Study of Heavy Quark Momentum Broadening in a Non-Abelian Plasma in- and out-of-equilibrium

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In collaboration with Sören Schlichting & Sayantan Sharma,
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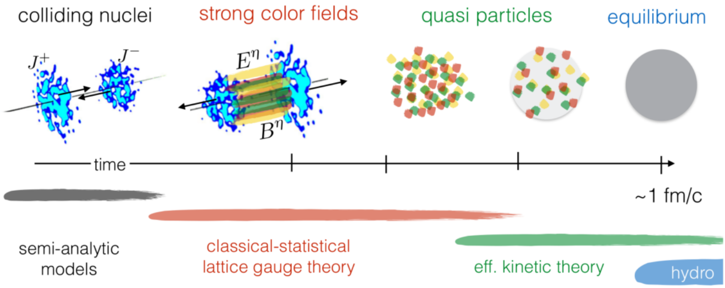
Why is studying heavy quark dynamics inside a non-Abelian plasma essential?

- In a heavy-ion collision, heavy quarks are formed in the very early stages $\sim$ 0.1 fm/c.
- Charm quarks (usually considered to be heavy) have shown collective behaviour similar to light quarks.

[ALICE Collaboration, S. Acharya et al., 18, Fig. courtesy S. Schlichting]



Why is studying heavy quark dynamics inside a non-Abelian plasma essential?

- To model their elliptic flow, heavy quark diffusion coefficient κ is an important ingredient. (See Dibyendu Bala's talk)

$$\tau \sim \frac{4\pi \; m \; T}{\kappa}$$

- A sizable contribution to flow comes from the non-eq. phase, where κ is calculated:
  1. Using Langevin dynamics,
  2. Using Wong Equations in Glasma, [Pooja, S. K. Das, L. Oliva, M. Ruggieri, 22] (See Pooja's talk)
  3. In the infinite mass limit from color electric field 2-point correlator.

     [K. Boguslavski, A. Kurkela, T. Lappi, J. Peuron, 20]

- We have developed a novel lattice technique to study heavy quark momentum broadening treating quarks as a Dirac particle.

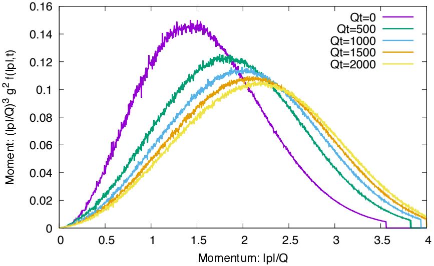
Initial Conditions: Non-Abelian SU(2) plasma in the self-similar regime

- Lattice parameters $\longrightarrow$ large volume $N_s^3 = 256^3$ lattice, with lattice spacing $Qa_s = 0.5$, with $N_c = 2$ and $N_f = 1$.

- Initial phase-space distribution of the gluons, motivated from Color Glass condensate effective theory [L. McLerran and R. Venugopalan, 94]

$$g^2 f_g(p) = n_0 \frac{Q}{p} e^{-\frac{p^2}{2Q^2}}$$

where $n_0/g^2 >> 1$.
We've chosen n_0 to be 0.2.



- Starting from this initial condition, we evolve gauge fields classically using Hamilton's equations.

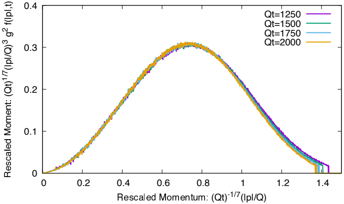
Initial Conditions: Non-Abelian SU(2) plasma in the self-similar regime

- To be deep within the self-similar scaling regime we have evolved the gauge fields till $Qt = 1500$ where the distribution is:

[J. Berges, K. Boguslavski, S. Schlichting, and R. Venugopalan, 14]

$$\left(\frac{\tilde{p}}{Q}\right)^3 f_S(\tilde{p}) = (Qt)^{\frac{1}{7}} \left(\frac{|\mathbf{p}|}{Q}\right)^3 f(|\mathbf{p}|, t)$$

(Here, $\tilde{p} = (Qt)^{-\frac{1}{7}} |\mathbf{p}|$.)



- In analogy to the equilibrium plasma, there is a clear separation of scales here as well

$$\underset{\sim Q(Qt)^{-3/10}}{\sqrt{\sigma(t)}} < \underset{\sim Q(Qt)^{-1/7}}{m_D(t)} \ll \underset{\sim Q(Qt)^{1/7}}{\Lambda(t)}$$

- Plasma in the self-similar regime represents a characteristic non-equilibrium state which we use as our initial state.

Evolving the heavy quarks

- We have implemented the evolution of heavy quarks as relativistic particles using Wilson-Dirac Hamiltonian on the lattice.

$$\hat{H}_f = \sum_{\mathbf{x}} \hat{\psi}^\dagger_{\mathbf{x}} \gamma^0 (-i \not{D}_W + m) \hat{\psi}_{\mathbf{x}}$$

$$i\gamma^0 \partial_{x^0} \hat{\psi}_{\mathbf{x}} = (-i \not{D}_W + m) \hat{\psi}_{\mathbf{x}}$$

- Our formalism is thus much more general in comparison to studies done earlier in the infinite-mass limit with non-relativistic quarks.

- Chose a wide set of quark masses, $m/Q = 0.006 - 12.0$.
  For $Q \sim 1\ \mathrm{GeV}$, the choice of $m/Q = 1.2$ represents a particle with mass close to that of the charm quark.

Momentum Broadening: How do we calculate it?

HP, S. Schlichting, S. Sharma, PRL 132, 222301 (2024)

- We start with a single quark in a fixed momentum ($\mathbf{P}$) and spin polarization (s) mode and let it evolve in the background of gauge fields. The quark field after a time t' is

$$\Psi(t',\mathbf{x}) = \frac{1}{\sqrt{N^3}} \sum_{\lambda,\mathbf{p}} \left[\phi^u_{\lambda,\mathbf{p}}(t',\mathbf{x}) b_\lambda(t'=0,\mathbf{p}) + \phi^v_{\lambda,\mathbf{p}}(t',\mathbf{x}) d^\dagger_\lambda(t'=0,\mathbf{p}) \right]$$

with initial conditions

$$\langle b^\dagger_\lambda(t=0,\mathbf{p}) b_{\lambda'}(t=0,\mathbf{p}')\rangle = \delta_{\lambda\lambda'}\ \delta_{\mathbf{pp}'}\ \delta_{\lambda s}\ \delta_{\mathbf{pP}}$$

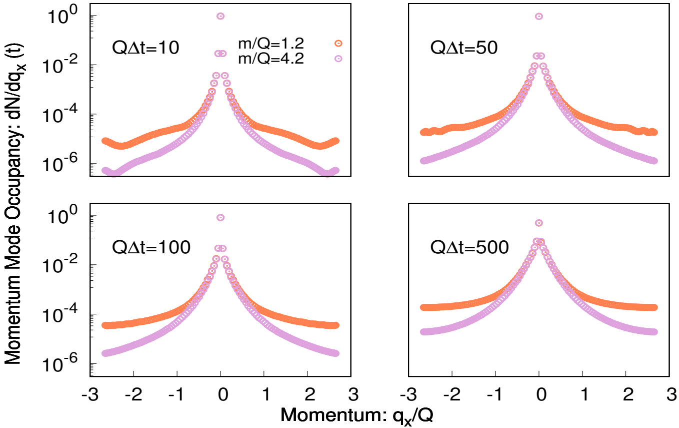
$$\langle d^\dagger_\lambda(t=0,\mathbf{p}) d_{\lambda'}(t=0,\mathbf{p}')\rangle = 0$$

- Momentum mode occupancy is then calculated as,

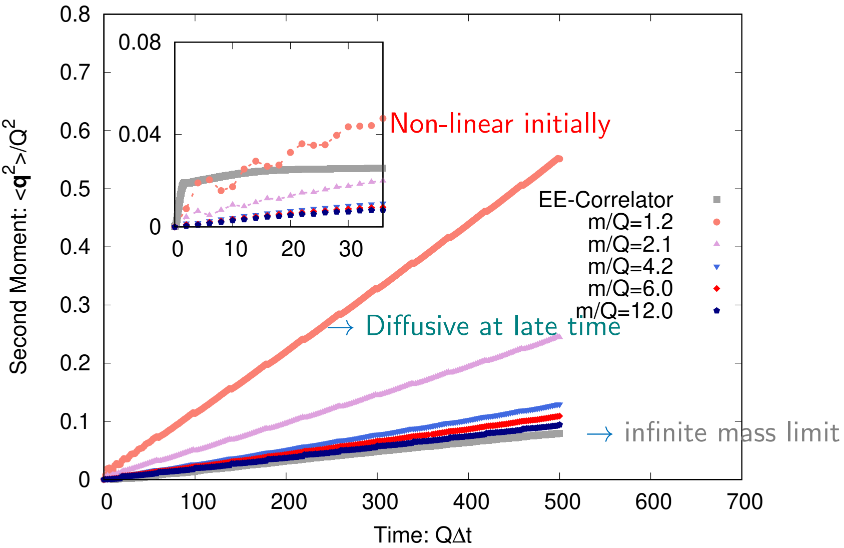
$$\frac{dN}{d^3\mathbf{q}} = \frac{1}{2N_c} \sum_{\lambda'} \langle b^\dagger_{\lambda'}(t',\mathbf{q}) b_{\lambda'}(t',\mathbf{q})\rangle = \frac{1}{2N_c} \sum_{\lambda,\lambda'} |u^\dagger_{\lambda'}(\mathbf{q}) \tilde{\phi}^u_\lambda(t',\mathbf{q})|^2$$

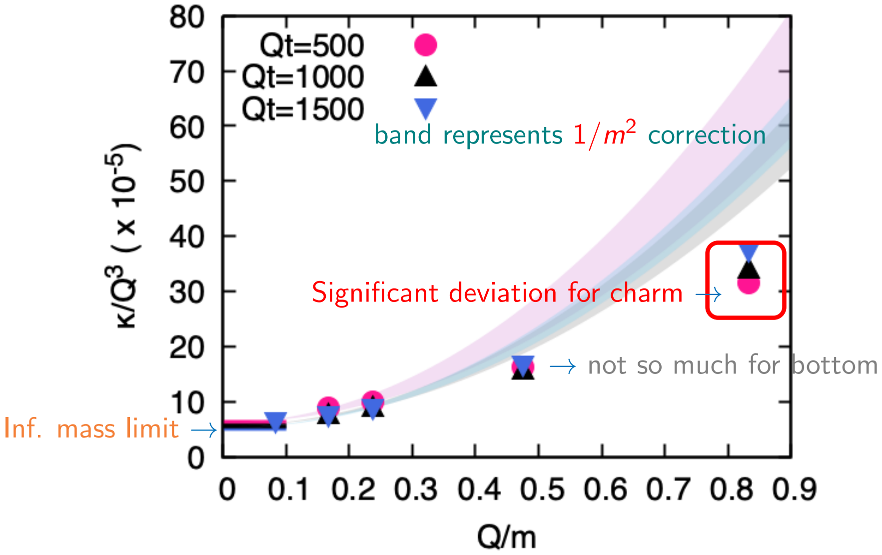
Momentum Broadening of heavy quarks: This is what it looks like! HP, S. Schlichting, S. Sharma, PRL 132, 222301 (2024)

- Starting with zero initial momenta, the momentum distribution broadens due to kicks it receives from the gluonic plasma.



Quantifying broadening through second moment of the momentum distribution HP, S. Schlichting, S. Sharma, PRL 132, 222301 (2024)




80
70
60
50
40
30
20
10
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Q/m
κ/Q³ (x 10⁻⁵)
Qt=500
Qt=1000
Qt=1500
band represents $1/m^2$ correction
Significant deviation for charm →
→ not so much for bottom
Inf. mass limit →

Estimation of kinetic equilibration time: Equilibrium vs. Non-equilibrium

- For a thermal plasma at $T = 600\ \mathrm{MeV}$,

  [N. Brambilla et al. 20, D. Banerjee et. al., 22, L. Altenkort et. al., 23]

$$(\kappa/T^3)_{eq} \approx 1$$

- For charm quark in our non-equilibrium plasma,

$$(\kappa/T_*^3)_{non-therm} \approx 3 \times 10^{-3}$$

- Hence, the ratio of kinetic equilibration times (since $\tau \propto 1/\kappa$),

$$\frac{\tau_{non-therm}}{\tau_{eq}} = \frac{\kappa_{eq}}{\kappa_{non-therm}} \approx 0.5$$

Summary and Outlook

- We've set up a new formalism to study heavy quark momentum broadening and extraction of heavy quark momentum diffusion coefficient inside a non-Abelian plasma off-equilibrium using first-principle lattice simulations.
- We find that there are large corrections to momentum broadening of charm quarks in full relativistic treatment without resorting to expansion in $1/m^2$ about infinite mass limit.
- The kinetic equilibration time comes out to be half of its thermal value when we consider the finite mass correction to the charm quark κ.
- Looking to extend this technique and calculate κ in equilibrium at high temperatures using Dietrich's effective theory.