Open Quantum System (OQS) approaches for quarkonium evolution in the QGP

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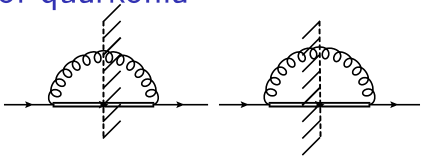
Quarkonia: energy scales

- Non-relativistic bound states with $M \gg \frac{1}{r} \gg E_b$. In the thermal medium, $T \sim E_b$ (few 100MeV for Bottomonia)
- pNRQCD *[Brambrilla, Pineada, Soto, Vairo (1999)]* is a non-relativistic EFT of quarkonia
- The lagrangian is

$$L_{\mathrm{pNRQCD}} = \int d^3\mathbf{r}\,\mathrm{tr}\Big(\mathcal{S}(\mathbf{r})^\dagger[i\partial_0 - h_s]\mathcal{S}(\mathbf{r}) \\ + \mathcal{O}(\mathbf{r})^\dagger[iD_0 - h_o]\mathcal{O}(\mathbf{r})\Big) \\ + \mathcal{O}^\dagger(\mathbf{r})\mathbf{r}\cdot g\mathbf{E}\mathcal{S}(\mathbf{r}) + \frac{1}{4}\{\mathcal{O}^\dagger(\mathbf{r})\{\mathbf{r}\cdot g\mathbf{E},\mathcal{O}(\mathbf{r})\}\} + \dots\Big)$$

- $\mathbf{r}$ is the relative separation between $Q\overline{Q}$, $\mathcal{S}$ is the singlet wavefunction and $\mathcal{O}$ is the octet wavefunction
- $\mathbf{E}$ is the chromo-electric field and $h_{o,s} = -\frac{\nabla^2}{M} + v_{o,s}(\mathbf{r})$, (See talk by Dibyendu Bala for a non-perturbative calculation of $v_{o,s}$.)

The decay-rate of quarkonia



- $\mathcal{S} \to \mathcal{O}$ via scattering or absorption of thermal gluons *[Brambilla, Ghiglieri, Vairo, Petreczky (2008)]*
- Incorporates well known processes like screening *[Matsui, Satz (1986)]*, time-like gluon absorption or Gluo-dissociation (GD) *[Bhanot, Peskin (1979)]*, space-like gluon scattering or Landau Damping (LD) *[Laine, Philipsen, Tassler, Romatschke (2007), Granchamp,Rapp (2001)]*
- The decay rate of a state ψ is given by

$$\Gamma = \frac{g^2}{3N_c} \sum_f |\langle f|\mathbf{r}|\psi\rangle|^2 f(k^0) \int \frac{d^3\mathbf{k}}{(2\pi)^3} \rho_{\mathbf{EE}}(k^0, \mathbf{k})|_{k^0 = E_f - E_\psi}$$

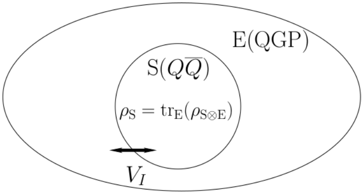
- The chromoelectric spectral function $\rho_{\mathbf{EE}}$ can be computed on the lattice (See talks by Dibyendu Bala, Saumen Datta.)

Quarkonium: a quantum system (S) in an environment (E)

- The $Q\bar{Q}$ pair is in a mixed state described by a density matrix obtained by tracing out the E: $\rho_{\mathrm{S}} = \mathrm{tr}_{\mathrm{E}}(\rho_{\mathrm{tot}})$
- The $\bar{Q}Q$ system (S) and the QGP environment (E) interact with each other. Their combined evolution is unitary

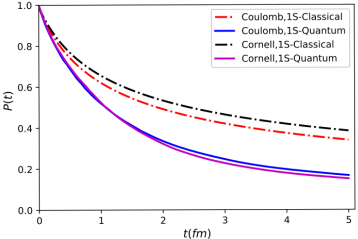
$$i\frac{d\rho_{\mathrm{tot}}}{dt} = [H_{\mathrm{tot}}, \rho_{\mathrm{tot}}]$$

- $H_{\mathrm{tot}} = H_{\mathrm{S}} + H_{\mathrm{E}} + V_{\mathrm{I}}$
- Tracing the environment gives ρ_{S}. OQS framework tracks ρ_{S}



Quantum mechanical evolution

- If $\Gamma \ll E_b$ then the quantum mechanical evolution is described by phase rotation between narrow states on time scales $\sim 1/E_b$ and transitions between them on scales $1/\Gamma$: Quantum Optical regime *[Borghini, Gombeaud (2011, 2011)]*
- Further, on a time scale $t \gg 1/E_b$, quantum effects are washed out and a semi-classical Boltzmann equation can be written *[Yao, Mehen (2018); Yao, Müller (2018); Yao, Ke, Xu, Bass, Müller (2021)]*
- For $\Upsilon(2/3S)$, $\Gamma \sim E_b$. Furthermore, the medium evolves on comparable timescales $\sim 1/T$. Hence, OQS *[Kajimoto et. al. (2017); Tiwari, Sharma (2019)]*



System Hamiltonian

- $$H_{\mathrm{S}} = \left(\frac{p^2}{M} + v_s(\mathbf{r})\right)|s\rangle\langle s| + \left(\frac{p^2}{M} + v_o(\mathbf{r})\right)|o_a\rangle\langle o_a|$$
  $$V_{\mathrm{I}} = -g\mathbf{r}\cdot\mathbf{E}^a\left(\frac{1}{\sqrt{2N_c}}|s\rangle\langle o_a| + \frac{1}{\sqrt{2N_c}}|o_a\rangle\langle s| + \frac{1}{2}d_{abc}|o_b\rangle\langle o_c|\right)$$
  Justified if $M \gg 1/r \gg E_b, T$
- $\rho_s(t) = \langle s|\rho_{\mathrm{S}}|s\rangle$, $\rho_o(t) = \langle o_a|\rho_{\mathrm{S}}|o_a\rangle$
- ρ_{S} is diagonal in s, o basis $\rho_{\mathrm{S}} = \mathrm{diag}\{\rho_s, \rho_o\}$. Similarly, $H_{\mathrm{S}} = \mathrm{diag}\{h_s, h_o\}$.
- Interaction between the system and the interaction comes from V_{I}

The master equation

- A master equation (in the interaction picture) up to $\mathcal{O}(V_{\mathrm{I}}^3)$

$$\frac{d\rho_{\mathrm{S}}}{dt} \approx -\int_0^t du\, [V_{\mathrm{I}}(t), [V_{\mathrm{I}}(u), \rho_{\mathrm{S}}(t)]]$$

[Breuer, Petruccione; Brambilla et. al. (2017)]

- Tracing over the environment gives the master equation in Schrödinger picture

$$i\frac{d\rho_{\mathrm{S}}}{dt} = [H_{\mathrm{S}}, \rho_{\mathrm{S}}] - i\int_0^t du\, \mathcal{C}(u) \sum_{i=1,3}^{n=\pm,d} \left\{ V_{ni}^{\dagger}(0) V_{ni}(u) \rho_{\mathrm{S}}(t) - V_{ni}(u)\rho_{\mathrm{S}}(t) V_{ni}^{\dagger}(0) + \mathrm{HC} \right\}$$

- $\mathcal{C}(t) = \frac{g^2}{3N_c} \mathrm{tr}_{\mathrm{E}}\{\mathbf{E}_i^a(t,0) U(t,0)_{ab} \mathbf{E}_i^b(0,0)\rho_{\mathrm{E}}\}\rangle$

The jump operators

- $\{V_{ni}(t)\}$ are time dependent operators corresponding to $s \to o$, $o \to s$, $o \to o$ transitions
- Explicitly,

$$V_{+i}(t) = e^{ih_s t}\mathbf{r}_i e^{-ih_o t}\sqrt{C_F}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$V_{-i}(t) = e^{ih_o t}\mathbf{r}_i e^{-ih_s t}\sqrt{\frac{1}{2N_c}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$V_{di}(t) = e^{ih_o t}\mathbf{r}_i e^{-ih_o t}\sqrt{\frac{N_c^2-4}{4N_c}}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\ .$$

Expansion in $\tau_{\mathrm{E}}/\tau_{\mathrm{S}}$ (LO, NLO expansion)

- $V_{ni}(t)$ evolves on a time scale of $\tau_{\mathrm{E}} \sim \frac{1}{E_b}$
- $\Gamma(t)$ is substantial only for $t < \tau_{\mathrm{E}} \sim \frac{1}{T}$. For example, in weak coupling, $\tau_{\mathrm{E}} \sim \frac{1}{gT}$
- The relaxation time (inverse of the thermal width) $\tau_R \sim 1/\Gamma$
- If $\tau_R, \tau_S \gg \tau_E$ then the correlations in the environment are lost rapidly on the system time scales, and hence the evolution equation of $\rho_S(t)$ does not depend on the history of the evolution: memoryless evolution
- If $\tau_{\mathrm{E}} \ll \tau_{\mathrm{S}}$ then

$$V_{ni}(t) \sim e^{ih_\alpha t}\mathbf{r}_i e^{-ih_\beta t} \approx \underbrace{\mathbf{r}_i}_{\mathrm{LO}} + \underbrace{it(h_\alpha \mathbf{r}_i - \mathbf{r}_i h_\beta)}_{\mathrm{NLO}} + \mathcal{O}\left[\left(\frac{\tau_{\mathrm{E}}}{\tau_{\mathrm{S}}}\right)^2\right]$$

- Both LO and NLO can be written in a Lindblad form (memoryless)
- In the memoryless approximation, only $\tilde{\mathcal{C}}(\omega = 0) = \frac{1}{2}(\kappa + i\gamma)$ is required

The Lindblad equation

- General Markovian evolution has the form

$$\frac{d\rho(t)}{dt} = \frac{1}{i}[H, \rho(t)] + \sum_n L_n \rho(t) L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, \rho(t)\}$$

- $[H, \rho]$ gives unitary evolution. The "jump operators", L_n give non time-reversible transitions between states
- Ensures $\mathrm{tr}(\rho(t)) = 1$ during evolution. Ensures that the eigenvalues of $\rho(t)$ are positive

The L_n to leading order (LO) in τ_E/τ_S

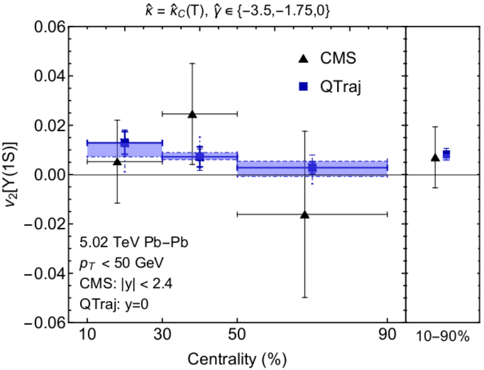
- LO jump operators, *Brambilla, Escobedo, Soto, Vairo (2016, 2017)*

$$L^0 = \mathbf{r}\sqrt{\frac{\kappa}{N_c^2-1}}\begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},\ L^1 = \mathbf{r}\sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- Open source solver for Lindblad equations for quarkonium (QTRAJ) - *Omar, Escobedo, Islam, Strickland, Thapar, Vander, Griend, Weber (2021)*
- Phenomenology (R_{AA} and v_2) - *Brambilla, Escobedo, Strickland, Vairo, Griend, Weber (2021, 2021)*

LO phenomenology (v_2)

- *Brambilla, Escobedo, Strickland, Vairo, Griend, Weber (2021)*
- $\hat{\kappa} = \kappa/T^3$, $\hat{\gamma} = \gamma/T^3$, can be treated as fit parameters. Aternatively, can constrain them from lattice measurements of the mass shift and spectral function width computed on the lattice (Talk by Dibyendu Bala)

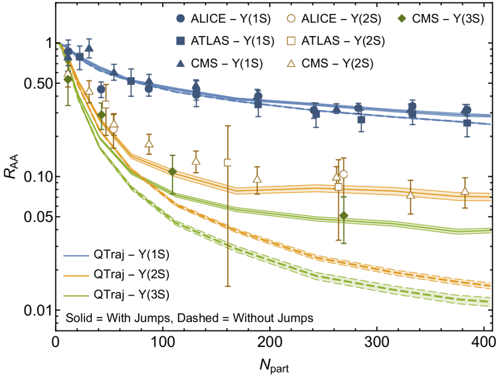


NLO formalism

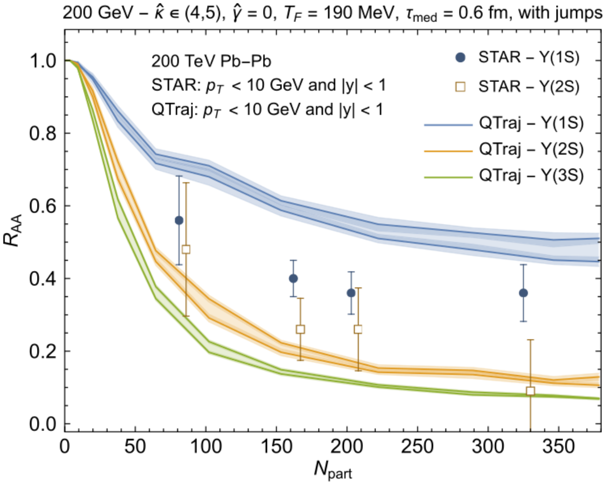
- Lindblad equations have been derived to next to leading order (NLO) in τ_S/τ_E
- Formalism and phenomenology
  - Derived and solved Boltzmann equations using the formalism - *Yao (2021)*
  - LO and NL0 equations in perturbative QCD - *Akamatsu (2017, 2020)*
  - NLO equations using pNRQCD derived and solved - *Brambilla, Escobedo, Islam, Strickland, Tiwari, Vairo, Griend (2022, 2023)*

NLO phenomenology (R_{AA})

- NLO equations using pNRQCD derived and solved - *Brambilla, Escobedo, Islam, Strickland, Tiwari, Vairo, Griend (2022, 2023)*
- Transitions associated with L_n essential in explaining $\Upsilon(2S)$, $\Upsilon(3S)$



Comparison with STAR results



- *[Strickland, Thapa (2023), STAR (2023)]*

Comparison with STAR results

- RHIC suppression larger than expected from models that fit LHC
- Regeneration not important for $\Upsilon(1S)$ *[Du, He, Rapp (2017)]*
- Possibly larger nuclear matter effects at RHIC compared to LHC?
- Finite frequency effects?

Expansion in $\tau_{\mathrm{E}}/\tau_{\mathrm{S}}$?

- However, $E_b \sim 500$MeV for $\Upsilon(1S)$, a little smaller for $\Upsilon(2S)$. On the other hand $T \lesssim 500$MeV
- For $\Upsilon(1S)$ in particular, it is worthwhile investigating whether further corrections in $\tau_{\mathrm{E}}/\tau_{\mathrm{S}}$ can have an effect on quantum dynamics
- Finite frequency affects the decay rates, *[Sharma, Singh (2023)]*. Therefore, worth investigating whether finite frequency effects modify the quantum evolution. (See talk by Vyshakh BR.)

OQS framework for charmonia

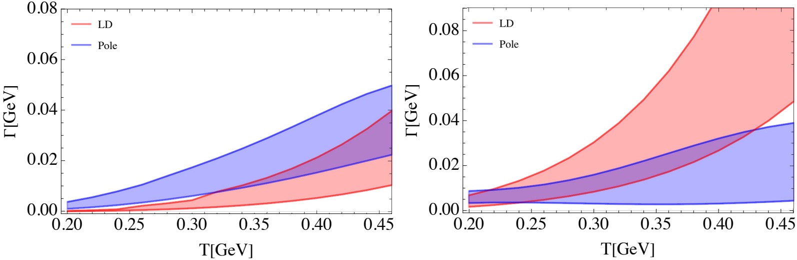
- The first challenge is that because $1/r \gtrsim T$ but not much larger, and pNRQCD is less controlled
- NRQCD formalism better suited (avoid making the multipole expansion) *[Blaizot, Escobedo (2018); Delorme, Katz, Gousset, Gossiaux, Blaizot (2024)]*. Factorization is not apparent
- Second, regeneration from uncorrelated charms plays an important role, especially at LHC
- Expensive and challenging problem

Summary

- For bottomonium, pNRQCD leads to the factorization of the non-perturbative quantity, $\langle \mathbf{EE} \rangle$ from the dynamics of the $Q\bar{Q}$
- In the hierarchy $\frac{\tau_{\rm E}}{\tau_{\rm S}} \ll 1$, Lindblad equations can be derived, and give a good description of LHC data but miss at RHIC
- Forgoing the expansion in $\frac{\tau_{\rm E}}{\tau_{\rm S}}$ leads to a master equation with memory, and this affects quantum evolution

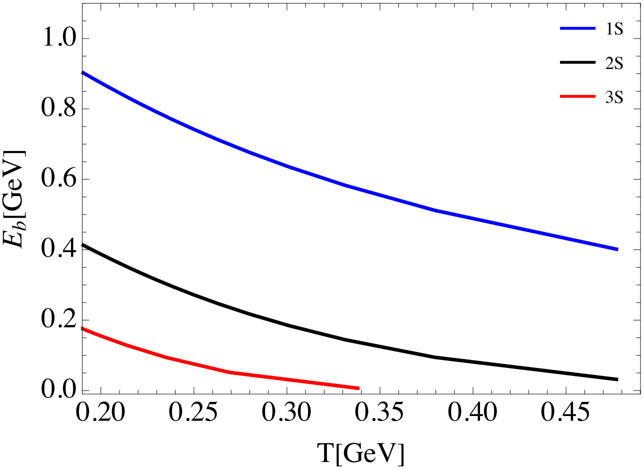
Backup slides

Decay widths



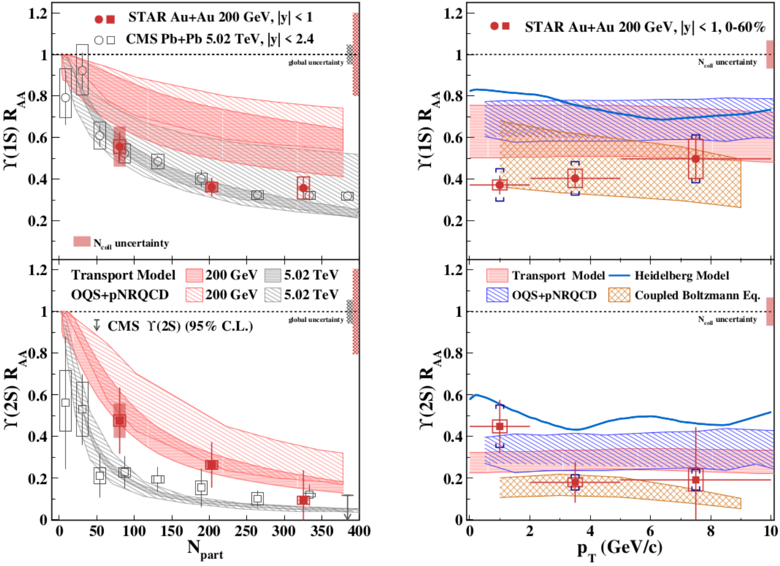
- Decay width with *T [Sharma, Singh (2023)]*

Binding energies



- Binding energy of the states with *T [Sharma, Singh (2023)]*

STAR results for Υ



- *[STAR (2023)]*

LO phenomenology (v_2)

- *Brambilla, Escobedo, Strickland, Vairo, Griend, Weber (2021)*