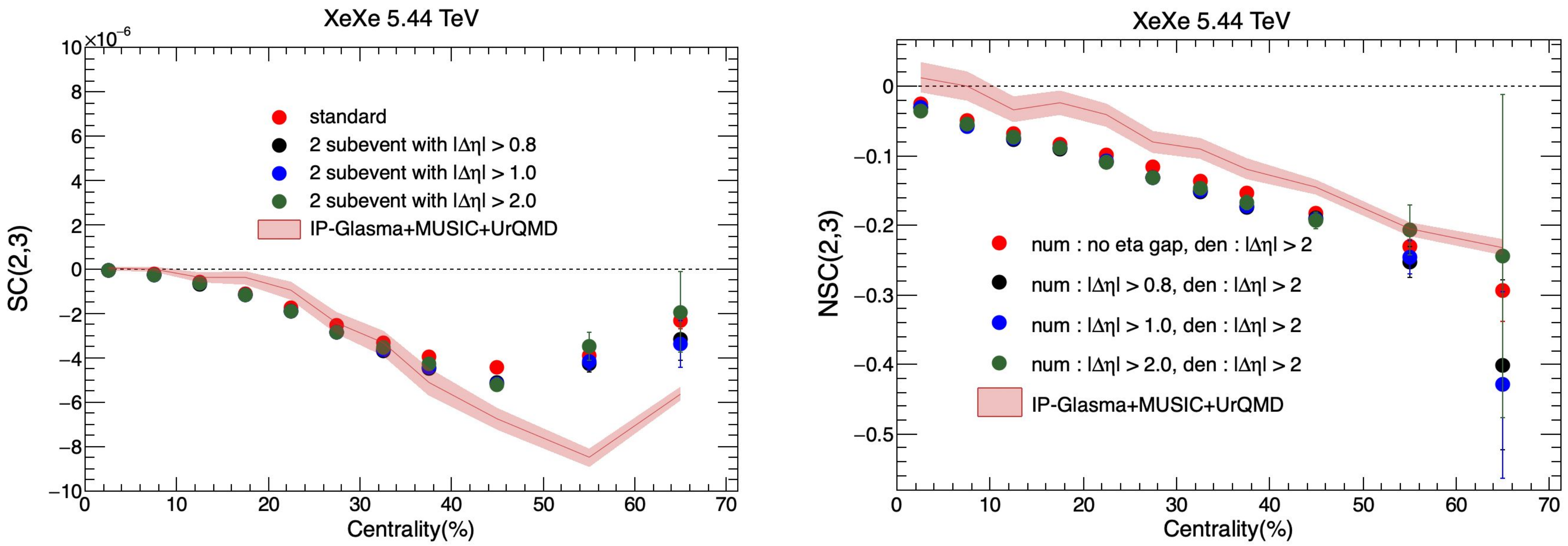
Effect of more non-flow removal in CMS (with 2 subevent for CMS)





- Considerable difference with 2 subevent with eta gap : non-flow still getting removed
- Surprisingly good match with model after taking 2-subevents, after normalization (paper plot)
- Error bar increases a little as we keep increasing eta gap - lower and lower statistics

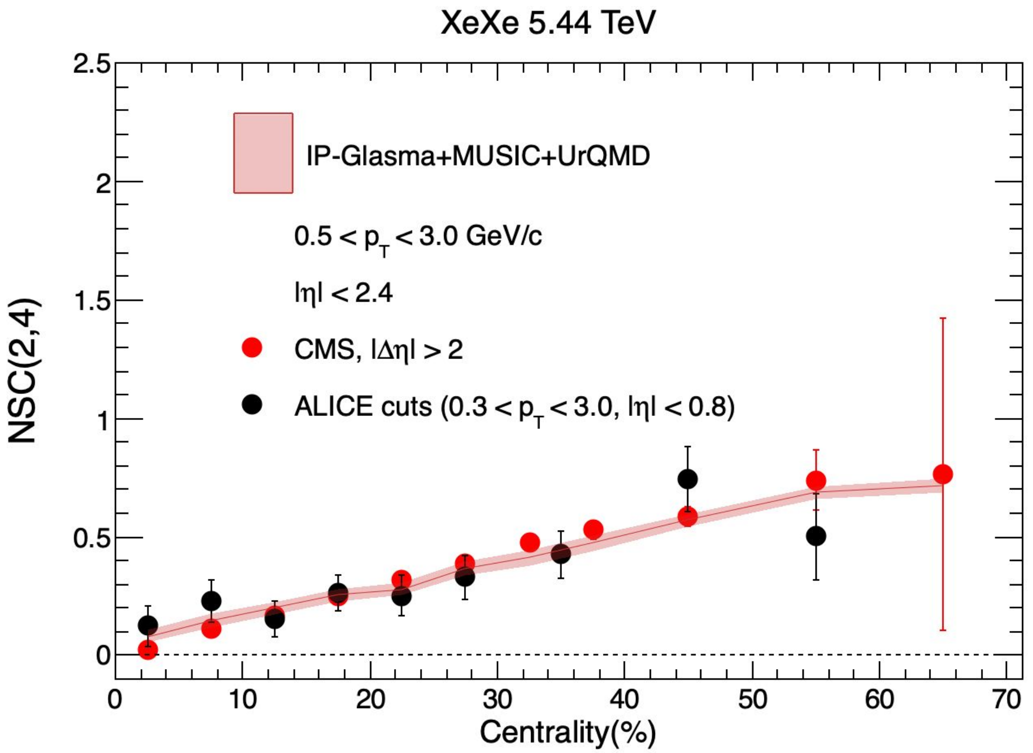
Effect of more non-flow removal in CMS (with 2 subevent for CMS)



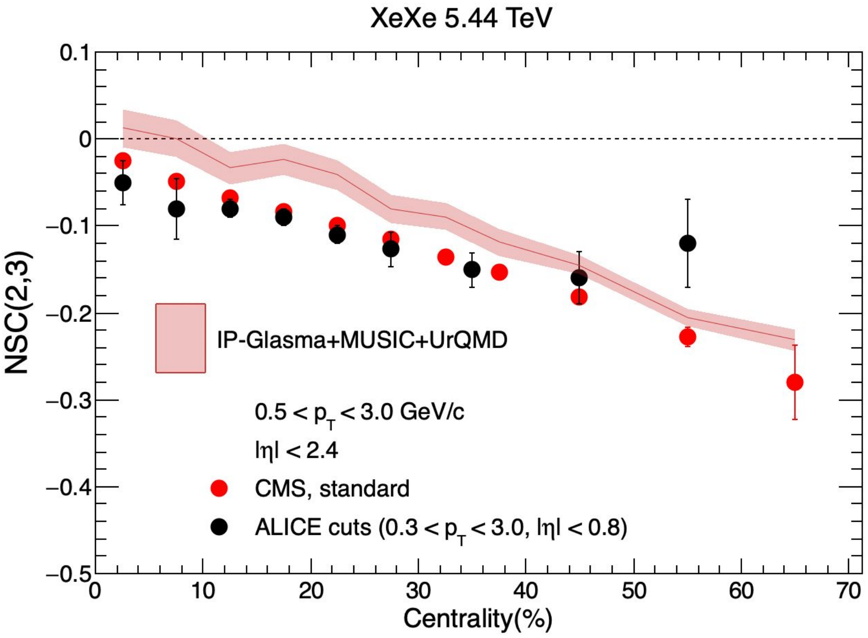
- No significant effect observed for SC(2,3) before or after applying subevents
- Error bar increases a little as we keep increasing eta gap - lower and lower statistics

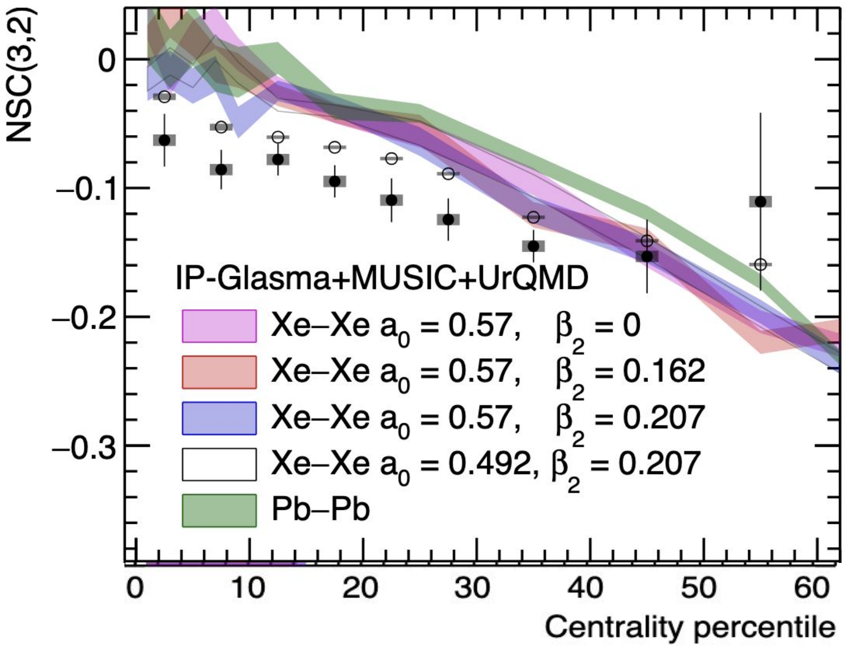
Similar results in backup

Effect of more non-flow removal in CMS (with 2 subevent for CMS) - Comparison with ALICE results



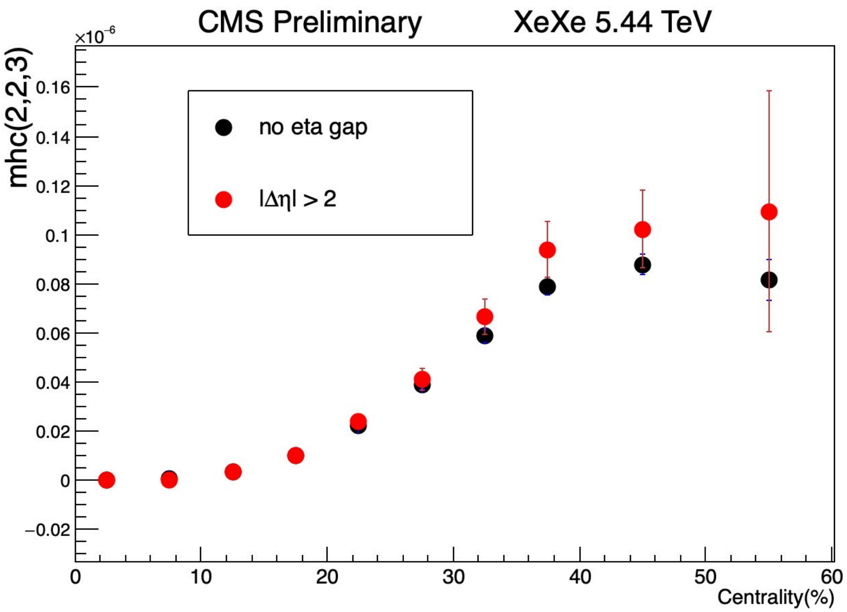
Effect of more non-flow removal in CMS (with 2 subevent for CMS) - Comparison with ALICE results

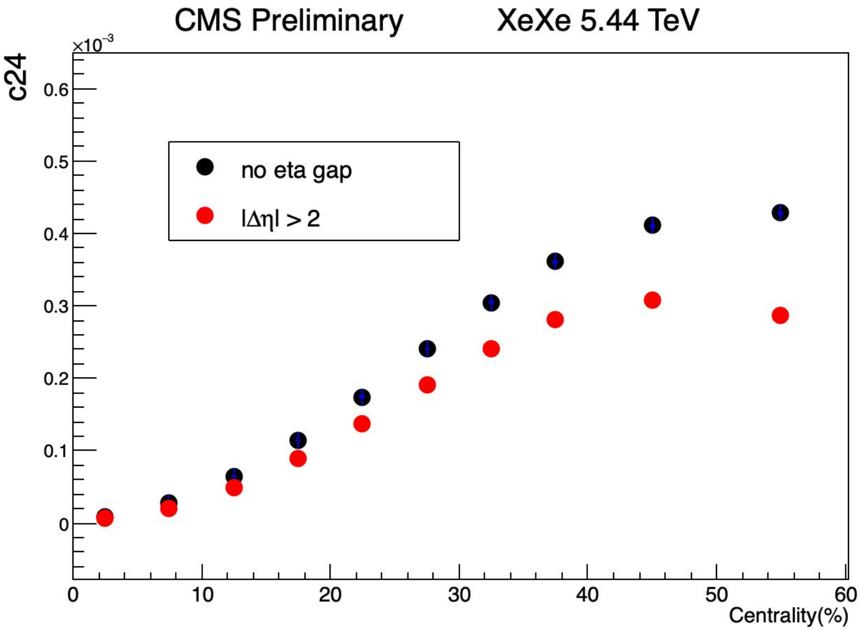




https://arxiv.org/pdf/2409.04343

Effect in 6- and 8-particle cumulants/correlators



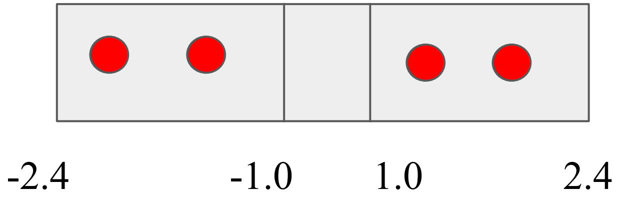


nMHC(2,2,3) = $MHC(v_2^4, v_3^2)/(\langle v_2^4\rangle * \langle v_3^2\rangle)$

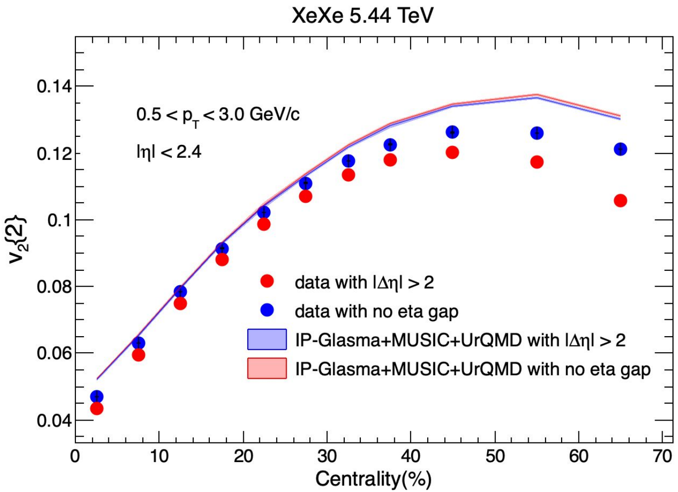
C24 = $\langle v_2^4 \rangle = \langle\langle \cos 2(\phi_{1A}+\phi_{2A}-\phi_{3B}-\phi_{4B}) \rangle\rangle$

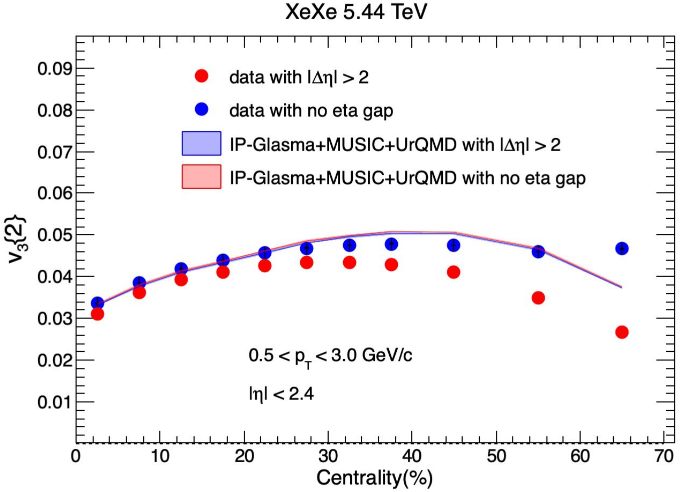
$$nMHC(v_m^k, v_n^l) = \frac{MHC(v_m^k, v_n^l)}{\langle v_m^k\rangle \langle v_n^l\rangle},$$

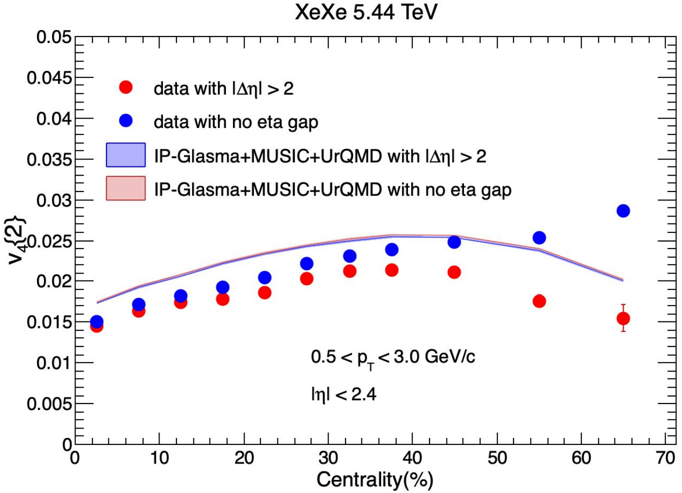
Already applied delta eta since the beginning



Comparison between data and model - with and without eta gap



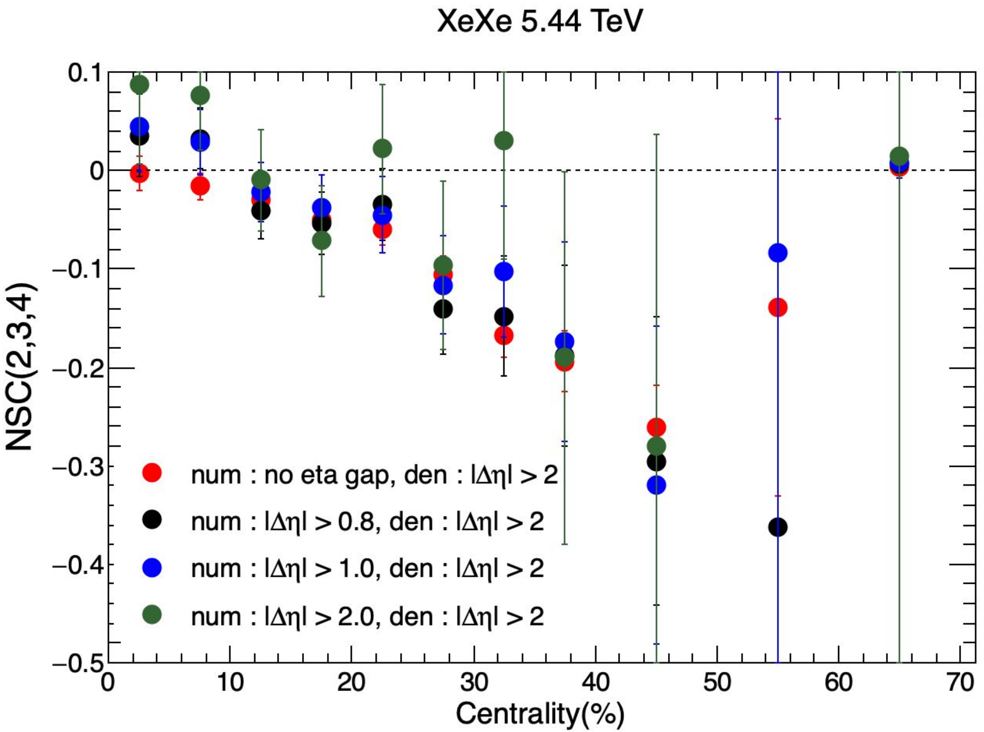




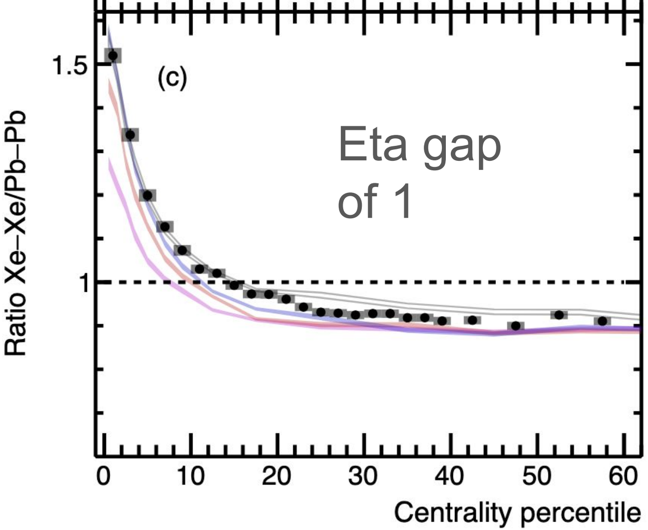
- Obvious difference in peripheral region (without eta gap) after ~ 40% centrality
- Drastic difference between data and model after introducing eta gap, right from the most central region in v_4

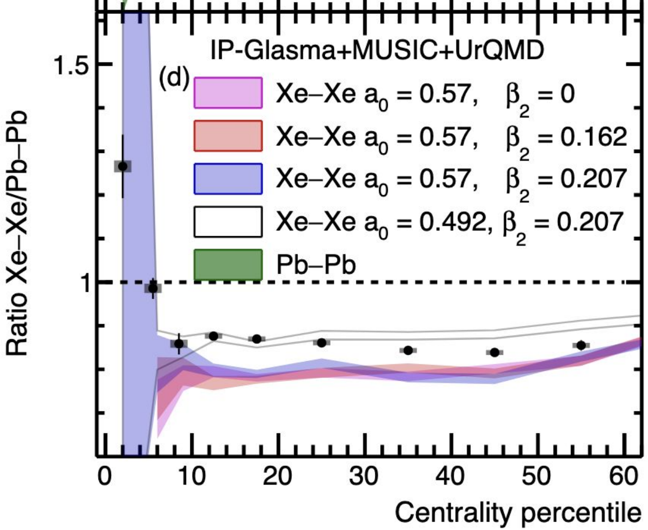
Effect in 6- and 8-particle cumulants

NSC(2,3,4) =
MHC(2,3,4)/($v_2^2 * v_3^2 * v_4^2$)



$$MHC(v_2^2, v_3^2, v_4^2) = \langle v_2^2 v_3^2 v_4^2 \rangle - \langle v_2^2 v_3^2 \rangle \langle v_4^2 \rangle - \langle v_2^2 v_4^2 \rangle \langle v_3^2 \rangle - \langle v_3^2 v_4^2 \rangle \langle v_2^2 \rangle + 2 \langle v_2^2 \rangle \langle v_3^2 \rangle \langle v_4^2 \rangle$$

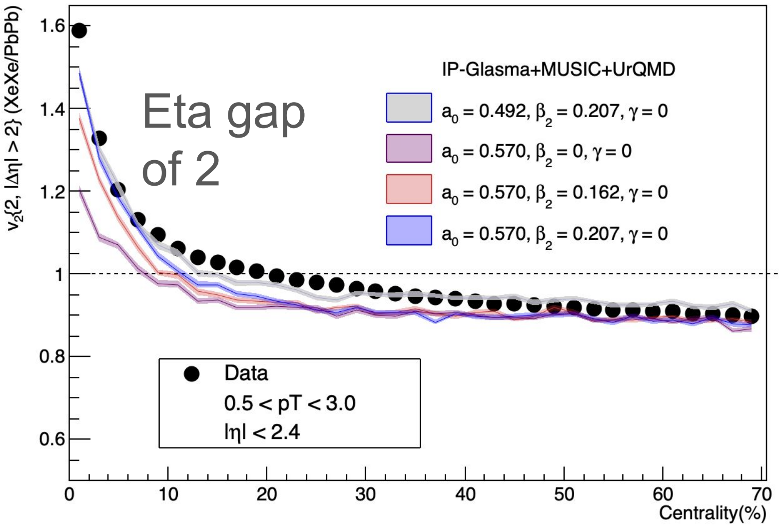


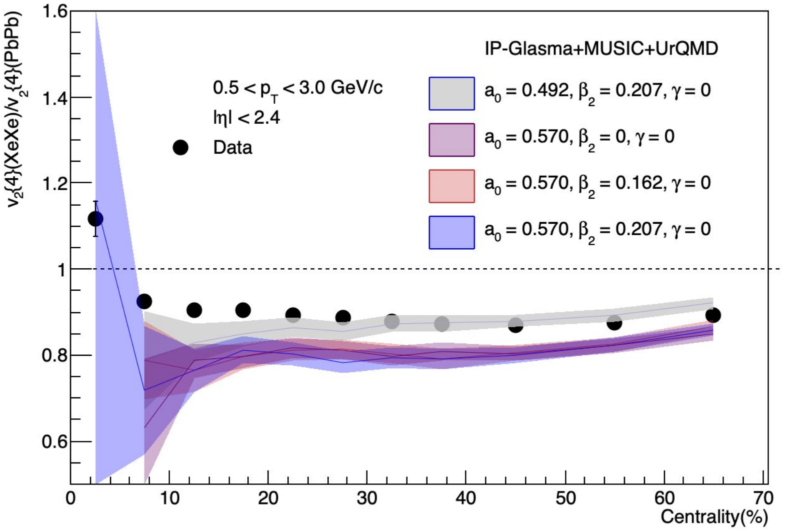


https://arxiv.org/pdf/2409.04343

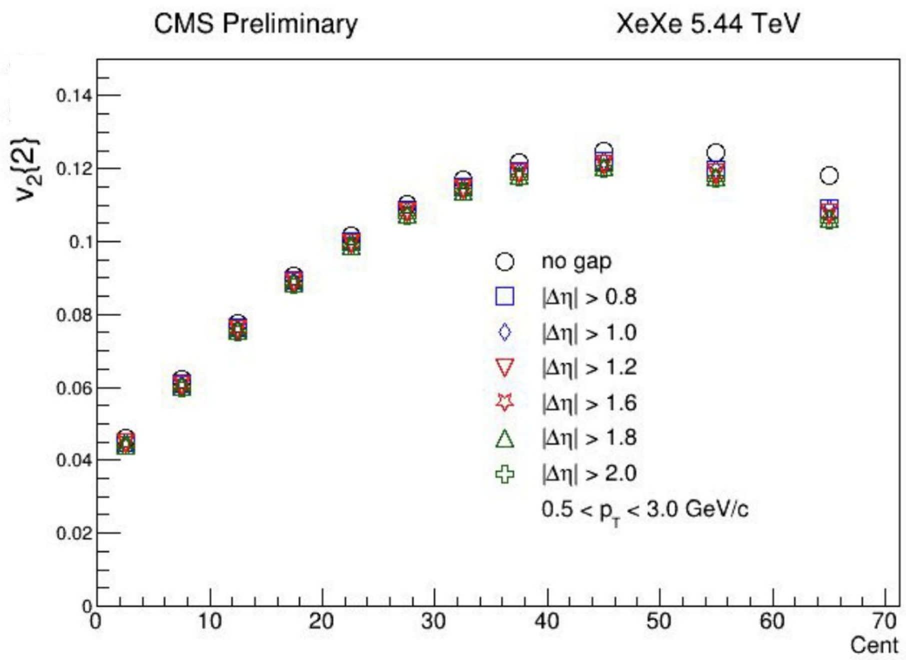
Main message - best fit with (0.492, 0.207) model

IMP : They have diff p_T, eta and eta gap cuts.



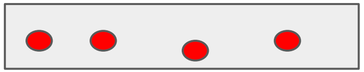


Non-flow removal with increasing eta gap



2 subevent method with eta gap for multiparticle correlations

Standard : $$\langle\langle\cos(m\varphi_1+n\varphi_2-m\varphi_3-n\varphi_4)\rangle\rangle_c = \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle \langle v_n^2\rangle \equiv \mathrm{SC}(m,n)\,.$$

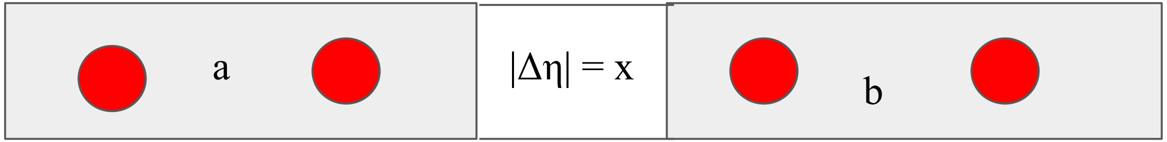


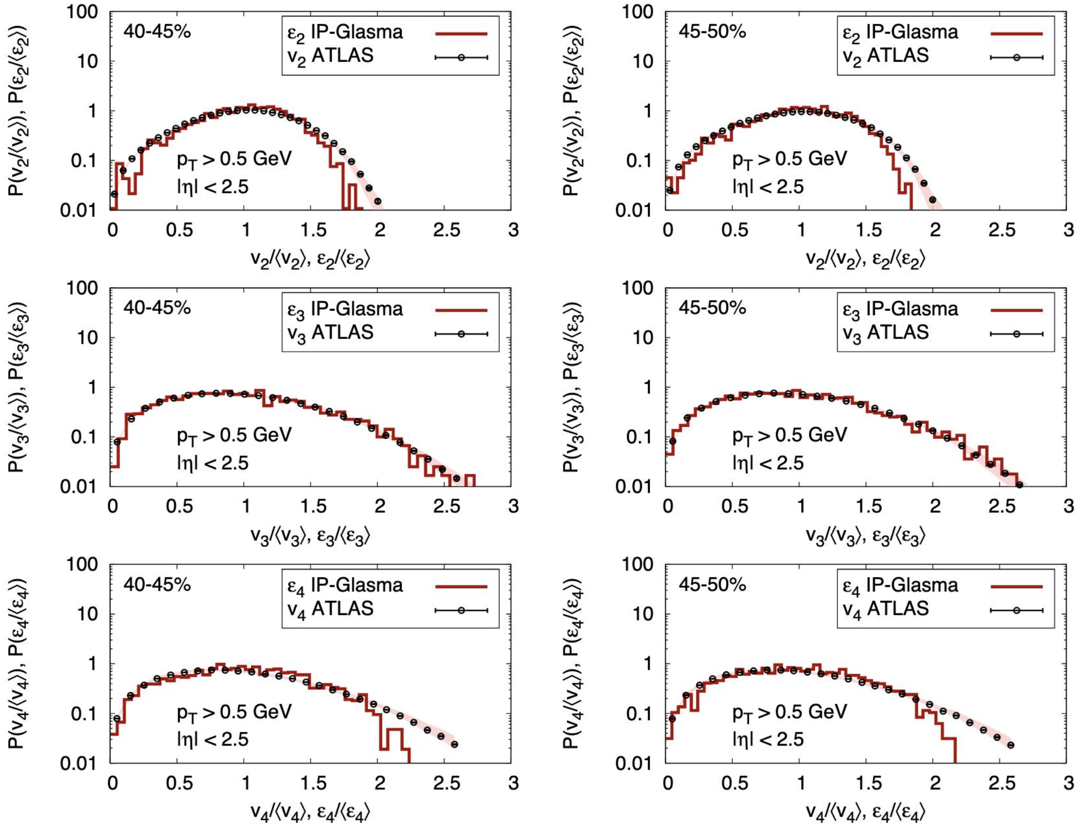
2 subevent method with eta gap : https://arxiv.org/pdf/1710.07567, https://arxiv.org/pdf/1705.04377, https://arxiv.org/pdf/2409.04343, https://arxiv.org/pdf/2102.12180

$$\mathrm{SC}(n,m)_{2-\mathrm{sub}} = \left\langle\!\left\langle \mathrm{e}^{in(\phi_1^a-\phi_2^b)+im(\phi_3^a-\phi_4^b)} \right\rangle\!\right\rangle - \left\langle\!\left\langle \mathrm{e}^{in(\phi_1^a-\phi_2^b)} \right\rangle\!\right\rangle \left\langle\!\left\langle \mathrm{e}^{im(\phi_1^a-\phi_2^b)} \right\rangle\!\right\rangle$$

We have explored 3 cases :

(i) |Δη| > 0.8 (ALICE)
(ii) |Δη| > 1.0
(iii) **|Δη| > 2.0**



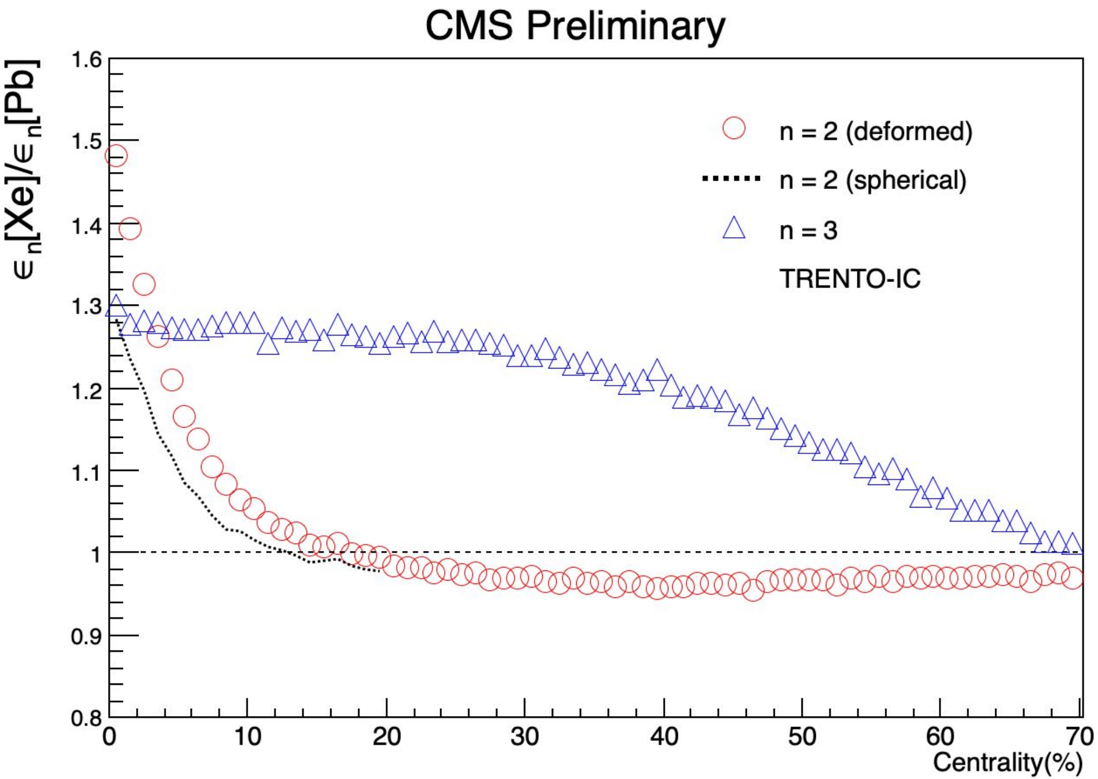


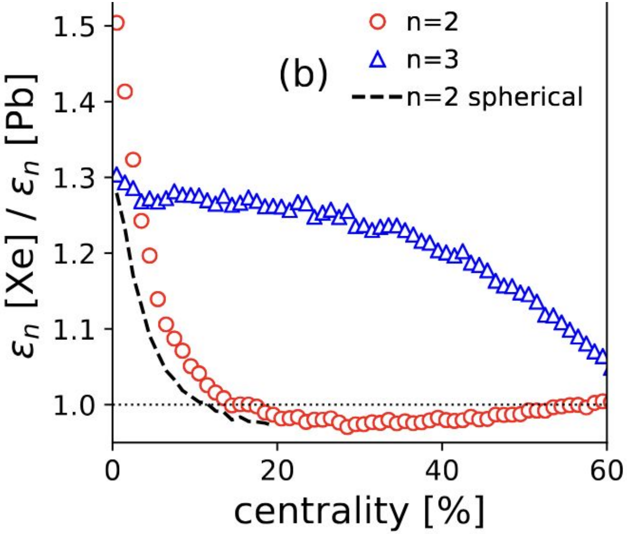
https://arxiv.org/pdf/1312.5588,
https://arxiv.org/pdf/1209.6330

- Important point to note for IC models :
  "v_2 and v_4 for centralities > 25% : The eccentricities miss the contribution to v_4 from non-linear coupling to the second harmonic and vice versa. Certain events with a large ε_2 will generate a large
  contribution to v_4 during the here neglected evolution."
  [https://arxiv.org/pdf/1312.5588]
- So no, non-linearity has not been included in the models. IS models assume - $v_n = a'\varepsilon_n$
- Actual dependence likely expected to form during evolution of the system :
  $v_4 = a\varepsilon_4 + b(\varepsilon_2)^2$
- Same observed for TRENTo-IC

Question 1 : In the simulations, what might happen if we consider the same shape for PbPb and XeXe, but different sizes? Good to compare that and then go to deformation to understand the effect to differentiate from final-state interaction effects. Try the v2/ε2 ratio to compare among XeXe and PbPb.

Solution :





Phys. Rev. C 97, 034904 (2018)

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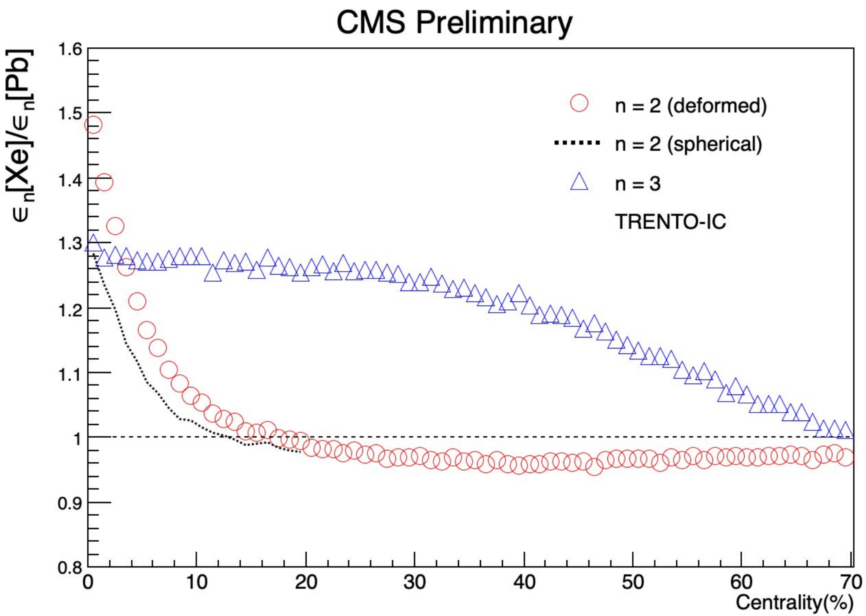


- Short-range fluctuations of the initial density profile - Their effect is typically proportional to $A^{-1/2} \simeq 1.27$ in most central region
- Explained well for v_3 and v_2 (spherical Xe)
- Significant deviation (upto 20%) in deformed Xe - non-zero β_2 in Woods-Saxon profile.
- After 20%, not much difference is seen.

$$\varepsilon_2 = \sqrt{\varepsilon_{\mathrm{RP}}^2 + \sigma^2},$$

Phys. Rev. C 97, 034904 (2018)

Woods-Saxon profile

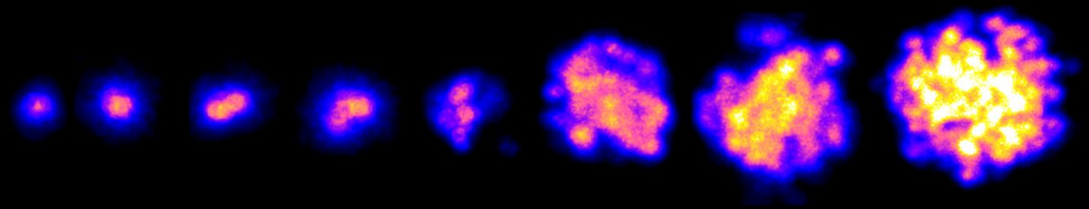
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{[r-R(\theta,\phi)]/a_0}},$$

$$R(\theta,\phi) = R_0(1+\beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^{3} \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^{4} \alpha_{4,m} Y_{4,m})$$

https://indico.cern.ch/event/975877/contributions/4118500/attachments/2185275/3692705/OO_and_pO.pdf

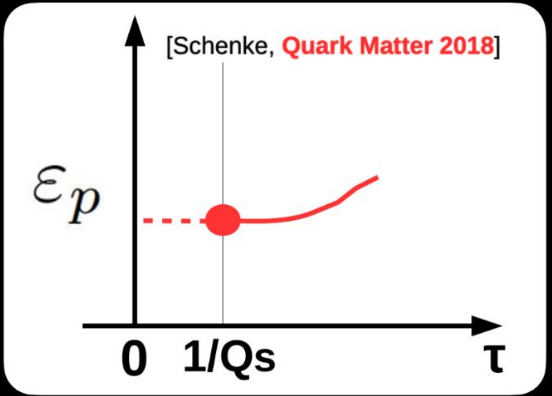
INITIAL STATE AND PRE-EQUILIBRIUM DYNAMICS: IP-GLASMA

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)



Includes fluctuations of:

Impact parameter, nucleon positions, quark positions, color charge normalization, color charges



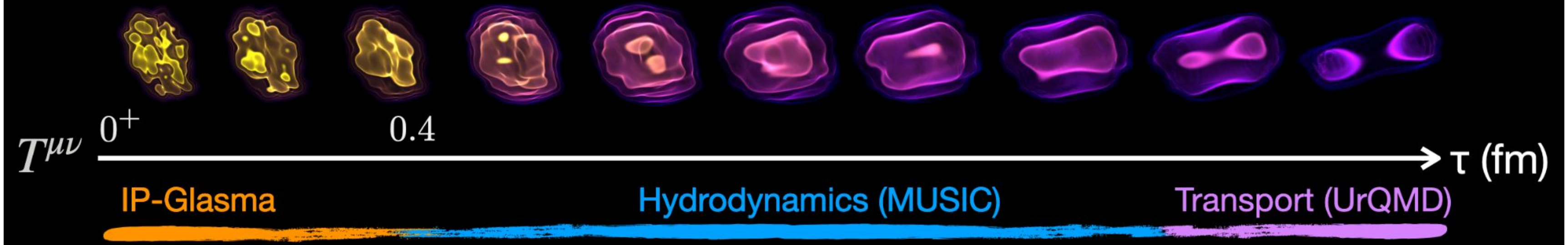
- The Color Glass Condensate (CGC) predicts anisotropic particle productions because of
  1. Local anisotropies in the color fields
  2. Local density gradients
  3. Quantum interference effects

$$\mathcal{E}_p = \varepsilon_p e^{i2\psi_2^p} = \frac{\langle T^{xx} - T^{yy}\rangle + i\langle 2T^{xy}\rangle}{\langle T^{xx} + T^{yy}\rangle}$$

https://indico.cern.ch/event/975877/contributions/4118500/attachments/2185275/3692705/OO_and_pO.pdf

THE HYBRID THEORETICAL FRAMEWORK

B. Schenke, C. Shen and P. Tribedy, Phys. Rev. C **102**, 044905 (2020)



$$T^{\mu\nu}_{\mathrm{CYM}} = T^{\mu\nu}_{\mathrm{hydro}}$$

+ Landau Matching with lattice EoS

Cooper-Frye particlization

- Continuously connect the system's energy-momentum tensor $T^{\mu\nu}$ between different stages
- Pb+Pb, O+O, p+Pb, and p+O collisions probe different phases with different weights

TRENTo-IC

- The TRENTo-IC model [http://www.arxiv.org/pdf/1412.4708] makes no assumption about specific physical mechanisms for entropy production, pre-equilibrium dynamics, or thermalization.

- It deposits the entropy proportional to the generalized (usually geometric) mean of nuclear overlap density between the two colliding nuclei and gives values of the initial-state eccentricities

Subgroup Method for Error Calculation

- n = 6 subgroups have been used.
- Following formula have been used :

$$\text{Mean : } x = (\sum x_i)/6$$

$$\text{Standard deviation : } \sigma = [(\sum(x_i - x)^2)/5]^{½}$$

$$\text{Error : } e = \sigma/(6)^{1/2}$$