Recent results from lattice QCD on the phase diagram

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Plenary

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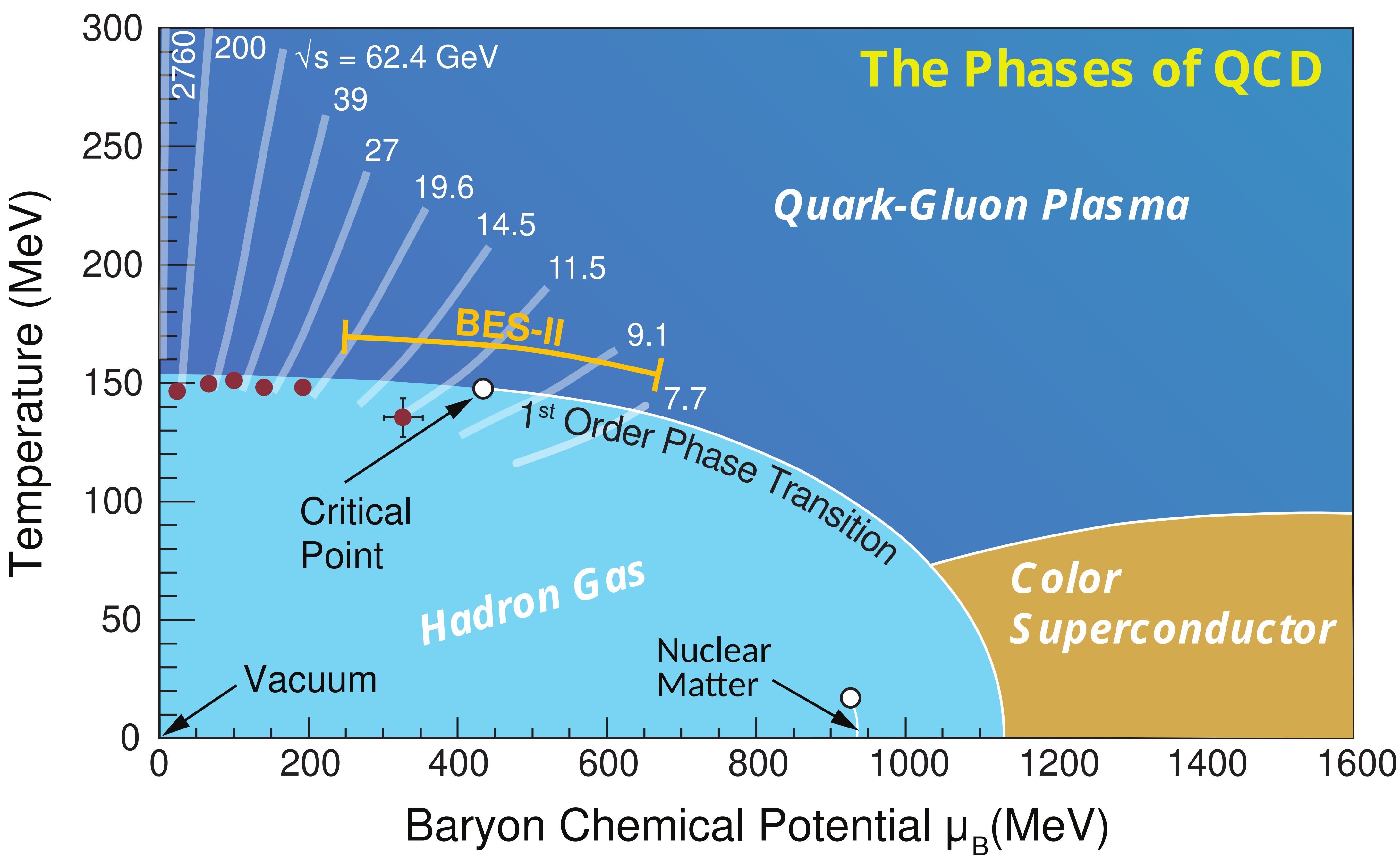
QCD phase diagram

“Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan”, Bzdaket al., Phys. Rept. ‘20



QCD phase diagram

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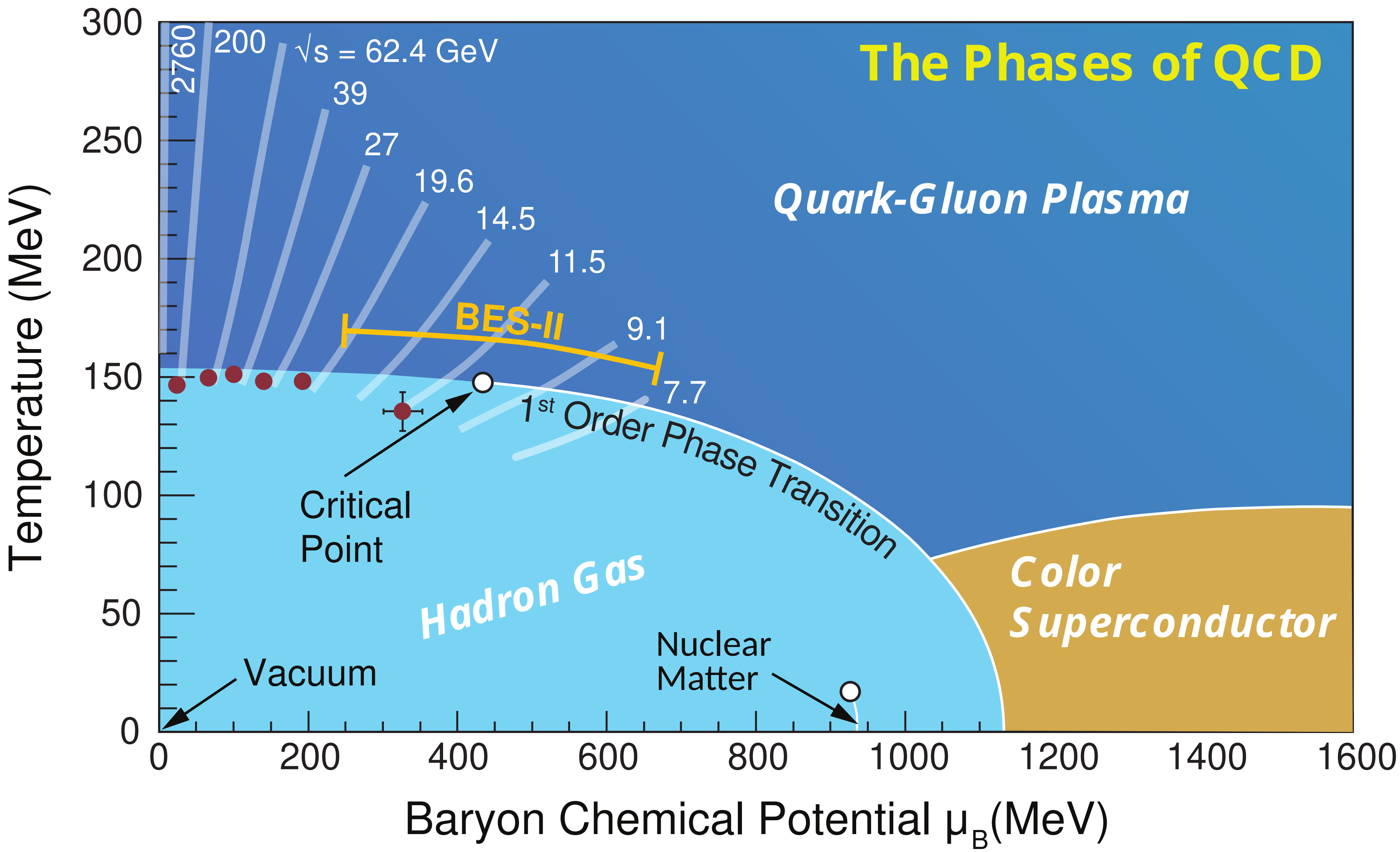


Motivation: Understand the thermodynamics at the QCD chiral transition and exploration of the QCD phase diagram with lattice QCD numerical simulation.

Chemical potential on the lattice

Partion function for (2+1)-flavor QCD,

$$\mathcal{Z}_{QCD} = \int \mathcal{D}U \; \det[M(m_u, \mu_u)]\det[M(m_d, \mu_d)] \; \det[M(m_s, \mu_s)] \; e^{-S_G(U)}$$

Continuum prescription, Divergence for the free fermion case in second order susceptibility: $\chi_2 \sim 1/a^2$

The prescription for chemical potential on the lattice,

$$(1 \pm \gamma_4)U_{\pm 4}(x) \rightarrow (1 \pm \gamma_4)e^{\pm \hat{\mu}}U_{\pm 4}(x)$$

P. Hasenfratz, F. Karsch, Phys.Lett.B 125 (1983) 308-310
R. V. Gavai,Phys. Rev. D 32, 519

No additional divergences appear in the interacting theory.

Steven Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. Lett. 59, 2247.
Rajiv V. Gavai, Sayantan Sharma, *Phys.Lett.B* 749 (2015) 8-13

Sign problem for, $\mu_f \neq 0, \; f = \{u, d, s\}$. We use Taylor expansions.

Rajiv V Gavai, Sourendu Gupta, Phys. Rev. D **71**, 114014

Saumen Datta, Rajiv V. Gavai, Sourendu Gupta, arXiv:1210.6784 **[hep-lat]**

Quark number susceptibility and conserved charge fluctuations in (2+1)-flavor QCD

In QCD with two light (u, d) and one strange flavor (s), pressure is expressed via a Taylor expansion as,

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k .$$

Generalized susceptibilities at zero chemical potential

In the context of heavy ion collision experiments there are 3 **conserved charges**, **B, Q and S** that couples to μ_B, μ_Q, μ_S,

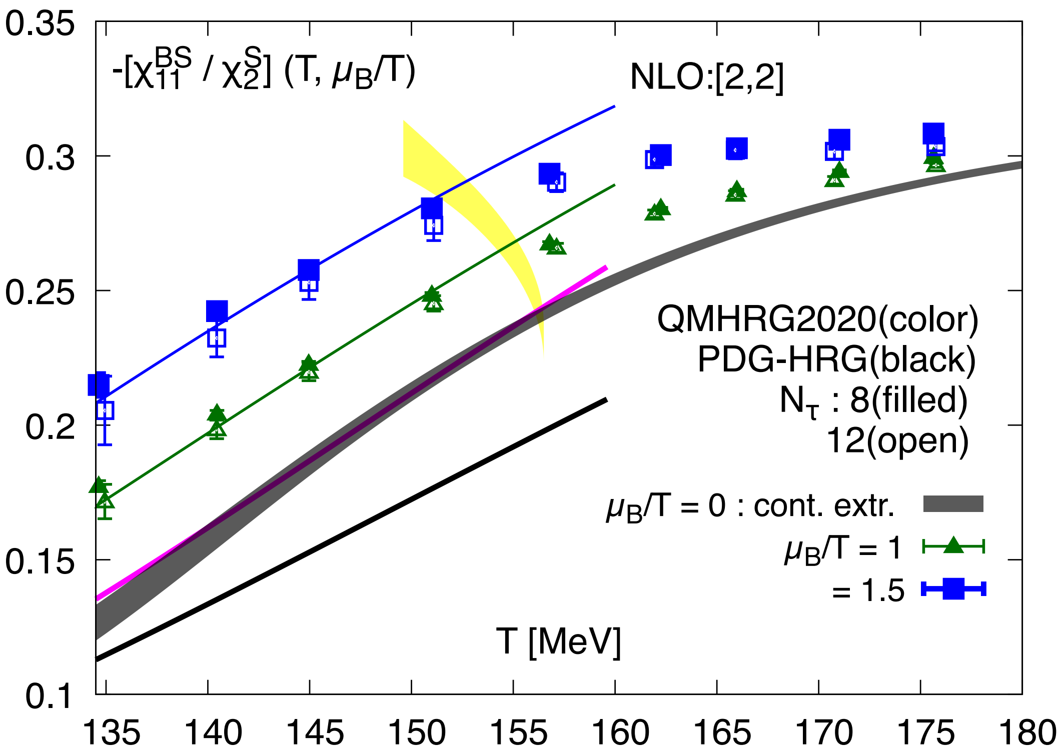
The condition satisfy : $n_S = 0$ (strangeness neutral) and $n_Q/n_B = 0.4$

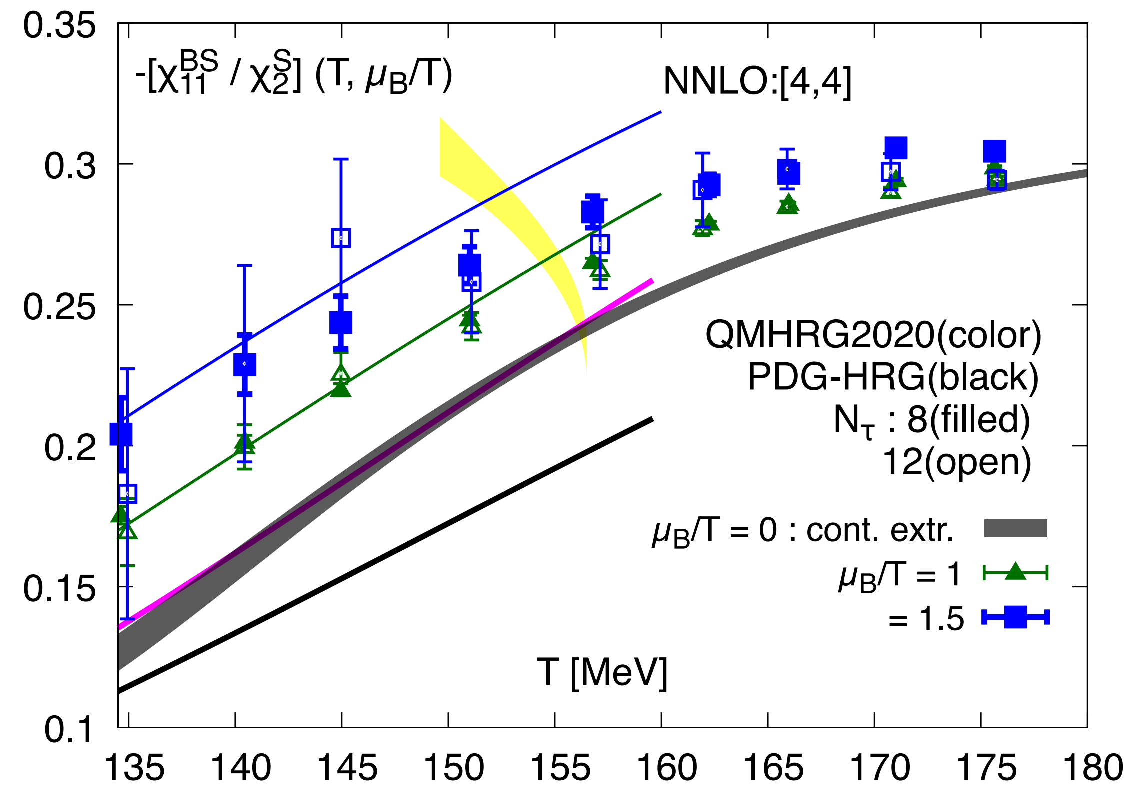
We satisfy this two conditions order by order:

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + ...$$

$$\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + ...$$

Baryon strangeness correlations in (2+1)-flavor QCD with HISQ fermions





D. Bollweg et al, Phys.Rev.D 110 (2024) 5, 054519

The pseudo critical line from lattice QCD,

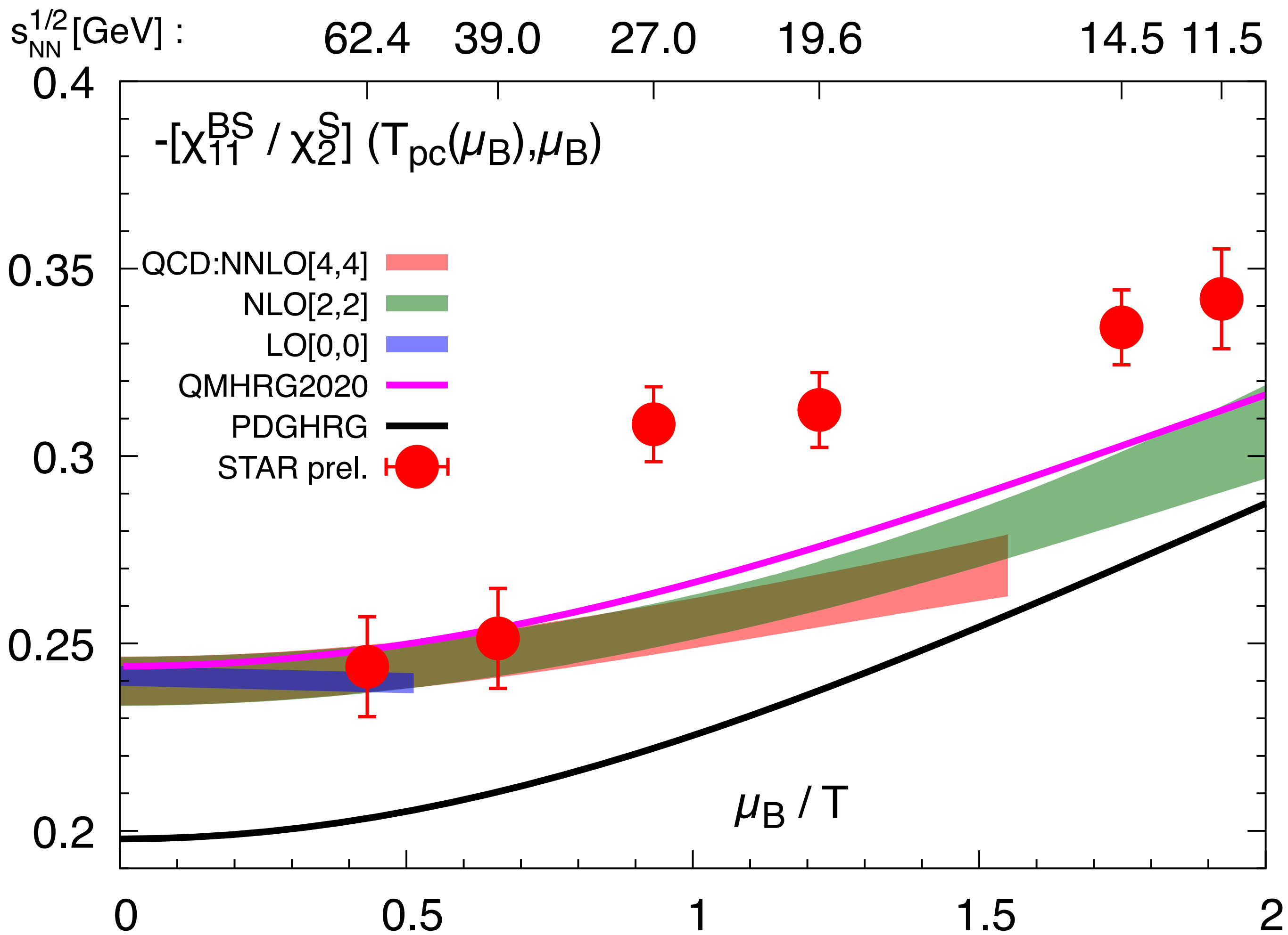
$$T_{pc}(\mu_B) = T_{pc,0}\left[1 - \kappa_2 \hat{\mu}_B^2 + \kappa_4 \hat{\mu}_B^4\right]$$

A. Bazavov et al. (HotQCD), Phys. Lett. B 795, 15 (2019);
B. S. Borsanyi et al, Phys. Rev. Lett. 125, 052001 (2020)

$$T_{pc,0} = (156.5 \pm 1.5)\ \text{MeV} \quad \text{and} \quad \kappa_2 = 0.012(4)$$

$$\frac{\chi_{11}^{BS}(T,\mu_B/T)}{\chi_2^S(T,\mu_B/T)} = \frac{\chi_{11}^{BS}(T,0) + ()\hat{\mu}_B^2 + ()\hat{\mu}_B^4 + \ldots}{\chi_2^S(T,0) + ()\hat{\mu}_B^2 + ()\hat{\mu}_B^4 + \ldots}$$

Baryon-strangeness correlations



QCD and STAR results are in good agreement for $\sqrt{s_{NN}} \geq 39$ GeV.

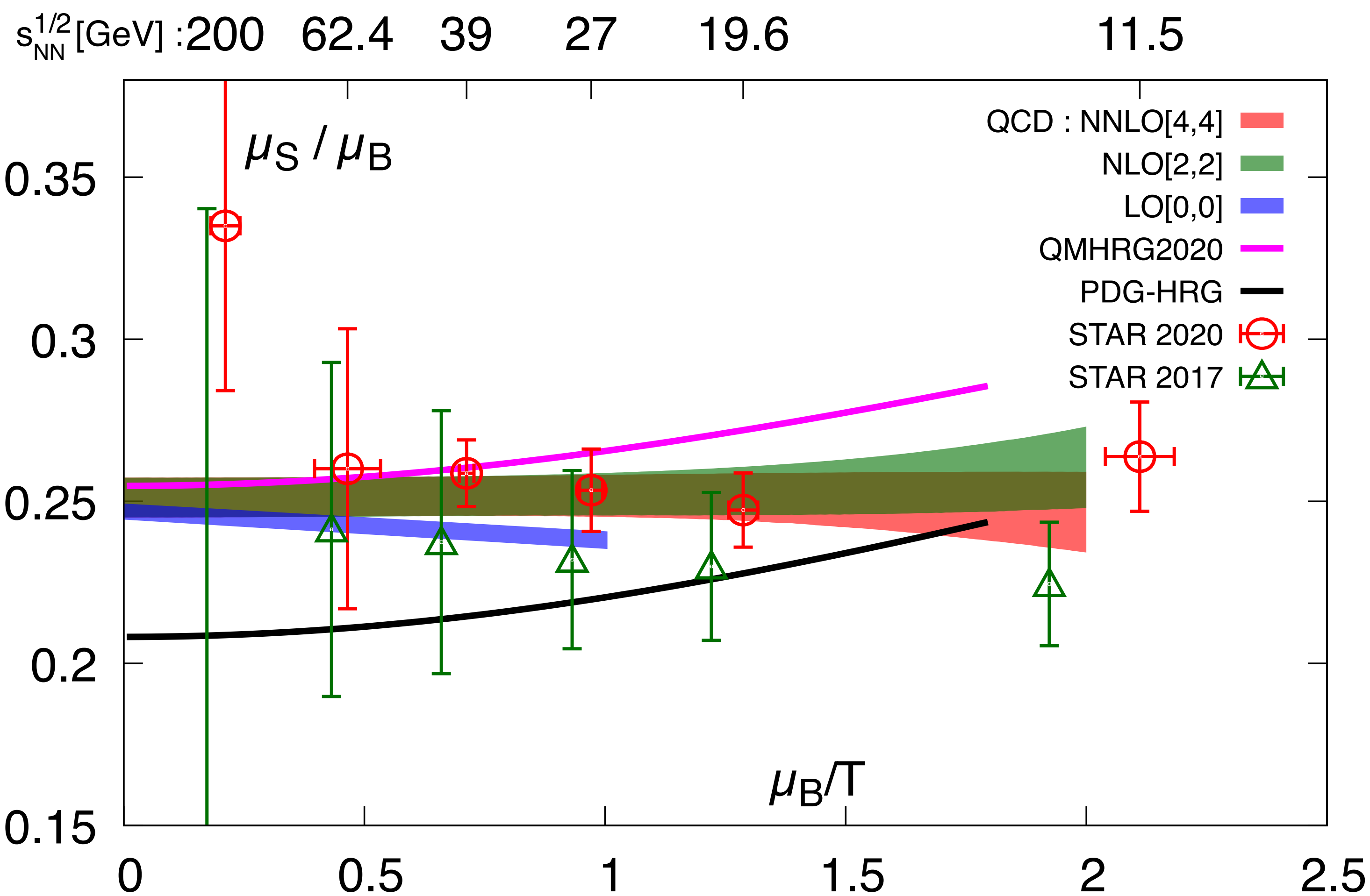
Significant differences between QCD and STAR results for $\sqrt{s_{NN}} \leq 27$ GeV.

The Lattice results agree more closely with the QMHRG2020 predictions.

HotQCD 2024 : *Phys.Rev.D* 110 (2024);

STAR results : CPOD2024

Ratio of μ_S/μ_B



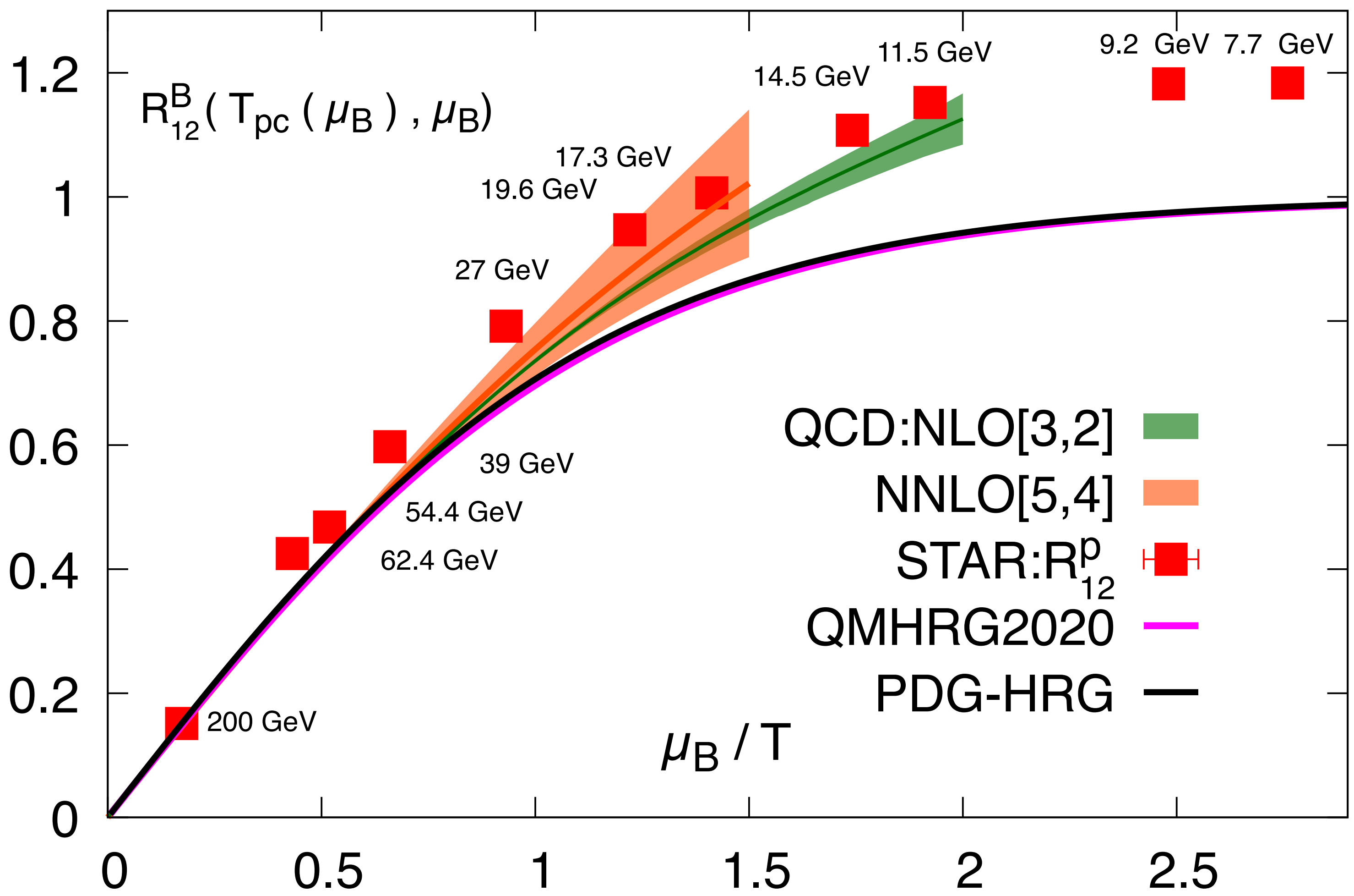
J. Adam et al. (STAR), Phys. Rev. C 102, 034909 (2020);
L. Adamczyk et al. (STAR), Phys. Rev. C 96, 044904 (2017)

QCD and STAR results are in good agreement for almost all the beam energies.

Significant differences between QCD and STAR results for $\sqrt{s_{NN}} = 200$ GeV.

The Lattice results agree more closely with the QMHRG2020 predictions.

Baryon number fluctuations



In good agreement with the STAR data down to $\sqrt{s_{NN}} \simeq 11.5$ GeV.

R^p_{12} becomes larger than unity for $\sqrt{s_{NN}} \simeq 17.3$ GeV.

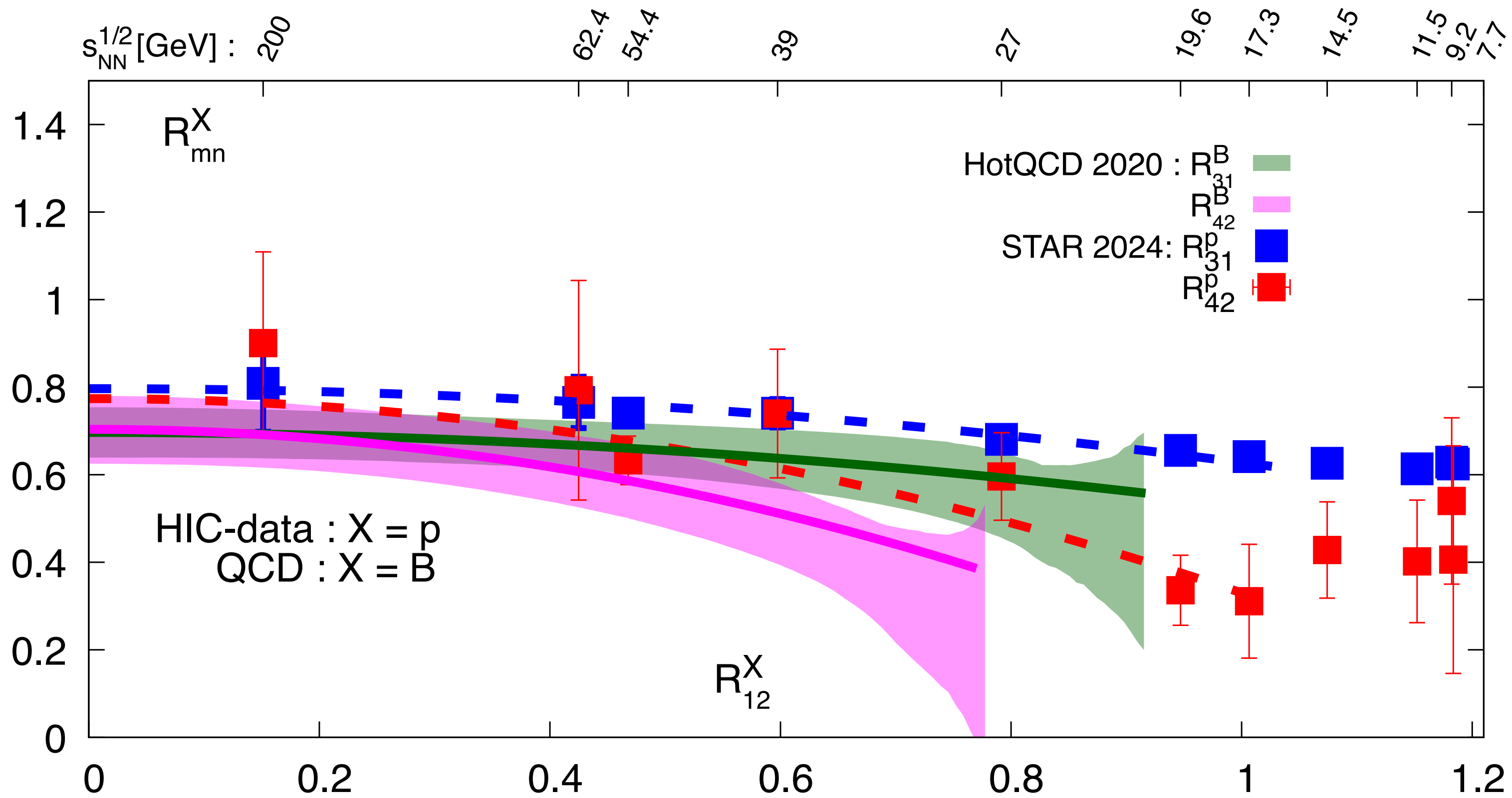
Consistent with QCD but not consistent with non interacting HRG.

$$R^B_{12} = M_B/\sigma^2_B = \frac{\chi^B_1}{\chi^B_2}$$

HotQCD 2017, 2020 : PRD; Goswami, Karsch , XQCD 2024

STAR results : CPOD2024

Higher order baryon number fluctuation



$$R_{31}^X = S_0^X + S_2^X \left(R_{12}^X\right)^2$$

$$R_{42}^X = K_0^X + K_2(R_{12}^X)^2$$

QCD and STAR results are in good agreement for $\sqrt{s_{NN}} \geq 19.6$ GeV.

$\mathcal{O}((R_{12}^B)^2)$ QCD results and quadratic fit to STAR results for R_{31}^p and R_{42}^p agree well on curvature coefficient.

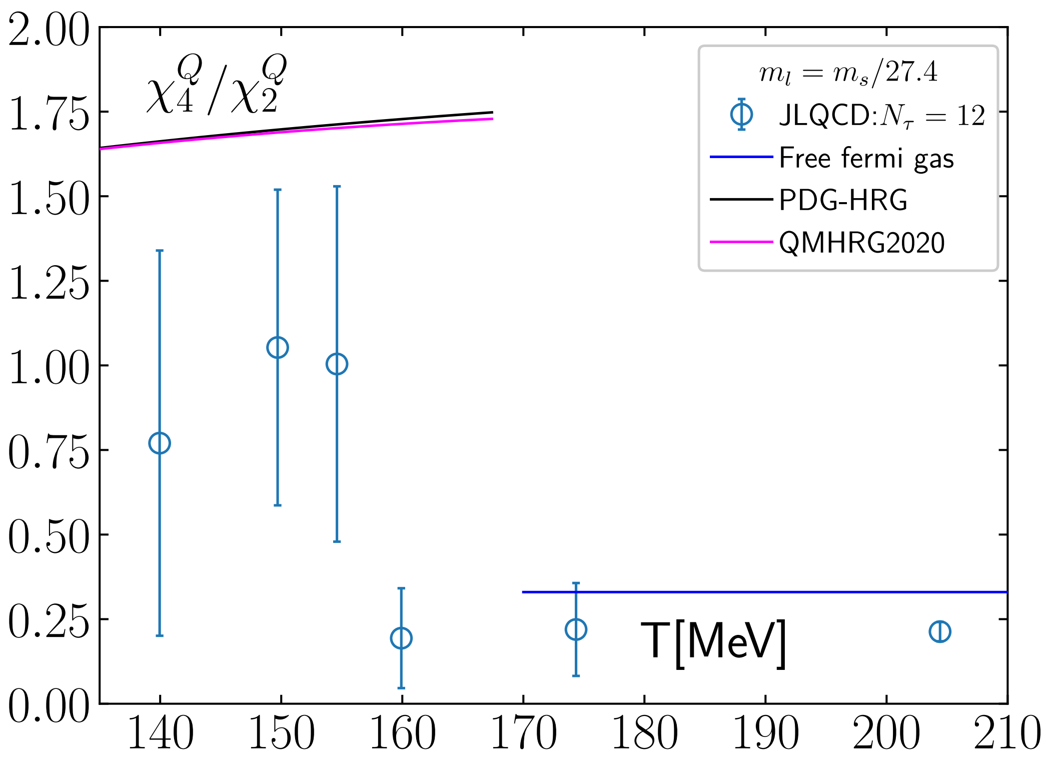
$$R_{31}^B = S_B\sigma_B^3 = \frac{\chi_3^B}{\chi_1^B} \quad R_{42}^B = \kappa_B\sigma_B^2 = \frac{\chi_4^B}{\chi_2^B}$$

$$S_0^p = 0.80(1)\ , S_0^B = 0.70(1)\ , K_0^p = 0.77(6)\ ; K_0^B = 0.705(1)$$

May suggest T_{pc} is slightly smaller than T_f

HotQCD 2017, 2020 : PRD; Goswami, Karsch , XQCD 2024
STAR results : CPOD2024

LO Kurtosis of electric charge and strangeness correlations



Jishnu Goswami et al, arXiv:2501.03509

Calculations with a chiral symmetric fermions, Möbius Domain Wall fermions.

$$R_{42}^Q = 1 \pm 0.53, \quad \text{for} \quad T = 154.6\ \text{MeV}.$$

$$R_{42}^Q = 1.05 \pm 0.49, \quad \text{for} \quad T = 149.7\ \text{MeV}$$

The results are preliminary and not continuum extrapolated.

Acknowledgments :

Supercomputer Fugaku(HPCIprojecthp240295, hp230207, hp200130, hp210165, hp220174 and Usability Research ra000001). MEXT as "Program for Promoting Researches on the Supercomputer Fugaku",*Simulation for basic science: from fundamental laws of particles to creation of nuclei*, JPMXP1020200105; "Simulation for basic science: approaching the quantum era" (JPMXP1020230411). JICFuS.

Searching for the QCD CEP with LYEs singularities

Partion function for (2+1)-flavor QCD,

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$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

Padé approximant : Possible extension of the Taylor series for exploring the low temperature and high density part of the QCD phase diagram.

EoS : **D. Bollweg et al(HotQCD coll.), *Phys.Rev.D* 108 (2023), JG (HotQCD coll.), *PoS* LATTICE2022 (2023) 149, JG QM2022**

Searching for CEP using Padé approximants

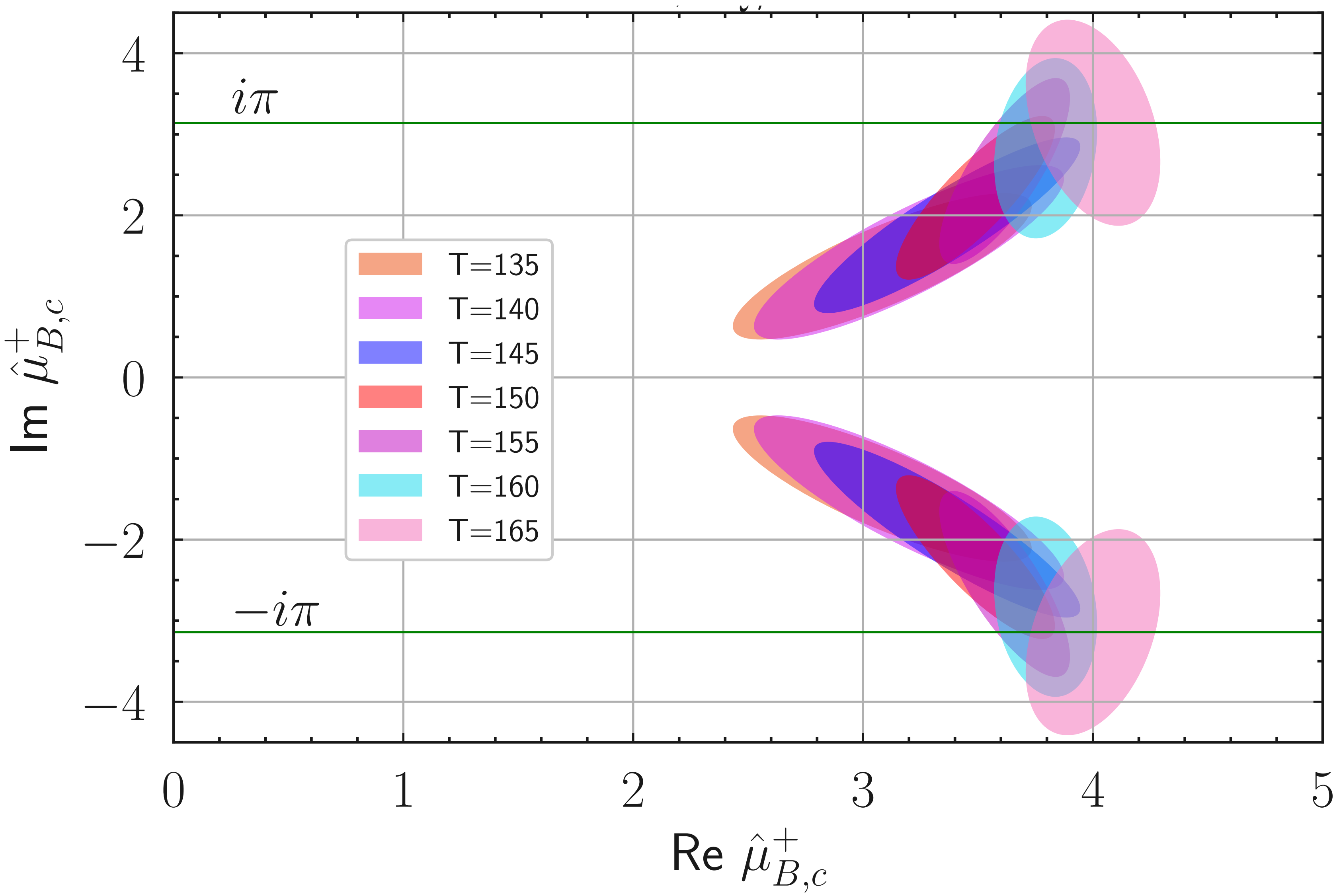
We only have finite number of Taylor coefficients.

$$f(x) = \sum_{i=0}^{n} c_i x^i$$

- Lee Yang : Phase transitions are related to singularities of the Taylor series on the real axis.
- Padé approximants : Rational functions of the form, $f(x) = \dfrac{\sum_{i=0}^{a} c_i x^i}{1 + \sum_{j=1}^{b} d_j x^j}$,
- Singularities : Solving the denominators.
- Furthermore, LYE singularities exhibit universal scaling behavior near a critical point

Complex zeros of the partition function → Investigate the universal scaling of the zeros of the partition function.

Location of the critical point at finite μ_B ??



Singularity of the pressure series using a [4,4] padè constructed from 8th order Taylor series

Expectation :

$$T_{CEP} < T_{chiral}$$

$$(T_{chiral} \sim 130\ \mathrm{MeV})$$

H.T. Ding *et al*, *Phys.Rev.Lett.* 123 (2019) 6, 062002

D. Bollweg et. al (HotQCD collaboration), Phys.Rev.D 105 (2022) 7, 074511,
J. G et. al (HotQCD collaboration), Acta Phys.Polon.Supp. 16 (2023) 1, 76

Bound for CEP :

$$T^{CEP} < 135\ \mathrm{MeV},\ \hat{\mu}_\mathrm{B}/\mathrm{T} \geq 2.5$$

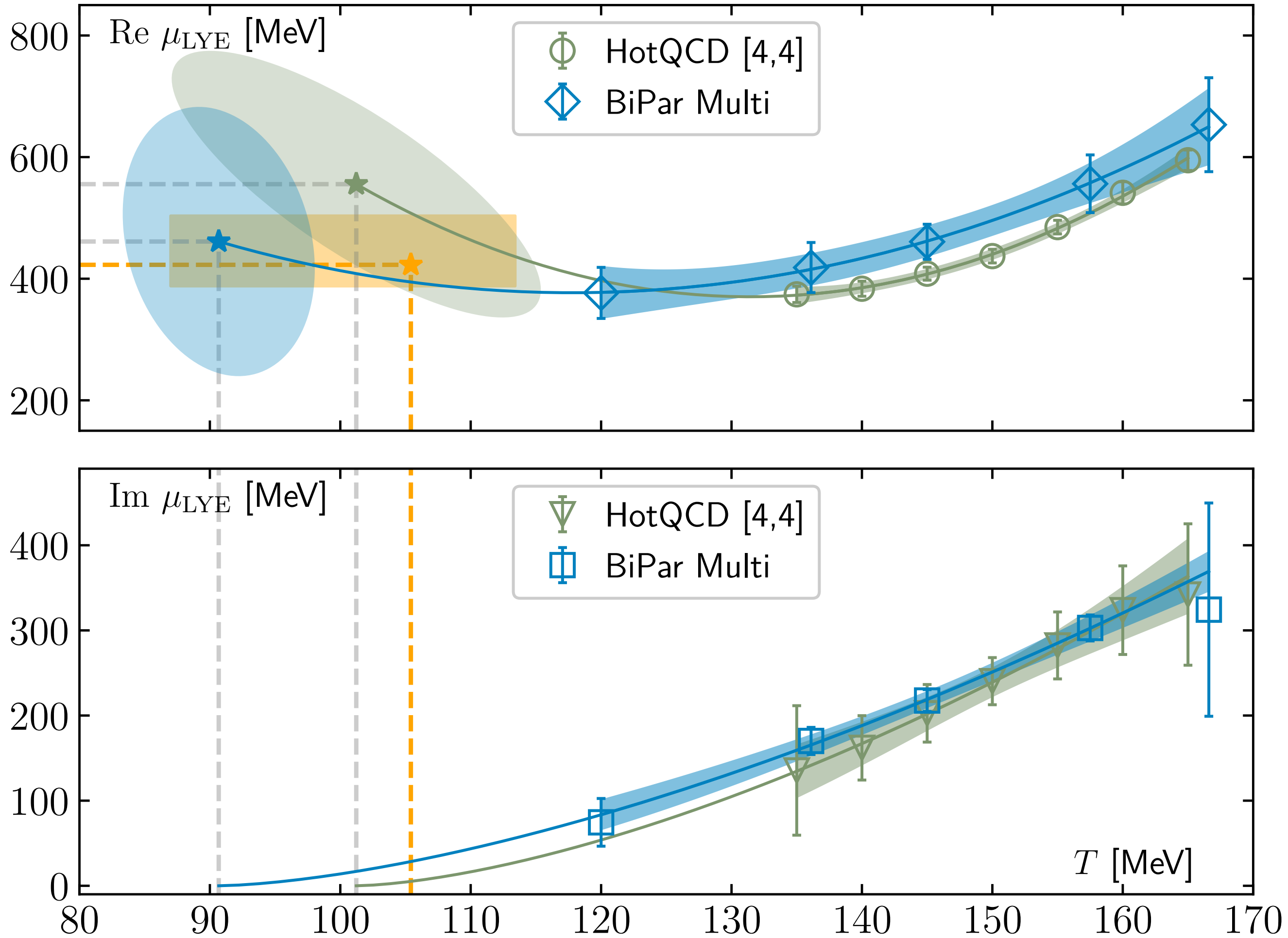
- Lee-yang theorem: Singularity in the real axis is a hint for a critical point.
- We find no indication of a CEP in almost the entire beam energy ($\sqrt{s}$) range covered by BESII in collider mode.

Searching for the QCD CEP with LYEs singularities

Partion function for (2+1)-flavor QCD,

$$\mathcal{Z}_{QCD} = \int \mathcal{D}U \det[M(m_u, \mu_u)]\det[M(m_d, \mu_d)] \det[M(m_s, \mu_s)]\, e^{-S_G(U)}$$

Sign problem for, real μ_f however, one can do calculations with purely imaginary $i\mu_f$.



$(T_{CEP}, \mu_{CEP}) = (105^{+8}_{-18}, 422^{+80}_{-35})$ MeV

David A. Clarke et al, arXiv:2405.10196 [hep-lat]

Caution :This results are not continuum extrapolated!!

Summary and Conclusions

- We present comparisons of conserved charge fluctuations using (2+1)-flavor lattice QCD and STAR results.
- We also present estimation of QCD CEP from LYEs.

Thank you for your attention !!