Chiral plasma instabilities in gauge theories

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13.01.2025

Asian Triangle Heavy-Ion Conference, ATHIC 2025, IISER Behrampur

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  [Tashiro, Vachaspati & Vilenkin 12, A. Boyarsky, O. Ruchayskiy, M. Shaposhnikov, 12]
- Crucial for understanding topological properties of non-Abelian gauge theories like QCD far away from equilibrium.

  [Y. Akamatsu & N. Yamamoto 13, Y. Hirono, D. Kharzeev, Y. Yin 15, K. Tuchin 17]

Plasma instabilities & Thermalization

- Non-Abelian plasmas with perturbative interactions realized within a kinetic theory will take a longer time ~ 2.5-3 fm/c to thermalize.

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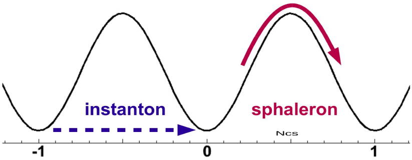
- Essentially the chaotic properties are inherent in the magnetic modes of the gauge theories ($|\mathrm{p}| < g^2 T/\pi$). [See talk by Ravi Shanker on the chaotic properties inherent in the eigenvalue spectrum of non-Abelian gauge theories.]

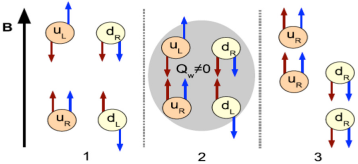
Chiral Magnetic effect and the role of instabilities

- Early stages of heavy-ion collisions provides a unique laboratory to study topological properties of QCD.
- Local regions of CP odd domains created due to sphaleron transitions in QCD $\rightarrow$ leads to a net axial charge j_0^a.
- In presence of an external $U(1)$ magnetic field $\vec{B}$, a vector current generated

[Kharzeev, McLerran, Warringa 07, Kharzeev, Fukushima, Warringa, 08]

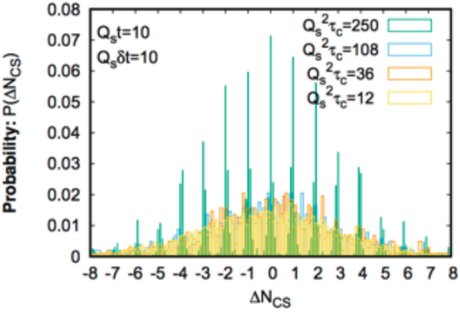
$$\vec{j}^v \propto j_0^a \; \vec{B} \qquad \text{Chiral Magnetic effect}$$





Chiral Magnetic effect and the role of instabilities

- Early stages of heavy-ion collisions provides a unique laboratory to study topological properties of QCD.
- Strong $SU(3)$ color fields at early times lead to enhancement of sphaleron rates in pre-equilibrium stages. [Mace, Schlichting and Venugopalan, 16].



Chiral Magnetic effect and the role of instabilities

- Crucial ingredients for understanding anomalous transport → sphaleron rates out of equilibrium [Mace, Schlichting, Venugopalan, 16]
  + anomalous charge production in the strong fields.

- Since axial charge is not conserved, just the knowledge of initial conditions are not sufficient to describe its subsequent evolution.

- It is important to accurately account for chiral charge some of which may get absorbed in the gauge fields.

The outline of this talk

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- Initial gauge field occupation numbers large $\rightarrow$ amenable to classical statistical lattice gauge theory techniques.
- Specifically study the effects of strong back-coupling between fermion and gauge fields on the efficiency of helicity transfer.
- We also want to study the entire time evolution of this process which is not possible using kinetic theory.

CLT for gauge theories on the lattice

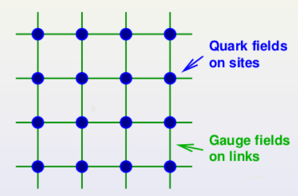
- Gauge theories are Constrained systems.
- The Hamiltonian in $A_0 = 0$ gauge is defined as

$$H_{YM} = \sum_{x,i} \frac{a_i^2}{g_c^2 a^3} \frac{E^i_{a,x} E^i_{a,x}}{2} + \sum_{x,i,j} \frac{a^3}{g_c^2 a_i^2 a_j^2} \, \mathrm{ReTr}\left[1 - U^{\square}_{ij}(x)\right] ,$$

- Evolution eq. for links and electric field variables are then derived from the lattice Hamiltonian yielding the following set of update rules

$$\partial_{x^0} U_{i,x} = -i \frac{a_i^2}{a^3} \, E^i_{a,x} t^a \; U_{i,x}$$

$$\partial_{x^0} E^i_{a,x} = \text{staples} + \left\langle \hat{j}^i_{a,x} \right\rangle$$



Introducing fermions on the lattice

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- Fermion operator $\hat{\psi}$ written in terms of mode-functions [Aarts, Smit, 97, 98]

$$\hat{\psi}_{\mathrm{x}}(t) = \frac{1}{\sqrt{V}} \sum_{\lambda} \left(\hat{b}_\lambda(0)\phi^u_\lambda(t,\mathrm{x}) + \hat{d}^\dagger_\lambda(0)\phi^v_\lambda(t,\mathrm{x}) \right) \ ,$$

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- On a N^3 lattice $\rightarrow 4N_cN^3$ modes, prohibitively expensive on realistic volumes. In small volumes lattice cut-off effects are more severe!

Implementation

- We start with non-interacting almost massless fermions with a large chiral imbalance $n_{u,v} = \frac{1}{\mathrm{e}^{\beta(\mathrm{E_p} \pm \mu_\mathrm{h})} + 1}$, $\beta\mu_h = 8$.

- At $t = 0$ the electric field and links are a random superposition of plane waves to describe their initial vacuum fluctuations.

- The helicity transferred from the fermion to gauge sector during the evolution of electric fields

$$\partial_{x^0} E^i_{a,x} = -\left(\frac{g_c a^3}{a_i}\right) \frac{\delta H}{\delta A^a_{i,x}} + e^2 N_f J^i_a.$$

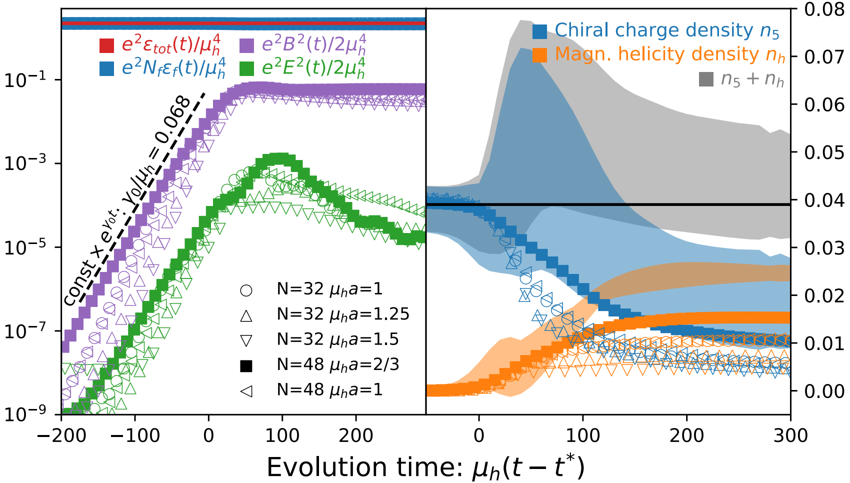
- We choose $e^2 N_f = 64$ to mimic the strong interactions.

[Mace, Mueller, Schlichting, SS, 19]

Chiral plasma instabilities in Abelian gauge theories

[Mace, Mueller, Schlichting, SS, PRL 124, 191604 (2020)]

Strong chirality transfer but not much energy lost in the process!



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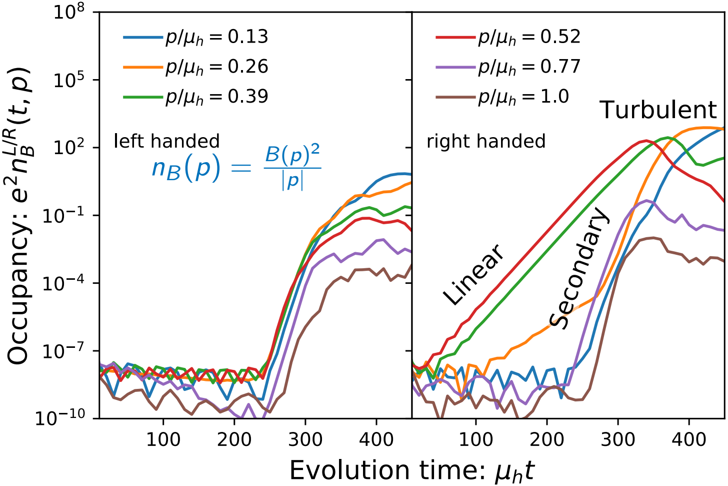
Linear growth of R-H modes starts immediately!



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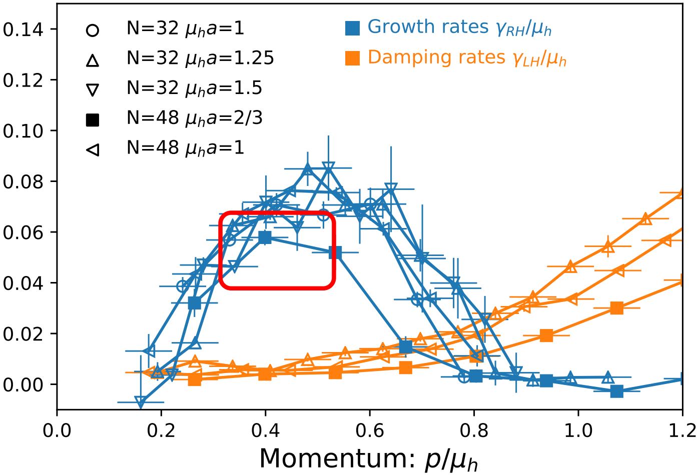
Intermediate R-H modes grows at the fastest rate!



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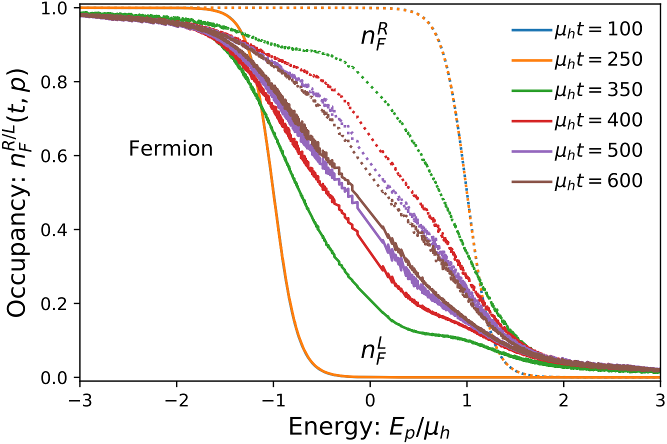
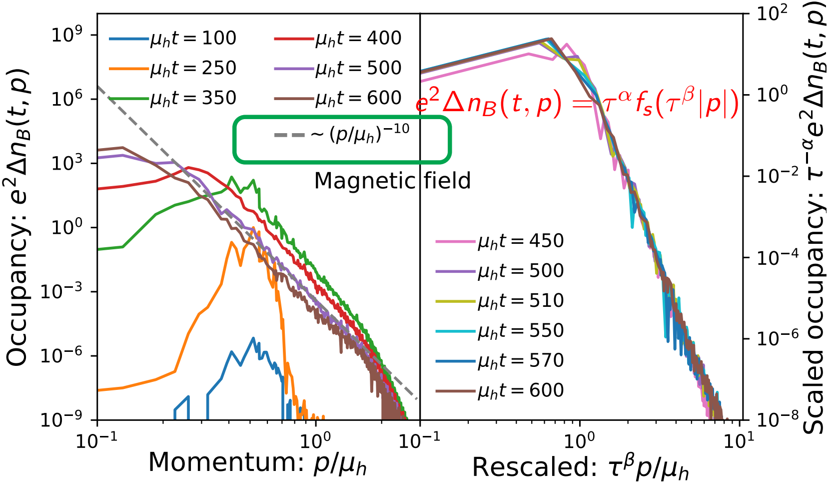
Fermions heated up significantly during growth of secondary instabilities.



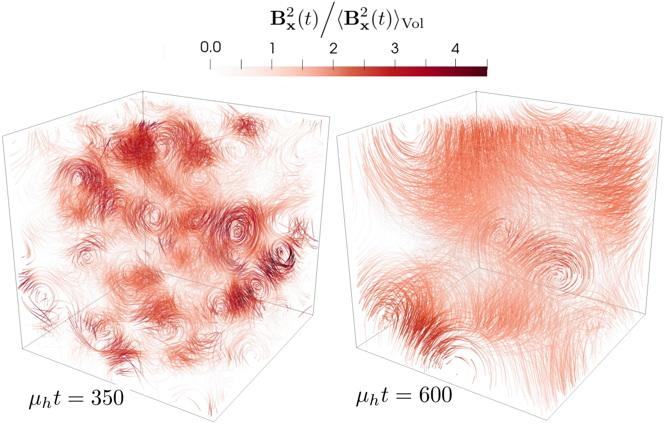
Self-similar cascade at late times

Scaling exponents $\alpha = 1.1(5)$, $\beta = 0.4(1)$, $\tau = \mu_h.t - 375$, Helicity consv.



Formation of large-scale magnetic fields

Inverse cascade from $l = \mu_h^{-1}$ to larger scales $l = \tau^{\beta}\mu_h^{-1}$.



Our results in the context of recent studies

- Within chiral kinetic theory the growth of unstable modes $0 \leq k \leq \alpha\mu_5/\pi$ are characterized by the exponent
  $\gamma = \frac{4\alpha\mu_5}{\pi^2 m_D^2} k^2 (1 - \frac{\pi k}{\alpha\mu_5})$ [Y. Akamatsu & N. Yamamoto 13].

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- Our study suggests that under strong back-coupling the onset of instabilities and turbulence is quicker.
- An earlier study observed that backreaction of fermions on the electromagnetic field prevents the system from acquiring chirality imbalance[P. V. Buividovich & M. V. Ulybyshev 16] $\rightarrow$ we observe an efficient transfer of chirality. This may be due to better control over chiral properties of fermions on a finite lattice.

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- For finite mass, time scale for chirality flipping may exceed the one needed to create helical magnetic fields. Cannot sustain the imbalance without an energy source, for example, via a turbulent mechanism
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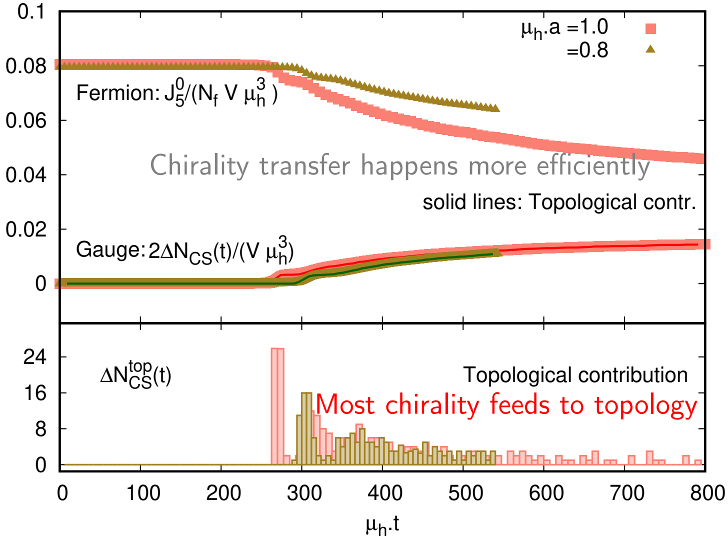
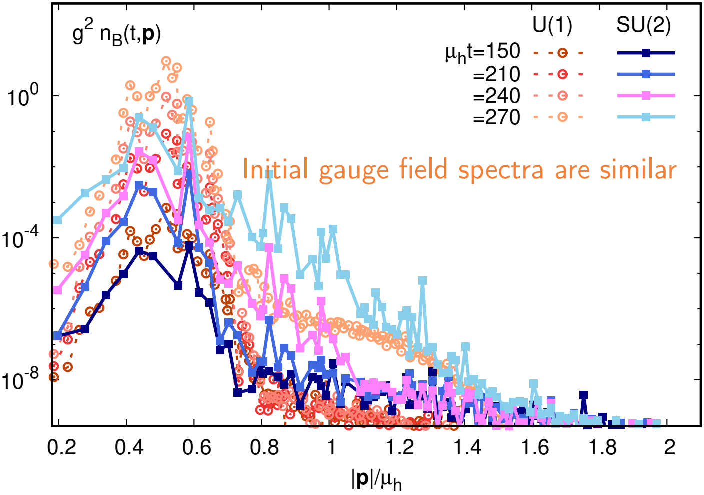
- Our study shows this could be a possibility but we need to perform runs with heavier quark mass.
- For Abelian plasma, collisions of fermions do not affect the instability when $\mu_5 \gg T$.
- In non-Abelian plasma the time scale of the onset of instabilities is expected to be comparatively faster [Y. Akamatsu & N. Yamamoto 13] $\rightarrow$ Need to perform a similar long-time evolution of chirally imbalance non-Abelian plasma with strong back-coupling.

What happens in a non-Abelian SU(2) plasma?



[Schlichting, SS, Phy. Rev. Lett. 131 (2023), 102303]

SU(2) vs U(1) plasma



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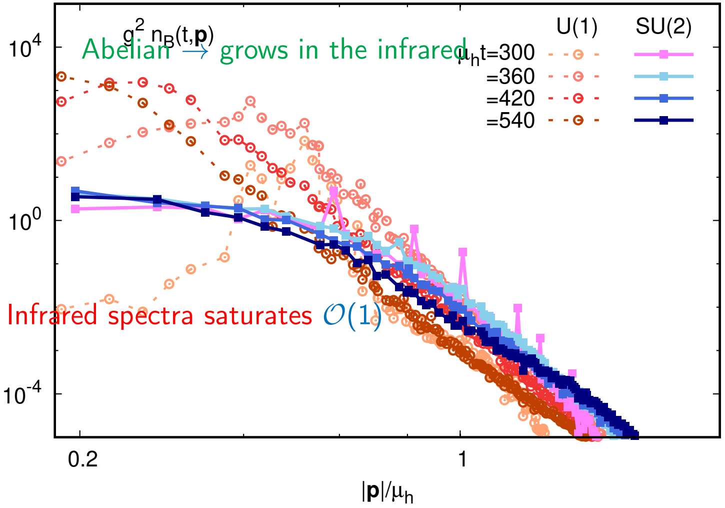

[Schlichting, SS, arxiv:2211.11365]

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- Hence the chirality washout is expected to be on timescales $\tau_{\rm sph} \sim 5/T$ which is not larger than the lifetime of the fireball.
- Erasure of the initial pockets of chirality imbalance will be compensated by creation of tiny regions of net-chirality due to thermal sphaleron transitions $\rightarrow$ could be done if one can cleanly disentagle the modes $|\vec{p}| \sim g^2 T$ from the modes at scale $\sim T$ [D. Bodeker, 98].

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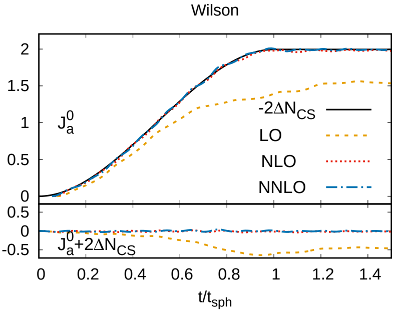
- Unlike the Abelian case, the non-Abelian modes do not show an infrared enhancement. Most of the initial chirality absorbed by the topology of the gauge fields.
- Including all orders quantum fluctuations in gauge sector is extremely challenging!

Backup slide: Anomaly balance in real-time

- Non-trivial check: how well is the anomaly relation satisfied on a finite lattice

$$\partial_\mu j_a^\mu(x) = 2m\eta_a(x) - 2\frac{g^2}{16\pi^2} trF_{\mu\nu}\tilde{F}^{\mu\nu}[W(x)] \rightarrow J_a^0(t) = -2\Delta N_{CS}(t)$$



- Improved version of Wilson fermions mimics very well the properties of chiral fermions! [Mace, Mueller, Schlichting, SS, 16]