

# Quantum kinetic theory and spin polarization in relativistic heavy ion collisions

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**University of Science and Technology of China**

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2025.01.16

# Outline

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- **Global and local polarization at AA systems**
- **Spin polarization at pA system**
- **Other related topics and discussion**
- **Summary**

# Discovery of spin in physics



Wolfgang Pauli

1924

“two valued-ness not describable classically”



Ralph Kronig

1925

self-rotation of the electron



George Uhlenbeck

1925

Published the paper about spin



Samuel Goudsmit



Llewellyn H. Thomas

1926

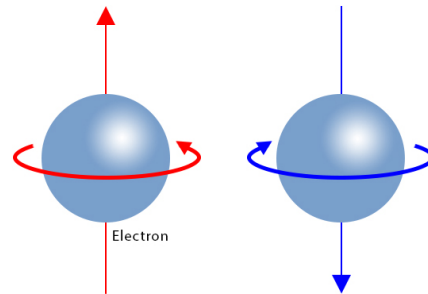
Thomas precession



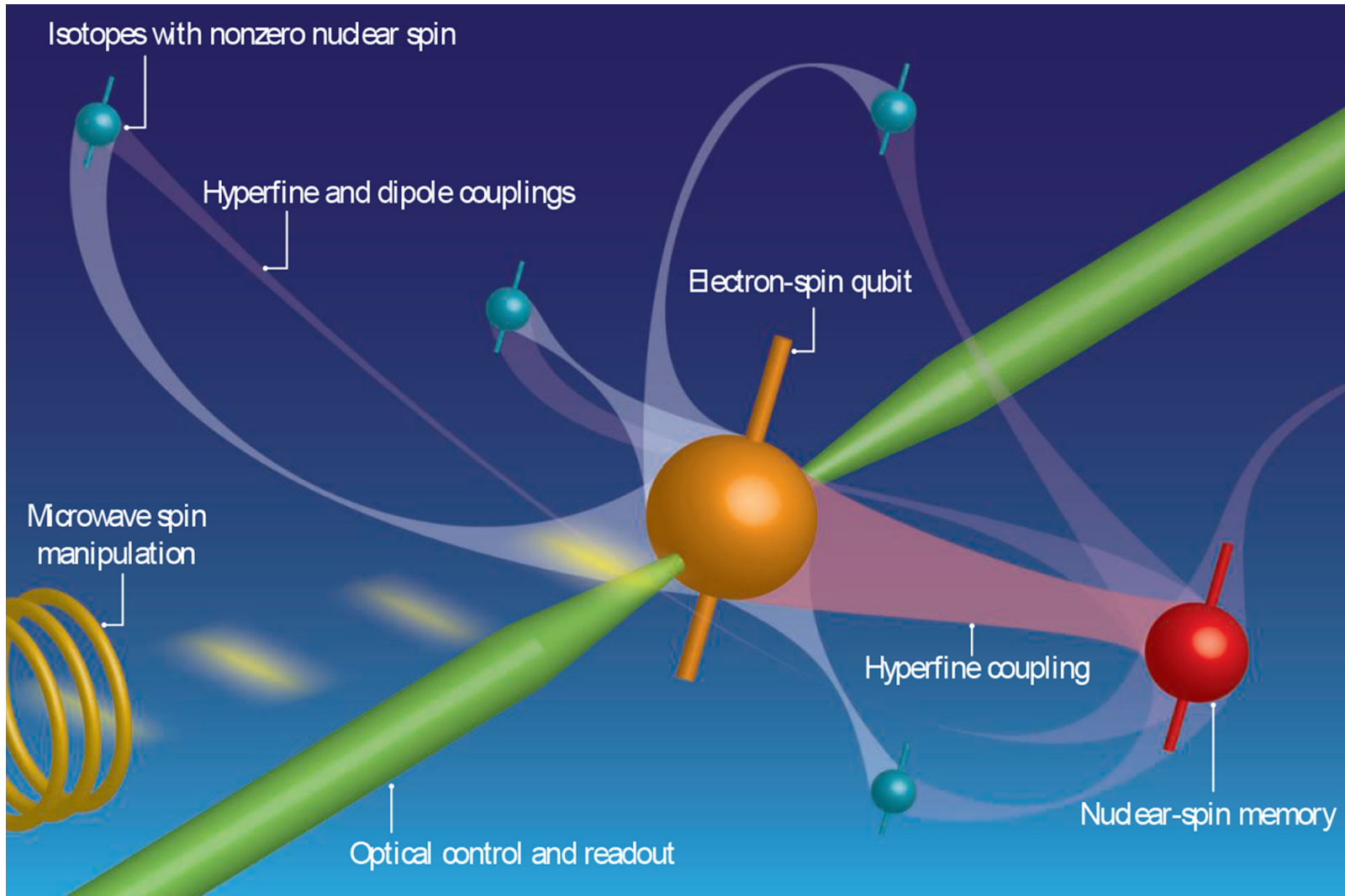
Wolfgang Pauli

1927

spin in quantum mechanics



# Spintronics





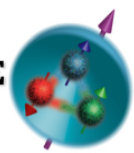
# Spin in high energy physics

Striking spin effects have been observed in high energy reactions since 1970s

## “Proton spin crisis” 质子自旋危机

夸克模型:

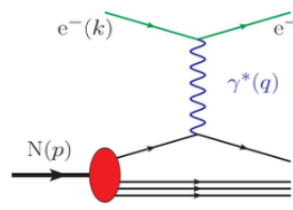
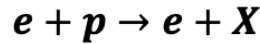
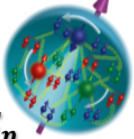
夸克自旋之和  
= 质子自旋  $S_p$



DIS实验:

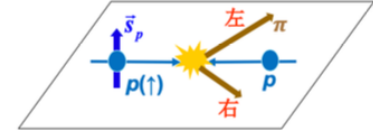
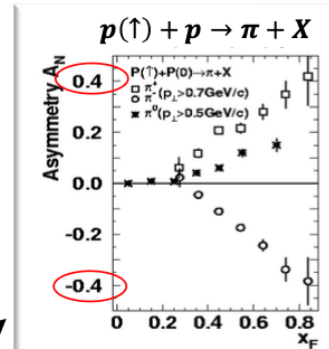
89年:  $\Sigma \sim 0$

目前:  $\Sigma \sim 20\% S_p$



EMC, PLB 206.364 (1988)

## “Single spin left-right asymmetry (SSA)”

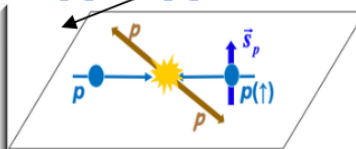
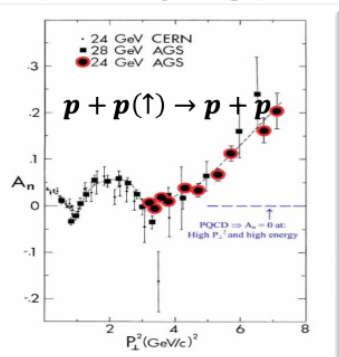


$$A_N \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

e.g. FNAL E704,  
PLB264, 462 (1991)

Predictions of pQCD  $\sim 0$

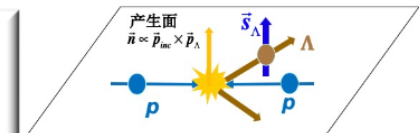
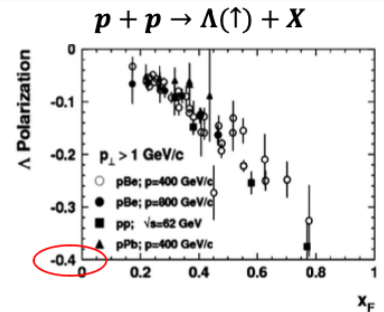
## “Spin analyzing power in $pp \rightarrow pp$ ”



$$A_N \equiv - \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

e.g. D. Grab et al.,  
PRL41, 1257 (1978)

## “Transverse polarization of hyperon in $pp \rightarrow \Lambda X$ ”

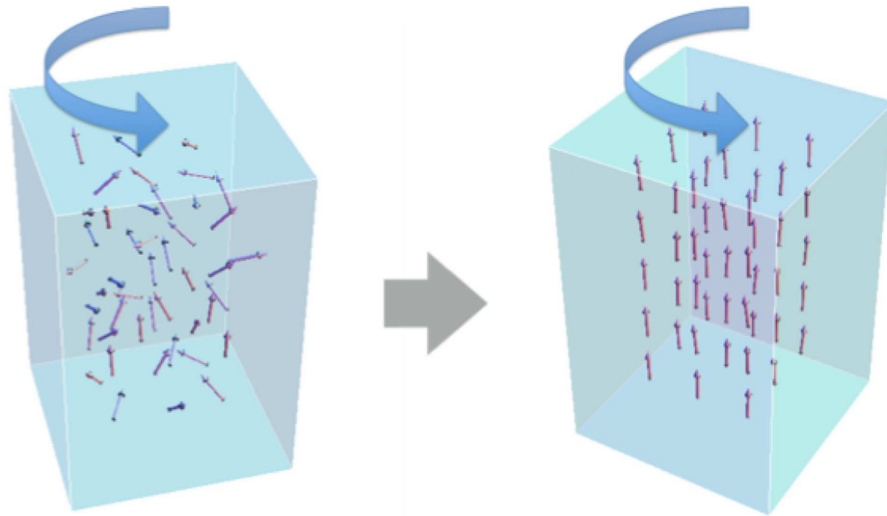


$$P_\Lambda \equiv \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$

e.g. S.A. Gourlay et al.,  
PRL56, 2244 (1986)

Slides copy from Prof. Zuo-tang Liang's review talk

# Barnett and Einstein-de Haas effects



## Barnett effect:

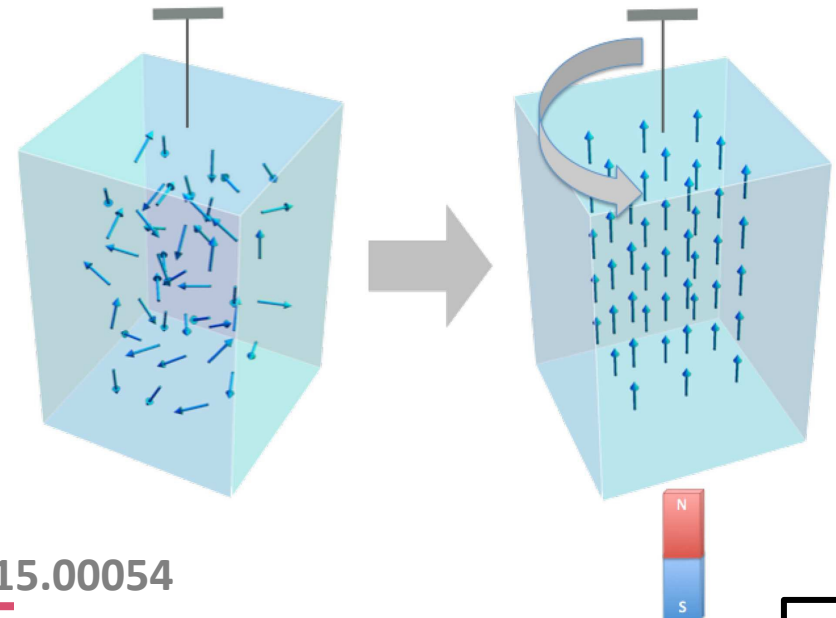
Rotation  $\implies$  Magnetization

*Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.*

## Einstein-de Haas effect:

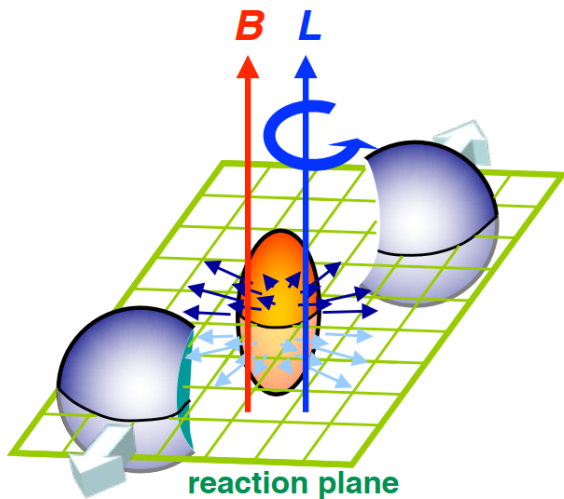
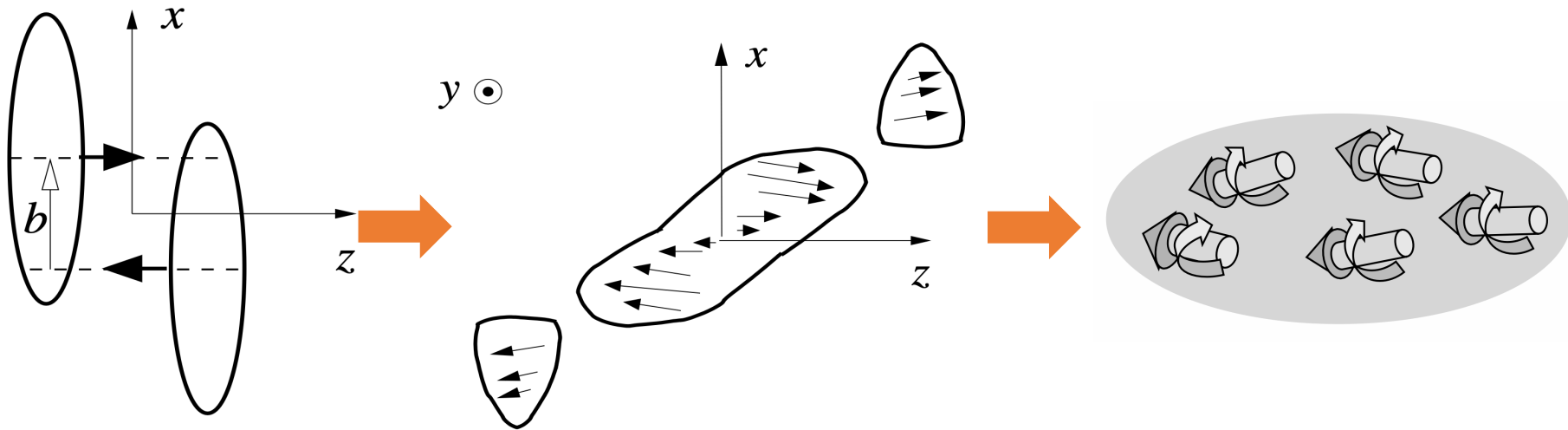
Magnetization  $\implies$  Rotation

*Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.*



Figures: copy from paper doi: [10.3389/fphy.2015.00054](https://doi.org/10.3389/fphy.2015.00054)

# OAM to spin polarization in HIC



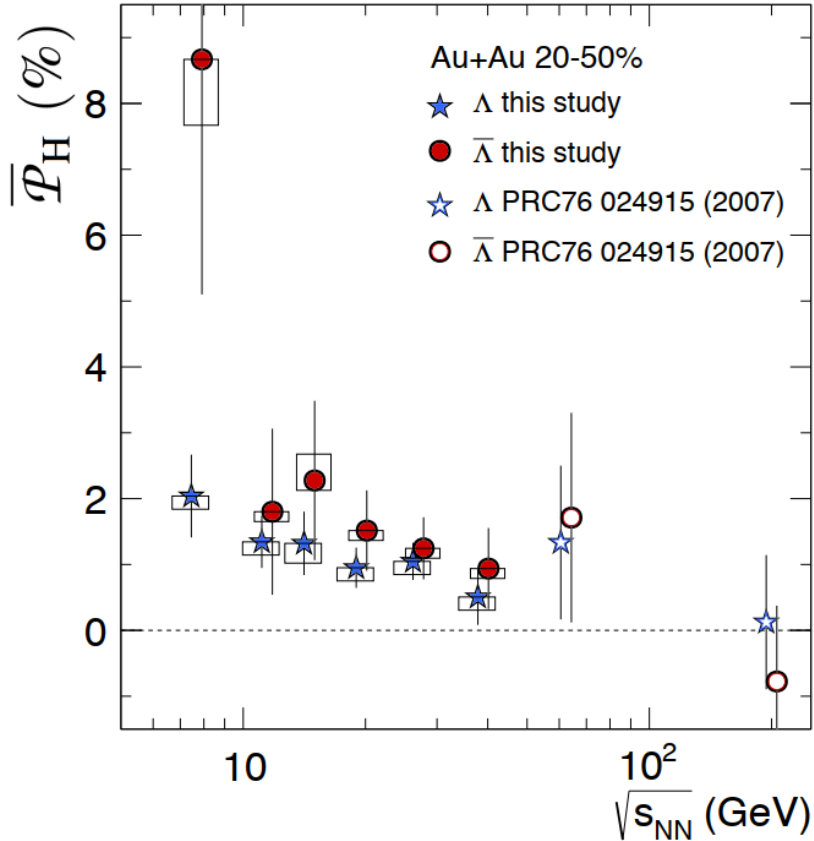
- Huge global orbital angular momenta ( $L \sim 10^5 \hbar$ ) are produced in HIC.
- Global orbital angular momentum leads to the polarizations of  $\Lambda$  hyperons and spin alignment of vector mesons through spin-orbital coupling.

Liang, Wang, PRL (2005); PLB (2005);  
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

# Global polarization at AA system

# Global polarization for $\Lambda$ and $\bar{\Lambda}$ hyperons

STAR, Nature 548, 62 (2017)



## parity-violating decay of hyperons

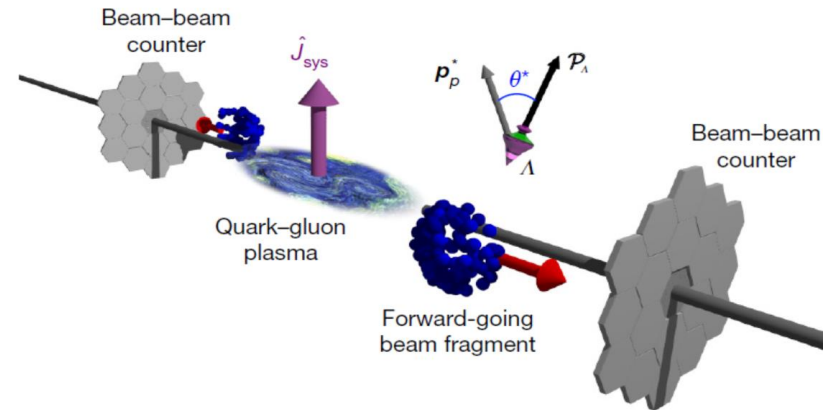
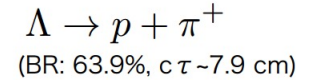
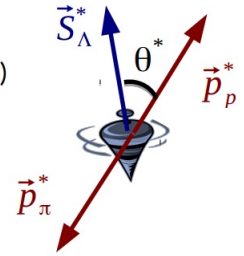
In case of  $\Lambda$ 's decay, daughter proton preferentially decays in the direction of  $\Lambda$ 's spin (opposite for anti- $\Lambda$ )

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} (1 + \alpha \mathbf{P}_\Lambda \cdot \mathbf{p}_p^*)$$

$\alpha$ :  $\Lambda$  decay parameter ( $=0.642 \pm 0.013$ )

$\mathbf{P}_\Lambda$ :  $\Lambda$  polarization

$\mathbf{p}_p^*$ : proton momentum in  $\Lambda$  rest frame



# Most vortical fluid

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- Estimation given by Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC95, 054902 (2017)

$$\mathbf{P}_\Lambda \simeq \frac{\omega}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T}$$
$$\mathbf{P}_{\bar{\Lambda}} \simeq \frac{\omega}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}$$

- $\omega = (9 \pm 1) \times 10^{21} / \text{s}$ , greater than previously observed in any system.
- QGP is **most vortical fluid** so far.

Liang, Wang, PRL (2005)

Betz, Gyulassy, Torrieri, PRC (2007)

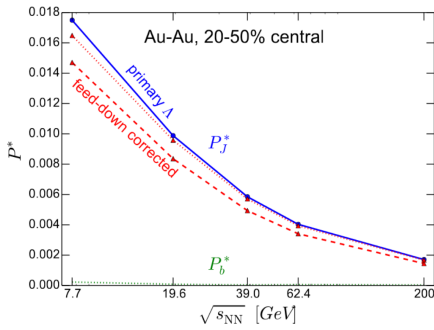
Becattini, Piccinini, Rizzo, PRC (2008)

Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)

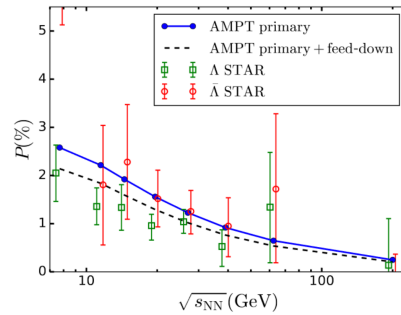
Fang, Pang, Q. Wang, X. Wang, PRC (2016)

...

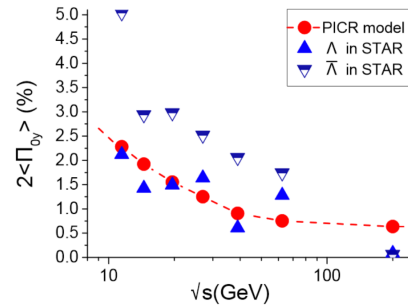
# Phenomenological models for global polarization



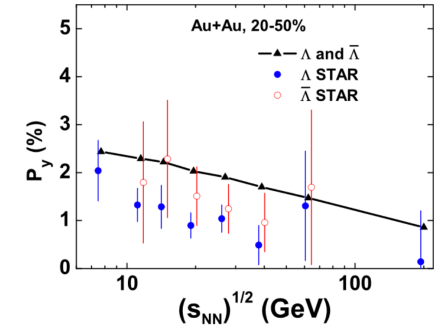
Karpenko, Becattini, EPJC(2017)



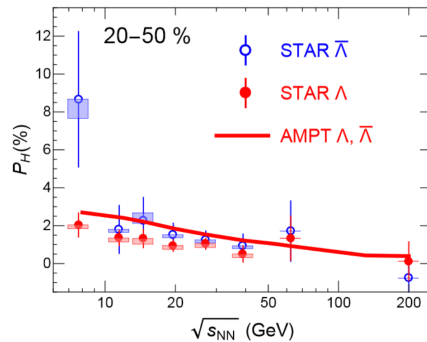
Li, Pang, Wang, Xia PRC(2017)



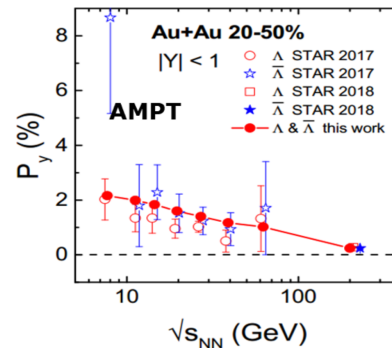
Xie, Wang, Csernai, PRC(2017)



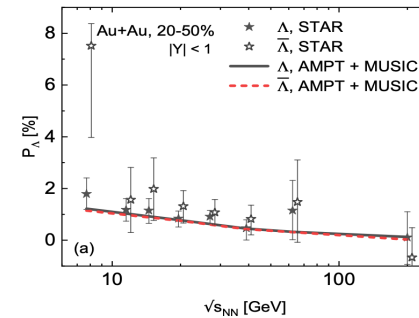
Sun, Ko, PRC(2017)



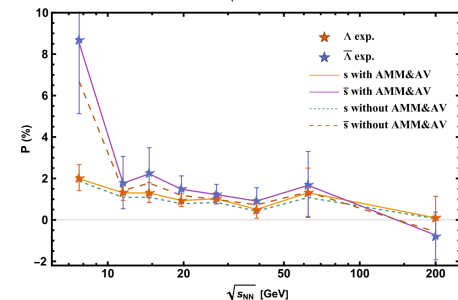
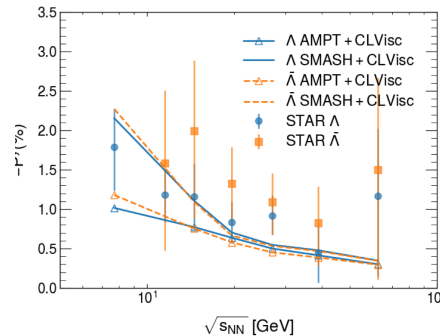
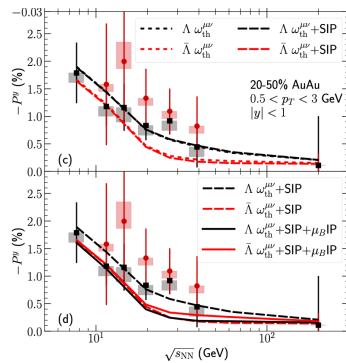
Shi, Li, Liao, PLB(2018)



Wei, Deng, Huang, PRC(2019)



Fu, Xu, Huang, Song, PRC (2021)

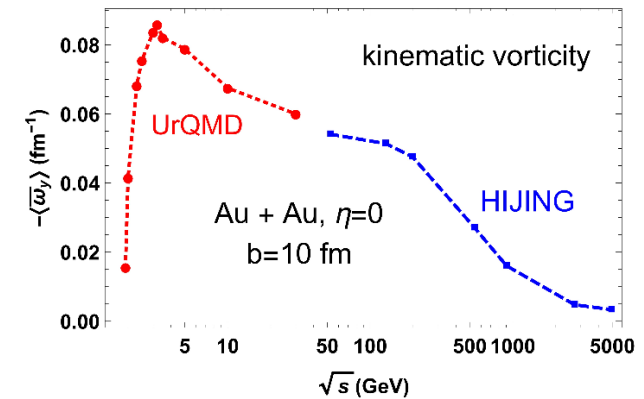
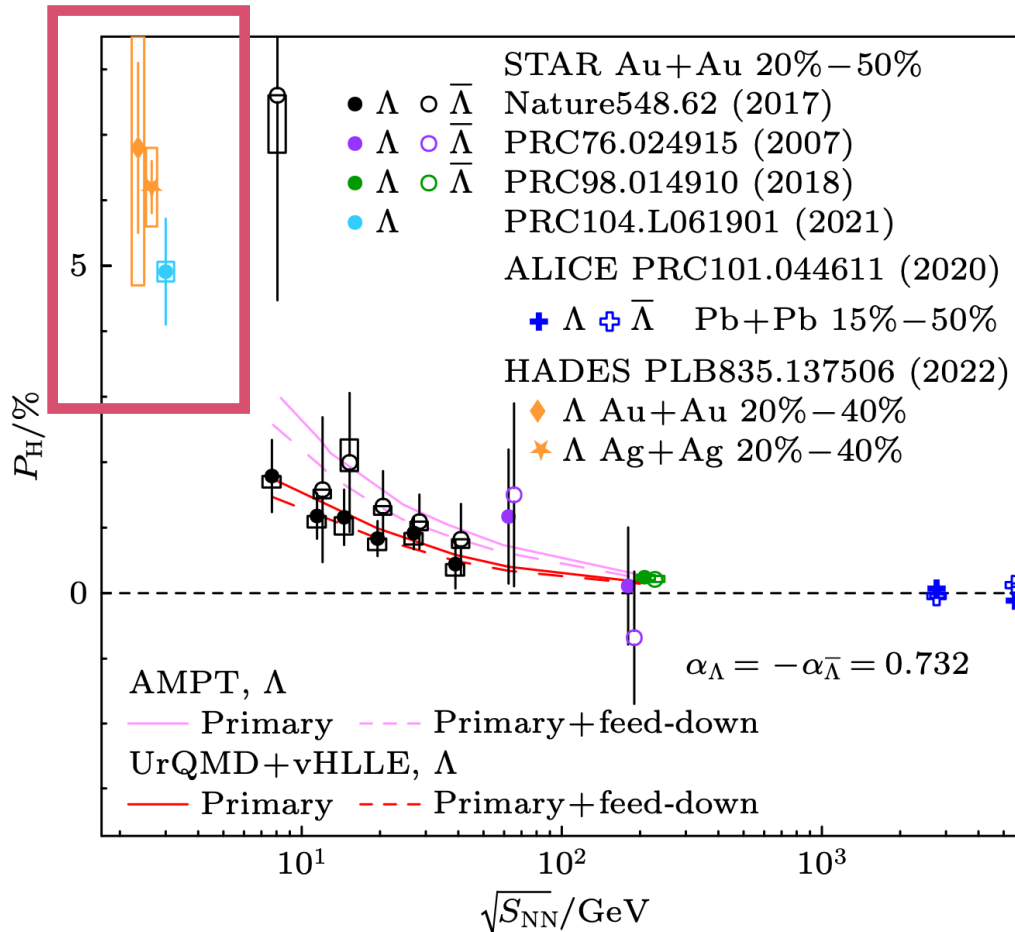


Xu, Lin, Huang, Huang, PRDL (2022)

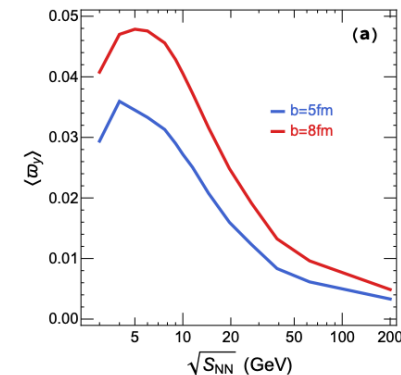
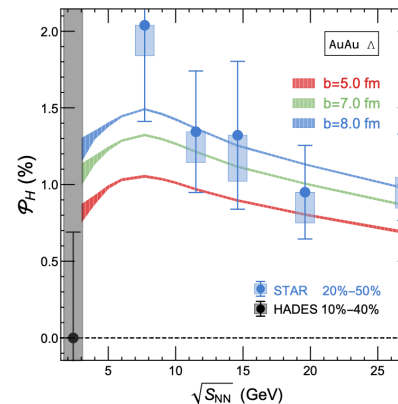
# Polarization at low energies

Will the polarization of Lambda be nonzero when  $\sqrt{s_{NN}} \rightarrow 0$ ?

If not, how large the “critical  $\sqrt{s_{NN}}$ ” will be?



Deng, Huang 2016; Deng, Huang, et. al 2020

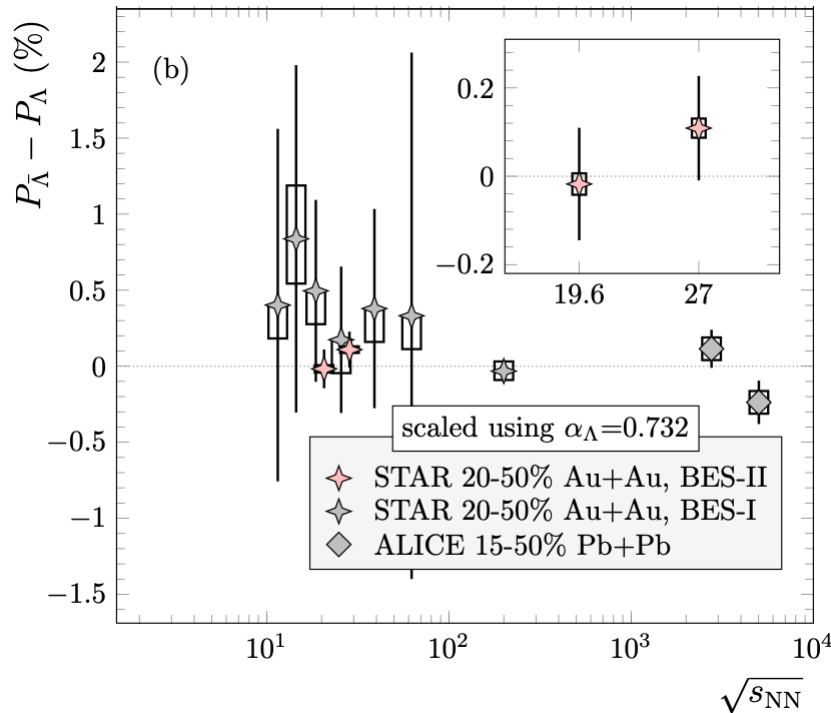


Sun Xu et al., Acta Phys. Sin. 72(7), 072401 (2023)

Guo, Liao, et. al, PRC 2021



# Global polarization splitting



- No splitting between the global polarization of  $\Lambda$  and  $\bar{\Lambda}$  hyperons.
- The splitting may come from the B fields.

$$P_{\Lambda} \approx \frac{\omega}{2T} + \frac{\mu_{\Lambda} B}{T}$$

$$P_{\bar{\Lambda}} \approx \frac{\omega}{2T} - \frac{\mu_{\Lambda} B}{T}$$

$$|B| \approx \frac{T_s |P_{\bar{\Lambda}} - P_{\Lambda}|}{2|\mu_{\Lambda}|}$$

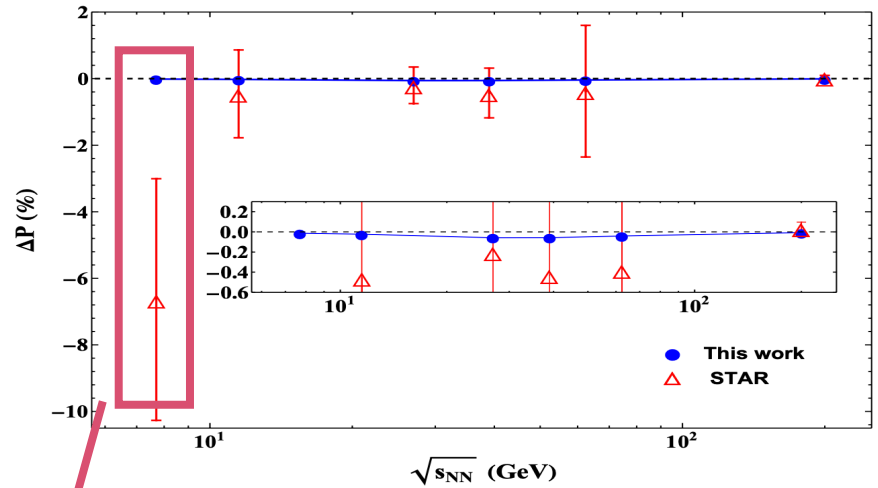
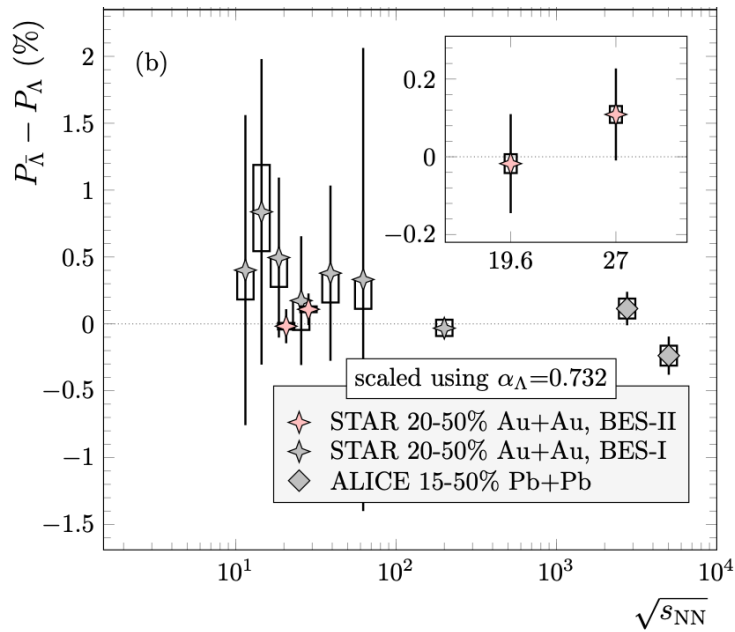
Estimation by STAR data:

$$B < 9.4 \times 10^{12} \text{ T} \quad (\sqrt{s_{NN}} = 19.6 \text{ GeV})$$

$$B < 1.4 \times 10^{13} \text{ T} \quad (\sqrt{s_{NN}} = 27 \text{ GeV})$$

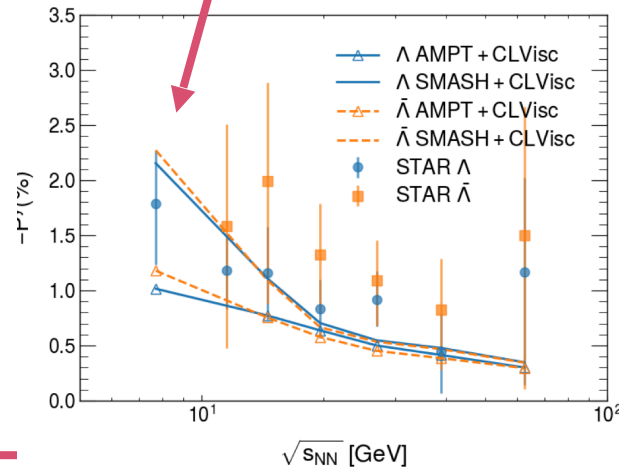
STAR, PRC 108 (2023) 1, 014910

# Global polarization splitting



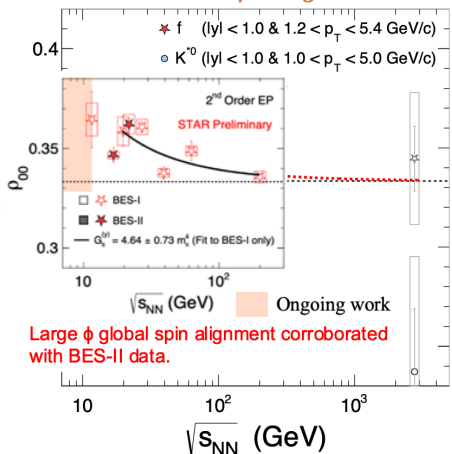
Estimation by Peng, Wu, Wang, She, Pu, PRD(2023)  
 UrQMD initial B fields (Sheng, Wang, PRC 2021)  
 + ideal Magnetohydrodynamics  
 + Modified Cooper-Frye formula

STAR, PRC 108 (2023) 1, 014910



The baryon diffusion plays an important role to the global polarization.  
 Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)  
 CLVis hydro + SMASH

# Lambda-(anti)-Lambda correlations



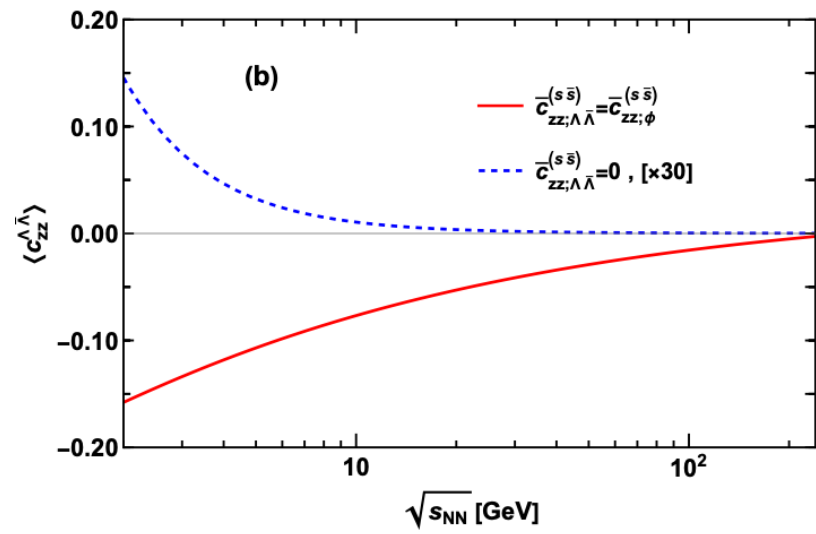
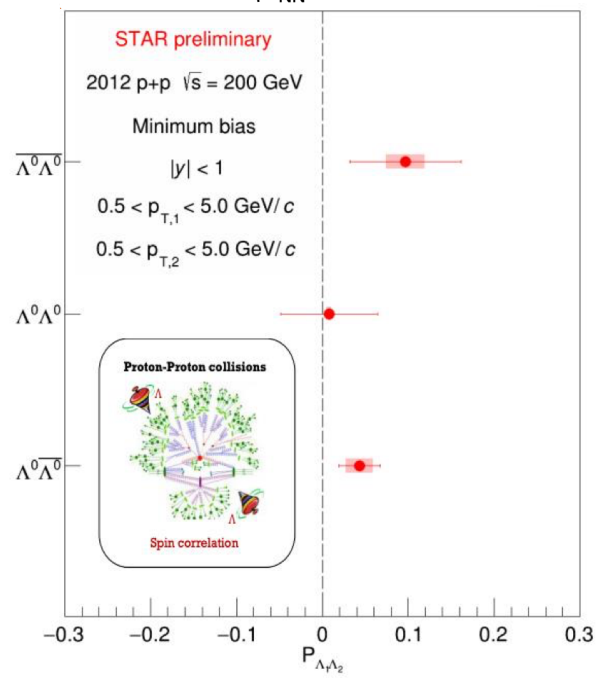
For  $\phi$  mesons, when we consider  $|P_s\rangle |P_{\bar{s}}\rangle \rightarrow |P_\phi\rangle$ , the spin density matrix has extra contributions

$$\langle P_i(\alpha_1) P_{2j}(\alpha_2) \rangle - \langle P_{1i}(\alpha_1) \rangle \langle P_{2j}(\alpha_2) \rangle$$

$$\langle P_\Lambda \rangle \sim \langle P_s \rangle,$$

$$\langle \rho_{00}^\phi \rangle \sim \frac{1 - \bar{c}_{zz;\phi}^{(s\bar{s})} - \langle P_s \rangle^2}{3 + \bar{c}_{zz;\phi}^{(s\bar{s})} + \langle P_s \rangle^2}, \quad c_{nn}^{H_1 \bar{H}_2} = \frac{f_{++}^{H_1 \bar{H}_2} + f_{--}^{H_1 \bar{H}_2} - f_{+-}^{H_1 \bar{H}_2} - f_{-+}^{H_1 \bar{H}_2}}{f_{++}^{H_1 \bar{H}_2} + f_{--}^{H_1 \bar{H}_2} + f_{+-}^{H_1 \bar{H}_2} + f_{-+}^{H_1 \bar{H}_2}},$$

$$\langle c_{zz}^{\Lambda \bar{\Lambda}} \rangle \sim \bar{c}_{zz;\Lambda \bar{\Lambda}}^{(s\bar{s})} + \langle P_s \rangle^2,$$

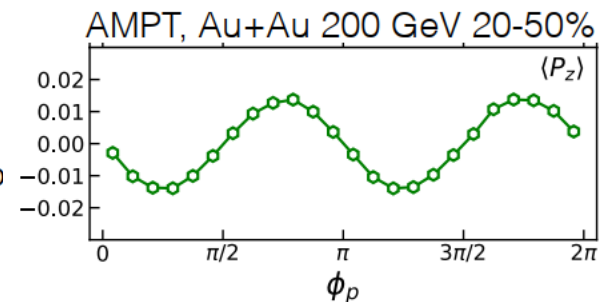
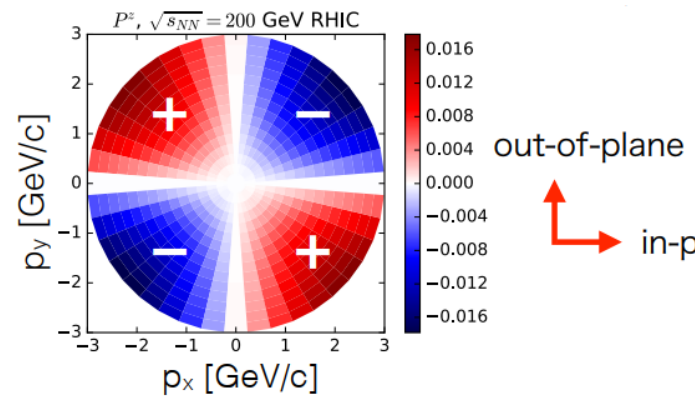
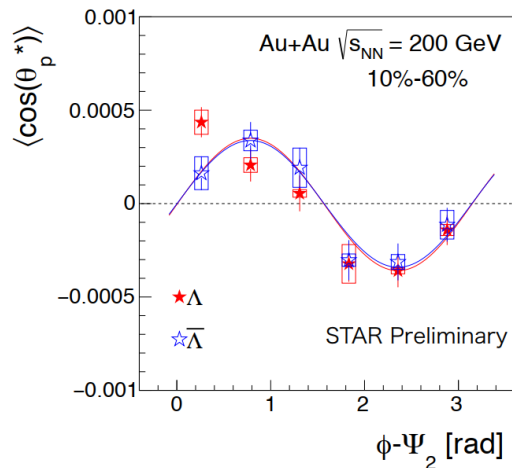
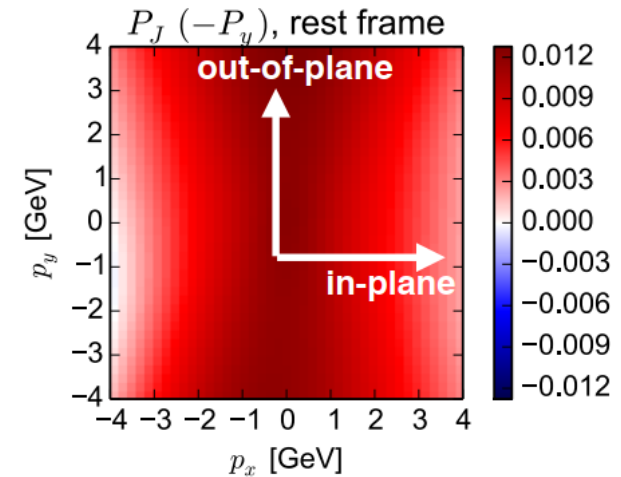
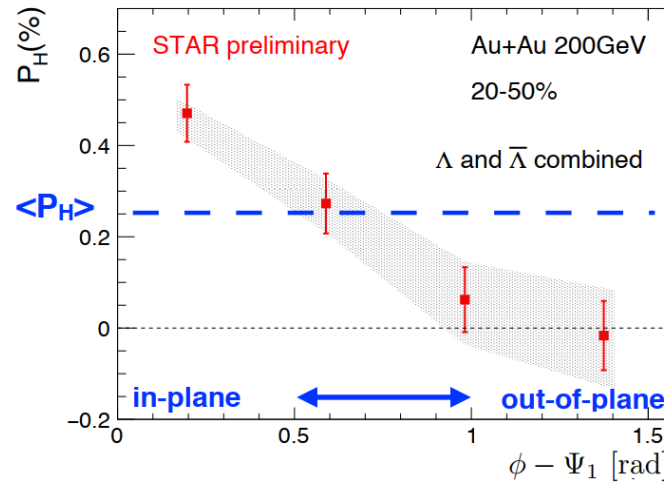
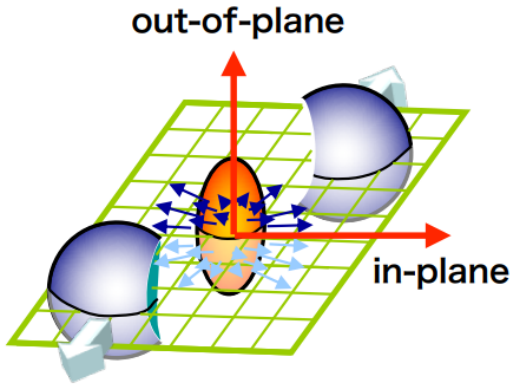


Results from Aihong's talk

Liang's group, PRD 109, 114003 (2024)

# Local polarization

# Local polarization and sign problem



- **UrQMD :** Becattini, Karpenko, PRL (2018)
- **AMPT:** Xia, Li, Tang, Wang, PRC (2018)

# Theoretical developments

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- **Spin hydrodynamics (macroscopic approach)**
  - Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  - Montenegro, Tinti, Torrieri (2017-2019);
  - Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  - Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022); ...
  - S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  - D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060
  - Weickgenannt, Wanger, Speranza, Rischke, PRD 2022; PRD 2022; Weickgenannt, Wanger, Speranza, PRD 2022; arXiv:2306.05936;
- **Quantum kinetic theory with collisions (microscopic approach)**
  - Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  - Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  - Liu, Mameda, Huang, arXiv:2002.03753.
  - Gao, Liang, PRD 2019
  - Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019); Z.Y. Wang, arXiv:2205.09334;
  - Li ,Yee, PRD100, 056022 (2019)
  - Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  - Fang, SP, Yang, PRD (2022)
- **Other approaches:**
  - Side-jump effect Liu, Sun, Ko PRL(2020)
  - Mesonic mean-field Csernai, Kapusta, Welle, PRC(2019)
  - Using different vorticity Wu, Pang, Huang, Wang, PRR (2019)
- **Recent reviews:**
  - Gao, Ma, SP, Wang, NST (2020)
  - Gao, Liang, Wang, IJMPA (2021)
  - Hidaka, SP, Yang, Wang, PPNP (2022)
  - F. Becattini, M. Buzzegoli, T. Niida, SP, A.H. Tang, Q. Wang, arXiv: 2402.04540

# Polarization and axial current

- The polarization tensor is connected to the axial current in phase space by modified Cooper-Frye formula

Karpenko, Becattini, EPJC. (2017); Fang, Pang, QW, Wang, PRC (2016)

$$\mathcal{S}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^\mu(p, X)}{2m_\Lambda \int d\Sigma \cdot \mathcal{N}(p, X)},$$

Polarization pseudo-vector ~ Spin tensor in phase space

- For massless fermions, the left and right handed currents can be derived by quantum kinetic theory,

$$\mathcal{J}_5^\mu = \mathcal{J}_{\text{thermal}}^\mu + \mathcal{J}_{\text{shear}}^\mu + \mathcal{J}_{\text{accT}}^\mu + \mathcal{J}_{\text{chemical}}^\mu + \mathcal{J}_{\text{EB}}^\mu,$$

Y. Hidaka, SP, and D.L. Yang, Phys. Rev. D97, 016004 (2018)

# Polarization induced by different sources

$$\mathcal{S}^\mu(\mathbf{p}) = \mathcal{S}_{\text{thermal}}^\mu + \mathcal{S}_{\text{shear}}^\mu + \mathcal{S}_{\text{accT}}^\mu + \mathcal{S}_{\text{chemical}}^\mu + \mathcal{S}_{\text{EB}}^\mu$$

Y. Hidaka, SP, D.L. Yang, PRD97, 016004 (2018); C. Yi, SP, D.L. Yang, PRC 2021

Thermal vorticity 热涡旋

$$\mathcal{S}_{\text{thermal}}^\mu(\mathbf{p}) = \frac{\hbar}{8m_\Lambda N} \int d\Sigma^\sigma p_\sigma f_V^{(0)} (1 - f_V^{(0)}) \epsilon^{\mu\nu\alpha\beta} p_\nu \partial_\alpha \frac{u_\beta}{T},$$

Shear viscous tensor 剪切粘滞张量

$$\mathcal{S}_{\text{shear}}^\mu(\mathbf{p}) = -\frac{\hbar}{4m_\Lambda N} \int d\Sigma \cdot p f_V^{(0)} (1 - f_V^{(0)}) \frac{\epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta}{(u \cdot p) T} \frac{1}{2} \{ p^\sigma (\partial_\sigma u_\nu + \partial_\nu u_\sigma) - D u_\nu \}$$

Fluid acceleration 流体加速

$$\mathcal{S}_{\text{accT}}^\mu(\mathbf{p}) = -\frac{\hbar}{8m_\Lambda N} \int d\Sigma \cdot p f_V^{(0)} (1 - f_V^{(0)}) \frac{1}{T} \epsilon^{\mu\nu\alpha\beta} p_\nu u_\alpha (D u_\beta - \frac{1}{T} \partial_\beta T),$$

Gradient of chemical potential 化学势/温度梯度

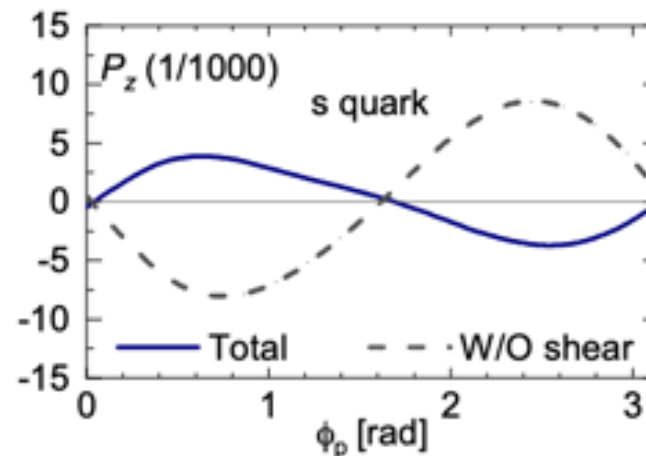
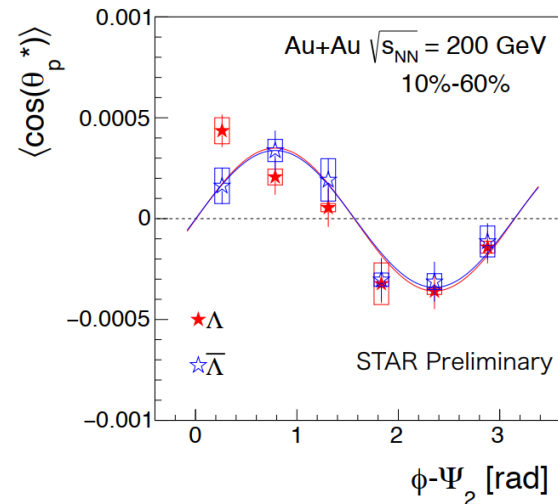
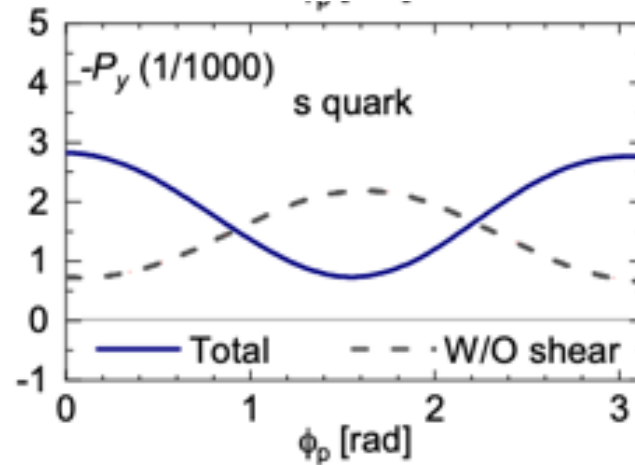
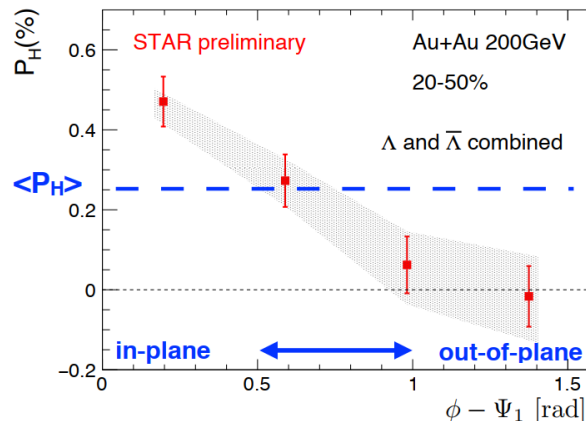
$$\mathcal{S}_{\text{chemical}}^\mu(\mathbf{p}) = \frac{\hbar}{4m_\Lambda N} \int d\Sigma \cdot p f_V^{(0)} (1 - f_V^{(0)}) \frac{1}{(u \cdot p)} \epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta \partial_\nu \frac{\mu}{T},$$

Electromagnetic fields 电磁场

$$\mathcal{S}_{\text{EB}}^\mu(\mathbf{p}) = \frac{\hbar}{4m_\Lambda N} \int d\Sigma \cdot p f_V^{(0)} (1 - f_V^{(0)}) \left( \frac{1}{(u \cdot p) T} \epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta E_\nu + \frac{B^\mu}{T} \right)$$



# Shear induced polarization: s quark scenario

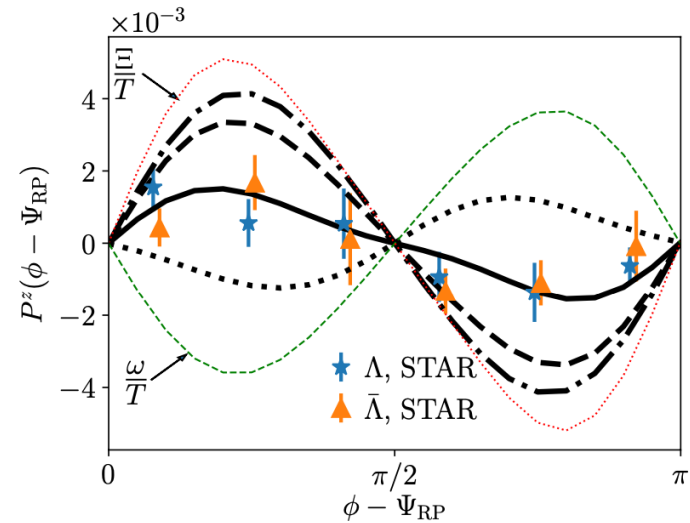
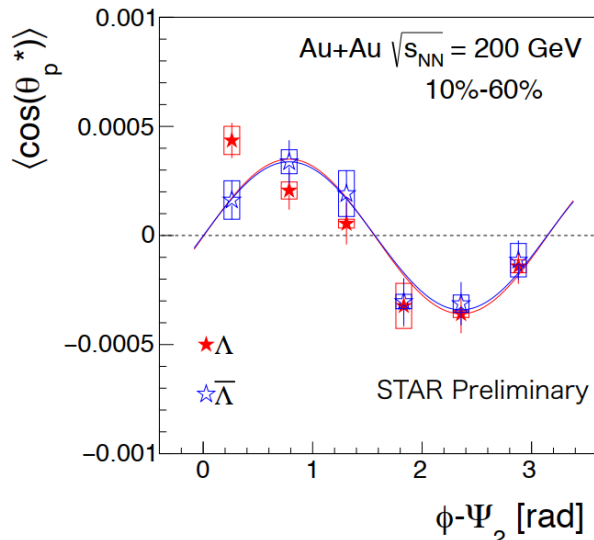
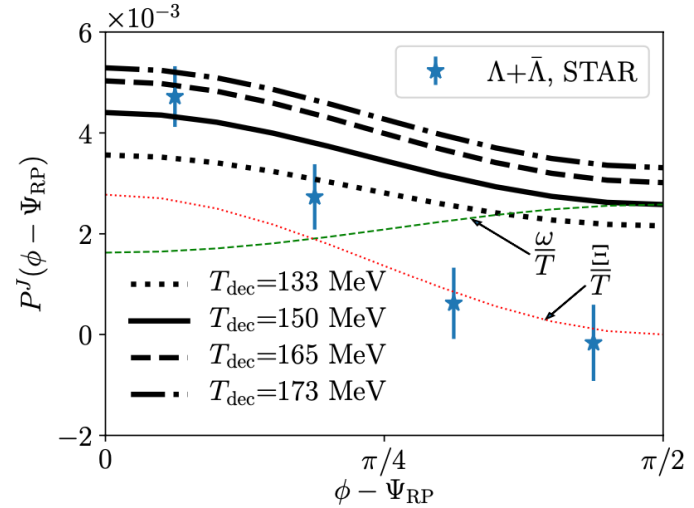
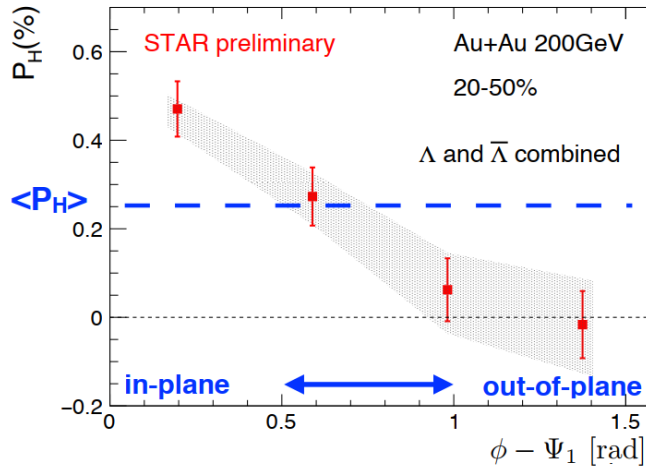


s quark scenarios (Thermal vorticity + shear)  
Fu, Liu, Pang, Song, Yin, PRL 2021

Also see:

Yi, Pu, Yang, PRC (2021); Yi, Wu, Qin, Pu, PRC (2022); Ryu, Jopic, Shen, PRC (2021)

# Shear induced polarization: isothermal equilibrium



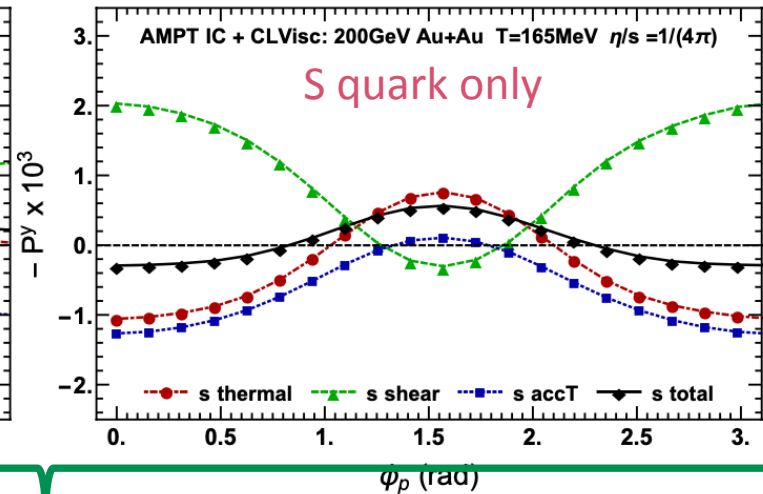
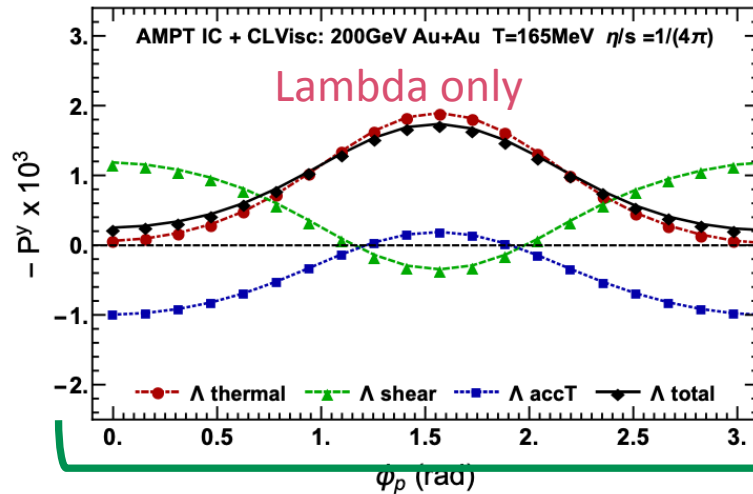
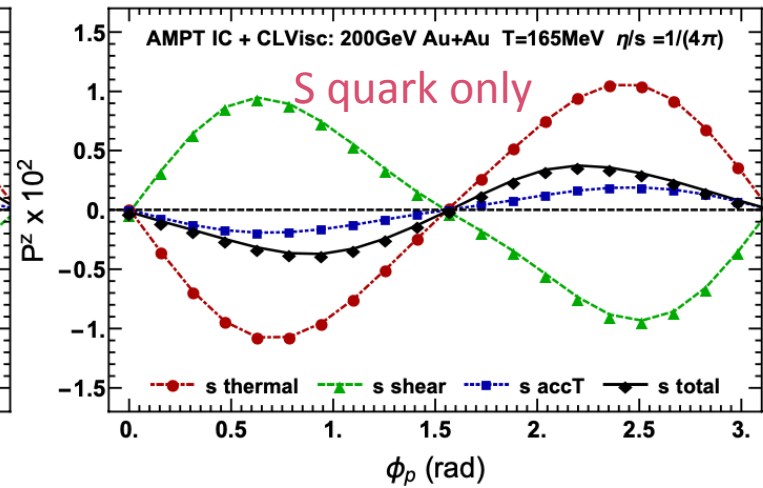
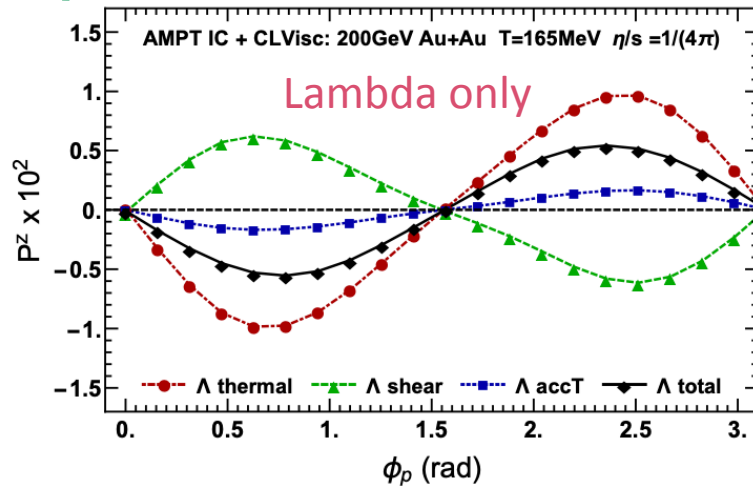
Isothermal equilibrium (Thermal vorticity + shear)

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

# Local spin polarization induced by shear tensor

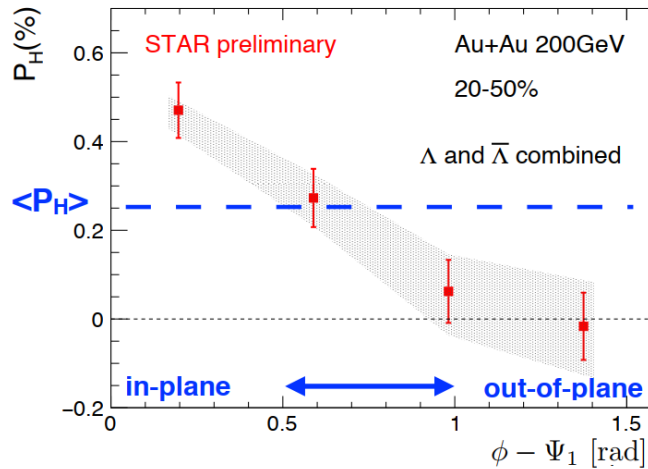
Polarization along beam direction

C. Yi, SP, D.L. Yang, PRC 2021

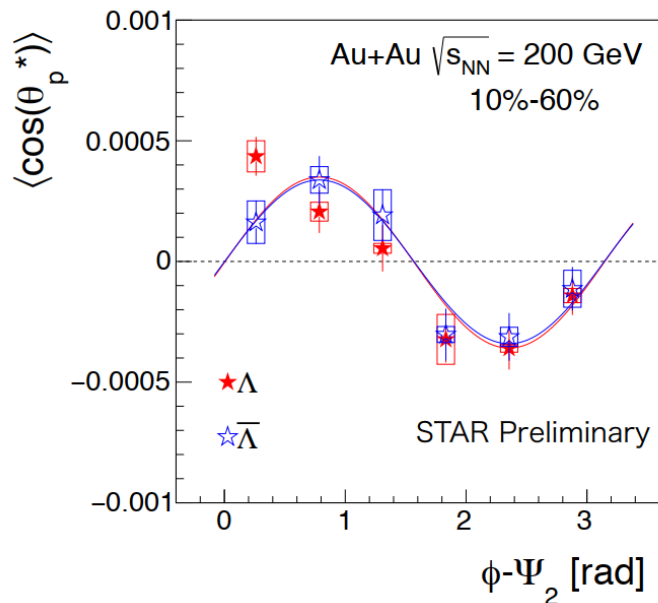


Polarization along out-of-plane direction

# $P_{2,y}$ and $P_{2,z}$ across BES



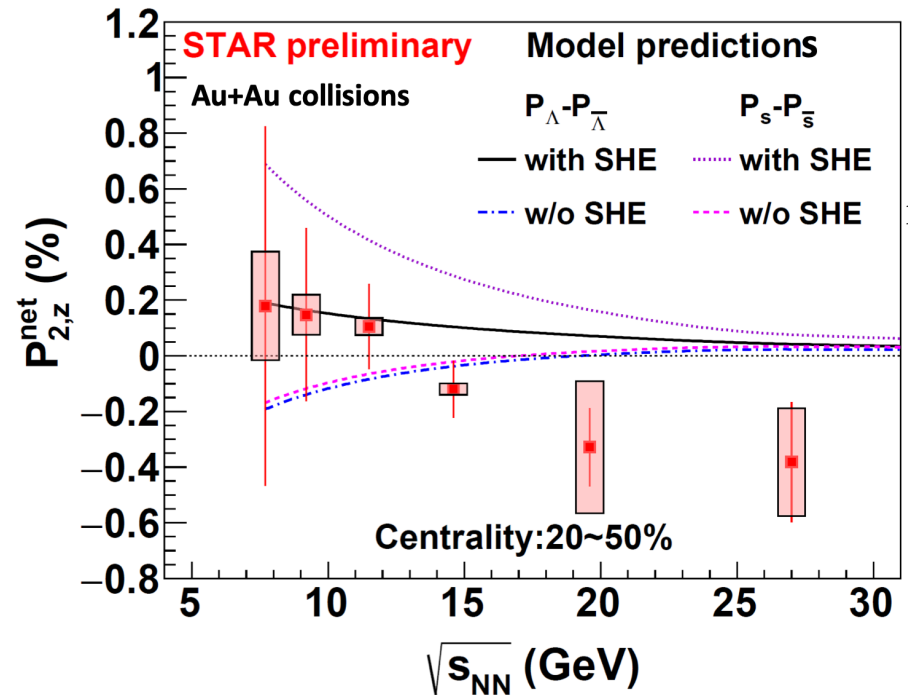
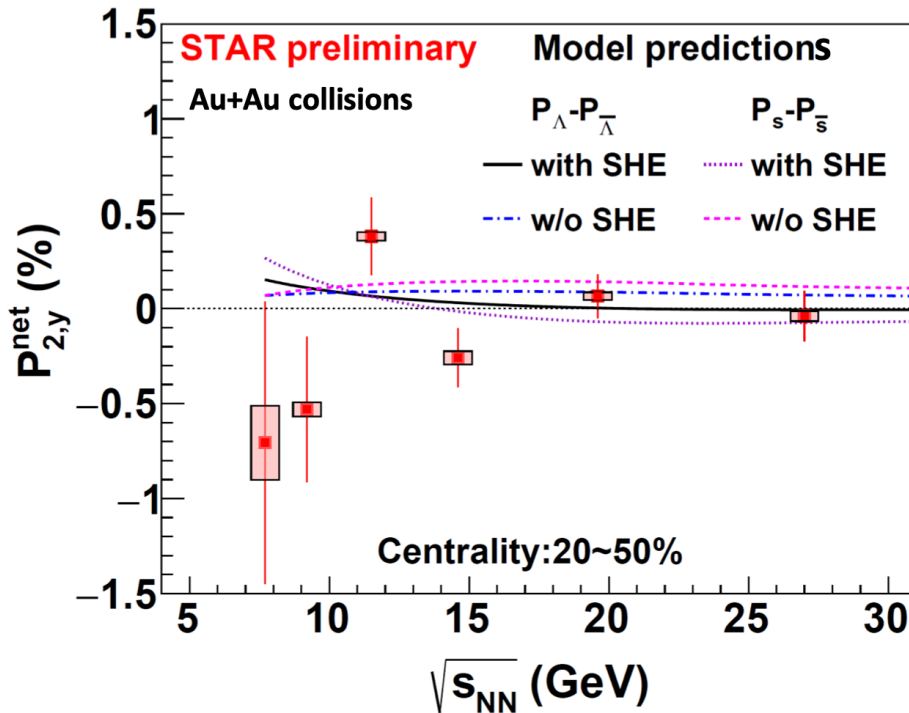
$$P_{2,y} \equiv \langle P_y \cos[2(\phi_\Lambda - \Psi_2)] \rangle$$



$$P_{2,z} \equiv \langle P_z \sin[2(\phi_\Lambda - \Psi_2)] \rangle$$

# Local polarization and spin Hall effect

$$S_{\text{chemical}}^{\mu}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N} \int d\Sigma \cdot p f_V^{(0)} (1 - f_V^{(0)}) \frac{1}{(u \cdot p)} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta} \partial_{\nu} \frac{\mu}{T},$$



Prediction: Fu, Pang, Song, Yin, 2208.00430

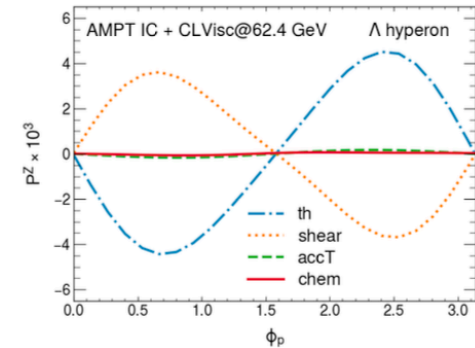
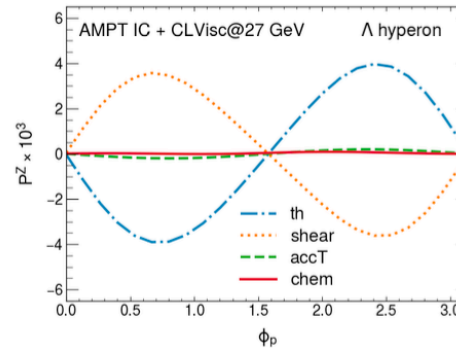
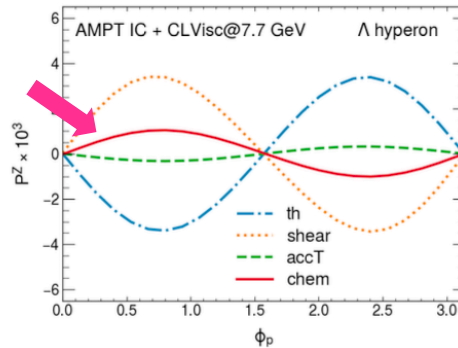
# Local polarization and spin Hall effect: SMASH VS AMPT

7.7 GeV

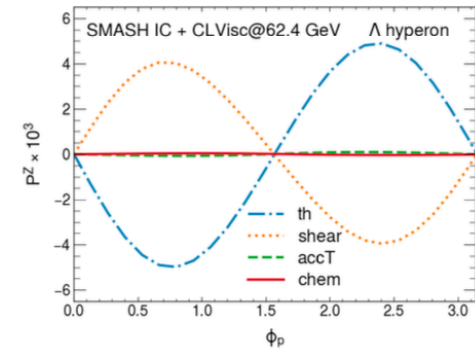
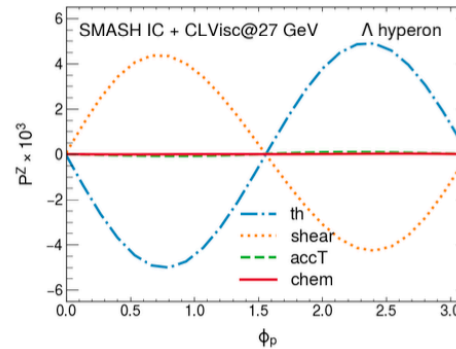
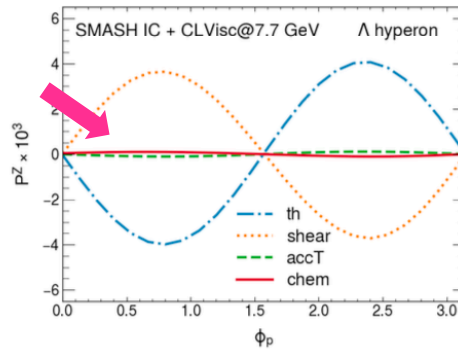
27 GeV

62.4 GeV

From **AMPT**  
Initial condition



From **SMASH**  
Initial condition



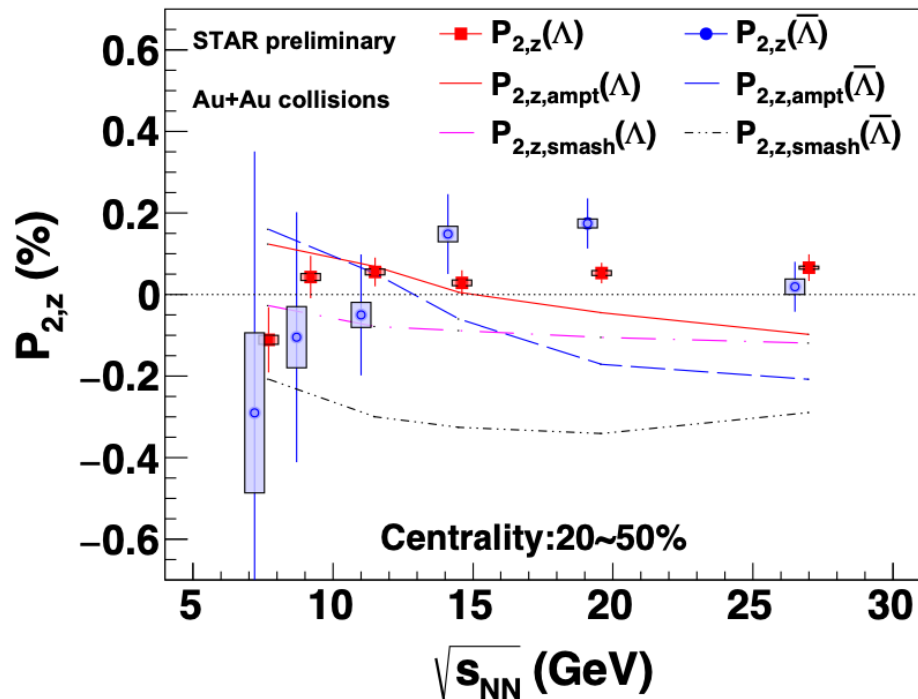
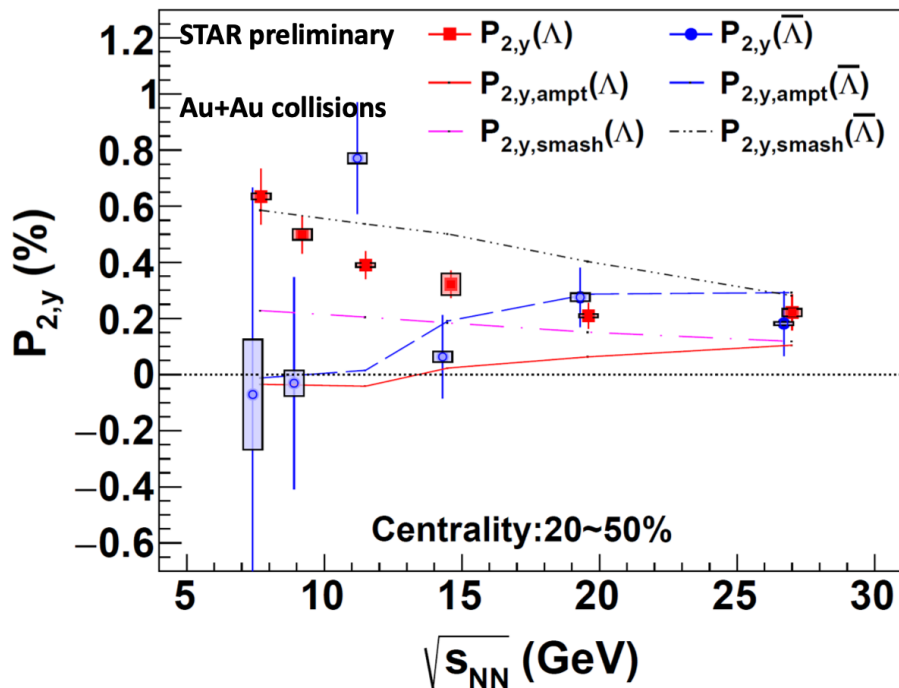
**Red lines:** contributions from spin Hall effect

Polarization induced by SHE is almost zero at 27, 62.4 GeV and it depends on the initial conditions at 7.7 GeV. For **SMASH**,  $P_z$  is still **almost vanishing** at 7.7 GeV.

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

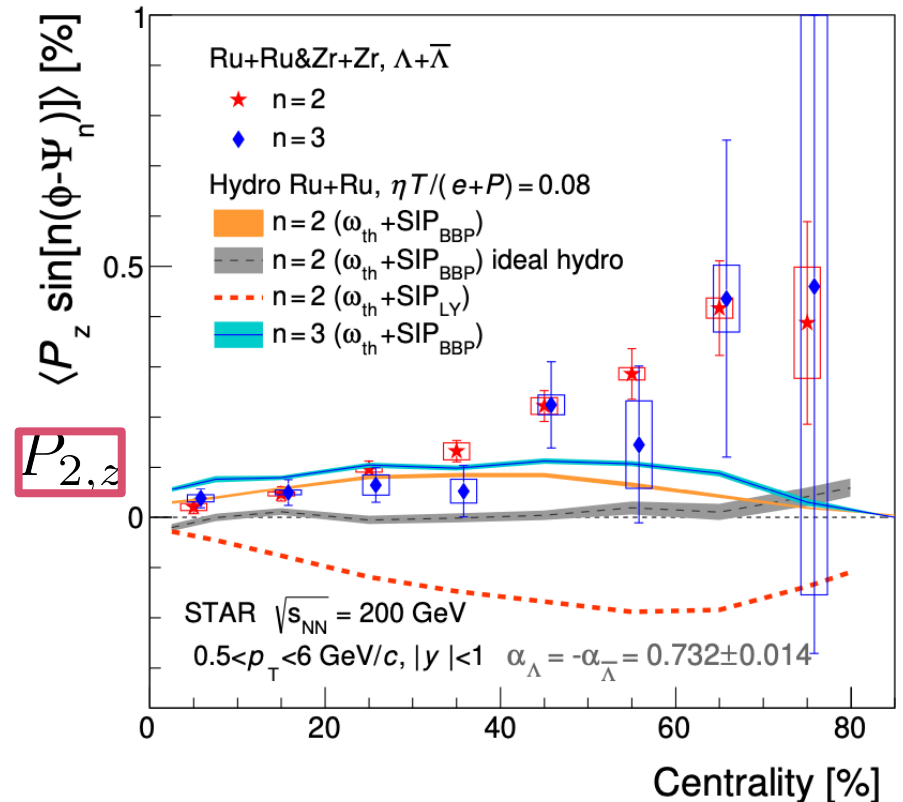
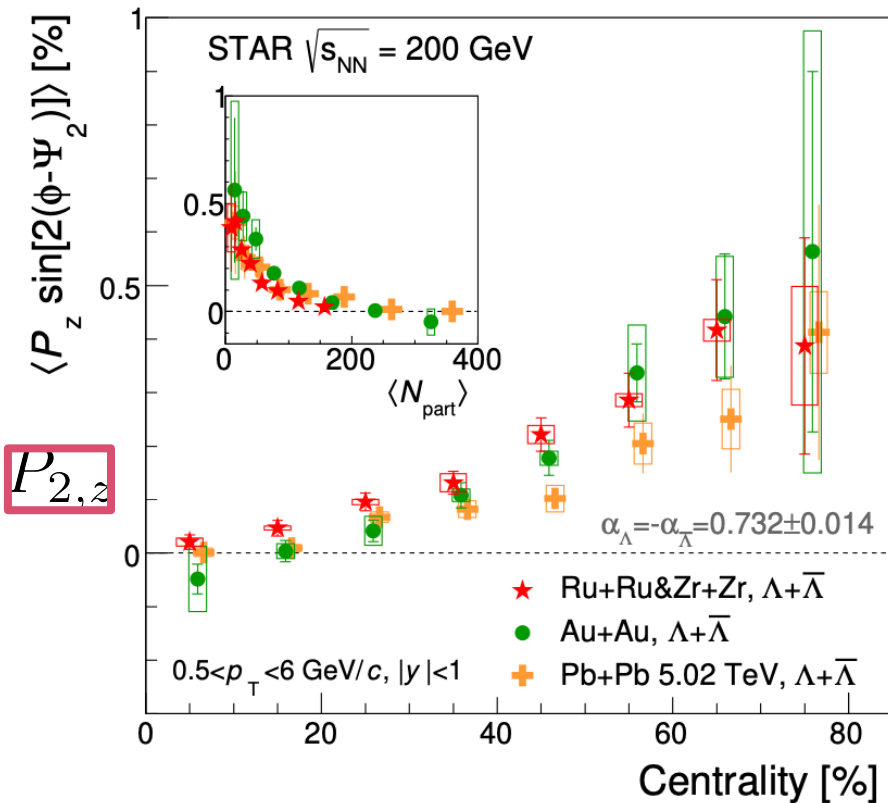
# Local polarization splitting

How can we understand the data in low energy collisions?



Model Prediction: X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

# Local polarization VS centrality



STAR, PRL 123, 132301 (2019)  
 ALICE, PRL 128, 172005 (2022)

STAR, PRL 131 (2023) 20, 202301

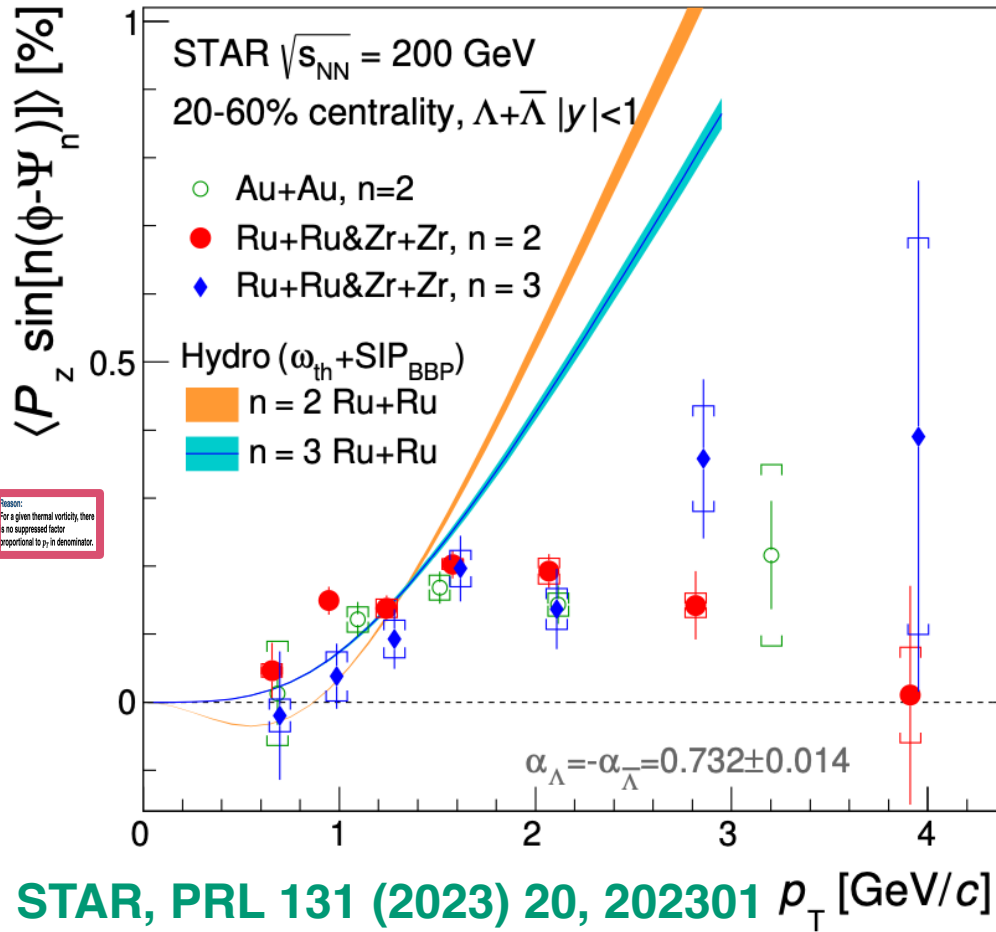
Model Calculation:

BBP: isothermal equilibrium; LY: s quark equilibrium

Simulation: Alzhrani, Ryu, Shen, PRC 106, 014905 (2022)



# Local polarization VS $p_T$



Similar question arises:

How large will the local polarization become when  $p_T$  is infinite?

$$\mathcal{S}_{\text{thermal}}^\mu(\mathbf{p}) = \frac{\hbar}{8m_\Lambda N} \int d\Sigma^\sigma p_\sigma f_V^{(0)} (1 - f_V^{(0)}) \epsilon^{\mu\nu\alpha\beta} p_\nu \partial_\alpha \frac{u_\beta}{T},$$

**Reason:**

For a given thermal vorticity, there is no suppressed factor proportional to  $p_T$  in denominator.

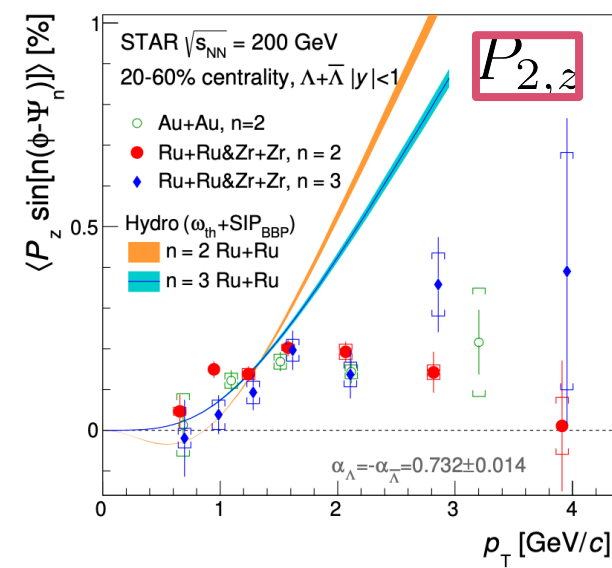
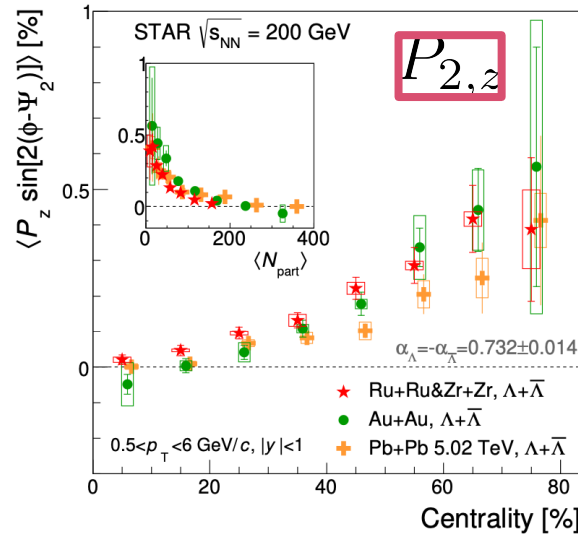
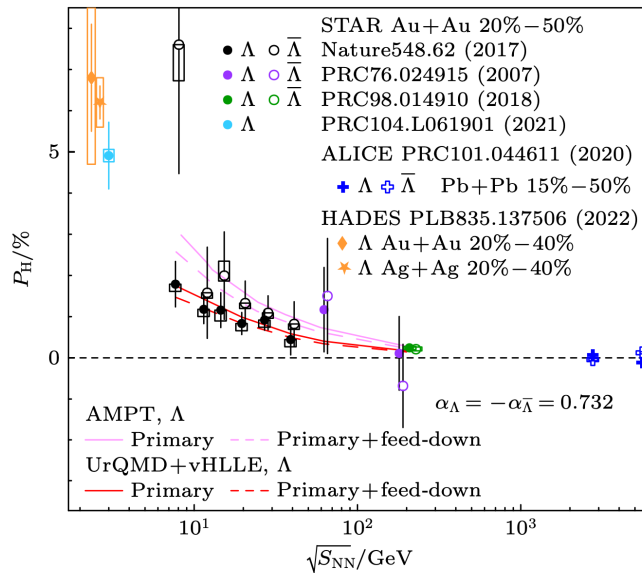
**Model Calculation:**

**BBP: isothermal equilibrium; LY: s quark equilibrium**

**Simulation: Alzhrani, Ryu, Shen, PRC 106, 014905 (2022)**

# Brief summary for polarization in AA systems

Question: when will the polarization stop growing?



# Spin polarization at pA system

C. Yi, X.Y. Wu, J. Zhu, SP, G.Y. Qin, arXiv: 2408.04296

# Setup (I)

- We follow the modified Cooper-Frye formula to compute the polarization pseudo-vector including the contribution from thermal vorticity and thermal shear tensor.

$$\mathcal{S}^\mu(\mathbf{p}) = \mathcal{S}_{\text{thermal}}^\mu(\mathbf{p}) + \mathcal{S}_{\text{th-shear}}^\mu(\mathbf{p})$$

$$\mathcal{S}_{\text{thermal}}^\mu(\mathbf{p}) = \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \varpi_{\alpha\beta},$$

$$\mathcal{S}_{\text{th-shear}}^\mu(\mathbf{p}) = \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{\epsilon^{\mu\nu\alpha\beta} p_\nu n_\beta}{(n \cdot p)} p^\sigma \xi_{\sigma\alpha}$$

**thermal vorticity**  $\varpi_{\alpha\beta} = \frac{1}{2} \left[ \partial_\alpha \left( \frac{u_\beta}{T} \right) - \partial_\beta \left( \frac{u_\alpha}{T} \right) \right],$

**thermal shear tensor**  $\xi_{\alpha\beta} = \frac{1}{2} \left[ \partial_\alpha \left( \frac{u_\beta}{T} \right) + \partial_\beta \left( \frac{u_\alpha}{T} \right) \right]$

# Setup (II)

---

- We consider three different scenarios:

- $\Lambda$  equilibrium:

- It is assumed that  $\Lambda$  hyperons reach the local (thermal) equilibrium at the freeze-out hyper-surface.

- $s$  quark equilibrium:

- The spin of  $\Lambda$  hyperons is assumed to be carried by the constituent  $s$  quark. **We take the  $s$  quark's mass instead of  $\Lambda$ 's mass in the simulation.**

- Isothermal equilibrium:

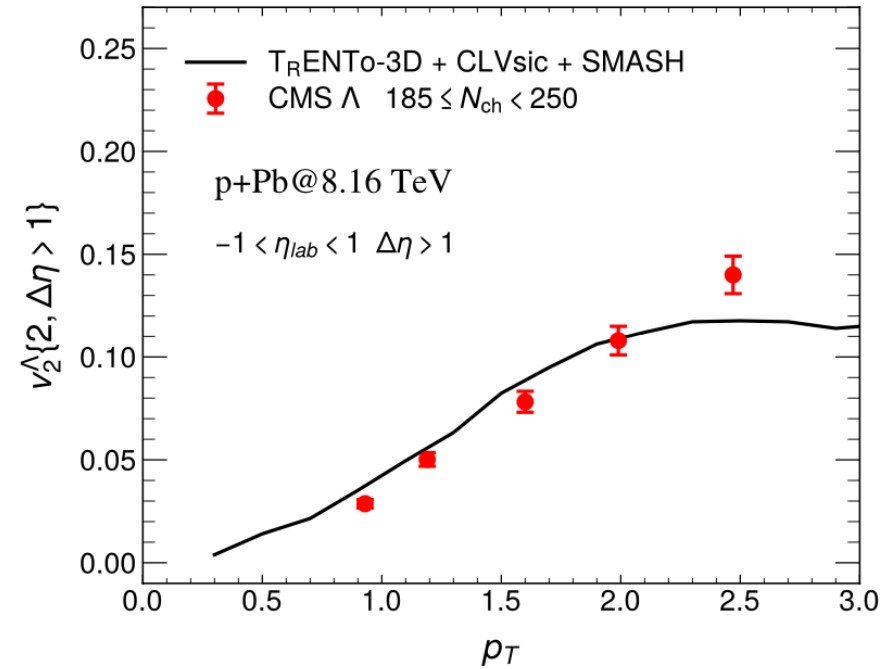
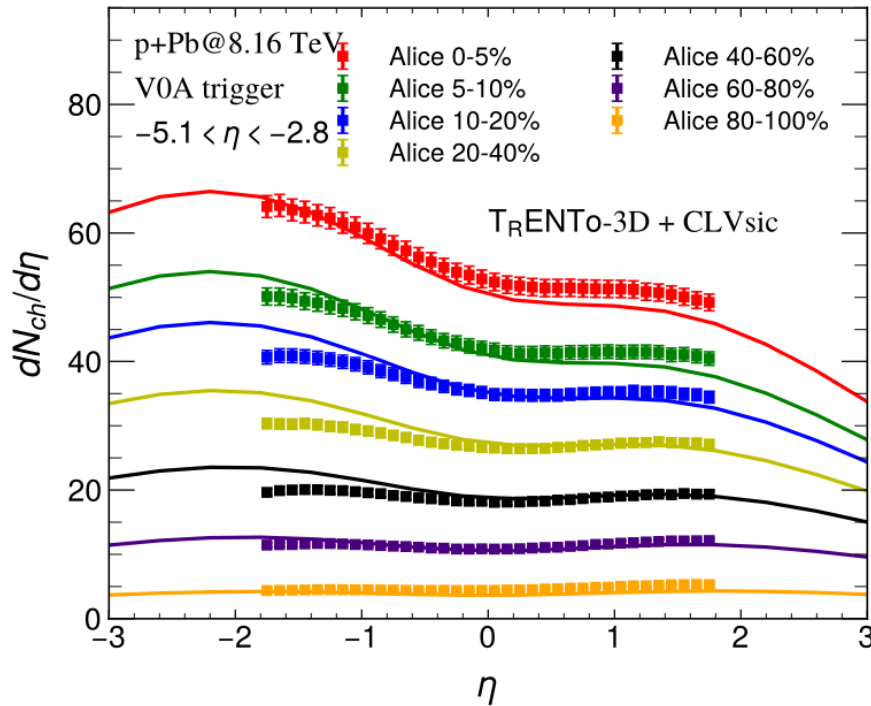
- The temperature of the system at the freeze-out hyper-surface is assumed to be constant. **The time unit vector is taken as fluid velocity for simplicity.**

# Setup (III)

---

- **We implement the 3+1D CLVisc hydrodynamics model**
  - Pang, Wang, Wang, PRC (2012)
  - Wu, Qin, Pang, Wang, PRC (2022)
- **Initial condition: TRENTo-3D model**
  - Soeder, Ke, Paquet, Bass, 2306.08665
  - Moreland, Bernhard, Bass, PRC (2015); PRC (2020)
  - Ke, Moreland, Bernhard, Bass, PRC (2017)
- **p+Pb collisions at  $\sqrt{s_{NN}} = 8.16$  TeV**

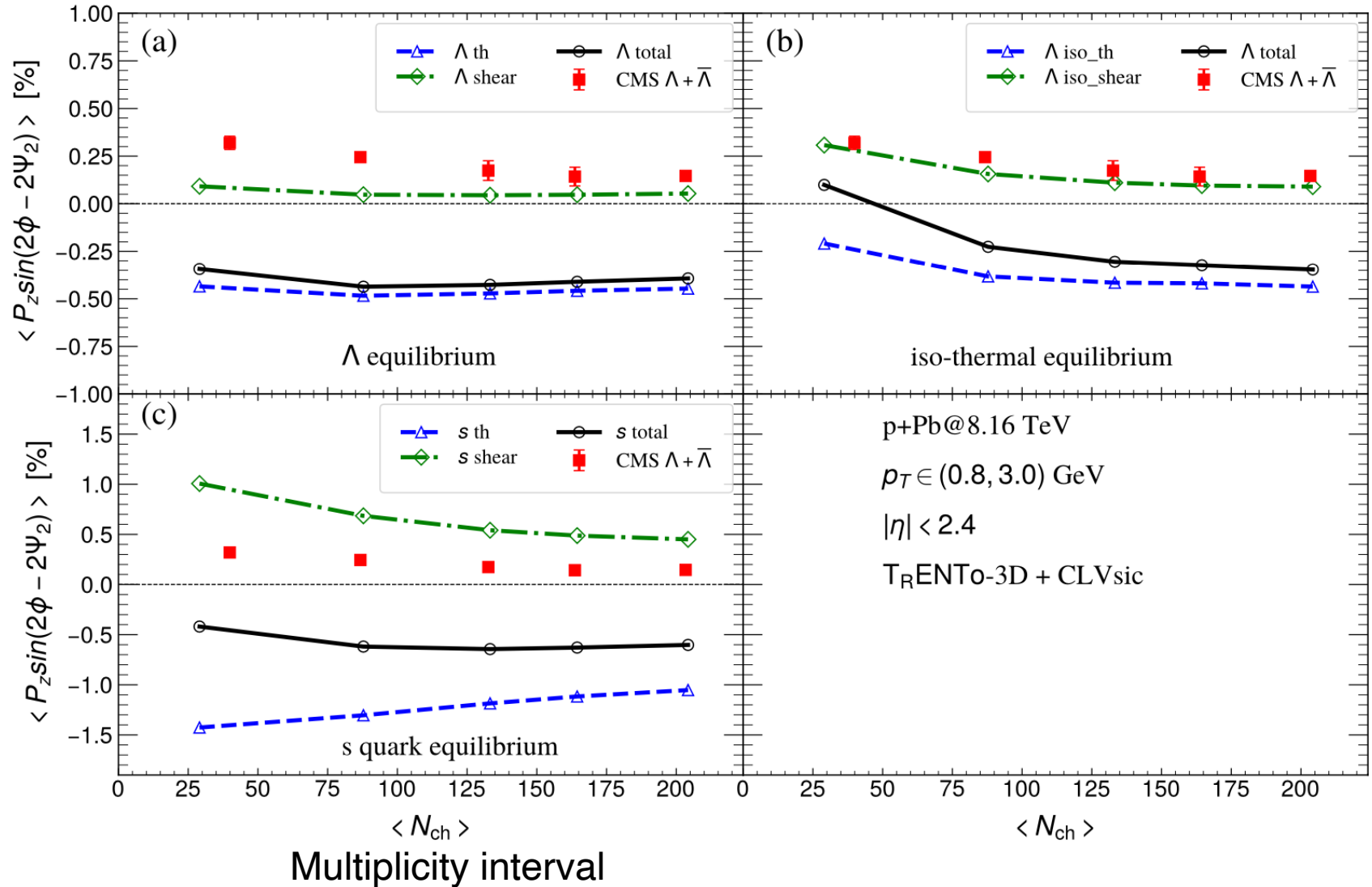
# Fit parameters and test v2 of $\Lambda$



Multiplicity intervals	$\langle N_{ch} \rangle_{\text{exp}}$	$\langle N_{ch} \rangle_{\text{CLVvisc}}$
[185,250)	203.3	204.2
[150,185)	163.6	164.5
[120,150)	132.7	133.57
[60,120)	86.7	87.7
[3,60)	40	29.3

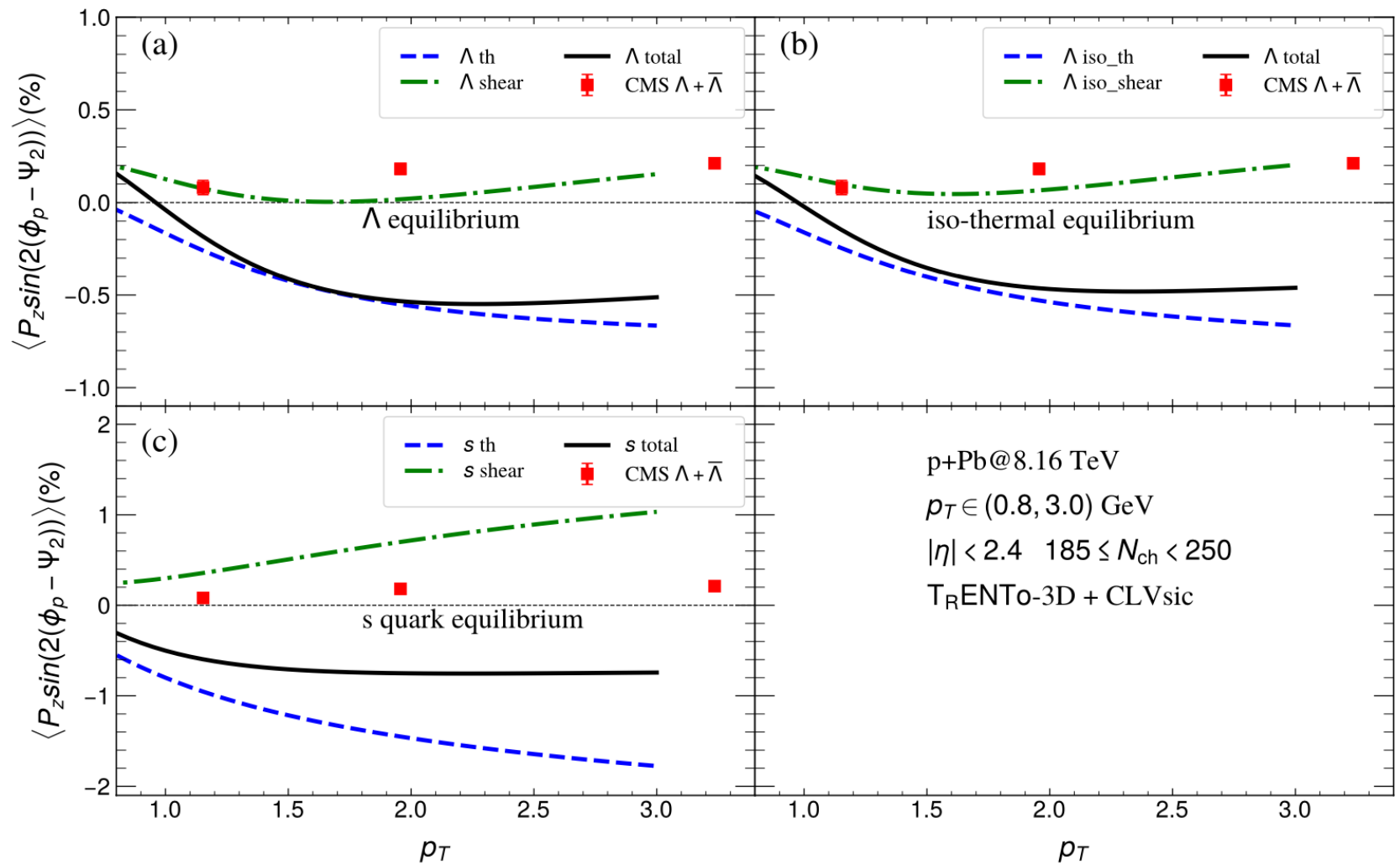
We have run  $10^5$  minimum bias events to divide the centrality. The centrality-dependent pseudo-rapidity distributions of charged hadrons and elliptic flow for  $\Lambda$  hyperons computed by our model are consistent with the experimental measurements.

# Multiplicity (centrality) dependence

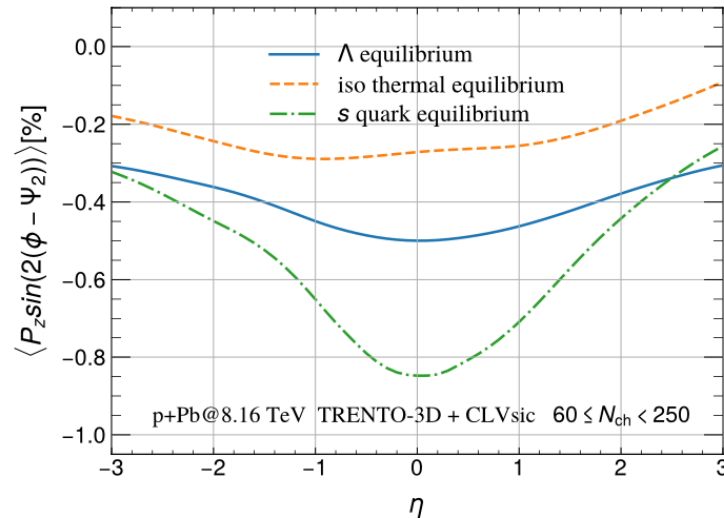
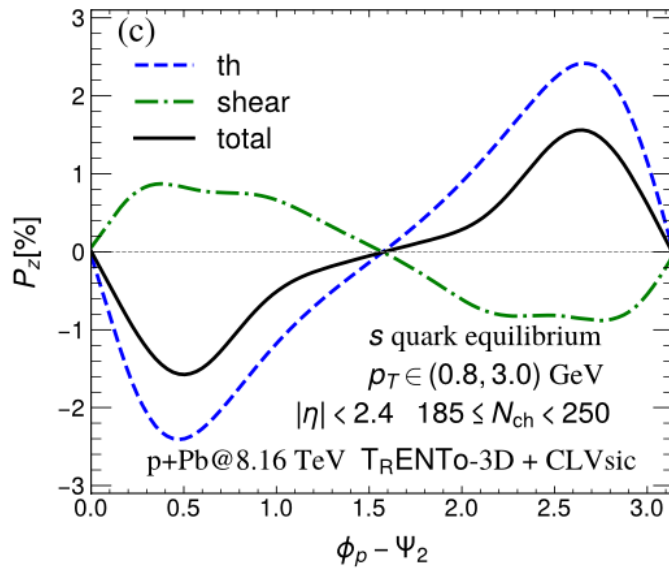
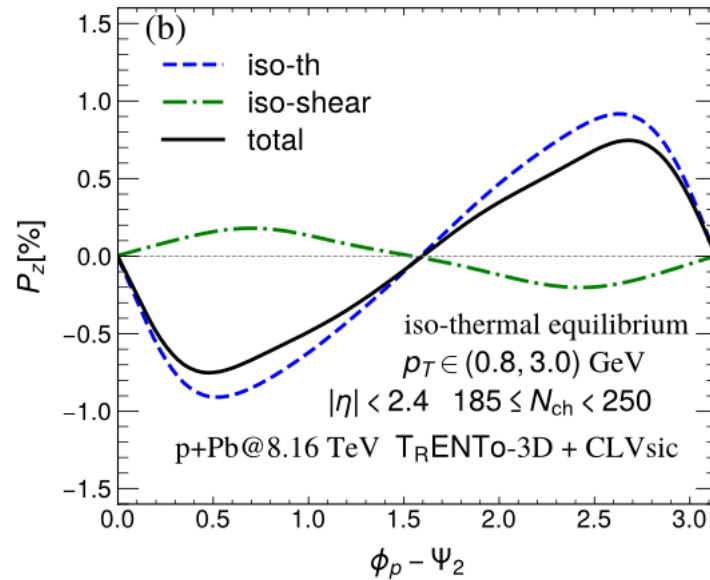
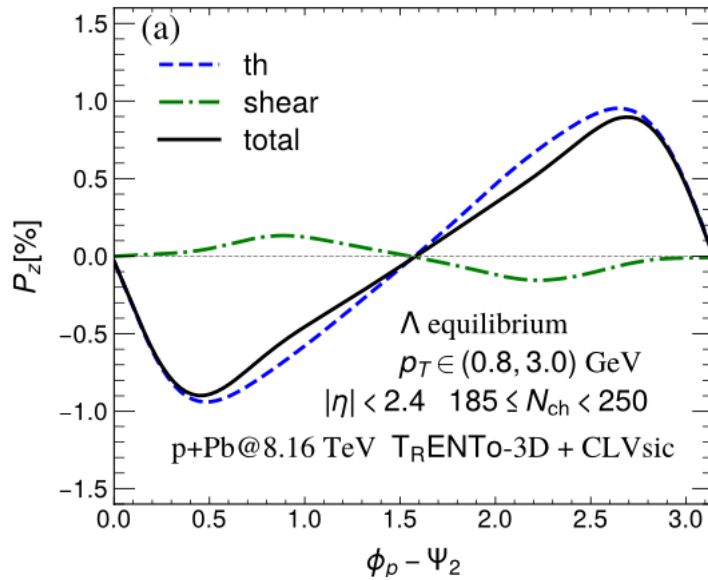




# $p_T$ dependence



# Azimuthal angle and pseudo-rapidity dependence



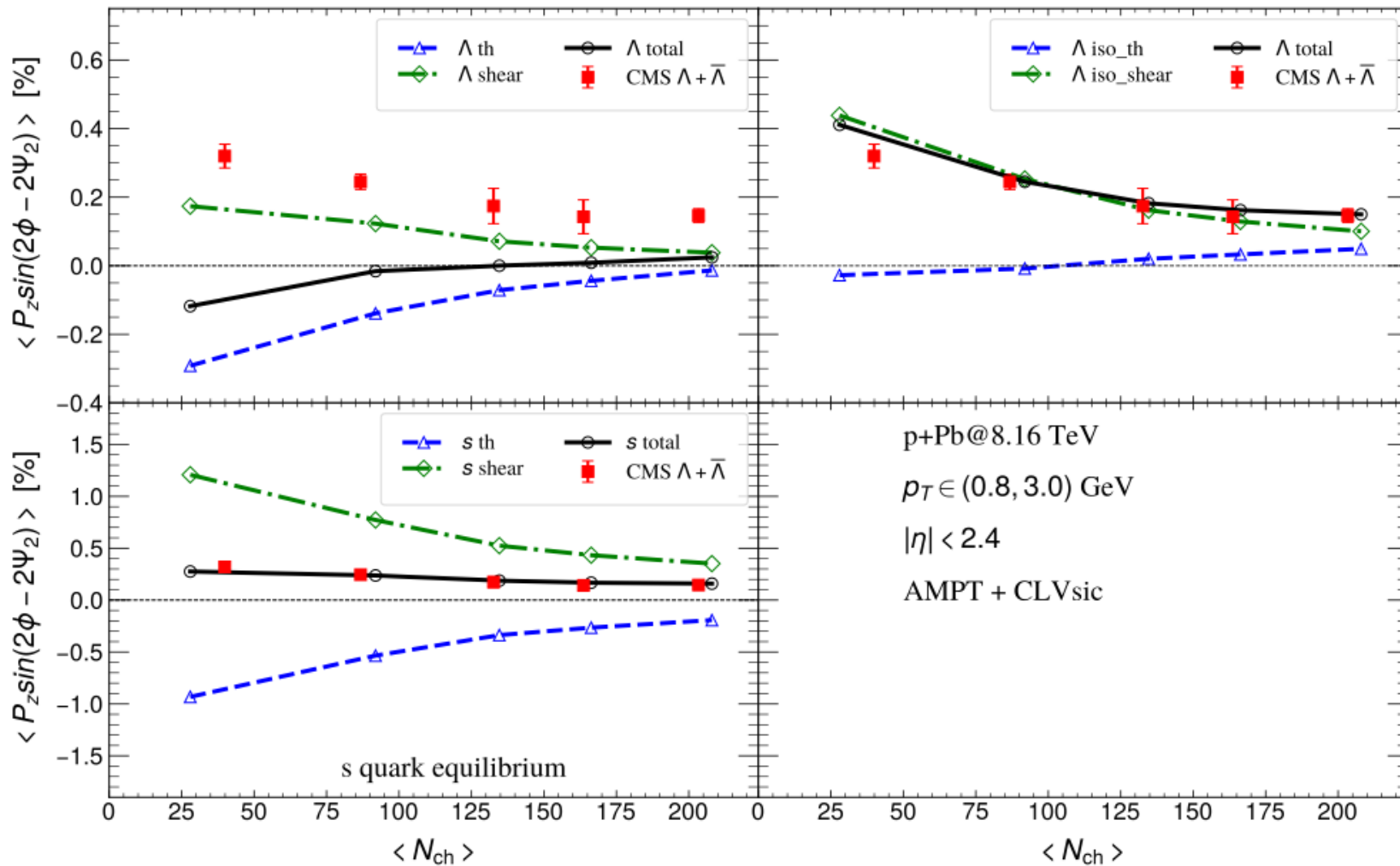
# Why?

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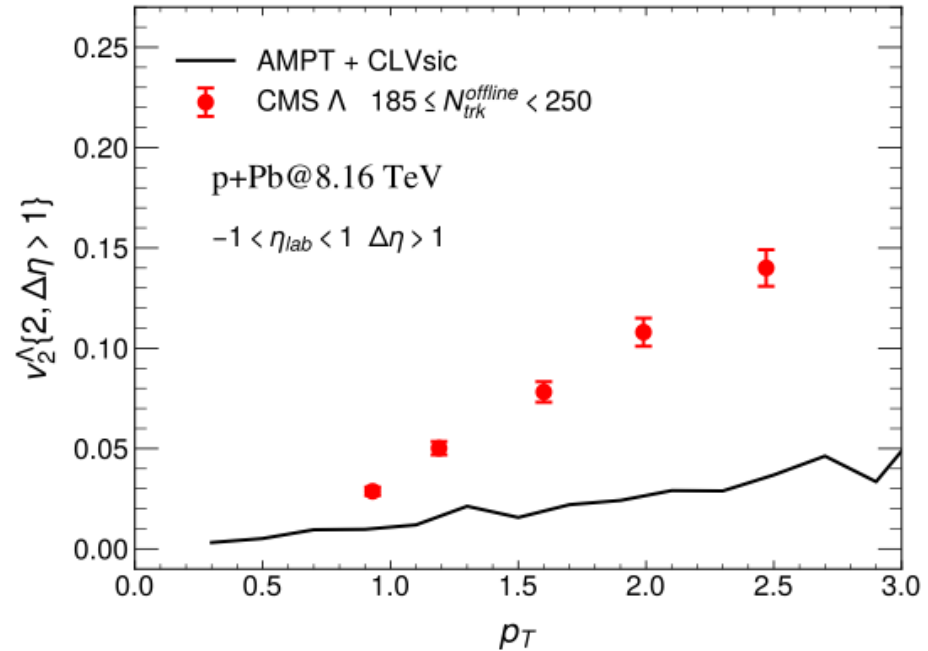
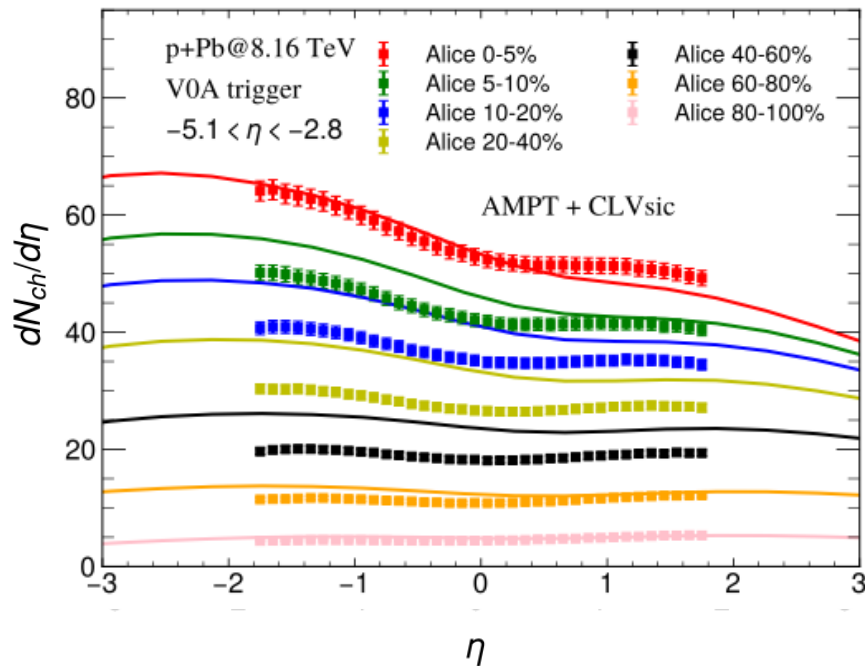
- We implement the 3+1D CLVisc hydrodynamics model
  - Pang, Wang, Wang, PRC (2012)
  - Wu, Qin, Pang, Wang, PRC (2022)
- Initial condition: **TRENTTo-3D model**
  - Soeder, Ke, Paquet, Bass, 2306.08665
  - Moreland, Bernhard, Bass, PRC (2015); PRC (2020)
  - Ke, Moreland, Bernhard, Bass, PRC (2017)
- p+Pb collisions at  $\sqrt{s_{NN}} = 8.16 \text{ TeV}$

# Test for AMPT initial conditions

It describes data well in s quark and isothermal equilibrium scenarios?

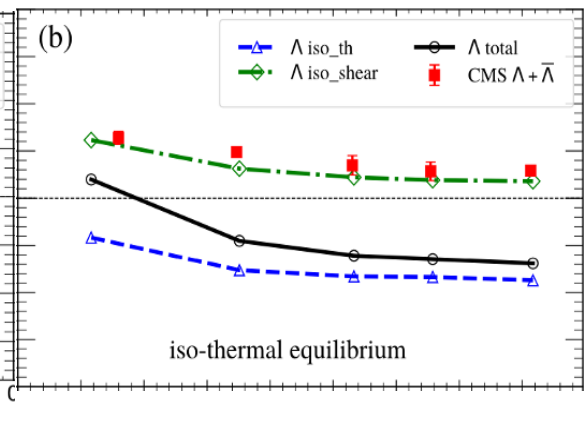
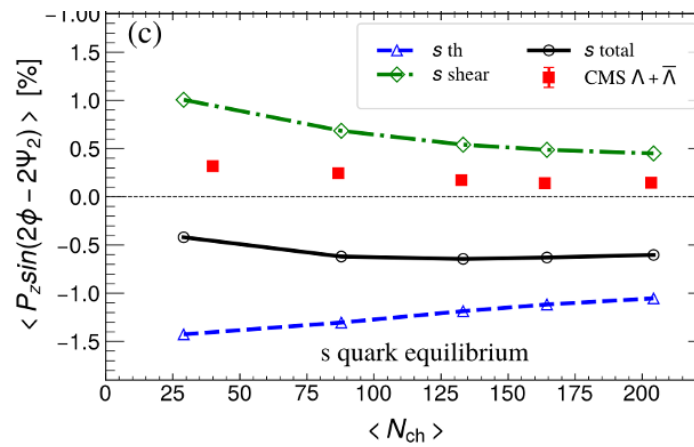
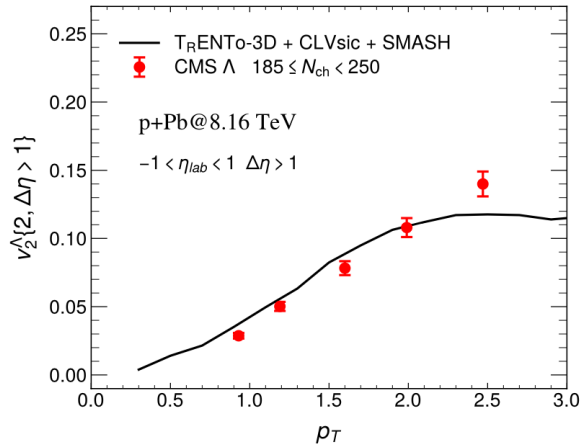


# Test for AMPT initial conditions



We fix the parameters in 3+1D CLVisc hydrodynamic model with AMPT initial conditions by the spectrum of charged hadrons. But, it **cannot** describe  $v_2$  well.

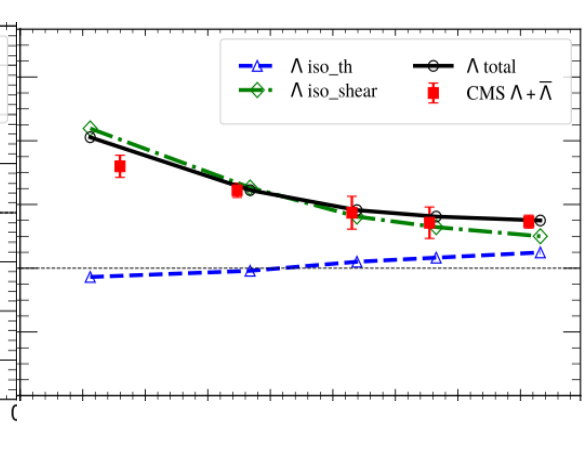
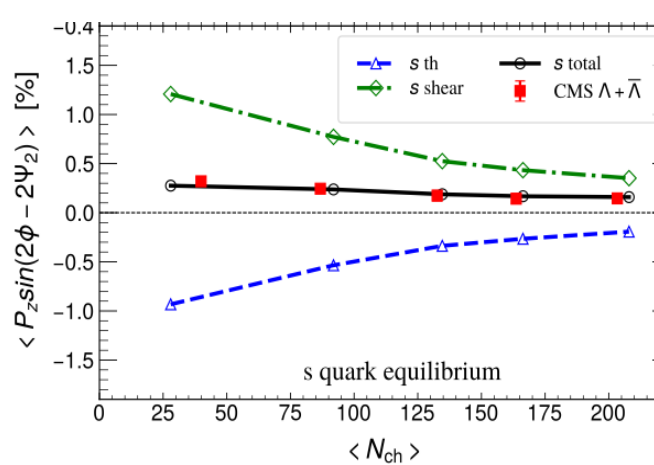
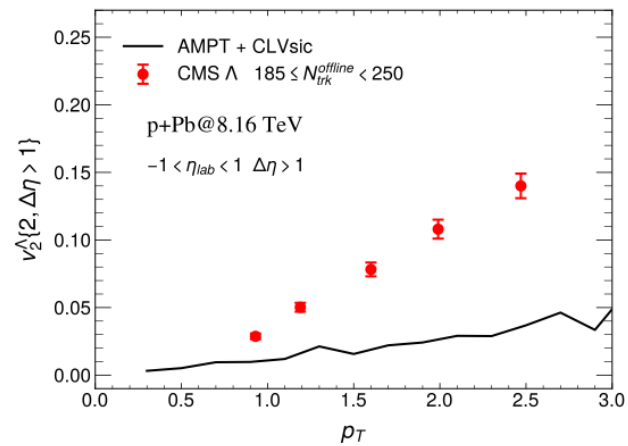
# However ...



Smaller  $v_2$  gives a larger polarization along beam direction ?

Smaller  $v_2$ , larger shear induced polarization, smaller thermal vortical induced polarization

Sensitive to initial conditions?



# Connection between $P_z$ and $v_2$

- Assuming we consider a Bjorken-like flow

$$\mathcal{S}_{\text{thermal}}^z = -\frac{1}{4m_\Lambda N} \frac{1}{T} \frac{dT}{d\tau} \Big|_\Sigma \partial_\phi \int d\Sigma_\alpha p^\alpha f_V^{(0)} \cosh \eta$$

since

$$\int d\Sigma_\lambda p^\lambda f_V^{(0)} = \frac{dN}{2\pi E_p p_T dp_T dY} \left[ 1 + \sum_{n=1}^{\infty} 2v_n(p_T, Y) \cos n\phi \right]$$

one can get

$$\mathcal{S}_{\text{thermal}}^z \approx \frac{1}{m_\Lambda} \frac{1}{T} \frac{dT}{d\tau} \Big|_\Sigma v_2(p_T, 0) \sin 2\phi.$$

Becattini, Karpenko, PRL (2018); C. Yi, SP, J.H. Gao, D.L. Yang, PRC (2022)

Beyond the picture about  $v_2$  induce  $P_z$ .

# Other related topics and discussion

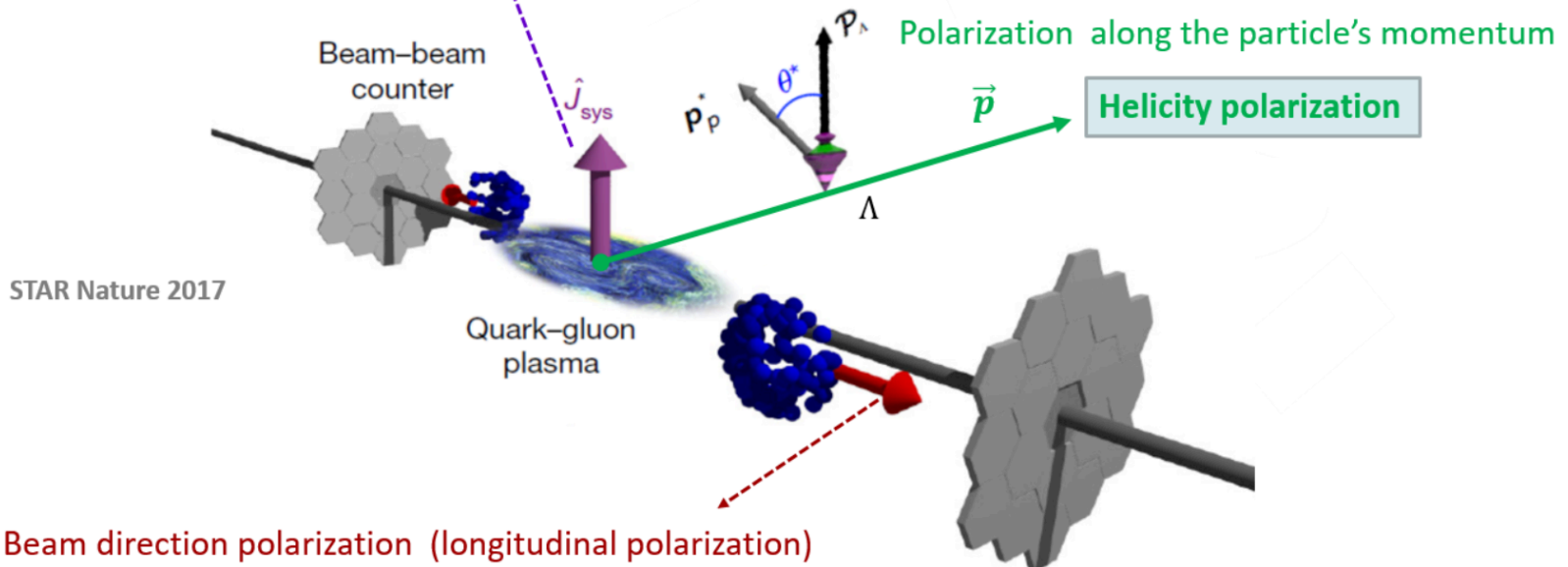


# (a) Helicity polarization

- The original idea for helicity polarization is proposed by Becattini, Buzzegoli, Palermo, Prokhorov, PLB(2021) and Gao, PRD(2021); Yi, Pu, Gao, Yang, PRC (2022) to probe the initial chiral chemical potential.
- Helicity instead of spin is widely-used in high energy spin physics.

$$S^h = \hat{\mathbf{p}} \cdot \mathbf{S}(\mathbf{p}) = \hat{p}^x S^x + \hat{p}^y S^y + \hat{p}^z S^z,$$

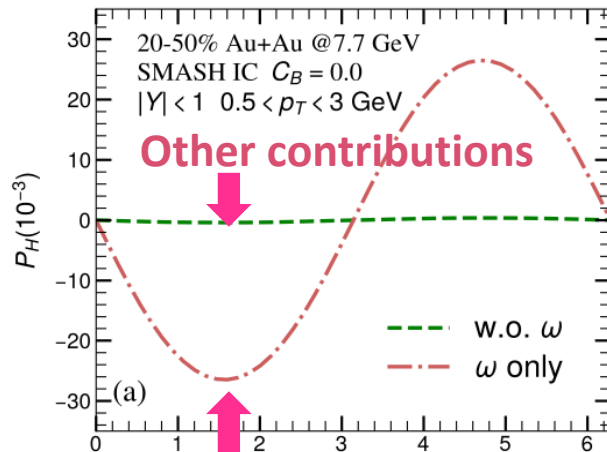
Out-plane direction polarization (transverse polarization)



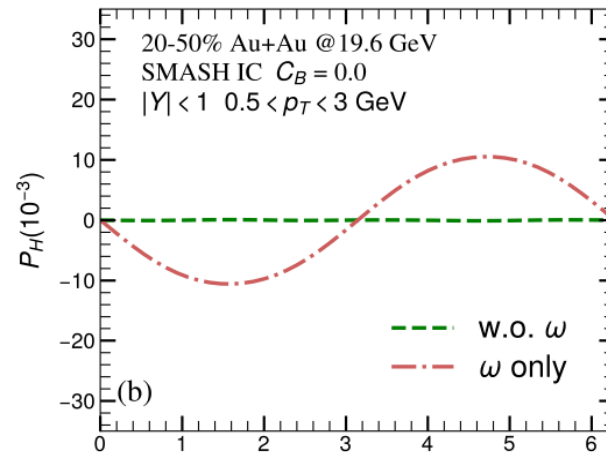
# Probe fine structure of vorticity by helicity polarization

$$S^h(\mathbf{p}) = S_{\text{thermal}}^h + S_{\text{shear}}^h + S_{\text{accT}}^h + S_{\text{chemical}}^h + S_{\text{EB}}^h$$

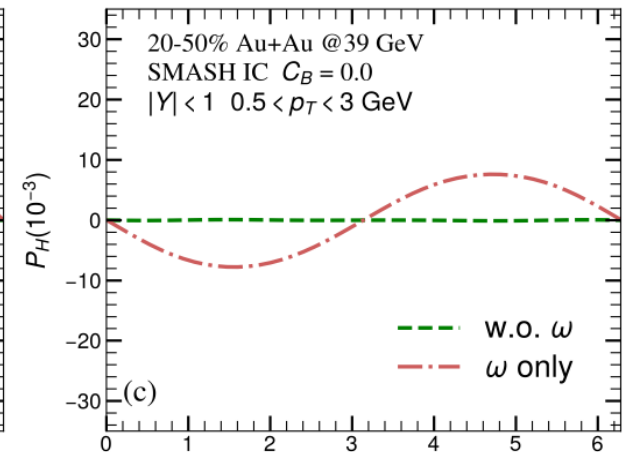
7.7 GeV



19.6 GeV



39 GeV



- Helicity polarization induced by **kinetic vorticity** dominates at low energy collisions.
- A possible way to **probe the fine structure of kinetic vorticity** by mapping the simulations of helicity polarization to the future measurements?  
Yi, Pu, Gao, Yang, PRC (2022); Yi, Wu, Yang, Gao, SP, Qin, PRC(Lett) (2023)

# (b) Collisional corrections

---

- **Collision term with quantum corrections**

Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD (2019); PRL (2021)

Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.

Liu, Mameda, Huang, arXiv:2002.03753.

Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019); Wang, Guo, Zhuang, EPJC (2021); Wang, Zhuang, arXiv:2105.00915

Li ,Yee, PRD100, 056022 (2019)

Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184

Fang, SP, Yang, PRD (2022)

Z.Y. Wang, arXiv:2205.09334; Lin, Wang, arXiv:2206.12573

...

## Recent reviews:

Gao, Ma, SP, Wang, NST 31 (2020) 9, 90

Gao, Liang, Wang, IJMPA 36 (2021), 2130001

Hidaka, SP, Yang, Wang, arXiv:2201.07644

**(b1) Corrections from self-energies**

**(b2) Corrections from scattering**

# (b1) Master equation for Wigner functions

$$\left[ \frac{i\hbar}{2} \gamma^\mu \nabla_\mu + \gamma^\mu \Pi_\mu - m + \underline{\bar{\Sigma}_g} \star \right] S^<(q, X) = -\frac{i\hbar}{2} (\Sigma_g^> \star S^< - \Sigma_g^< \star S^>),$$
$$S^< \left( -\frac{i\hbar}{2} \gamma^\mu \overleftarrow{\nabla}_\mu + \gamma^\mu \overleftarrow{\Pi}_\mu - m \right) + S^< \star \underline{\bar{\Sigma}_g} = -\frac{i\hbar}{2} (S^> \star \Sigma_g^< - S^< \star \Sigma_g^>),$$

\* denotes the Moyal product

$S^<$ : Wigner function

$$\bar{\Sigma}_g(q, X) = \Sigma^\delta(X) + \text{Re}\Sigma_g^r$$

For a long time, we always neglect the **self-energy terms** for simplicity. Now, we consider the contributions from them carefully.

# Applications to spin polarization

- We consider effects from the thermal QCD background. After a heavy calculation, we get the corrections to polarization vectors from self-energies:

Corrections to polarization induced by:

$$\begin{aligned} \delta\mathcal{P}_{\text{therm}}^\mu(t, \mathbf{q}) &= -\frac{\hbar^2}{2mN} \int_{\Sigma} q \cdot d\sigma G_{\text{T}}(E_q, \mathbf{q}) \frac{m_f^2 T}{E_q^3} \epsilon^{\mu\nu\alpha\beta} q_\nu \partial_\alpha \left( \frac{u_\beta}{T} \right), & \longleftrightarrow & \text{Thermal vorticity} \\ \delta\mathcal{P}_{\text{shear}}^\mu(t, \mathbf{q}) &= -\frac{\hbar^2}{4mN} \int_{\Sigma} q \cdot d\sigma G_{\omega_1}(E_q, \mathbf{q}) \frac{m_f^2}{E_q^3} \frac{\epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma}{E_q} q^\gamma \sigma_{\nu\gamma}, & \longleftrightarrow & \text{Shear tensor} \\ \delta\mathcal{P}_{\text{chem}}^\mu(t, \mathbf{q}) &= -\frac{\hbar^2}{4mN} \int_{\Sigma} q \cdot d\sigma G_{\text{T}}(E_q, \mathbf{q}) \frac{C_{\text{F}} g^2 \mu T}{4\pi^2 E_q^2} \frac{\epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma}{E_q} \nabla_\nu \left( \frac{\mu}{T} \right), & \longleftrightarrow & \text{Gradient of chemical Potential over temperature} \\ \delta\mathcal{P}_{\text{acc}}^\mu(t, \mathbf{q}) &= \frac{\hbar^2}{4mN} \int_{\Sigma} q \cdot d\sigma G_{\text{T}}(E_q, \mathbf{q}) \frac{3m_f^2}{E_q^3} \epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma \underline{D}u_\nu, & \longleftrightarrow & \text{Fluid acceleration} \\ \delta\mathcal{P}_{\text{vor}}^\mu(t, \mathbf{q}) &= \frac{\hbar^2}{4mN} \int_{\Sigma} q \cdot d\sigma \frac{m_f^2}{E_q^2} \left[ \underline{\omega}^\mu \left( 4G_{\text{T}}(E_q, \mathbf{q}) - \frac{|q_\perp|^2}{E_q^2} G_{\omega_1}(E_q, \mathbf{q}) + 2G_{\omega_2}(E_q, \mathbf{q}) \right) \right. \\ &\quad \left. - \frac{(\underline{\omega} \cdot \mathbf{q})}{E_q} \left( 6u^\mu G_{\text{T}}(E_q, \mathbf{q}) + \frac{q_\perp^\mu}{E_q} G_{\omega_1}(E_q, \mathbf{q}) \right) \right], & \longleftrightarrow & \text{Kinetic vorticity} \end{aligned}$$

Shuo Fang, Shi Pu, Di-Lun Yang, PRD (2024), arXiv: 2311.15197

# Estimation

It can contribute **30%** to the original polarization vector in the case of **low momentum**.

	$ q_{\perp}  = 0.5 \text{ GeV}$	$ q_{\perp}  = 1.0 \text{ GeV}$	$ q_{\perp}  = 2.0 \text{ GeV}$
$ \delta \mathcal{J}_{\text{therm}}^{5,\mu} / \mathcal{J}_{\text{therm,leq}}^{5,\mu} $	0.325	0.098	0.024
$ \delta \mathcal{J}_{\text{shear}}^{5,\mu} / \mathcal{J}_{\text{shear,leq}}^{5,\mu} $	0.081	0.028	0.007
$ \delta \mathcal{J}_{\text{vor}}^{5,\mu} / \mathcal{J}_{\text{therm,leq}}^{5,\mu} $	0.177	0.103	0.030

We have chosen temperature  $T = 0.165 \text{ GeV}$ , chemical potential  $\mu = 0.01 \text{ GeV}$  and constituent s quark mass  $m = 0.3 \text{ GeV}$ .

## (b2) Spin Boltzmann equations (II)

- We derive the spin Boltzmann equation incorporating Møller scattering process using hard thermal loop approximations.

$$p^\mu \partial_\mu f_A^<(p) + \hbar \partial_\mu S^{(u),\mu\alpha}(p) \partial_\alpha f_V^<(p) = C_A + \hbar \partial_\mu \left( S_{(u)}^{\mu\alpha} C_{V,\alpha}[f_V^<] \right)$$

S. Fang, SP, D.L. Yang, PRD (2022); S. Fang, SP, arXiv:2408.09877

- **Scenario (I): particle distribution function is off-equilibrium**

$$\partial \sim \lambda^{-1} \text{Kn} \ll \lambda^{-1}$$

Kn: Knudsen number  
 $\lambda$ : mean free path

- **Scenario (II): particle distribution function is at local equilibrium**  
Similar to standard kinetic theory, e.g. AMY

# New corrections from scattering

Let us start from the kinetic theory for massless fermions.

$$p \cdot \partial f_0 = C_{pp' \rightarrow kk'} [\delta f],$$

We consider the system close to the global equilibrium,

$$f = f_0 + \delta f,$$

We can estimate

$$\delta f \sim A p_\mu p_\nu \pi^{\mu\nu}, \quad A \sim 1/C_{pp' \rightarrow kk'} [f] \sim 1/e^4,$$

Recalling the **spin** current in phase space,

$$j^\mu(p) = p^\mu f + S^{\mu\nu} \partial_\nu f + \int_{p', k, k'} C_{pp' \rightarrow kk'} [f] \Delta^\mu,$$

$$j^\mu \sim \int_{p', k, k'} C_{pp' \rightarrow kk'} [\delta f] \Delta^\mu \sim \int_{p', k, k'} C_{pp' \rightarrow kk'} [A p_\mu p_\nu \pi^{\mu\nu}] \Delta^\mu$$

$$\sim \cancel{e^4} \frac{1}{e^4} \pi^{\mu\nu} p_\mu p_\nu.$$

- Leading order in gradient expansion!
- Corrections from scatterings but do not depend on coupling constant

It can also be derived by Kubo formula.



# Connection to condensed matter physics

- After we finish this work, we find that the same discoveries have been derived in condensed matter physics in their quantum kinetic theory.

T. Valet, R. Raimondi, arXiv:2410.08975

$$\hat{\rho}_{nm}^{(1)} = -\hbar \frac{f_n^{(0)} - f_m^{(0)}}{\varepsilon_n - \varepsilon_m} e\mathbf{E} \cdot \hat{\mathcal{A}}_{nm} - \hbar \partial_x \frac{f_n^{(1)} + f_m^{(1)}}{2} \cdot \hat{\mathcal{A}}_{nm}$$

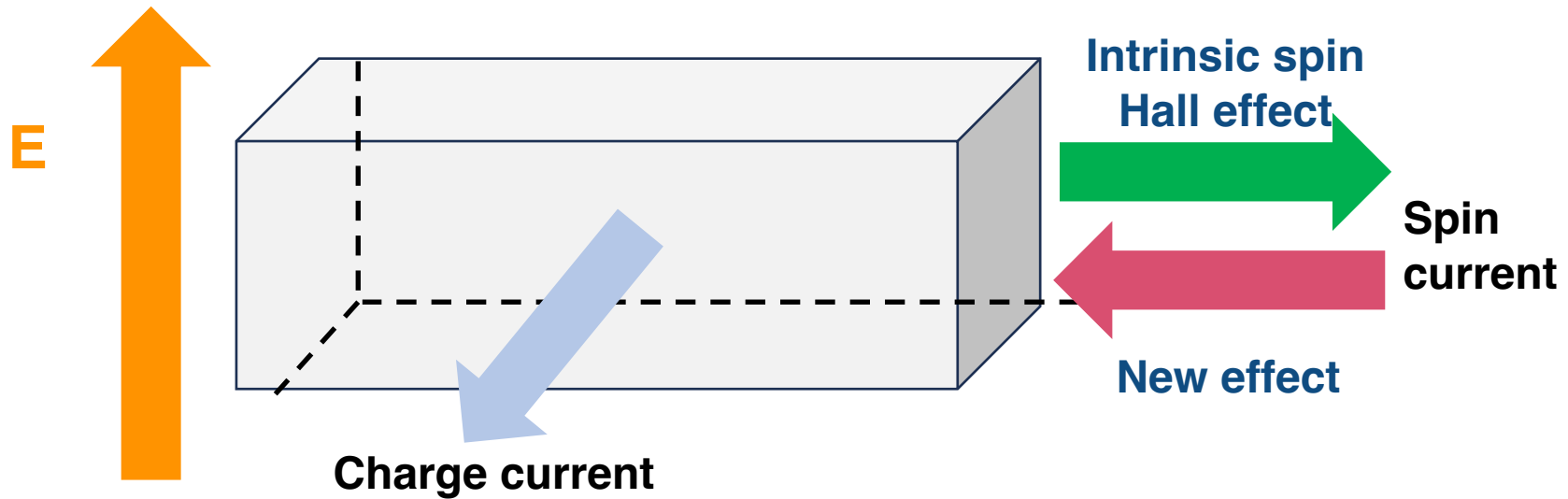
$$- \frac{i\hbar}{2(\varepsilon_n - \varepsilon_m)} \frac{2\pi n_i v_0^2}{\hbar} \sum_q \int \frac{d^d p}{(2\pi\hbar)^d} \hat{P}_n \hat{P}_q \hat{P}_m \times \dots \quad (1)$$

$$\dots \times \left[ \delta(\varepsilon_q - \varepsilon_n) \left( f_q^{(1)} - f_n^{(1)} \right) + \delta(\varepsilon_q - \varepsilon_m) \left( f_q^{(1)} - f_m^{(1)} \right) \right],$$

Replacing the electric force by shear force, the results are consistent with what we found.

Private communication and check with T. Valet in workshop at Hangzhou.

# Physical meaning of new effects



$$\hat{\rho}_{nm}^{(1)} = -\hbar \frac{f_n^{(0)} - f_m^{(0)}}{\varepsilon_n - \varepsilon_m} e \mathbf{E} \cdot \hat{\mathbf{A}}_{nm} - \hbar \partial_x \frac{f_n^{(1)} + f_m^{(1)}}{2} \cdot \hat{\mathbf{A}}_{nm}$$

$$- \frac{i\hbar}{2(\varepsilon_n - \varepsilon_m)} \frac{2\pi n_i v_0^2}{\hbar} \sum_q \int \frac{d^d p}{(2\pi\hbar)^d} \hat{P}_n \hat{P}_q \hat{P}_m \times \dots \quad (1)$$

$$\dots \times \left[ \delta(\varepsilon_q - \varepsilon_n) \left( f_q^{(1)} - f_n^{(1)} \right) + \delta(\varepsilon_q - \varepsilon_m) \left( f_q^{(1)} - f_m^{(1)} \right) \right],$$

# New corrections to shear induced polarization

- **Scenario (I):**

$$\delta\mathcal{P}_{(I)}^\mu(\mathbf{p}) = \delta\mathcal{P}_{(I),\text{shear}}^\mu + \delta\mathcal{P}_{(I),\text{chem}}^\mu + \mathcal{O}(\hbar^2\partial^2)$$

$$\delta\mathcal{P}_{(I),\text{shear}}^\mu = -\frac{\hbar^2}{4N} \int_{\Sigma} \frac{d\Sigma \cdot p}{E_{\mathbf{p}}} \beta_0 g_2(E_{\mathbf{p}}) \epsilon^{\mu\nu\rho\sigma} p_{\rho} u_{\sigma} \sigma_{\nu\alpha} p^{\alpha},$$

$$\delta\mathcal{P}_{(I),\text{chem}}^\mu = -\frac{\hbar^2}{4N} \int_{\Sigma} \frac{d\Sigma \cdot p}{E_{\mathbf{p}}} \beta_0 g_1(E_{\mathbf{p}}) \epsilon^{\mu\nu\rho\sigma} p_{\rho} u_{\sigma} \nabla_{\nu} \alpha_0.$$

**They come from scatterings but do not depend on coupling constant explicitly. They correspond to anomalous spin Hall conductivity in condensed matter.**

**Also see the similar findings: S. Lin and Z. Wang, arXiv:2406.10003.**

# Other second order corrections

- Scenario (II):

$$\delta\mathcal{P}_{(\text{II})}^\mu(\mathbf{p}) = \mathcal{P}_{(\text{II}),\omega-\nabla T}^\mu + \mathcal{P}_{(\text{II}),\nabla\omega}^\mu + \mathcal{P}_{(\text{II}),\omega-\text{chem}}^\mu + \mathcal{P}_{(\text{II}),\omega-\text{shear}}^\mu \\ + \mathcal{P}_{(\text{II}),\text{chem}-\nabla T}^\mu + \mathcal{P}_{(\text{II}),\text{shear}-\nabla T}^\mu + \mathcal{P}_{(\text{II}),\text{shear}-\text{chem}}^\mu + \mathcal{O}(\hbar^2\partial^3)$$

$$\mathcal{P}_{(\text{II}),\omega-\nabla T}^\mu = -\hbar^2 \int_{\Sigma} d\Sigma \cdot pa_{(\text{II})}^\mu \left[ d_2 \left( E_{\mathbf{p}} - d_1 \frac{1}{\beta_0} \right) \omega^\alpha \nabla_\alpha \beta_0 - d_6 \beta_0 p_{\langle\alpha} p_{\rho\rangle} \omega^\alpha \nabla^\rho \beta_0 \right],$$

$$\mathcal{P}_{(\text{II}),\nabla\omega}^\mu = \hbar^2 \int_{\Sigma} d\Sigma \cdot pa_{(\text{II})}^\mu \left[ \left( E_{\mathbf{p}} - d_1 \frac{1}{\beta_0} \right) d_2 \beta_0 \nabla^\alpha \omega_\alpha + d_6 \frac{1}{2} \beta_0^2 \nabla^\alpha \omega^\rho p_{\langle\alpha} p_{\rho\rangle} \right],$$

$$\mathcal{P}_{(\text{II}),\omega-\text{chem}}^\mu = \hbar^2 \int_{\Sigma} d\Sigma \cdot pa_{(\text{II})}^\mu \left[ \left( E_{\mathbf{p}} - d_1 \frac{1}{\beta_0} \right) d_3 \beta_0 \omega^\alpha \nabla_\alpha \alpha_0 - d_8 \beta_0^2 \omega^\alpha \nabla^\rho \alpha_0 p_{\langle\alpha} p_{\rho\rangle} \right],$$

$$\mathcal{P}_{(\text{II}),\omega-\text{shear}}^\mu = -\hbar^2 \beta_0 \int_{\Sigma} d\Sigma \cdot pa_{(\text{II})}^\mu \left[ -d_4 \omega^\rho \sigma_\rho^\alpha p_{\langle\alpha\rangle} + d_9 \beta_0^2 \omega^\beta \sigma^{\alpha\lambda} p_{\langle\beta} p_\alpha p_{\lambda\rangle} \right]$$

$$\mathcal{P}_{(\text{II}),\text{chem}-\nabla T}^\mu = \hbar^2 \int_{\Sigma} d\Sigma \cdot pa_{(\text{II})}^\mu d_5 \epsilon^{\rho\nu\alpha\beta} u_\beta \nabla_\nu \alpha_0 \nabla_\rho \beta_0 p_{\langle\alpha\rangle},$$

$$\mathcal{P}_{(\text{II}),\text{shear}-\nabla T}^\mu = \hbar^2 \beta_0 \int_{\Sigma} d\Sigma \cdot pa_{(\text{II})}^\mu d_6 \epsilon^{\beta\nu\sigma\rho} \sigma_\beta^\alpha u_\sigma \nabla_\nu \beta_0 p_{\langle\alpha} p_{\rho\rangle},$$

$$\mathcal{P}_{(\text{II}),\text{shear}-\text{chem}}^\mu = -\hbar^2 \beta_0^2 \int_{\Sigma} d\Sigma \cdot pa_{(\text{II})}^\mu d_7 \epsilon^{\mu\nu\sigma\rho} \sigma_\mu^\alpha u_\sigma \nabla_\nu \alpha_0 p_{\langle\alpha} p_{\rho\rangle},$$

# Corrections from space-time dependent EM fields

- We derived the corrections to Wigner function and polarization from varying EM fields.

$$\mathcal{S}_{(2)}^\mu = \frac{1}{8mN} \sum_{m=1,2,3} \int d\Sigma^\sigma p_\sigma X_{(m)}^\mu f_5^{(m)},$$

**Corrections for 2nd order constant EM fields**

$$\begin{aligned} X_{(0)}^\mu &= \frac{1}{p_u^3} \left( u^\mu u^\nu u_\lambda - \frac{1}{p_u} p^\mu u^\nu u_\lambda - \frac{2}{p_u} u^\mu u^\nu p_\lambda - \frac{1}{2p_u^2} u^\mu p^\nu p_\lambda + \frac{1}{p_u^2} p^\mu u^\nu p_\lambda + u_\lambda g^{\mu\nu} \right) F_{\nu\rho} F^{\lambda\rho}, \\ X_{(1)}^\mu &= \frac{1}{p_u^2} \left( \frac{1}{2} p^\mu u^\nu u_\lambda + u^\mu u^\nu p_\lambda - \frac{1}{4p_u} u^\mu p^\nu p_\lambda - \frac{1}{2p_u} p^\mu u^\nu p_\lambda - p_u g^{\mu\nu} u_\lambda \right) (F_{\nu\gamma} \Omega^{\lambda\gamma} + \Omega_{\nu\gamma} F^{\lambda\gamma}) \\ &+ \frac{\beta}{p_u^3} \left( p^\mu u^\nu u_\lambda + 2u^\mu u^\nu p_\lambda - \frac{1}{p_u} u^\mu p_\mu p_\lambda - \frac{2}{p_u} p^\mu u^\nu p_\lambda + \frac{1}{2p_u^2} p^\mu p^\nu p_\lambda - 2p_u g^{\mu\nu} u_\lambda + g^{\mu\nu} p_\lambda \right) F_{\nu\rho} F^{\lambda\rho} \end{aligned}$$

$$\begin{aligned} X_{(2)}^\mu &= \frac{1}{p_u} \left( \frac{1}{2} u^\mu p^\nu p_\lambda + p^\mu u^\nu p_\lambda - p_u g^{\mu\nu} p_\lambda \right) \Omega_{\nu\rho} \Omega^{\lambda\rho} \\ &+ \frac{\beta}{p_u^2} \left( p^\mu u^\nu p_\lambda + \frac{1}{2} u^\mu p^\nu p_\lambda - \frac{1}{4p_u} p^\mu p^\nu p_\lambda - p_u g^{\mu\nu} p_\lambda \right) (F_{\nu\gamma} \Omega^{\lambda\gamma} + \Omega_{\nu\gamma} F^{\lambda\gamma}) \\ &+ \frac{\beta^2}{2p_u^3} \left( u^\mu p_\mu p_\lambda + 2p^\mu u^\nu p_\lambda - \frac{1}{p_u} p^\mu p_\mu p_\lambda - 2p_u g^{\mu\nu} p_\lambda \right) F_{\nu\rho} F^{\lambda\rho}, \\ X_{(3)}^\mu &= \frac{\beta}{2p_u} p^\mu p^\nu p_\lambda \Omega_{\nu\rho} \Omega^{\lambda\rho} + \frac{\beta^2}{4p_u^2} p^\mu p^\nu p_\lambda (F_{\nu\gamma} \Omega^{\lambda\gamma} + \Omega_{\nu\gamma} F^{\lambda\gamma}) + \frac{\beta^3}{3p_u^3} p^\mu p^\nu p_\lambda F_{\nu\rho} F^{\lambda\rho}. \end{aligned}$$

$$\mathcal{S}_{\partial,EM}^\mu = \frac{1}{8mN} \sum_{m=0,1,2,3} \int d\Sigma^\sigma p_\sigma Y_{(m)}^\mu f_5^{(m)},$$

**Corrections for varying EM fields**

$$\begin{aligned} Y_{(0)}^\mu &= -\frac{2}{3p_u^2} \left( u_\lambda u_\nu - \frac{1}{2p_u} u_\lambda p_\nu - \frac{1}{2p_u} p_\lambda u_\nu \right) \partial^\lambda F^{\mu\nu} \\ &+ \frac{1}{3p_u^2} \left( 2u^\mu u^\nu - \frac{1}{p_u} p^\mu u^\nu - \frac{1}{p_u} u^\mu p^\nu \right) \partial^\lambda F_{\lambda\nu}, \\ Y_{(1)}^\mu &= +\frac{2\beta}{3p_u} \left( u_\lambda u_\nu - \frac{1}{p_u} u_\lambda p_\nu - \frac{1}{p_u} p_\lambda u_\nu + \frac{1}{2p_u^2} p_\lambda p_\nu \right) \partial^\lambda F^{\mu\nu} \\ &+ \frac{\beta}{6p_u} \left( u^\mu u^\nu + \frac{4}{p_u} p^\mu u^\nu + \frac{4}{p_u} u^\mu p^\nu - \frac{2}{p_u^2} p^\mu p^\nu \right) \partial^\lambda F_{\lambda\nu} \\ &- \frac{\beta}{3p_u^2} \left( u^\mu u^\lambda p^\nu - \frac{1}{2p_u} p^\mu u^\lambda p^\nu - \frac{1}{2p_u} u^\mu p^\lambda p^\nu \right) u^\rho \partial_\lambda F_{\nu\rho} \\ &\frac{4\beta}{3p_u} \partial_\lambda F^{\mu\lambda} + \frac{\beta}{3p_u} u^\nu u^\rho \partial^\mu F_{\nu\rho}, \end{aligned}$$

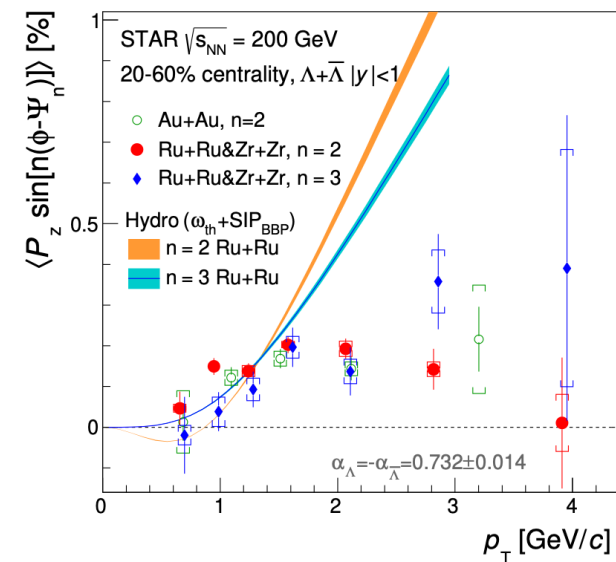
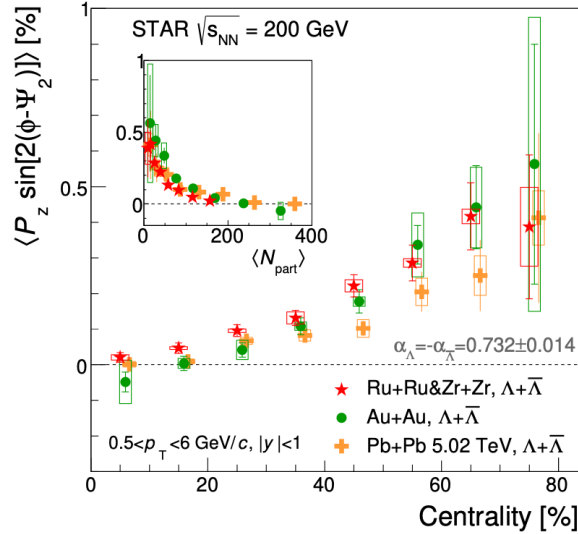
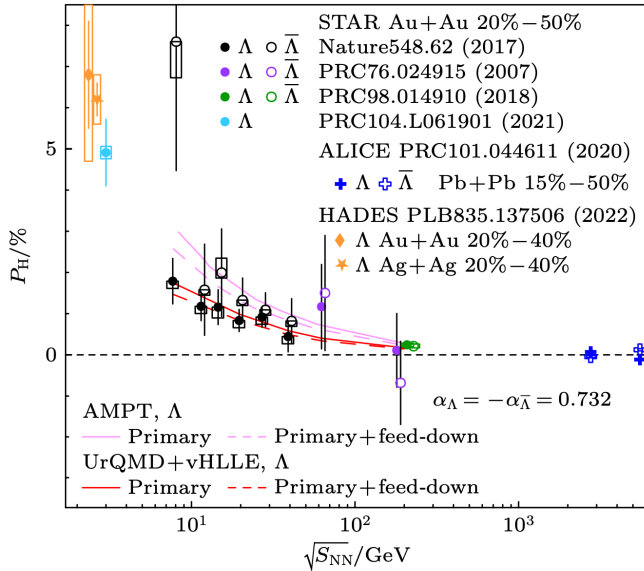
$$\begin{aligned} Y_{(2)}^\mu &= -\frac{\beta^2}{3} \left( u_\lambda u_\nu - \frac{2}{p_u} u_\lambda p_\nu + \frac{1}{p_u^2} p_\lambda p_\nu \right) \partial^\lambda F^{\mu\nu} \\ &+ \frac{\beta^2}{6} \left( \frac{1}{p_u^2} p^\mu u^\nu + \frac{2}{p_u^2} p^\mu p^\nu \right) \partial^\lambda F_{\lambda\nu} + \frac{\beta^2}{3p_u} p^\nu u^\rho \partial^\mu F_{\nu\rho} \\ &+ \frac{\beta^2}{3p_u} \left( u^\mu u^\lambda p^\nu - \frac{1}{p_u} p^\mu u^\lambda p^\nu - \frac{1}{p_u} u^\mu p^\lambda p^\nu + \frac{1}{2p_u^2} p^\mu p^\lambda p^\nu \right) u^\rho \partial_\lambda F_{\nu\rho}, \\ Y_{(3)}^\mu &= \frac{\beta^3}{3p_u} \left( p^\mu u^\lambda p^\nu - \frac{1}{2p_u} p^\mu p^\lambda p^\nu \right) u^\rho \partial_\lambda F_{\nu\rho}. \end{aligned}$$

**S. Z. Yang, J.H. Gao, SP, arXiv: 2409.00456**

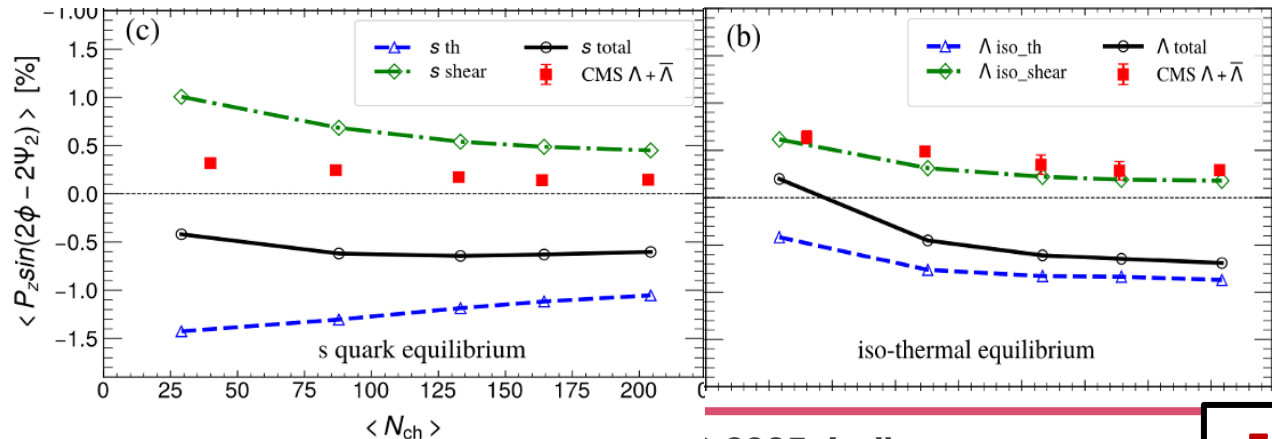
# Summary and outlook

# Summary (I)

## When will the polarization stop growing?



Smaller  $v_2$  gives a larger polarization along beam direction?  
 Sensitive to initial conditions?



# Summary (II)

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New corrections to shear induced polarization come from scatterings but do not depend on coupling constant explicitly.

$$\delta\mathcal{P}_{(I)}^\mu(\mathbf{p}) = \delta\mathcal{P}_{(I),\text{shear}}^\mu + \delta\mathcal{P}_{(I),\text{chem}}^\mu + \mathcal{O}(\hbar^2\partial^2)$$

$$\delta\mathcal{P}_{(I),\text{shear}}^\mu = -\frac{\hbar^2}{4N} \int_{\Sigma} \frac{d\Sigma \cdot p}{E_{\mathbf{p}}} \beta_0 g_2(E_{\mathbf{p}}) \epsilon^{\mu\nu\rho\sigma} p_{\rho} u_{\sigma} \sigma_{\nu\alpha} p^{\alpha},$$

$$\delta\mathcal{P}_{(I),\text{chem}}^\mu = -\frac{\hbar^2}{4N} \int_{\Sigma} \frac{d\Sigma \cdot p}{E_{\mathbf{p}}} \beta_0 g_1(E_{\mathbf{p}}) \epsilon^{\mu\nu\rho\sigma} p_{\rho} u_{\sigma} \nabla_{\nu} \alpha_0.$$



**Thank you for your time!**

**Any comments and suggestions are  
welcome!**

# Backup

# Puzzle : T-odd/T-even VS dissipative/non-dissipative

Are shear induced polarization or spin alignment non-dissipative?

Q1: If the coefficient is T-even, it is non-dissipative.

**CME**  $\mathbf{j} \sim C\mathbf{B}$   $\xrightarrow{\text{Taking T transformation}}$   $\mathbf{j} \rightarrow -\mathbf{j}$   
 $\mathbf{B} \rightarrow -\mathbf{B}$  **C: T-even** ✓

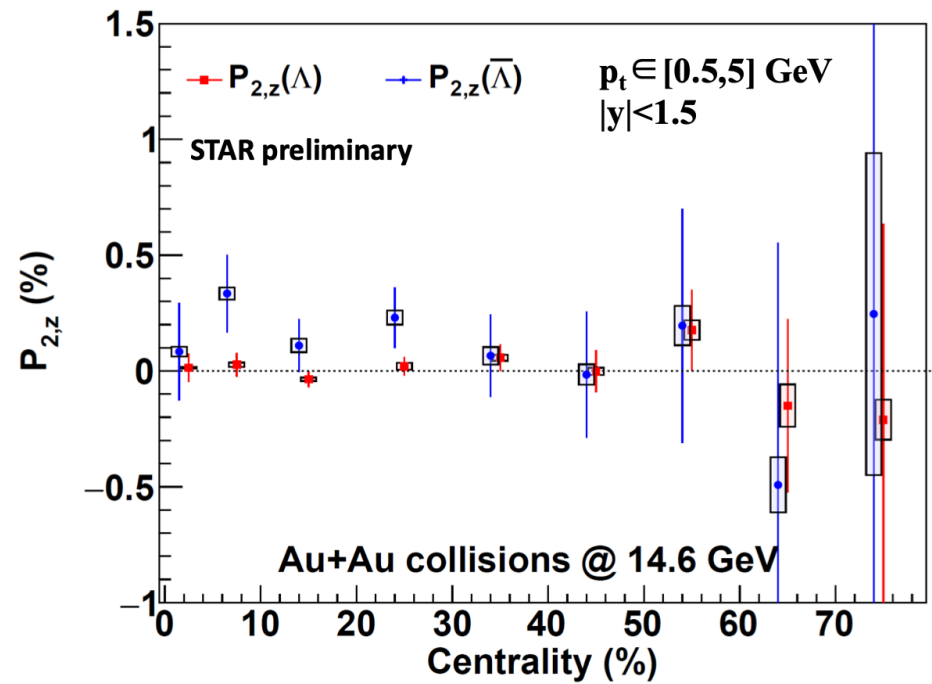
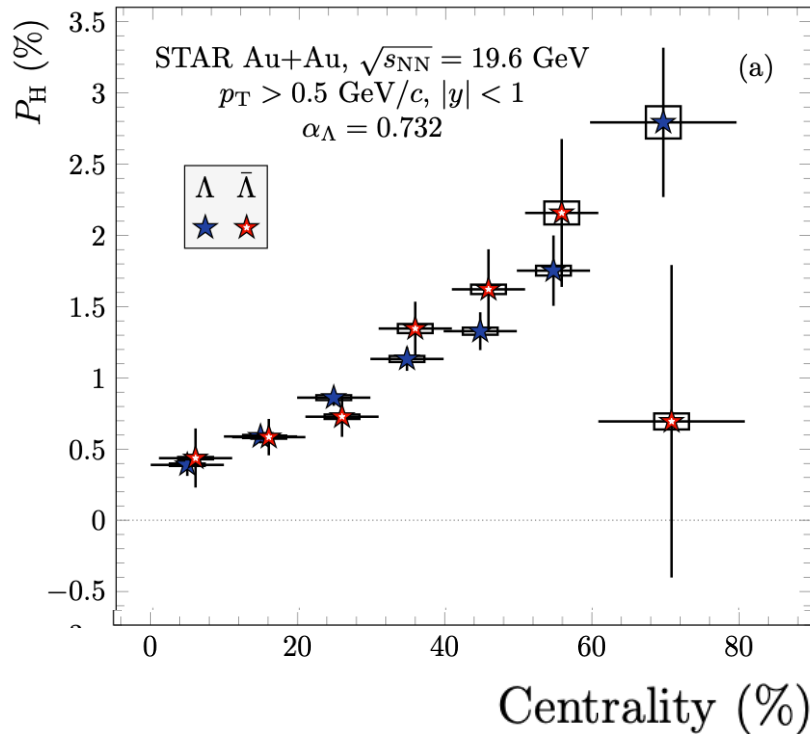
**Spin polarization vector**  $\mathcal{S}^i \sim C^{ijk}(\partial_j u_k + \partial_k u_j)$   $\xrightarrow{\hspace{2cm}}$   $\mathcal{S}^i \rightarrow -\mathcal{S}^i$   
 $\partial_j u_k \rightarrow -\partial_j u_k$  **C: T-even**  
**Non-dissipative?**

**Spin alignment**  $\epsilon^i(\lambda)\epsilon^{*j}(\lambda')\rho_{\lambda\lambda'} \sim a\pi^{ij}$   $\xrightarrow{\hspace{2cm}}$   $\rho_{\lambda\lambda'} \rightarrow (-1)^{\lambda+\lambda'}\rho_{\lambda\lambda'}$   
 $\epsilon_\mu^{s*}(\mathbf{p}) \rightarrow -(-1)^s \tilde{\delta}_\mu^\alpha \epsilon_\alpha^{-s*}(-\mathbf{p})$   
 $\pi^{ij} \rightarrow -\pi^{ij}$

For  $\rho_{00}$ , the coefficient “a” is **T-odd**. **Dissipative?**

In Zubarev approach, the non-dissipative means the results does NOT depend on hypersurface. But, shear tensor comes from local equilibrium operators and should always depends on hypersurface. So, shear induced something is always dissipative?

# Local polarization VS centrality



STAR, Phys.Rev.C 108 (2023) 1, 014910

Hu's talk at SQM 2024