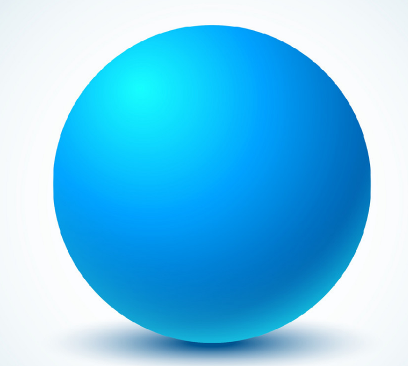
Spin polarization in heavy ion collisions and relativistic spin hydrodynamics

Amaresh Jaiswal

School of Physical Sciences, NISER Bhubaneswar, Jatni, India

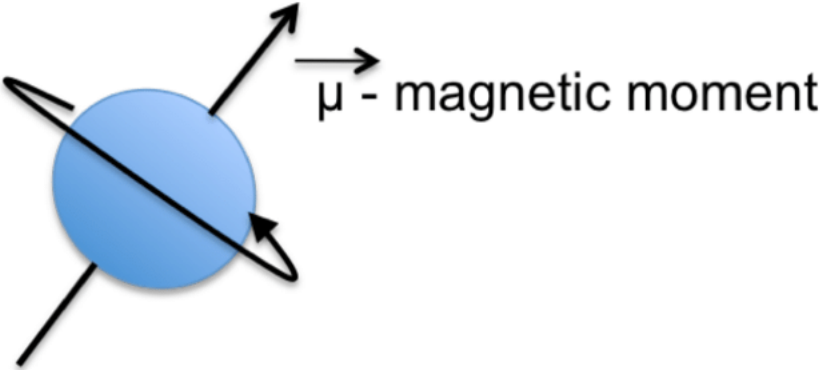
ATHIC 2025

Decay of scalar particles



No anisotropy in the rest frame: isotropic decay products.

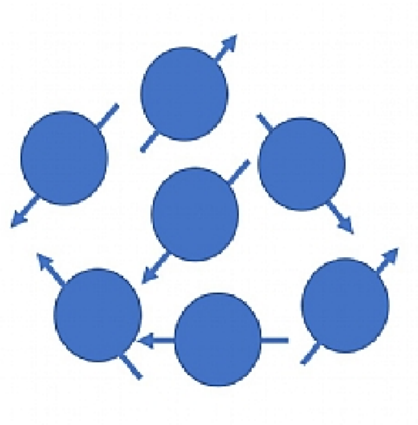
Decay of particles with spin



Preferred direction due to spin: anisotropic decay products

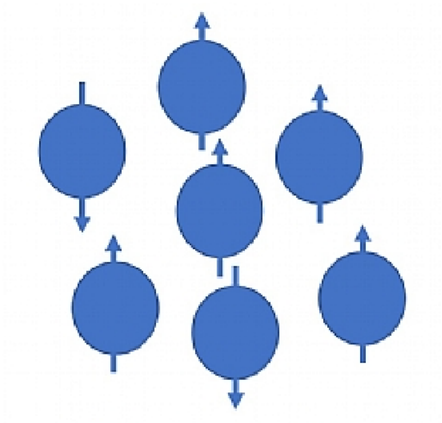
Basis for polarization observables.

Several random decays



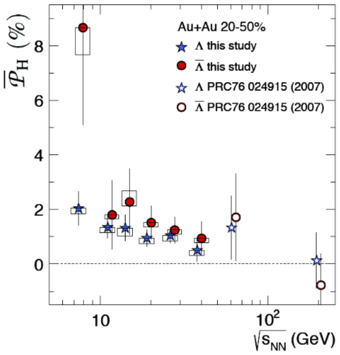
Averaging over random decays should lead to isotropic decay products.

Decay of spin polarized particles



Averaging over decay of spin-polarized particles should lead to anisotropic decay products.

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017



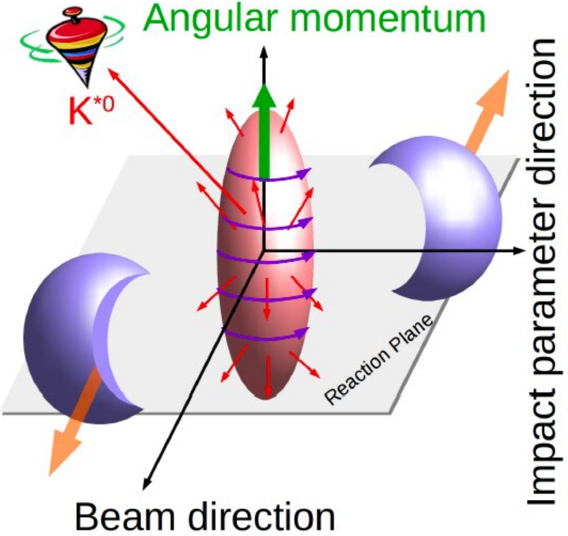
First evidence of a quantum effect in (relativistic) hydrodynamics

Adapted from F. Becattini
'Subatomic Vortices'

Spin polarization of hadrons in heavy-ion collisions

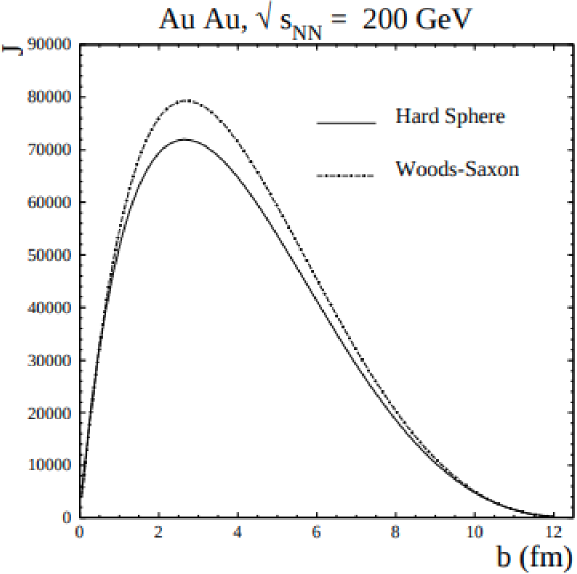
- Spin polarization is a relatively new topic in heavy ion collisions.
- Provides unique opportunity to probe QGP properties.
- Several measurements of spin polarization of hadrons.
- In baryon sector:
  - Λ (spin 1/2): STAR, Nature, 548, 62–65 (2017); HADES; ALICE.
  - Ω (spin 3/2): STAR, Phys. Rev. Lett. 126, 162301 (2021).
  - Ξ (spin 1/2): STAR, Phys. Rev. Lett. 126, 162301 (2021).
- In meson sector:
  - K^{*0} : ALICE, PRL 125, 012301 (2020); STAR, Nature, 614, 244-248 (2023).
  - ϕ : ALICE, PRL 125, 012301 (2020); STAR, Nature, 614, 244-248 (2023).
  - Heavy quarkonium, J/ψ and Υ(1S) : ALICE, PLB 815, 136146 (2021).
- Global and local polarization measurements.

Global angular momentum in heavy ion collisions

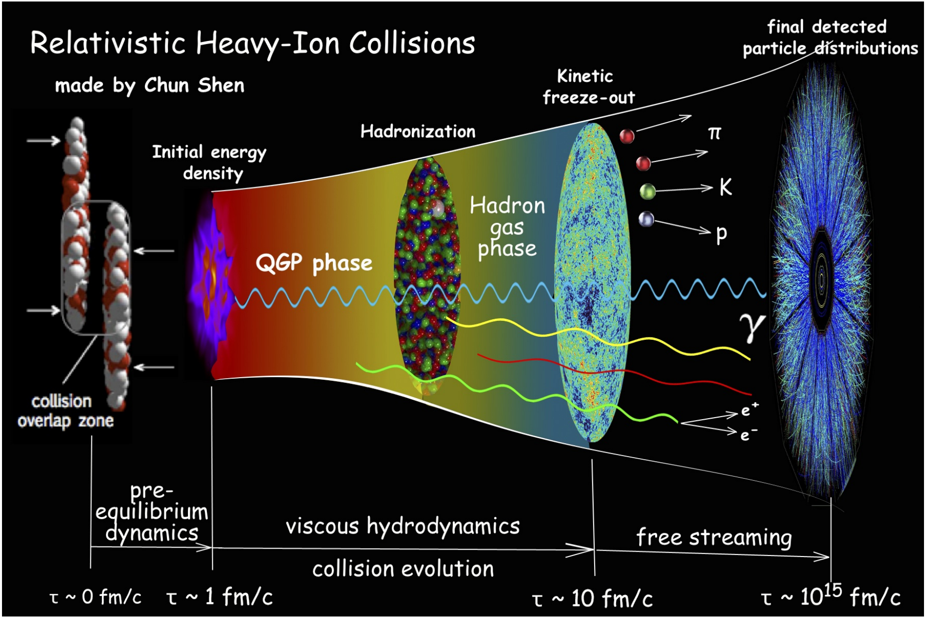


[B. Mohanty, ICTS News 6, 18-20 (2020).]

Angular momentum generation in non-central collisions



[F. Becattini, et al., Phys. Rev. C77, 024906 (2008)]


Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected
particle distributions
Kinetic
freeze-out
Hadronization
Initial energy
density
Hadron
gas
phase
QGP phase
π
K
p
γ
e⁺
e⁻
collision
overlap zone
pre-
equilibrium
dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Relativistic spin-hydrodynamics

Angular momentum conservation: particles

- Angular momentum of a particle with momentum $\vec{p}$:

$$\vec{L} = \vec{x} \times \vec{p} \quad \Rightarrow \quad L_i = \varepsilon_{ijk}\, x_i\, p_j$$

- One can obtain the dual tensor:

$$L_{ij} \equiv \varepsilon_{ijk}\, L_k \quad \Rightarrow \quad L_{ij} = x_i\, p_j - x_j\, p_i$$

- We know that both definitions are equivalent.
- In absence of external torque, $\dfrac{d\vec{L}}{dt} = 0$, we also have: $\partial_i L_{ij} = 0$.
- Relativistic generalization: $L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$ and $\partial_\mu L^{\mu\nu} = 0$.
- This treatment valid for point particles.
- For fluids, particle momenta $\rightarrow$ "generalized fluid momenta"

The energy-momentum tensor

Angular momentum conservation: fluid

- The orbital angular momentum for relativistic fluids is defined as

$$L^{\lambda,\mu\nu} = x^{\mu}T^{\lambda\nu} - x^{\nu}T^{\lambda\mu}$$

- Keeping in mind the energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$:

$$\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$$

- Obviously, for symmetric $T^{\mu\nu}$, orbital angular momentum is automatically conserved. Classically $T^{\mu\nu}$ symmetric.
- For medium constituent with intrinsic spin, different story

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

- Ensure total angular momentum conservation: $\partial_\lambda J^{\lambda,\mu\nu} = 0$.
- Basis for formulation of spin Hydrodynamics.

  [Florkowski et. al., Prog.Part.Nucl.Phys. 108 (2019) 103709; Bhadury et. al., Eur.Phys.J.ST 230 (2021) 3, 655-672]

Pseudo-gauge transformations

- Total angular momentum is

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

- With $\partial_\mu T^{\mu\nu} = 0$, and $\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$,

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \quad \Longrightarrow \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Hence the final hydrodynamic equations can be written as

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Also holds with the following redefinition

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} - \Phi^{\mu,\lambda\nu} - \Phi^{\nu,\lambda\mu}\right)$$

$$\tilde{S}^{\lambda,\mu\nu} = S^{\lambda,\mu\nu} - \Phi^{\lambda,\mu\nu} + \partial_\rho Z^{\mu\nu,\lambda\rho}$$

- Polarization observables are independent of pseudo-gauge freedom.

[Gallegos et. al., SciPost Phys. 11, 041 (2021); Hongo et. al., JHEP 11 (2021) 150]

Pseudo-gauge transformations and transport

- Different forms of conserved currents used:
  1. Canonical: $S^{[\lambda\mu\nu]},\ T^{(\mu\nu)} + T^{[\mu\nu]}$
  2. Belinfante: $S^{\lambda,\mu\nu} = 0,\ T^{(\mu\nu)}$
  3. de Groot, van Leuween and van Weert (GLW): $S^{\lambda,[\mu\nu]},\ T^{(\mu\nu)}$
  4. Hilgevoord and Wouthuysen (HW): $S^{\lambda,[\mu\nu]},\ T^{(\mu\nu)}$
  5. Phenomenological: $S^{\lambda,\mu\nu} \sim u^\lambda \omega^{\mu\nu},\ T^{(\mu\nu)} + T^{[\mu\nu]}$
- Belinfante does not retain information about evolution of spin.
- Canonical is not most general: anti-symmetry in all three indices.
- Phenomenological not related to canonical via PG transformation.
- Derivative terms are generated in conserved currents by PG trans.
- Redistribution of spin evolution between $S^{\lambda,\mu\nu}$ and $T^{[\mu\nu]}$.
- Issues in counting of transport coefficients for spin evolution.

Extended phase-space for spin degrees of freedom

- The phase-space for single particle distribution function gets extended $f(x, p, s)$.
- The equilibrium distribution for Fermions is given by

$$f_{eq}(x,p,s) = \frac{1}{\exp\left[\beta\cdot p - \alpha - \frac{1}{2}\,\omega : s\right] + 1} \qquad \begin{cases} \beta\cdot p \equiv \beta_\mu p^\mu \\ \omega : s \equiv \omega_{\mu\nu} s^{\mu\nu} \end{cases}$$

- Quantities $\beta^\mu = u^\mu/T,\ \alpha = \mu/T,\ \omega_{\mu\nu}$ are functions of x.
- $\alpha,\ \beta^\mu,\ \omega^{\mu\nu}$: Lagrange multipliers for conserved quantities.
- $s^{\mu\nu}$: Particle spin, on equal footing with particle momenta p^μ.
- Hydrodynamics: average over particle momenta and spin.
- Like T, μ, u^μ, solve for $\omega^{\mu\nu}$ with appropriate initial conditions.
- Current state-of-art: Thermal vorticity used as a proxy for $\omega^{\mu\nu}$.

Boltzmann equation and global equilibrium

- Boltzmann equation for distribution function is

$$p^\mu \partial_\mu f = C[f]$$

- In equilibrium, $C[f] = 0$. Global equilibrium condition:

$$p^\mu \partial_\mu f_{eq} = 0$$

- For $f_{eq} = \left[\exp\left(\beta \cdot p - \alpha - \frac{1}{2}\,\omega : s\right) + 1\right]^{-1}$, one obtains

$$\partial_\mu \alpha = 0; \quad \partial^\mu \beta^\nu + \partial^\nu \beta^\mu = 0; \quad \partial_\mu \omega_{\rho\sigma} = 0$$

- A solution can be obtained as

$$\alpha = \text{const.}; \quad \beta^\mu = \beta_0^\mu + x_\lambda\, \omega_0^{\mu\lambda}; \quad \omega_{\rho\sigma} = \text{const.}$$

- The last two solutions leads to

$$\omega_0^{\mu\nu} = -\frac{1}{2}\left(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu\right); \quad \omega_{\mu\nu} \to \omega_0^{\mu\nu} \equiv \varpi_{\mu\nu}$$

- This assumption avoids solving spin-hydro equations.

Pauli-Lubanski and Polarization

- On freeze-out hypersurface: $\langle P(\phi_p)\rangle = \dfrac{\int p_T dp_T E_p \frac{d\Pi^z(p)}{d^3p}}{\int d\phi_p p_T dp_T E_p \frac{dN(p)}{d^3p}}$

- $E_p \dfrac{dN(p)}{d^3p} = \dfrac{4\cosh\xi}{(2\pi)^3}\displaystyle\int \Delta\Sigma_\lambda p^\lambda e^{-\beta\cdot p}, \qquad \xi = \mu/T,\ \beta^\mu = u^\mu/T$

- $E_p \dfrac{d\Delta\Pi_\tau(x,p)}{d^3p} = -\dfrac{1}{2}\epsilon_{\tau\mu\nu\beta}\,\Delta\Sigma_\lambda E_p \dfrac{dS^{\lambda,\mu\nu}(\omega)}{d^3p}\dfrac{p^\beta}{m}$

  [Florkowski et. al., Prog.Part.Nucl.Phys. 108 (2019) 103709]

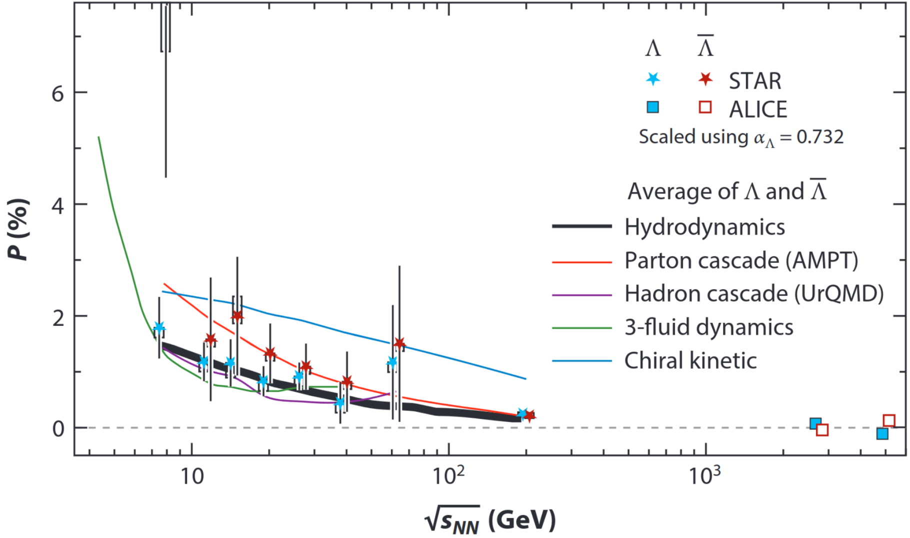
- The spin tensor can be defined as

$$S^{\lambda,\mu\nu}(\omega) = \int dPdS\; p^\lambda s^{\mu\nu}\left[f(x,p,s) + \bar{f}(x,p,s)\right]$$

- In absence of hydrodynamic evolution, one uses the ansatz:

$$\omega_{\mu\nu} \to \varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

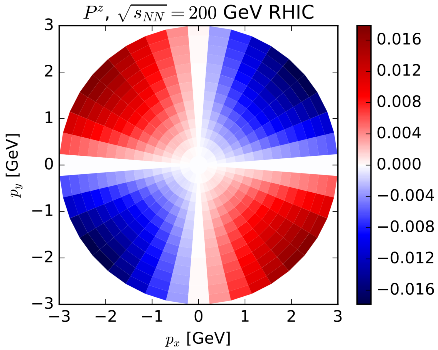
Success of thermal vorticity: Global polarization



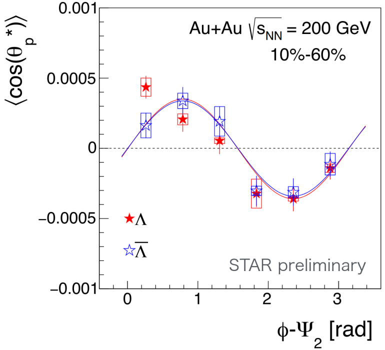
[F. Becattini and M. Lisa, Annu. Rev. Nucl. Part. Sci. 2020. 70:395-423]

Longitudinal/local polarization and sign problem

vHLLE+Glissando IS



Preliminary STAR data: Takafumi Niida, talk at Chirality workshop 2018

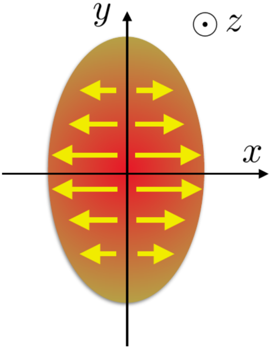


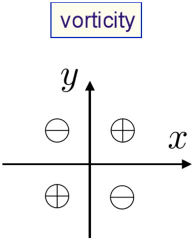
Similar $\sin(2\phi)$ structure is observed, with opposite sign!

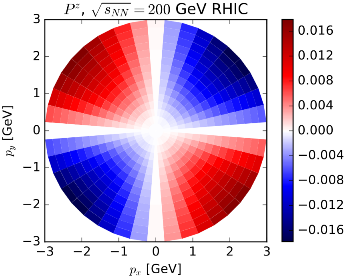
[Iurii Karpenko, Lambda polarization from RHIC BES to LHC]

Simplified explanation of the quadrupole structure

(c) Sergei Voloshin, SQM2017







Polarization depends on the the thermal vorticity:

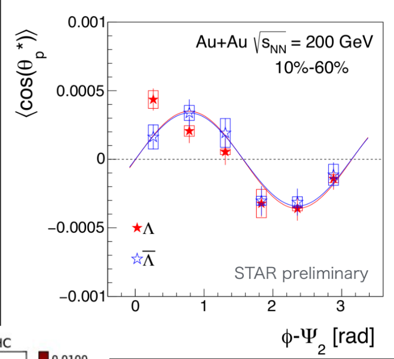
$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

[Iurii Karpenko, Lambda polarization from RHIC BES to LHC]

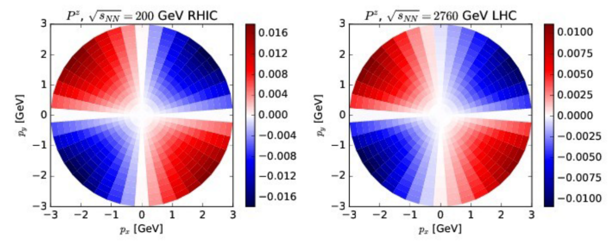
A *sign problem* for the longitudinal component

Quadrupolar structure of longitudinal polarization in the transverse momentum plane, as predicted.
Spectacular confirmation of hydro predictions... yet with a flipped sign!

- Hydro initial conditions? (polarization is a sensitive probe of the initial flow)
- Incomplete local thermodynamic equilibrium for the spin degrees of freedom (spin kinetic theory)?
- Effect of spin dissipative transport coefficients?
- Effect of initial state fluctuations?
- Effect of decays?
- Error in the calculation



Z. Ye, T. Niida, Quark Matter 2018



F. B., I. Karpenko, Phys. Rev. Lett. 120 (2018) 012302
S. Voloshin, in SQM 2017

Same pattern found in AMPT+thermal vorticity calculation X. L. Xia, H. Li, Z. B. Tang and Q. Wang, 1803.00867

Global equilibrium and thermal vorticity

- Global equilibrium may not be achievable: short fireball lifetime.
- Large spin equilibration time [1907.10750, 2405.00533, 2405.05089, ...].
- Spin hydrodynamic evolution necessary with appropriate initial conditions [Singh et. al., 2411.08223].
- Thermal vorticity is a robust prediction of spin-hydrodynamics.
- Alternate systems for signature of thermal vorticity solution.
- Electrons in graphene near Dirac point: "relativistic" dispersion.
- No issues with short lifetime for graphene: global equilibrium.
- Analog of Barnett effect: *Thermovortical magnetization* [2409.07764].

Our work in spin hydrodynamics within kinetic theory

- Non-dissipative spin-hydrodynamics:
  - W. Flokowski, B. Friman, A. Jaiswal and E. Speranza, Physical Review C 97, 041901 (2018).
  - W. Flokowski, B. Friman, A. Jaiswal, R. Ryblewski and E. Speranza, Physical Review D 97, 116017 (2018).
- Dissipative spin-hydrodynamics:
  - S. Bhadury, W. Flokowski, A. Jaiswal, A. Kumar, and R. Ryblewski, Physics Letters B 814, 136096 (2021).
  - S. Bhadury, W. Flokowski, A. Jaiswal, A. Kumar, and R. Ryblewski, Physical Review D 103, 014030 (2021).

- Relativistic Spin Magnetohydrodynamics: S. Bhadury, W. Flokowski, A. Jaiswal, A. Kumar, and R. Ryblewski, Phys. Rev. Lett., 129, 192301 (2022).

Ongoing work from field theory and geometry

- Starting from the symmetries of the Lagrangian of a given theory, one can construct conserved currents using Noether's theorem.
- Energy-momentum tensor-variation of Lagrangian with metric $g^{\mu\nu}$: conservation is a consequence of diffeomorphism invariance.
- Conserved charge current-variation with gauge field A^{μ}: consequence of local gauge symmetry.
- Spin-current can be constructed similarly.
- Price to pay: introduce torsion in metric, non-Riemannian geometry.
- Spin current: variation w.r.t torsion.
- Angular momentum conservation: consequence of local Lorentz invariance.
- Kubo relations for dissipation in spin current.
- PG and SO(3) algebra of spin [S. Dey *et. al.*, PLB 843 (2023) 137994].

Heavy-ion phenomenology with spin-hydrodynamics

- Global and local (longitudinal) polarization measured in HICs; talk by Radoslaw Ryblewski.
- Global polarization relatively well understood from spin hydrodynmaic prediction of thermal vorticity.
- Sign problem in longitudinal spin polarization of the Λ hyperons.
- Thermal vorticity employed at freezeout.
- Assumes global equilibrium: not a good approximation.
- Large spin-relaxation time obtained from fit with experimental data of longitudinal polarization [S. Banerjee *et. al.*, arXiv:2405.05089].
- Hydrodynamic evolution required for $\omega^{\mu\nu}$.
- Numerical solution of relativistic spin-hydrodynamics.

Summary

- Pseudogauge freedom in the formulation of spin hydrodynamics.
- Polarization observables independent of pseudogauge freedom.
- Pseudogauge freedom and counting of spin transport not settled.
- Sign problem in longitudinal component of spin polarization.
- Thermal vorticity ansatz for polarization tensor: not good.
- Evolution with spin-hydrodynamics necessary, some progress.
- Polarization and spin hydrodynamics: exciting times.
- Opportunities for exciting new physics.

Thank you!