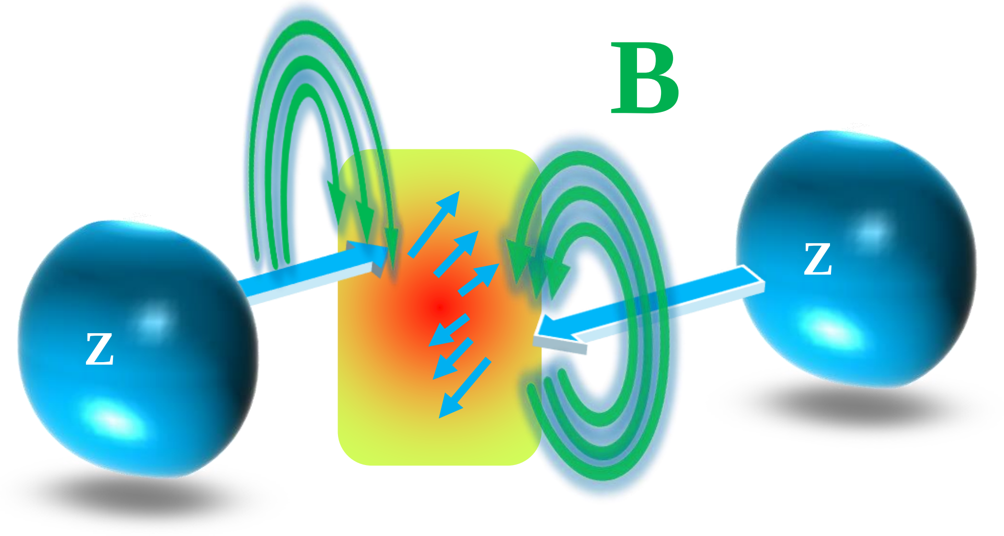
# The physics of strong electromagnetic fields in heavy ion collisions



$\boldsymbol{\omega} = \nabla \times \mathbf{v}$

Koichi Hattori
Zhejiang University

10th Asian Triangle Heavy Ion Conference (ATHIC 2025), Jan. 13

# Hard probes in magnetic fields

- Photons/dileptons from vacuum to medium effects
- Heavy quarks

“Linearly Polarized Photon Collisions,” STAR collaboration,
PRL 127, 052302 (2021)
“Directed Flow of $D^0$ and $^{-}D^0$ ,” STAR collaboration,
PRL 123, 162301 (2019)

# Soft probes in magnetic fields

- Magnetohydrodynamics (MHD)
- Spin magnetohydrodynamics
- Magneto-vortical matter

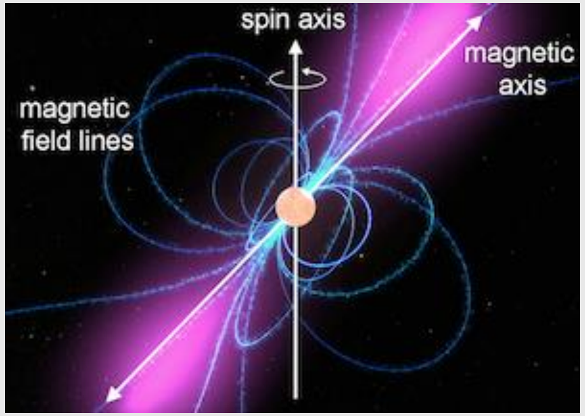
“Global Lambda polarization,” STAR Collaboration
Nature 548, 62-65 (2017); PRC108,014910(2023)
“Charge-Dependent Directed Flow,” ALICE, PRL 125 (2020)
STAR collaboration, PRX 14, 011028 (2024)

# Hard probes in magnetic fields

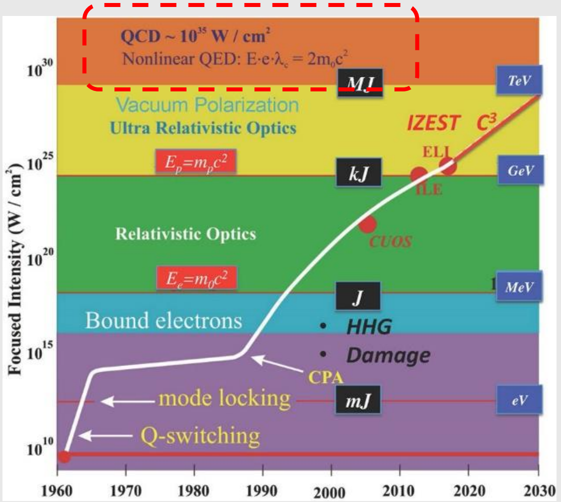
- Photons/dileptons from vacuum to medium effects
- Heavy quarks

# Strong fields and EM probes in nature and laboratory

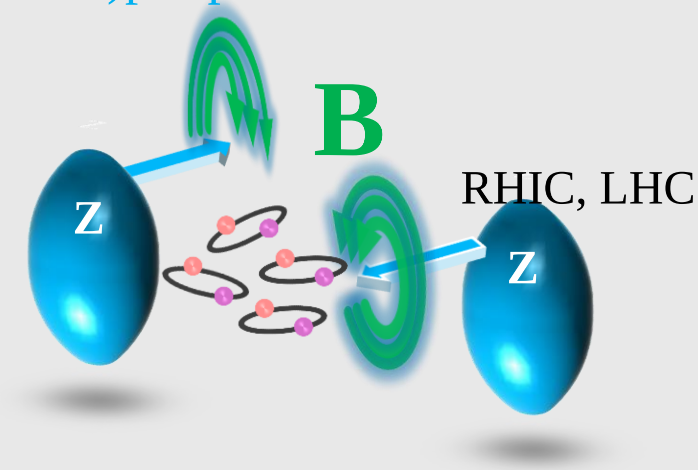
## Magnetospheres of neutron stars/magnetars



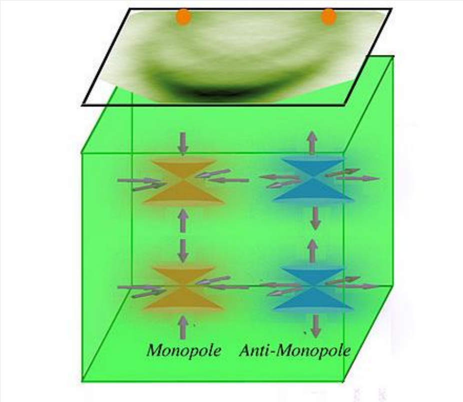
## High-intensity laser field



## (Ultra)peripheral collisions

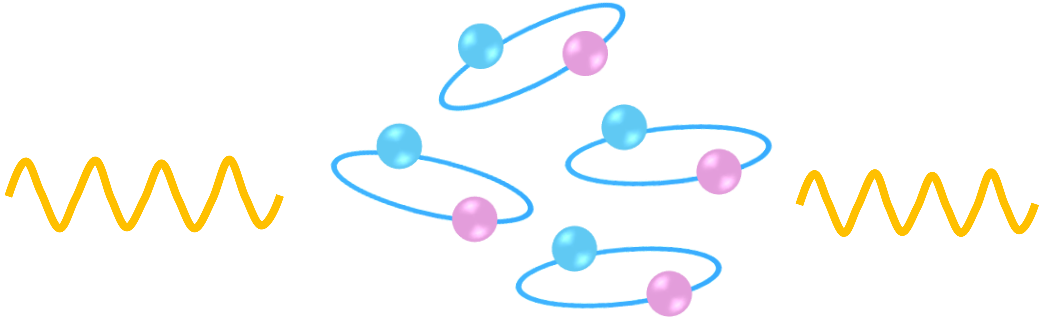


## Cond. Matt. materials



# Effects of B on photon propagation

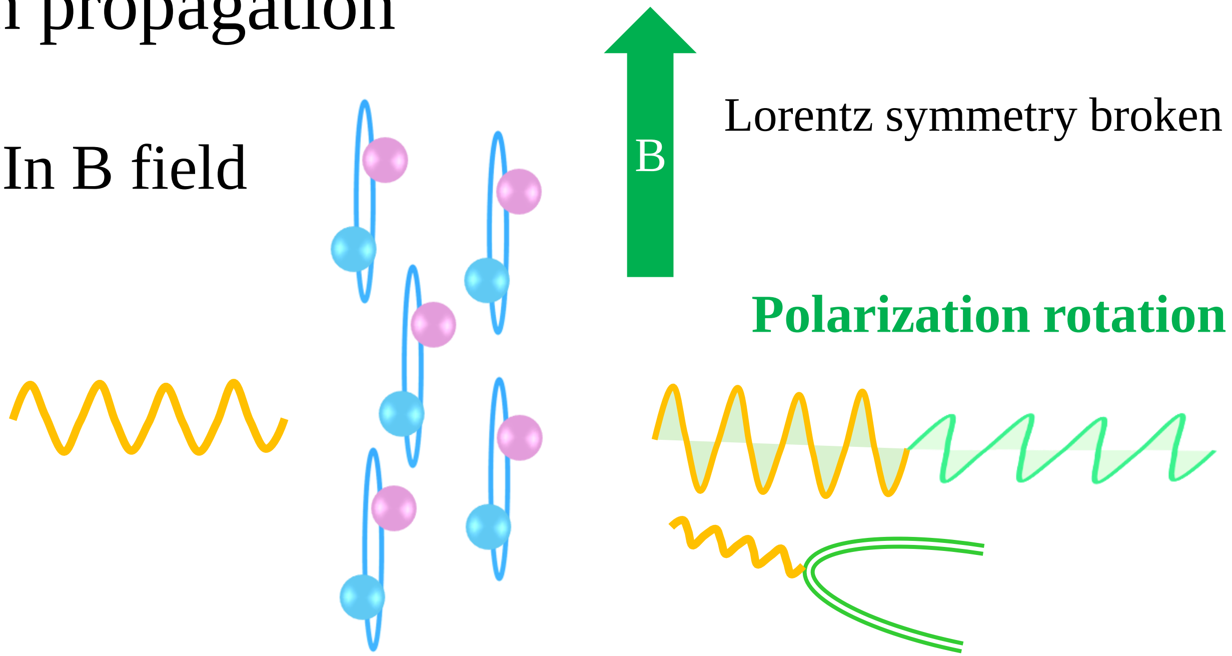
Without B field



In medium: Refractive index

In vacuum: No change in refractive index
(Gauge and Lorentz symmetries)

In B field



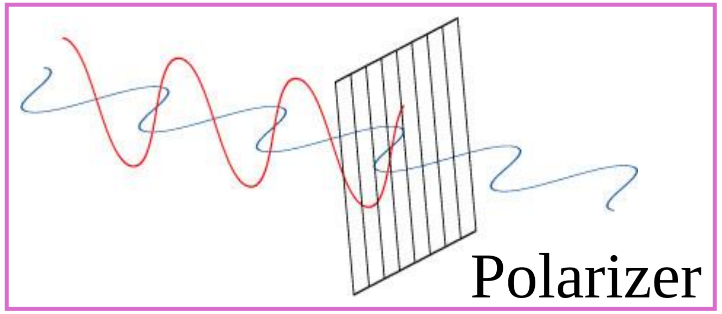
Lorentz symmetry broken

**Polarization rotation**

**Real-photon decay**
**(Dichroism)**

Not experimentally confirmed yet

**Birefringence**

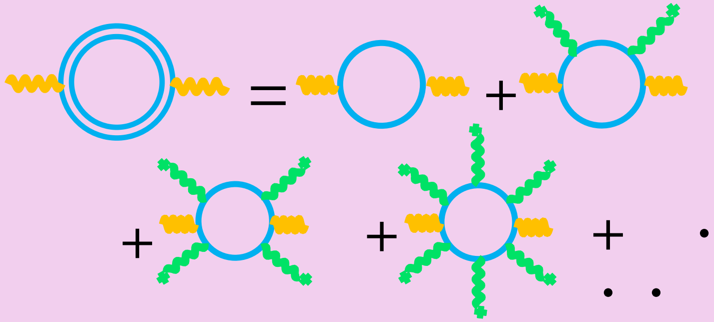


Polarizer

Cf. Magneto-optic effects
Cotton-Mouton effect, Faraday effect, etc in optics

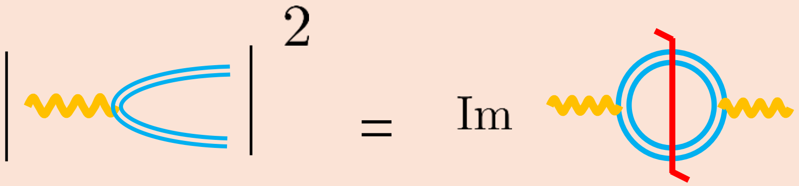
# Complex refractive indices

Refraction (real part of the index of refraction)



Decay cross-section

Optical theorem for the imaginary part



B-field opens 1-to-2 kinematics

Photon refraction

Both sides of a coin

Dispersion integral

Di-fermion production

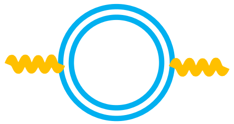
Imaginary part = On-shell processes

→ Simpler to understand (than the real part)

# Refractive index from the Maxwell equation

$$[q^2 g^{\mu\nu} - q^\mu q^\nu - \Pi^{\mu\nu}]A_\nu(q) = 0$$

$$\Pi^{\mu\nu} \;=$$



Ward identity:
Constraint by the gauge symmetry

$$q_\mu \Pi^{\mu\nu} = 0$$

$$\Pi^{\mu\nu} = -[\chi_0 P_0^{\mu\nu} + \chi_1 P_1^{\mu\nu} + \chi_2 P_2^{\mu\nu}]$$

$$P_0^{\mu\nu} = q^2 \eta^{\mu\nu} - q^\mu q^\nu$$

$$P_1^{\mu\nu} = q_\parallel^2 \eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu$$

$$P_2^{\mu\nu} = q_\perp^2 \eta_\perp^{\mu\nu} - q_\perp^\mu q_\perp^\nu$$

B-induced anisotropic structures

Preferred orientation in B

$$B = (0, 0, B)$$

$$\eta_\parallel^{\mu\nu} = \mathrm{diag}(1, 0, 0, -1)$$

$$\eta_\perp^{\mu\nu} = \mathrm{diag}(0, -1, -1, 0)$$

$$q_\parallel^\mu = (q^0, 0, 0, q^3)$$

$$q_\perp^\mu = (0, q^1, q^2, 0)$$

Medium effects may induce more structures.

# Birefringence

Cf. KH, Itakura, 1209.2663

Definition of refractive index $n = \dfrac{|\boldsymbol{q}|}{\omega}$

Two distinct solutions for the Maxwell equation:

$$
\begin{aligned}
n_\parallel^2 &= \frac{1+\chi_0+\chi_1}{1+\chi_0+\chi_1\cos^2\theta} \to 1 \\
n_\perp^2 &= \frac{1+\chi_0}{1+\chi_0+\chi_2\sin^2\theta} \to 1
\end{aligned}
$$

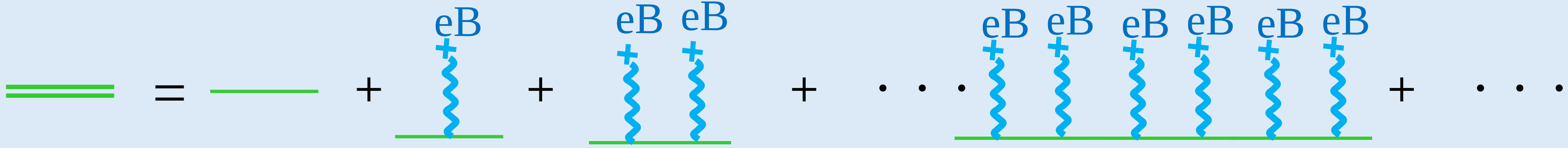
Direct consequence of the gauge symmetry and the breaking of one spatial rotational symmetry.

Vanishing B limit: $\chi_0 \to \Pi_{\rm vac}\,,\quad \chi_{1,2} \to 0$

Theoretical issues boil down to computation of χ.

# Computation of the polarization tensor

## Resummed propagator

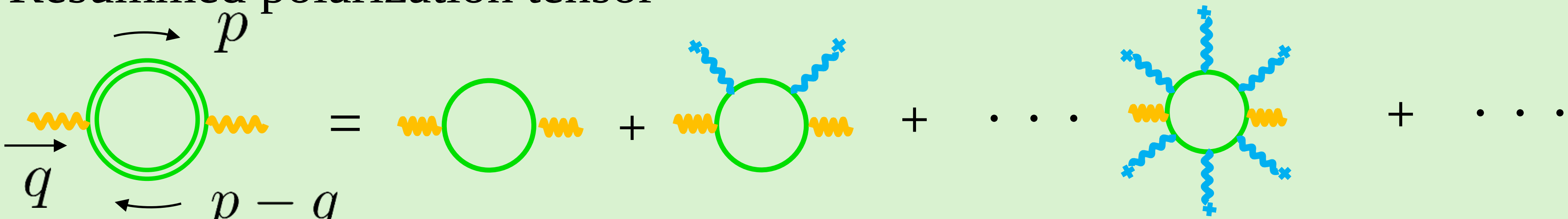


$$S(p) \quad = \quad i(\not{D} + m) \times i^{-1} \int_0^\infty \boxed{d\tau} e^{i\tau(\not{D}\not{D} - m^2 + i\epsilon)}$$

Fock (1937), Schwinger (1951)

$\tau$ : proper-time

## Resummed polarization tensor



$$i\Pi^{\mu\nu} = e^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}[\gamma^\mu S_A(p) \gamma^\nu S_A(p - q)]$$

Options: vacuum or medium; strong or weak field, etc.

# Rich products from the polarization tensor

Refractive index/Photon decay rate

Vacuum: KH, Itakura (2012), Ishikawa, Kimura, Shigaki, Tsuji (2012)
KH, Taya, Yoshida (2021)

Medium: KH, Itakura (2022), Fukushima, Hidaka, Uji (2024)

Photon emission rate

Wang, Shovkovy (2020-2024)

Debye screening/plasma frequency

Fukushima, KH, Yee, Yin (2016), Bandyopadhyay, Ismam, Mustafa (2017), KH, Itakura (2022)

Electric conductivity

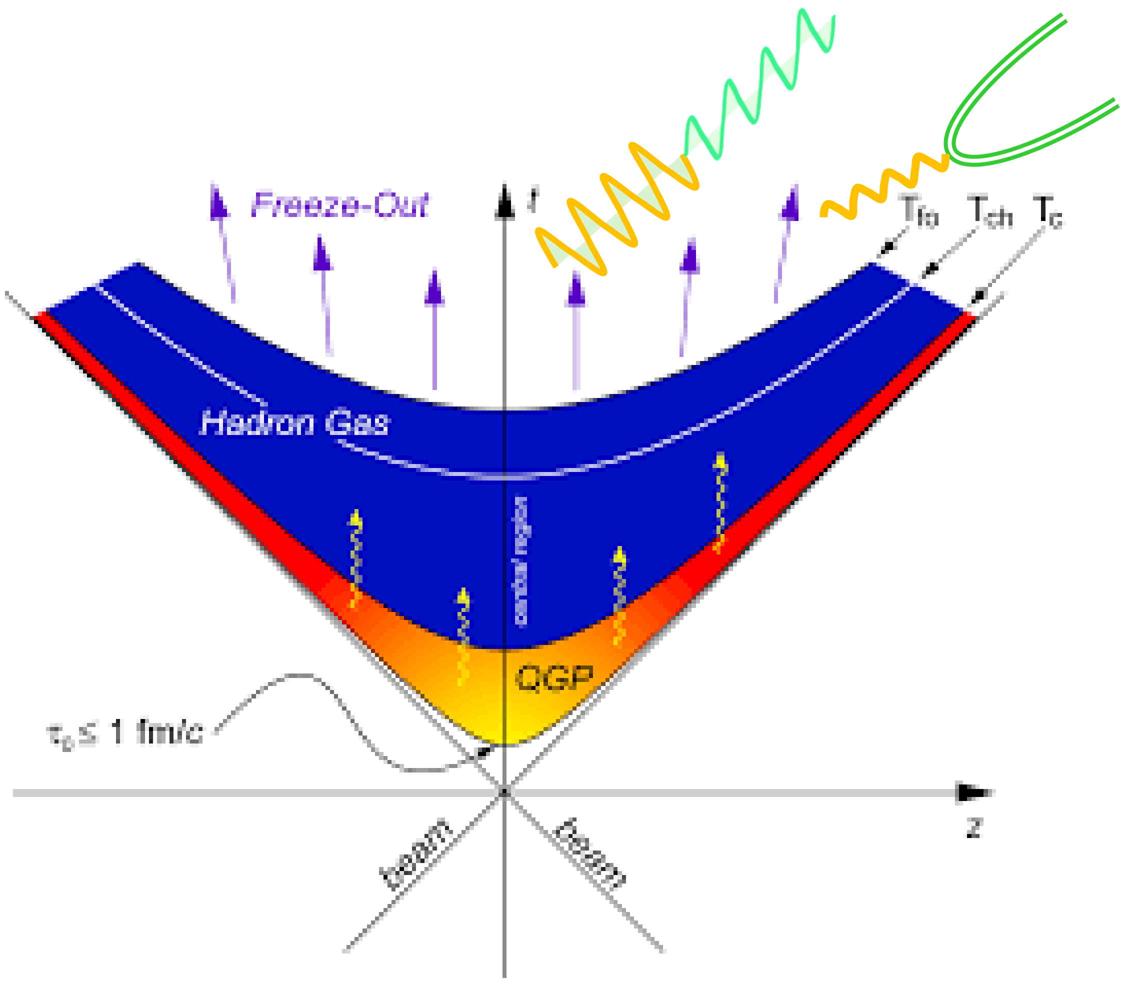
KH, Li, Satow, Yee (2016), Fukushima, Hidaka (2018), Ghosh, et al. (2020), Das, et al. (2020), Satapathy, Ghosh, Ghosh (2021), Peng, et al (2023), Ghosh, Shovkovy (2024)

In-medium axial Ward identity in strong B

KH, Itakura (2022)

Studied in other communities as well.

# Theory + Phenomenology + Experiment



Kimura, Benoit, Ishikawa, Nonaka, Shigaki (2024)

Talk by C. Nonaka (Same session)

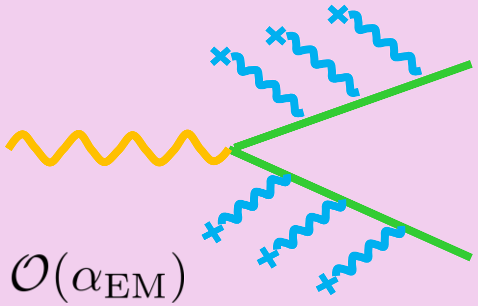
Polarization measurement is challenging in other communities too. (Cosmology, astrophy, laser physics)

# Pair production in a magnetic field

## LO w/ strong B-fields

$$\gamma \to (f\bar{f})_B$$

$$\gamma^* \to (f\bar{f})_B$$



$$|\mathcal{M}|^2 \sim \mathcal{O}(\alpha_{\mathrm{EM}})$$

Nonperturbatively dressed fermions

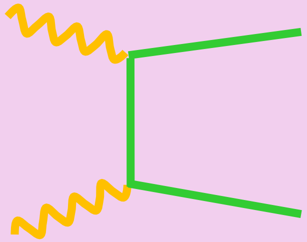
Both <u>on-shell and off-shell photons</u> can decay in B.

## LO w/o B-fields

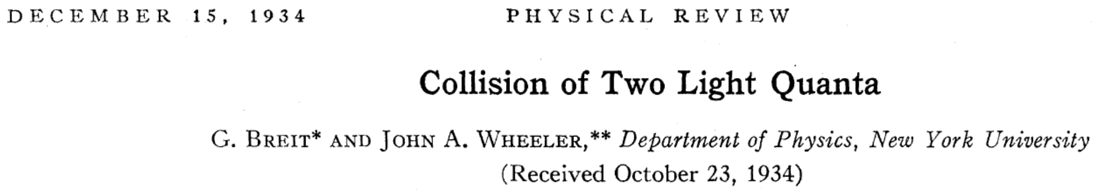
No kinematical window for
<u>a single on-shell photon when B = 0.</u>

→ Starts only from 2 photons

$$|\mathcal{M}|^2 \sim \mathcal{O}(\alpha_{\mathrm{EM}}^2)$$



Breit-Wheeler process in UPC

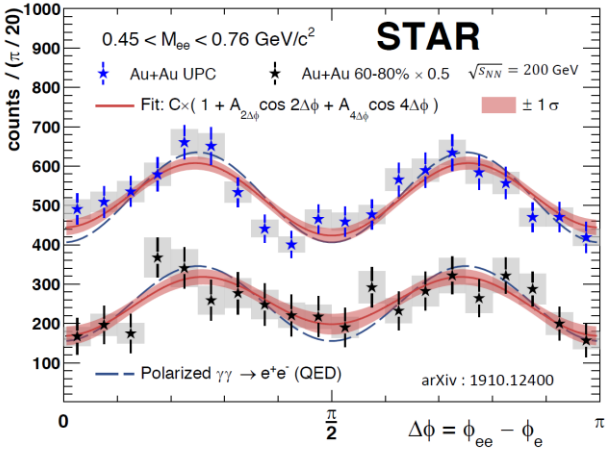


DECEMBER 15, 1934 PHYSICAL REVIEW

**Collision of Two Light Quanta**

G. Breit* and John A. Wheeler,** *Department of Physics, New York University*

(Received October 23, 1934)



# Muon-pair excess over electron pairs

KH, Taya, Yoshida [2010.13492]

## "Helicity suppression" in pion decay

$\pi^- \to \mu^- \bar{\nu}_\mu$ 99.9877 %

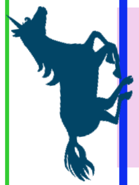
$\pi^- \to e^- \bar{\nu}_e$ $10.23 \times 10^{-4}$ %

PDG

Opposite momentum

Opposite spin

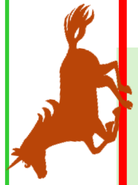
Same helicity



$\ell^-$ $S_\ell = \pm\frac{1}{2}$ $h_\ell = \pm\frac{1}{2}$

$\pi^-$ $S = 0$

Needs chirality mixing by a finite mass.



$\bar{\nu}$ $S_{\bar{\nu}} = \mp\frac{1}{2}$ $h_{\bar{\nu}} = \pm\frac{1}{2}$

## The LLL (= soft photon decay)

$$\frac{N_{\mu^+\mu^-}}{N_{e^+e^-}} \propto \frac{m_\mu^2}{m_e^2} \sim 4.4 \times 10^4$$

Opposite spin along B-field

Same helicity

$S_{\ell^-} = -\frac{1}{2}$ $h_{\ell^-} = -\frac{1}{2}$

$S_\gamma = 0$

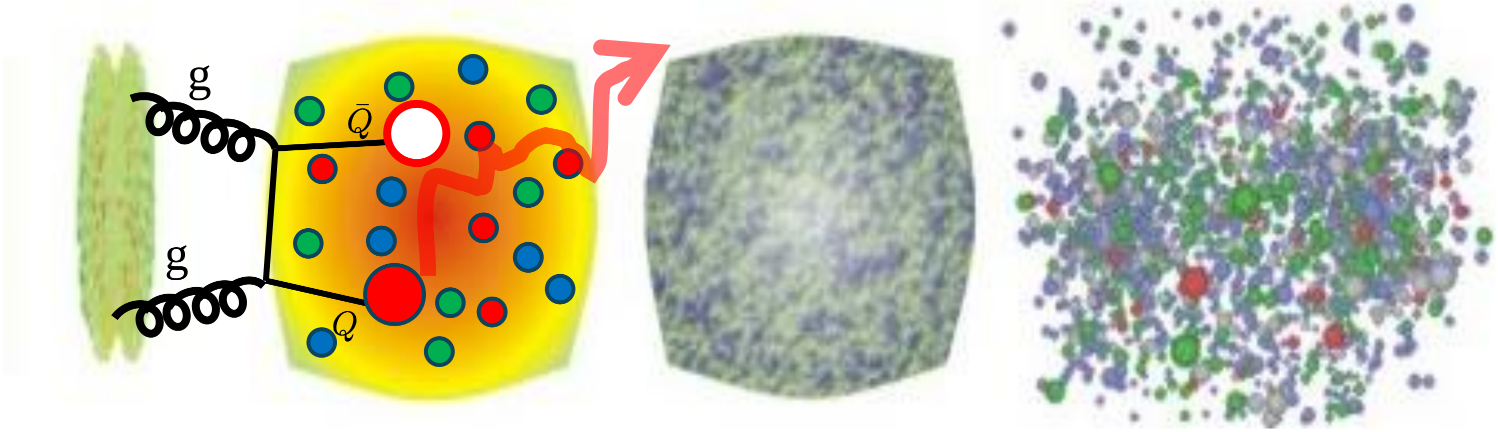
Superposition of $(\pm, L)$

Needs chirality mixing by a finite mass.

$S_{\ell^+} = +\frac{1}{2}$ $h_{\ell^+} = -\frac{1}{2}$

R neutrino (L antineutrino) does not exist at the QCD scale.
However, this is not an essential reason for the helicity suppression.

# Heavy quarks

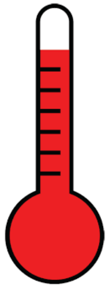


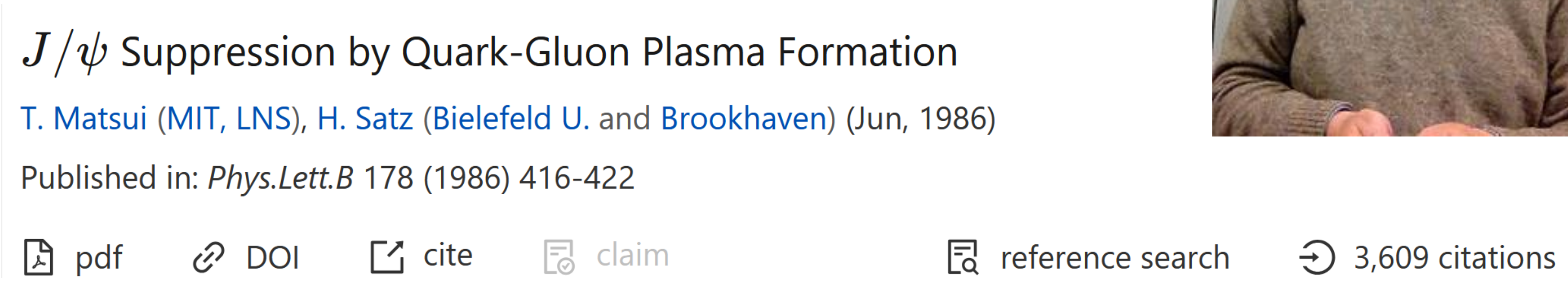
HQ production

Dissociation
Diffusion

Recombination
Open HQs

Thermometer of QGP





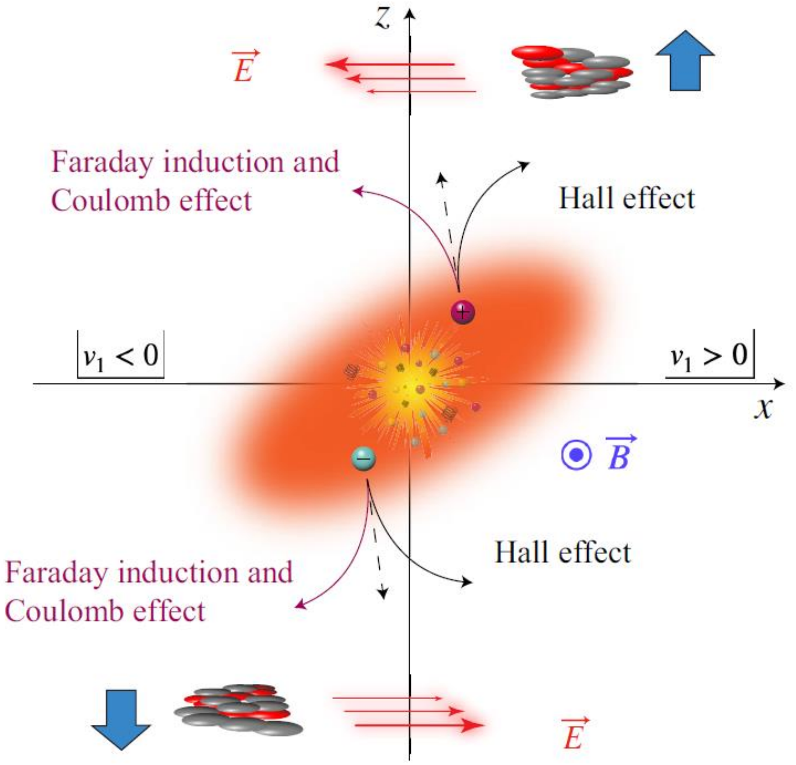
$J/\psi$ Suppression by Quark-Gluon Plasma Formation

T. Matsui (MIT, LNS), H. Satz (Bielefeld U. and Brookhaven) (Jun, 1986)

Published in: *Phys.Lett.B* 178 (1986) 416-422

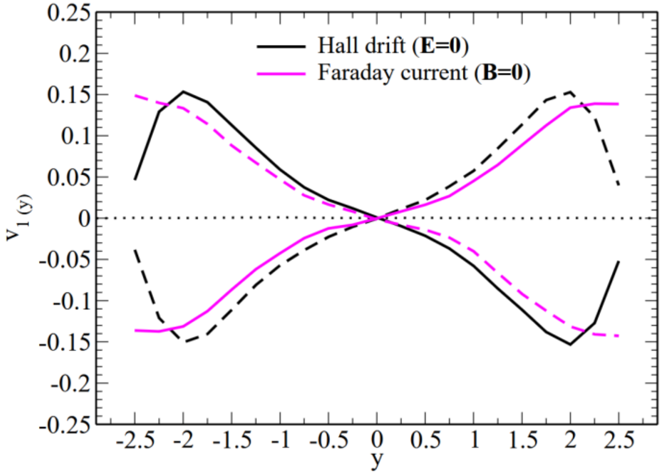
# Early-time dynamics

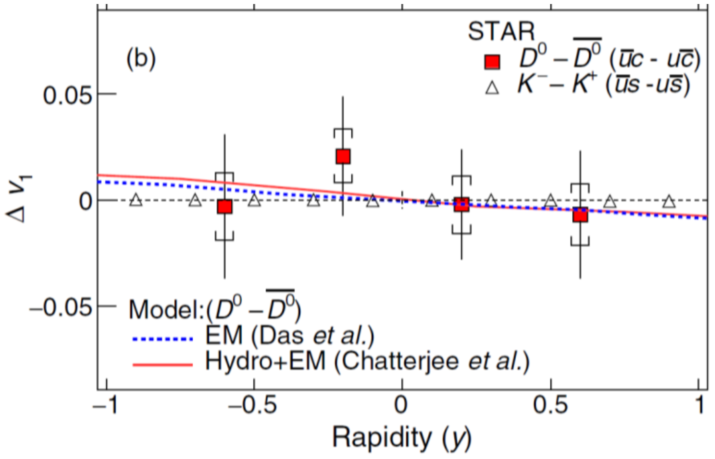
## Directed flow (v1)



From STAR, PRX (2024)
(for light hadrons)

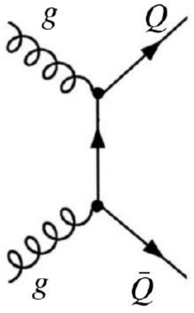
Das, Plumari, Chatterjee, Alam, Scardina, Greco, 1608.02231
Sun, Plumari, Das, 2304.12792



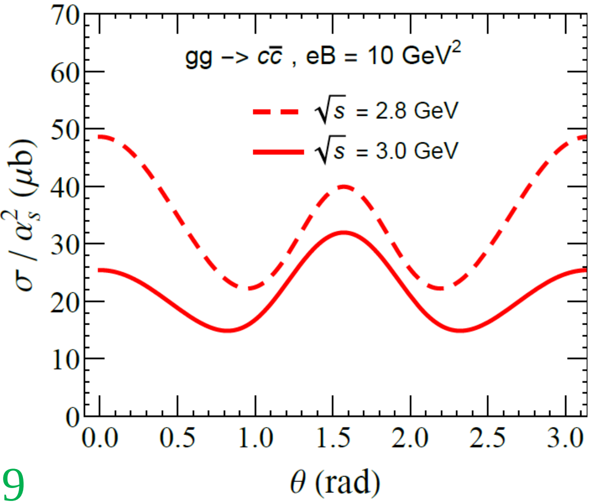


STAR, PRL 123 (2019)

Gluon fusion in B
“Gluonic Breit-Wheeler”



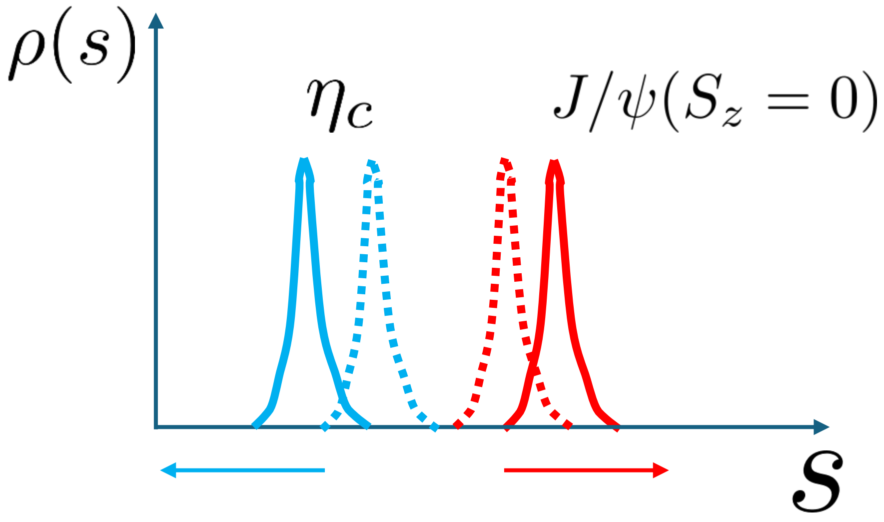
Chen, Zhao, Zhuang, 2401.17559



# Spectral modification and dissociation

In anisotropic systems, spin size S is not a good quantum number.

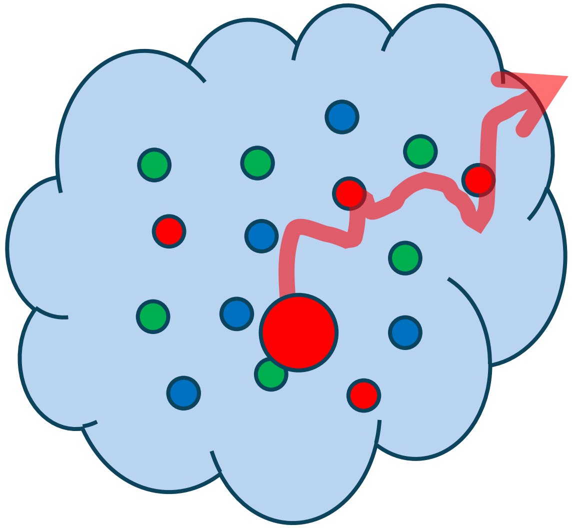
→ Spin singlet and triplet ($S_z = 0$) are mixed, and the energy levels repel with each other.



Marasinghe, Tuchin, 1103.1329
Machado, Navarra, de Oliveira, Noronha, Strickland, 1305.3308
Alford, Strickland, 1309.3003
Cho, KH, Lee, Morita, Ozaki, 1406.4586; 1411.7675
Guo, Shi, Xu, Xu, Zhuang, 1502.04407
Gubler, KH, Lee, Ozaki, Suzuki, 1512.08864
Suzuki and Lee, 1610.09853
Iwasaki, Oka, Suzuki, Yoshida, 1802.04971
Singh, Thakur, Mishra, 1711.03071
Jahan, Dhale, Reddy, Kesarwani, Mishra, 1803.04322
Mishra, Misra, 2004.01007
Sebastian, Thakur, Mishra, Haque, 2308.04410

Lattice measurement by Pisa group
1403.6094, 1607.08160, 1807.01673, 2111.11237

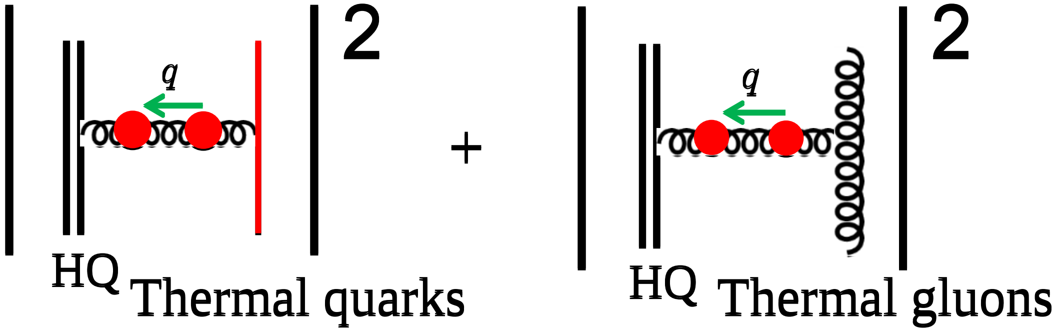
# Brownian motion in QGP



Momentum diffusion constant $\kappa_i = \int d^3\boldsymbol{q}\, q_i^2 \frac{d\Gamma}{d^3\boldsymbol{q}}$

Momentum transfer rate in LO Coulomb scatterings

$$\frac{d\Gamma}{d^3\boldsymbol{q}} =$$

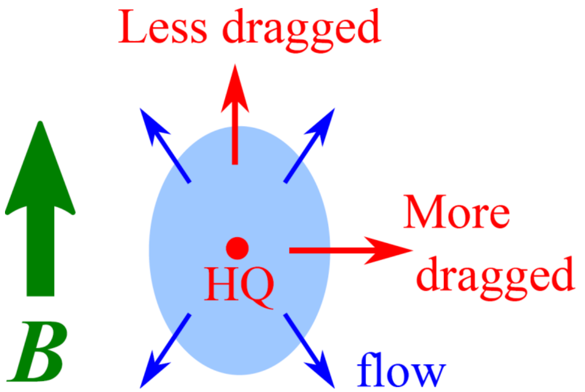


Fukushima, KH, Yee, Yin, 1512.03689
Cf. KH and Huang, 1609.00747

c.f.) LO and NLO without B (Moore & Teaney, Caron-Huot & Moore)

Anisotropy in a magnetic field

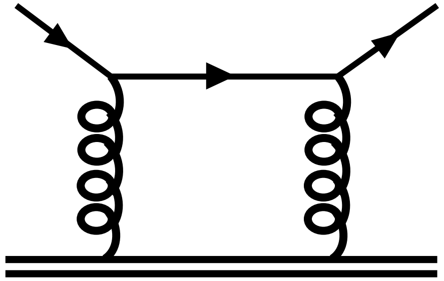
$$\kappa_\perp^{\rm quark} \sim \alpha_s^2 T \times eB \times \log 1/\alpha_s$$
$$\kappa_\parallel^{\rm gluon} \sim \alpha_s^2 T \times T^2 \times \log 1/\alpha_s$$



Effects on heavy-quark spin alignment   Liu, Bai, Zhen, Huang, Chen, 2404.02032

# Large log corrections from quantum effects

Beyond the LO may be interesting.

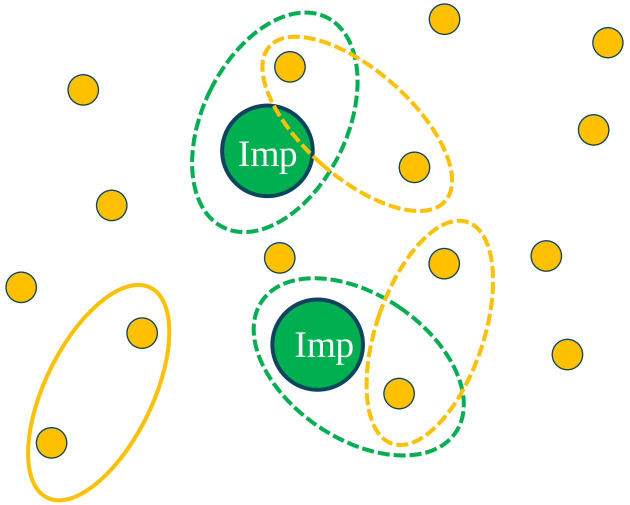


Light quark

Heavy quark

$$\sim \log \frac{\Lambda}{\Lambda - d\Lambda}$$

This log is the origin of the Kondo effect.
→ Strong correlation between a HQ impurity and light quarks.



KH, Itakura, Ozaki, Yasui, 1504.07619
Ozaki, Itakura, Kuramoto, 1509.06966
KH, Huang, Pisarski, 1903.10953
KH, Suenaga, Suzuki, Yasui, 2211.16150
etc.

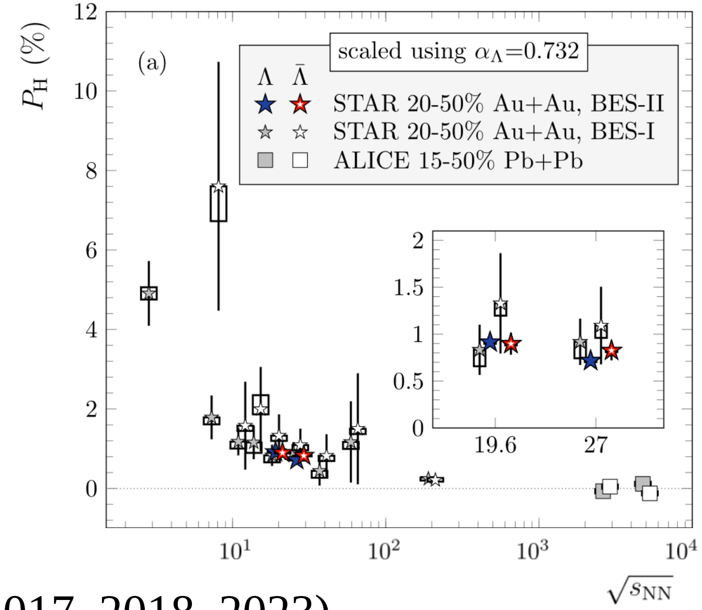
# How hydro evolves in magnetic and vortical fields

- Magnetohydrodynamics (MHD)

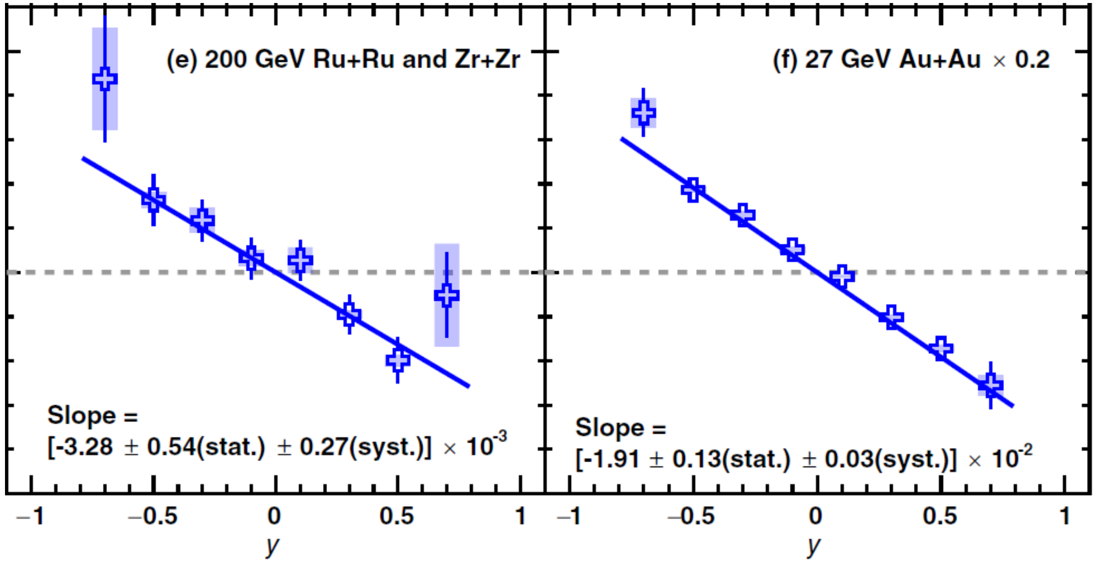
Talk by C. Nonaka and A. Dash

- Spin magnetohydrodynamics
- Magneto-vortical matter

Talk by H. Z. Huang and S. Sharma

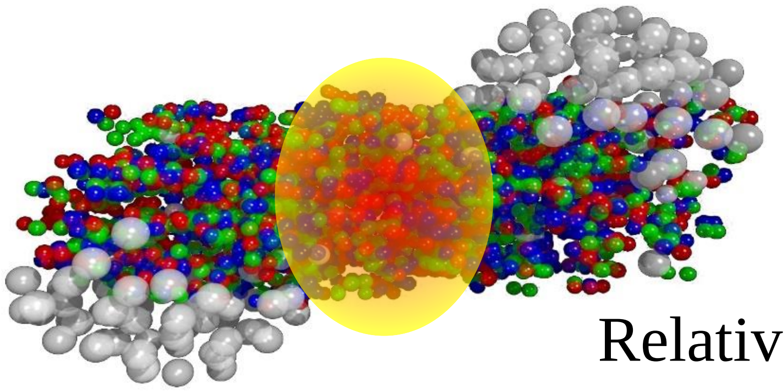


STAR (2007 – 2017, 2018, 2023)
“Global Lambda polarization,” STAR Collaboration
Nature 548, 62-65 (2017); PRC108,014910(2023)



“Charge-Dependent Directed Flow,”
ALICE, PRL 125 (2020), STAR, PRX 14 (2024)

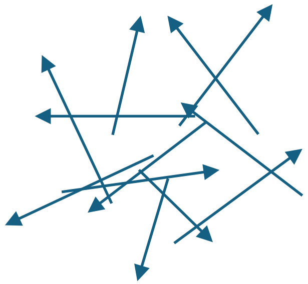
# Hydrodynamics behavior of the quark-gluon plasma



Relativistic heavy-ion
collisions at RHIC and LHC

✗ Free-streaming limit?
(Asymptotic-free limit)

✗ Random momentum distribution



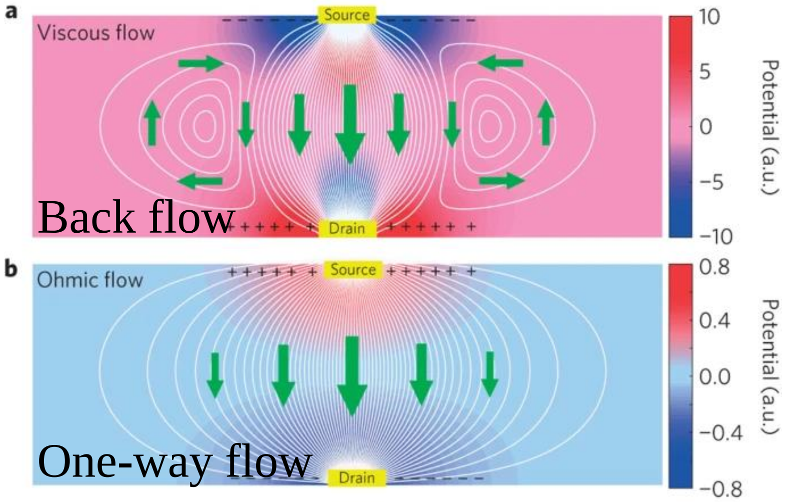
✓ Strongly correlated matter
→ Collective flow at RHIC and LHC

✓Correlated momentum angle distribution

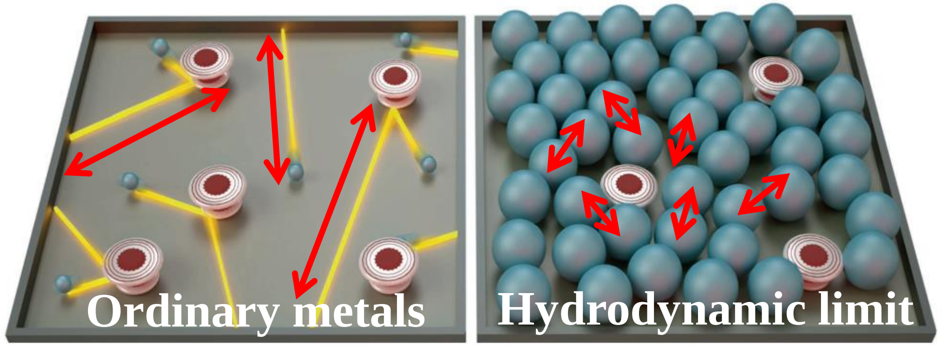
Frequent interactions induce strong correlations (even if the coupling constant is perturbatively small.)

# Collective “hydrodynamic” motion in various systems

## Condensed matter systems

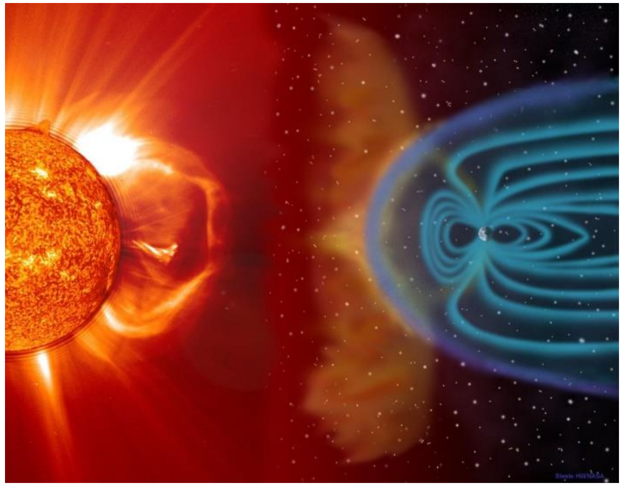


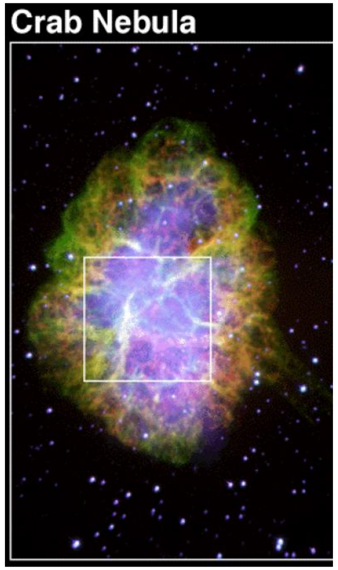
Graphene
Nature Phys., Levitov & Falkovich

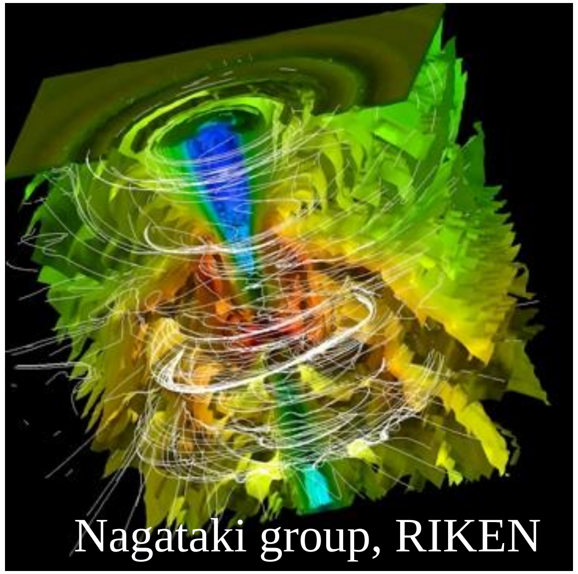


Science, Zaanen

## Astrophysics







Nagataki group, RIKEN

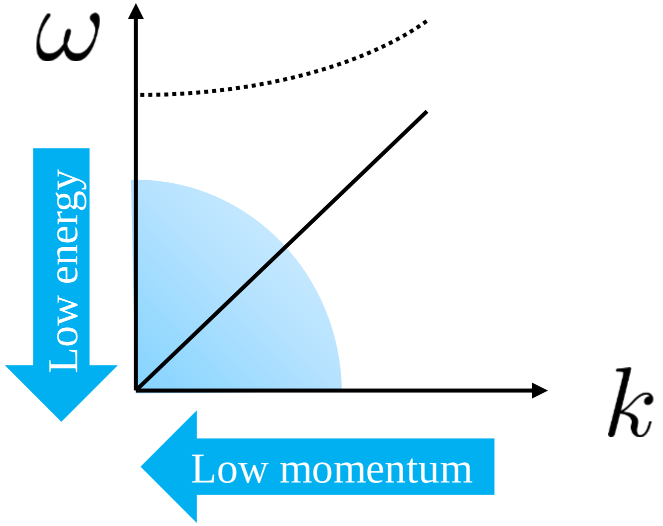
# Hydrodynamics as universal low-energy EFT

Relevant dof. in low energy regime = Gapless modes
= Conserved charges surviving in a long spacetime scale.

A set of conservation laws
= Equations of motion

E.g., Translational symmetry

$$\partial_\mu T^{\mu\nu} = 0$$



Hydrodynamics = Universal low-energy EFT based on symmetries

# Dynamics of conserved charges

## Hydrodynamic variables

{Total energy density, fluid flow velocity}

$$\{\epsilon,\ u^\mu\}$$

## Conservation laws

Translational symmetries

$$\partial_\mu T^{\mu\nu}_{\text{total}} = 0$$

## Electromagnetism

$$\boldsymbol{B}$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

A static charge distribution does not screen the B-field.
(No “magnetic Coulomb field” without a magnetic monopole).

$$B \not\to 0$$

- Conservation of magnetic flux.

E-field is screened by a static charge distribution, i.e., the Debye screening effect.

$$E \to 0$$

E(u, B) and j(u, B) are induced at off-equilibrium

# Derivation of constitutive equations

See a review, KH, Huang, Hongo (2022)

## Statistical description of underlying theory

Weak coupling

Strong coupling

Kinetic theory

Operator method (QFT)

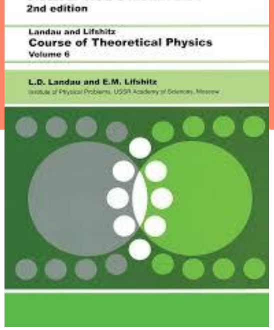
Grad, Israel-Stewart, Frankfurt group

Zubarev, Becattini, Hayata-Hidaka-Noumi-Hongo, Hongo-KH (for MHD), Hongo-Huang-Stephanov-Yee (for spin)

Dynamics of averaged charges
(Course graining)

## Thermodynamics

Constraints on constitutive eqs. from the 2nd law
aka the "entropy-current analysis"



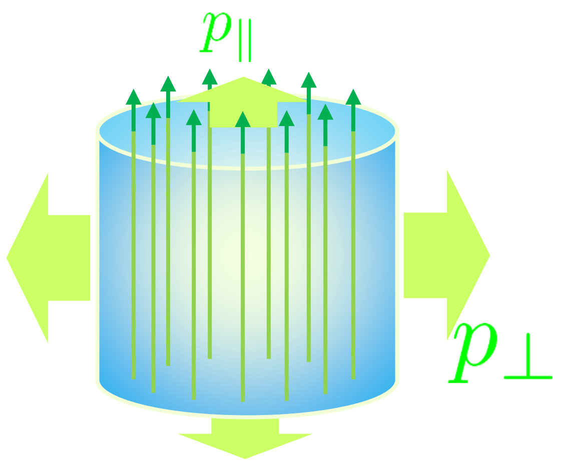
Fluid Mechanics
2nd edition
Landau and Lifshitz
Course of Theoretical Physics
Volume 6
L.D. Landau and E.M. Lifshitz
Institute of Physical Problems, USSR Academy of Sciences, Moscow

## Hydrodynamics

# Anisotropies in constitutive equations

$$\Xi^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu - b^\mu b^\nu$$

## Anisotropic pressure



$$\Theta^{\mu\nu}_{(0)} \quad = \quad \epsilon u^\mu u^\nu - p_\perp \Xi^{\mu\nu} + p_{||} b^\mu b^\nu$$

$p_\perp$ and $p_{||}$ are different by the Maxwell stress.

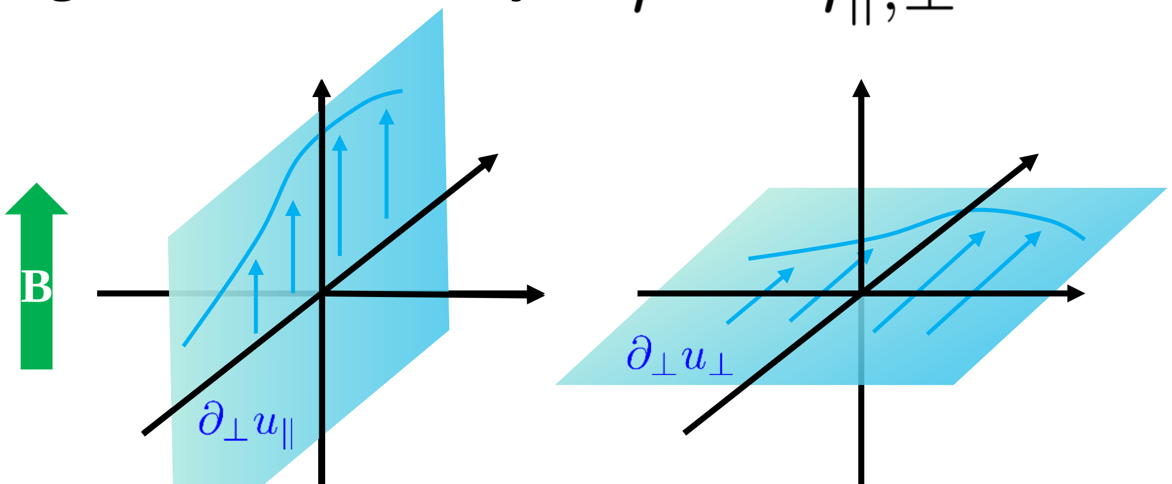
## Anisotropic viscosities

$$\delta\Theta^{(\mu\nu)} = -T\eta^{\mu\nu\rho\sigma}\partial_{(\rho}\beta_{\sigma)}$$

Splitting of shear viscosity $\eta \rightarrow \eta_{||,\perp}$

2 shear viscosities

3 bulk viscosities



See a review, KH, Huang, Hongo (2022)

# Spin as a quasi-hydro variable

- Total AM conservation from rotational symmetry : $\partial_\mu J^{\mu\alpha\beta} = 0$
- Spin-orbit coupling for relativistic constituents

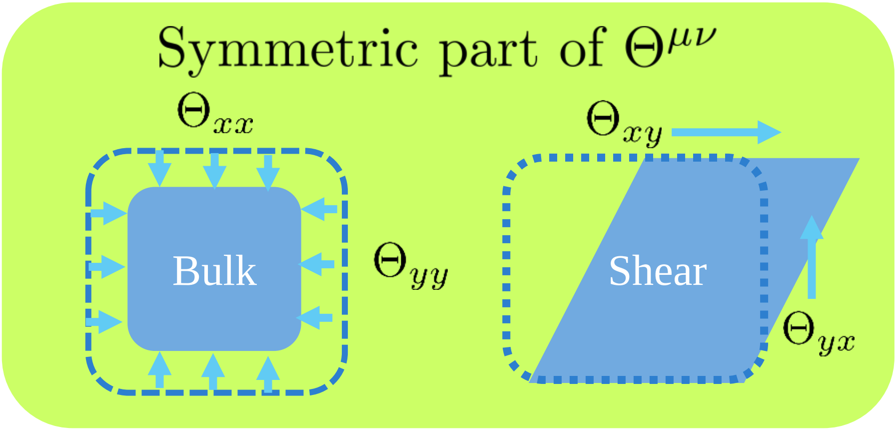
$$\partial_\mu \left( L^{\mu\alpha\beta} + \Sigma^{\mu\alpha\beta} \right) = 0 \qquad L^{\mu\alpha\beta} = x^\alpha \Theta^{\mu\beta} - x^\beta \Theta^{\mu\alpha}$$

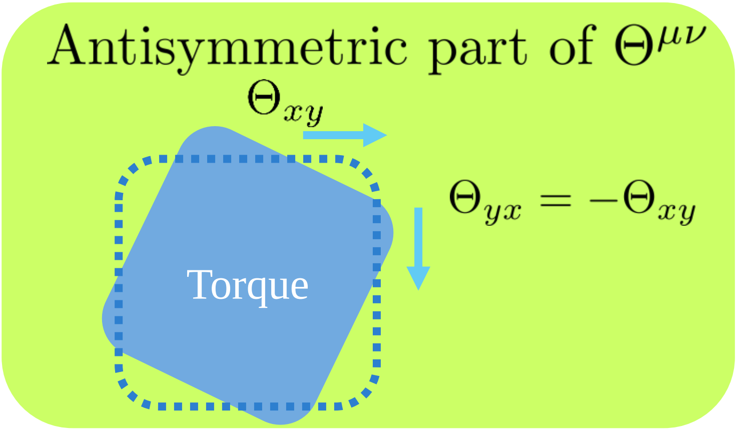
Orbital AM Spin

→ Spin "non-conservation" equation

$$\begin{aligned} \partial_\mu \Sigma^{\mu\alpha\beta} &= -\partial_\mu L^{\mu\alpha\beta} \\ &= -\left( \Theta^{\alpha\beta} - \Theta^{\beta\alpha} \right) \end{aligned}$$

No symmetry protecting spin conservation.





# Rotational viscosities (Anti-symmetric part)

$$\delta\Theta^{[\mu\nu]} \sim -T\gamma^{\mu\nu\rho\sigma}(\partial_{[\rho}\beta_{\sigma]} - \beta\mu_{\rho\sigma})$$

Fang, KH, Hu, 2409.07096

Thermal vorticity Spin potential

- Extended first law

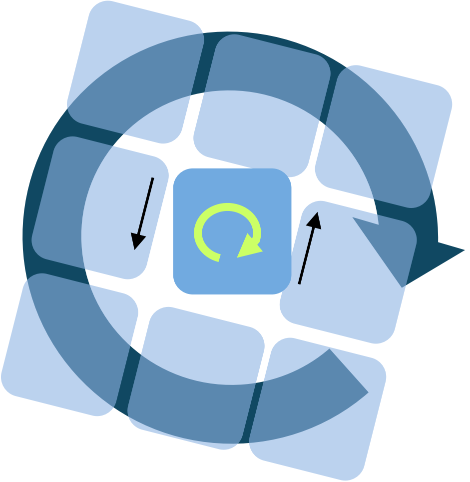
$$Tds = de - \frac{1}{2}\mu^{\nu\rho}d\sigma_{\nu\rho} - H_\mu dB^\mu$$

Energy density Spin density Magnetic flux

$$\beta\mu^{\nu\rho} \equiv -2\frac{\partial s}{\partial\sigma_{\nu\rho}}$$

Spin potential

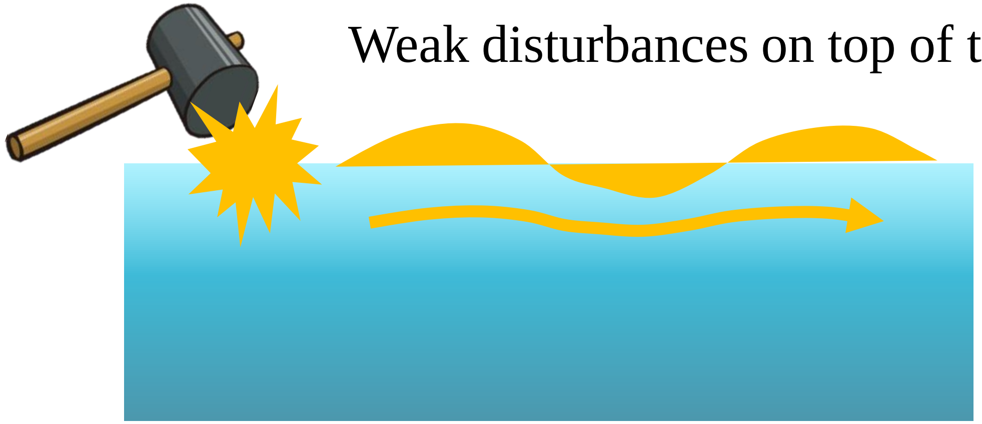
“No-slip condition” with the rotational fluid motion



Isotropic limit
KH, Hongo, Huang, Matsuo, Taya, 1901.06615

Spin hydrodynamics
KH, Hongo, Huang, Matsuo, Taya, 1901.06615
Fukushima, Pu, 2010.01608
Li, Stephanov, Yee, 2011.12318
She, Huang, Hou, Liao, 2105.04060
Hongo, Huang, Kaminski, Stephanov, Yee, 2107.14231
Biswas, Daher, Das, Florkowski, Ryblewski, 2304.01009

# *Solving spin MHD within linear-mode analysis*



Weak disturbances on top of the equilibrium state.

For numerical methods beyond linear regime, see talk by C. Nonaka and A. Dash

$$e \to e + \delta e(x), \quad u^\mu \to u^\mu + \delta u^\mu(x),$$
$$B^\mu \to B^\mu + \delta B^\mu(x), \quad S^{\mu\nu} \to S^{\mu\nu} + \delta S^{\mu\nu}(x)$$

- Energy density (1)
- Flow velocity (3)
- Magnetic field (2)
- Spin (3)

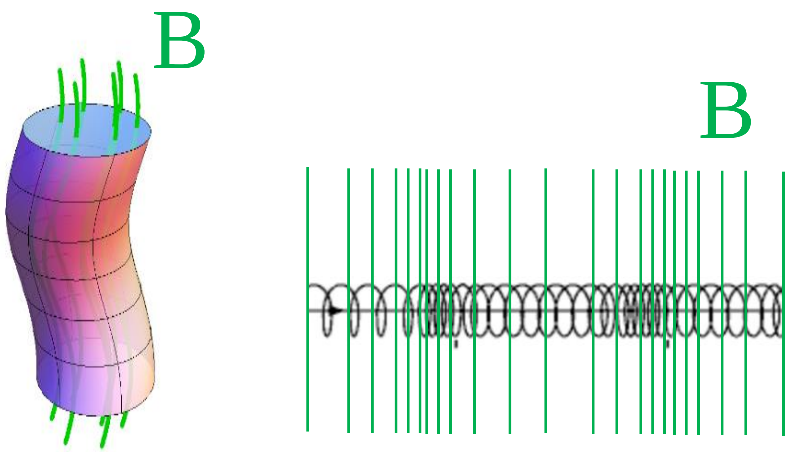
9 × 9 matrix !!

$$\begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \delta u_y \\ \delta B_y \\ \delta S_x \\ \delta S_z \\ \cdot \\ \cdot \end{pmatrix} = 0$$

Exactly diagonalized with an analytic algorithm.

Fang, KH, Hu, 2402.18601; 2409.07096

# An issue in preceding works



Magnetic tension and pressure

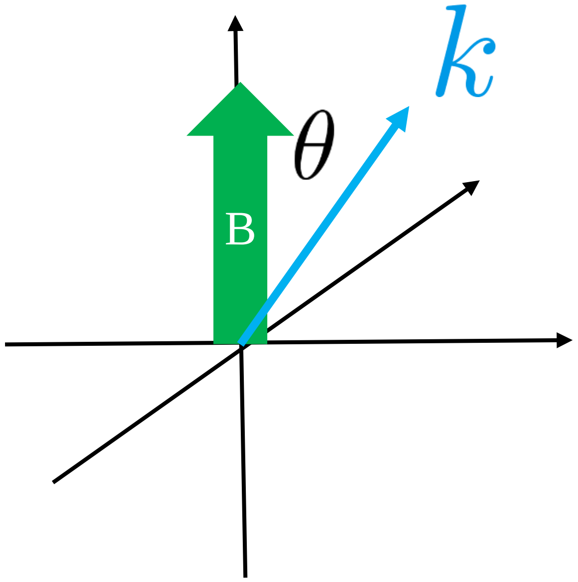
- Ideal MHD (well-known)

Alfven waves and Magneto-sonic waves

- First-order MHD (Not known due to strong anisotropy)

Solutions had been known only at θ = 0 or π/2.

Moreover, there was disagreement
between those known solutions at θ = π/2.



Grozdanov et al. (2017)
Armas and Camilloni (2022) ⟷ Hernandez and Kovtun (2017)

# “Critical angle”

Fang, KH, Hu, 2402.18601

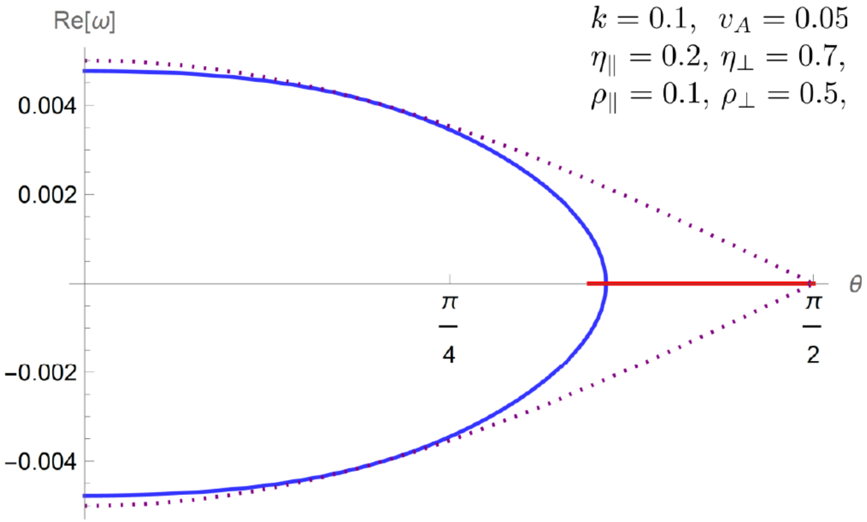
Solutions at θ = 0 and π/2 are not smoothly connected.

$$\omega = \pm vk - i\Gamma k^2 + O\left(\frac{k^n}{\cos^n\theta}\right)$$

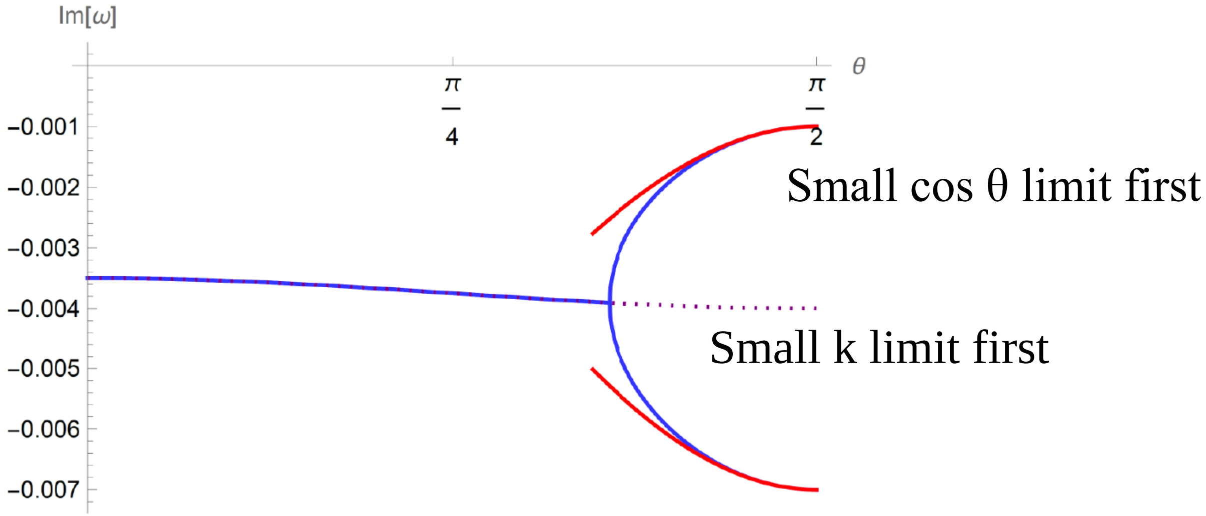
Small k expansion breaks down due to the anisotropy.
→ Small k and cos θ limit do not commute.

Small cosine expansion $\neq$ Small k expansion (Dotted lines)
Without any expansion

Phase velocities



Damping rates



Small cos θ limit first

Small k limit first

Purely diffusive near the transverse direction.

Split of the diffusion rates

# Split of damping spin modes

$$\begin{aligned}\omega &= -i\Gamma_\perp - i\frac{\gamma_\perp}{h}k_\perp^2 \\ \omega &= -i\Gamma_\parallel - i\frac{(\gamma_\parallel - \xi)^2}{h\gamma_\parallel}k_\parallel^2 \\ \omega &= -i\Gamma_\parallel - iw_3 k^2\end{aligned}$$

$\Gamma_{\parallel,\perp} = \frac{8}{\chi}\gamma_{\parallel,\perp}$

Spin susceptibility: $S^{\mu\nu} = \chi\mu^{\mu\nu}$

$$w_3 = \frac{(\gamma'_\parallel - \xi')^2}{\gamma'_\parallel}\frac{1 - v_A^2\cos^2\theta}{1 - v_A^2} + 4\xi'\frac{\sin^2\theta}{1 - v_A^2}$$

Spins are not conserved → Gapped modes at k = 0.

All solutions are always damping.

# Anomalous quantum transport

Dirac equation in the chiral representation
→ Chirality-spin-momentum locked in the massless limit

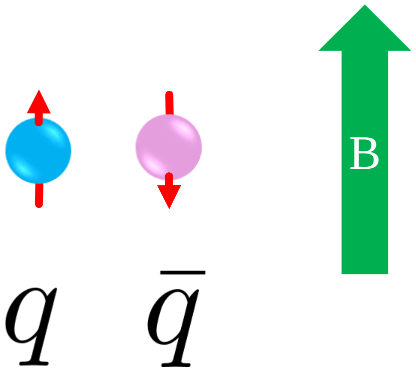
Talk by H. Z. Huang and S. Sharma

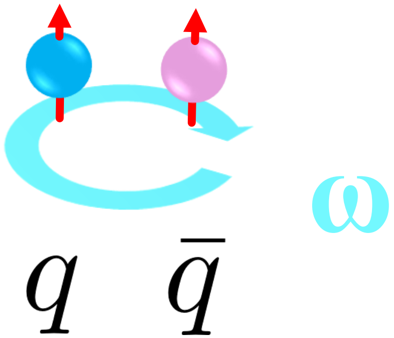
{Chirality, Spin, Momentum}

$$\begin{pmatrix} -m & p_\mu \sigma^\mu \\ p_\mu \bar{\sigma}^\mu & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

Spin polarization in B or ω

Chirality-momentum locking
→ Finite currents when one of chirality is favored.





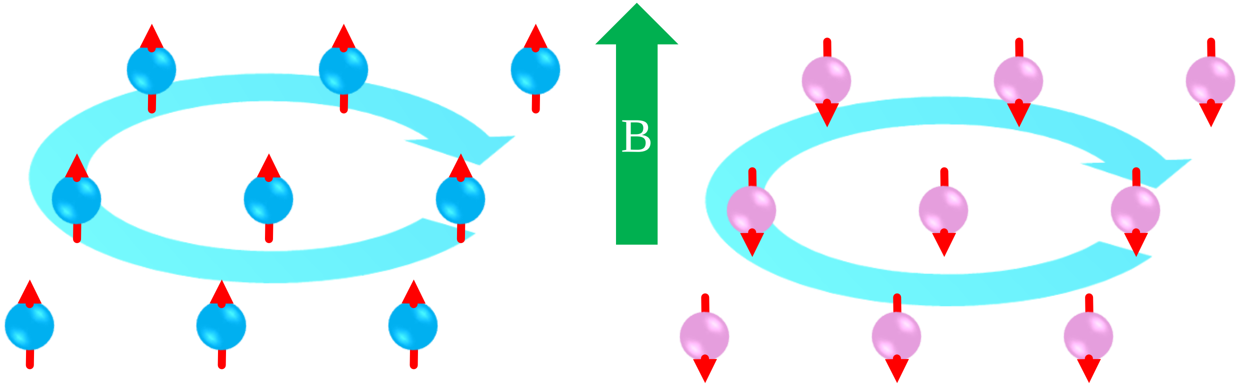
$$\boldsymbol{j}_{\rm CME} \propto \mu_A \boldsymbol{B}$$

$$\boldsymbol{j}_{\rm CVE} \propto \mu \mu_A \boldsymbol{\omega}$$

$$\mu_A = (\mu_R - \mu_L)/2, \quad \mu = (\mu_R + \mu_L)/2$$

# Magneto-vortical matter

## - Anomalous transport w/o chirality imbalance



$$j^0 = C_A \frac{1}{2} \boldsymbol{\omega} \cdot \boldsymbol{B}$$

KH and Yin, 1607.01513

$$\text{Charge-dependent energy shift: } \Delta E = \boldsymbol{S} \cdot \boldsymbol{\omega} = \left( \pm \tfrac{1}{2} \hat{\boldsymbol{B}} \right) \cdot \boldsymbol{\omega}$$

→ Effective chemical potential

- Chiral kinetic theory
Yang, et al., 2003.04517, Mameda, 2305.02134,
Yang, et al., arXiv:2409.00456

$$\text{Landau degeneracy: } \frac{|eB|}{2\pi}$$

- Effective theory as the Chern-Simons current
Yamamoto and Yang, 2103.13208

But some technical issue is recognized in QFT approach. Ebihara, Fukushima, Mameda, 1608.00336

# Sign inversion by the orbital contribution in the strong-field regime

Fukushima, KH, Mameda 2409.18652

$$j^0 = C_A\left(\frac{1}{2} - 1\right)\boldsymbol{\omega}\cdot\boldsymbol{B} = -C_A\frac{1}{2}\boldsymbol{\omega}\cdot\boldsymbol{B}$$

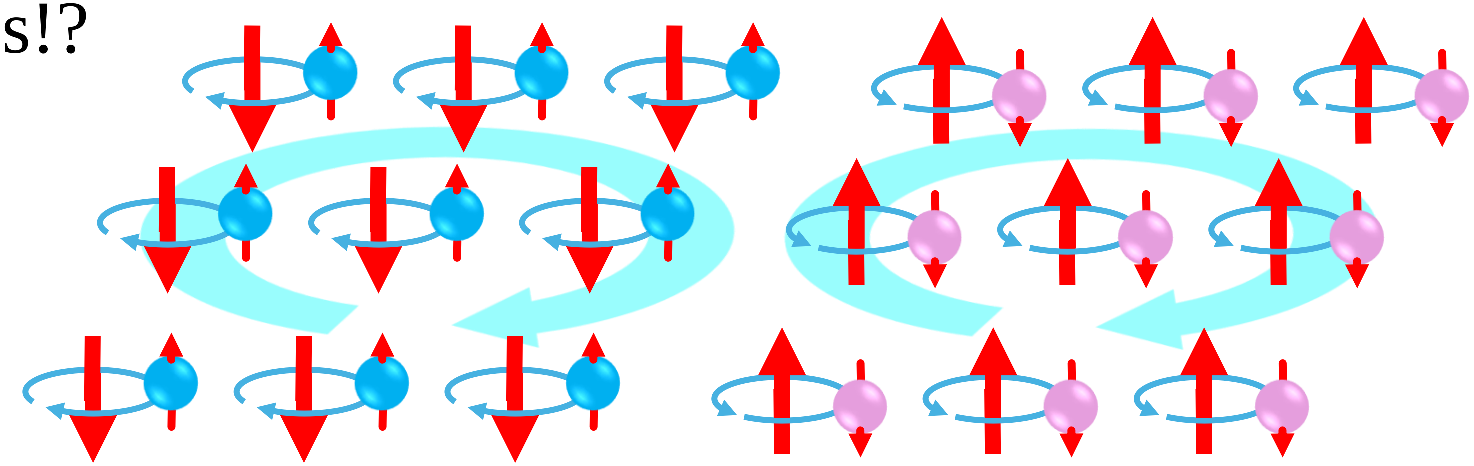
Spin - Orbital

Each cyclotron orbit has a kinetic angular momentum that is additive to spin.

$$\langle n|M_{\rm kin}|n\rangle = -{\rm sgn}(eB)(2n+1)$$

Cf. KH, Itakura, Ozaki, 2305.03865

Orbitronics!?



# Summary

Physics opportunities both in hard and soft observables.

EM probes and anomalous transport have been pursued in other communities as well (laser, cond. matt., cosmology/astro)

Collaborations among theory, pheno., and exp. are developing.

Theoretical challenge: Transition from strong to weak fields.