Effects of memory on quarkonium evolution in QGP

ATHIC 2025
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Introduction

- Matsui and Satz (1986) proposed quarkonium suppression as a signal of QGP
- Quarkonia measurements can provide information about QGP medium.
- This requires for the accurate modeling of quarkonia dynamics in the medium.
- Open Quantum System framework gives a natural way to model this
- There have been several studies using OQS with NRQCD[1] and pNRQCD[2]

[1] Akamatsu, "Heavy quark master equations in the Lindblad form at high temperatures"; Blaizot et al., "Heavy quark bound states in a quark–gluon plasma: Dissociation and recombination"; Sharma and Tiwari, "Quantum evolution of quarkonia with correlated and uncorrelated noise".

[2] Brambilla, Escobedo, Strickland, et al., "Bottomonium suppression in an open quantum system using the quantum trajectories method".

Density matrix approach

- System+Environment can be characterised by density matrix

$$i\frac{d\rho_{tot}}{dt} = [H_{tot}, \rho_{tot}] \quad \rho_{tot} = \sum_k p_k |\psi_k(t)\rangle\langle\psi_k(t)| \tag{1}$$

$$H_{tot} = H_{\mathrm{S}} + H_{\mathrm{E}} + H_I \tag{2}$$

- System evolution is described by the reduced density matrix ρ_S[3]

$$\rho_{\mathrm{S}} = \mathrm{Tr}_{\mathrm{E}}(\rho_{tot}) \tag{3}$$

- Consider the following master equation, accurate upto $O(H_I^3)$

$$\frac{\partial \rho_{\mathrm{S}}}{\partial t} = -\int_0^t ds\,[H_I(t), [H_I(t-s), \rho_{\mathrm{S}}(t)\cdot\rho_{\mathrm{E}}]]$$

[3] H.-P. Breuer and Francesco Petruccione, *The Theory of Open Quantum Systems*.

- $M \gg 1/r \sim Mv \gg E_b \sim Mv^2 \rightarrow$ pNRQCD to model the quarkonium
- In COM of the quarkonia, system Hamiltonian is given by,

$$\begin{aligned} H_{\mathrm{S}} &= h_s|s\rangle\langle s| + h_o|a\rangle\langle a|, \quad \text{where,} \quad h_{s,o}(r) = \frac{p^2}{M} + v_{s,o}(r), \\ H_{\mathrm{Int}} &= -\, g\mathbf{E}^a \cdot \, \mathbf{r}\left[\frac{1}{\sqrt{2N_c}}|s\rangle\langle a| + \frac{1}{\sqrt{2N_c}}|a\rangle\langle s| + \frac{d_{abc}}{2}|b\rangle\langle c|\right] \end{aligned} \quad (4)$$

- Define singlet and octet blocks of the density matrix.

$$\rho_S(t) = \langle s|\rho|s\rangle \qquad \rho_O(t) = \langle a|\rho|a\rangle$$

- We get for $\rho_{\rm S} = (\rho_s \quad \rho_o)$

$$\frac{d\rho_{\rm S}}{dt} = -iH_{\rm eff}\rho_{\rm S} + \int_0^t ds \left\{ \Gamma(t,s) \sum_{n=1}^{3} \mathbf{V}_n(s)\rho_{\rm S}(t)\mathbf{V}_n^\dagger(0) \right\} + \mathrm{H.C} \quad (5)$$

where,

$$H_{\rm eff} = H_{\rm S} - i \int_0^t ds\, \Gamma(t,s) \sum_{n=1}^{3} \mathbf{V}_n^\dagger(0)\mathbf{V}_n(s) \;. \quad (6)$$

- $\{V_{ni}(t)\}$ are transition operators corresponding to $s \to o$, $o \to s$ and $o \to o$ transitions.

$$V_{ni}(t) \sim e^{ih_u t} r_i e^{-ih_v t} |u\rangle\langle v|$$

- All the information about the environment goes into the correlator

$$\Gamma(t,s) = \frac{g^2}{6N_c} \mathrm{Tr}_{\rm E}\Big(E_i^a(t,\vec{0}) E_i^a(s,\vec{0}) \rho_{\rm E} \Big) \;. \quad (7)$$

- Relevant scales are: system time scale τ_S and environment relaxation scale τ_E

$$\tau_s \sim E_b^{-1} \quad \tau_E \sim \frac{1}{T}$$

- If one assumes $\tau_{\mathrm{E}} \ll \tau_{\mathrm{S}}$, one can make the following expansion:

$$\mathbf{V}_n(t) \sim e^{-ih_\alpha t}\mathbf{r}e^{ih_\beta t} \approx \mathbf{r} - it(h_\alpha \mathbf{r} - \mathbf{r}h_\beta) + \mathcal{O}\left[\left(\frac{\tau_{\mathrm{E}}}{\tau_{\mathrm{S}}}\right)^2\right]. \quad (8)$$

- Truncation at the first term and second term gives leading order (LO)[4] & next-to-leading order (NLO)[5] equations.
- For bottomonium $\tau_s^{-1} \sim 450\ MeV$ ($Coulomb\, 1S$) $\rightarrow$ hierarchy fails

[4]Brambilla, Escobedo, Strickland, et al., "Bottomonium suppression in an open quantum system using the quantum trajectories method".

[5]Brambilla, Escobedo, Islam, et al., "Heavy quarkonium dynamics at next-to-leading order in the binding energy over temperature".

- Hence we need to solve the general non-Markovian equation.
- $\rho_{\mathrm{S}}(t)$ can be reduced into block diagonal form in angular momentum basis.

$$\rho^{l}_{\mathrm{S}}(t) = \sum_{m} \langle l, m|\rho_{\mathrm{S}}(t)|l, m\rangle$$

- In angular momentum basis,

$$\frac{\partial \rho^{l}_{\mathrm{S}}}{\partial t} = -ih^{l}_{eff}\rho_{\mathrm{S}}(t) + \int_{0}^{t} ds \sum_{n}\sum_{l'} T_n(l \to l'|s)\rho^{l'}_{\mathrm{S}}(t) T^{\dagger}_{n}(l \to l'|0) + h.c$$

- T_n's are transition operators that change $l \to l'$ and also the color states.
- As the interaction is $\sim r.E$, only transitions that take $l \to l \pm 1$ are allowed.

Results

- Numerical solution is generally through stochastic methods such as Unravelling, Quantum diffusion etc.
- Computationally intensive to solve the full density matrix equation
- Shown in[6], H_{eff} gives reasonable estimates for $\Upsilon(1S)$.
- In this study the correlation function used is

$$\Gamma(t,s) = \frac{\kappa}{2\tau_{\mathrm{E}}} e^{-|t-s|/\tau_{\mathrm{E}}}$$

  For the plots shown we take $\kappa/T^3 = 4.0$
- First we show a comparison with LO and NLO with a Bjorken background

[6] Brambilla, Escobedo, Islam, et al., "Heavy quarkonium dynamics at next-to-leading order in the binding energy over temperature".

Results

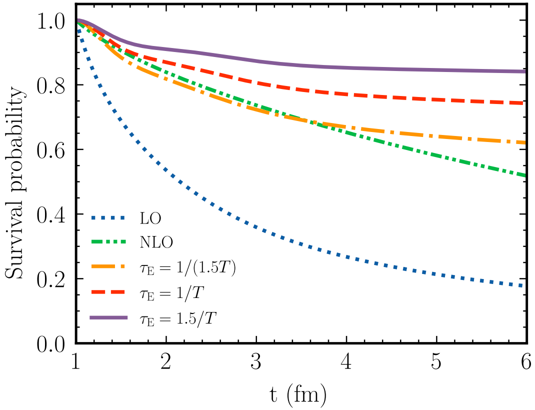

Figure: $\Upsilon(1S)$ evolution with a Bjorken medium: $T_0 = 330$ MeV, $\kappa/T^3 = 4.0$

- We use AMPT[7] + (2+1) viscous hydrodynamics to model the background.
- We present $\Upsilon(1S)$ survival probabilities at $\sqrt{s}$ 5.02 TeV and 2.76 TeV

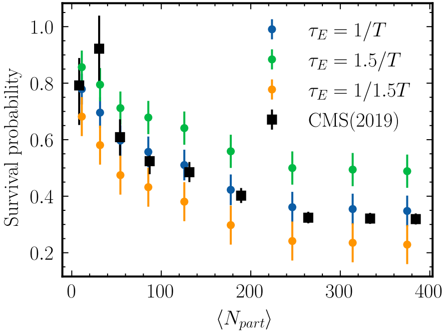

Figure: $\Upsilon(1S)$ survival probability for PbPb@ 5.02 TeV: Comparison of OQS wi CMS(2017)[8]

[7] Lin et al., "Multiphase transport model for relativistic heavy ion collisions".
[8] Sirunyan et al., "Measurement of nuclear modification factors of Υ(1S), Υ(2S), and Υ(3S) mesons in PbPb collisions at sNN=5.02 TeV".

Results

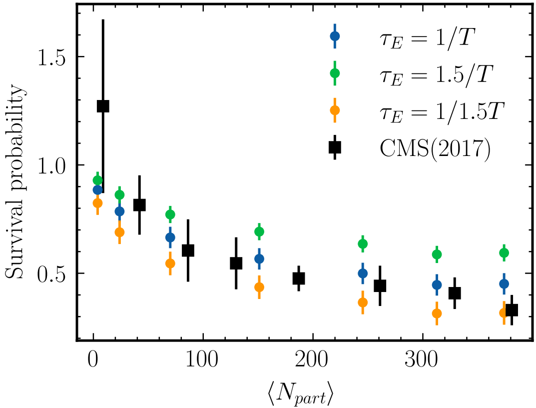

Figure: $\Upsilon(1S)$ survival probability PbPb@2.76 TeV: Comparison of OQS with CMS(2017)[9] data.

[9] Khachatryan et al., "Suppression of Υ(1S), Υ(2S), and Υ(3S) quarkonium states in PbPb collisions at sNN=2.76TeV".

Results

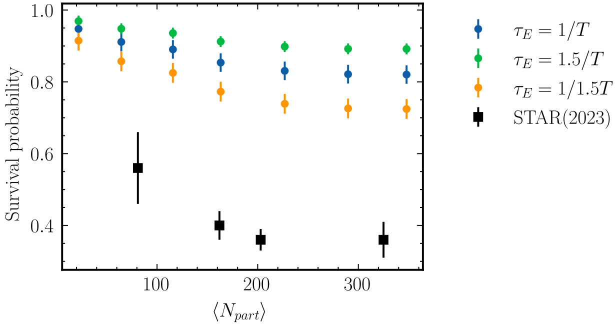

Figure: $\Upsilon(1S)$ survival probability AuAu@200 GeV: Comparison of OQS with STAR(2023)[10] data.

[10] Aboona et al., "Measurement of Sequential Υ Suppression in $\mathrm{Au} + \mathrm{Au}$ Collisions at $\sqrt{s_{NN}} = 200\ \mathrm{GeV}$ with the STAR Experiment".

Conclusion

- We have derived a general master-equation in non-Markovian limit
- Calculated survival probabilities for $\Upsilon(1S)$ at various collision energies with a realistic background.
- Survival probability of 1S is sensitive to hierarchy between τ_{S} and τ_{E}
- τ_{S}, τ_{E} hierarchy is crucial for $\Upsilon(1S)$ and can receive large contributions.
- Inclusion of memory decreases suppression.

Acknowledgements

We would like to thank Prof. Subrata Pal (TIFR, Mumbai) for providing the hydrodynamics temperature profiles for 200 GeV, 2.76 TeV and 5.02 TeV energies.

- Hamiltonian of system + Interactions

$$H = \frac{p^2}{M} - \frac{C_F \alpha_s}{r} |s\rangle\langle s| + \frac{\alpha_s}{2N_c r} |a\rangle\langle a| \\ - \vec{r} \cdot \; g\vec{E}^a \left[\frac{1}{\sqrt{2N_c}} |s\rangle\langle a| + \frac{1}{\sqrt{2N_c}} |a\rangle\langle s| + \frac{d_{abc}}{2} |b\rangle\langle c| \right]$$

- Lindblad

$$\frac{\partial \rho_{sys}}{\partial t} = -i(H_{eff}\,\rho_{sys} - \rho_{sys} H_{eff}^\dagger) + \sum_i C_i \rho_{sys} C_i^\dagger$$

Backup: Numerical simulation

- The Non-Markovian density matrix equation can be solved using quantum trajectories method.
- Consider a general master equation

$$\frac{\partial \rho_{sys}}{\partial t} = A(t)\rho_{sys} + \rho_{sys}B^\dagger(t) + \sum_i C_i(t)\rho_{sys}D_i^\dagger(t)$$

- The idea[11] is to define a two component wavefunction:

$$|\psi(t)\rangle = (|\phi_1(t)\rangle \;\; |\phi_2(t)\rangle)$$

- Then evolve the wavefunction using H_{eff} inter spaced with quantum jumps $J_i|\psi\rangle$

$$Heff = diag(A(t) \;\; B(t)) \quad J_i = diag(C_i(t) \;\; D_i(t))$$

- Averaging $|\phi_1\rangle\langle\phi_2|$ over different realisations gives the density matrix

[11] H. P. Breuer, Kappler, and F. Petruccione, "Stochastic wave function method for non-markovian quantum master equations".