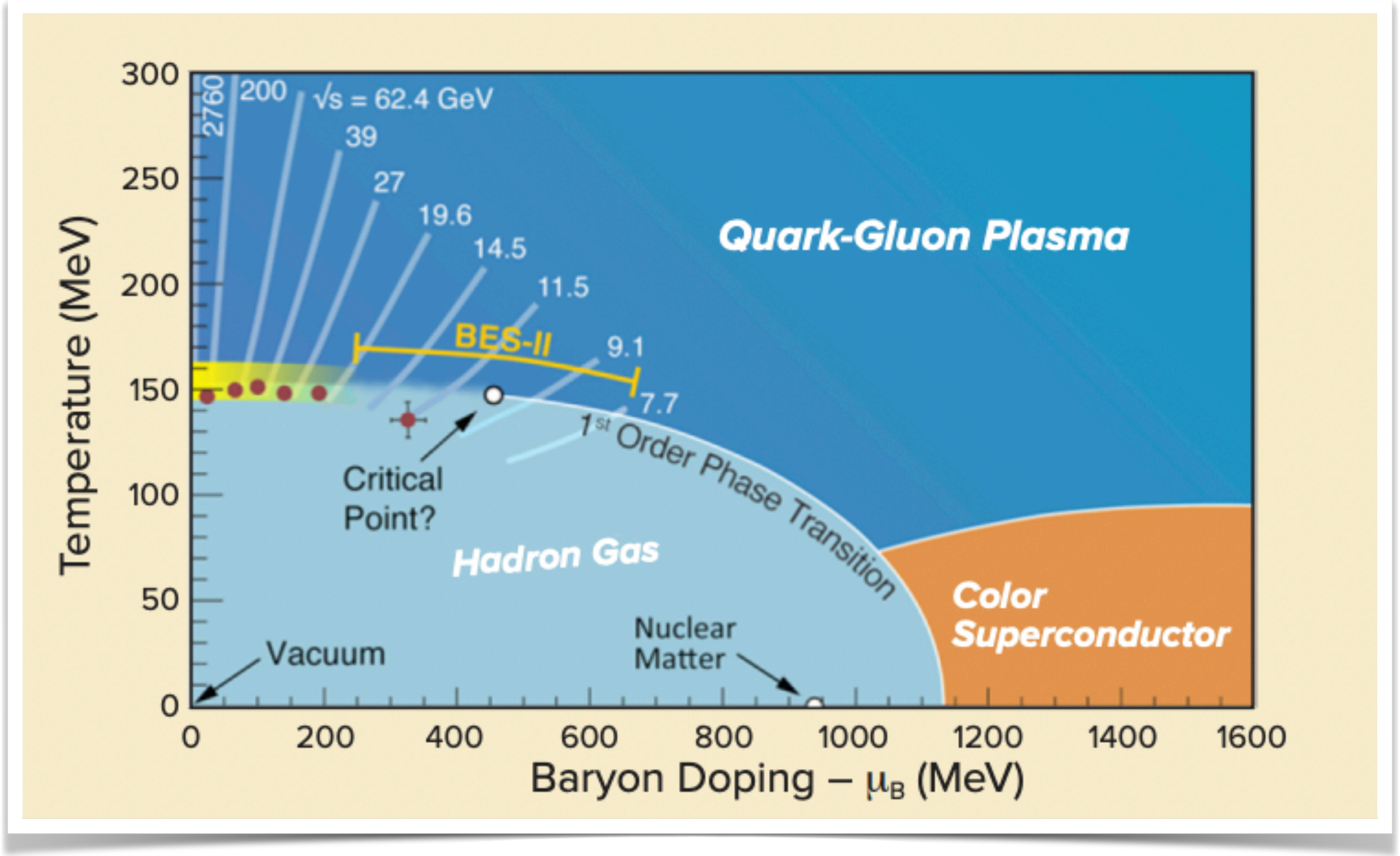
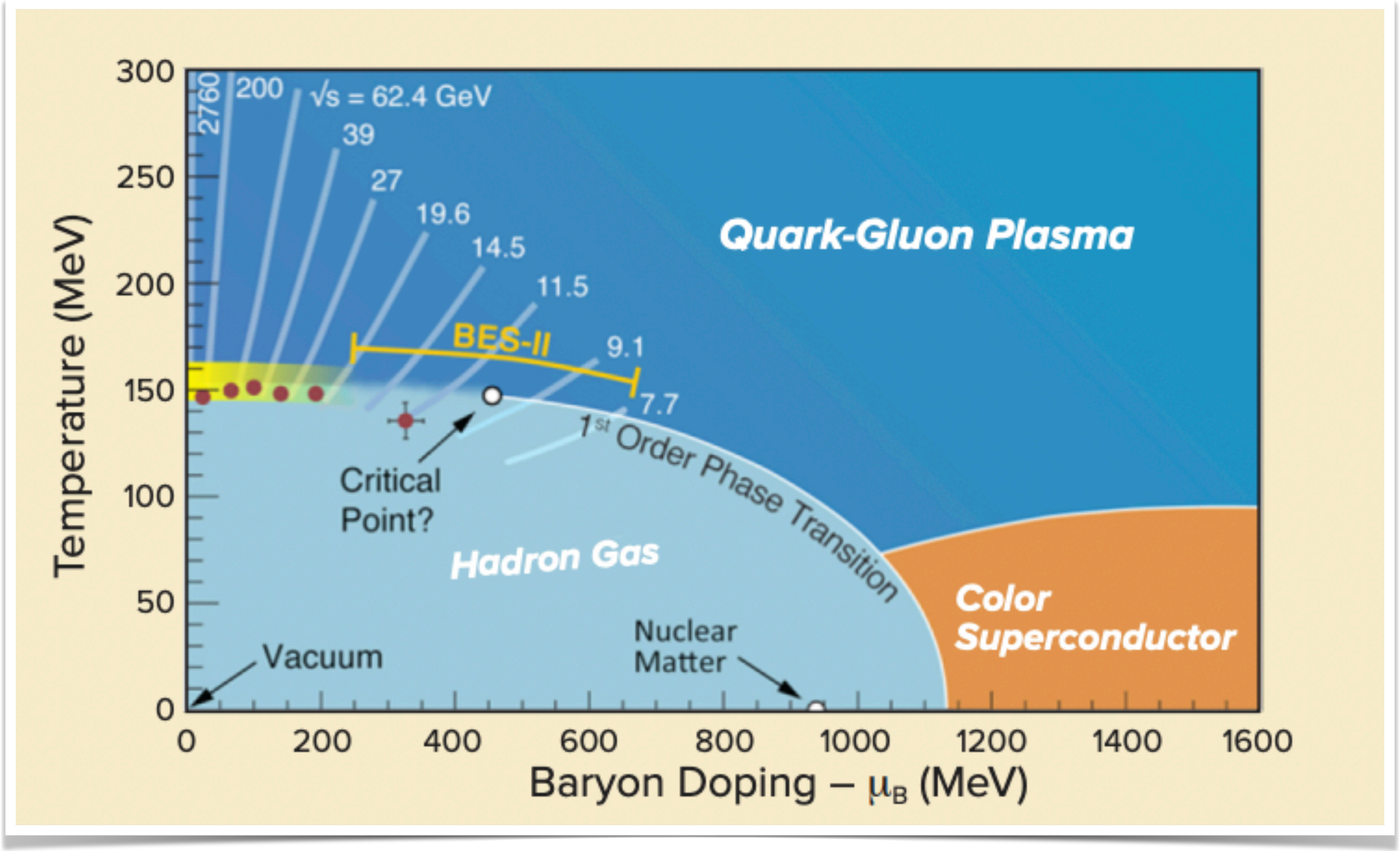


Studying QCD phase diagram via fluctuations

ATHIC, 2025
Gopalpur, India

Maneesha Pradeep
University of Maryland at College Park

Is there a critical point on the QCD phase diagram?



- At $\mu_B = 0$: It is a cross-over. — Lattice QCD simulations
- At T= 0 : Models predict a first order phase transition — Not known from first principles

Current constraints on the location of the critical point from Lattice QCD

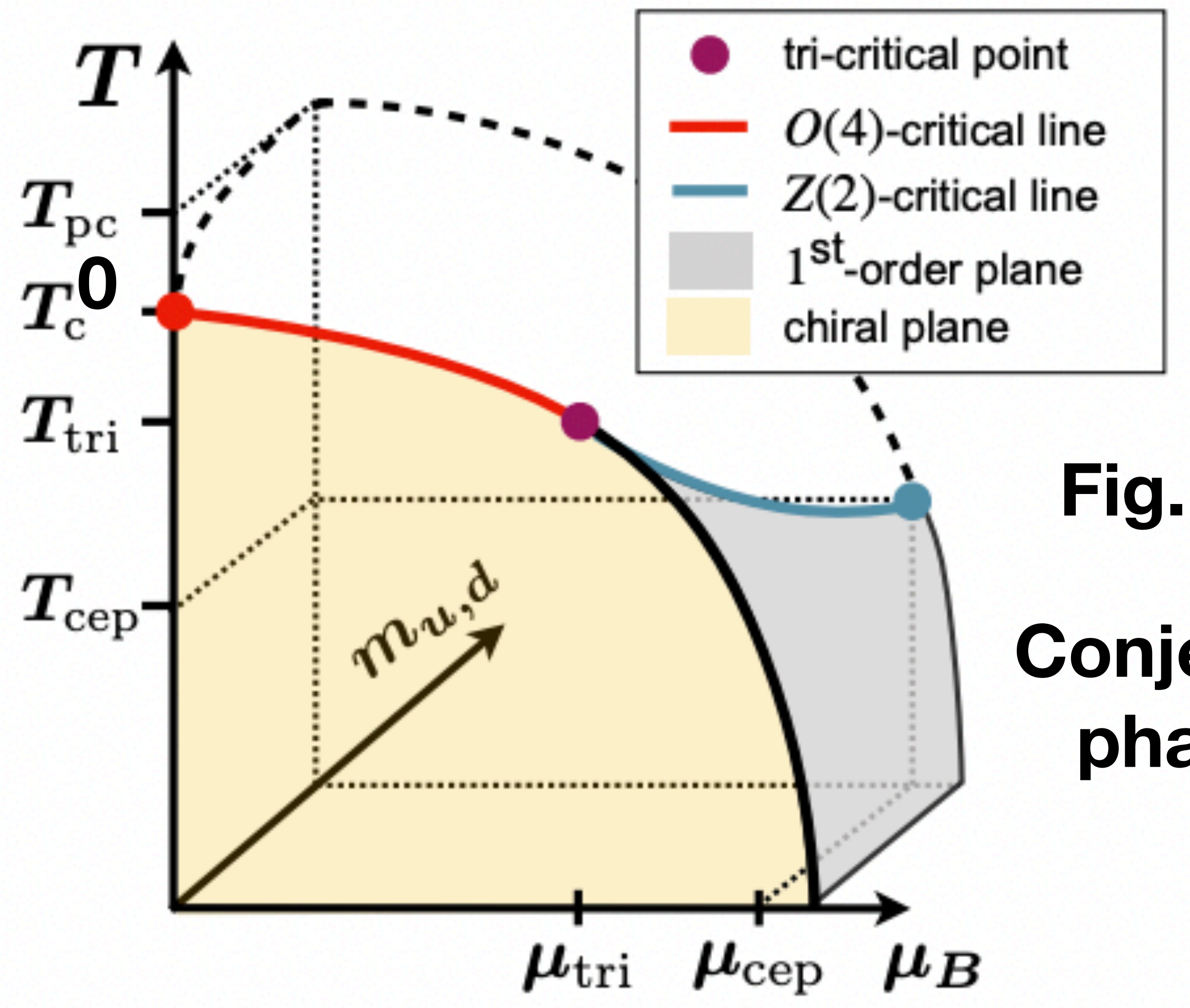


Fig. : Ding et al , 24

Conjectured chiral phase diagram

$T_c^0 > T_{\rm tri} > T_c$

Hatta, Ikeda, 03 $T_c^0 = 132^{+3}_{-6}\,\mathrm{MeV}$ **HotQCD, Ding et al, 19**

Pseudo-critical curve at physical quark mass

$$T_{\rm pc}(\mu_B) = T_{\rm pc}(0)\left[1 + \kappa_2\left(\frac{\mu_B}{T_{\rm pc}}\right)^2\right]$$

Bazavov et al, 19, Ding et al,24

$$T_{\rm pc}(0) = 156 \pm 1.5\,\mathrm{MeV}\,,\ \kappa_2 = 0.012(4)$$

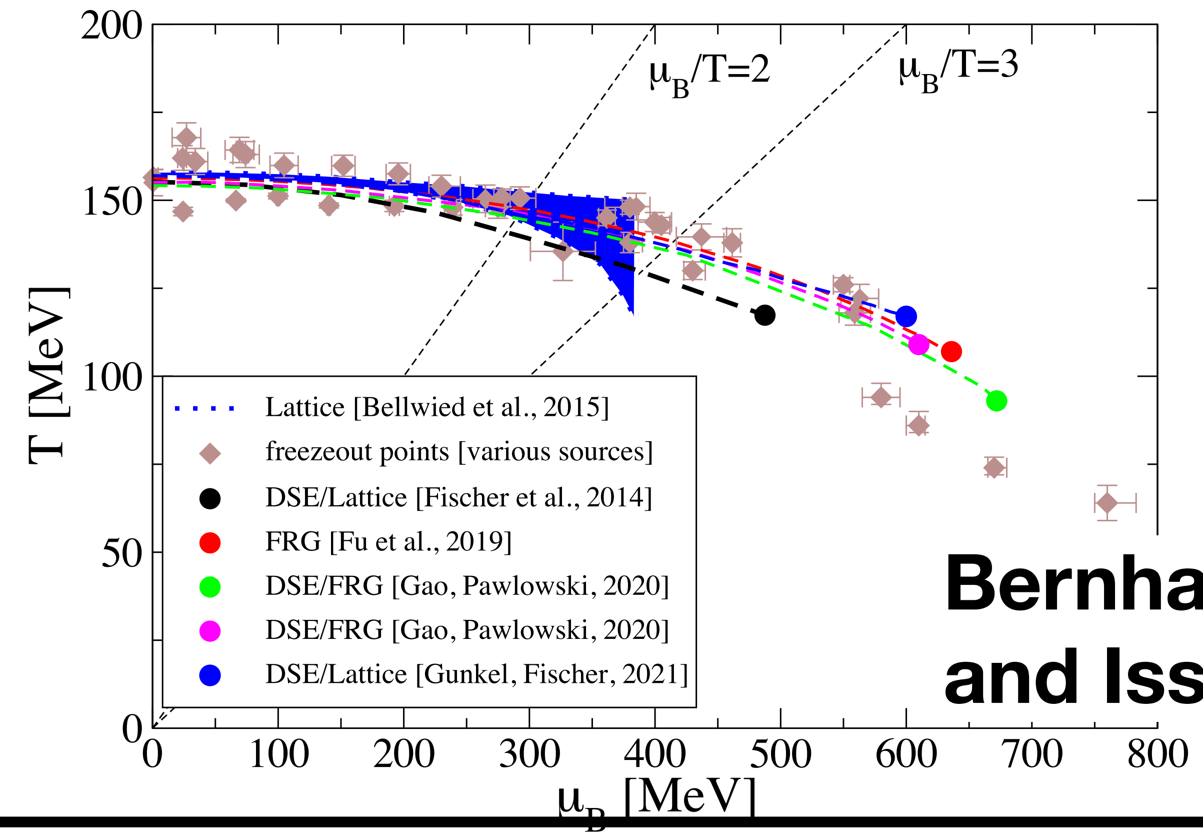
Borsanyi et al,20

$$T_{\rm pc}(0) = 158 \pm 0.6\,\mathrm{MeV}\,,\ \kappa_2 = 0.0153(18)$$

Critical point disfavored for $\mu_B/T < 3$

Jishnu Goswami's talk today

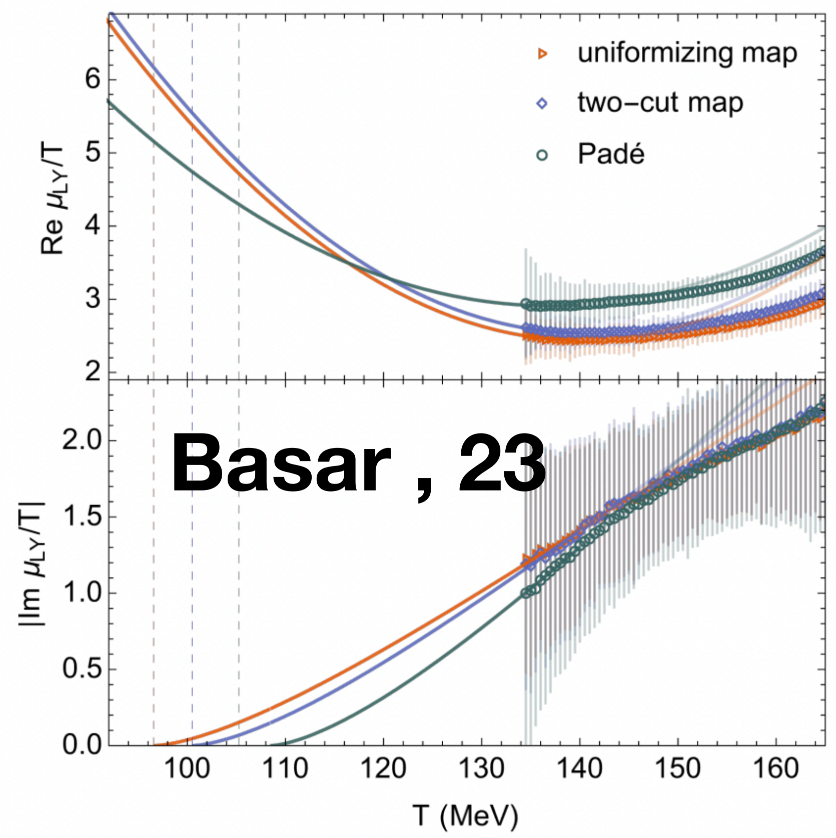
Recent theory guidances for the location of the critical point



FRG/ DSE

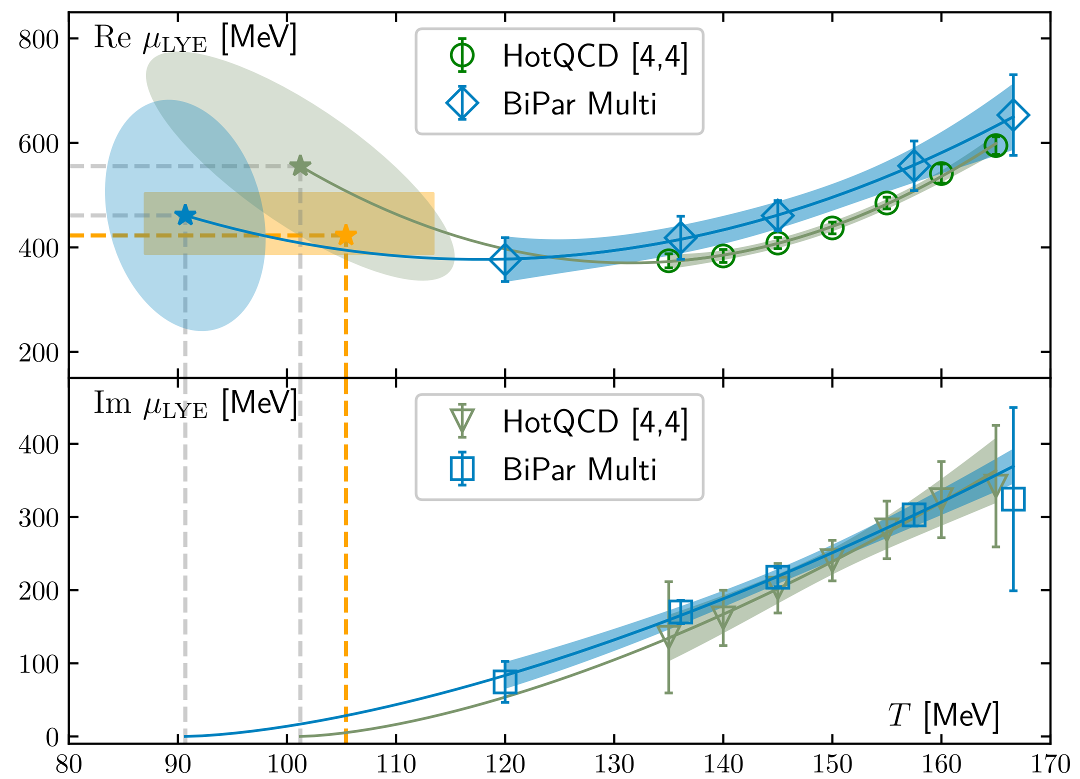
Bernhardt, Fischer and Isserstedt, 23

$$(\mu_{BC}, T_c) = (495 - 654, 108 - 119)\ \text{MeV}$$



Basar , 23

$$(\mu_{BC}, T_c) \approx (580, 100)\ \text{MeV}$$



Clarke et al., 24

$$(\mu_{BC}, T_c) = (422^{+80}_{-35}, 105^{+8}_{-18})\ \text{MeV}$$

Extrapolations of Lee-Yang edge singularities to real axis

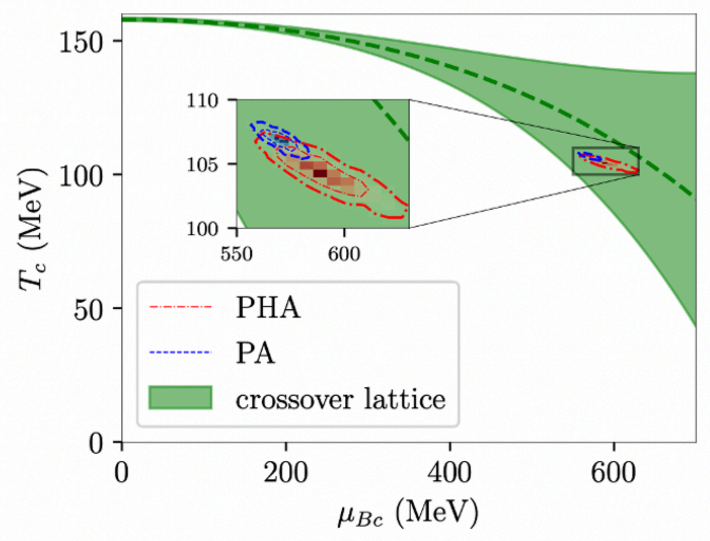
Bayesian Holography + Lattice input at $\mu = 0$

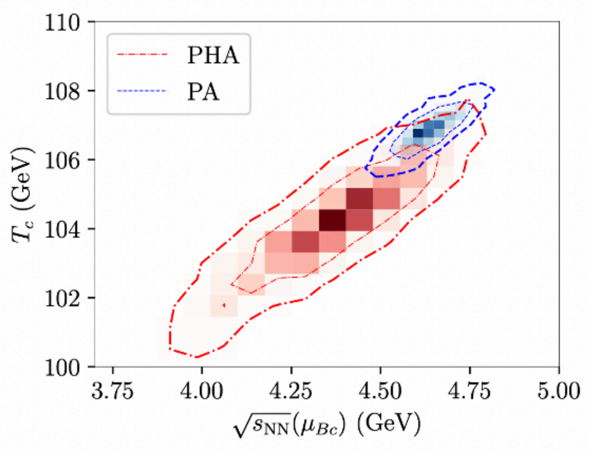
Hippert et al, e-Print: 2309.00579 [nucl-th]

Predict CEP (95% confidence level):

$$T_c = 101 - 108\ \text{MeV} \quad \mu_c = 560 - 625\ \text{MeV}$$

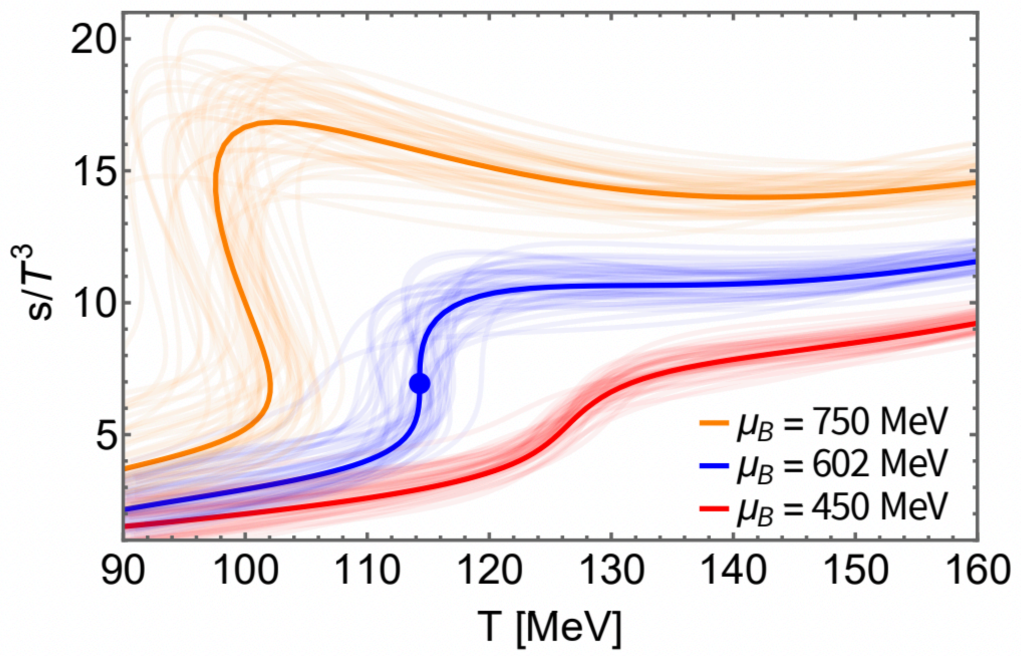
$$\sqrt{s} = 4.0 \ - \ 4.8\ \text{GeV}$$





Thermodynamic analysis of a lattice QCD extrapolated EoS

Shah et al., 24



$$(\mu_{BC}, T_c) = (602.1 \pm 62.1\ ,\ 114.3 \pm 6.9)$$

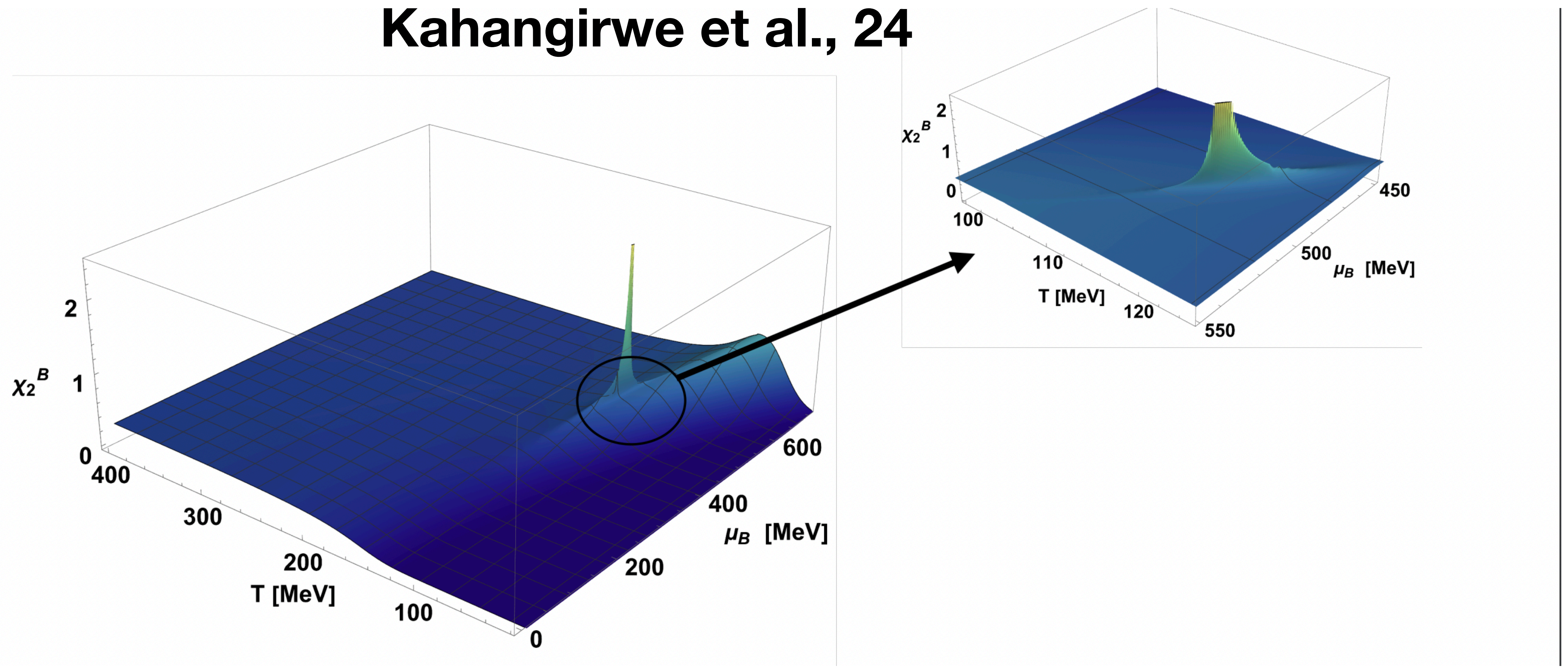
Thermodynamic fluctuations as signatures of critical point

Diverges at CP

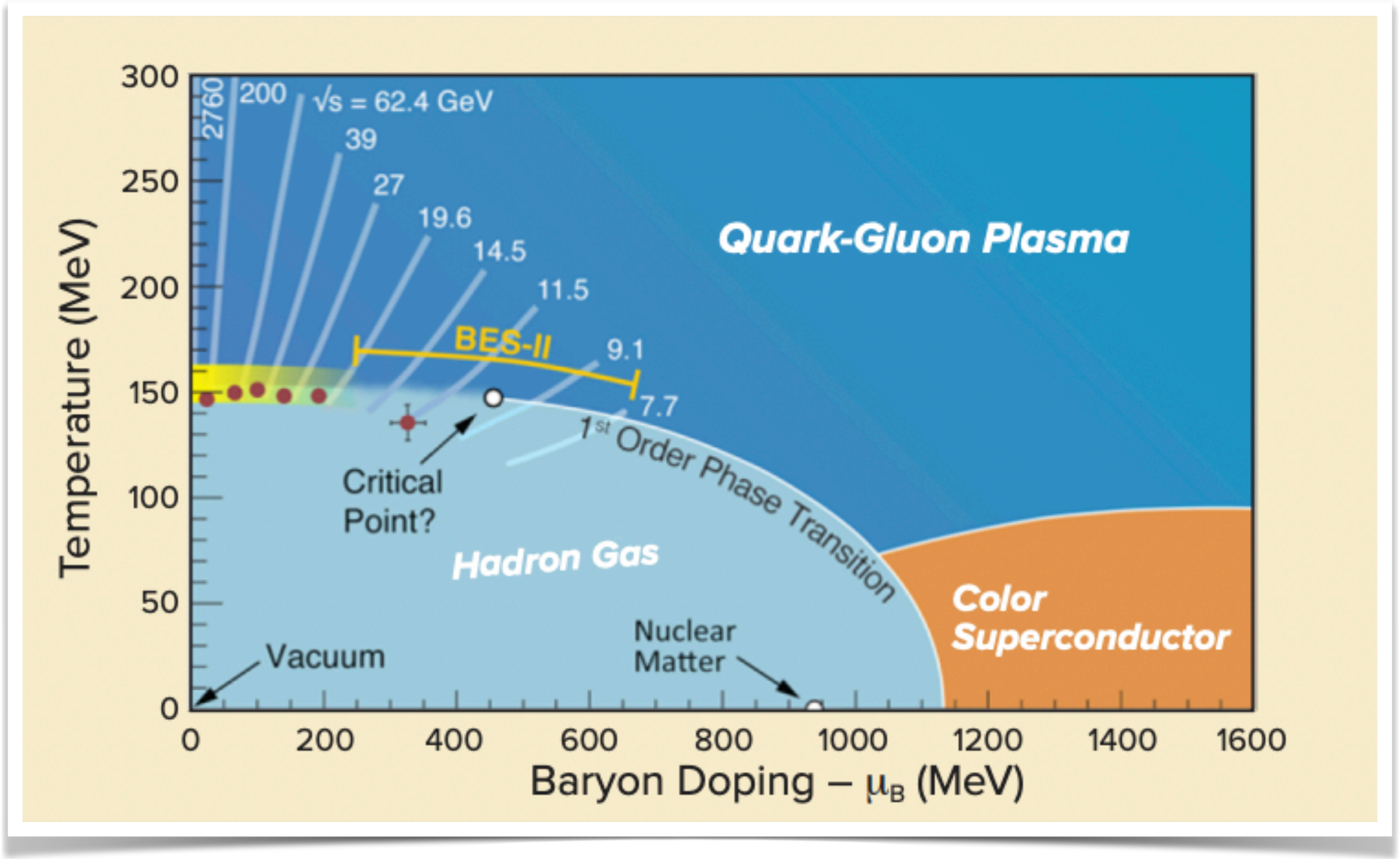
$$C_k \equiv \langle \delta N_B^k \rangle_c \overset{\text{in eq.}}{=} V T^{k-1} \frac{\partial^k P}{\partial \mu^k}$$

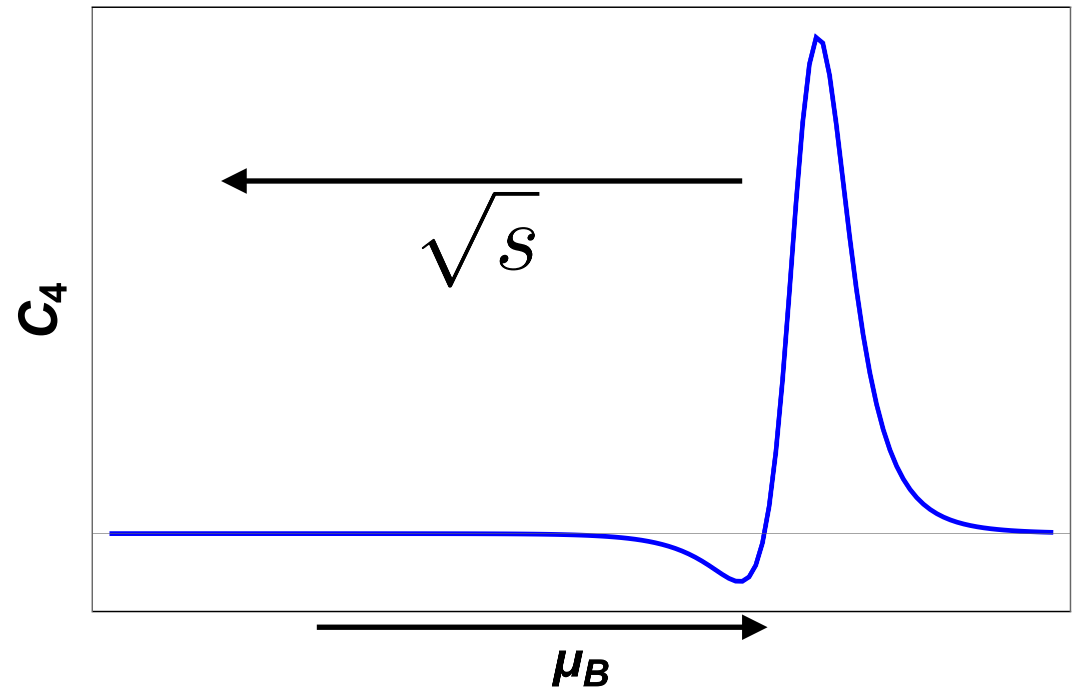
Fluctuations are enhanced near CP

Kahangirwe et al., 24



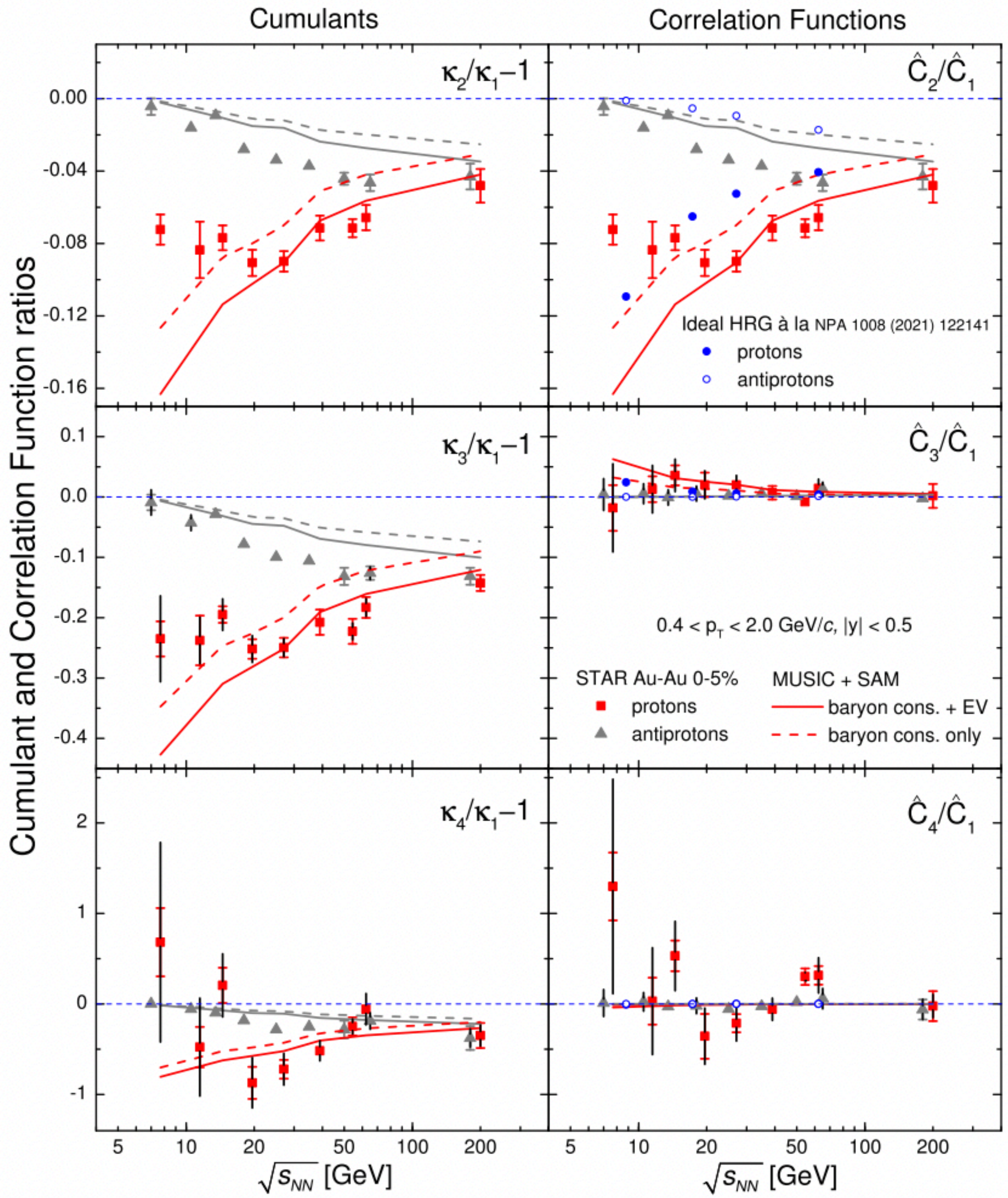
Susceptibilities that diverge at CP





Rajagopal, Shuryak, Stephanov, 98, 99, Stephanov, 09

Deviation of cumulants of proton multiplicities relative to hydrodynamic (non-critical) baseline



Cumulants of proton multiplicities are expected to be highly sensitive to the critical point. **Stephanov, 09**

A *clear excess* of scaled proton-number variance from non-critical baseline reported for $\sqrt{s_{NN}} \leq 10\,\mathrm{GeV}$

Vovchenko, Koch, Shen, 22

Ashish Pandav's talk today

Quantitative analysis requires

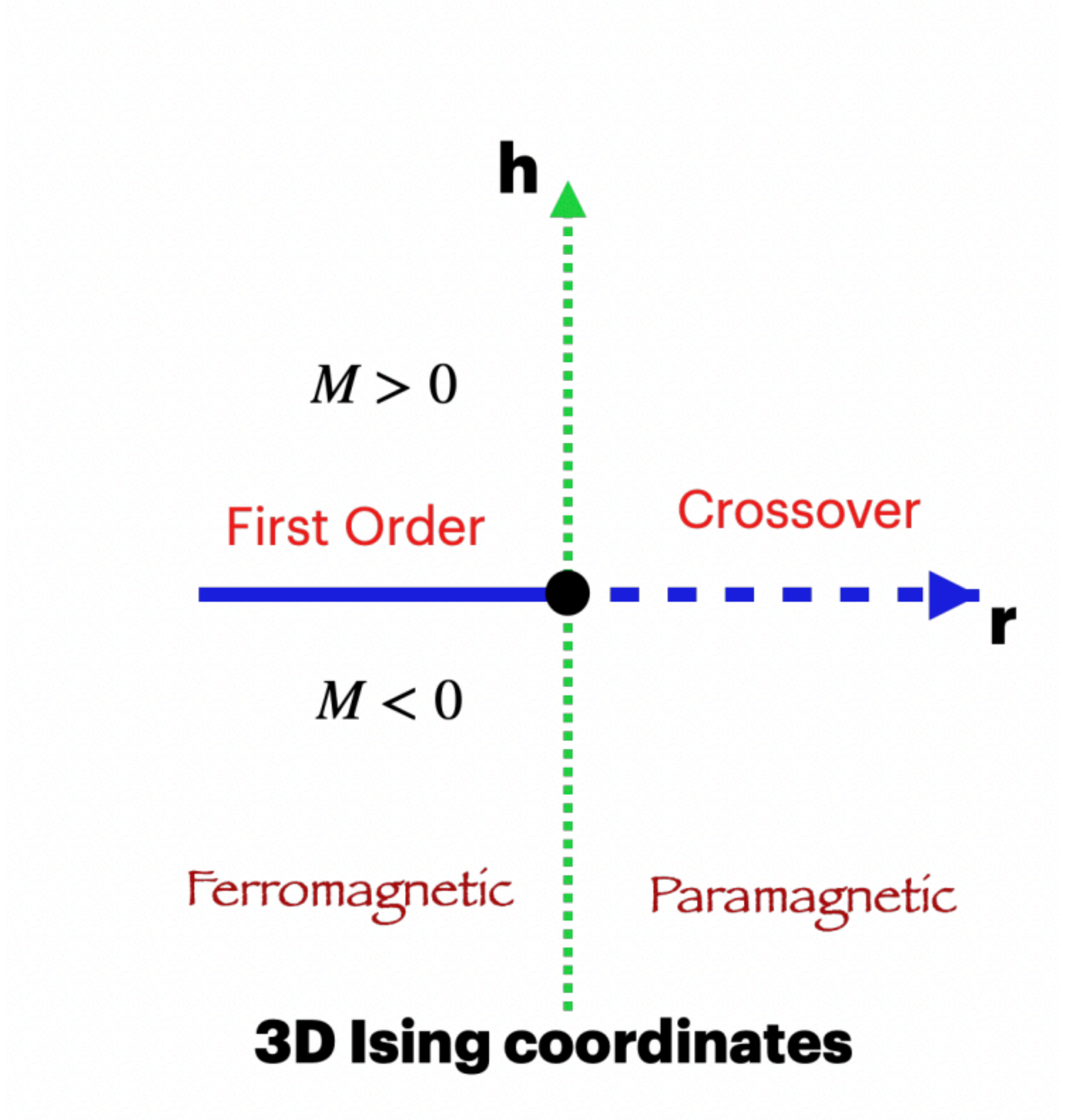
- Knowledge about QCD EoS
- Procedure to relate EoS to proton multiplicity cumulants
- Understanding of dynamics of hydrodynamic fluctuations near CP

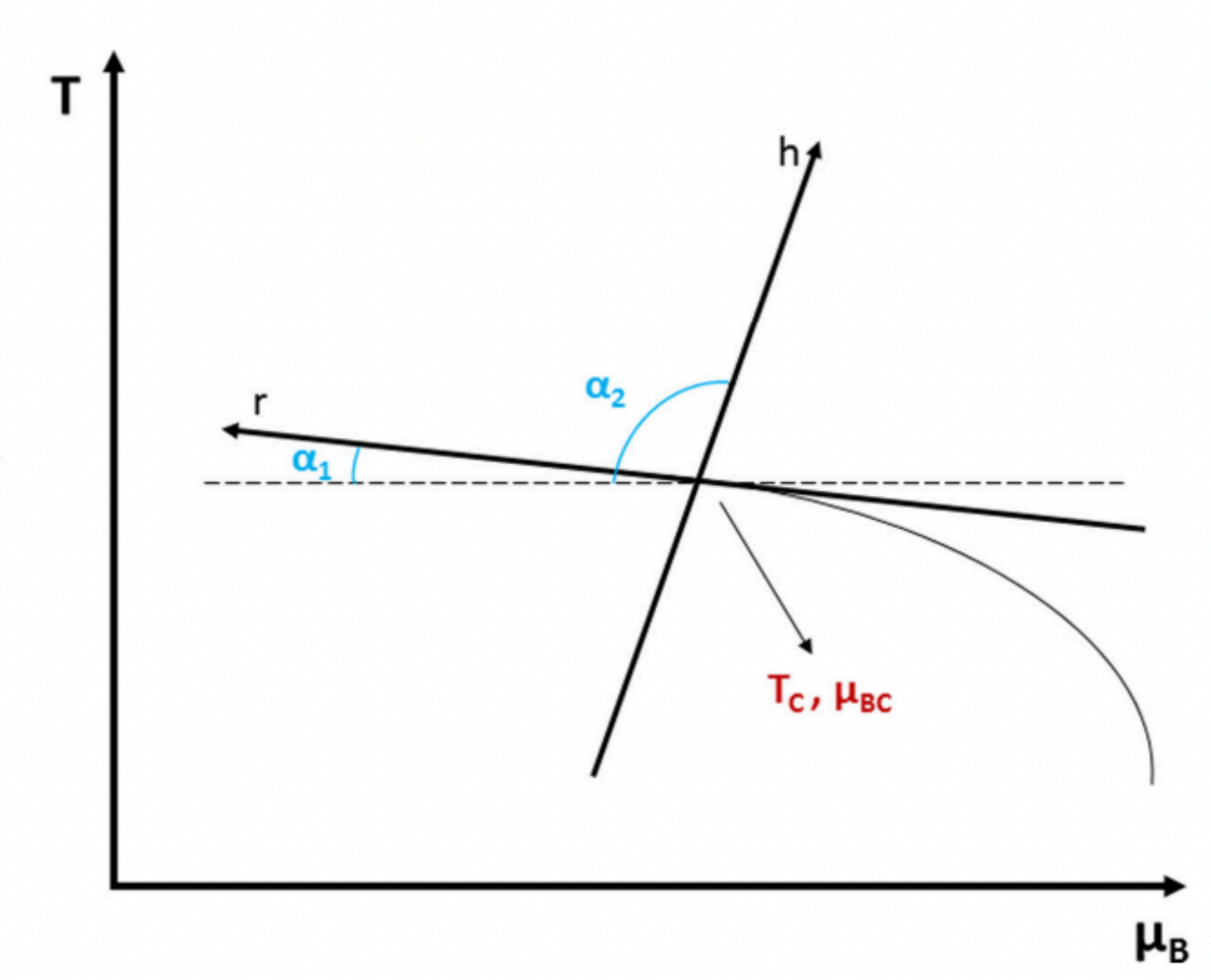
Equation of States with a QCD critical point

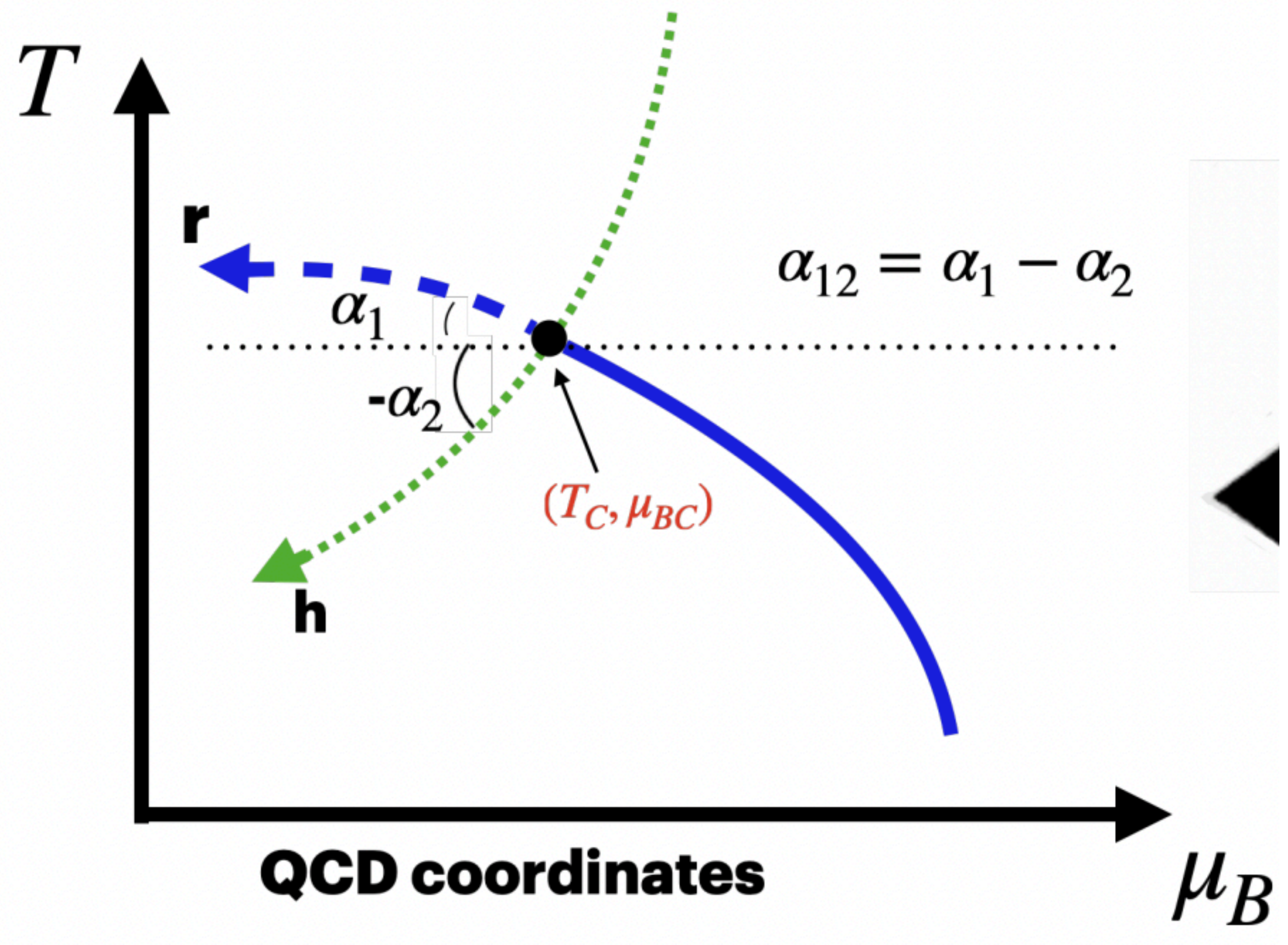
- Must agree with the Taylor Expanded EoS from lattice
- Compatible with other limits : PQCD, HRG
- Critical point in the 3D Ising universality class

Examples of recently developed EoSs that have a CP in the Ising universality class but differ in their implementation: Parotto et al, 19, Karthein et al.,21, Grefa et al., 21,Kapusta&Welle,22, Kahangirwe et al.,24

QCD EoS near the Critical Point







Parotto et al., 18, Karthein et al., 21

Kahangirwe et al., 24

Summation scheme by WB collaboration

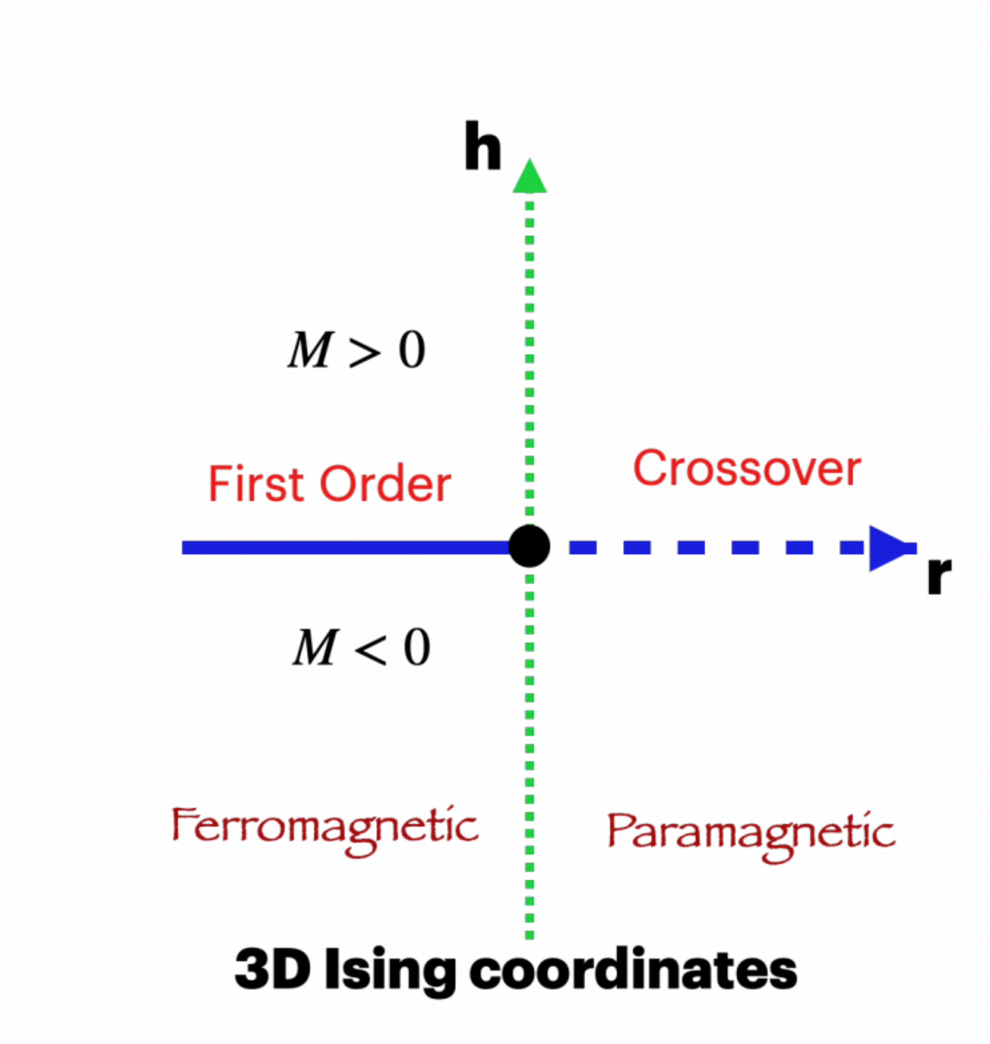
Borsanyi et al,21

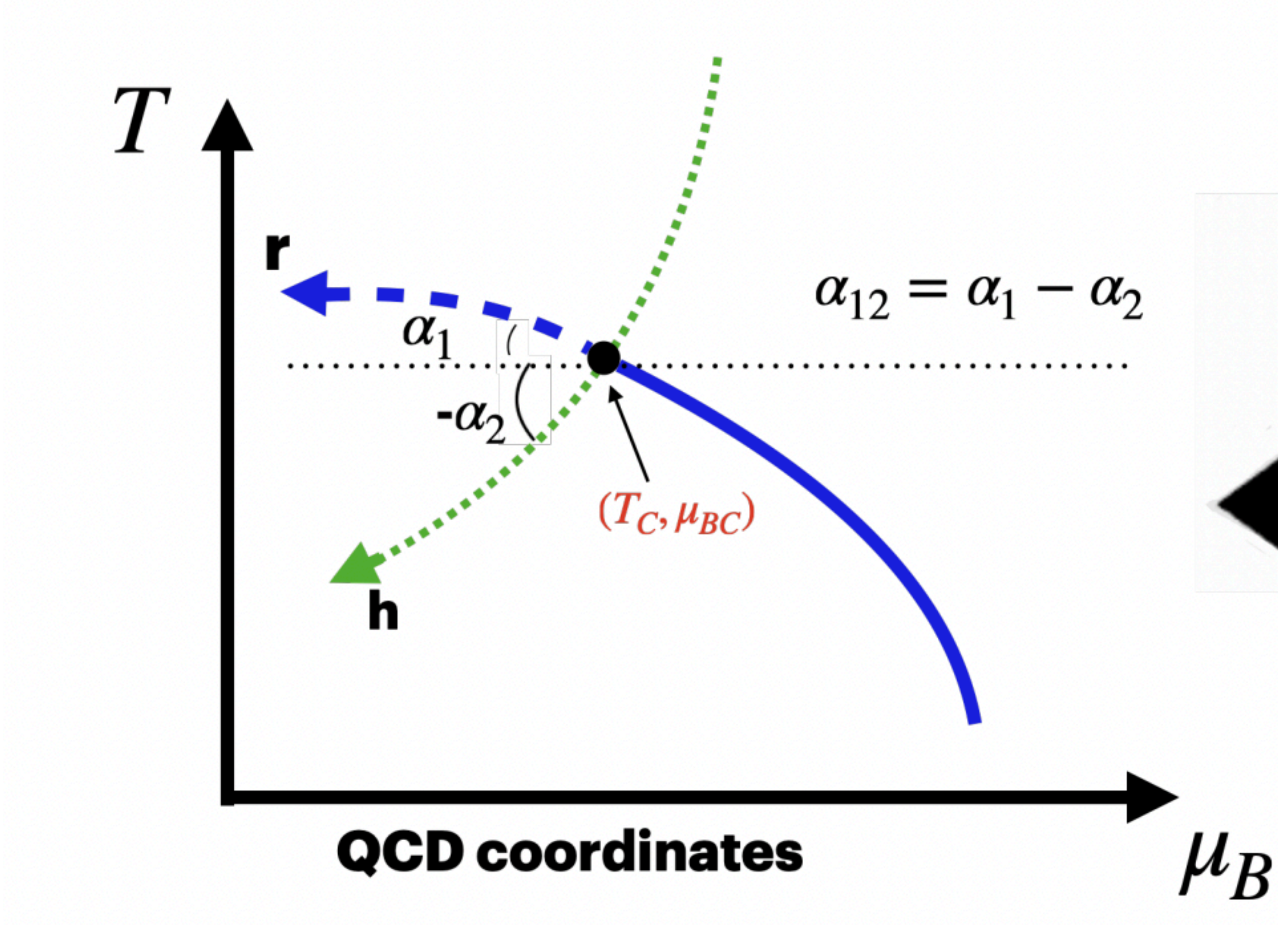
Non-universal map from QCD to Ising variables

$$P_{\mathrm{QCD}}(\mu, T) = P_{\mathrm{BG}}(\mu, T) + A\, G(r(\mu, T), h(\mu, T))$$

A general class of candidate EoSs

$$P_{\mathrm{QCD}}(\mu, T) = P_{\mathrm{BG}}(\mu, T) + A\, G(r(\mu, T), h(\mu, T))$$





Kahangirwe et al., 24

Independent & non-universal parameters

$$\mu_c \,,\; \alpha_{12} \,,\; \rho \,,\; w$$

Weakly constrained in the chiral limit

MP, Stephanov, 19

Range of Validity improved

$$0 \leq \mu_B \leq 700\,\mathrm{MeV}\,,\; 25\,\mathrm{MeV} \leq T \leq 800\,\mathrm{MeV}$$

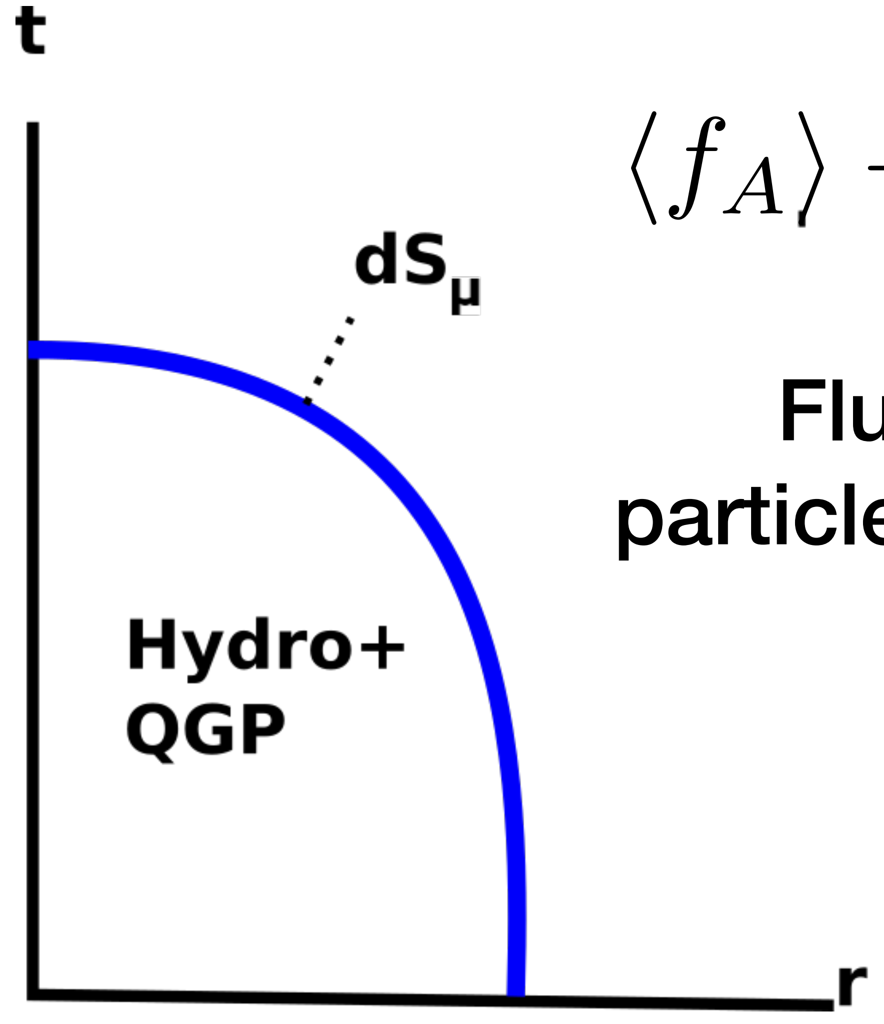
The new construction is causal and stable for a larger range of ρ and w

Generalization to Cooper-Frye (74) freeze-out based on maximum entropy principle

MP, Stephanov, 23

$$\text{Hydrodynamics} + \text{Correlation Functions} \to \text{HRG Ensemble with non-trivial phase space correlations}$$

- Hydrodynamics + Correlations contain information about critical EoS
- Infinitely many ensembles that would match with hydro+correlations
- Cooper-Frye (74) maximizes thermodynamic entropy of HRG
- We maximize the *nPI entropy* of the HRG ensemble with fluctuations subject to matching conditions



$\langle f_A \rangle + \delta f_A$

Fluctuating particle distribution

ME gives least biased ensemble of HRG that matches with hydro+correlations

MP, Stephanov, 23

- Natural generalization of factorial cumulants (IRCs, or irreducible relative cumulants)

Depends only on reference distribution

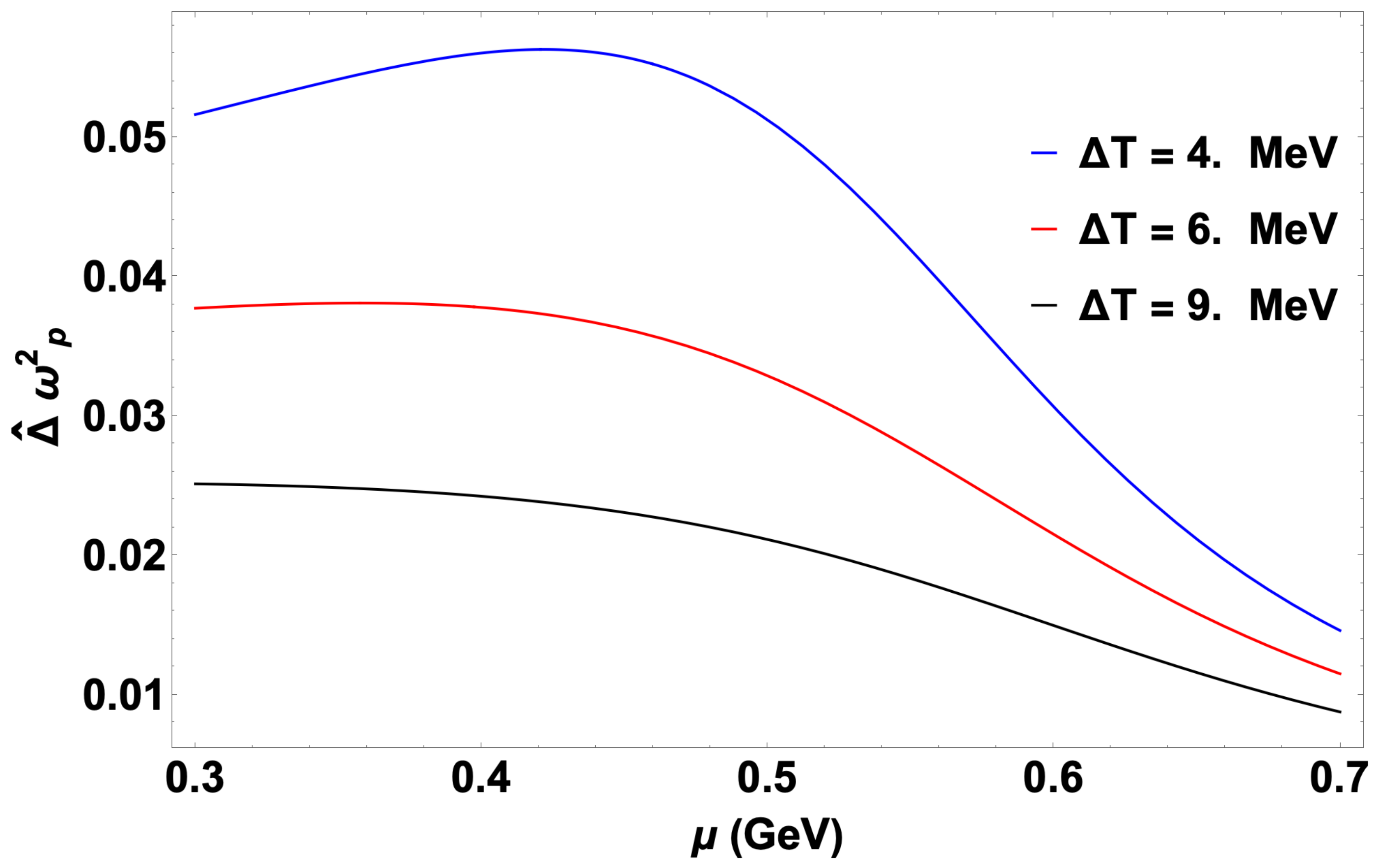
$$\hat{\Delta} G_{ABC\ldots} = \mathcal{F}(\bar{H}, \bar{G})\, \hat{\Delta} H_{ABC\ldots}$$

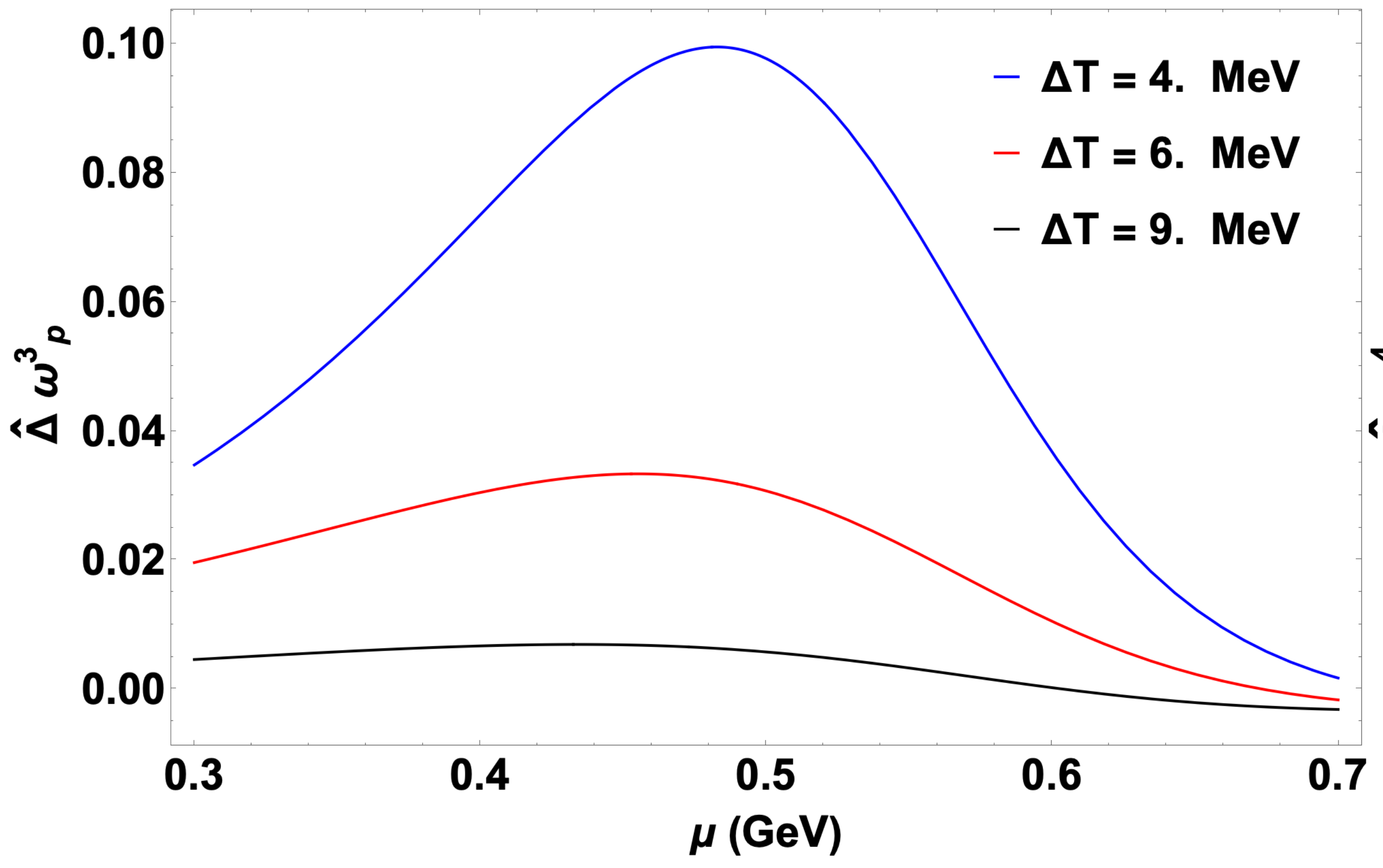
Phase space correlation functions of the gas variables (IRCs)

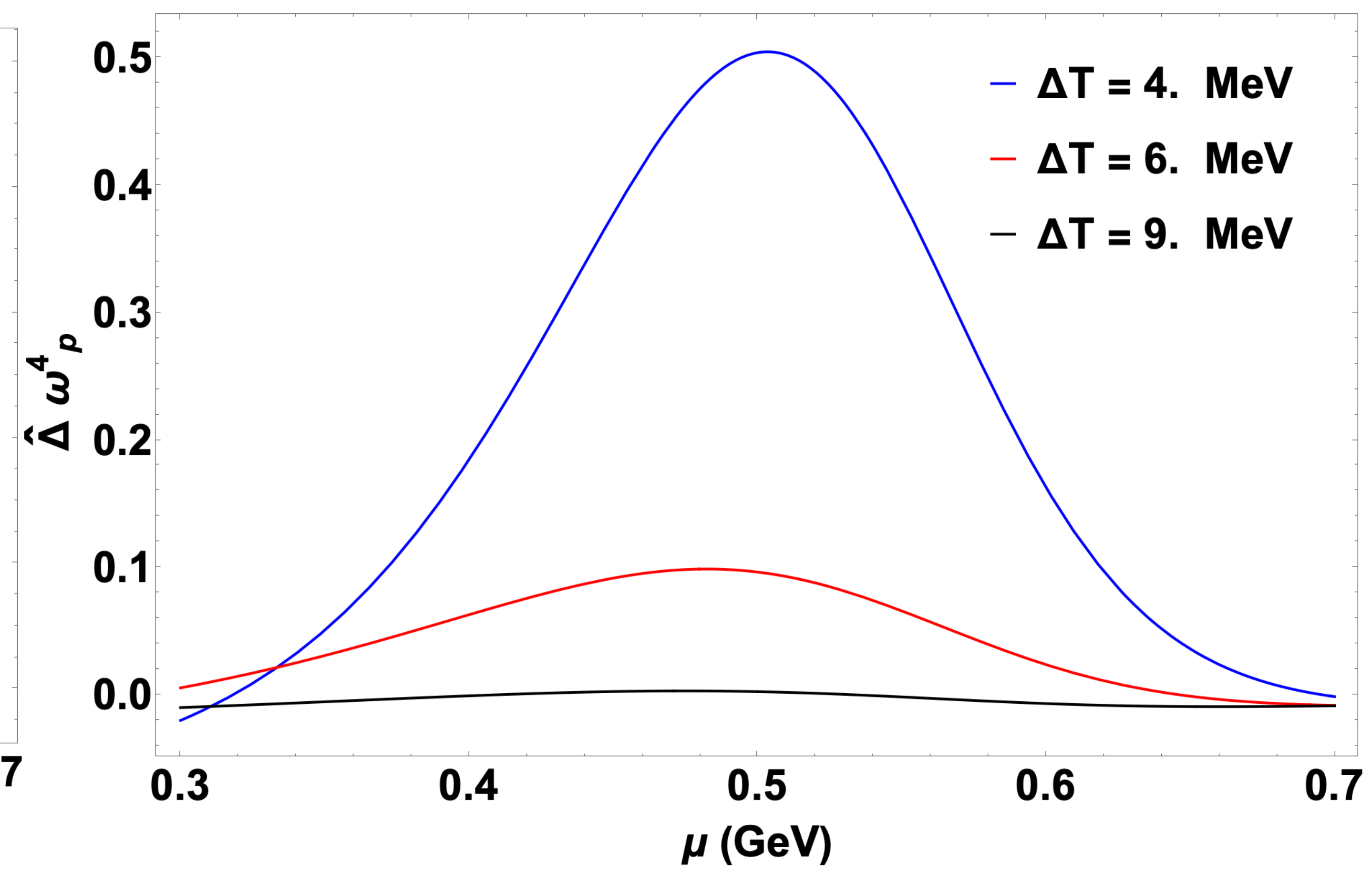
Correlation functions of the hydrodynamic variables (IRCs)

Equilibrium estimates for critical contribution to factorial cumulants of proton multiplicities

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)





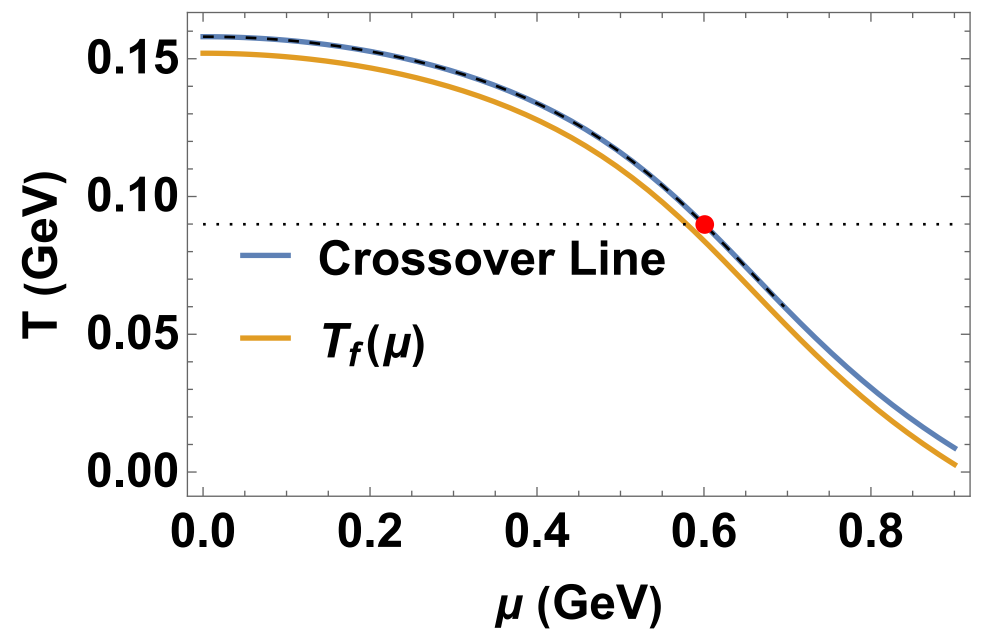


$$T_f(\mu) = T_{\rm co}(\mu) - \Delta T$$

$$\mu_c = 600\,\mathrm{MeV}\,,\ \alpha_2 = 0^\circ\,,\ \rho = 1\,,\ w = 20$$

Example **choice of Mapping Parameters**

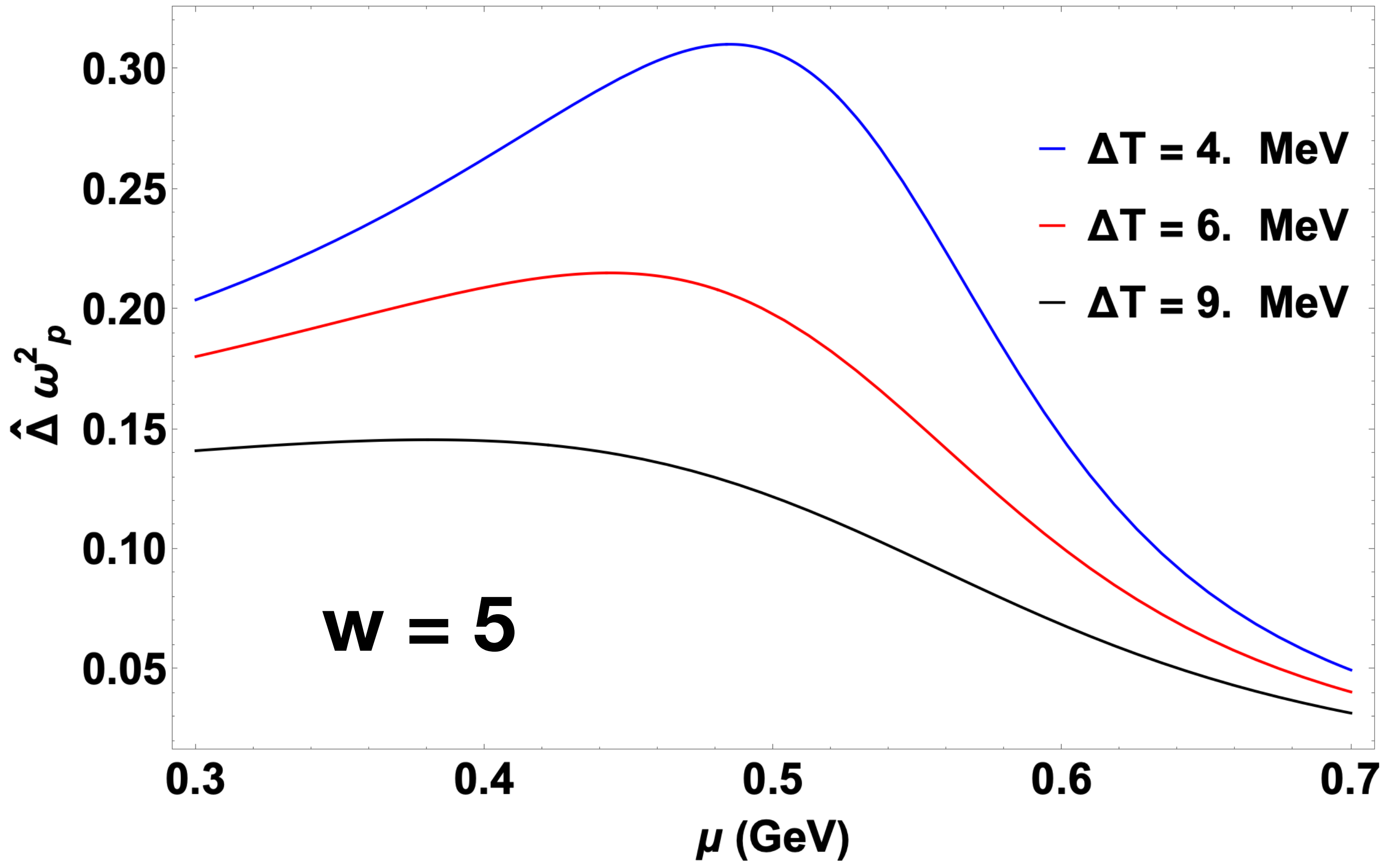
$$\hat{\Delta}\omega_p^k = \frac{\hat{\Delta}\,\langle \delta N_p^k \rangle}{\langle N_p \rangle}$$

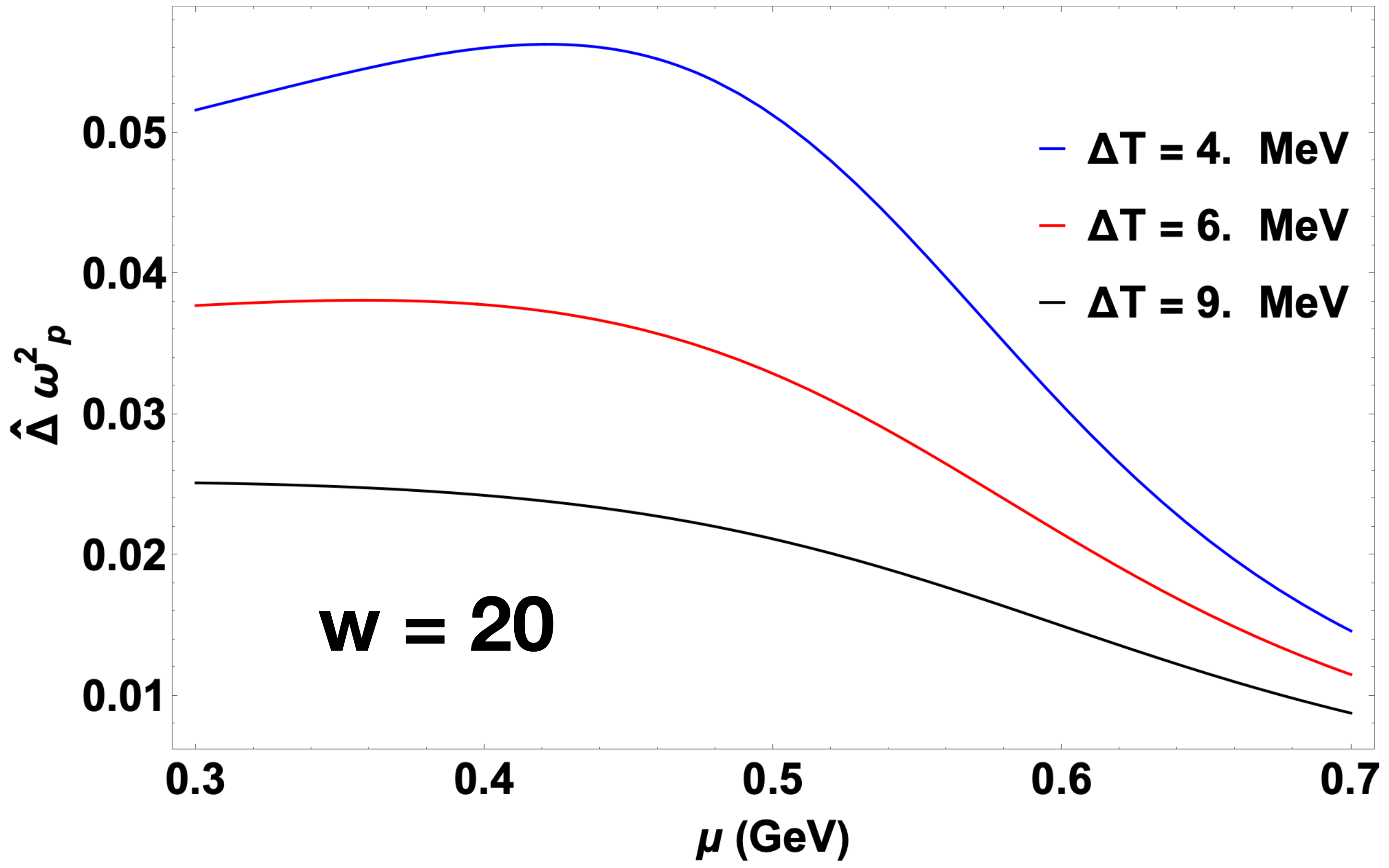


Freeze-out curve

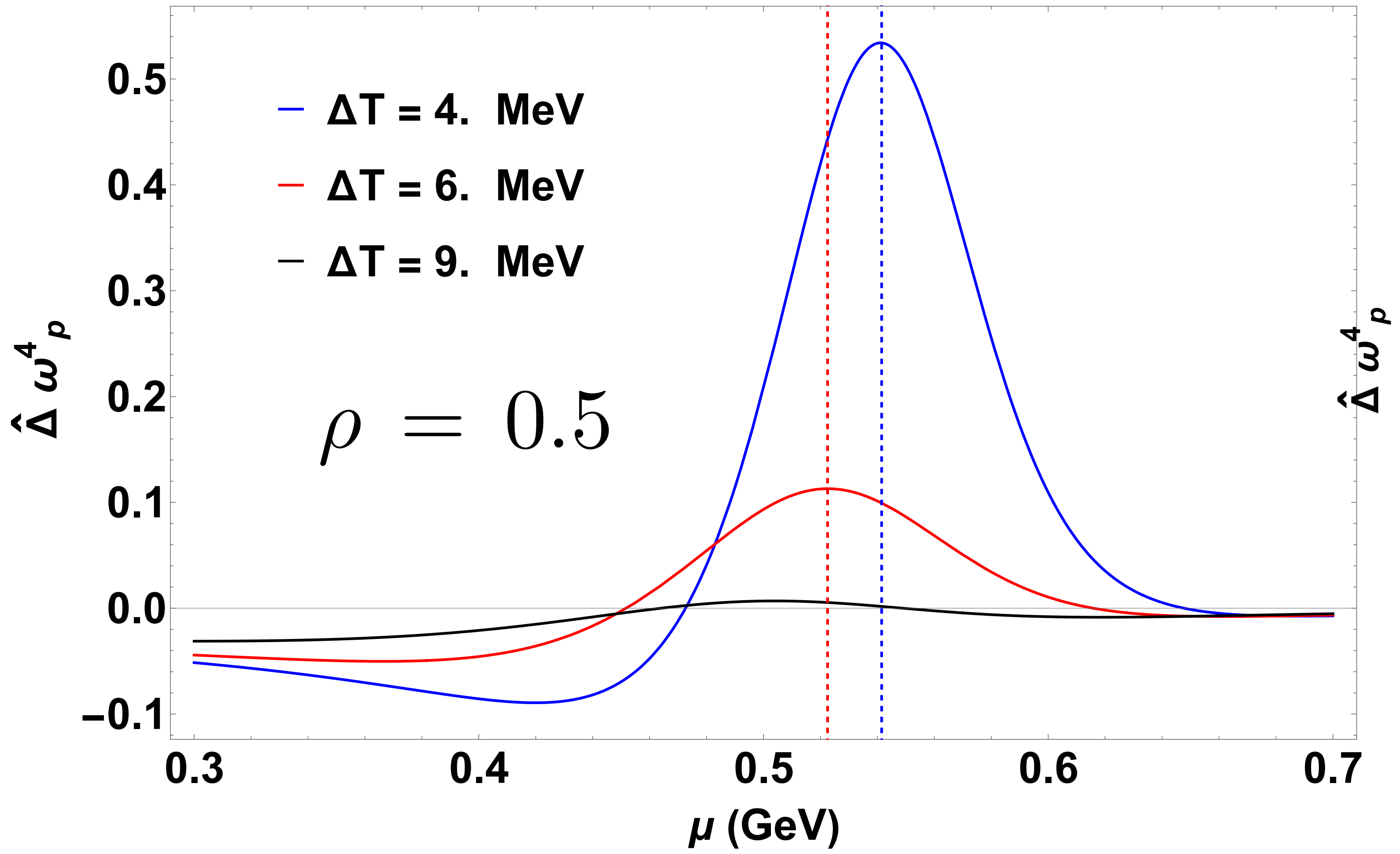
Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)

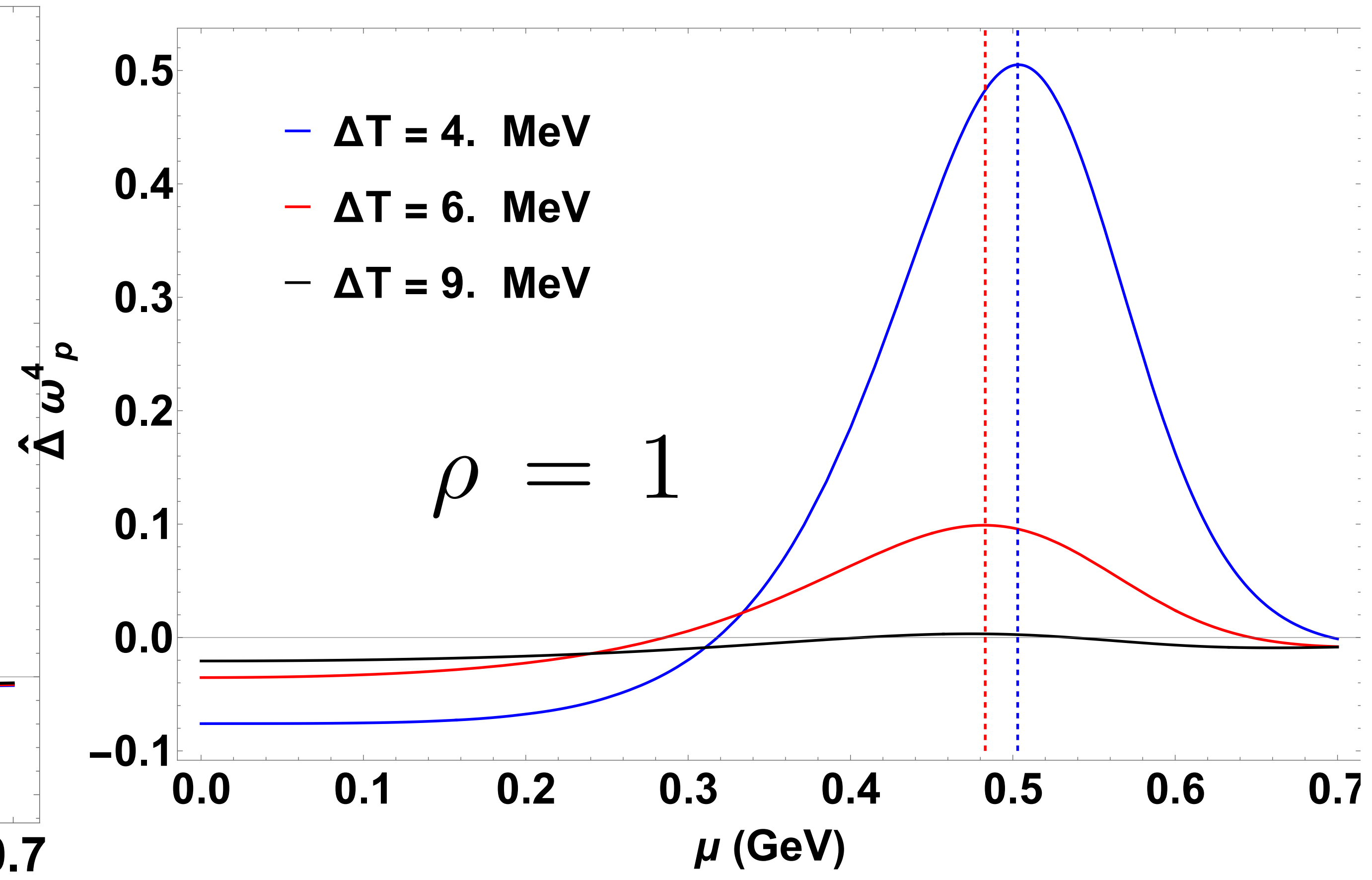
Role of scale factors

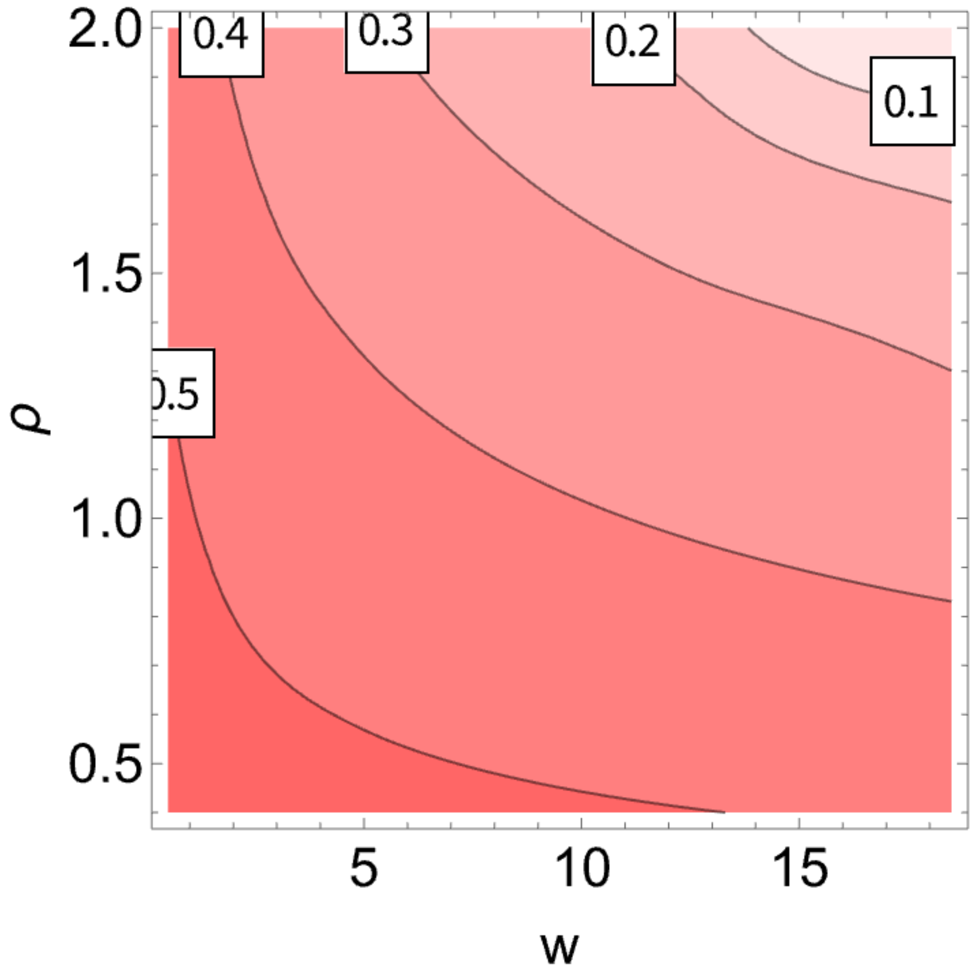


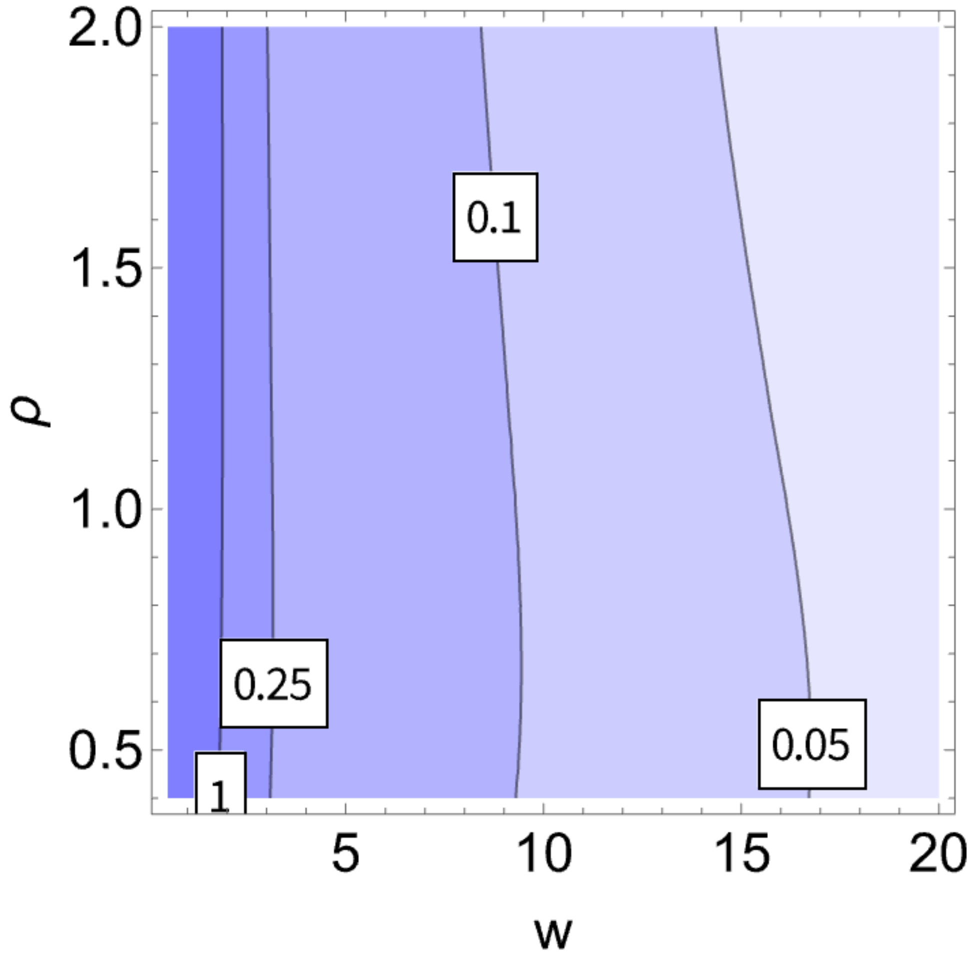


- Peak value of cumulants along the freeze-out curve is fixed by w
- Location of the maxima on the freeze-out curve fixed by ρw









Fluctuation dynamics

Hydrodynamic evolution of the fireball and collectivity

Stochastic Approach

- Stochastic differential equations for hydro fields
- Implementation using Metropolis algorithm developed and applied to various models, including Model H

Schaefer and Skokov, 22, Chattopadhyay et al, 23, Florio et al, 21, Basar et al, 24,Chattopadhyay et al, 24…

Deterministic Approach

- Deterministic evolution for hydro correlation functions
- Semi-realistic estimates for two-point correlation functions computed and connected to observables

Teaney, Akamatsu, Mazeliauskas, 16 along with Yan,Yin , 19, Stephanov, Yin,17, An, Basar, Stephanov,Yee, 19,20,22,24…

An, Basar, Stephanov,Yee, 22,24

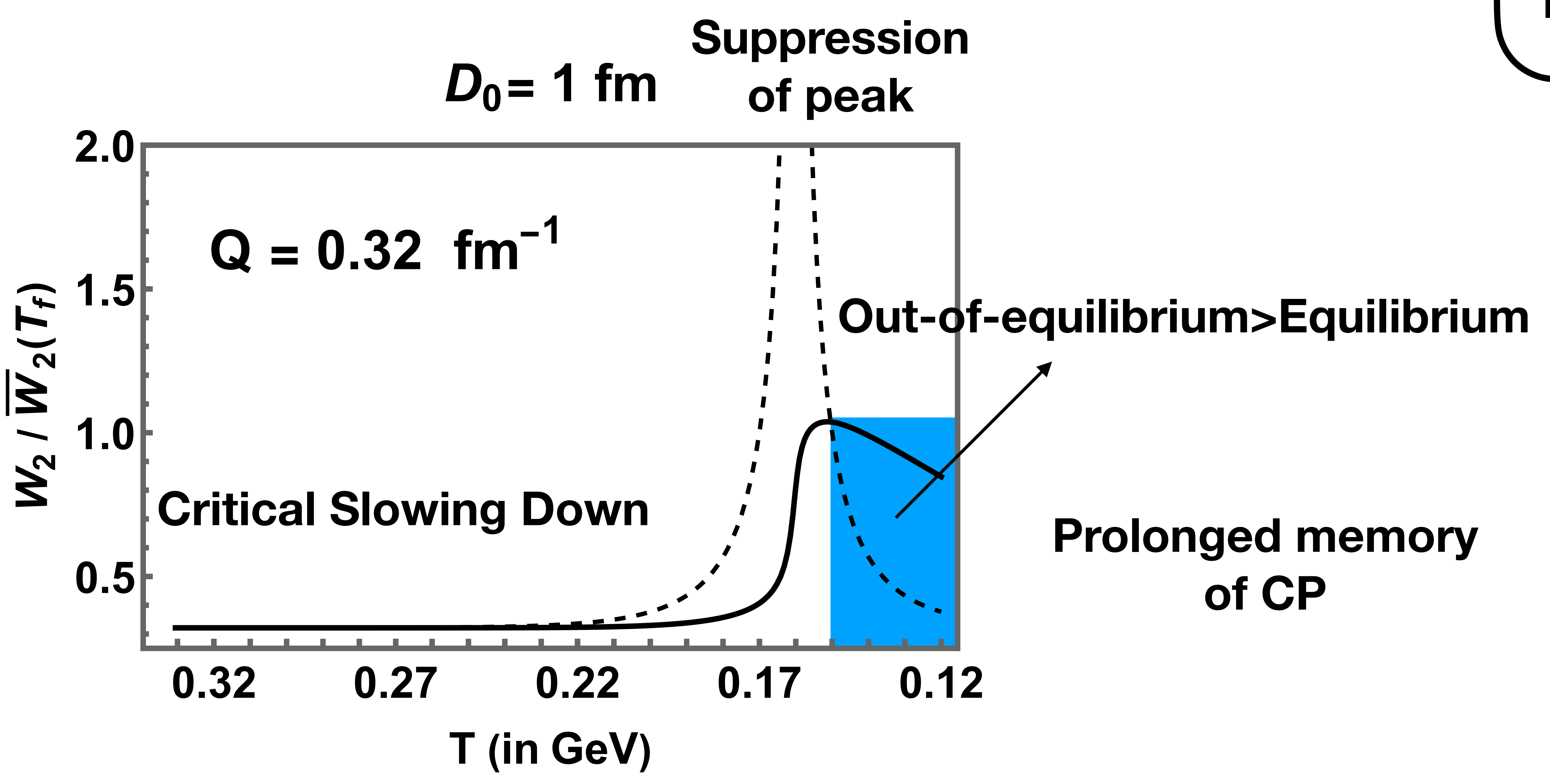
Rajagopal,Ridgway,Weller,Yin,19,Du,Heinz,Rajagopal, Yin,20
MP, Rajagopal, Stephanov, Yin, 22,Mukherjee, Venugpalan, Yin 15

Chandrodoy Chattopadhyay's talk today

Out-of-equilibrium effects of fluctuations near the CP

$$\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-)\rangle = \int e^{i\mathbf{Q}\cdot\mathbf{\Delta x}}\, W_2(\mathbf{Q})\,,\ \mathbf{\Delta x} = x_+ - x_-$$

Persistence of critical imprints in the fluctuation observables until freeze-out



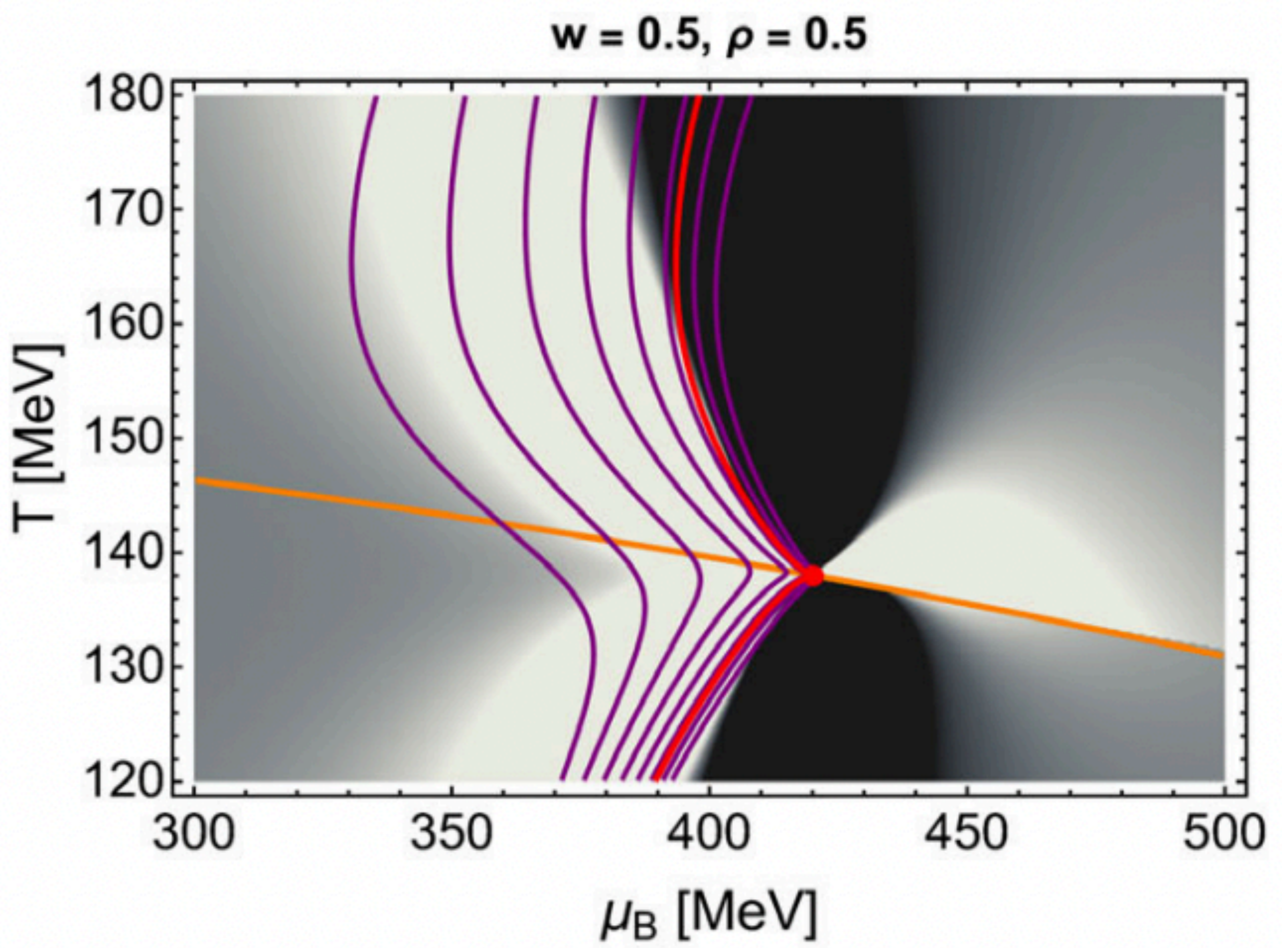
Rajagopal,Ridgway,Weller,Yin,19
Du,Heinz,Rajagopal, Yin,20
MP, Rajagopal, Stephanov, Yin, 22
Mukherjee, Venugopalan, Yin 15

CP fluctuations in MD simulations : V. Kuznietsov et al

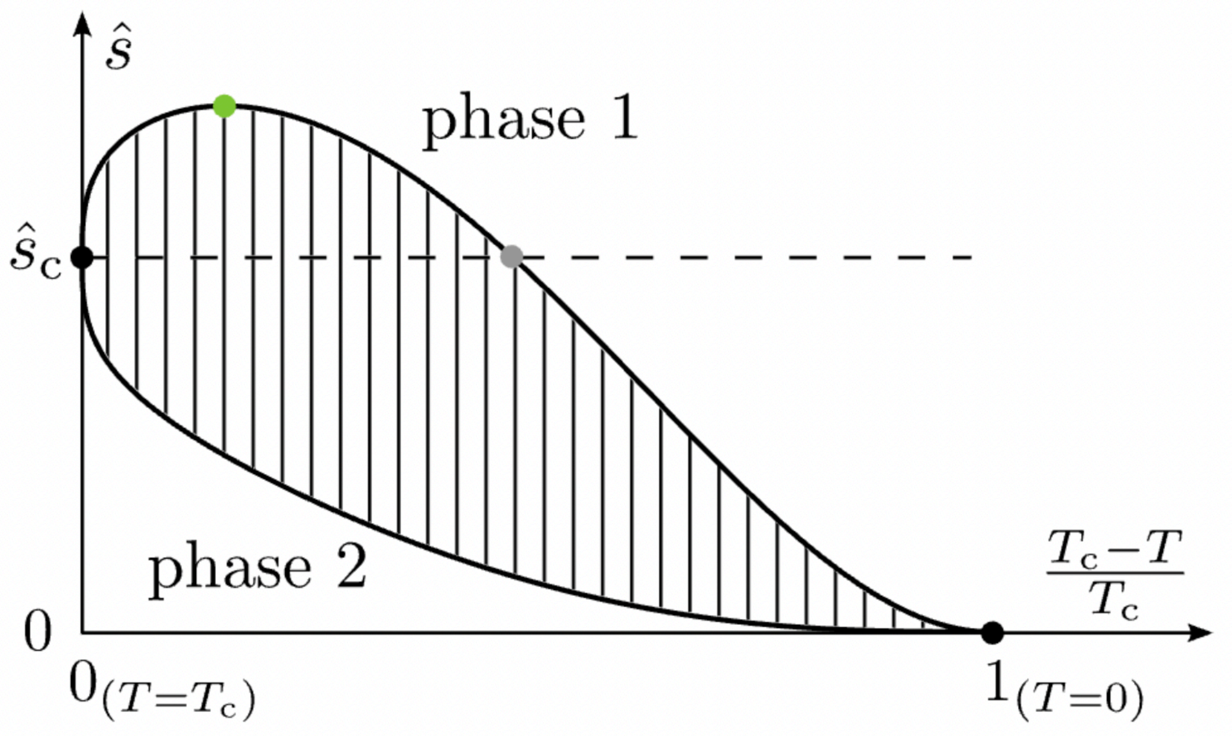
Deformation of hydrodynamic trajectories near CP

MP, Sogabe, Stephanov, Yee,24

Deformations can be broadly classified based on the value of the mapping parameter α_2



Universal scaling near CP

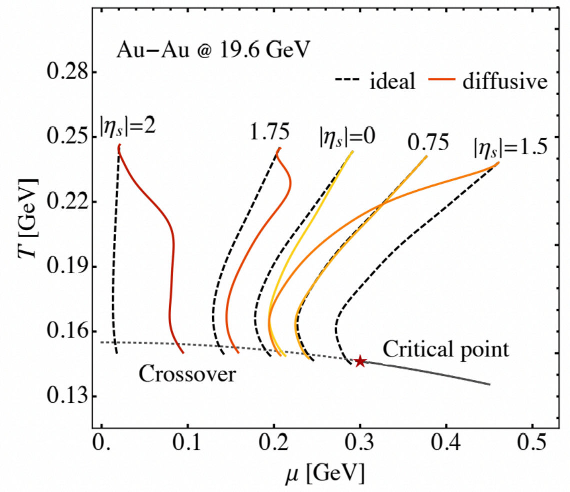


Third Law

- Specific entropy is non-monotonic along one of the branches on the first-order curve

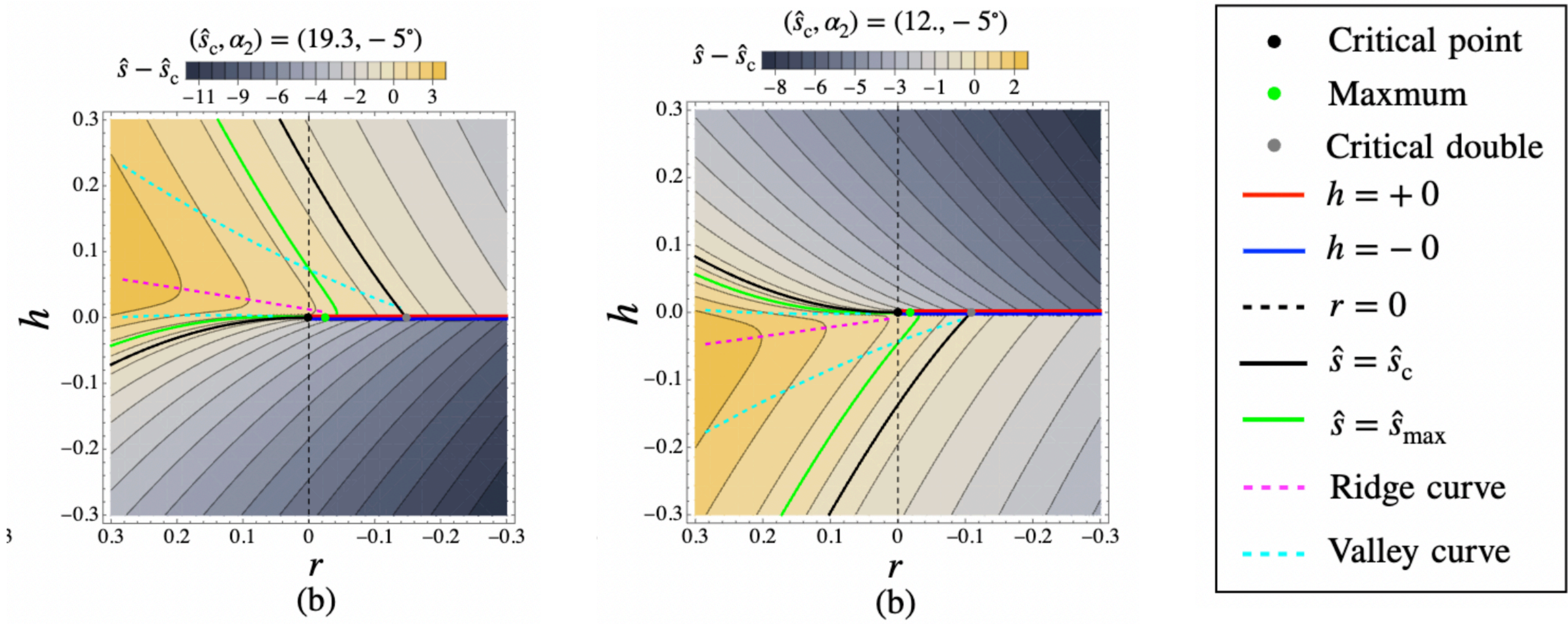
Critical lensing~Dore et al,22, Nonaka&Asakawa, 05

- Consequence of universal ridge-like structure of the isentropes near CP

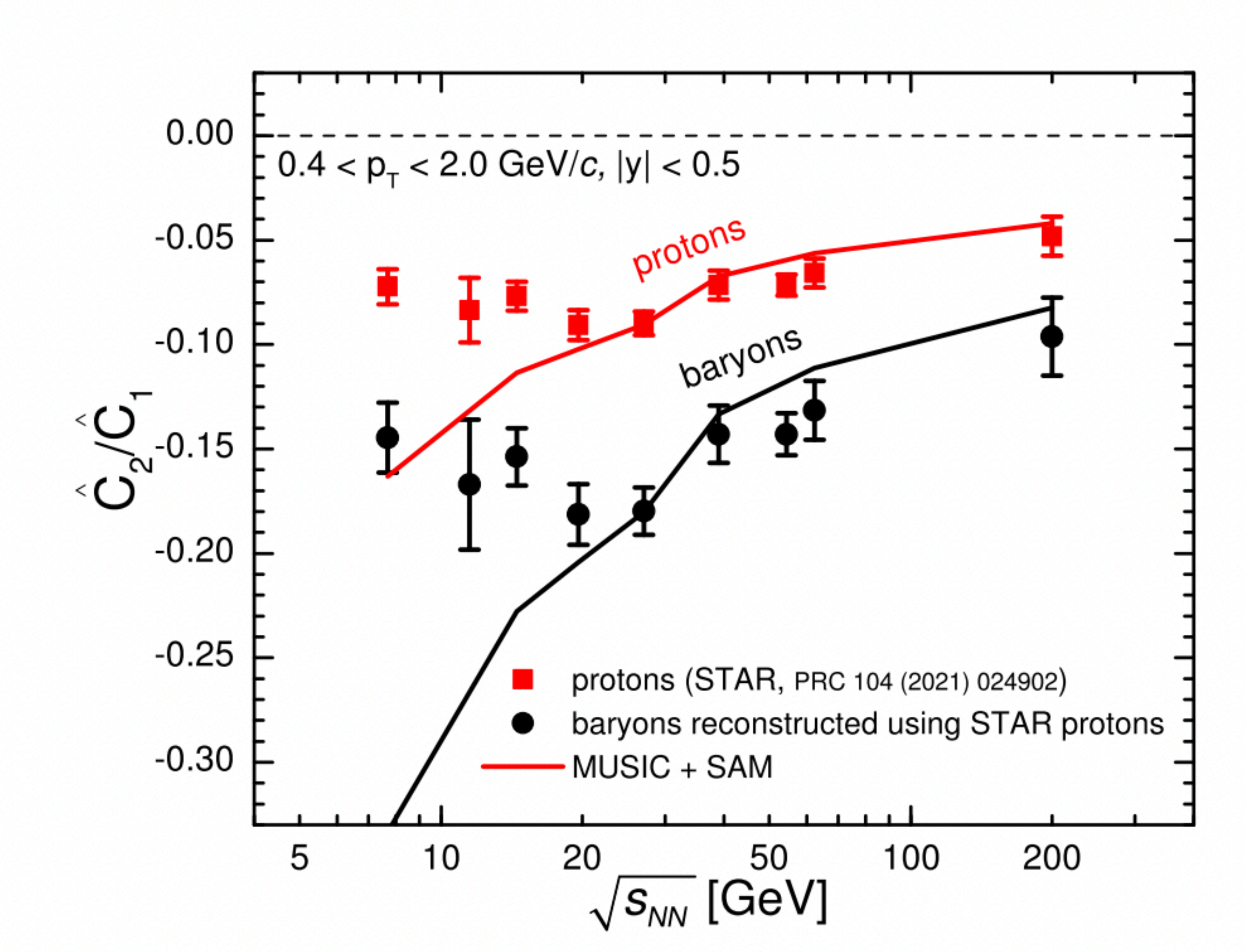


Smearing effect - Du et al, 22

- Phenomenological implications



Summarizing & Looking forward



Vovchenko,Koch,Shen,22

Non-critical ***baselines*** important!

A ***family of candidate EoSs with a CP*** that match with the lattice have been developed

$$\chi_j(\mu, T; \mu_c, \alpha_{12}, w, \rho) \overset{\text{ME}}{\rightarrow} C_A^k(\mu_F(T_F); \mu_c, \alpha_{12}, w, \rho, \Gamma)$$

Dynamics

Semi-***Quantitative*** estimates for ***out-of equilibrium*** corrections to higher order cumulants - needed

Bayesian Analysis of experimental data pertaining to ***multiple observables*** with the theoretical framework may possibly help us learn about QCD EoS near CP, if it exists in the regime scanned by HICs

Thank you!

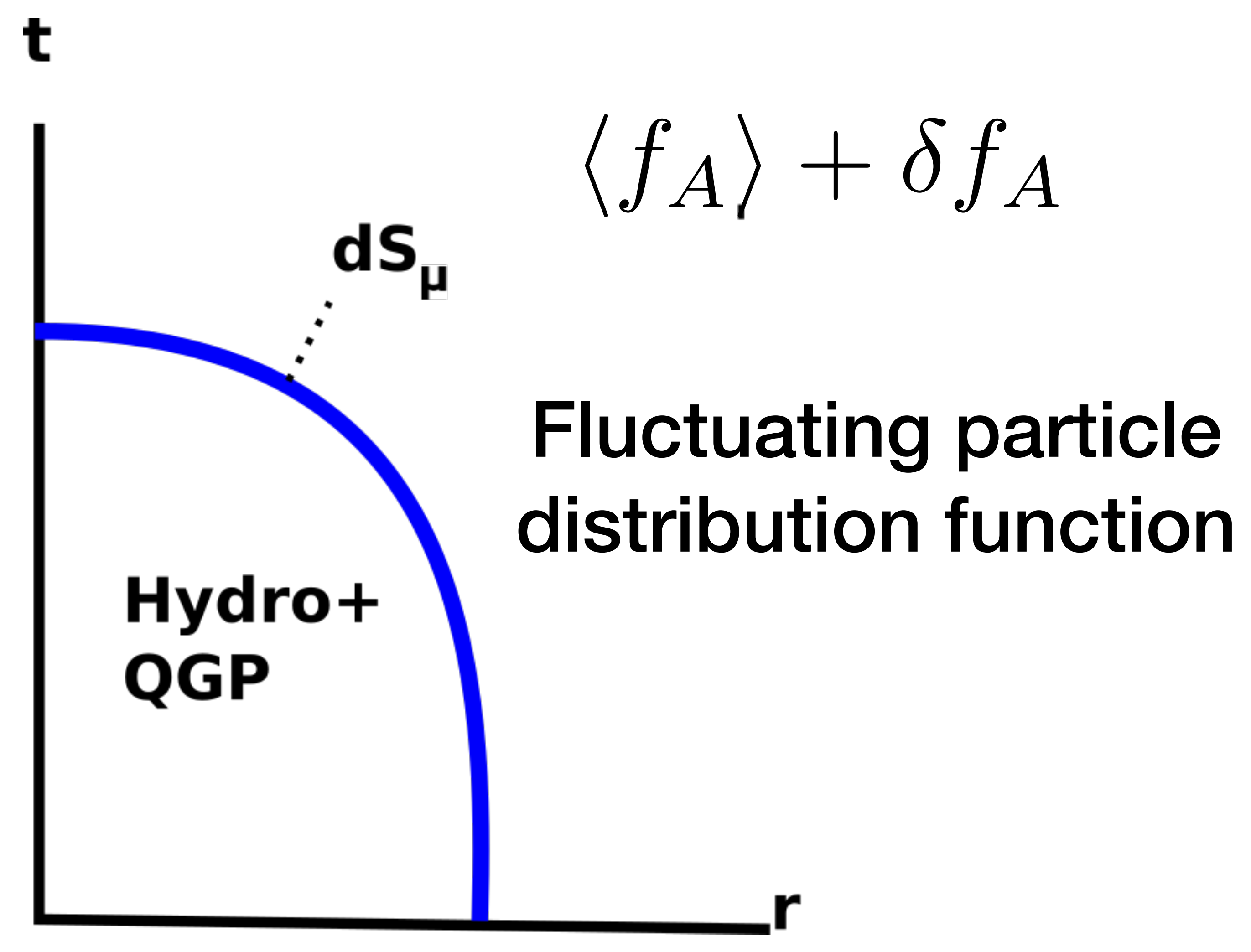
BACK UP SLIDES

Freeze-out : Transition from hydrodynamics to hadron gas

Hydrodynamic mean densities

$$\{\langle \epsilon u^{\mu} \rangle \, , \, \langle n \rangle\} \equiv \Psi^a$$

Conserved energy, momentum and charge densities and their correlations



Hydrodynamic correlations

$$\Psi^a, \, \left\langle \delta\Psi^a \delta\Psi^b \right\rangle \equiv H^{ab}, \ldots \, H^{abc\ldots}$$

Particle distribution function at freeze-out

$$\langle f_A \rangle = \bar{f}_A \, , \, \langle \delta f_A \delta f_B \rangle = G_{AB} \, , \, \langle \delta f_A \delta f_B \delta f_C \rangle = G_{ABC} \, \ldots$$

f_A Is the phase space distribution function for species A

Matching conditions at freeze-out

$$\langle \epsilon\, u^\mu \rangle = \sum_A \int_{p_A} \bar{f}_A\, p_A^\mu \,, \quad \langle n \rangle = \sum_A q_A \int_{p_A} \bar{f}_A$$

$$\Psi^a = \sum_A \int_{p_A} \bar{f}_A P_A^a$$

$$P_A = \begin{bmatrix} p_A^\mu \\ q_A \end{bmatrix}$$

$$H^{abc\ldots} = \sum_{A,B,C,\ldots} \int_{p_A p_B p_C \ldots} G_{ABC\ldots} P_A^a P_B^b P_C^c \ldots$$

- Matching conditions for averages of conserved densities
- Infinitely many sets of distribution functions that satisfy these matching conditions
- Freeze-out prescription corresponds to choosing one of these sets - **How to choose**?

Equilibrium

$$S[\bar{f}]$$

Which entropy to maximize?

Generalized

$$S[P(f)]$$

$$S[P(f)] = S_0\left[\bar{f}\right] + \int_f P(f)\log\frac{P_{\mathrm{eq}}(f)}{P(f)}$$

$$S[\bar{f}, G_2, G_3, \ldots]$$

G s are the correlation functions in the Hadron Gas description

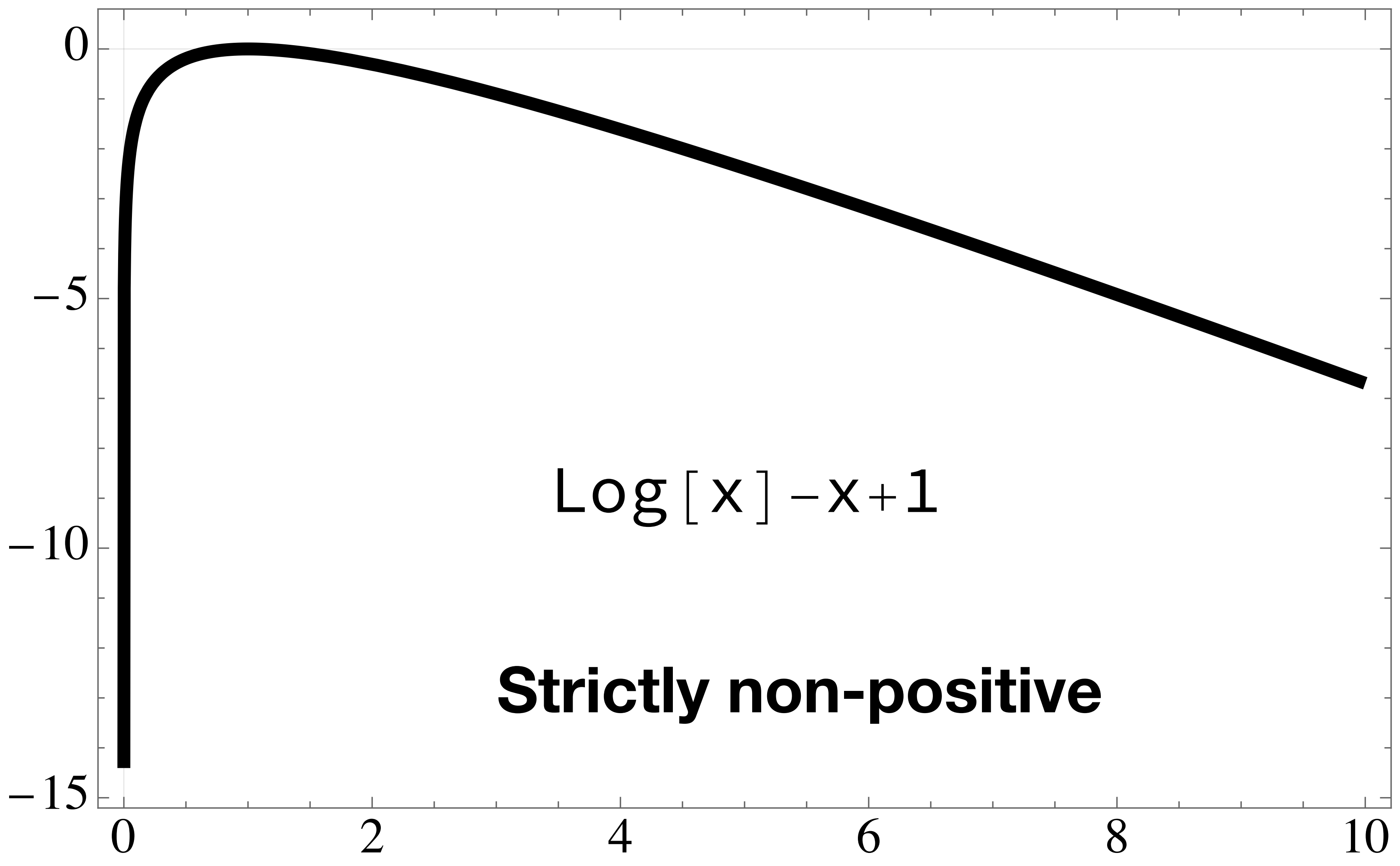
$$S_0[\bar{f}] = -\int_f P_{\mathrm{eq}}(f)\log P_{\mathrm{eq}}(f)$$

- Maximize the relative entropy when correlations are out of equilibrium
- Constraints from matching conditions

Entropy to describe out-of equilibrium two-point correlations in ideal HRG

$$S_2 = S_0 + \frac{1}{2}\mathrm{Tr}\left[\log G\bar{G}^{-1} - G\bar{G}^{-1} + 1\right] \,,$$

Equilibrium



Similar 2-PI action

Berges, 04, Stephanov, Yin, 17…

2-PI entropy

Upon maximizing the 2PI entropy, subject to constraints of conservation

$$G^{-1}_{AB} = \bar{G}^{-1}_{AB} + (H^{-1} - \bar{H}^{-1})^{ab}(\bar{H}^{-1}P\bar{G})^a_A(\bar{H}^{-1}P\bar{G})^b_B$$

When all but two-point correlations are in equilibrium, the solution given above is exact.

Linearizing,

$$G_{AB\ldots} = \bar{G}_{AB} + \Delta H_{ab}(\bar{H}^{-1}P\bar{G})^a_A(\bar{H}^{-1}P\bar{G})^b_B + \ldots,$$

Self correlations

Contribution of self correlations to hydrodynamics is subtracted

Contribution of self correlations to hydrodynamics

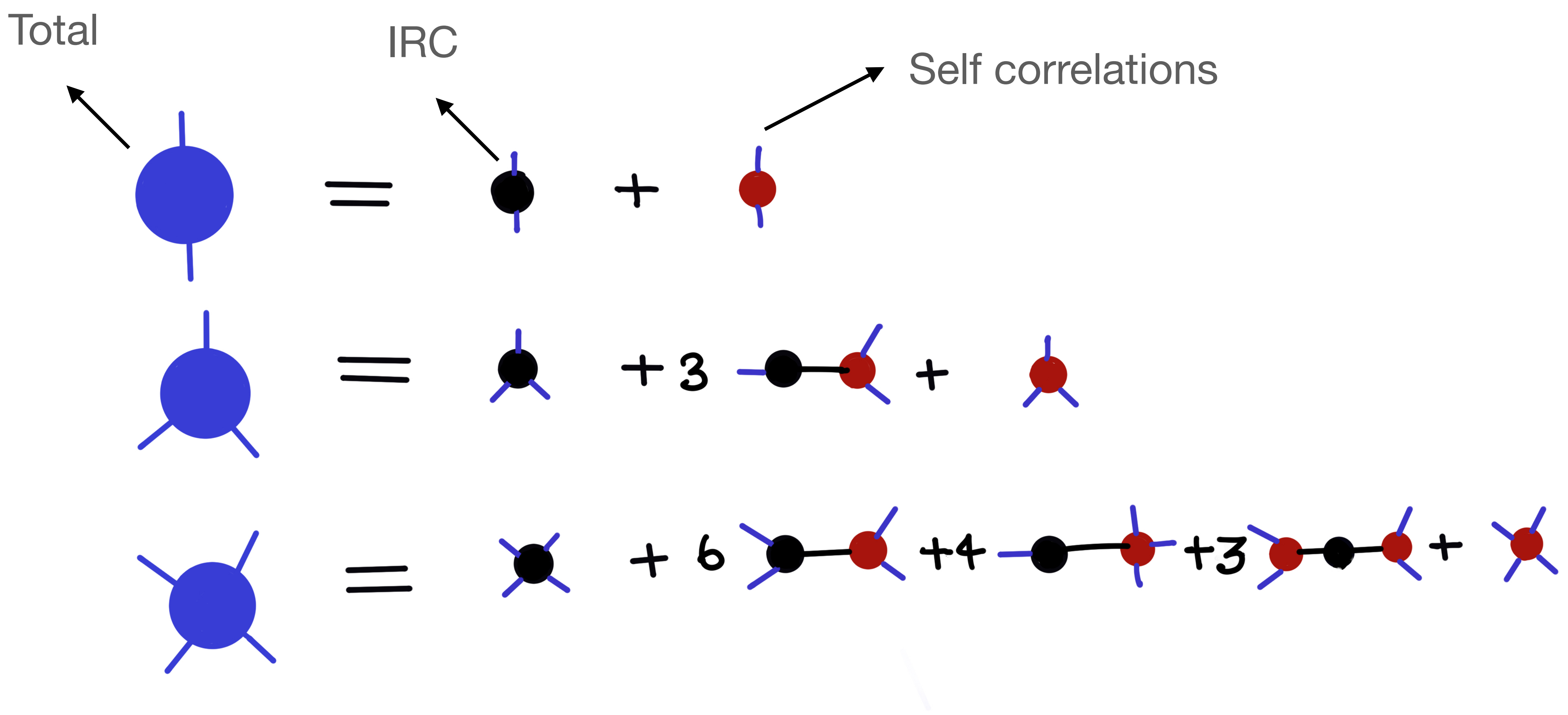
$$\Delta H^{ab} = H^{ab} - \bar{H}^{ab}$$

$$\bar{H}_{ab} = \sum_{A,B} \int \bar{G}_{AB} P^a_A P^b_B$$

Generalization to Non-Gaussian Correlations

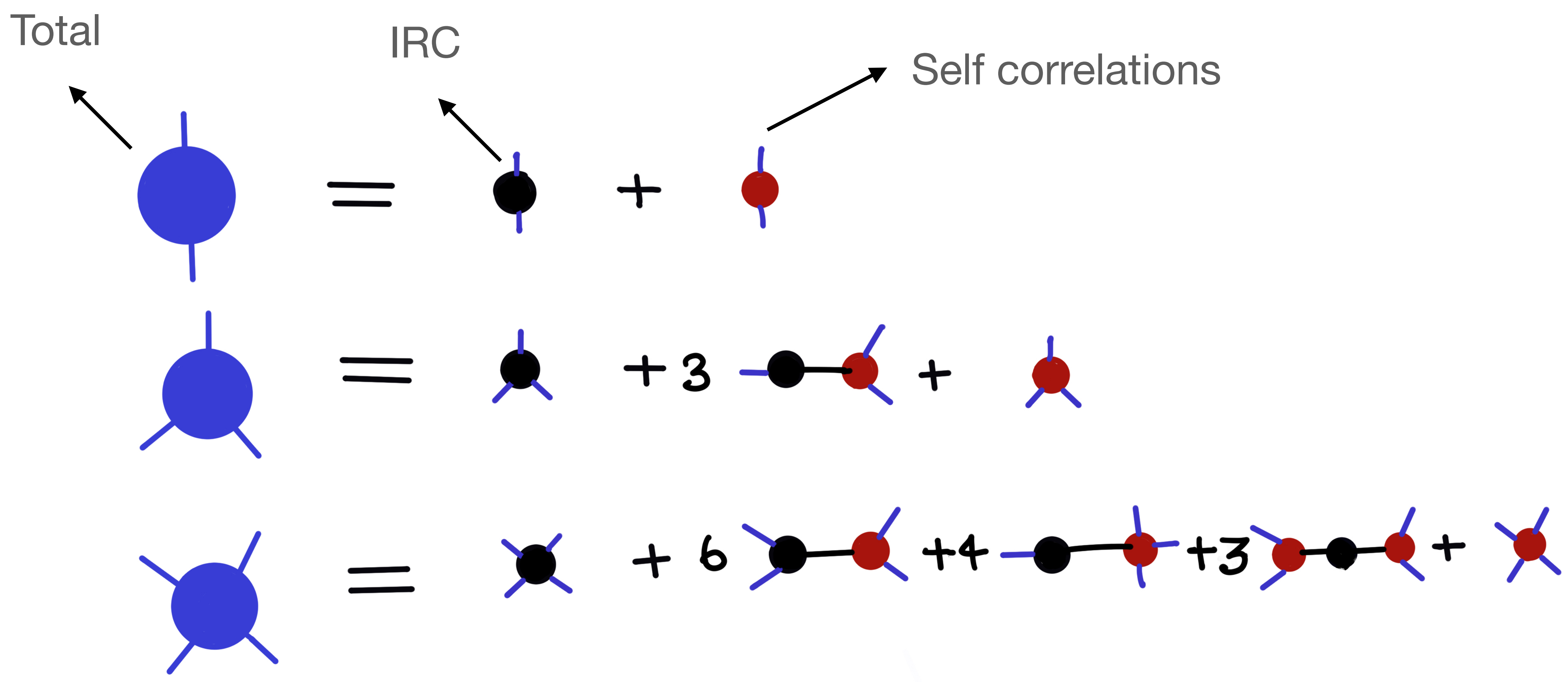
$$\overset{\text{IRC}}{\hat{\Delta} G_{ABC\ldots}} = \underbrace{\mathcal{F}(\bar{H}, \bar{G})}_{\text{Equilibrium}} \, \overset{\text{IRC}}{\hat{\Delta} H_{ABC\ldots}}$$

For classical gas, irreducible relative cumulants (IRCs) reduce to so called "factorial cumulants".



Irreducible relative cumulants

For classical gas, irreducible relative cumulants (IRCs) reduce to so called “factorial cumulants”.



- For gases obeying different statistics, IRCs quantify the non-trivial correlations
- Non-trivial correlations relative to any specified baseline distribution

Equilibrium estimates for the critical contribution to the factorial cumulants of proton multiplicity

$$\omega_p^k = \frac{C_k}{C_1}_{\text{crit}} \approx \frac{T^3}{n_p}\left(\frac{X^T \bar{H}^{-1} P}{w \sin(\alpha_1 - \alpha_2)}\right)^k \kappa_k(\mu, T)$$

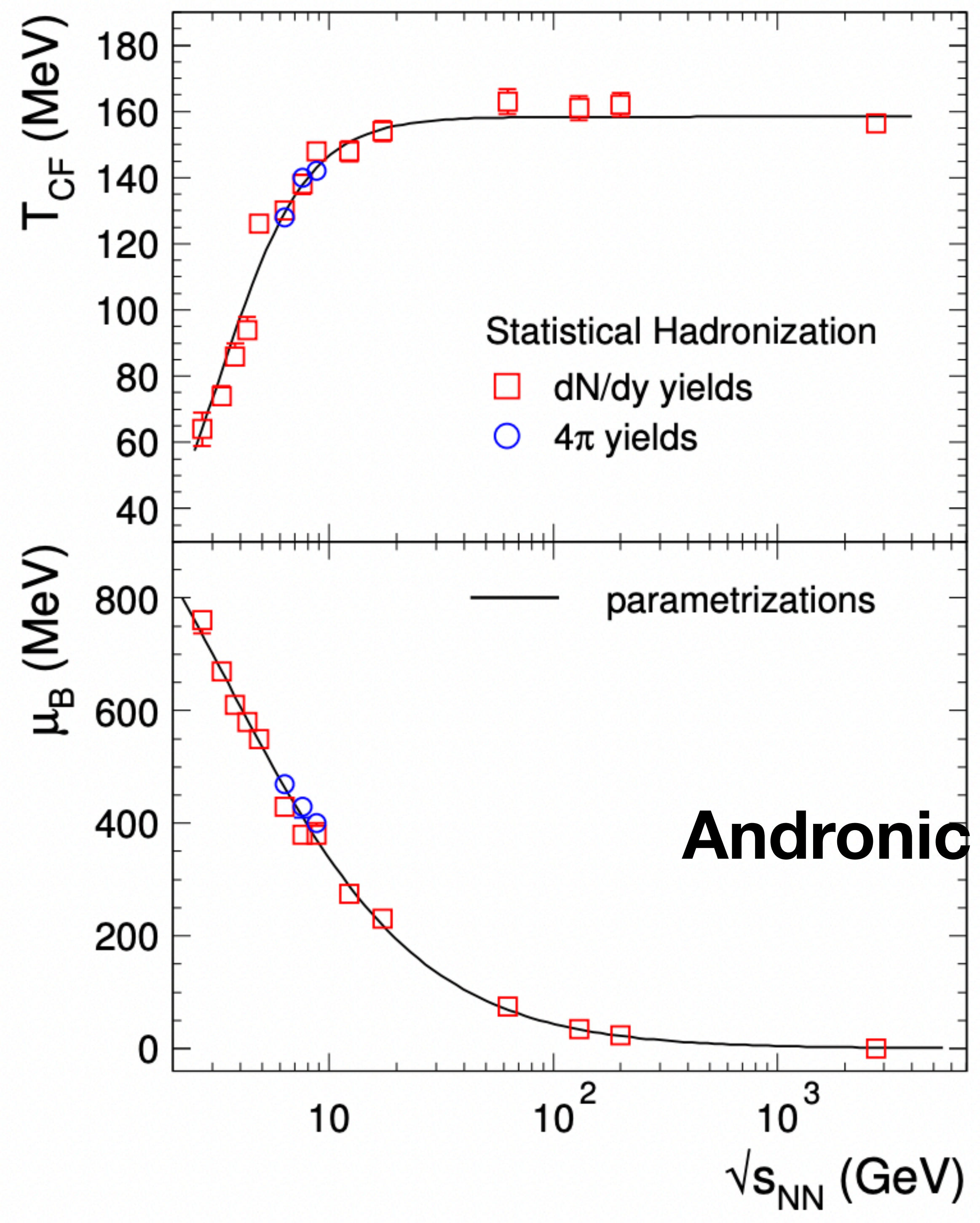
Cumulants in Ising model mapped to QCD

$\bar{H}^{-1}P$ HRG

X depends on the mapping to Ising

$$X = \begin{pmatrix} s_1 \\ c_1 + \mu_c s_1/T_c \end{pmatrix}, \bar{H} = \int_H f'_H \begin{pmatrix} (E_H/T_c)^2 & (E_H/T_c)q_H \\ (E_H/T_c)q_H & q_H^2 \end{pmatrix}, P_A = \int_A f'_A \begin{pmatrix} E_A/T_c \\ q_A \end{pmatrix}$$

Freeze-out parametrization



Andronic et al, 18

We use freeze-out parametrization from Andronic et al, 18 and add a variable additive constant such that

$$T_c - T_f(\mu_c) = \Delta T$$

We study sensitivity of observables to ΔT

Novel summation scheme from WB collaboration

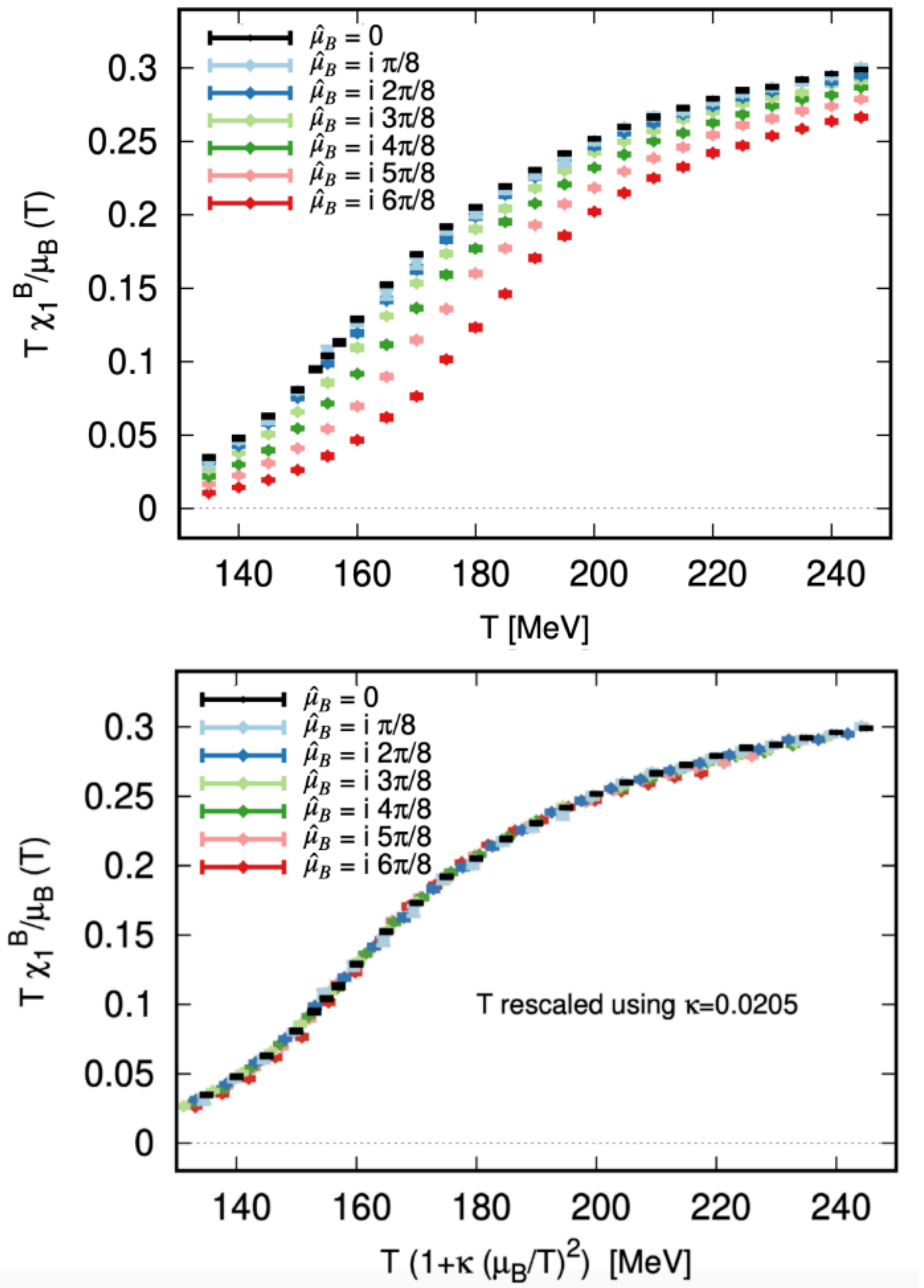


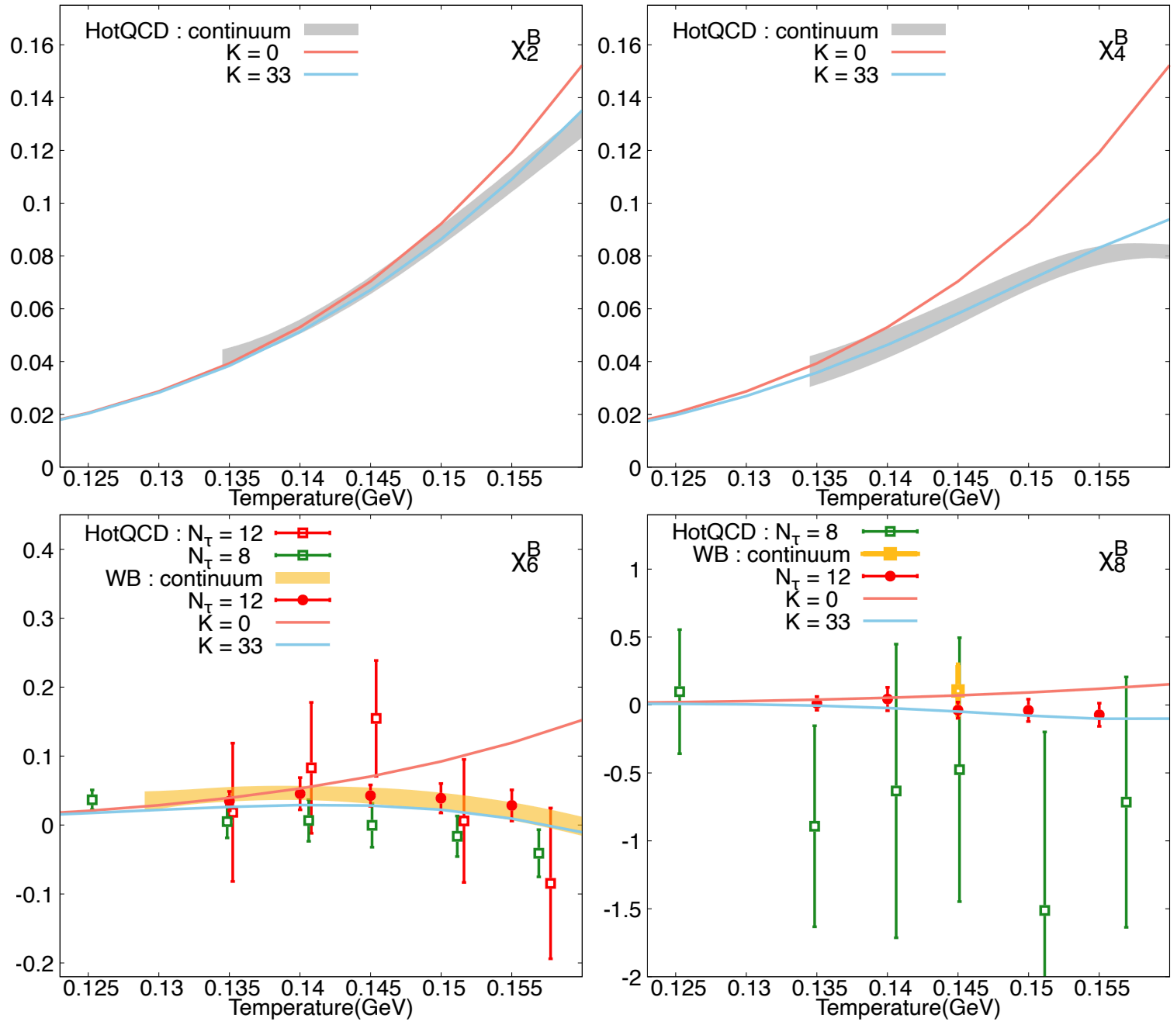
FIG. 1. Upper panel: scaled baryon density $\chi_1^B(T,\mu_B)/\hat{\mu}_B$, as a function of temperature for different values of scaled imaginary baryon chemical potential $\hat{\mu}_B \equiv \mu_B/T$ (labeled using different colors). Lower panel: the same quantity, but with the temperature rescaled by a factor $1+\kappa\hat{\mu}_B^2$, with $\kappa = 0.0205$. In terms of the rescaled temperature the curves representing different $\hat{\mu}_B$ collapse onto the same curve. The points labeled $\hat{\mu}_B = 0$ correspond to the limit $\mu_B \to 0$ which is the baryon number susceptibility $\chi_2^B(T,0)$ (The figure is taken from Ref. [13]).

$$\frac{T\chi_1(\mu_B,T)}{\mu_B} = \chi_2(T',0)$$

Borsanyi et al,21,

Kahangirwe et al., 24

Baryon density fluctuations away from CP using lattice and FRG methods



At zero chemical potentials from lattice
Biswas, Petrezcky, Sharma., 24, Bazavov et al, 20, Borsanyi et al, 18,23

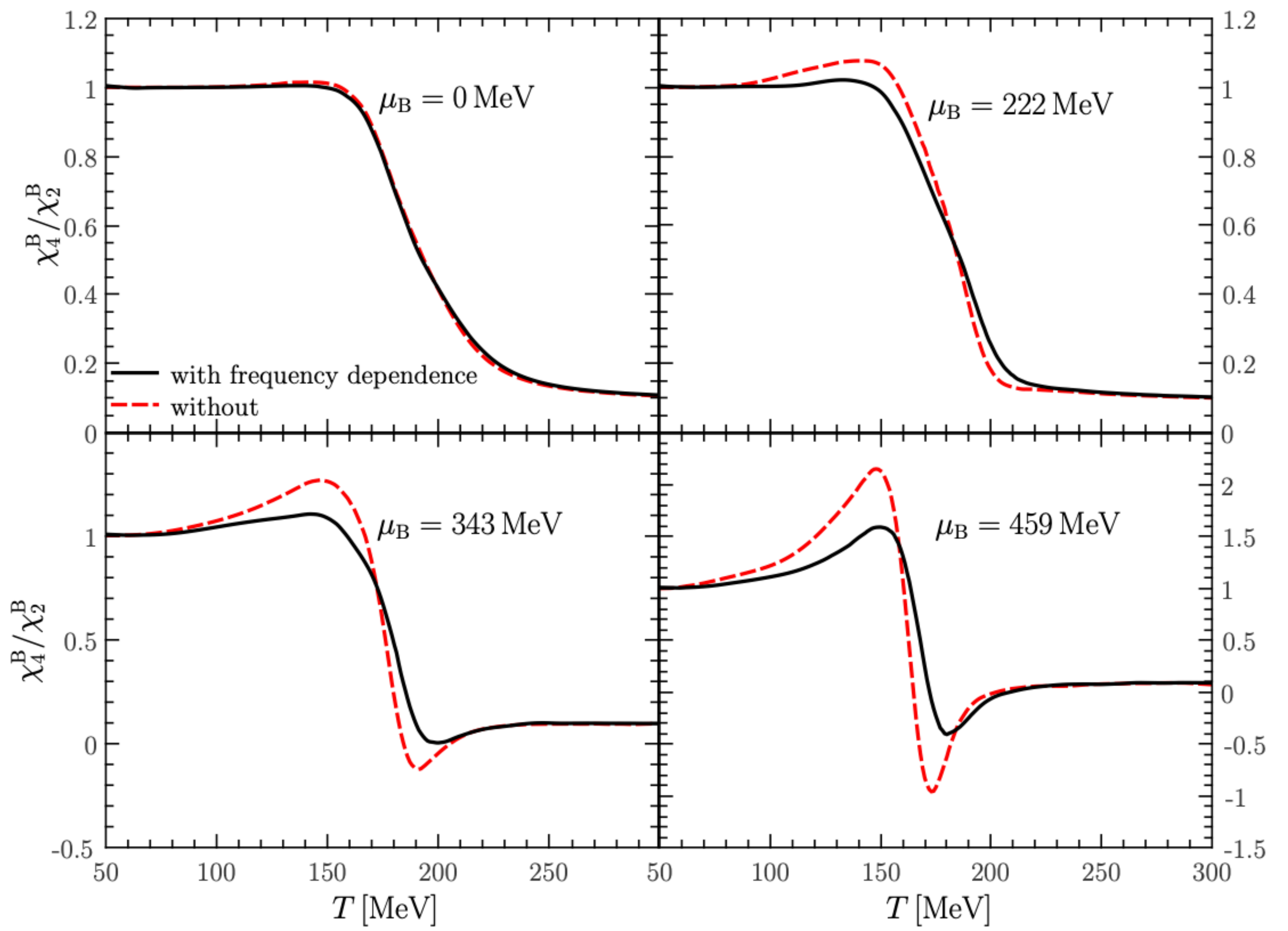


FIG. 4. Kurtosis as a function of the temperature for different baryon chemical potentials. Solid lines include the frequency dependence of the quark anomalous dimension and dashed lines not.

Baryon density fluctuations from FRG

Fu et al, 2016, 2022

Current constraints on the location of the critical point from Lattice QCD

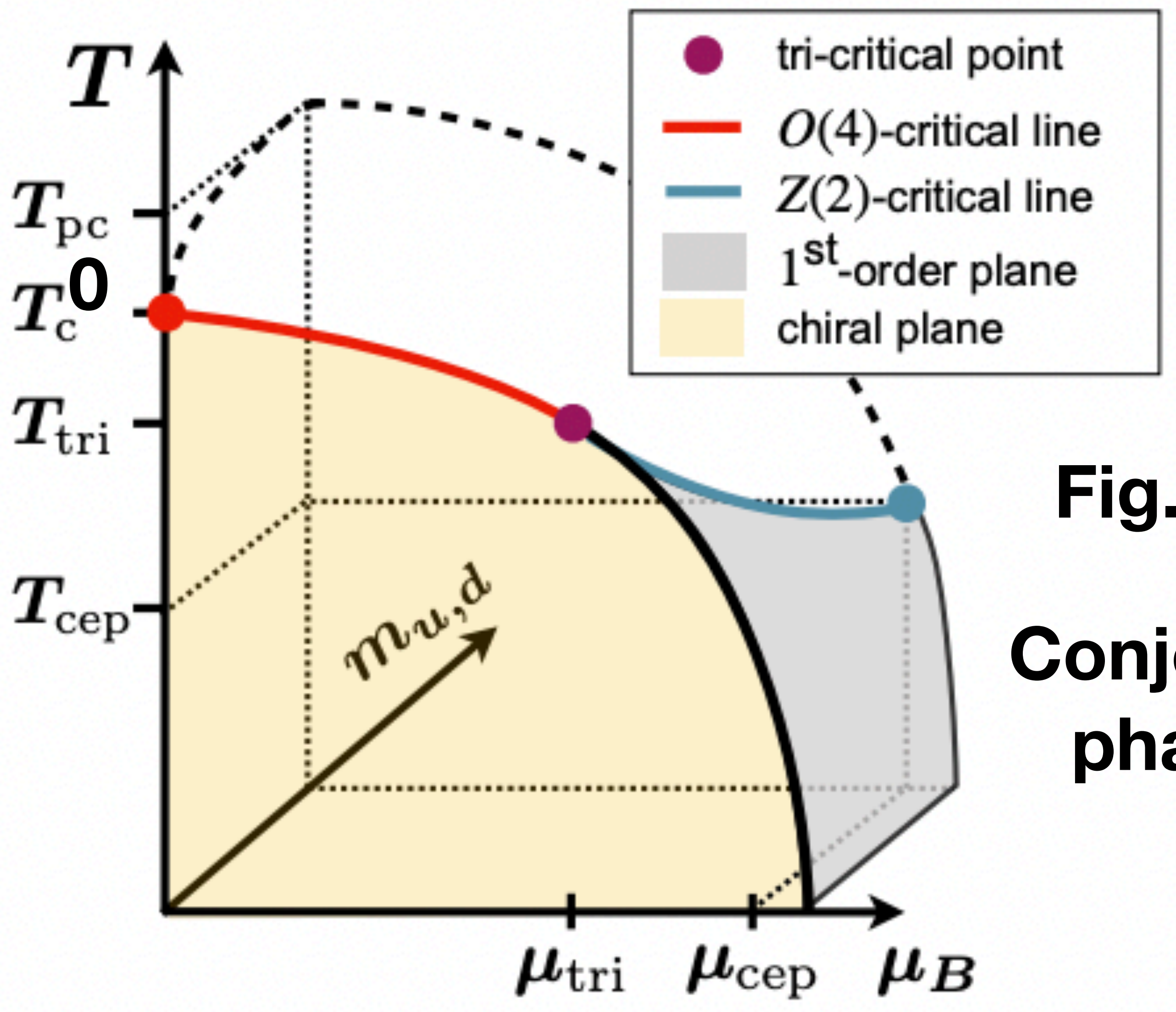


Fig. : Ding et al , 24

Conjectured chiral phase diagram

$T_c^0 > T_{\rm tri} > T_c$

Hatta, Ikeda, 03 $T_c^0 = 132^{+3}_{-6}\,\mathrm{MeV}$ **HotQCD, Ding et al, 19**

Pseudo-critical curve at physical quark mass

$$T_{\rm pc}(\mu_B) = T_{\rm pc}(0)\left[1+\kappa_2\left(\frac{\mu_B}{T_{\rm pc}}\right)^2\right]$$

Bazavov et al, 19, Ding et al,24

$$T_{\rm pc}(0) = 156 \pm 1.5\,\mathrm{MeV}\,,\ \kappa_2 = 0.012(4)$$

Borsanyi et al,20

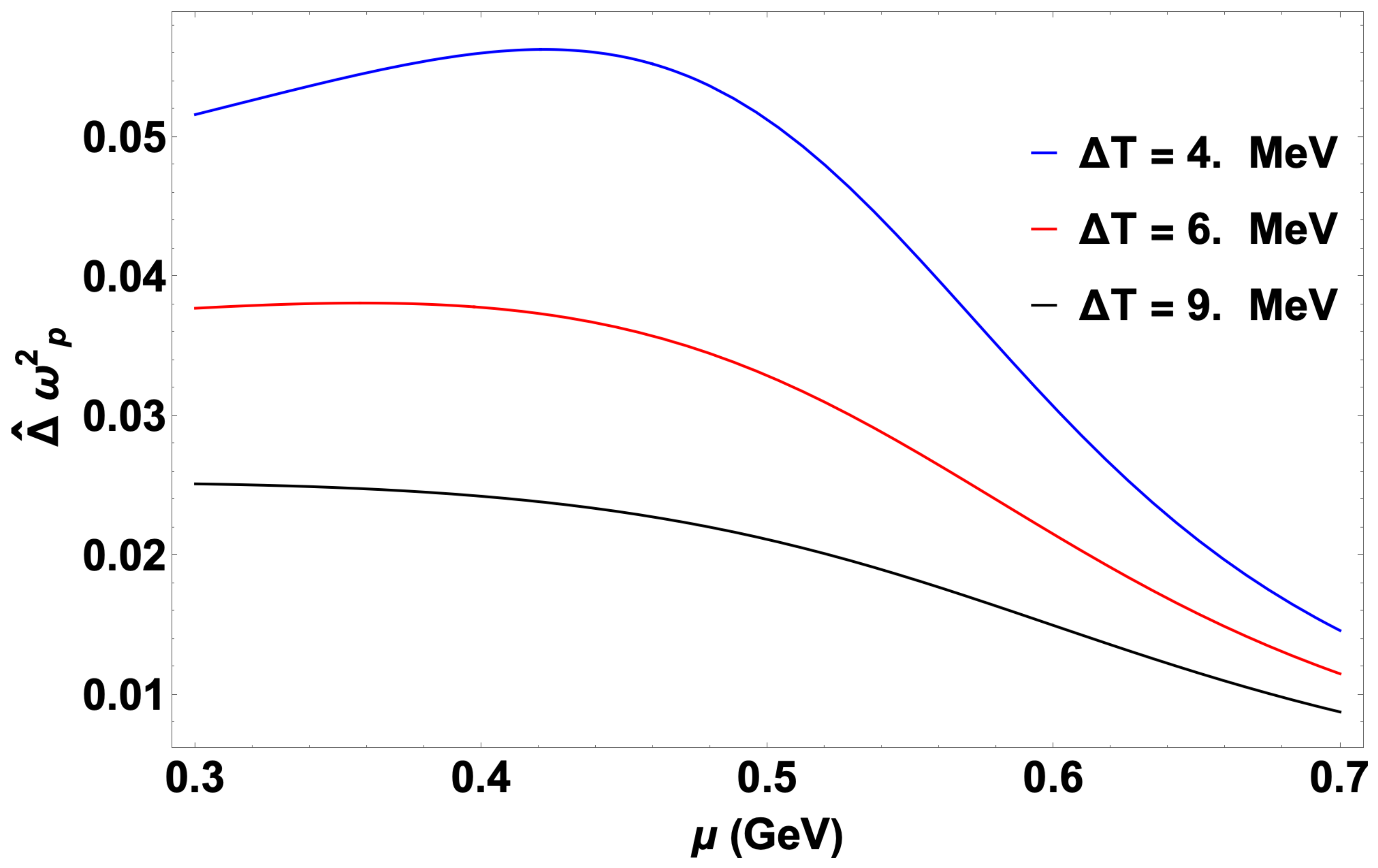
$$T_{\rm pc}(0) = 158 \pm 0.6\,\mathrm{MeV}\,,\ \kappa_2 = 0.0153(18)$$

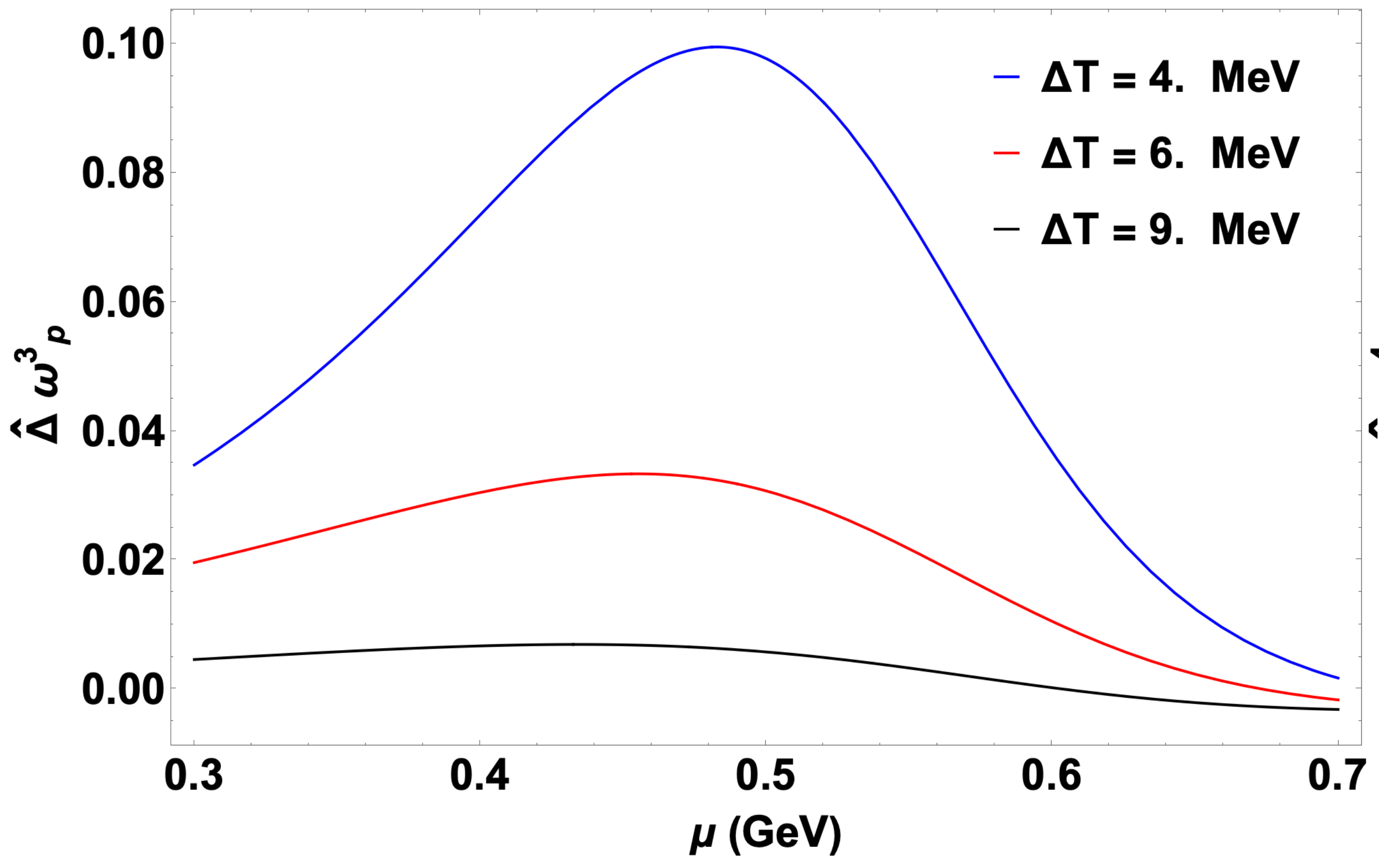
Critical point disfavored for $\mu_B/T < 3$

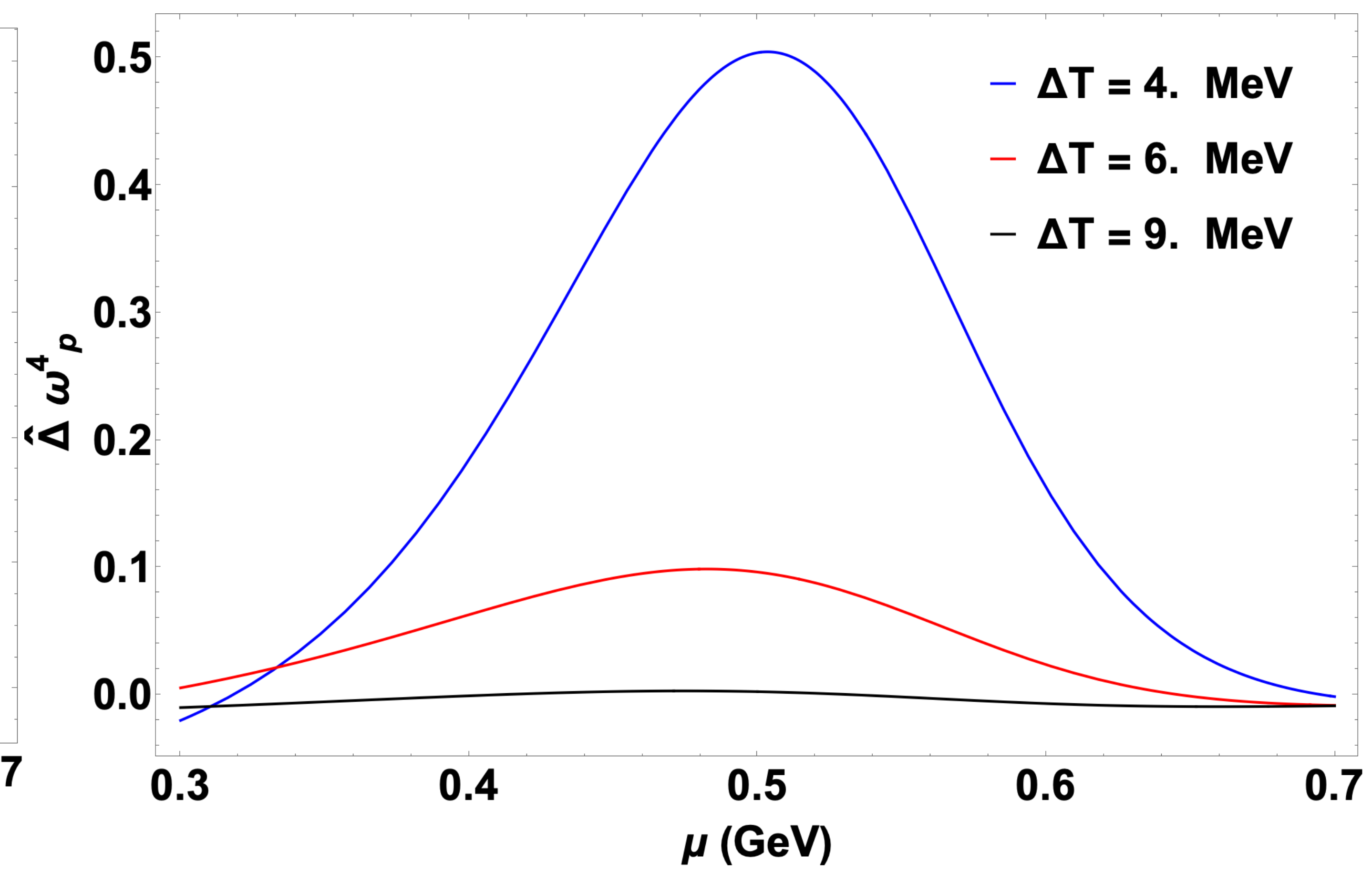
Discussions on Columbia Plot - S. Gupta, Thurs, 2:20 pm

Equilibrium estimates for critical contribution to factorial cumulants of proton multiplicities

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)





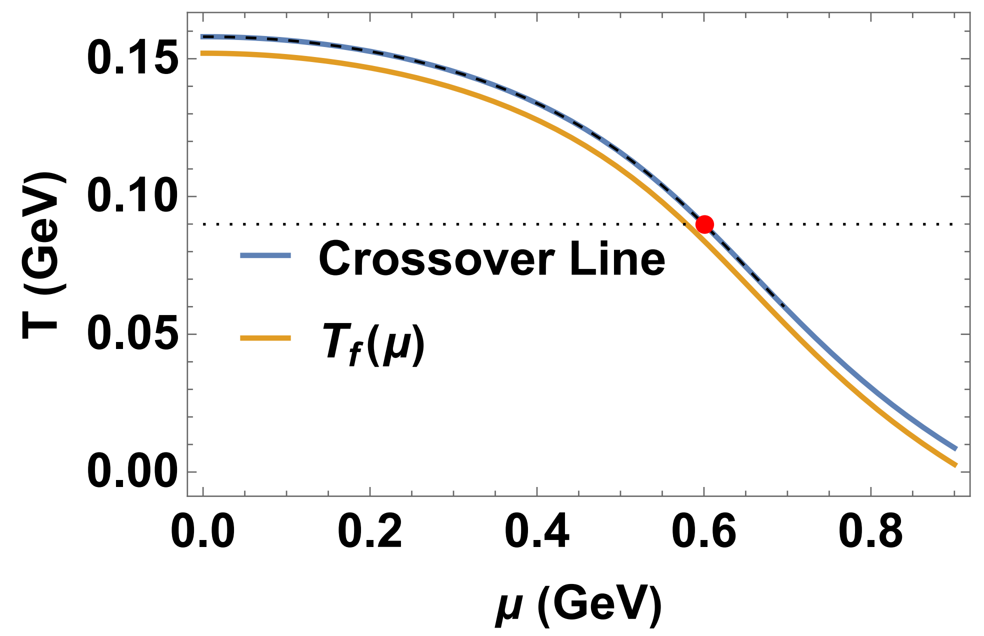


$$T_f(\mu) = T_{\rm co}(\mu) - \Delta T$$

$$\mu_c = 600\,\mathrm{MeV}\,,\ \alpha_2 = 0^\circ\,,\ \rho = 1\,,\ w = 20$$

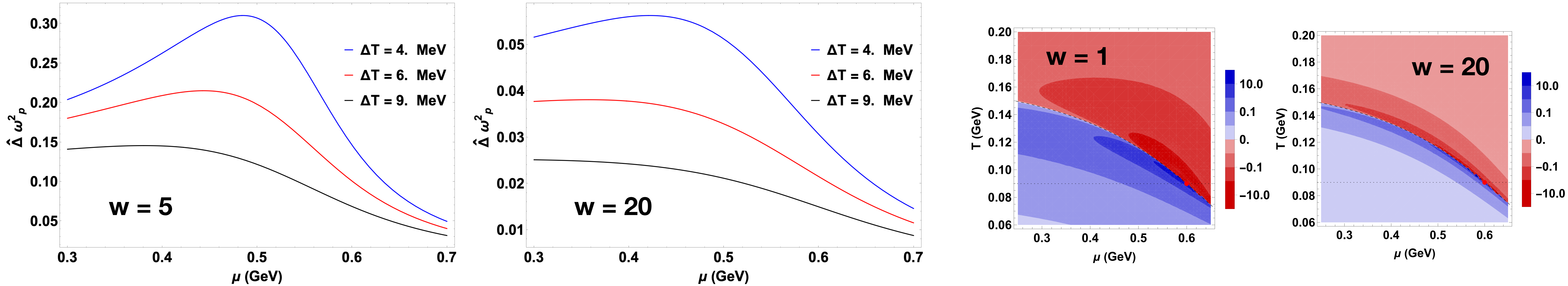
Example **choice of Mapping Parameters**

$$\hat{\Delta}\omega_p^k = \frac{\hat{\Delta}\,\langle \delta N_p^k \rangle}{\langle N_p \rangle}$$



Freeze-out curve

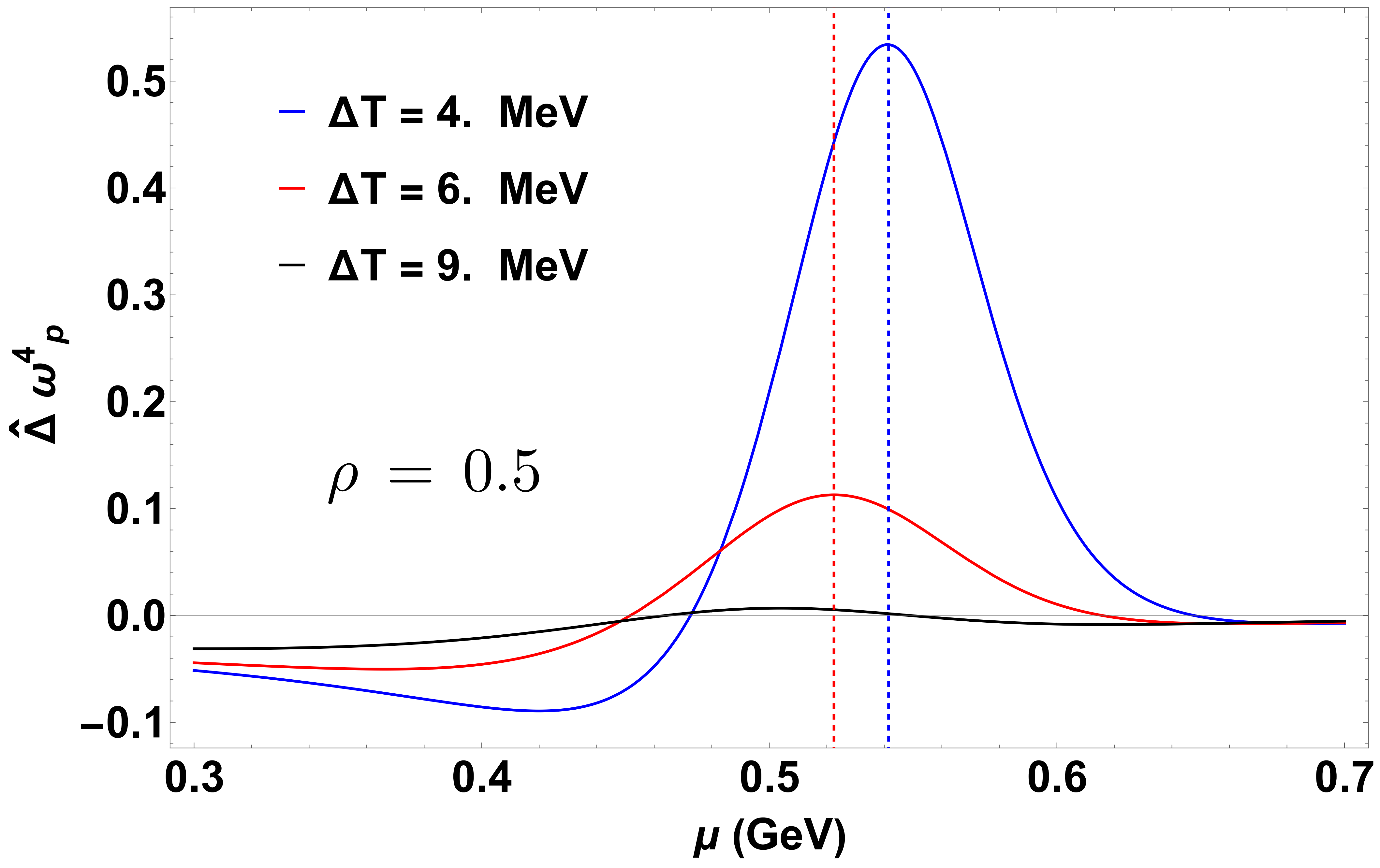
Peak value depends on w

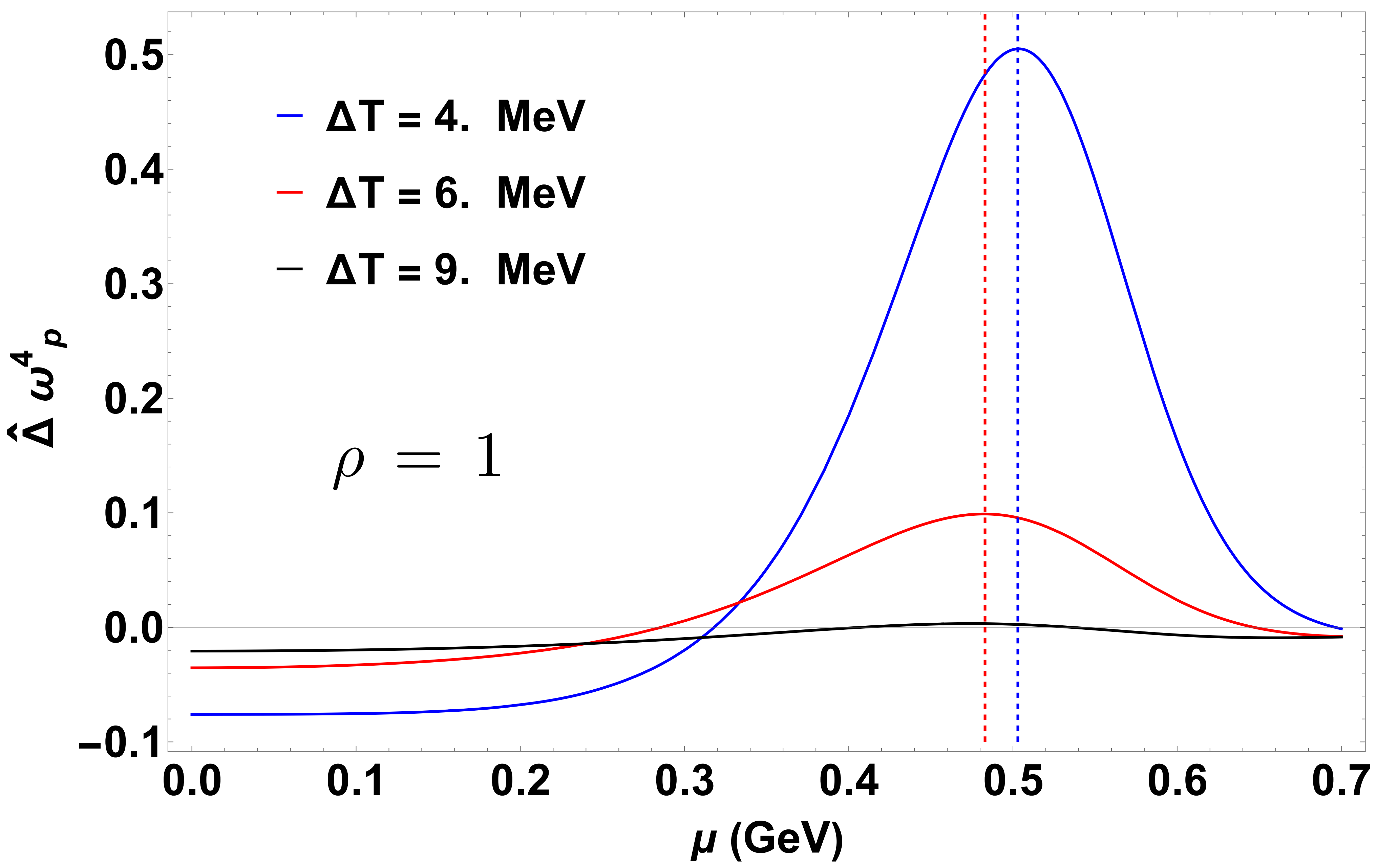


$$\hat{\Delta}\omega_p^k(\mu\,,\,T_f(\mu);w) \approx \left(\frac{w^{'}}{w}\right)^{1+\frac{1}{\delta}} \hat{\Delta}\omega_p^k\left(\sqrt{(\mu^2-\mu_c^2)\frac{w^{'}}{w}+\mu_c^2}\,,\,T_f(\mu);w^{'}\right)$$

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)

Role of ρ



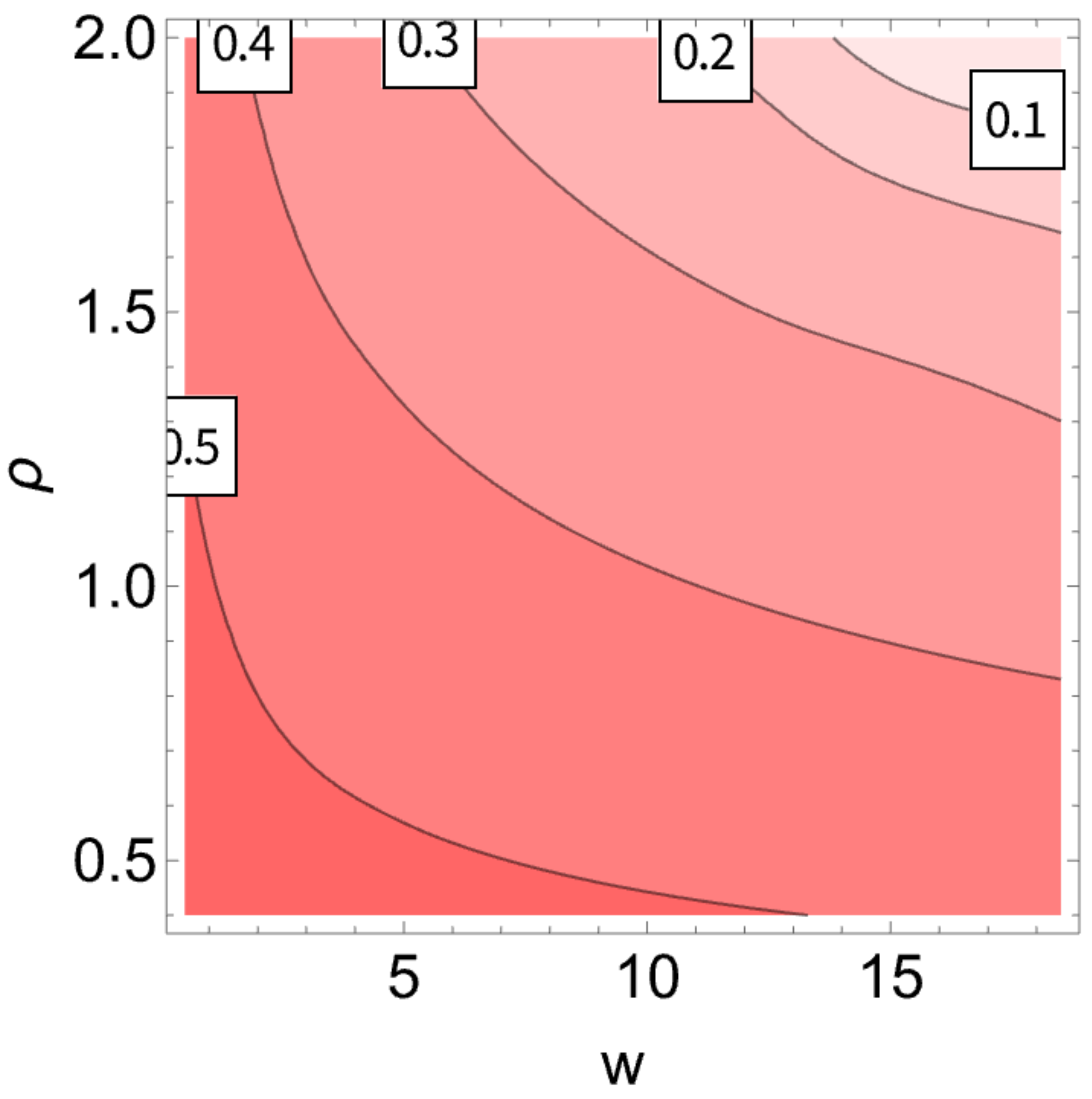


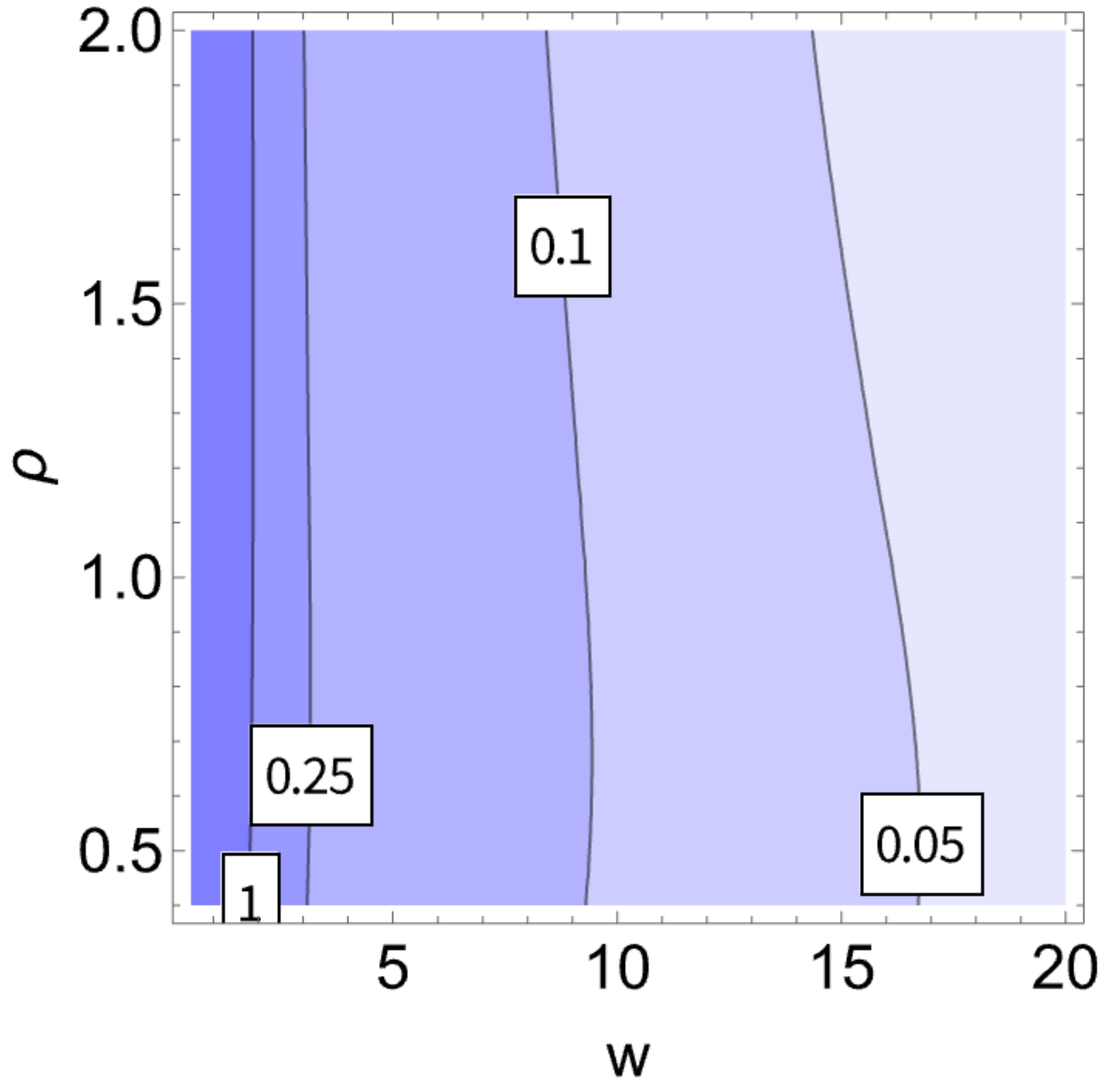
$$\hat{\Delta}\omega_p^k(\mu\,,\,T_f(\mu);\rho) \approx \hat{\Delta}\omega_p^k\left(\sqrt{(\mu^2-\mu_c^2)\frac{\rho^{'}}{\rho}+\mu_c^2}\,,\,T_f(\mu);\rho^{'}\right)$$

Example **choice of Mapping Parameters**

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)

Role of mapping parameters





- Peak value of cumulants along the freeze-out curve is fixed by w
- Location of the maxima on the freezout curve fixed by $\rho\, w$

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)