UNIVERSITÄT BIELEFELD

CRC-TR 211

Some Recent Results on Heavy Quark Systems from Lattice

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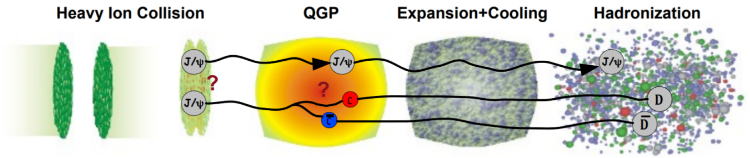
ATHIC 2025

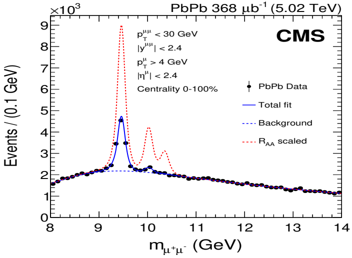
Outline

- **Thermal potential**
- **Fate of Quarkonia bound states**
- **Heavy Quark Diffusion**



Matsui and Satz, PLB 178, (1986) 416

- QGP cause suppression of Quarkonia (bound states of heavy $q\bar{q}$), an important probes to study properties of QGP.
- Need to know real time dynamics to understand experimental observation.

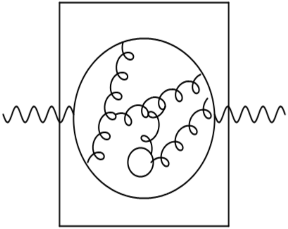


CMS Collaboration, PLB 790 (2019) 270

- Theoretically, the dilepton production rate from a thermally equilibrated plasma is given by:

$$\frac{d\Gamma_{\mu+\mu-}}{d^4Q} \sim \frac{e^2}{Q^2} n_b \, \rho_V(Q)$$

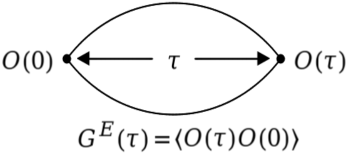
- ρ_V vector channel spectral function.

- 

$$\rho_{\mu\nu}(\omega, \vec{k}) = Im[\pi_{\mu\nu}(\omega, \vec{k})]$$

- Lattice calculate correlation function in imaginary time at a given temperature.

- Need analytic continuation.

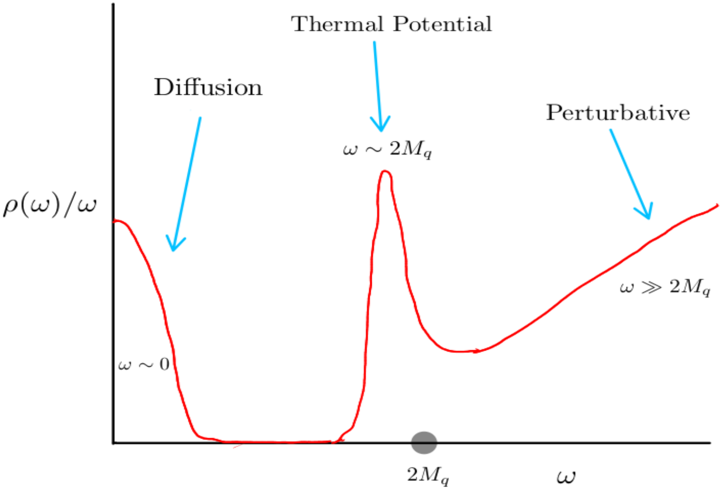


$$G^E_\Gamma(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_\Gamma(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically ill-posed

- Needs further physics input

The broad picture



$$|\phi_n^M(\vec{r},t)\rangle = \underbrace{\bar{\psi}(-\vec{r}/2,t)U(-\vec{r}/2,\vec{0})\mathrm{M}U(\vec{0},\vec{r}/2)\psi(\vec{r}/2,t)}_{O^M(\vec{r},t)}|n\rangle$$

$$\mathrm{M} = \{1\,(\text{singlet}),\ \mathrm{T}^a\,(\text{octet})\}$$

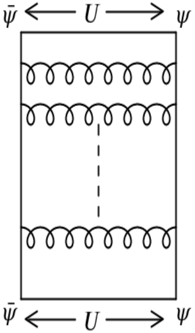
- Correlation: $C^M(t;r) = \langle O^M(r,t)O^M(r,0)\rangle_T$
- $M_q \to \infty$, $C^M(t;r) \propto \langle W^M(r,t)\rangle_T$
- Static potential:

$$V(r) = i \lim_{t\to\infty} \frac{\partial \log W^M(r,t)}{\partial t}$$

- Non-perturbative formulation:

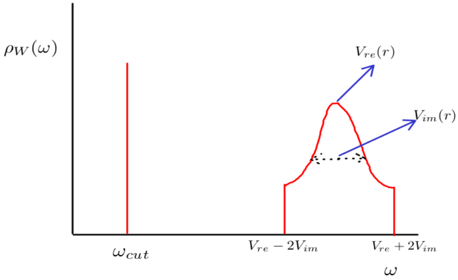
$$W_L^M(r,\tau) = \int_{-\infty}^{\infty} d\omega\, \rho^M(\omega,T)\exp(-\omega\,\tau)$$

$$W^M(r,t) = \int_{-\infty}^{\infty} d\omega\, \rho^M(\omega,T)\exp(-i\omega\,t)$$



A. Rothkopf et al., PRL. 108 (2012) 162001

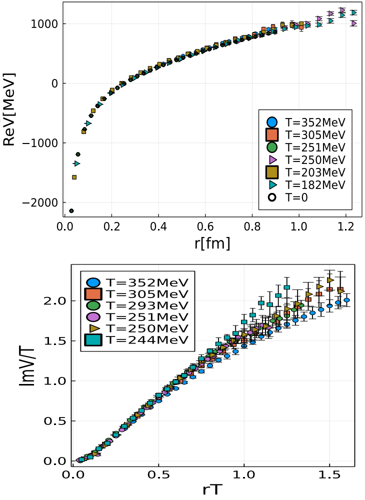
Cut-Lorentzian



- **Ad-hoc cut-off applied on the Lorentzian.**
- **Existence of low-ω delta function.**

$$V(r) = i \lim_{t \to \infty} \frac{\partial \log W^M(r,t)}{\partial t} = \omega_{cut}!$$

- **Unstable fit.**



R. Larsen et al., PRD. 109, 074504

HTL Inspired Spectral Function

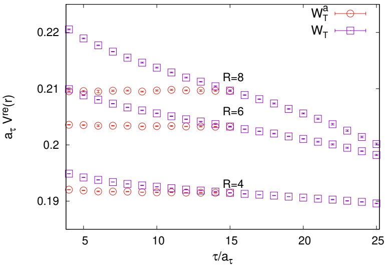
Perturbatively

$$W_E(r,\tau) = W_p(r,\tau)\exp(-C\tau)$$

$$W_p(r,\tau) = W_p\left(r, \tau - \frac{1}{T}\right)$$

$$m_{eff}(r,\tau) = \log\left(\frac{W(r,\tau)}{W(r,\tau+a)}\right)$$

$$V^{re}(r,\tau) = \frac{1}{\frac{\beta}{2}-\tau}\ \log\left[\frac{W(r,\tau)}{W(r,\beta-\tau)}\right]$$



This behavior also holds non-perturbatively.

$$i\lim_{t\to\infty}\frac{\partial \log W(r,\tau\to it)}{\partial t} = V_{re}(r) - iV_{im}(r)$$

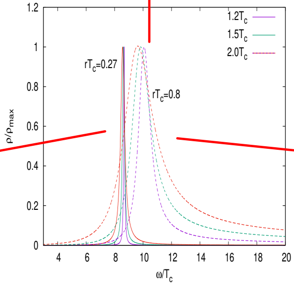
These two conditions put strong constraints on the spectral function.

DB and S. Datta, PRD 101, 034507

HTL Inspired Spectral Function: $\omega \sim V_{re}$:

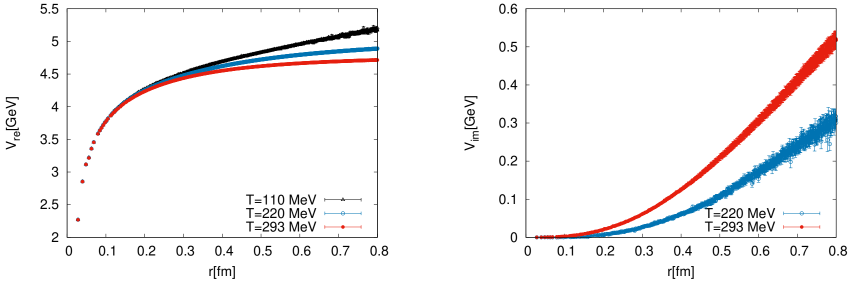
$$\rho_{\text{low}}(r;\omega \sim V_{re}) \approx \sqrt{\frac{2}{\pi}}\frac{V_{\text{im}}}{(V_{\text{re}}-\omega)^2+V_{\text{im}}^2}$$

$$\approx \exp(-(V_{re}-\omega)/T)$$



$$\approx \frac{1}{(\omega - V_{re})^{\alpha}}$$

DB and S. Datta, PRD 103, 014512 DB, O. Kaczmarek et al., PRD 105, 054513



Lattice QCD data supports existence of color screening.

DB, O. Kaczmarek et al, arxiv:2412.17570

DB and S. Datta, PRD 101, 034507

DB, O. Kaczmarek et al., PRD 105, 054513

Octet Potential

- Color octet potential input to open-quantum system approach to quarkonia. see also talk by Rishi Sharma
- Gauge invariance issue on lattice.
- Octet operator:

$$O^a = \bar{\psi} U T^a U \psi$$

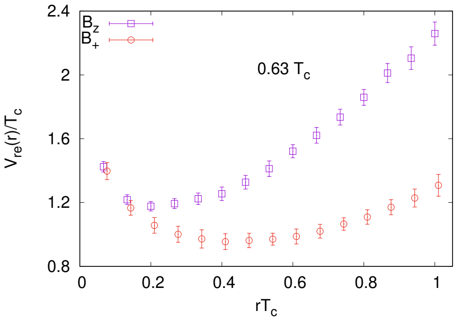
- Hybrid operator:

$$O = \bar{\psi} U T^a P^a U \psi$$

- Attached operator decouples at leading order.

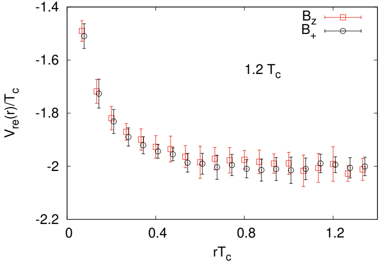
$$V_O(r) = \frac{\alpha}{6r}$$

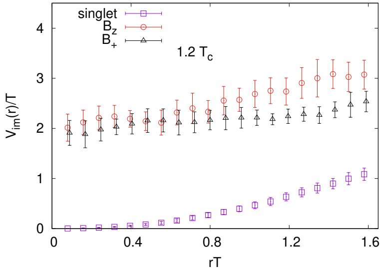
- $P^a = B_z, \; B_x + iB_y$



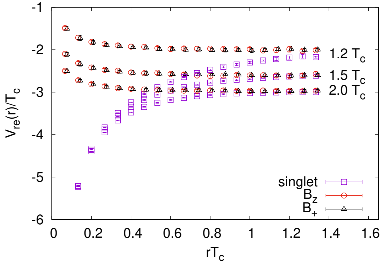
- Octet potential = operator-independent part of hybrid potential.

DB and S. Datta, PRD 103, 014512





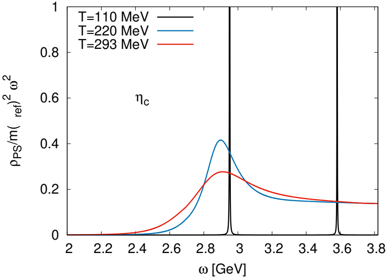
- The operator decouples at all distances in the real part.
- The imaginary part is independent up to $rT \sim 1$ and is non-zero at short distances.
- The real parts of the octet and singlet match at large distances, providing strong evidence of color screening.

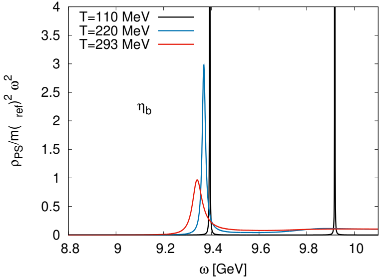


DB and S. Datta, PRD 103, 014512

Quarkonia Spectral function

$$\left\{ i\partial_t - \left[2M + V_T(r) - \frac{\nabla^2_{\vec{r}}}{M} \right] \right\} C_>(t; \vec{r}, \vec{r'}) = 0$$

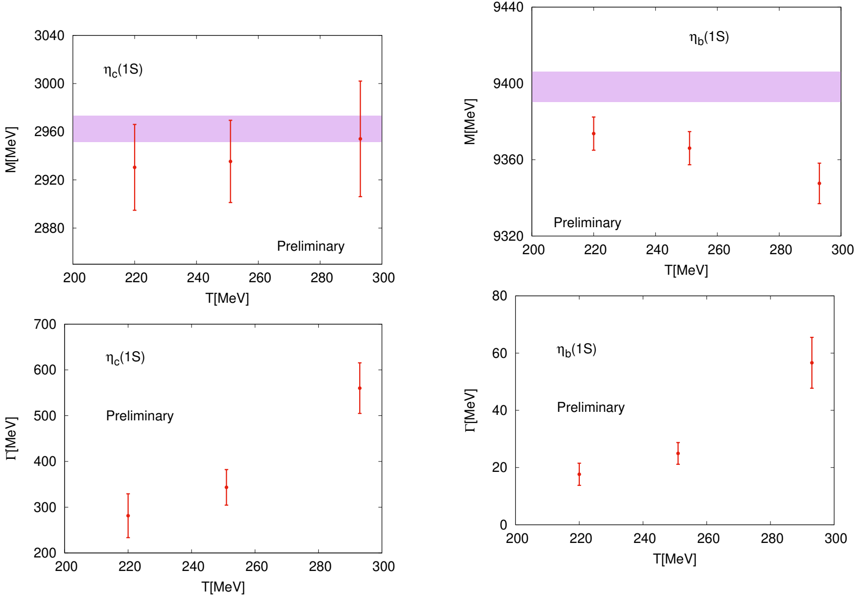




- (1S) state for bottom melts much after T_c ($T_c = 180MeV$)
- Significant thermal effects on charmonium state.

Quarkonia spectral function at finite momentum talk by Pavan
Determination of screening mass from spatial potential talk by Swagatam Tah

Thermal mass and Decay width



- $\Gamma_c(1S) \gg \Gamma_b(1S)$

Diffusion of Heavy Quarks

Diffusion scale $M/T^2 \gg 1/T$. Possible to describe by Langevin equation:

$$\frac{dp_i}{dt} = -\eta_D p + F_i(t)$$

$$\langle F_i(t)F_j(t')\rangle = \kappa\,\delta_{ij}\,\delta(t-t')$$

$$\kappa = \frac{1}{3}\int_0^\infty dt\langle F_i(t)F_i(0)\rangle_T$$

$$\vec{F} = g(\vec{E} + \vec{v}\times\vec{B})$$

$$\kappa = \frac{g^2}{3}\int_0^\infty dt(\underbrace{\langle E_i(t)E_i(0)\rangle_T}_{O(M_q^0)} + \frac{2}{3}v^2\underbrace{\langle B_i(t)B_i(0)\rangle_T}_{O(M_q^{-1})})$$

$$\kappa = \kappa_E + \frac{2}{3}v^2\kappa_B$$

M. Laine et al,JHEP 0904:053,2009
M. Laine et al, JHEP 12 (2020) 150

$$G_O(\tau) = -\frac{\mathrm{ReTr}(U(\beta,\tau)O_i(\tau)U(\tau,0)O_i(0))}{3\mathrm{Tr}(U(\beta,0))}$$

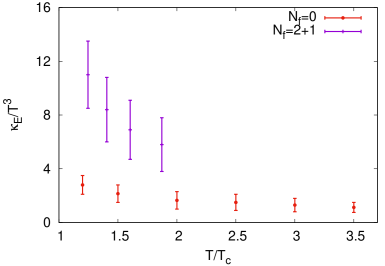
A similar kind of observable also appears in the gluon dissociation width Talk by Saumen Datta

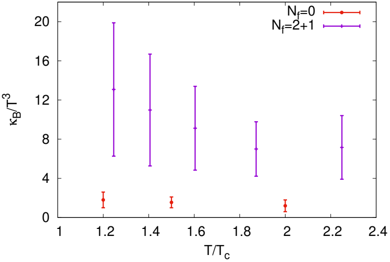
$$G_O(\tau) = \int_0^\infty \frac{d\omega}{\pi}\rho_O(\omega)\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

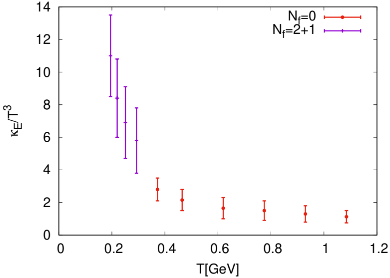
$$k_E = 2T\lim_{\omega\to 0}\frac{\rho_E(\omega)}{\omega}$$

Banerjee et al, PRD, 85, 014510
Altenkort et al, PRD, 103.014511

$$k_B = 2T\lim_{\omega\to 0}\frac{\rho_B(\omega)}{\omega}$$





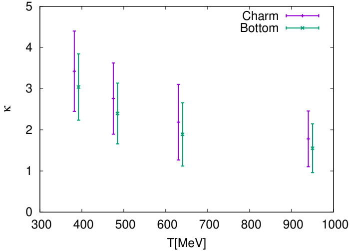


$N_f = 0, \kappa_B$
Banerjee et al, JHEP 12 (20) 150
$N_f = 2+1, \kappa_B$
Altenkort et al, PRL, 132.(2024).051902

- Much of difference between $N_f = 0$ and $N_f = 2+1$ originates from different value T_c and different coupling strength.
- Estimate of k in pre-equlibrium stage. talk by Harshit Pandey

$N_f = 0, \kappa_E$ Banerjee et al, Nuclear Physics A, 2023.122721
$N_f = 2+1, \kappa_E$ Altenkort et al, PRL, 132 (2024) 5, 051902

$\kappa = \kappa_E + \frac{2}{3} v^2 \kappa_B$
$v^2 = 3T/M_q$
Banerjee et al, Nuclear Physics A, 2023.122721



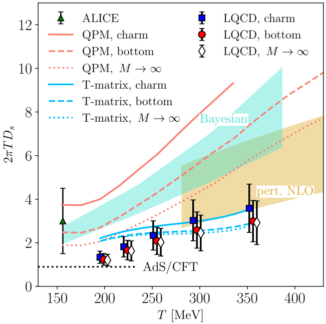
Position space diffusion coefficient:

$$D\nabla^2 n(\vec{x}, t) = \frac{\partial n(\vec{x}, t)}{\partial t}$$

Average distance covered $\langle x(t)^2 \rangle = 6Dt$
Fluctuation Dissipation theorem:

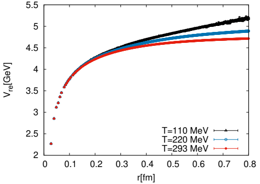
$$D = \frac{2\,T^2}{\kappa}$$

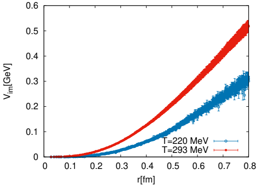
Here
Altenkort et al, PRL, 132 (2024) 5, 051902



Summary

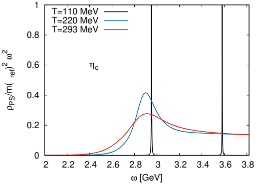
Thermal Potential

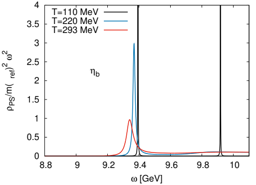




Color Screening exist

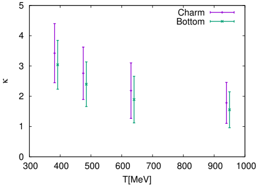
Fate of Quarkonia

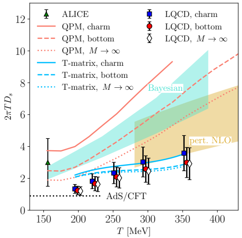




The charm state has a much larger thermal decay width.

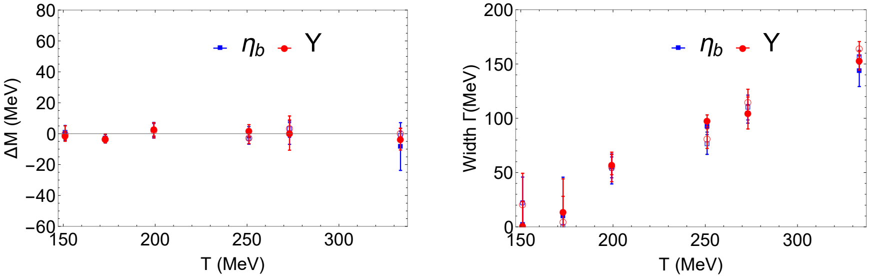
Heavy quark diffusion



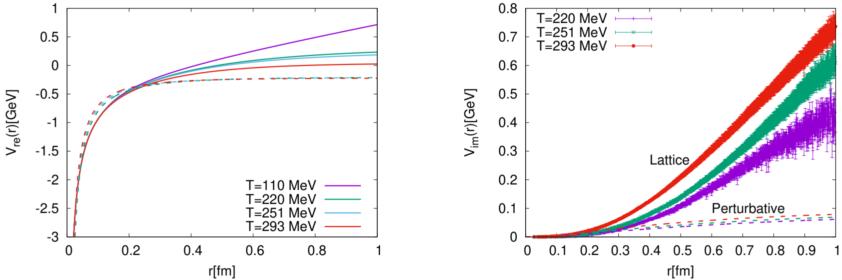


Large momentum transport coefficient

Lattice NRQCD correlator fitted with Gaussian spectral function.



- Why the peak position is identified with mass, as Gaussian is an entire function.
- What does the width of Gaussian has to do with the thermal decay width.



$$V_T^{re}(r) = -\frac{g^2}{4\pi} C_F \left[m_d + \frac{\exp(-m_d r)}{r} \right]$$

$$V_T^{im}(r) = \frac{g^2}{4\pi} C_F T \int_0^\infty \frac{zdz}{(z^2+1)^2} \left[1 - \frac{\sin(z m_d r)}{z m_d r} \right]$$

- Non-perturbative thermal potential is very much different from the perturbative potential.

$$\log(W(r,\tau)) = -V_{re}(r)\tau - \int_{-\infty}^{\infty} du\, \sigma(r,u) \left[\exp(u\,\tau) + \exp(u\,(\beta-\tau))\right] + ..$$

- $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t} = \text{finite} \implies \lim_{u\to 0} \sigma(r,u) \sim \frac{1}{u^2}$
- Following HTL PT, $\sigma(r,u) = n_B(u)\left[\frac{V_{im}}{u} + c_1\, u + c_3\, u^3 + ...\right]$
- Parametrization

$$W(r,\tau) \;=\; A \exp\left[-V_{re}(r)\tau - \frac{\beta V_{im}(r)}{\pi} \log\left(\sin\left(\frac{\pi\tau}{\beta}\right)\right) + ...\right]$$

DB and S. Datta, PRD 101, 034507