Non-equilibrium evolution near the QCD critical point

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Plan

- The QCD critical point
- Static nature – fluctuations
- Dynamic nature – critical soft modes
- Out-of-equilibrium nature – A schematic demo
- Remarks

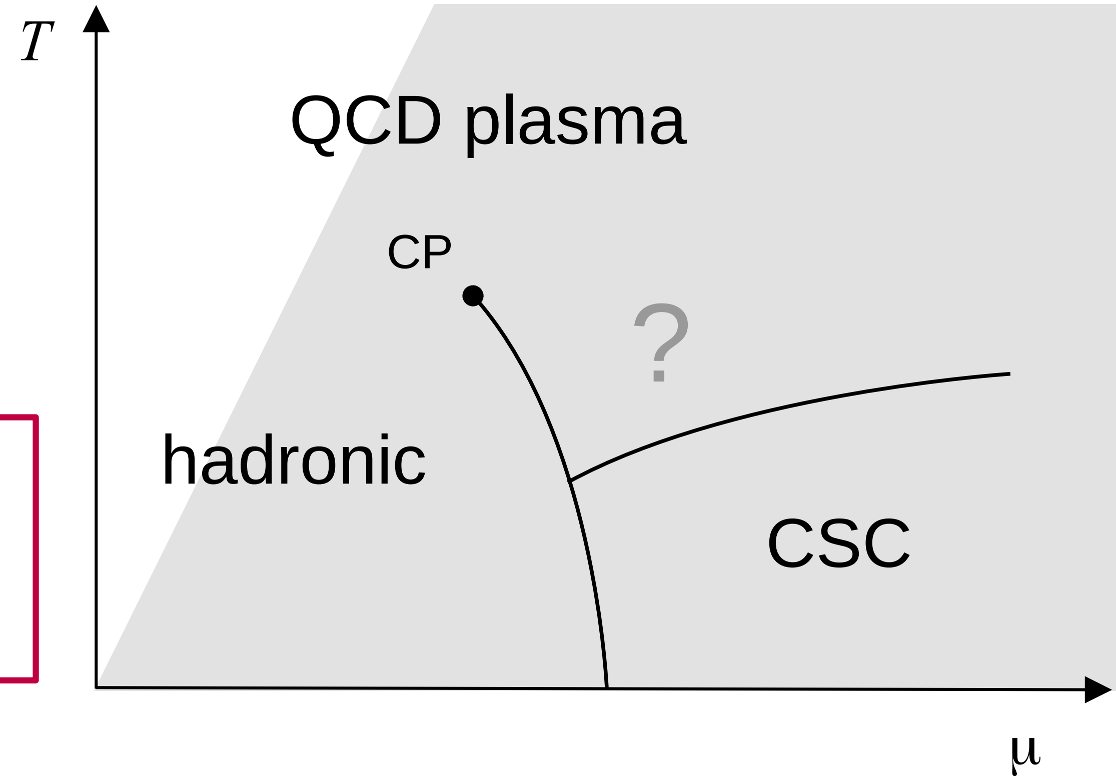
So many works on the QCD critical point, and cited refs are biased due to my limited knowledge

QCD critical point

Asakawa-Yazaki, ‘89, Barducci+, ‘89, ...

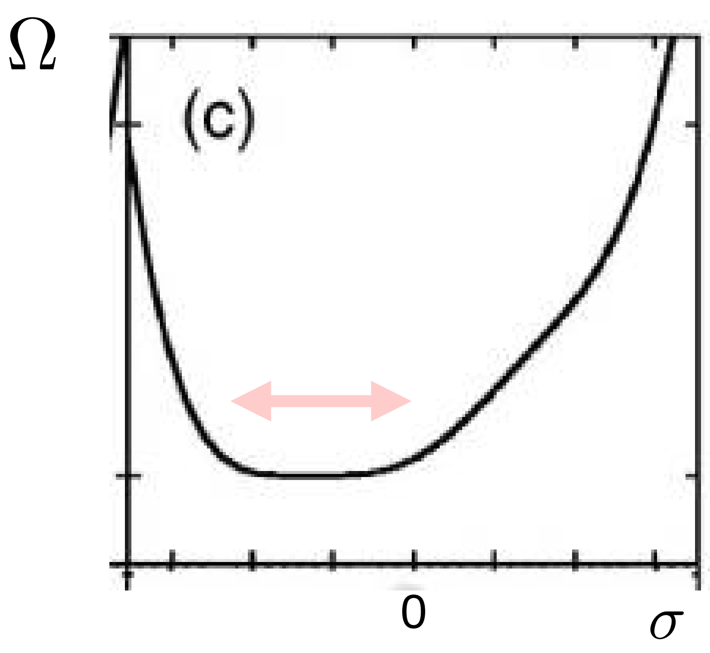
- Important landmark predicted by many effective models
- Currently, difficult to access by LQCD methods: sign problem
- Critical phenomena should manifest in relevant observables

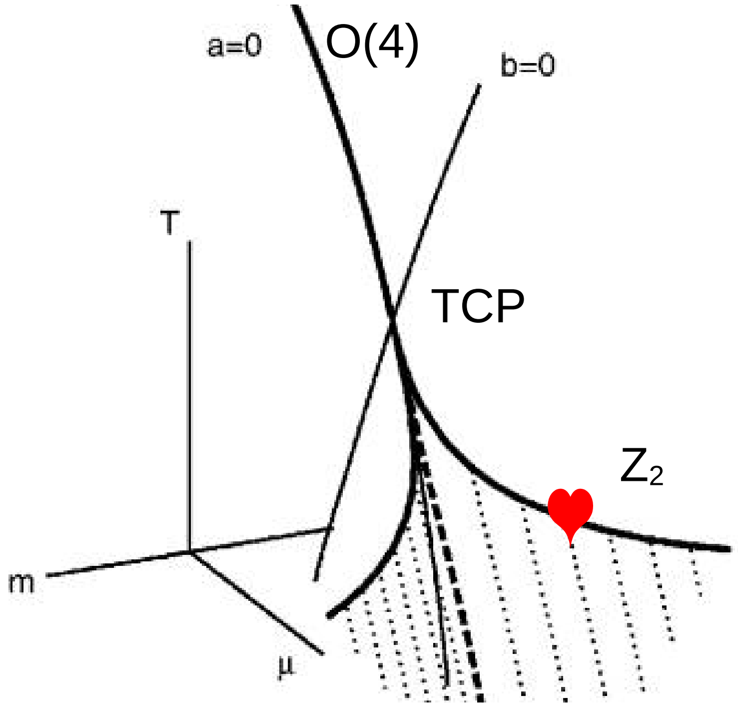
Study of its observable consequences is crucially important.



massless σ?

- The order parameter of chiral symmetry
- Non-zero m != 0 explicitly breaks the symmetry
- Yet the potential becomes flat at the critical point





HF-M.Ohtani, PRD70, 014016 ('04)

Static nature

- described by Landau-Ginzburg free energy in terms of σ
- in the same universality class with Ising model

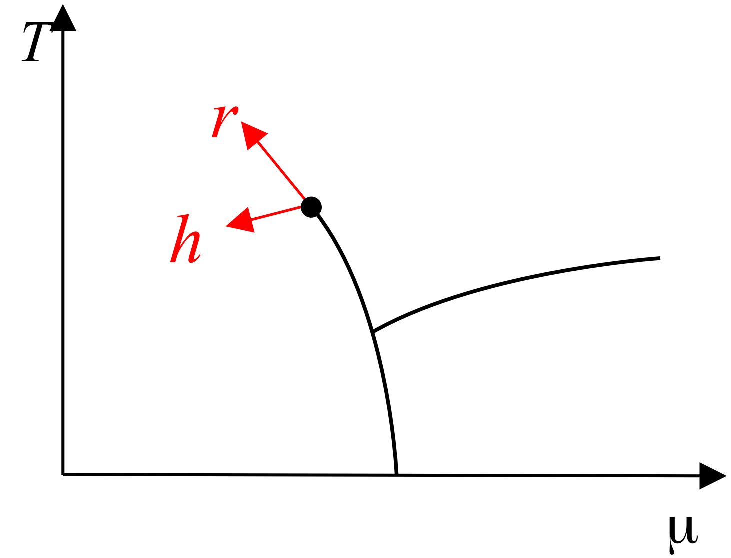
$$f = \frac{1}{2} r \phi^2 + \frac{1}{4} \lambda \phi^4 - h\phi$$

but, more precisely, ...

Static nature

- Mapping to Z_2 universality

$$\frac{\partial}{\partial h} \to \phi \qquad \frac{\partial}{\partial r} \to \phi^2$$



$$\frac{\partial}{\partial h} = c_1 \frac{\partial}{\partial \beta} + c_2 \frac{\partial}{\partial(\beta\mu)} \to c_1 \delta e + c_2 \delta n$$

$$\frac{\partial}{\partial r} = c_3 \frac{\partial}{\partial \beta} + c_4 \frac{\partial}{\partial(\beta\mu)} \to c_3 \delta e + c_4 \delta n \qquad \beta = 1/T$$

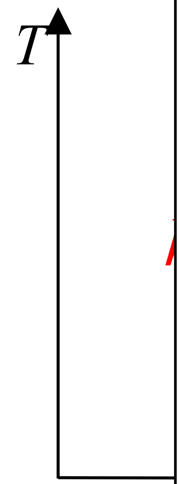
$m_q \neq 0$ $\qquad \delta\sigma \sim \delta e \sim \delta n$ $\qquad$ sigma ~ energy ~ baryon #

- Any of {$\delta\sigma$, δe, δn} can be chosen as the order param
- Susceptibilities χ's of them share the same criticality

Static nature

- Mapping to Z_2 universality

$$\frac{\partial}{\partial h} \to \phi \qquad \frac{\partial}{\partial r} \to \phi^2$$



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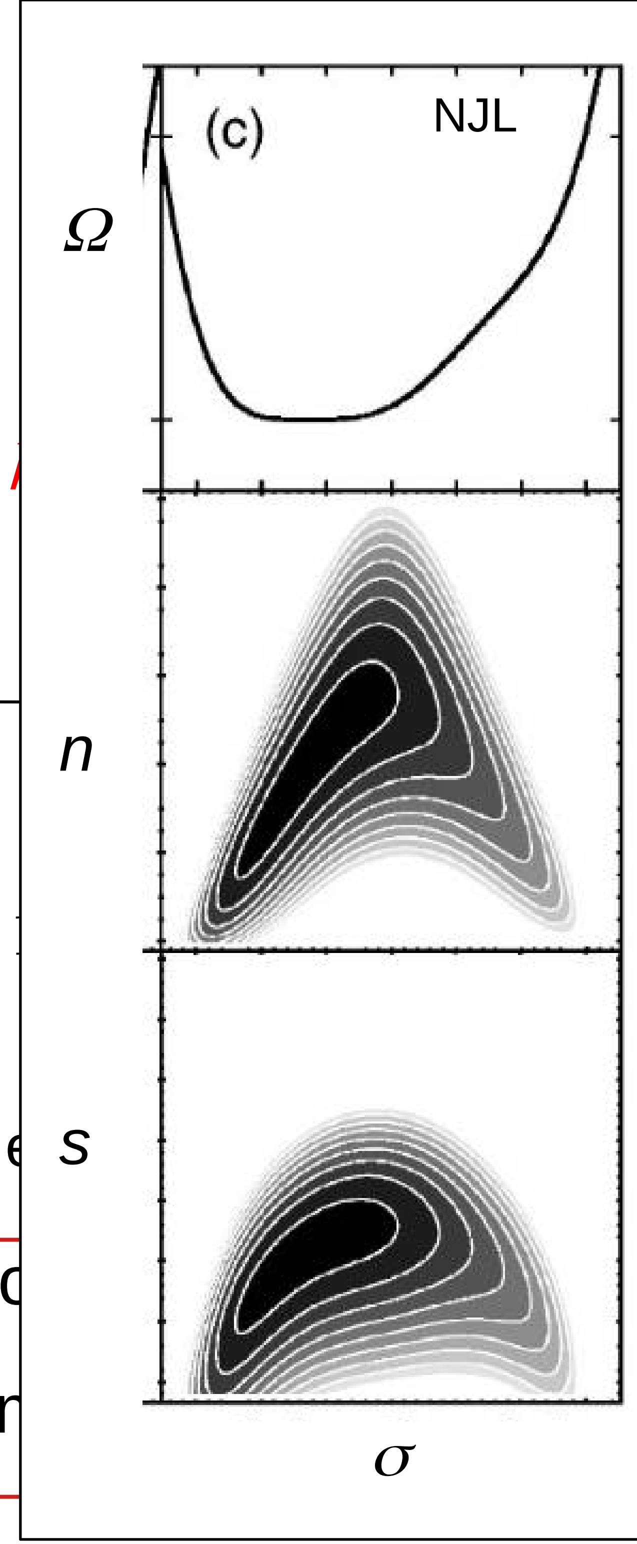
$$\frac{\partial}{\partial r} = c_3 \frac{\partial}{\partial \beta} + c_4 \frac{\partial}{\partial(\beta\mu)} \to c_3 \delta e + c_4 \delta n$$

$\beta =$

$m_q \neq 0$ $\qquad \delta\sigma \sim \delta e \sim \delta n$ $\qquad$ sigma ~ e



- Any of {δσ, δe, δ*n*} can be chosen as the c
- Susceptibilities χ's of them share the sar

Higher order fluctuations κ_n

Also, M.Asakawa, S. Ejiri,M. Kitazawa, '09

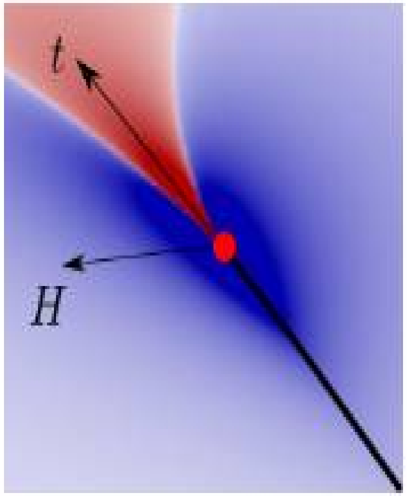
- More sensitive to critical correlation length ξ (suppressed by V)

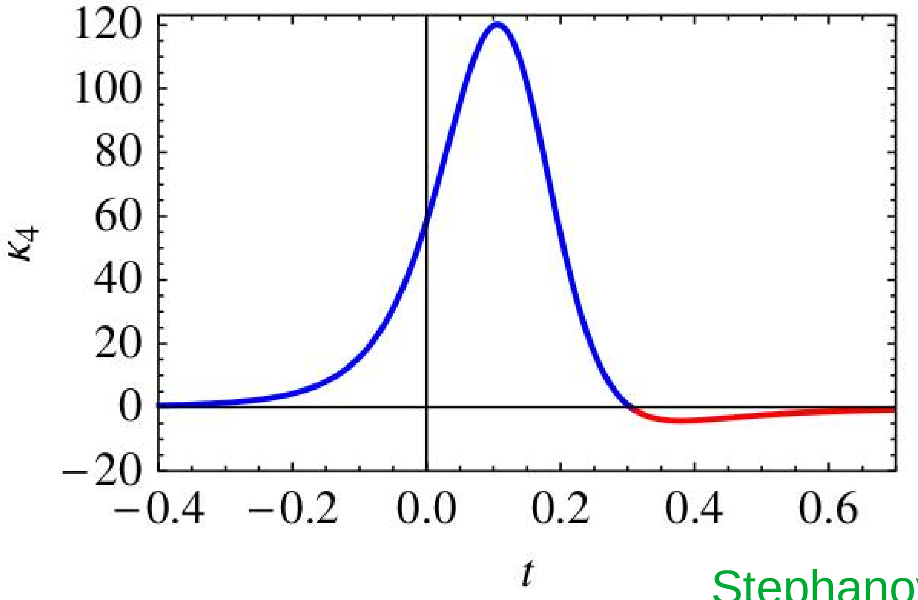
$$V^{n-1}\kappa_n = \frac{1}{V}\int_{x_1,\dots x_n}\langle\phi(x_1)\cdots\phi(x_n)\rangle \sim T^{n/2}\xi^{5n/2-3}$$

Stephanov ('09)

- Sign change of 4th-order fluctuation is predicted

See Nu Xu's lecture for exp't status





Stephanov ('11)

Dynamic nature – model H

Hohenberg-Halperin

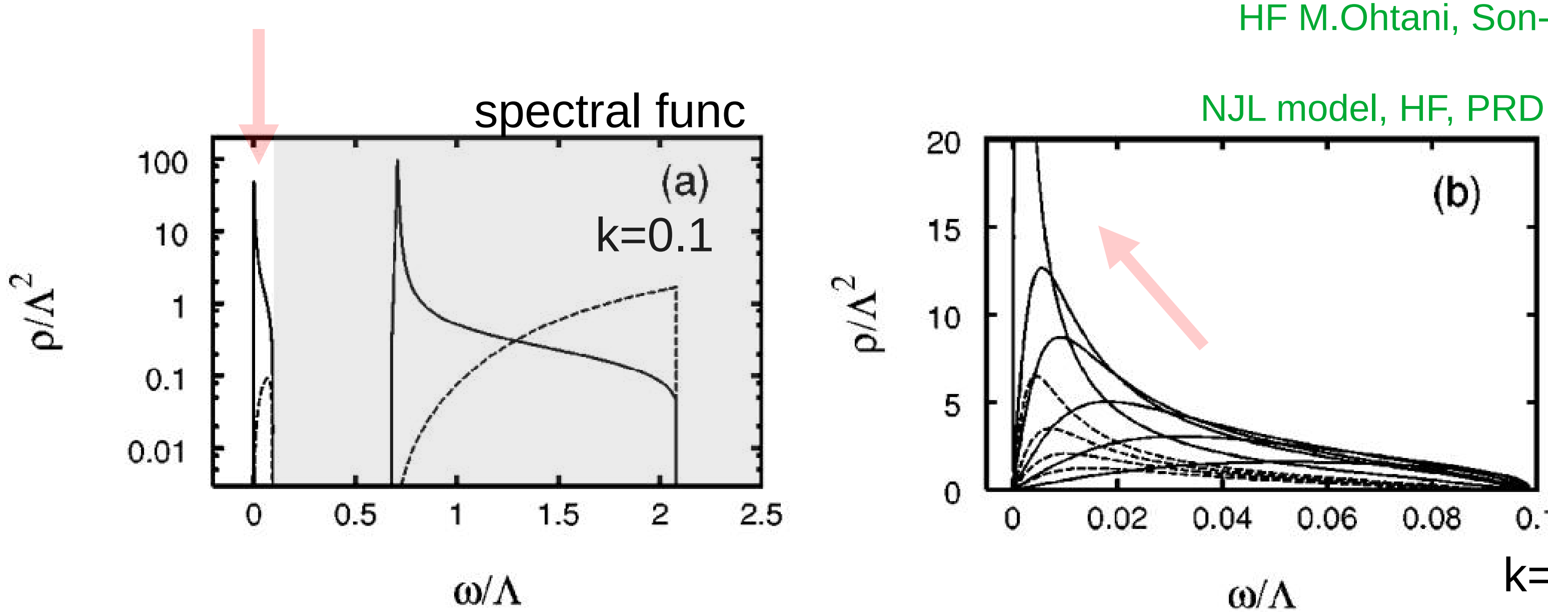
Chattopadhyay's lecture

- Long-time behavior is governed by a set of hydrodynamic modes
  - Susceptibilities of baryon# and energy densities come solely from hydro-dynamic poles, *not by "σ "-meson pole*
  - In quark picture, scatterings among quarks with space-like mom-exch are enhanced as the critical point is approached
  - The set of hydro modes of QCD critical point – energy, baryon number + momentum densities = model H

HF M.Ohtani, Son-Stephanov ('04)

NJL model, HF, PRD 67, 094018 ('03)

spectral func

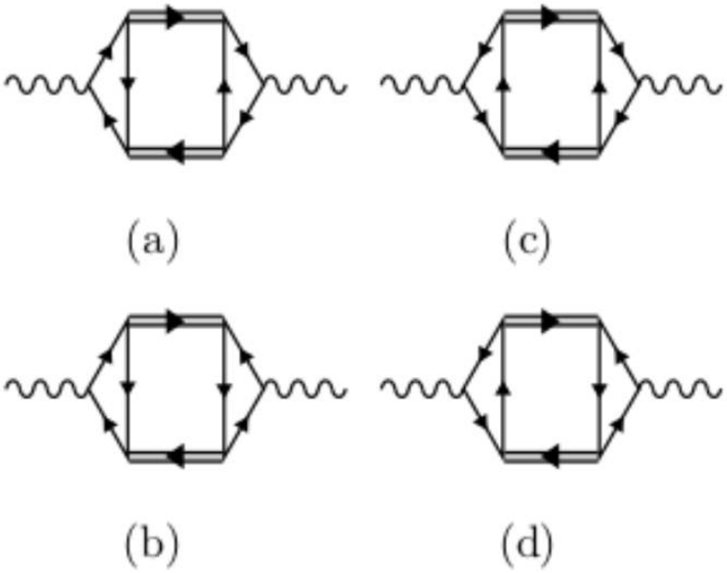


k=0.1

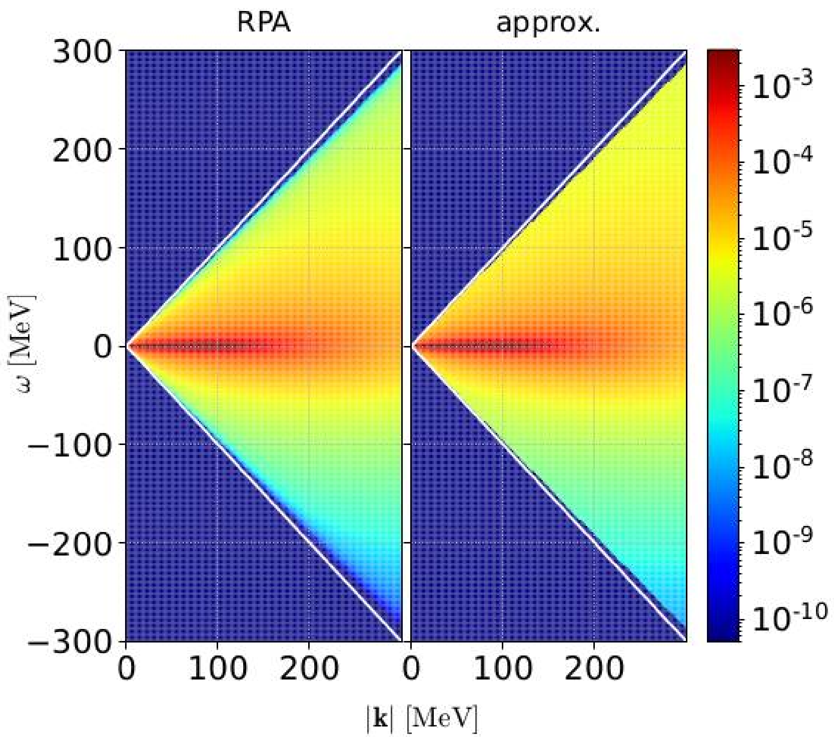
diplepton production

- soft mode couples to photons via quark loop

Nishimura-Kunihiro-Kitazawa (‘23)



+MT term + DOS term

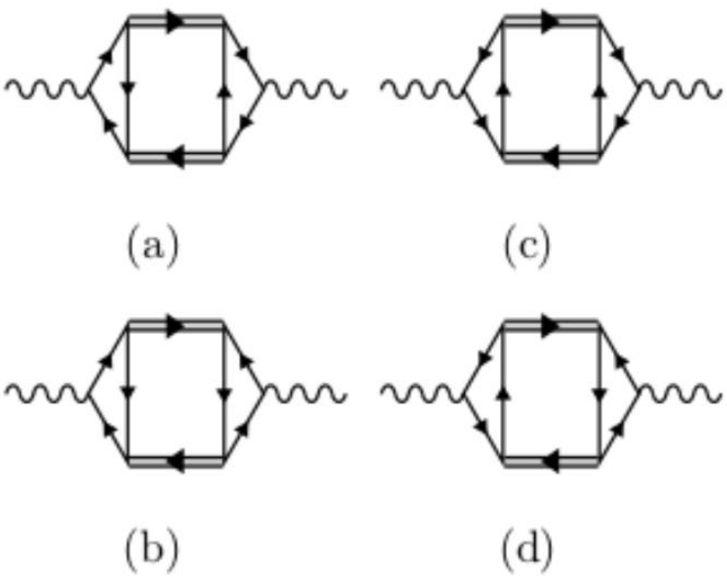


dynamical structure factor of soft mode (NJL & its TDGL approx)

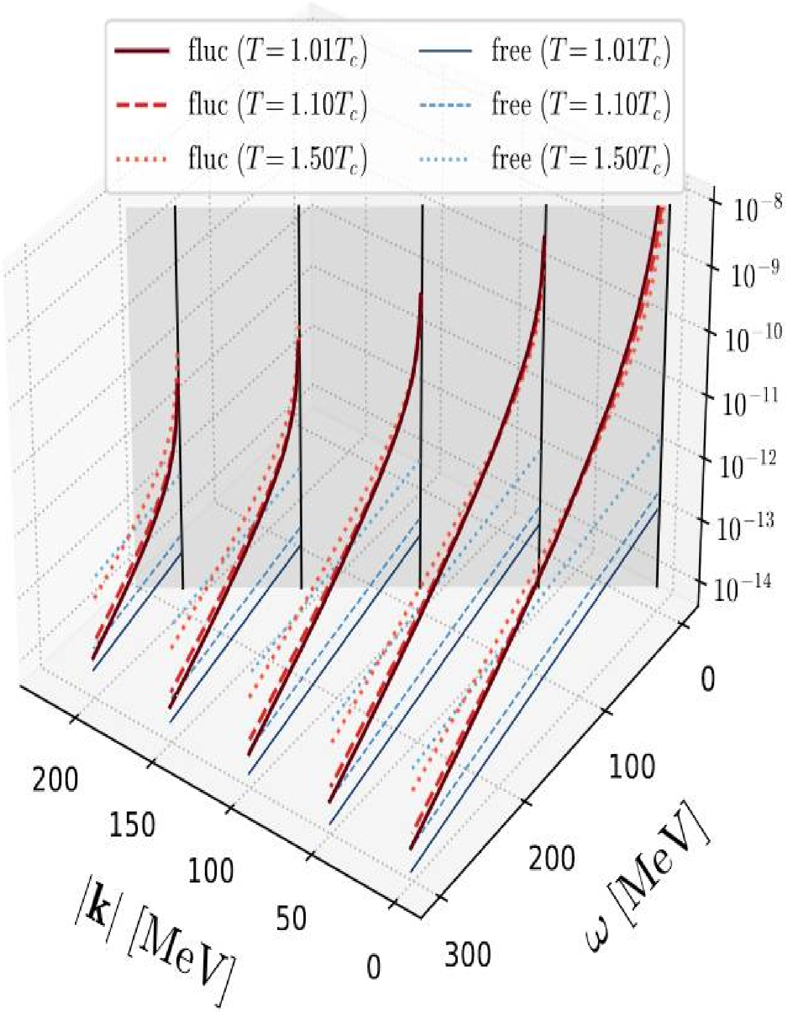
diplepton production

- soft mode couples to photons via quark loop

Nishimura-Kunihiro-Kitazawa ('23)



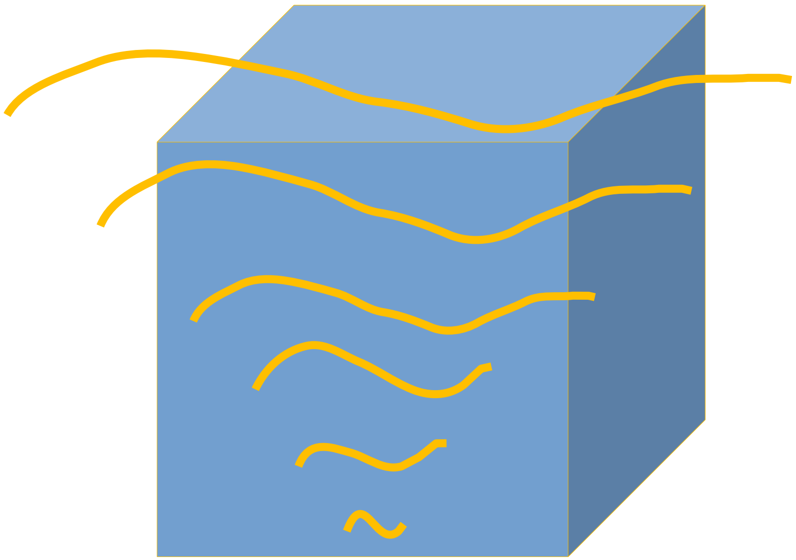
+MT term + DOS term



Limitations on long-range fluctuations

- Critical phenomena appear at long distance and long time
- So far we've assumed a large system close to equilibrium

- Heavy-ion collisions
  - finite system size
  - finite event duration
  - and a dynamic system

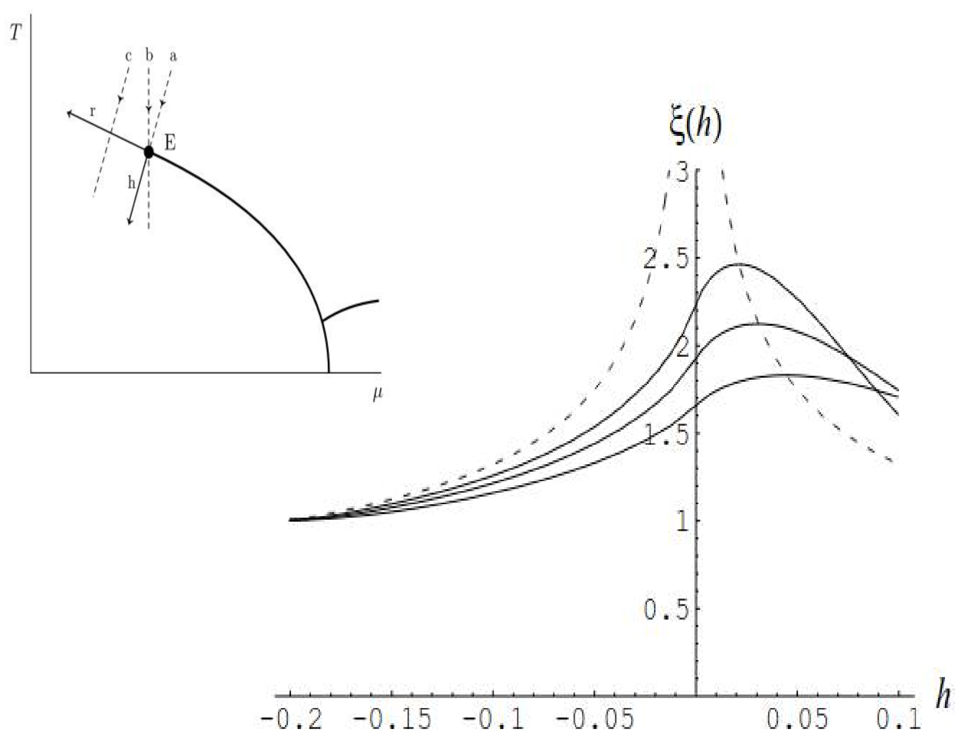


- Long-range, long-time modes are likely out of equilibrium

Finite duration

Problematic?

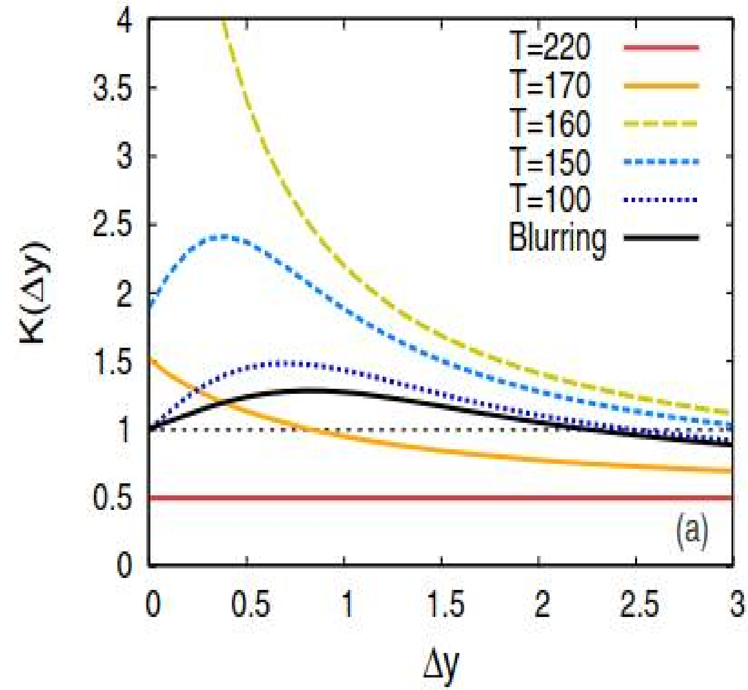
Divergence of correlation length near critical point is rounded off



Berdnikov-Rajagopal, PRD61 (2000) 105017
Nonaka-Asakawa, PRC71 (2005) 044904

Useful?

Baryon # fluctuation remembers the past as a correlation btw rapidity gap & time



M. Kitazawa, M. Asakawa, H. Ohno, PLB728, 386(2014);
Sakaida, Asakawa, HF, Kitazawa, PRC95, 064905 (2017), …

Sakai, Murase, HF, Hirano, in preparation,

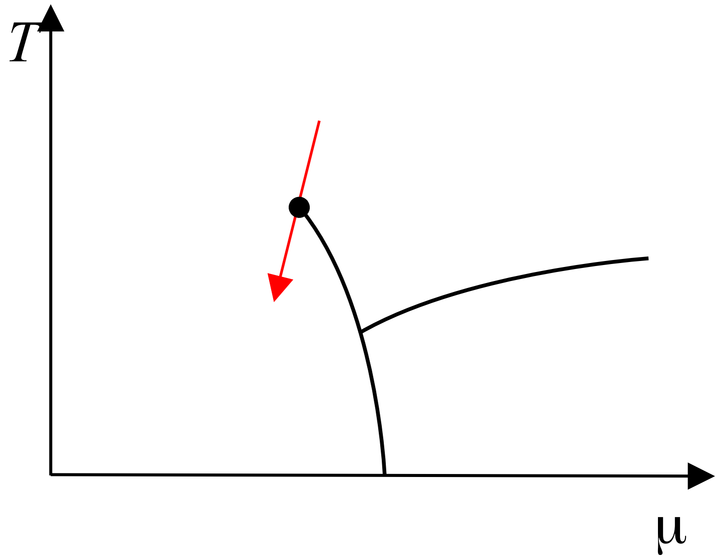
Non-equilibrium evolution of soft mode
- a simple demo -

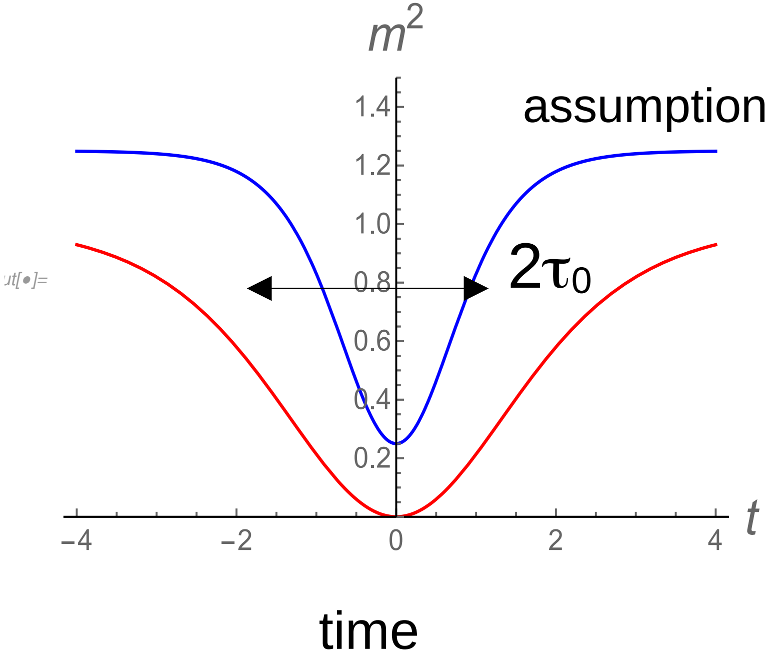
Time-dep GL equation with time-dep coeffs. for the soft mode

Assumption = separation of soft mode from others

$$\left(\frac{1}{\lambda k}\frac{\partial}{\partial t} + m_0^2 \tanh^2(t/\tau_0) + k^2\right) G_R(t,t',k) = \delta(t-t')$$

kinetic coeff. motivated by NJL model





Fluctuation and spectrum fn

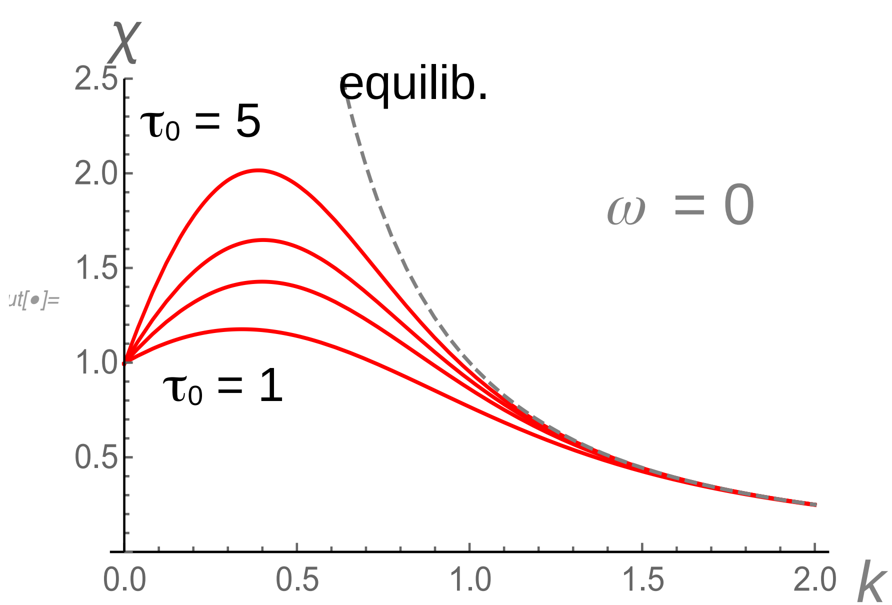
Real and imag parts of G_R of the soft mode

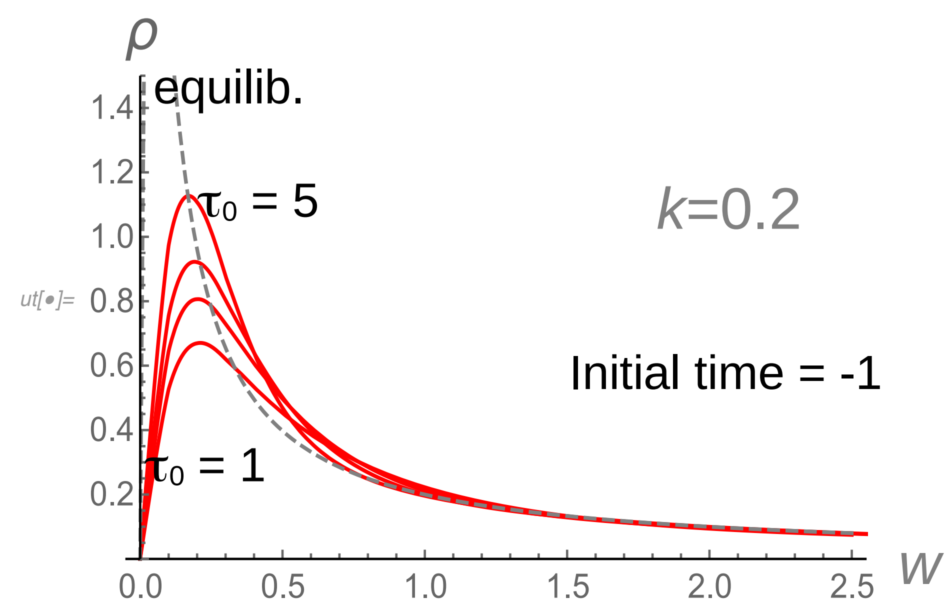
Duration of passing critical region τ_0 = 1, 2, 3, 5 $m_0 = \lambda = 1$

Both low-mom and low-freq parts need more time to approach equilibrium values

May affect low *k* and ω observables

see also Akamatsu et al. ('19)
Kibble-Zurek scaling





Concluding Remarks

- The nature of QCD critical point is recapped
  - σ field is a fast mode and integrated out in equilibrium
  - falls in the dynamic universality class of model H
- Critical enhancements of fluctuations are intensively studied in the HIC community
- In reality, HIC events are very dynamic, and scale separation btw slow and fast modes are non-trivial
- Non-equilibrium evolution effects on low *k* and low ω modes are to be studied more explicitly and seriously