

# Effects of Collision Dynamics on $p\phi$ Femtoscopy

arXiv:2410.01204 [hep-ph]

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In collaboration with

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# Basics of Femtoscopy

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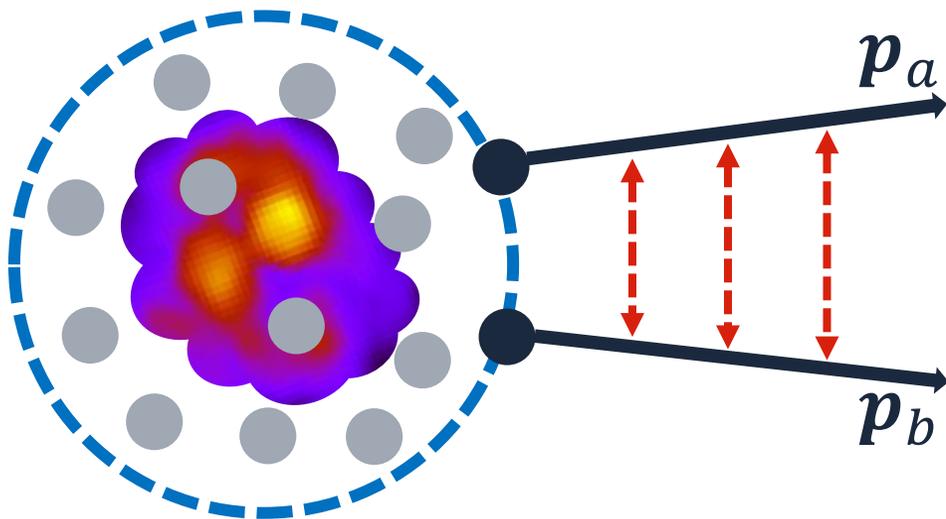
- Correlation Function
- Koonin-Pratt Formula

Momentum correlations in **high-energy nuclear collisions**  
→ Useful for studying **low-energy hadron interactions**

**Correlation Function (CF)** at Pair Rest Frame ( $\mathbf{P} = 0$ )

$$C(\mathbf{q}) := \frac{N_{\text{pair}}(\mathbf{p}_a, \mathbf{p}_b)}{N_a(\mathbf{p}_a) N_b(\mathbf{p}_b)}$$

Total momentum:  $\mathbf{P} = \mathbf{p}_a + \mathbf{p}_b$   
Relative momentum:  $\mathbf{q} = \frac{m_b \mathbf{p}_a - m_a \mathbf{p}_b}{m_a + m_b}$   
Two-particle momentum dist.:  $N_{\text{pair}}(\mathbf{p}_a, \mathbf{p}_b)$   
One-particle momentum dist.:  $N_a(\mathbf{p}_a)$



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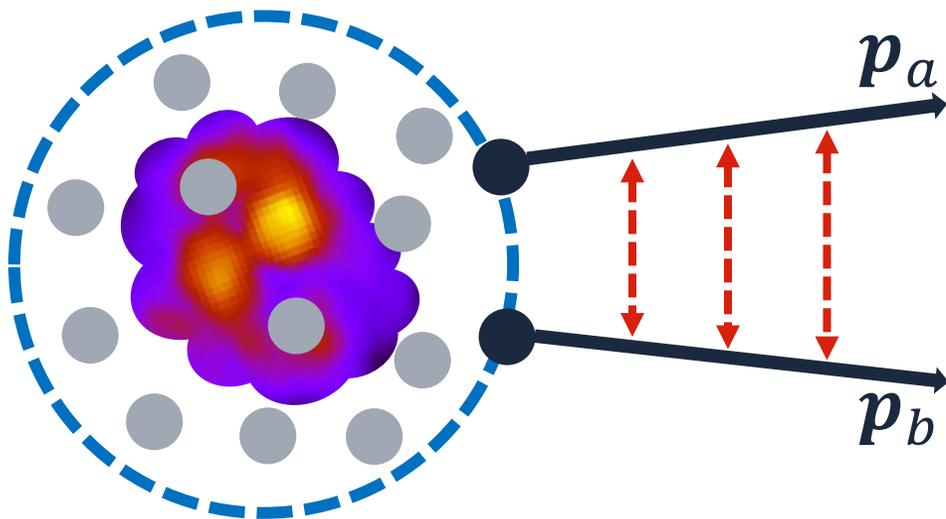
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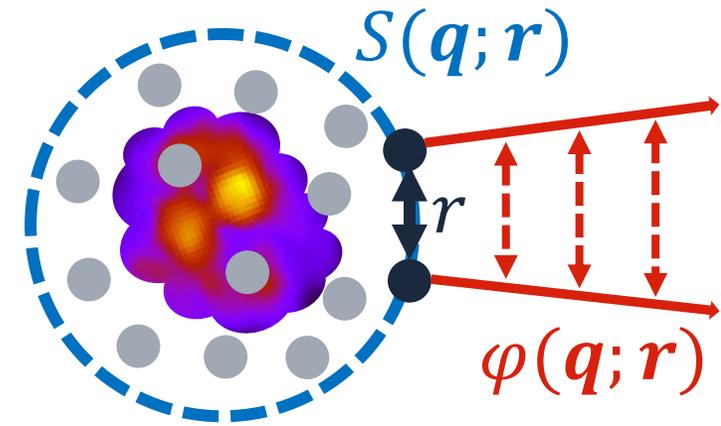
**Hadron CF** provides insights into

- **Space-time structure of the matter**
- **Final state hadron interactions**

## Koonin-Pratt formula S. E. Koonin, PLB 70, 43 (1977); S. Pratt, PRL 53, 1219 (1984)

Under several assumptions,

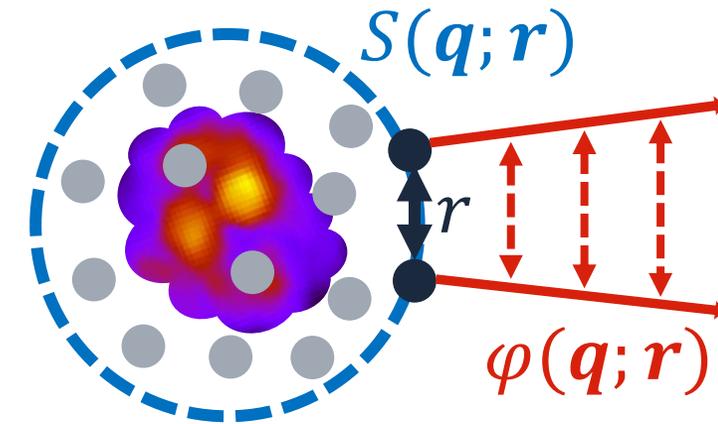
$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}; \mathbf{r}) |\varphi(\mathbf{q}; \mathbf{r})|^2$$



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From experimental correlation function

- Input: hadron interaction → Output: source function
- Input: source function → Output: hadron interaction

Recent active studies have demonstrated its usefulness and powerfulness

L. Fabbietti *et al.*, Ann. Rev. Nucl. Part. Sci. **71**, 377 (2021)

Assuming **static Gaussian SF**

**Actual SF should reflect the complex dynamics of nuclear collisions**

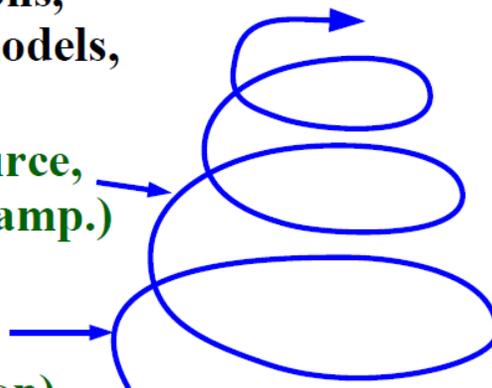
A. Ohnishi, talk at RHIC-BES On-line seminar IV (2022)

- For more realistic estimate of hh interactions, we need reliable interactions and source models, together with more data.

**2nd round**  
(dynamical source,  
 $C(q) \rightarrow$  scatt. amp.)

**1st round**  
(simple source,  
existing interaction)

**State-of-the-art**



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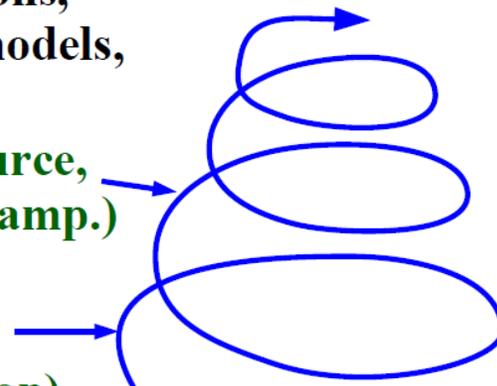
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State-of-the-art



To explore less understood hadron interactions,

**Femtoscopy using dynamical models**

# $p\phi$ Femtoscopy using Dynamical Model

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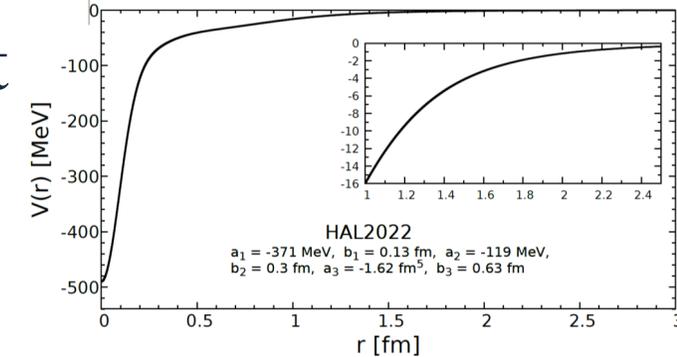
- Interaction
- Source Function
- Correlation Function
- Effects of Collision Dynamics

Consider only  $s$ -wave scattering  $\rightarrow$  2 channels:  ${}^4S_{3/2}$  &  ${}^2S_{1/2}$

${}^4S_{3/2}$ : HAL QCD potential Y. Lyu *et al.*, PRD **106**, 074507 (2022)

(2+1)-flavor lattice QCD at near physical point

➤ Overall attraction w/o bound states



${}^2S_{1/2}$ : Parametrized potential E. Chizzali *et al.*, PLB **848**, 138358 (2023)

Motivated by HAL QCD  ${}^4S_{3/2}$  potential

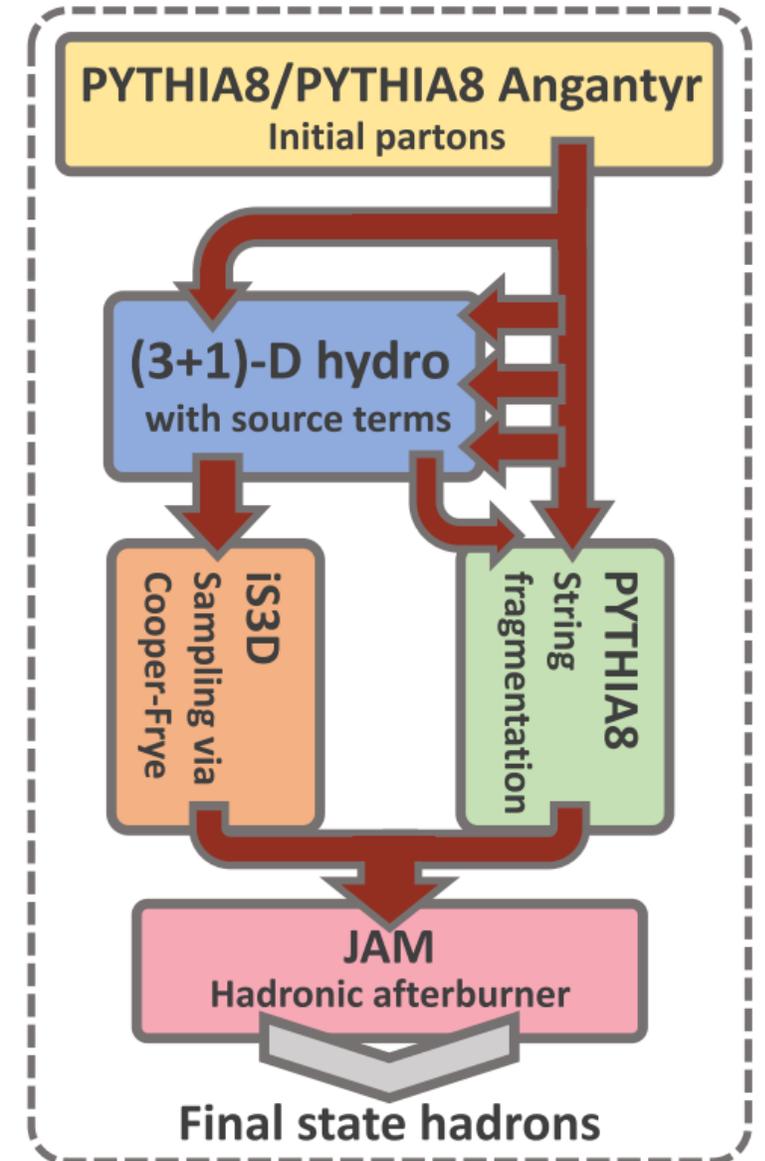
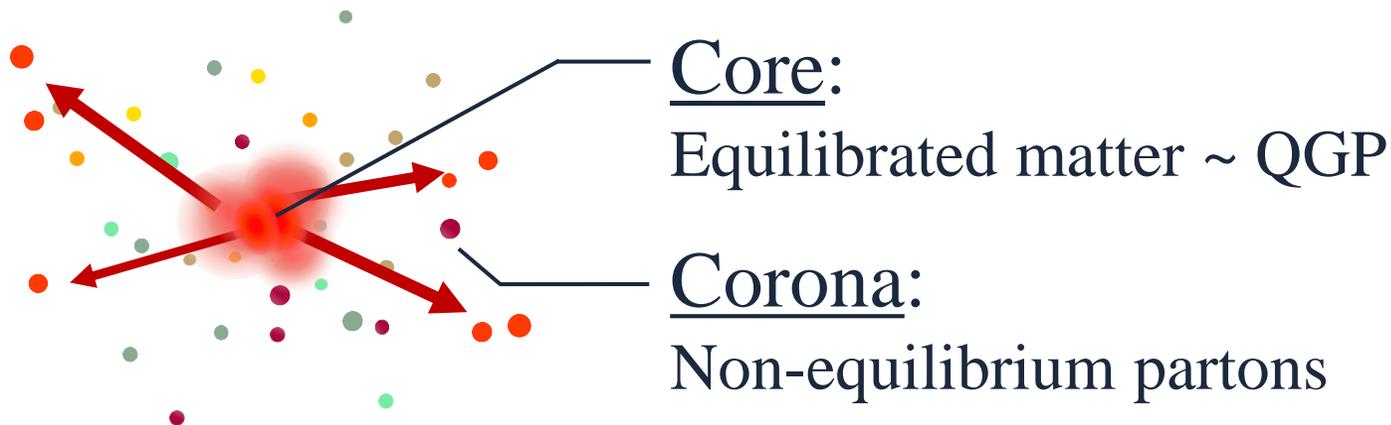
➤ Should be constrained phenomenologically via femtoscopy

Femtoscopy using Gaussian SF  $\rightarrow$  Indication of a bound state

## Dynamical Core–Corona Initialization model (DCCI2)

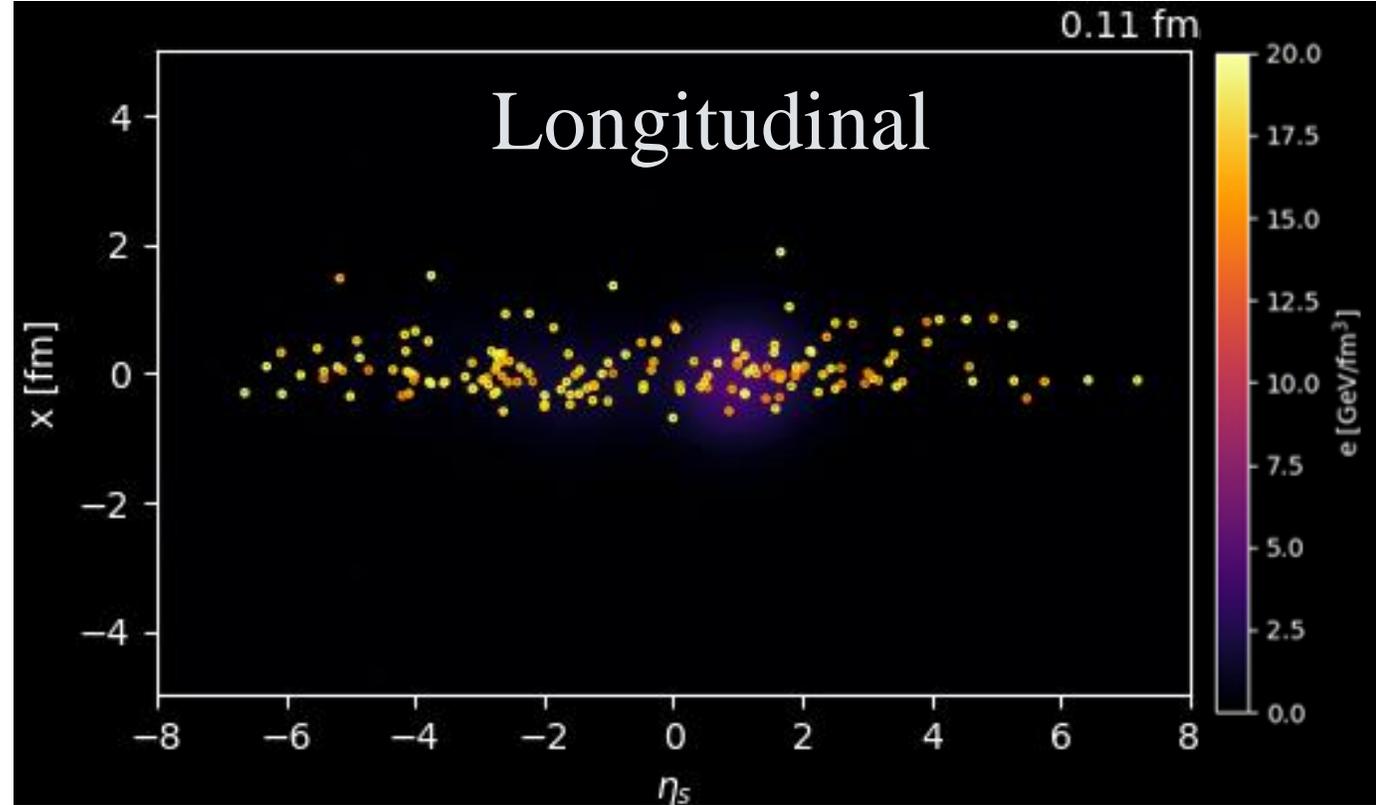
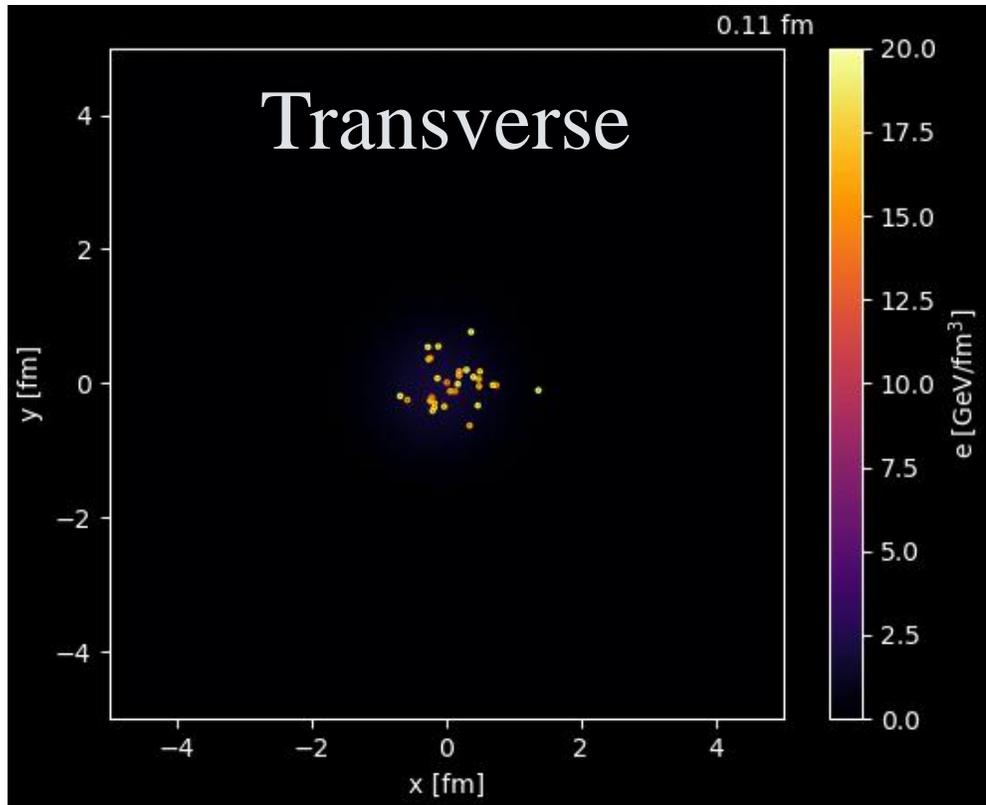
Y. Kanakubo, Y. Tachibana, and T. Hirano, PRC **105**, 024905 (2022)

A cutting-edge dynamical model  
based on **core–corona picture**



High-multiplicity p+p collisions at  $\sqrt{s} = 7$  TeV

Movies provided by Y. Kanakubo

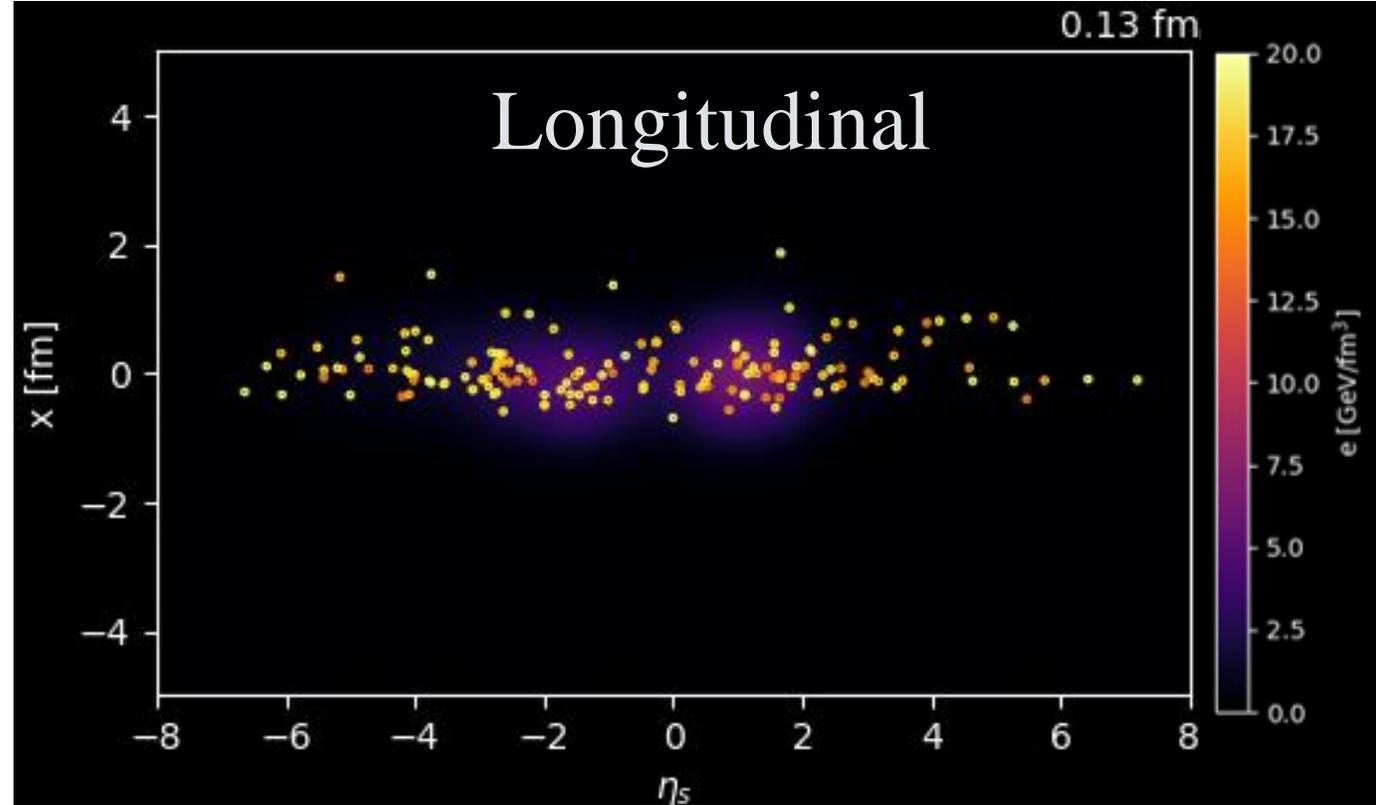
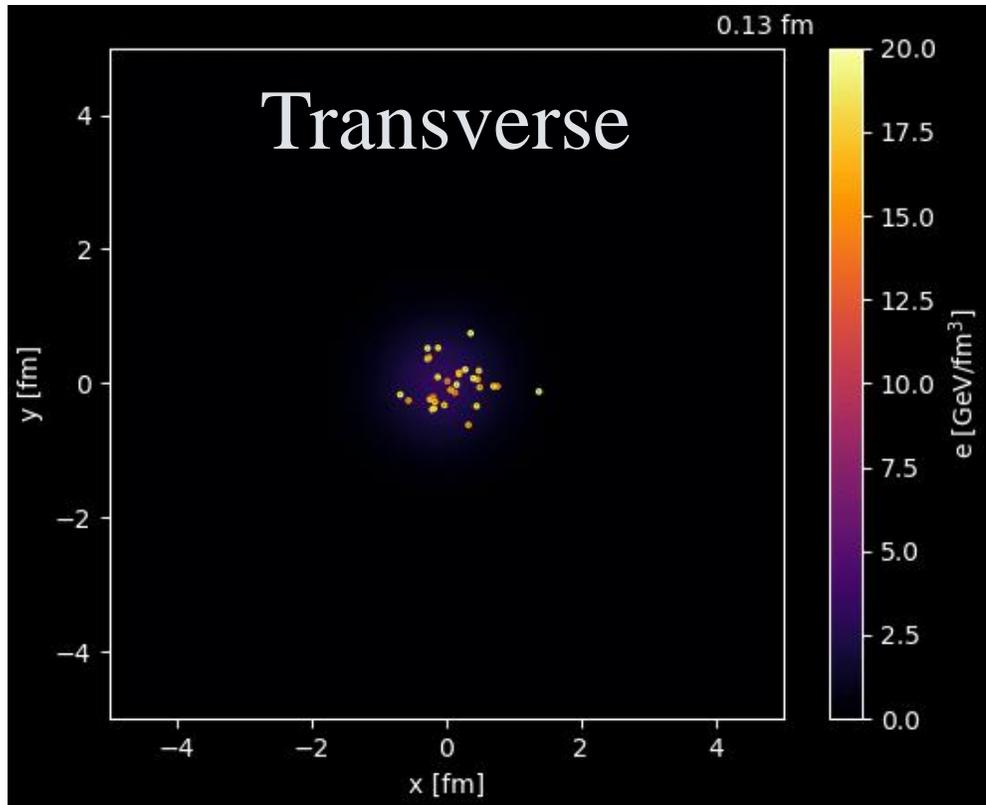


Describes the entire evolution of nuclear collisions  
→ SF that reflects collision dynamics

# Space-Time Evolution by DCCI2

High-multiplicity p+p collisions at  $\sqrt{s} = 7$  TeV

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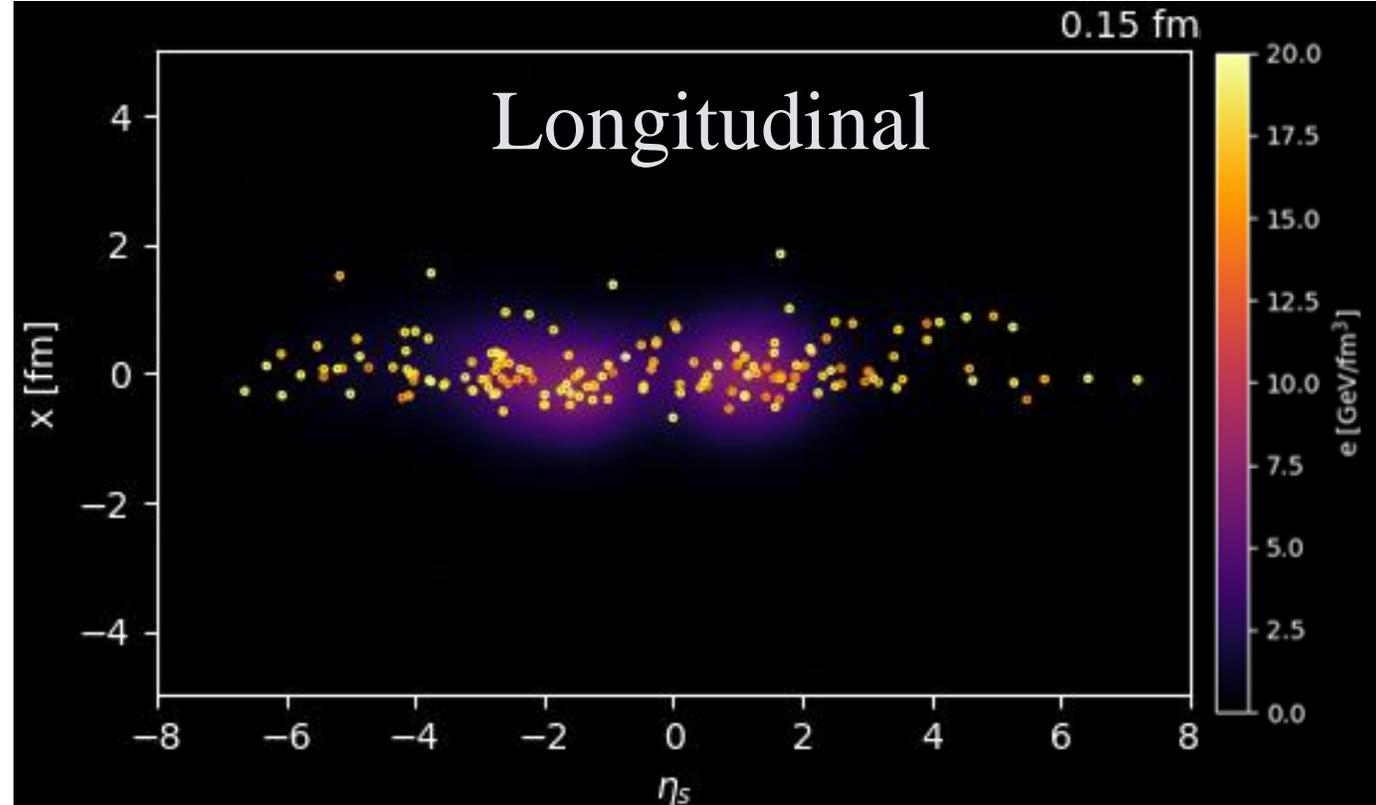
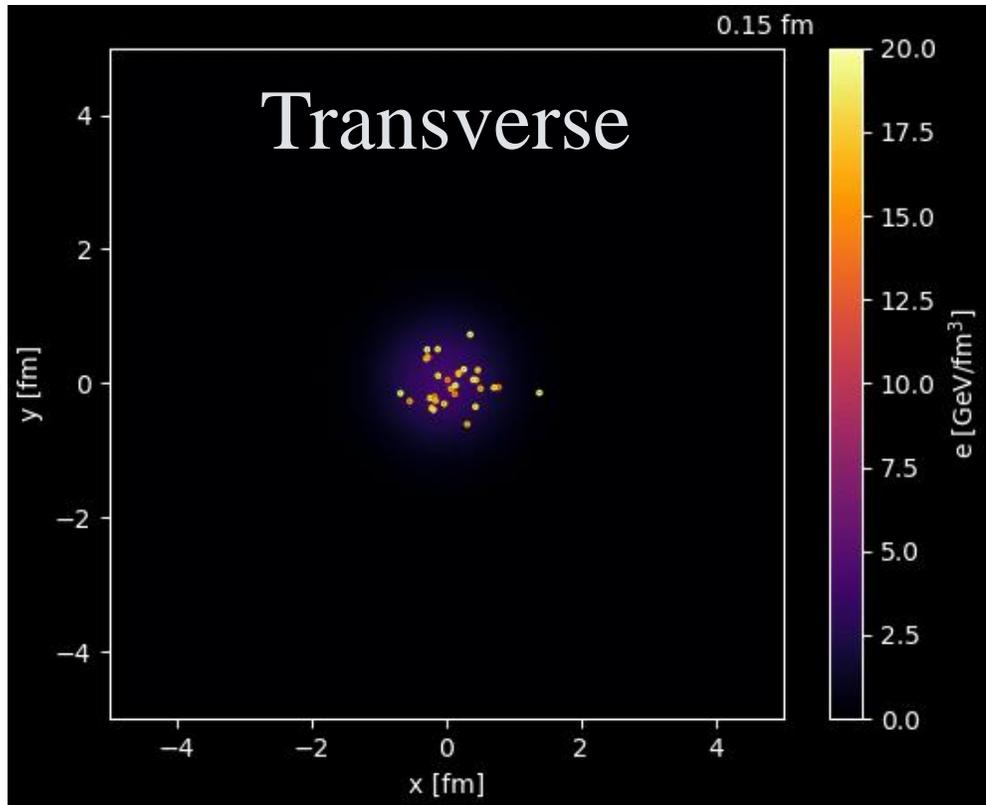


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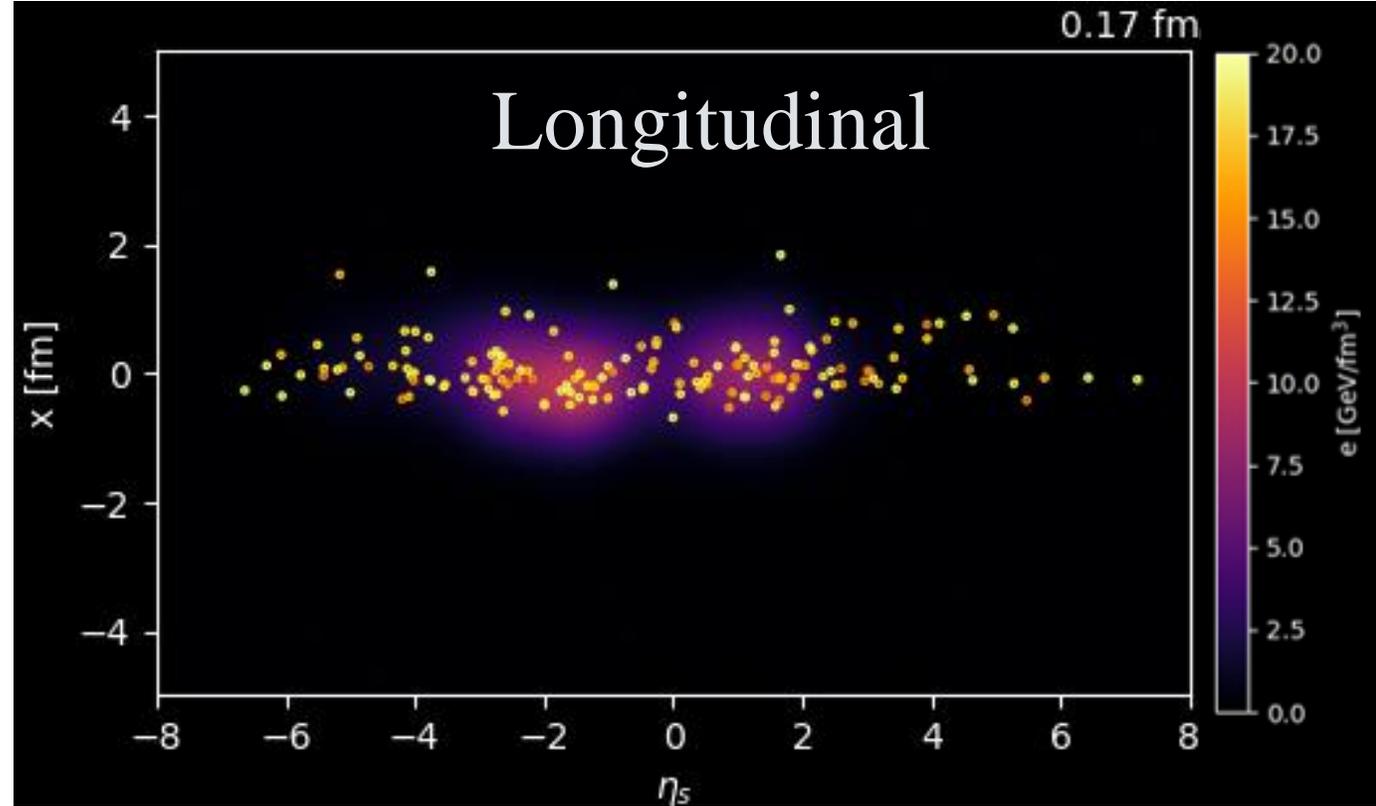
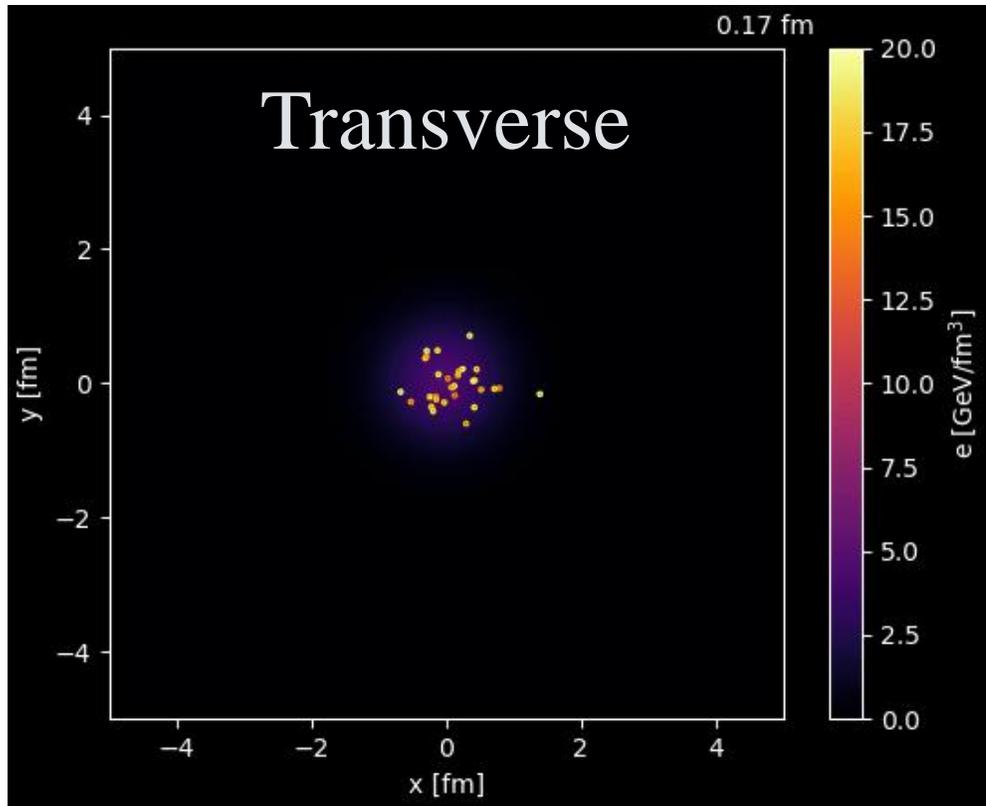
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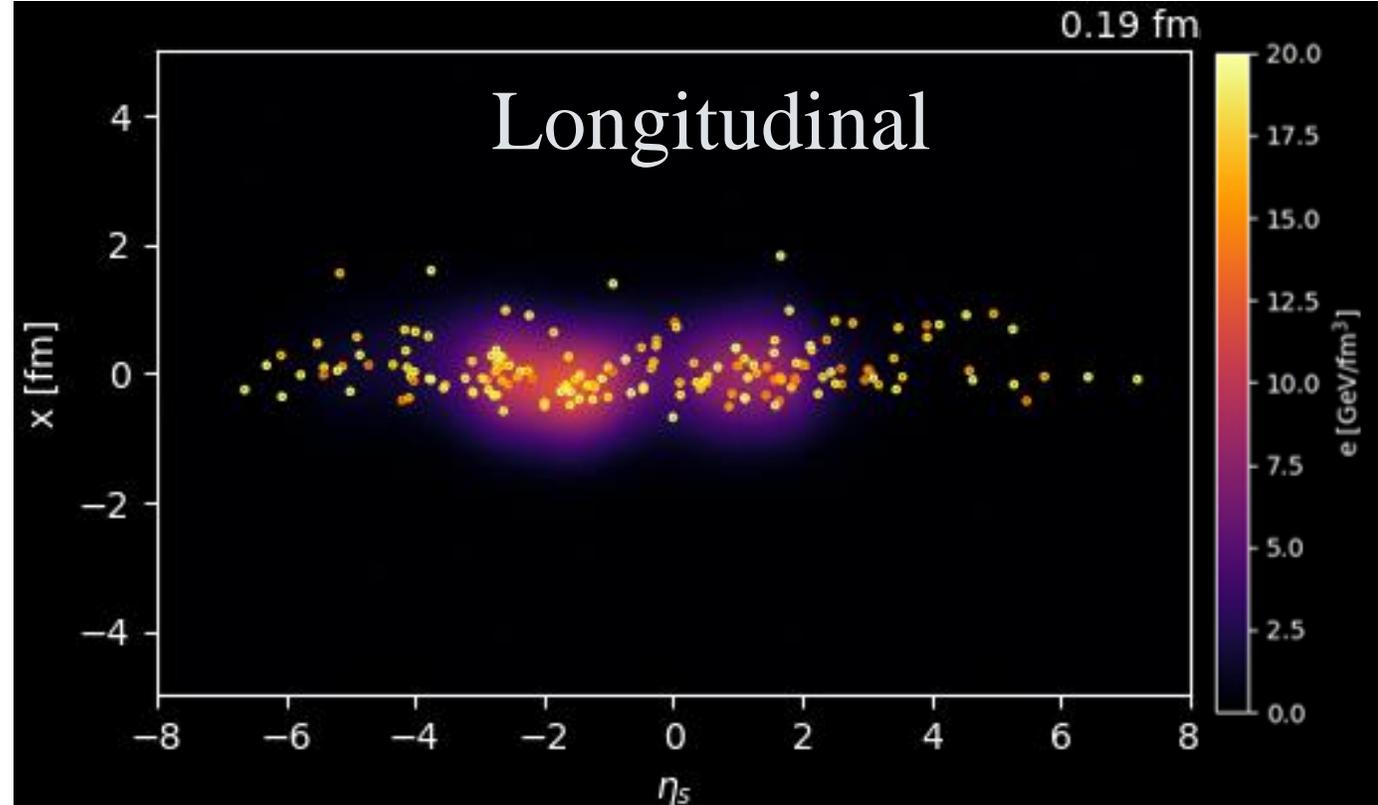
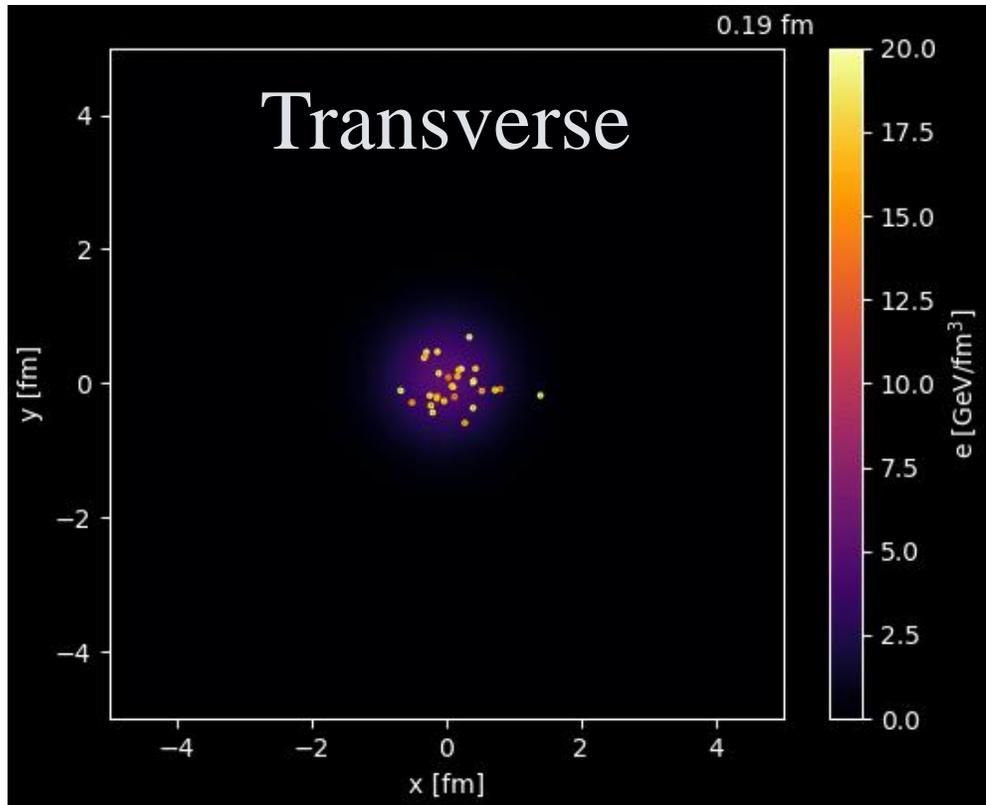
Describes the entire evolution of nuclear collisions  
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# Space-Time Evolution by DCCI2

6

High-multiplicity p+p collisions at  $\sqrt{s} = 7$  TeV

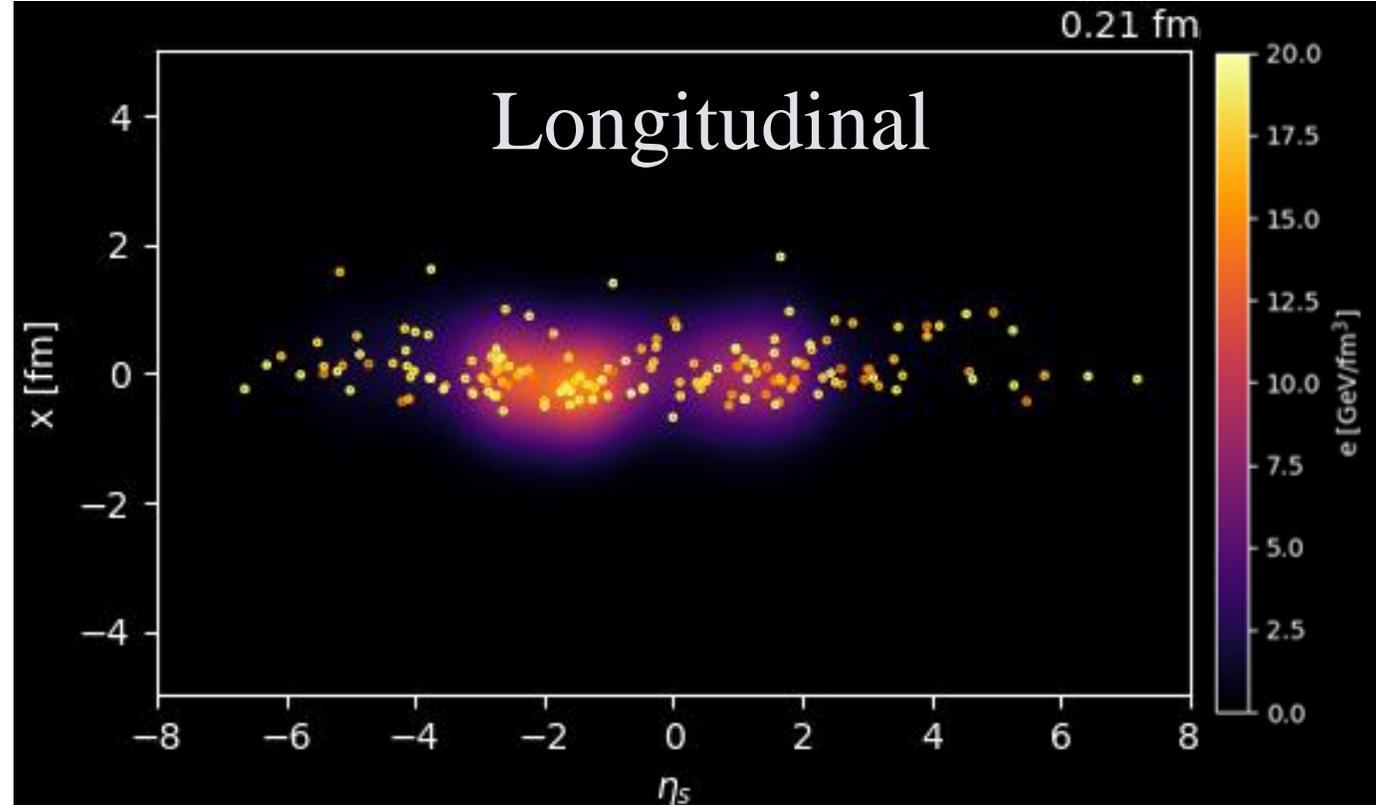
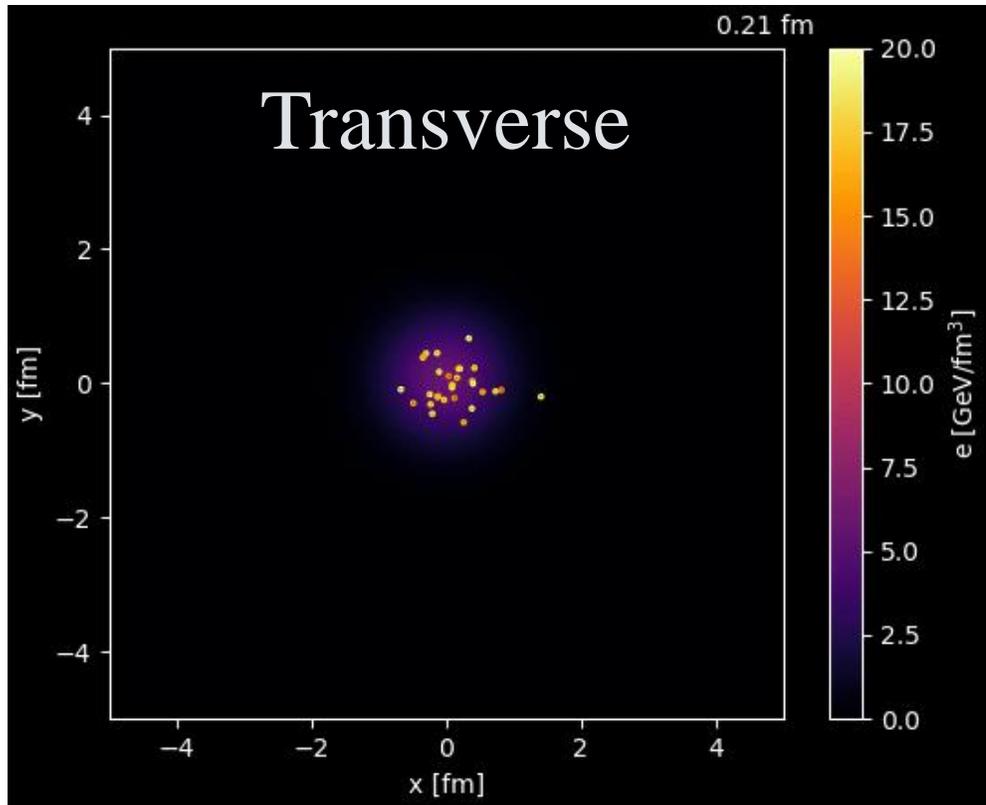
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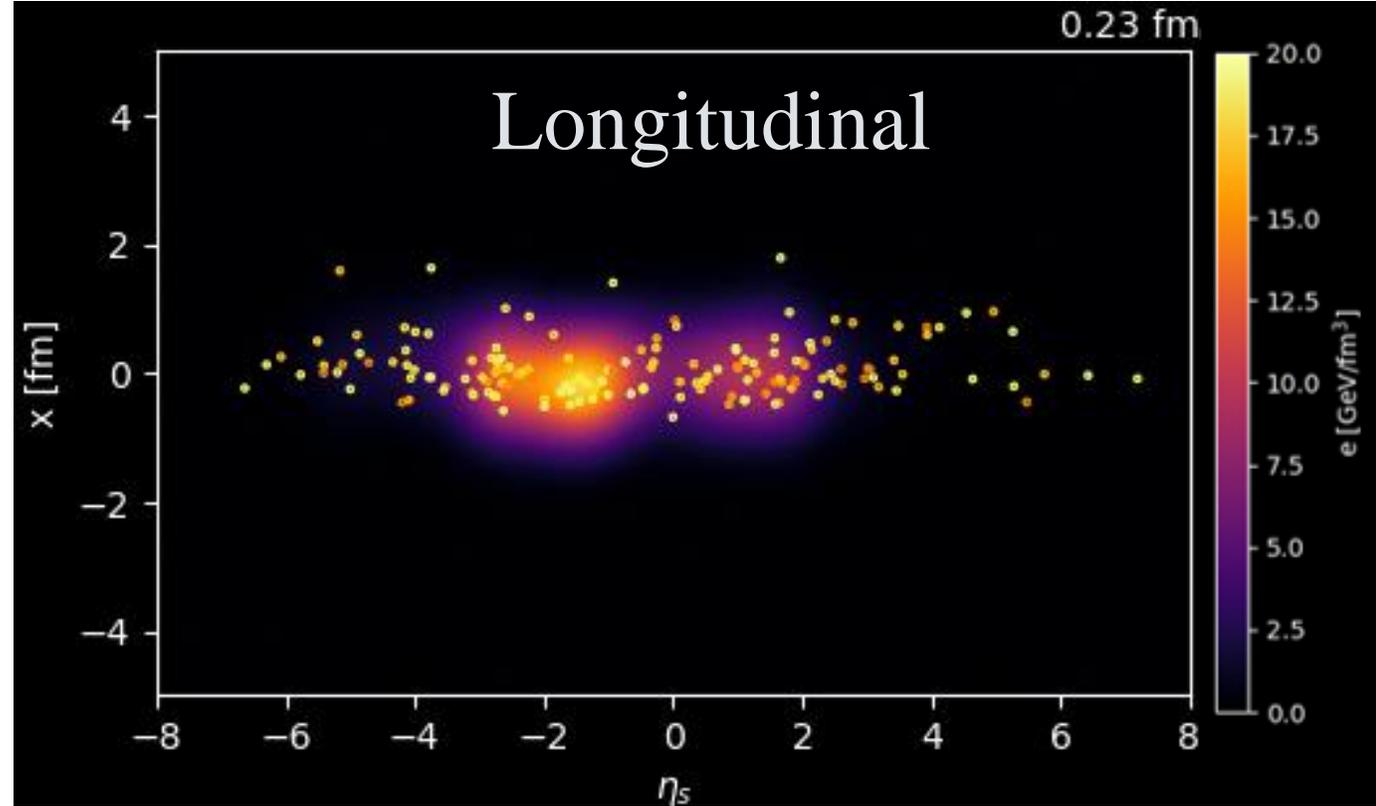
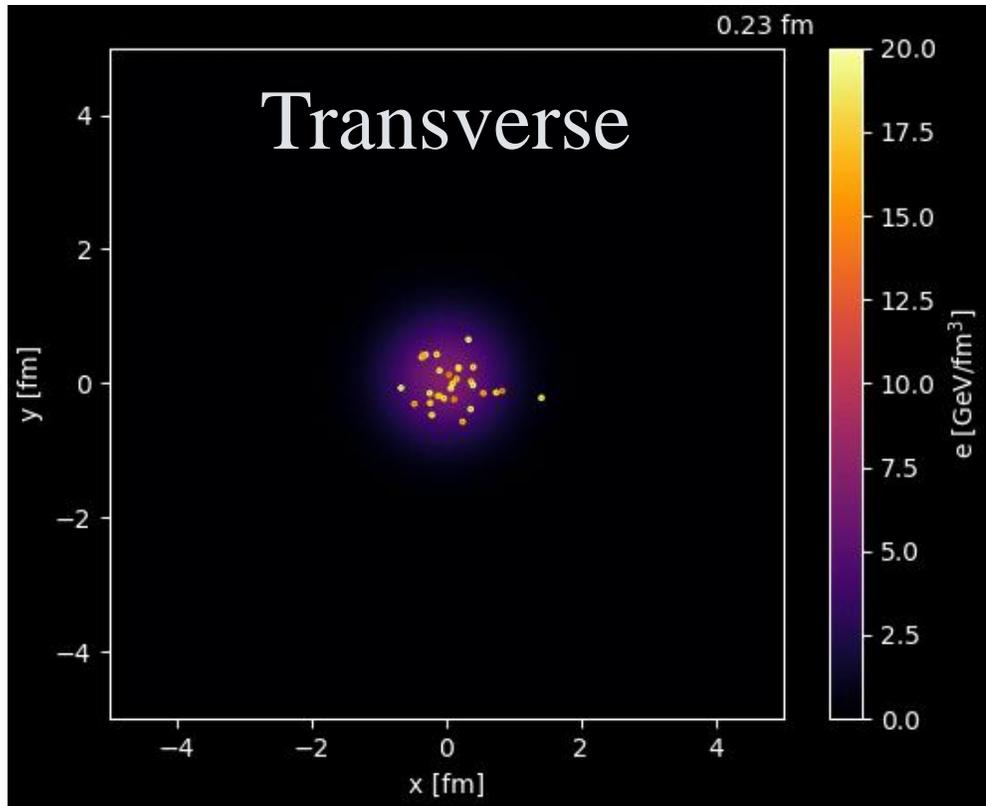


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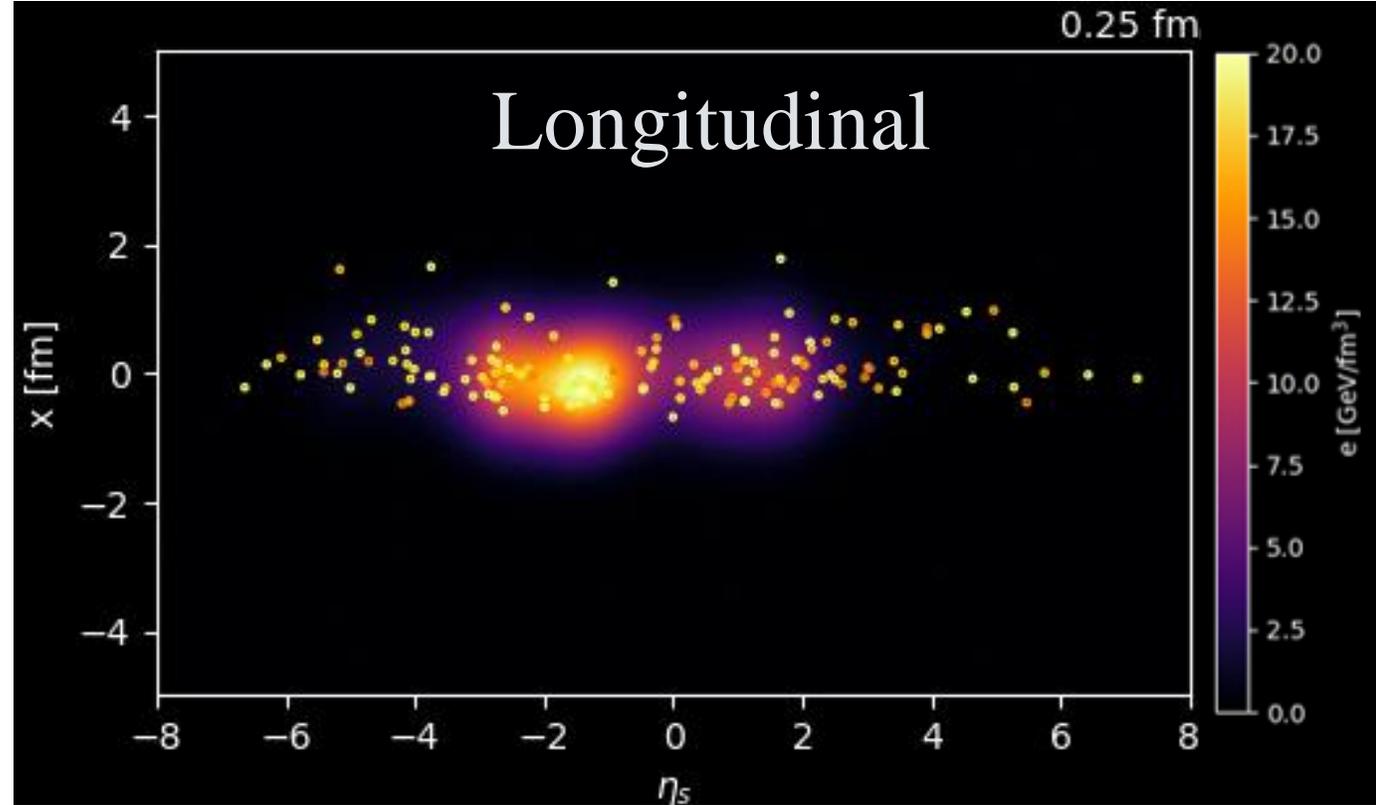
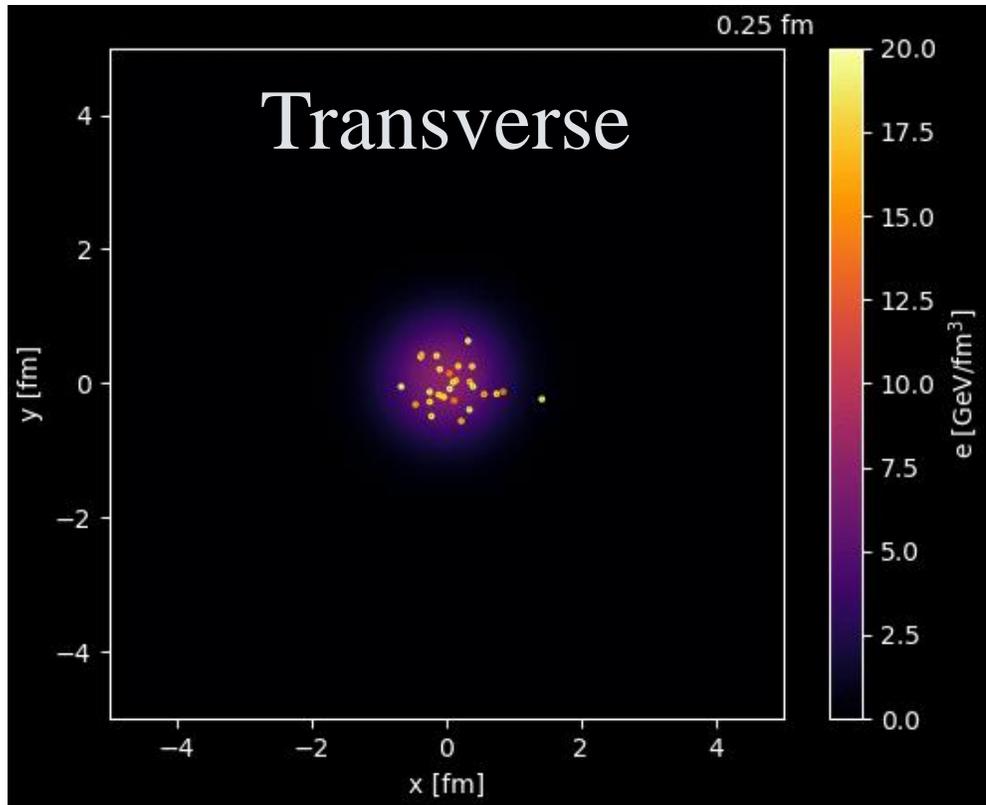


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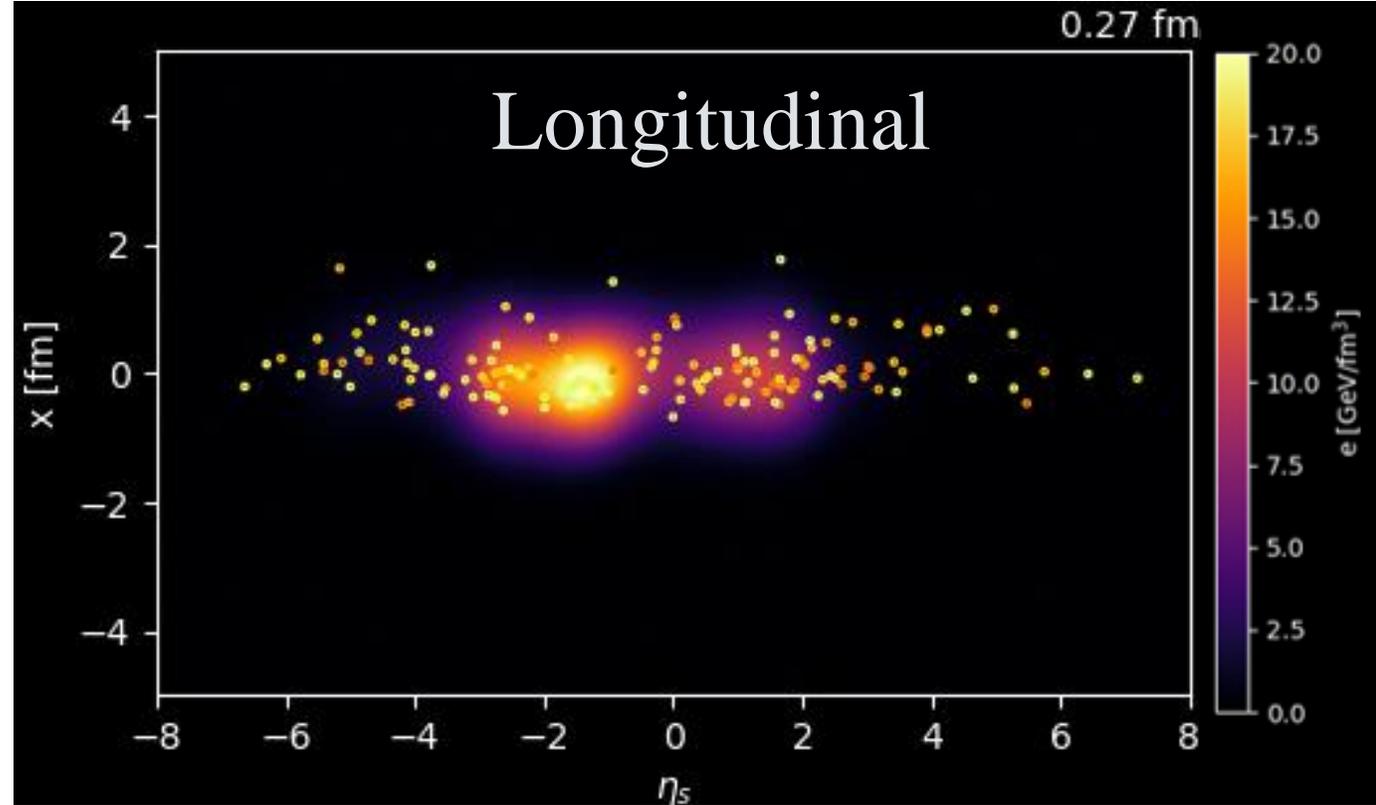
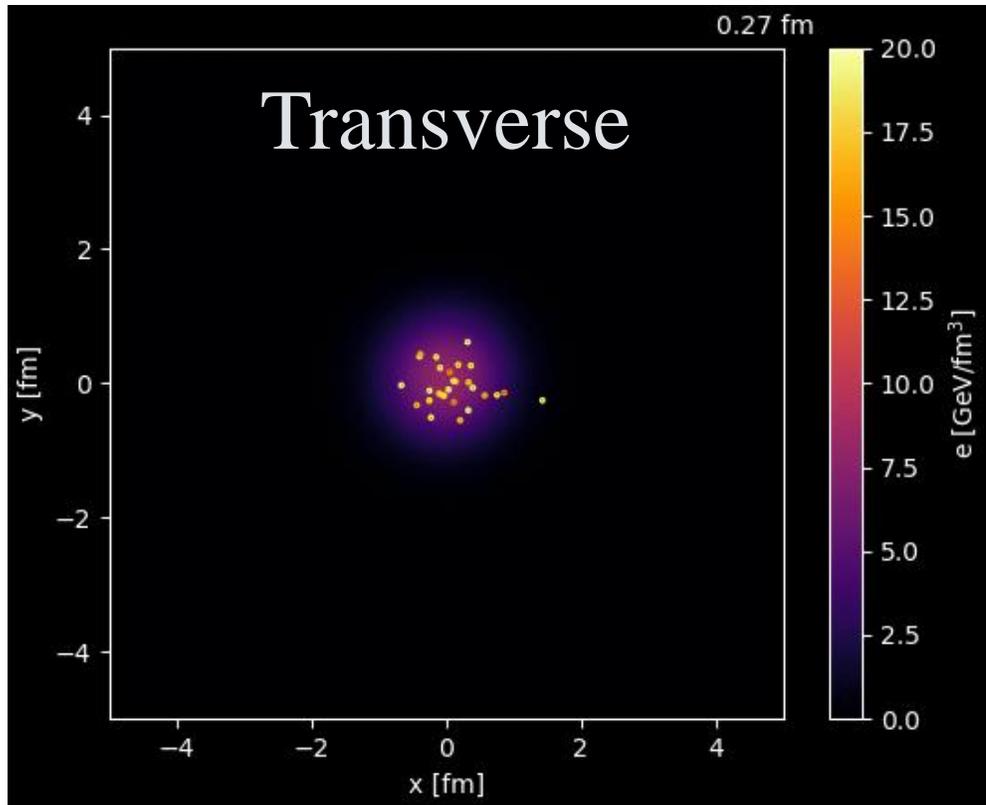
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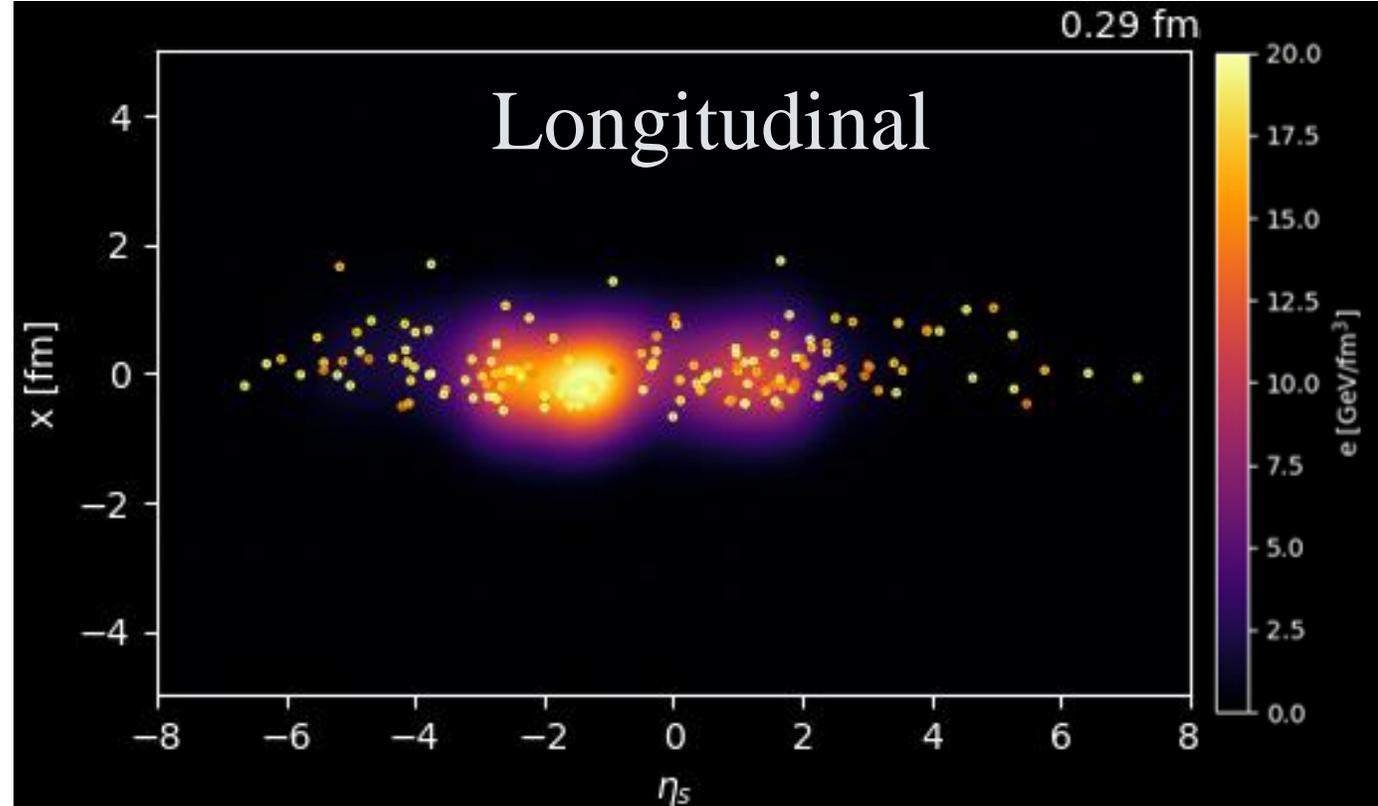
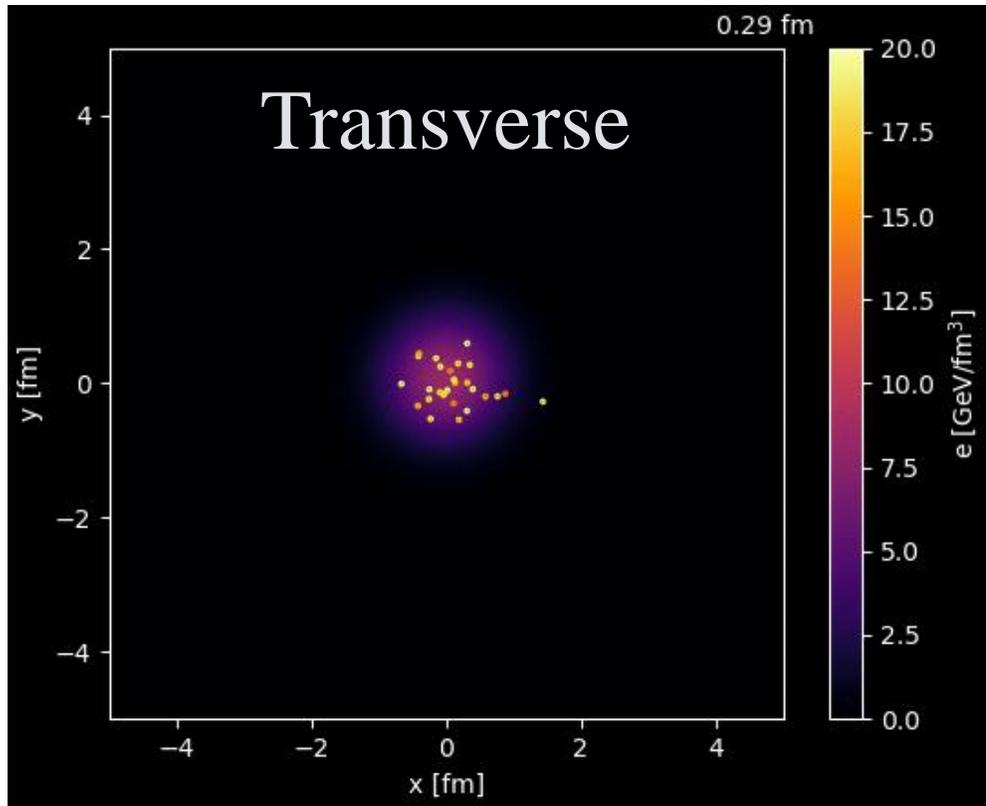
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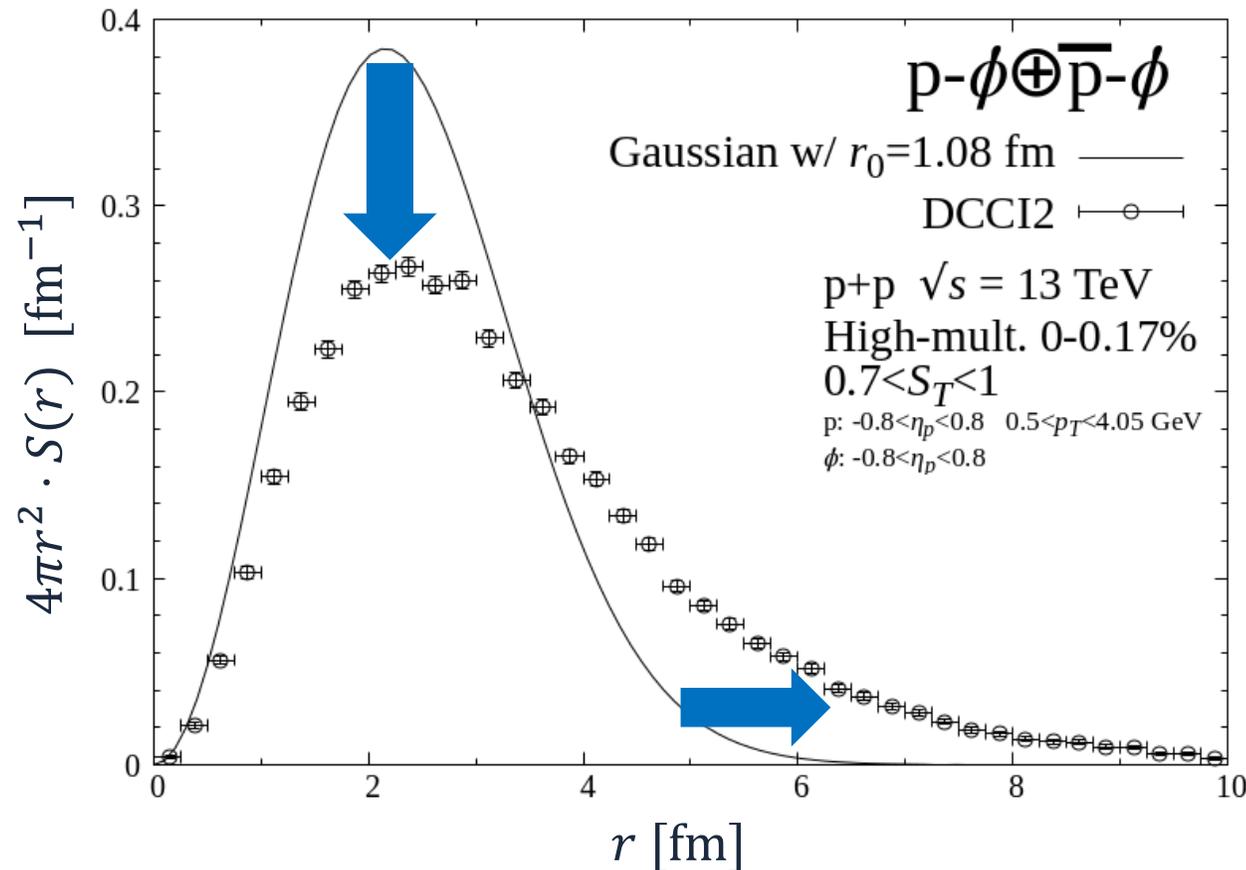
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Describes the entire evolution of nuclear collisions  
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High-multiplicity 0-0.17% p+p collisions at  $\sqrt{s} = 13$  TeV

Plot: DCCI2 SF, Line: Gaussian SF  $S(r) \propto \exp(-r^2/4r_0^2)$  w/  $r_0 = 1.08$  fm



**Non-Gaussian long-tail**

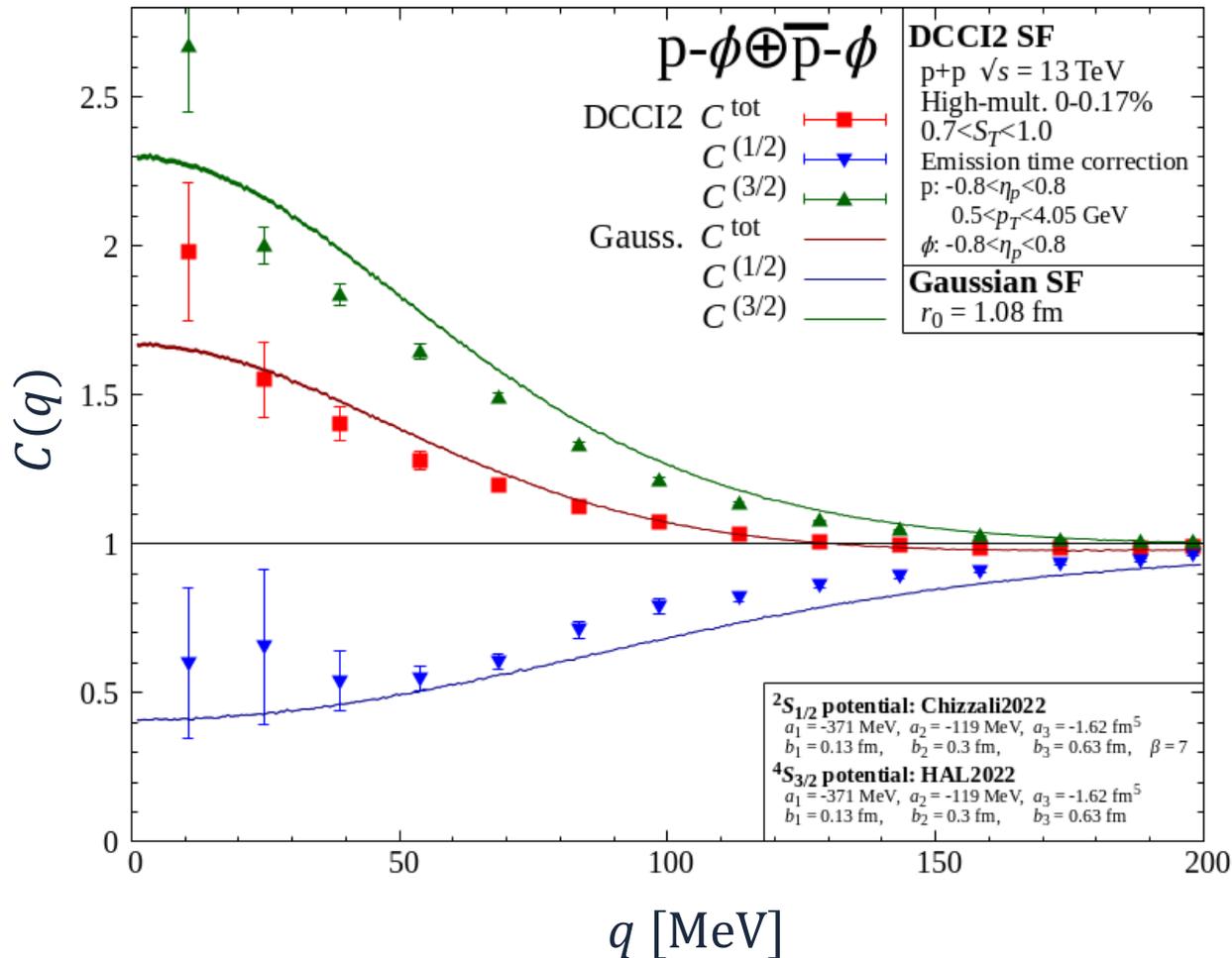
→ **Larger source size  $\langle r^2 \rangle$**

Mainly due to p rescatterings  
with surrounding pion gas  
“Pion wind”

**Hadronic rescatterings  
even in p+p collisions**

Green:  $C^{(3/2)}$ , Blue:  $C^{(1/2)}$ , Red:  $C^{\text{tot}} = \frac{2}{3}C^{(3/2)} + \frac{1}{3}C^{(1/2)}$

Plots: **DCCI2 SF**, Lines: Gaussian SF w/  $r_0 = 1.08$  fm



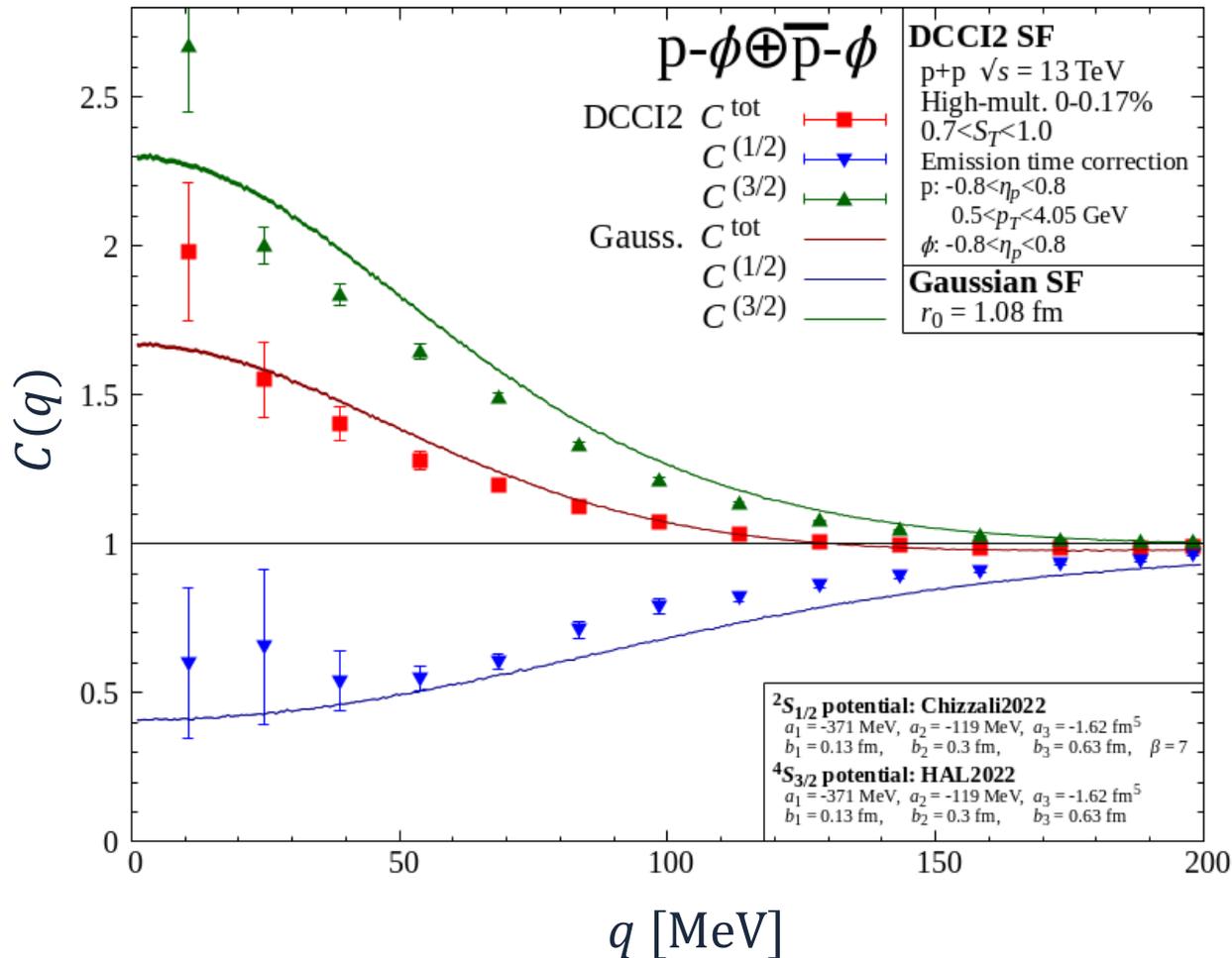
DCCI2 vs. Gaussian

- Slightly weaker correlation  
 Due to non-Gaussian long-tail
- Non-trivial behavior at small  $q$

**A small but statistically significant difference**

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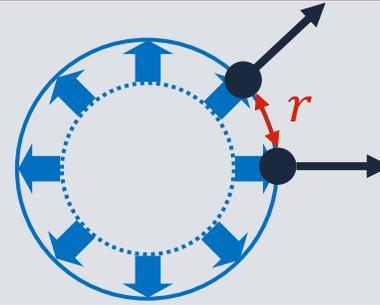
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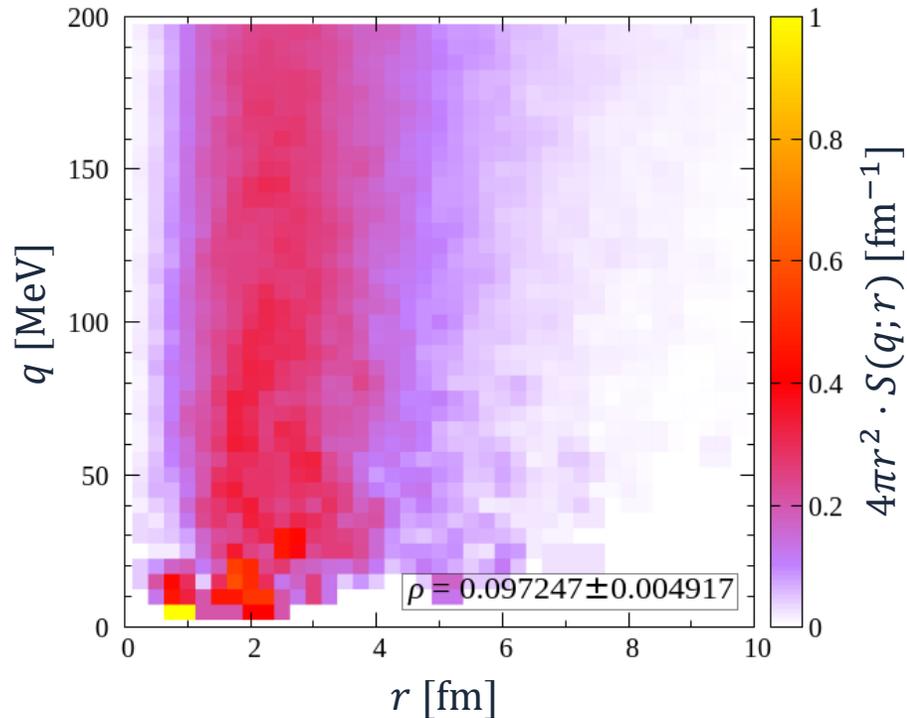
A small but statistically significant difference

From comparison w/ ALICE data  
 ALICE, PRL 127, 172301 (2021)  
**Indication of a bound state in  ${}^2S_{1/2}$**   
 ( $E_B \cong 10-70$  MeV)

**SF generally depends on  $q$   
due to e.g., collectivity**

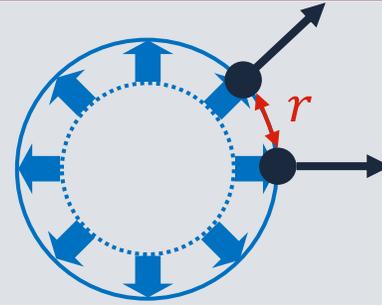


Close in position space  
 $\Updownarrow$   
Close in momentum space

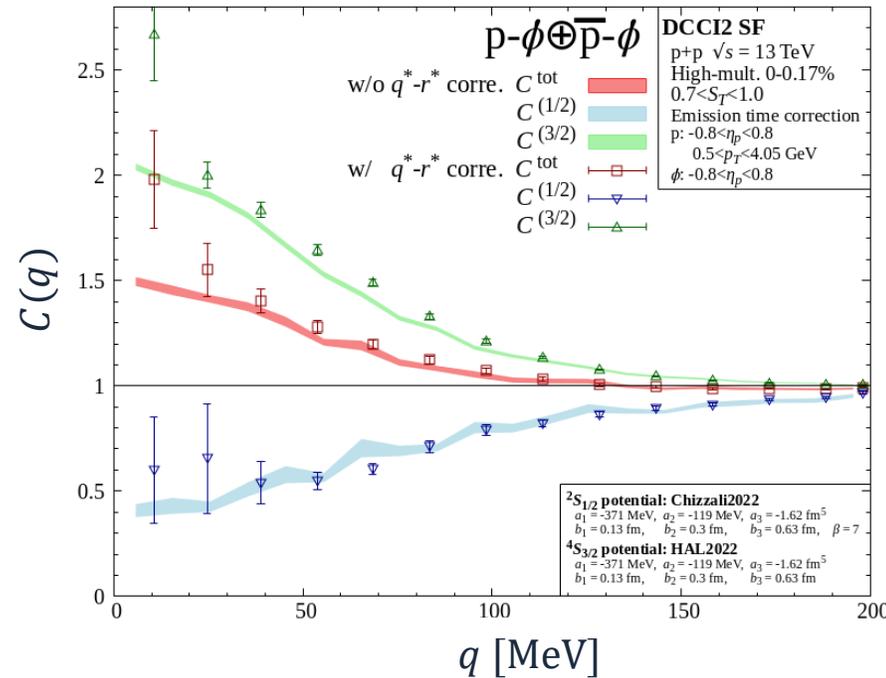
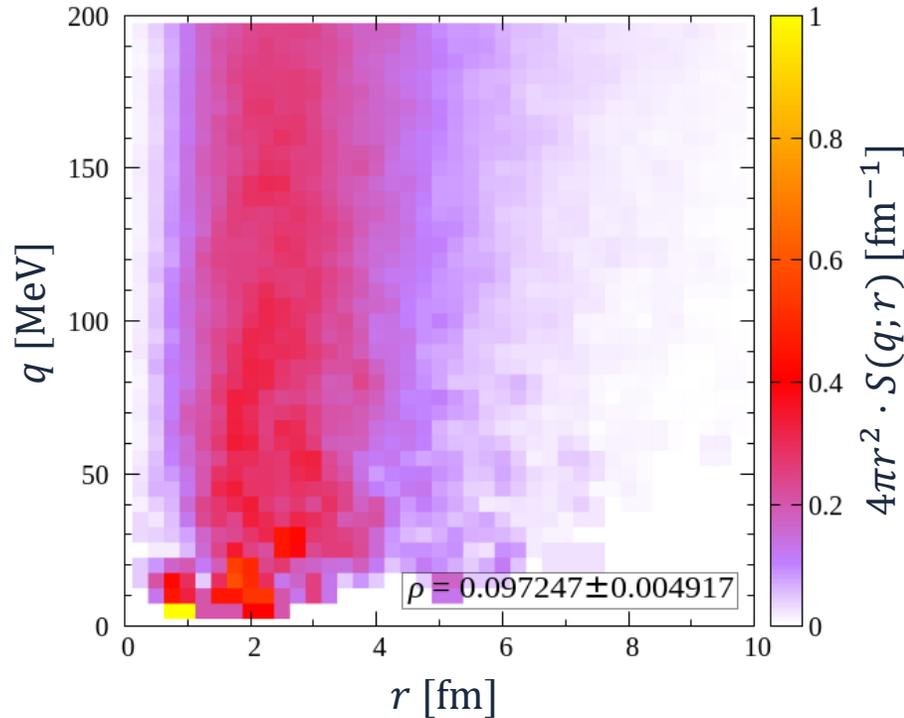


- Slightly positive  $q$ – $r$  correlation
- Significant small source at small  $q$

SF generally depends on  $q$  due to e.g., collectivity



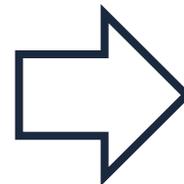
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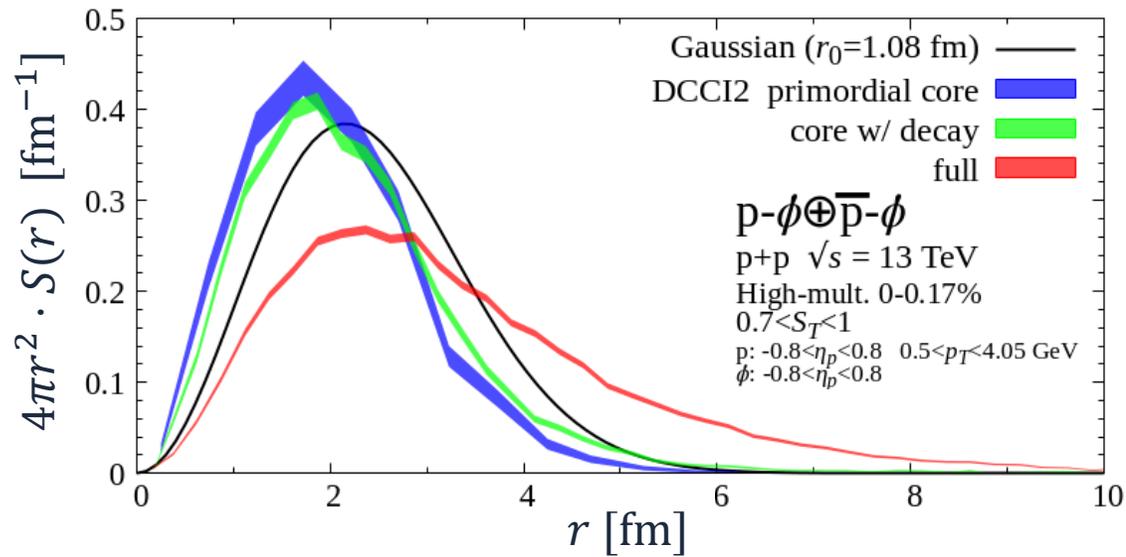
Plots:  
 W/  
 $q-r$  correlation

Bands:  
 W/o  
 $q-r$  correlation

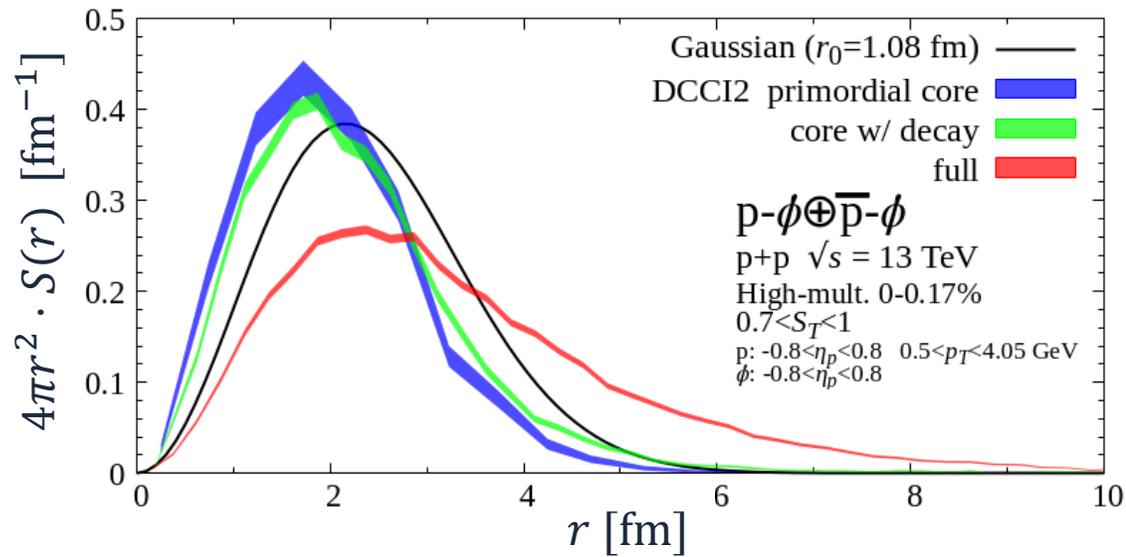
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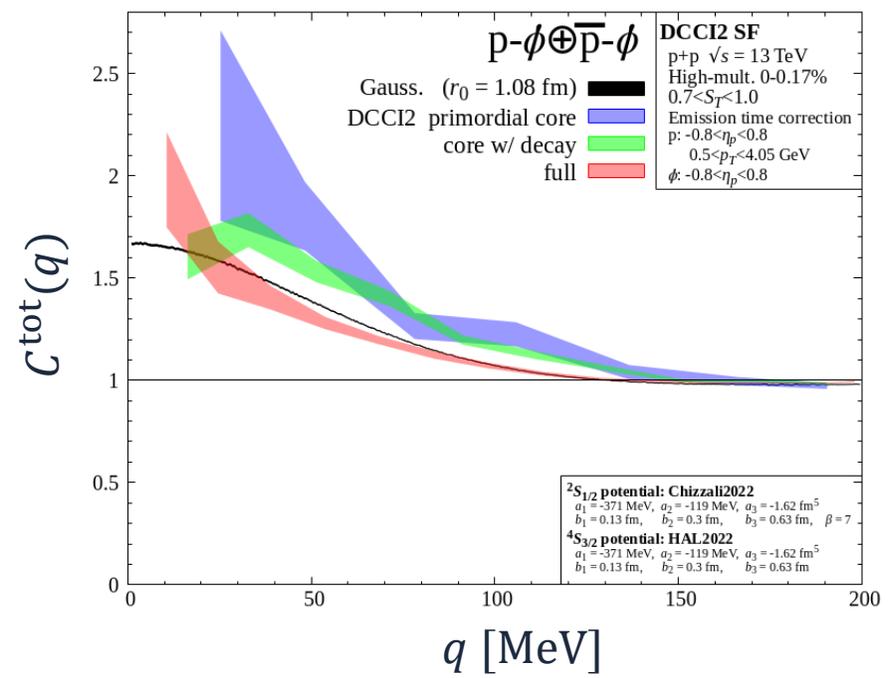
CF at small  $q$  is sensitive to the WF in the scattering region



- Distribution at hypersurface  $\sim$  Gaussian
- Resonance decay  $\rightarrow$  A little long-tail
- Hadronic rescatterings  $\rightarrow$  Long-tail and larger source size

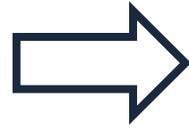


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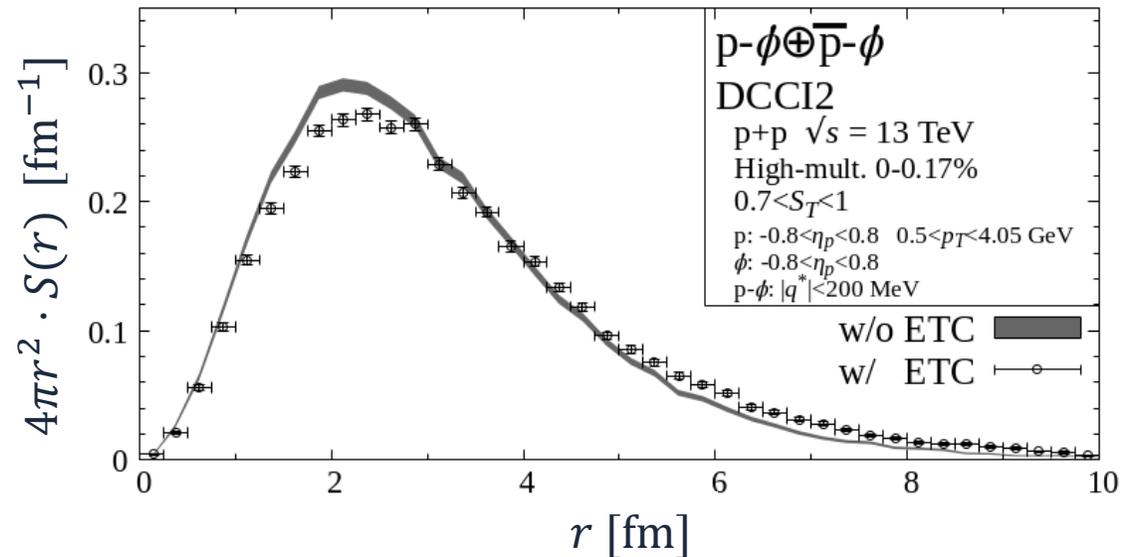
Larger effects of **hadronic rescatterings** than **resonance decay** on SF & CF

Emission time difference of the pair  
from a dynamical model



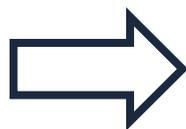
**Emission Time Correction (ETC)**

Plots: w/ ETC, Bands: w/o ETC



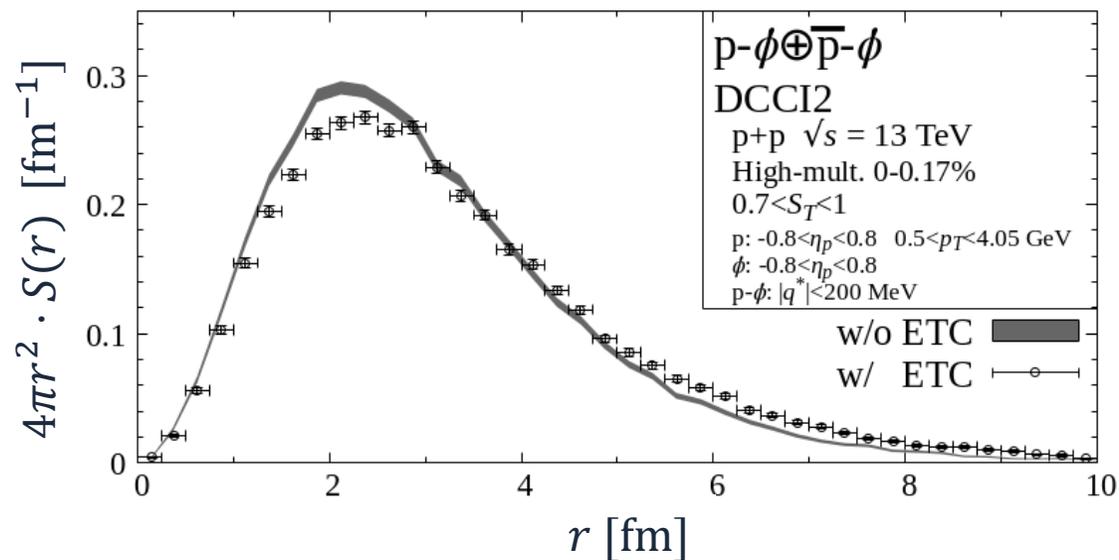
**ETC slightly enlarges source size**

Emission time difference of the pair from a dynamical model

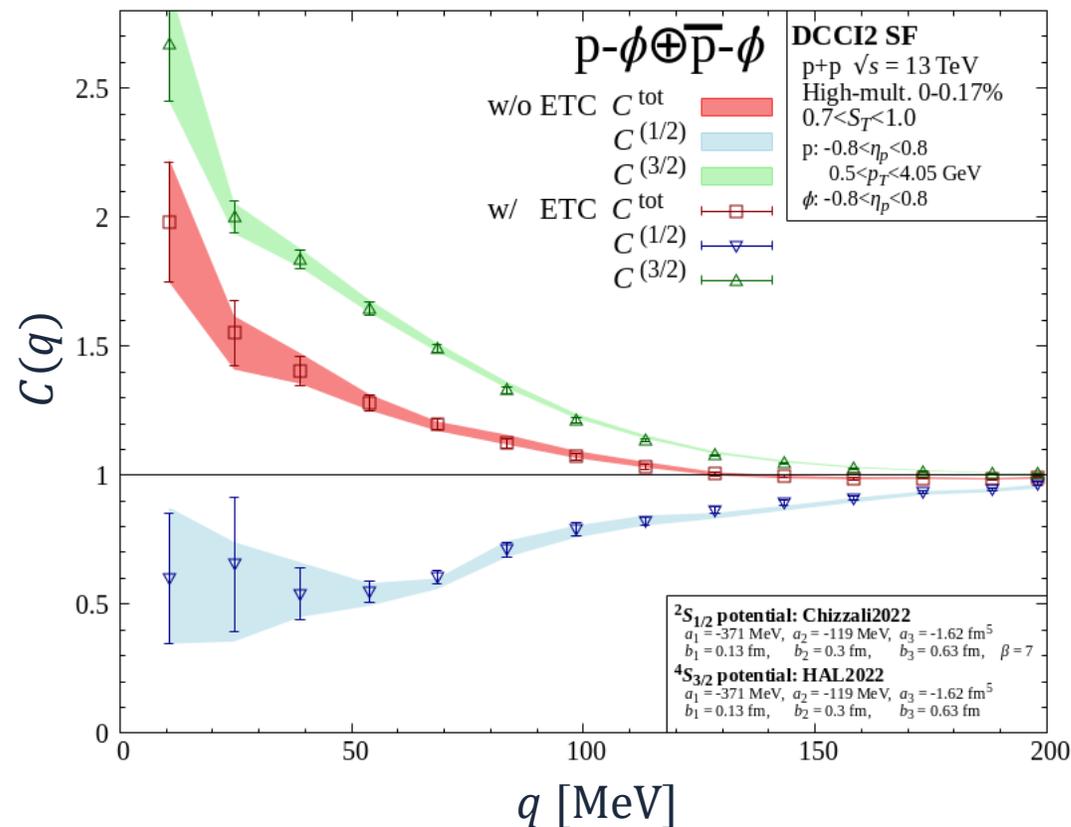


**Emission Time Correction (ETC)**

Plots: w/ ETC, Bands: w/o ETC



**ETC slightly enlarges source size**



**No statistically significant effects on CF in this particular case**

## **p $\phi$ femtoscopy using SF from a dynamical model (DCCI2)**

### Effects of collision dynamics

#### **Small but statistically significant**

- ✓ Slightly larger source size mainly due to **hadronic rescatterings**
- ✓ SF depends on relative momentum due to e.g., **collectivity**

### Phenomenological constraint on interaction

- ✓ Indication of a bound state in  $^2S_{1/2}$  channel ( $E_B \cong 10\text{--}70$  MeV)  
Slightly different but qualitatively consistent w/ that using Gaussian SF

**Importance of using SF that reflects collision dynamics**  
for precise studies of hadron interactions via femtoscopy

# Backup

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- Assumptions
- **Chaotic source** ~ thermal equilibrium
  - Same time approximation
  - On-shell approximation
  - **Closed system after emission** ~ in vacuum propagation

$$C(\mathbf{q}, \mathbf{P}) = \frac{\int d^4x_a d^4x_b S_a(\mathbf{p}_a; x_a) S_b(\mathbf{p}_b; x_b) |\varphi(\mathbf{q}; \mathbf{r})|^2}{\int d^4x_a S_a(\mathbf{p}_a; x_a) \int d^4x_b S_b(\mathbf{p}_b; x_b)}$$

**Pair Rest Frame ( $\mathbf{P} = \mathbf{0}$ )**

Integrate out CM

$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}; \mathbf{r}) |\varphi(\mathbf{q}; \mathbf{r})|^2$$

**Spherical SF**  
 $S(q; r)$

**Only s-wave scattering**

$$\varphi(\mathbf{q}; \mathbf{r}) = \exp(i\mathbf{q} \cdot \mathbf{r}) - j_0(qr) + \varphi_0(q; r)$$

Plane-wave
Plane-wave
WF  
(s-wave)
(s-wave)
(s-wave)

$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}; \mathbf{r}) |\varphi(\mathbf{q}; \mathbf{r})|^2$$

$$= 1 + \int_0^\infty dr \underbrace{4\pi r^2 S(q; r)}_{\text{SF w/ Jacobian}} \underbrace{[|\varphi_0(q; r)|^2 - |j_0(qr)|^2]}_{\text{s-wave Change}}$$

**SF**  
 w/ Jacobian

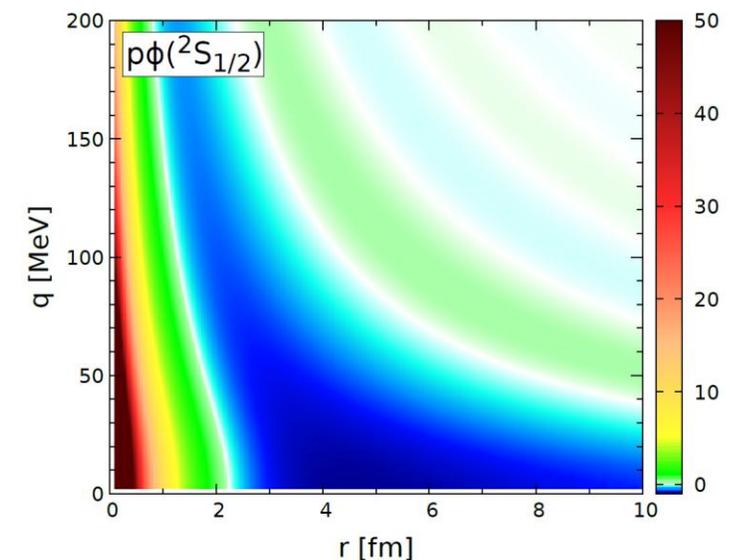
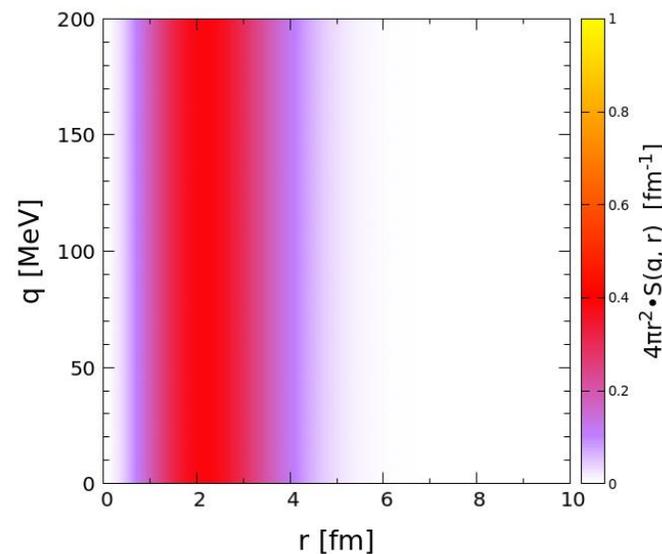
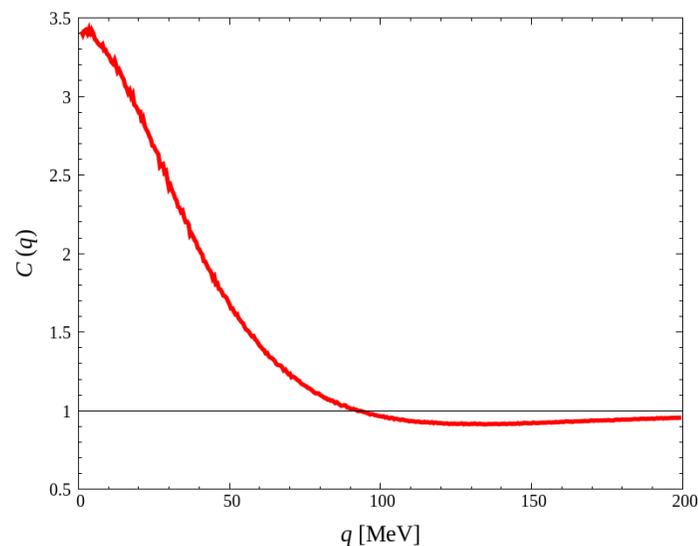
**s-wave Change**  
 Increase/Decrease of WF by **FSI**



Considering only ***s*-wave scattering** together with **spherical SF**

$$C(q) = 1 + \int_0^{\infty} dr \underbrace{4\pi r^2 S(q; r)}_{\substack{\text{SF} \\ \text{with Jacobian}}} \underbrace{[|\varphi_0(q; r)|^2 - |j_0(qr)|^2]}_{\substack{\text{s-wave Change} \\ \text{Increase/Decrease in WF by interaction}}}$$

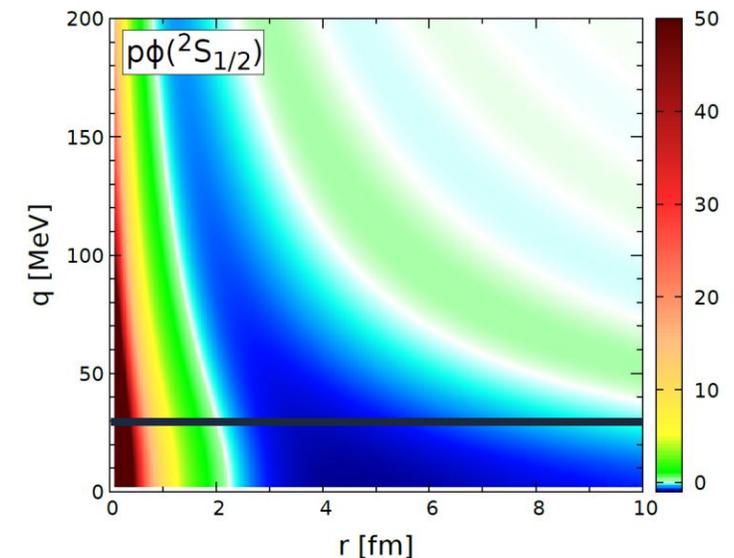
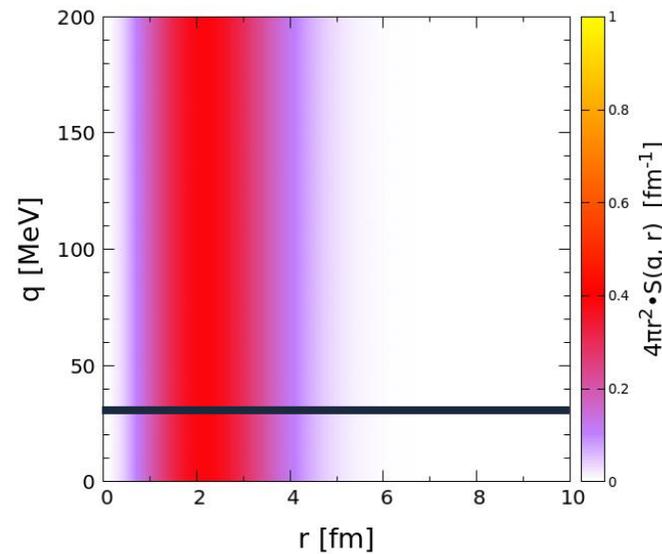
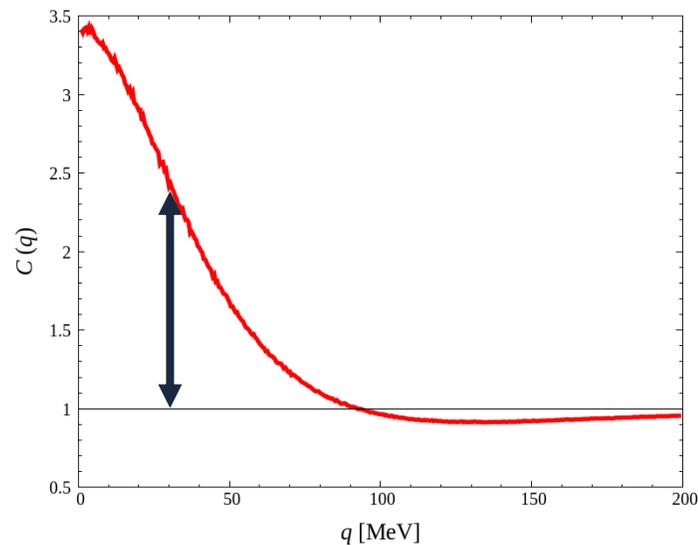
Deviation of  $C(q)$  from 1 = How much **SF** “picks up” **WF change**



Considering only ***s-wave scattering*** together with **spherical SF**

$$C(q) = 1 + \int_0^{\infty} dr \quad \underbrace{4\pi r^2 S(q; r)}_{\substack{\text{SF} \\ \text{with Jacobian}}} \quad \underbrace{[|\varphi_0(q; r)|^2 - |j_0(qr)|^2]}_{\substack{\text{s-wave Change} \\ \text{Increase/Decrease in WF by interaction}}}$$

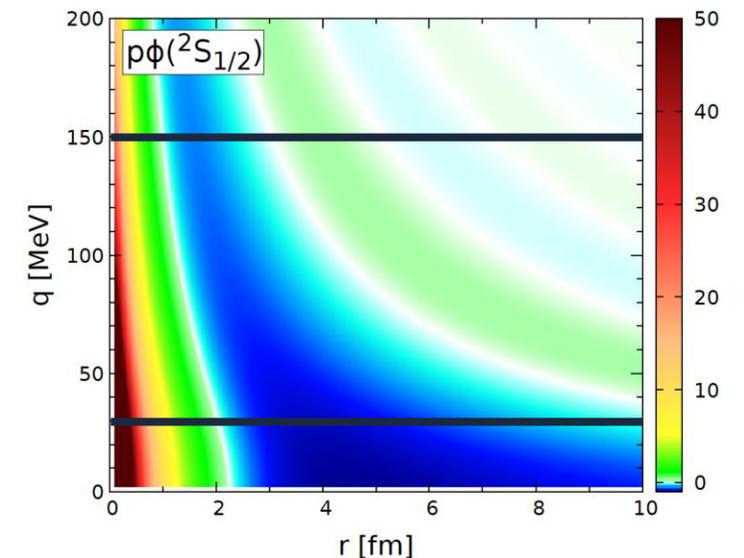
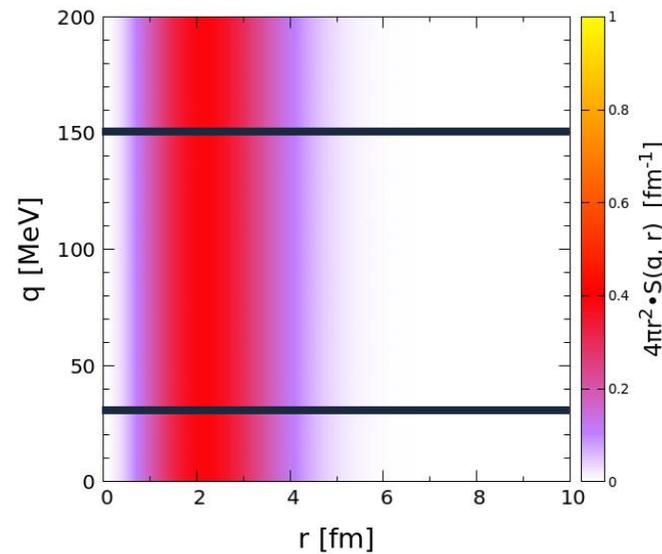
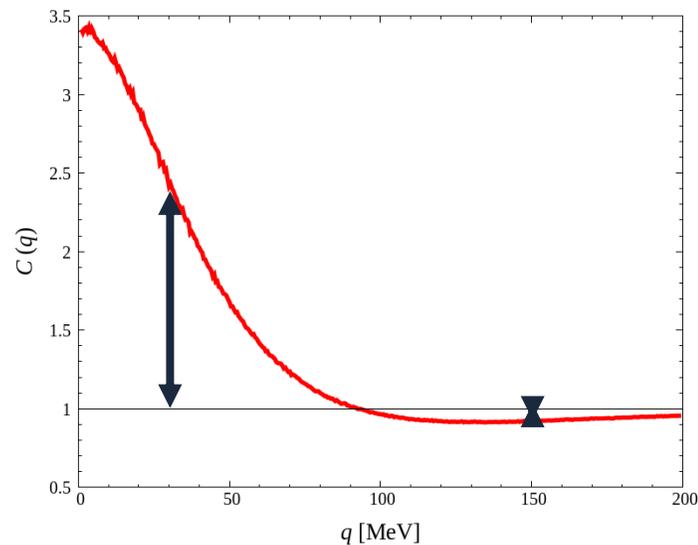
Deviation of  $C(q)$  from 1 = How much **SF** “picks up” **WF change**



Considering only **s-wave scattering** together with **spherical SF**

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Deviation of  $C(q)$  from 1 = How much **SF** “picks up” **WF change**



**WF in KP formula** = Weighted average of WF in each  $^{2S+1}L_J$  channel

$$|\varphi|^2 = \sum_{\text{states}(S,L,J)} \omega_{(S,L,J)} |\varphi^{(S,L,J)}|^2$$

$$\omega_{(S,L,J)} = \frac{2S + 1}{(2s_a + 1)(2s_b + 1)} \frac{2J + 1}{(2L + 1)(2S + 1)}$$

## Koonin-Pratt formula

**Spin-independent SF** ← chaotic source

### Spin-averaged CF

$$C^{\text{tot}}(\mathbf{q}) = \sum_{\text{states}(S,L,J)} \omega_{(S,L,J)} C^{(S,L,J)}(\mathbf{q})$$

Comparable  
w/ exp. CF

Focusing on low- $q$  region w/ chaotic source and closed system assumptions  
→ **Steady-state Schrödinger eq. w/ central force**

## Partial-wave expansion

$$\varphi(\mathbf{q}; \mathbf{r}) = \sum_{l=0}^{\infty} (2l + 1) i^l \varphi_l(q; r) P_l(\cos\theta)$$

For each  $^{2S+1}L_J$  channel,

$$\left[ -\frac{1}{2\mu} \frac{d^2}{dr^2} + V(r) + \frac{1}{2\mu} \frac{l(l+1)}{r^2} \right] u_l(q; r) = \frac{q^2}{2\mu} u_l(q; r)$$

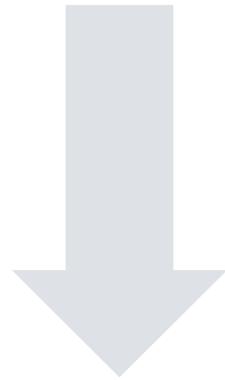
$$u_l := r \varphi_l$$

Reduced mass:

$$\mu = \frac{m_a m_b}{m_a + m_b}$$

R. Lednický and V. L. Lyuboshits, Yad. Fiz. **35**, 1316 (1981)

$$C(q) = 1 + \int_0^\infty dr 4\pi r^2 S(q; r) [|\varphi_0(q; r)|^2 - |j_0(qr)|^2]$$



## Assumptions

- **Gaussian SF**:  $S(q; r) \approx S(r) \propto \exp\left(-\frac{r^2}{4r_0^2}\right)$
- **Asymptotic WF** (+ effective range correction)

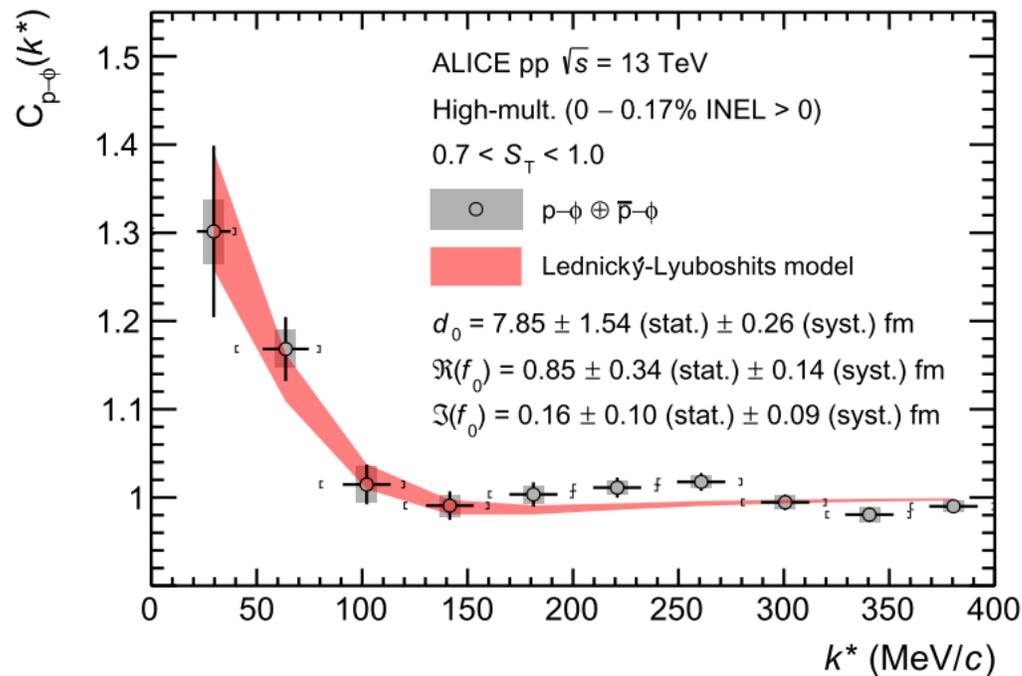
$$C(q) = 1 + \frac{|f_0(q)|^2}{2r_0^2} F_3\left(\frac{r_{\text{eff}}}{r_0}\right) + \frac{2\text{Re}f_0(q)}{\sqrt{\pi}r_0} F_1(2qr_0) - \frac{\text{Im}f_0(q)}{r_0} F_2(2qr_0)$$

$$F_1, \dots, F_3: \text{Known functions, } f_0(q) = \frac{1}{q \cot \delta_0(q) - iq} \approx \frac{1}{-\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 - iq}$$

**CF becomes a function of  $a_0$ ,  $r_{\text{eff}}$ , and  $r_0$**

## Experimental CF ALICE, PRL 127, 172301 (2021)

High-multiplicity (0–0.17%) p+p collisions at  $\sqrt{s} = 13$  TeV



## Lednický-Lyuboshits fit

R. Lednický and V. L. Lyuboshits, Yad. Fiz. **35**, 1316 (1981)

Gaussian source size:  $r_0 = 1.08$  fm

Scattering length:  $a_0 \cong -0.85 - 0.16i$  fm

Effective range:  $r_{\text{eff}} \cong 7.85$  fm

Attractive  $p\phi$  interaction as a spin-average

## Spin-channel-by-channel femtoscopy E. Chizzali *et al.*, PLB 848, 138358 (2023)

Gaussian source size:  $r_0 = 1.08$  fm

$^4S_{3/2}$ : HAL QCD potential Y. Lyu *et al.*, PRD 106, 074507 (2022)

$$a_0^{(3/2)} \cong -1.43 \text{ fm}, \quad r_{\text{eff}}^{(3/2)} \cong 2.36 \text{ fm}$$

Attraction without bound states

$^2S_{1/2}$ : Parametrized potential ← Constrain by **experimental CF**

$$a_0^{(1/2)} \cong 1.54 - i0.00 \text{ fm}, \quad r_{\text{eff}}^{(1/2)} \cong 0.39 + i0.00 \text{ fm}$$

■ Strong attraction

■ Small effects of channel-coupling

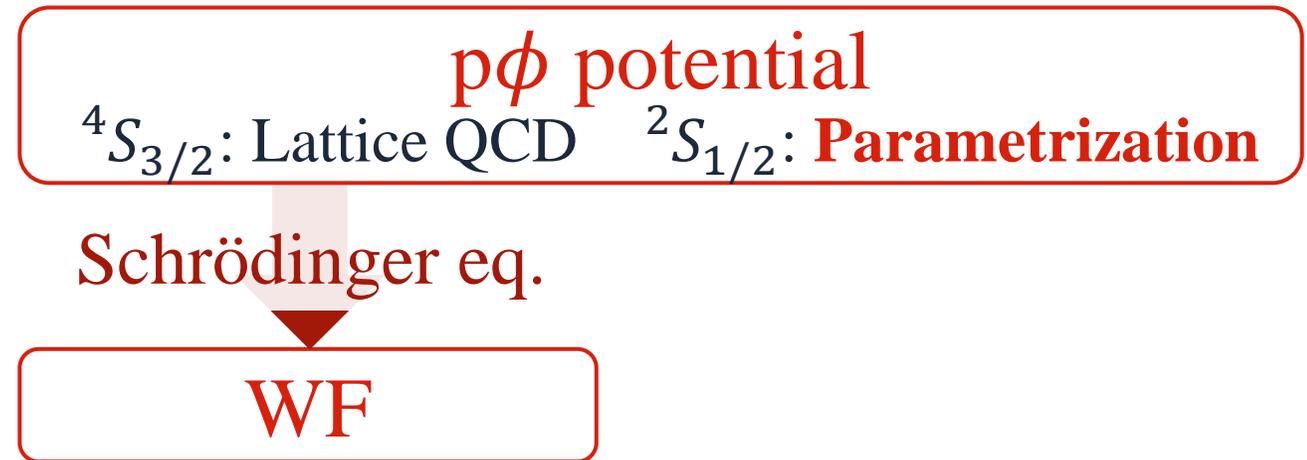
Indication of a  $p\phi$  bound state

SF should reflect the complex dynamics of nuclear collisions

This study: **Femtoscscopy using SF from a dynamical model**

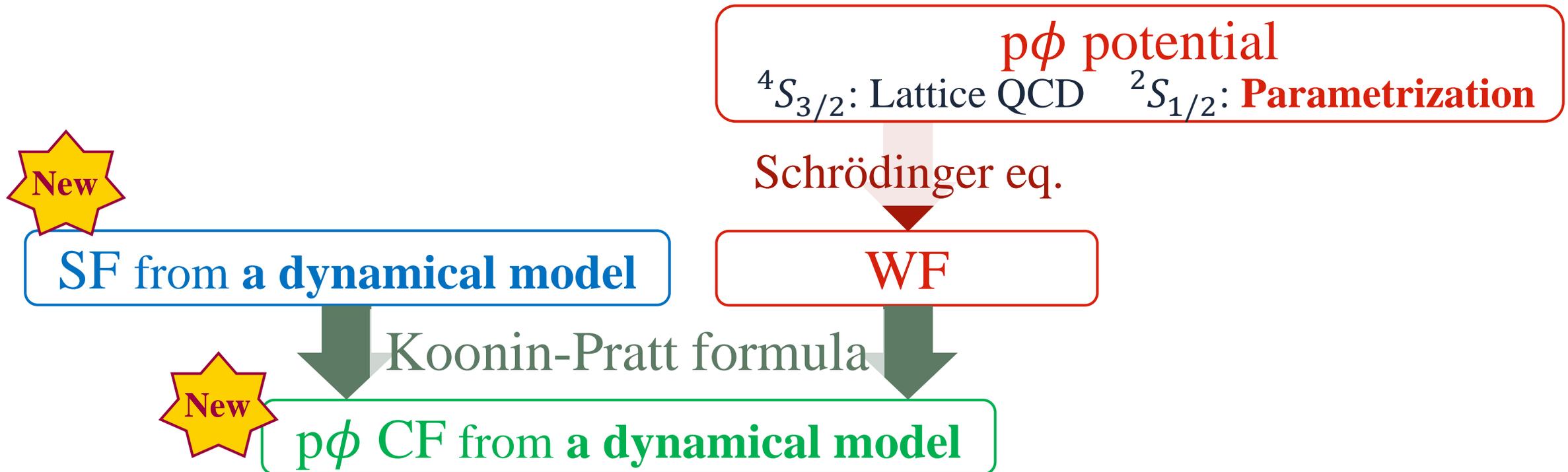
SF should reflect the complex dynamics of nuclear collisions

This study: **Femtoscscopy using SF from a dynamical model**



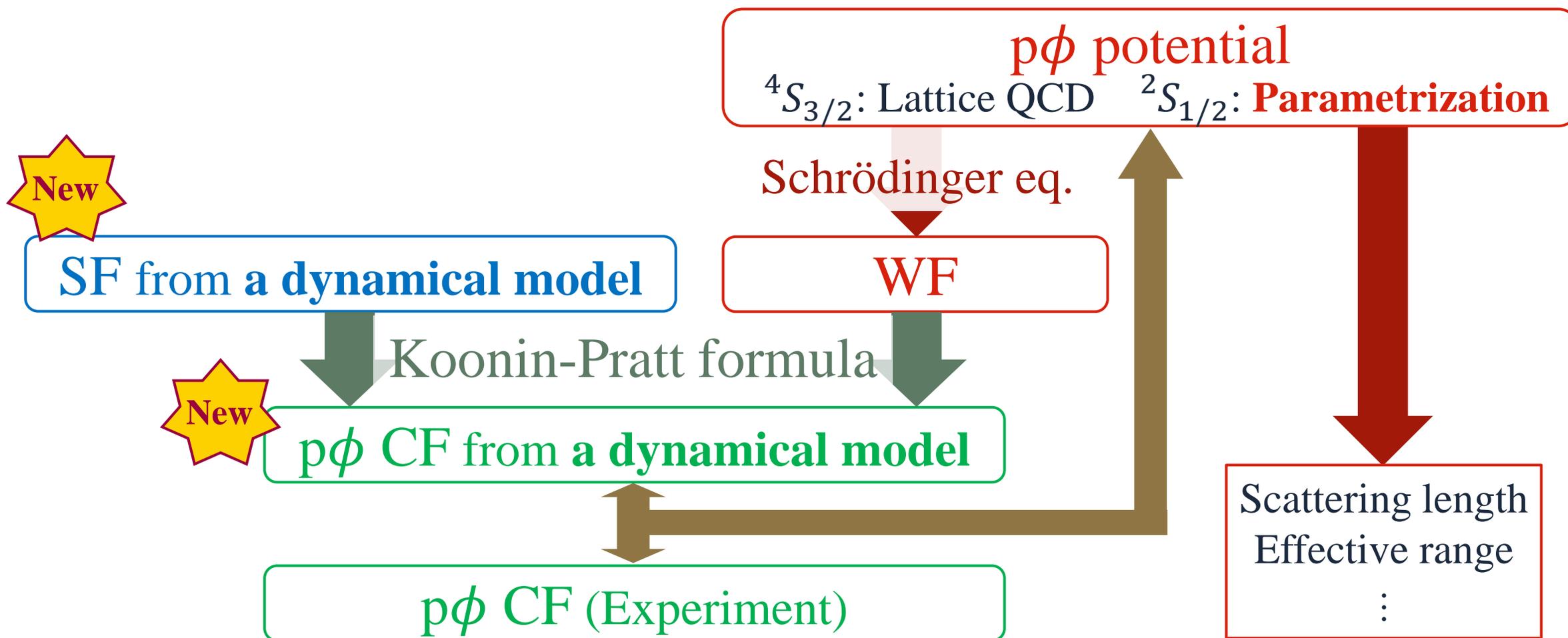
SF should reflect the complex dynamics of nuclear collisions

This study: **Femtoscopy using SF from a dynamical model**



SF should reflect the complex dynamics of nuclear collisions

This study: **Femtoscscopy using SF from a dynamical model**



## HAL QCD potential Y. Lyu *et al.*, PRD **106**, 074507 (2022)

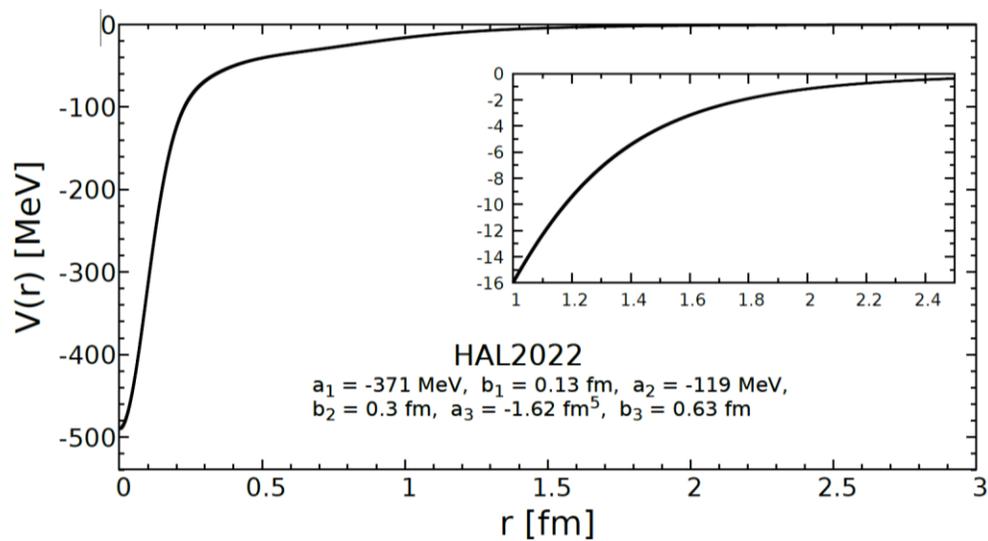
Lattice QCD at nearly physical point ( $m_\pi = 146.4$  MeV)

$$V^{(3/2)}(r) = \underbrace{a_1 e^{-(r/b_1)^2} + a_2 e^{-(r/b_2)^2}}_{\text{Short-range attraction}} + \underbrace{a_3 m_\pi^4 f(r; b_3) \frac{e^{-2m_\pi r}}{r^2}}_{\text{TPE}}$$

Argonne-type form factor:

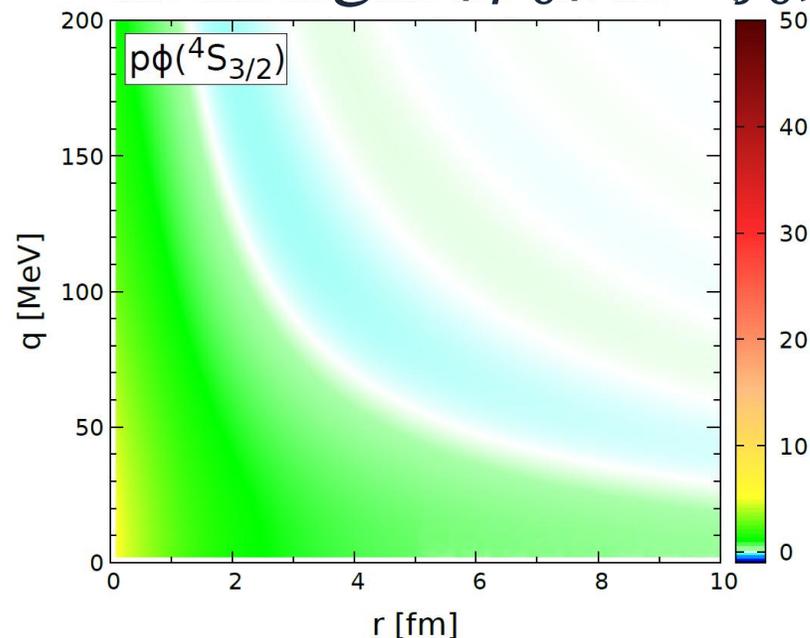
$$f(r; b_3) = [1 - e^{-(r/b_3)^2}]^2$$

Parameter	Fitted value
$a_1$ [MeV]	$-371 \pm 27$
$b_1$ [fm]	$0.13 \pm 0.01$
$a_2$ [MeV]	$-119 \pm 39$
$b_2$ [fm]	$0.30 \pm 0.05$
$a_3$ [fm <sup>5</sup> ]	$-1.62 \pm 0.23$
$b_3$ [fm]	$0.63 \pm 0.04$



No bound state

WF change:  $|\varphi_0|^2 - (j_0)^2$



**Enhancement  
at small  $qr$   
due to attraction**

## Parametrized potential E. Chizzali *et al.*, PLB 848, 138358 (2023)

Channel-couplings are neglected for simplicity

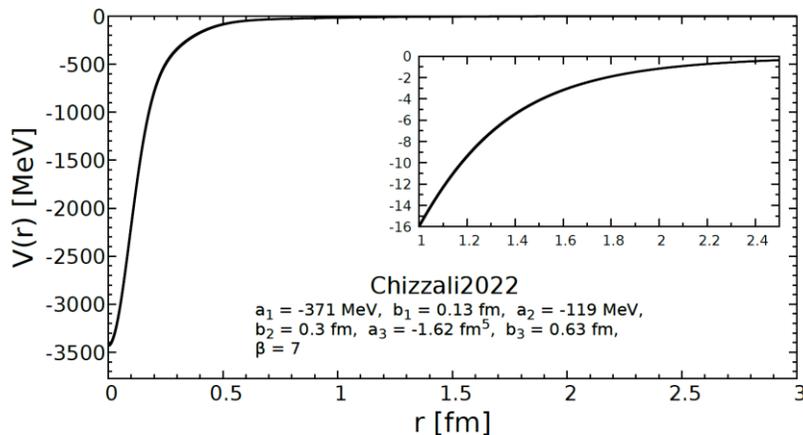
$$V^{(1/2)}(r) = \beta \left[ a_1 e^{-(r/b_1)^2} + a_2 e^{-(r/b_2)^2} \right] + a_3 m_\pi^4 f(r; b_3) \frac{e^{-2m_\pi r}}{r^2}$$

Short-range interaction                      TPE

**Only one adjustable parameter**

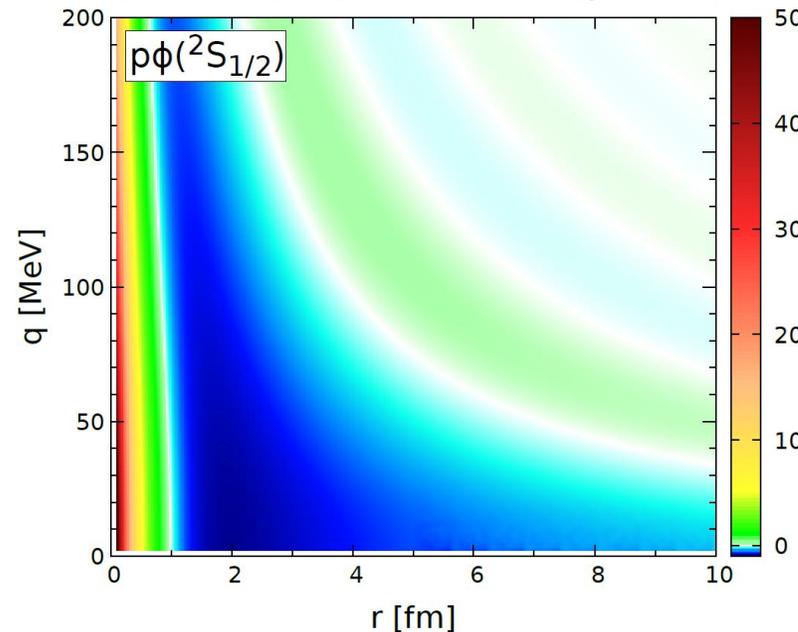
$\beta$

default:  $\beta = 7$



$a_0 = 1.99$  fm  
 $r_{\text{eff}} = 0.46$  fm  
**A bound state**

WF change:  $|\varphi_0|^2 - (j_0)^2$



■ **Strong enhancement at small  $qr$**

■ **“Negative valley” around  $a_0$**

Weak

Attractive potential w/ a bound state

Strong

$$\beta = 6$$

$$a_0 = 4.54 \text{ fm}$$

$$E_B = 2.3 \text{ MeV}$$

$$\beta = 7$$

$$a_0 = 1.99 \text{ fm}$$

$$E_B = 13.3 \text{ MeV}$$

$$\beta = 8$$

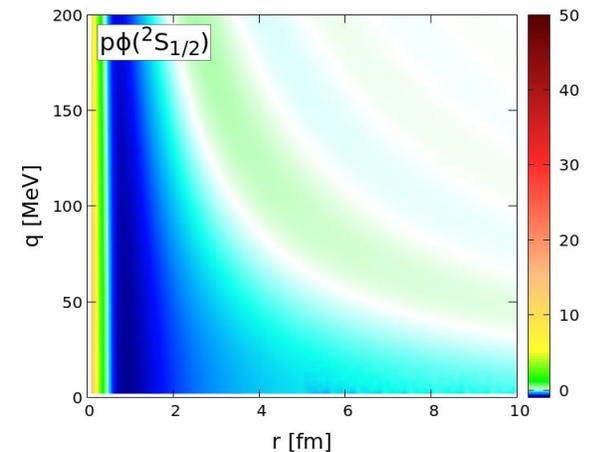
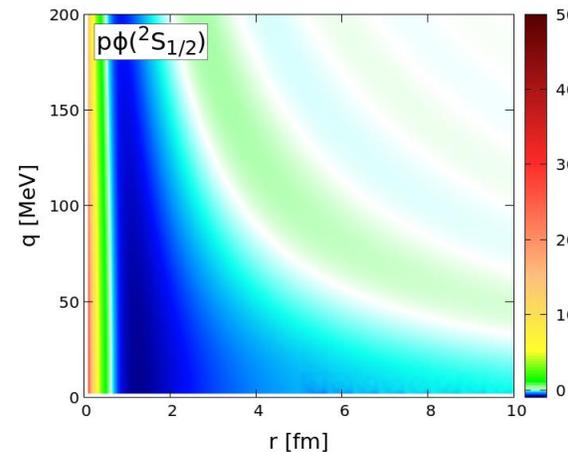
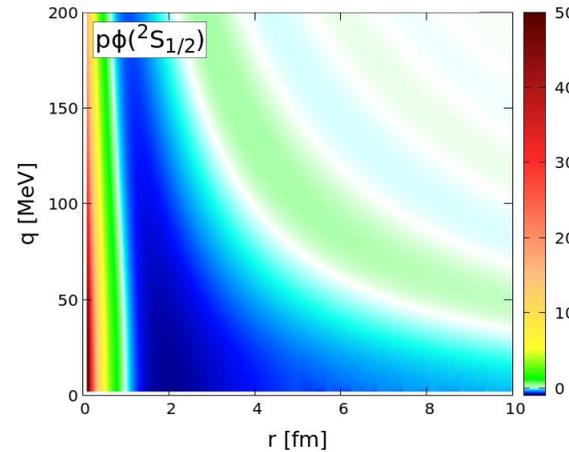
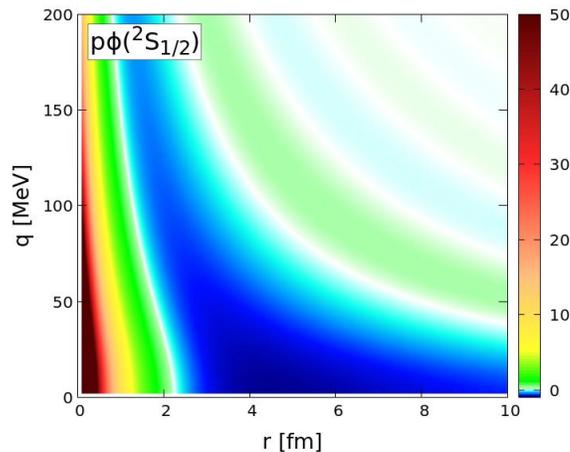
$$a_0 = 1.23 \text{ fm}$$

$$E_B = 37.5 \text{ MeV}$$

$$\beta = 9$$

$$a_0 = 0.85 \text{ fm}$$

$$E_B = 93.1 \text{ MeV}$$

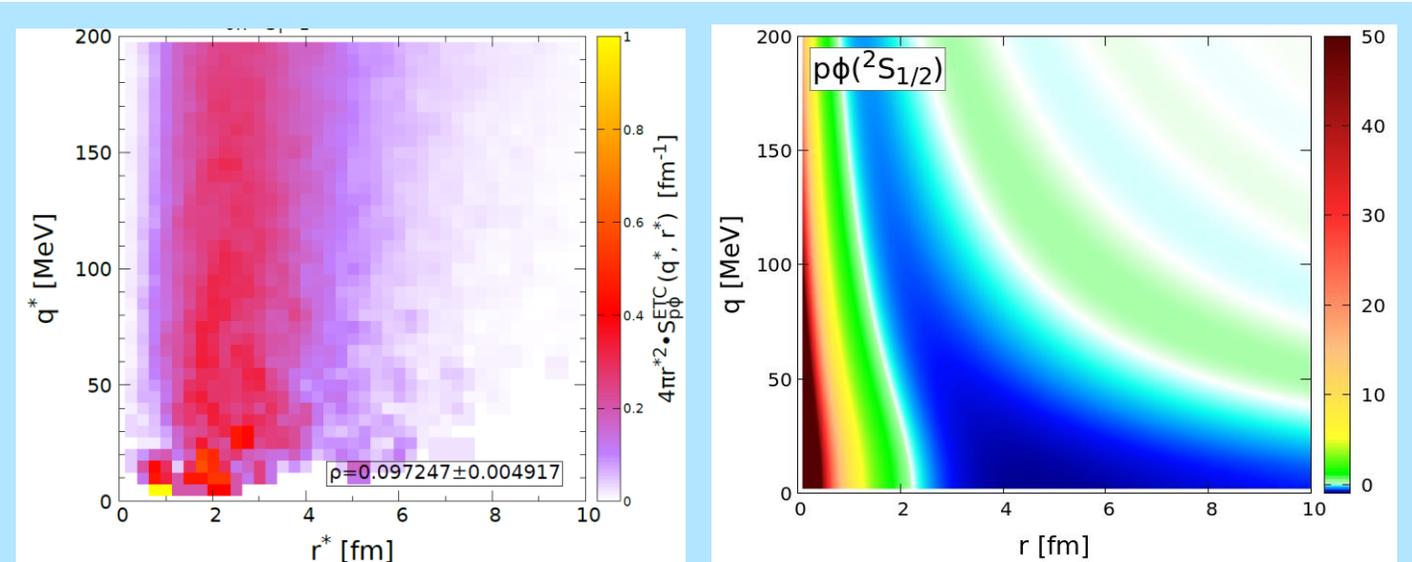
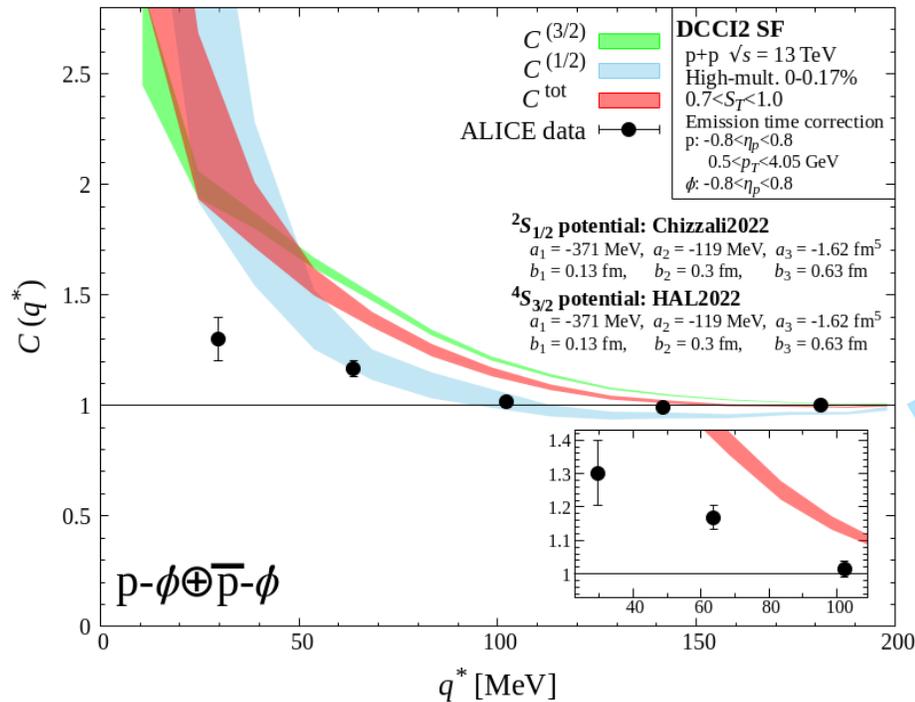


The negative valley moves towards the small  $r$  region

$C^{(3/2)}$ : Fixed,  $C^{(1/2)}$ : Change with  $\beta$

Compare  $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$  with ALICE data

$\beta = 6$

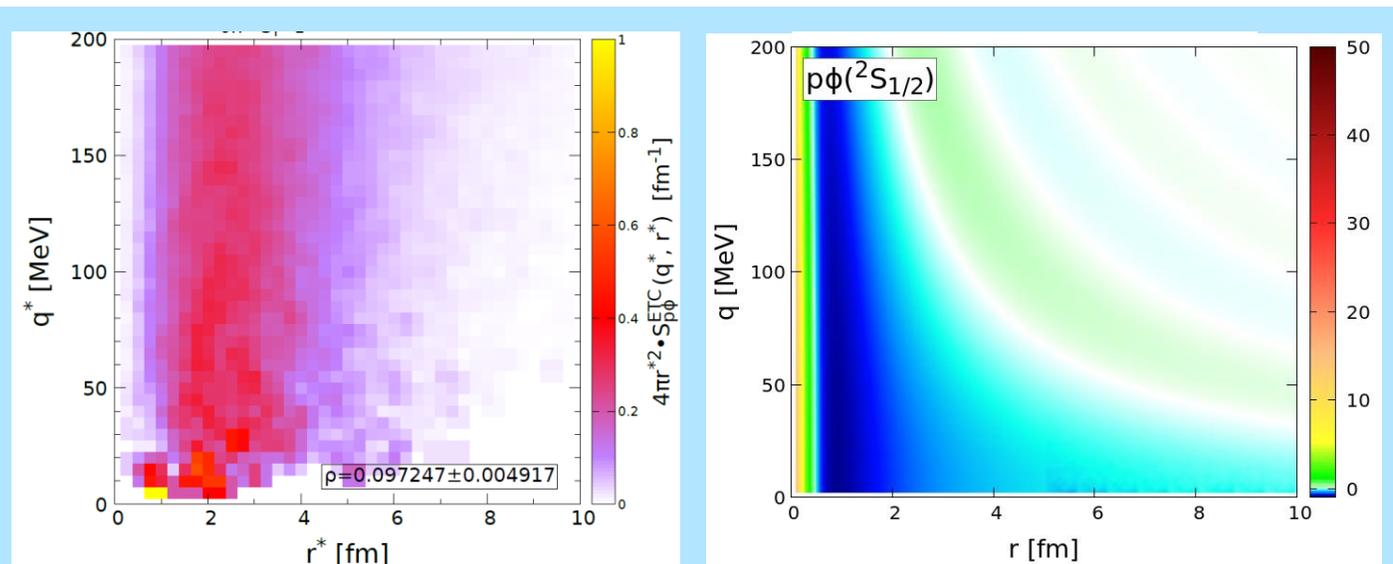


**SF picks up strong positive region of WF**

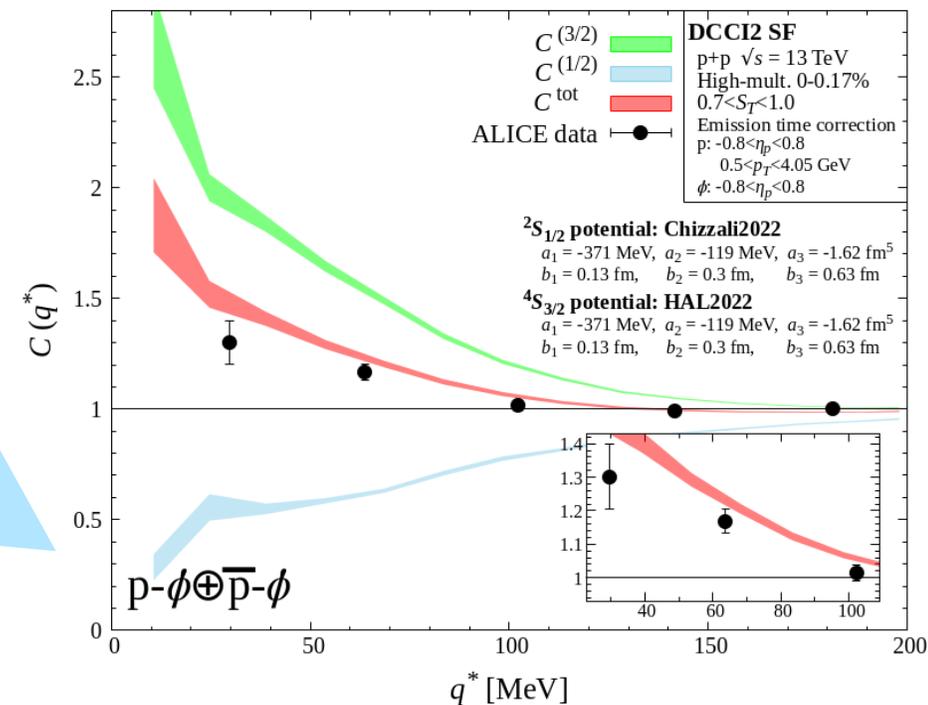
$C^{\text{tot}} > C^{\text{exp}}$

$C^{(3/2)}$ : Fixed,  $C^{(1/2)}$ : Change with  $\beta$   
 Compare  $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$  with ALICE data

$\beta = 9$



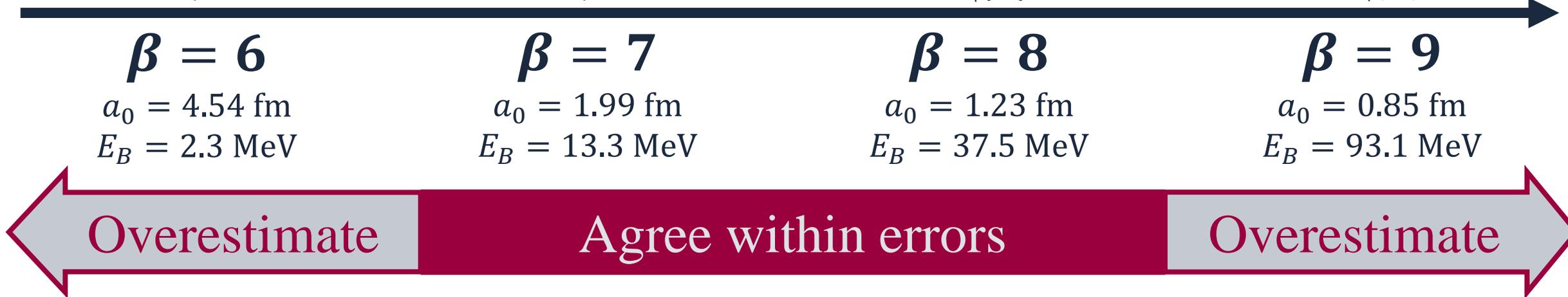
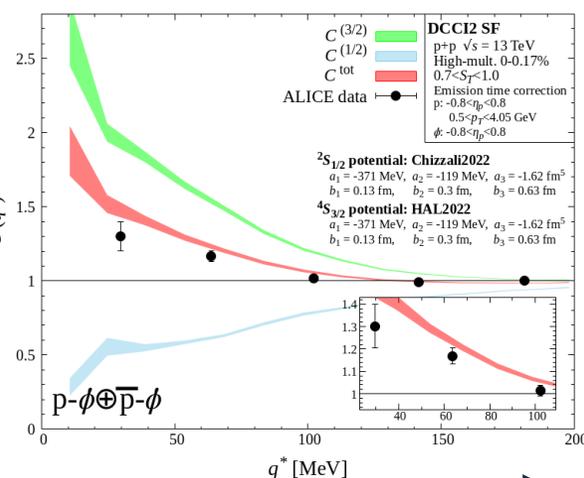
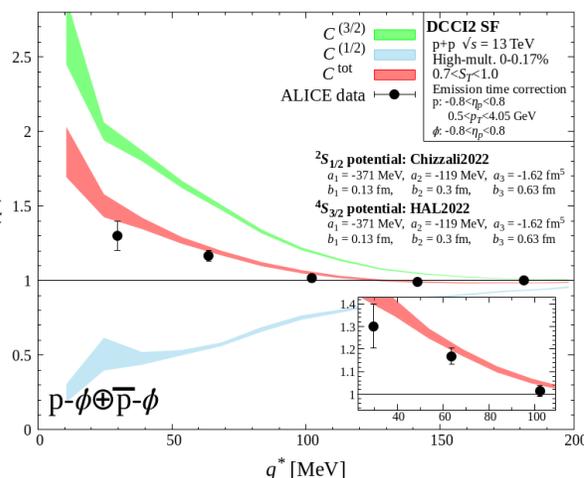
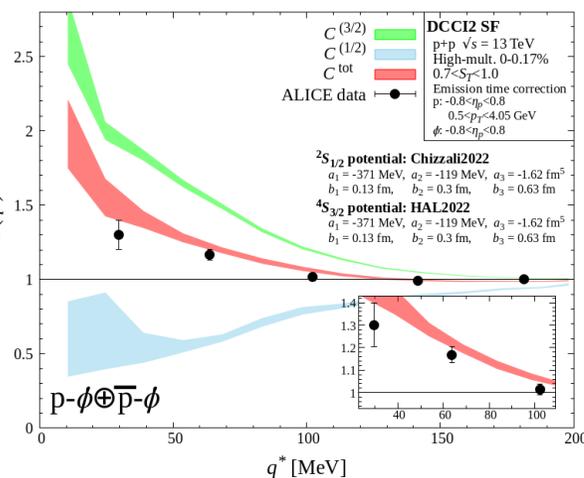
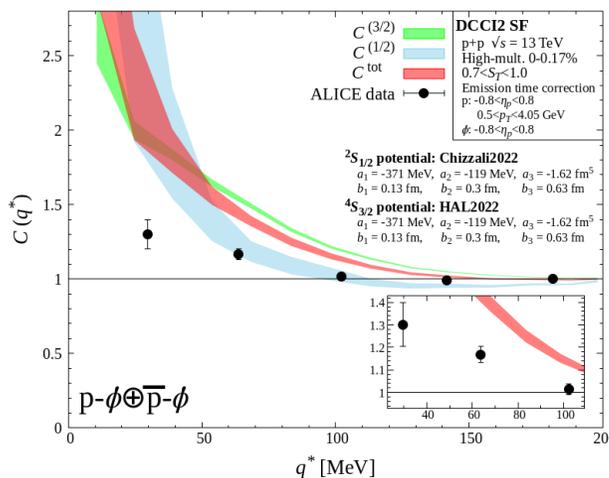
SF cannot pick up negative valley efficiently



$C^{\text{tot}} > C^{\text{exp}}$

$C^{(3/2)}$ : Fixed,  $C^{(1/2)}$ : Change with  $\beta$

Compare  $C^{\text{tot}} = \frac{2}{3} C^{(3/2)} + \frac{1}{3} C^{(1/2)}$  with ALICE data

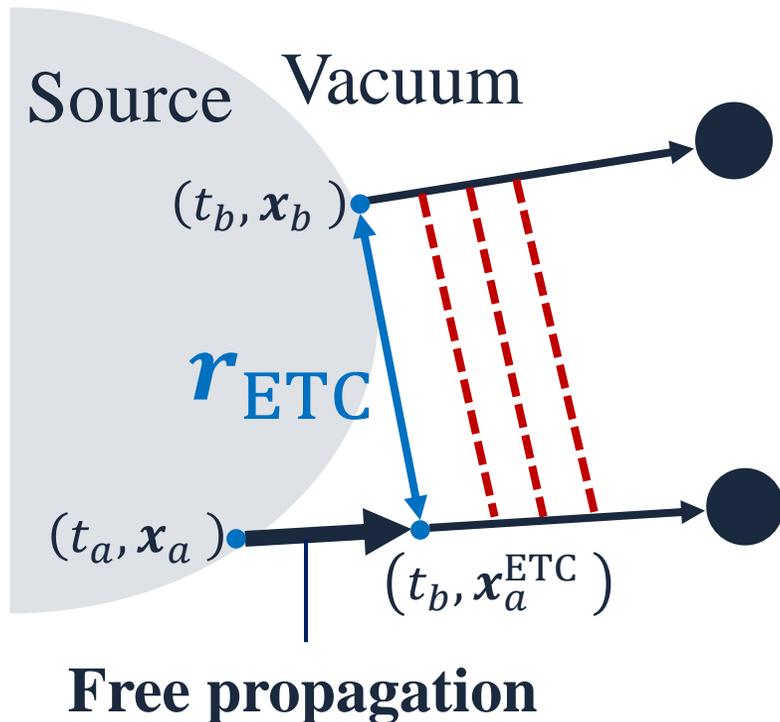


## Problem

Dynamical model  $\rightarrow$  Emission time difference:  $S(\mathbf{q}; r^0 \neq 0, \mathbf{r})$

Violates **same time approximation** in KP formula

## Free propagation until the other's emission



$$S(\mathbf{q}; r^0 \neq 0, \mathbf{r})$$

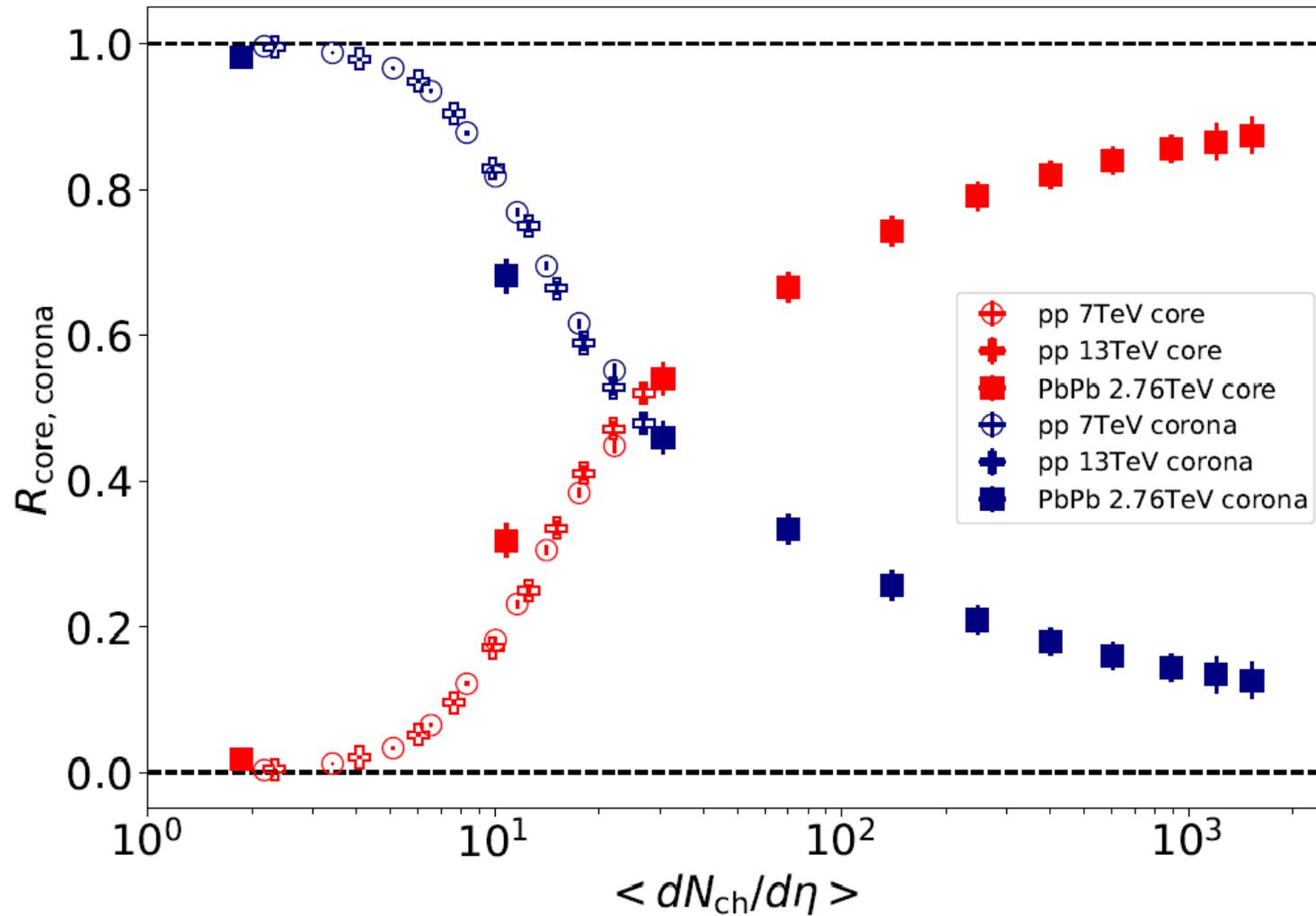
**ETC**  $\rightarrow$

$$S^{\text{ETC}}(\mathbf{q}; \mathbf{r}_{\text{ETC}}) \delta(r^0)$$

$$\mathbf{r}_{\text{ETC}} = \mathbf{r} + \frac{\mathbf{p}_a}{E_a} (t_b - t_a) \theta(t_b - t_a) - \frac{\mathbf{p}_b}{E_b} (t_a - t_b) \theta(t_a - t_b)$$

**Correction**

Y. Kanakubo, Y. Tachibana, and T. Hirano, PRC **105**, 024905 (2022)



According to DCCI2

$$R_{\text{core}} \sim 0.5$$

in high-multiplicity  
p+p collisions  
at  $\sqrt{s} = 13$  TeV