Overview of neutron stars and their connection to QCD phase diagram

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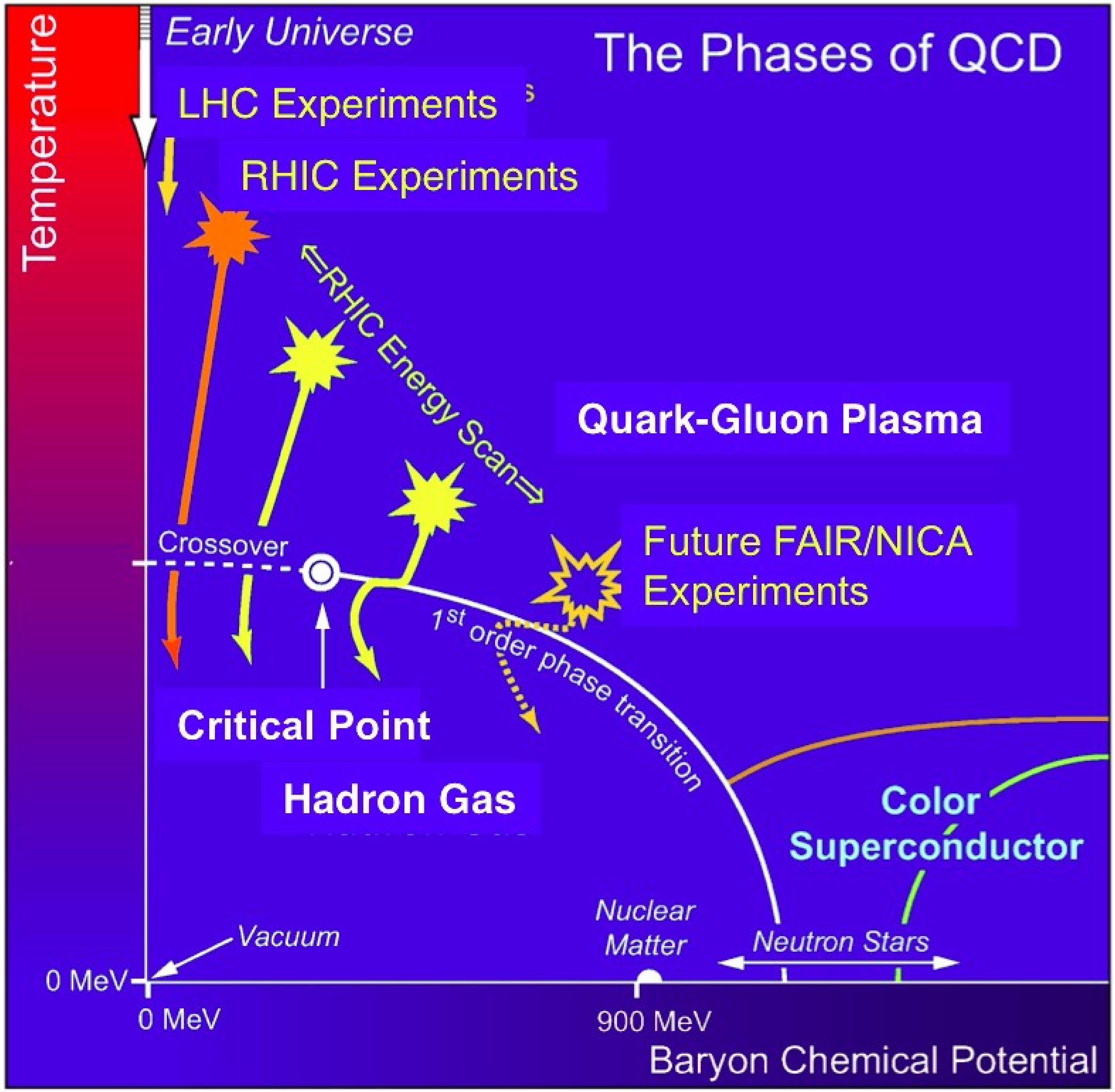
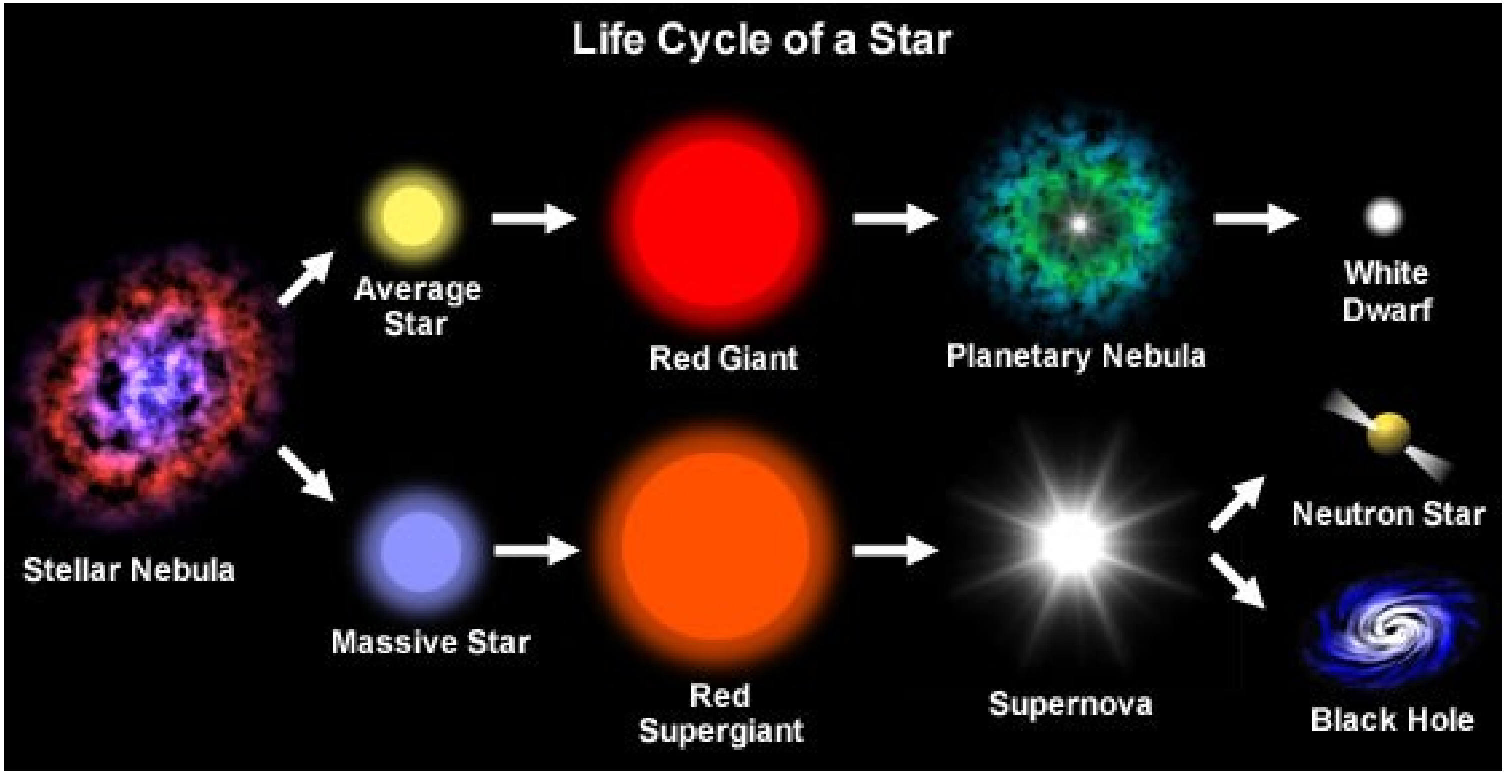
Introduction



Introduction to Neutron Stars

Neutron stars are dead stars, produced via the gravitational collapse of massive stars ($8M_\odot < M < 25M_\odot$) via supernova.



Basic features

- **Mass (M)** ~ $1\text{–}2\, M_\odot$
- **Radius (R)** ~ **10 – 15** km; R_{Earth}~ 6000 km, R_{Sun} ~ 7 × 10^5 km
- **Density (ρ)** ~ 10^{15} g cm^{-3} .
  - All of humanity could be squashed down to a sugar cube-sized piece of a neutron star.
- Rotates very fast, **Period** ~ 1 ms −10 s

  ⇒ as fast as blenders.
- Strongest magnetic fields: B_{surface} ~ 10^8 – 10^{12} G;

  B_{Earth} ~ 0.5 G. Strongest magnet produced on earth B=1.2×10^7 G.

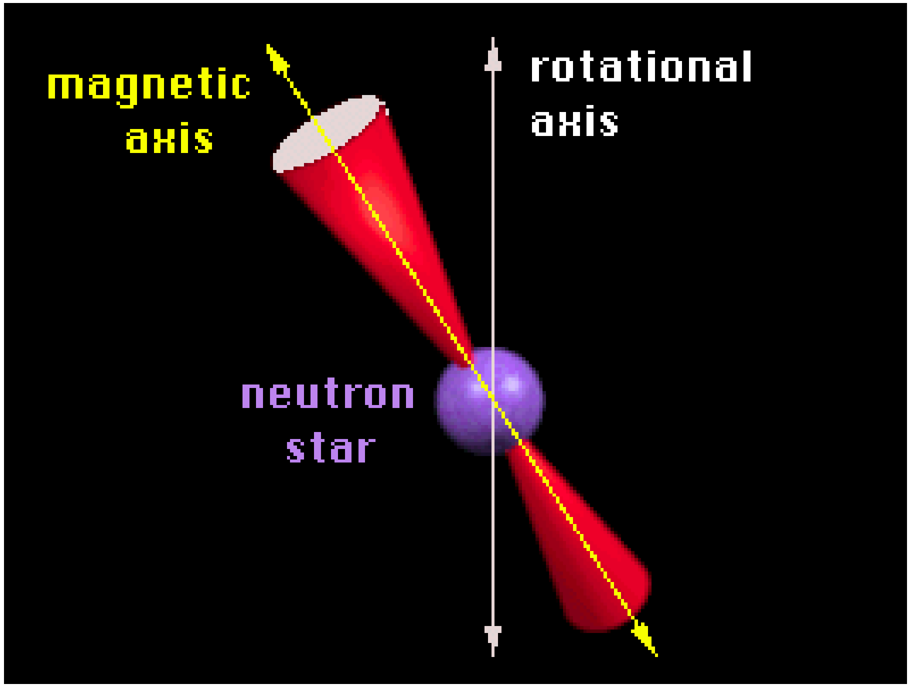
  ⇒ Magnetars have even higher magnetic fields:

  B_{surface} ~ 10^{13} – 10^{15} G.

➢ Unique laboratories for studying matter under extreme conditions.

Discovery

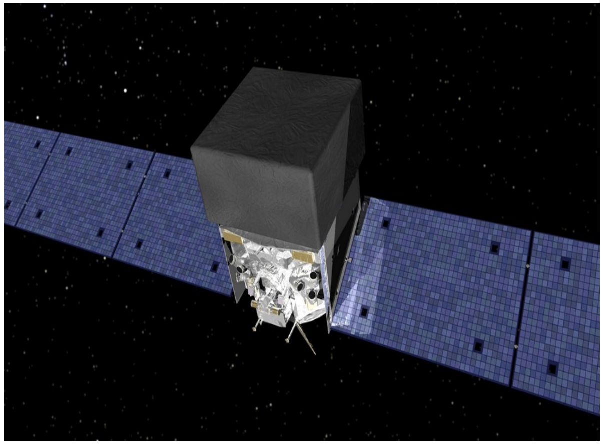
- Neutron stars are mostly observed as radio pulsars.
- Jocelyn Bell and Anthony Hewish (Nobel prize, 1974) discovered the first radio pulsar in 1967.
- Soon identified as a **highly-magnetized rotating neutron star**.

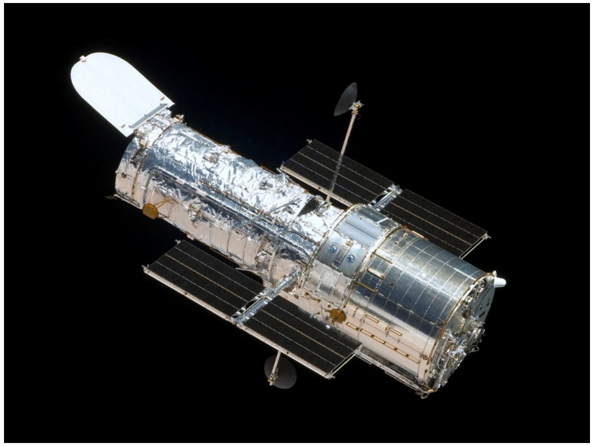


- More than 3300 pulsars are discovered so far.

Observation


Chandra (x-ray)


Fermi (γ- ray)


Hubble (UV, NIR, Visible)

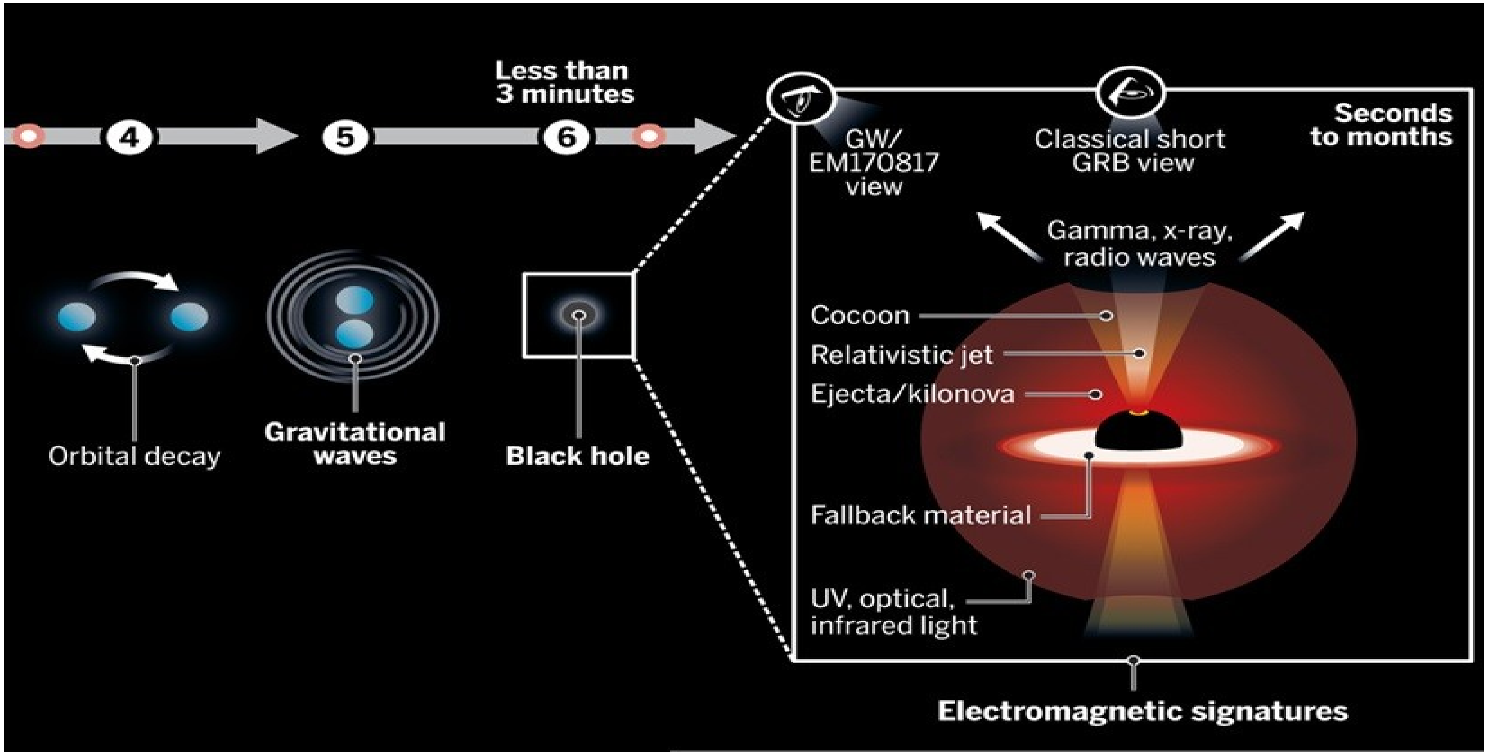

VLT (optical)


Arecibo (radio)


GMRT (radio)

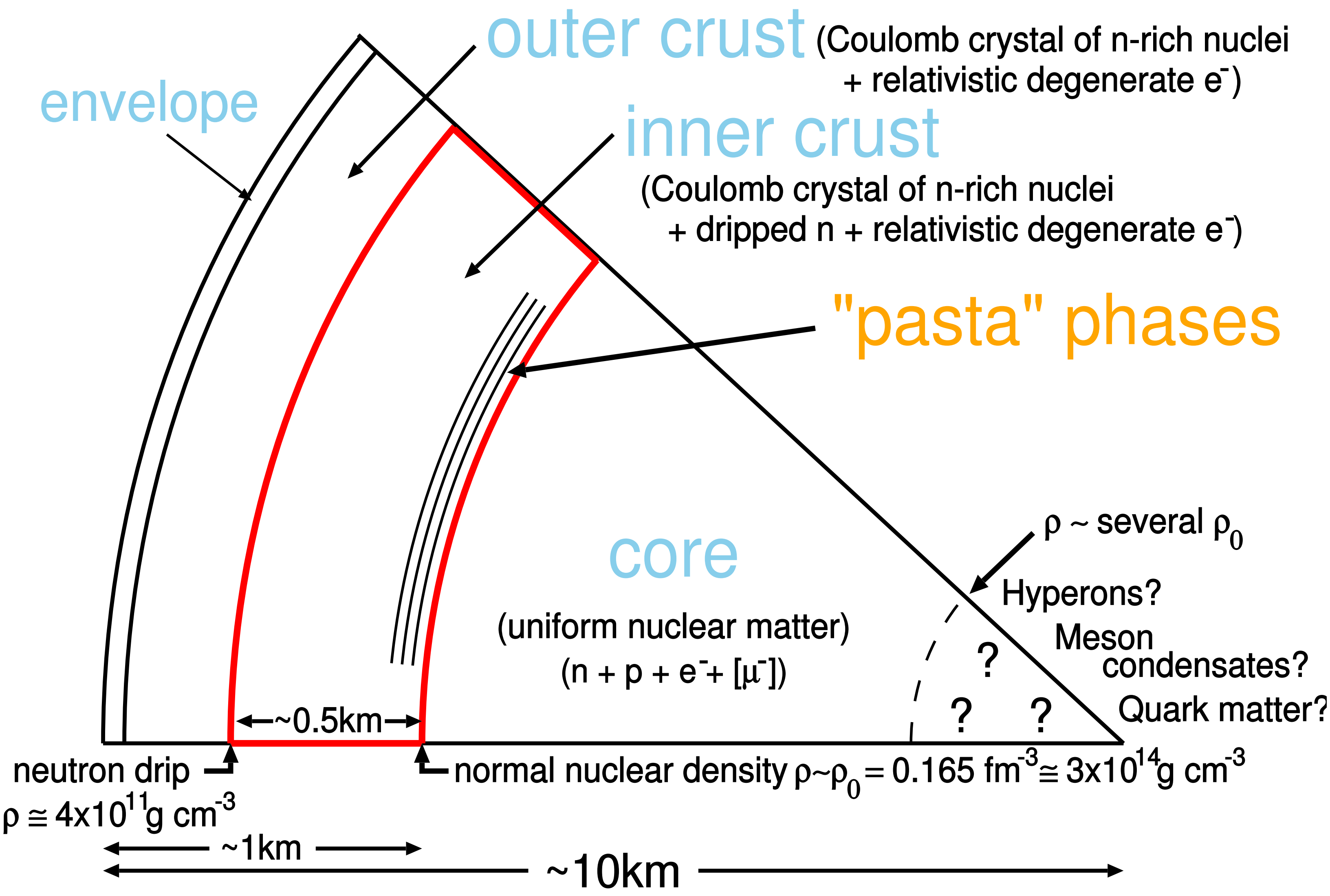
GW: A new window

- On August 2017, LIGO-Virgo collaboration detected first ever GW from a binary neutron star merger event: GW170817.
- Subsequent electromagnetic counterparts were detected by ~70 observatories.



- Marks the beginning of multi-messenger era of astronomy.

Neutron Star Interior



envelope

outer crust (Coulomb crystal of n-rich nuclei + relativistic degenerate e^-)

inner crust (Coulomb crystal of n-rich nuclei + dripped n + relativistic degenerate e^-)

"pasta" phases

core

(uniform nuclear matter)

$(n + p + e^- + [\mu^-])$

$\rho \sim$ several ρ_0

Hyperons?

Meson condensates?

Quark matter?

? ? ?

~0.5km

neutron drip

$\rho \cong 4 \times 10^{11}$ g cm^{-3}

normal nuclear density $\rho \sim \rho_0 = 0.165$ fm$^{-3} \cong 3 \times 10^{14}$ g cm^{-3}

~1km

~10km

Schematic picture (Watanabe and Maryuama)

Building a NS

- The structure of a static (i.e., non-rotating) star with spherical symmetry in General Relativity is described by the Tolman-Oppenheimer-Volkoff (TOV) eqns (G=c=1):

$$\frac{dp}{dr} = -G\frac{m(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{c^2\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)m(r)}{c^2}\right)\left(1-\frac{2Gm(r)}{c^2 r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon(r)$$

P = pressure , $\varepsilon(r)$ = energy density

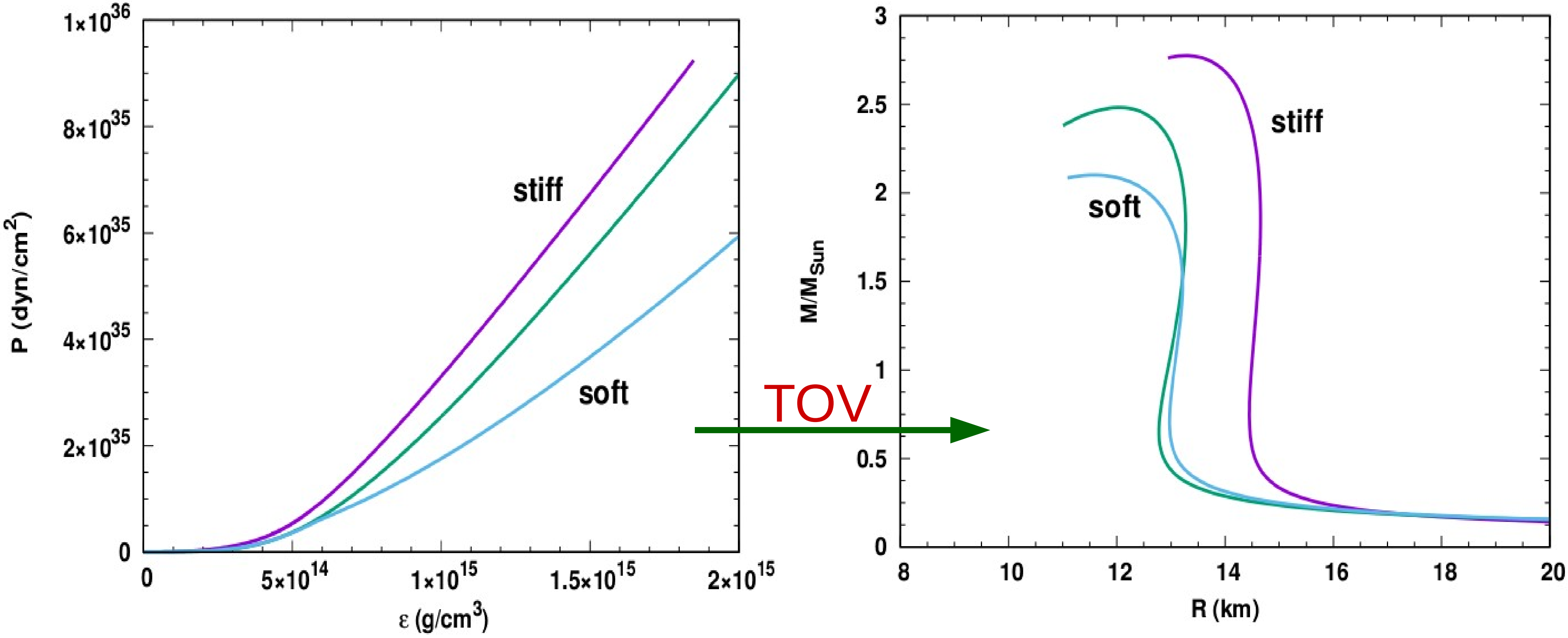
- Boundary Conditions:

$$\begin{aligned} P(r=0) &= P_c, \quad m(r=0) = 0 \\ P(r=R) &= 0, \quad m(r=R) = M \end{aligned}$$

Equation of State

- Essential ingredient to solve TOV

$$p = p(\varepsilon) \qquad \text{or} \qquad \varepsilon = \varepsilon(p)$$

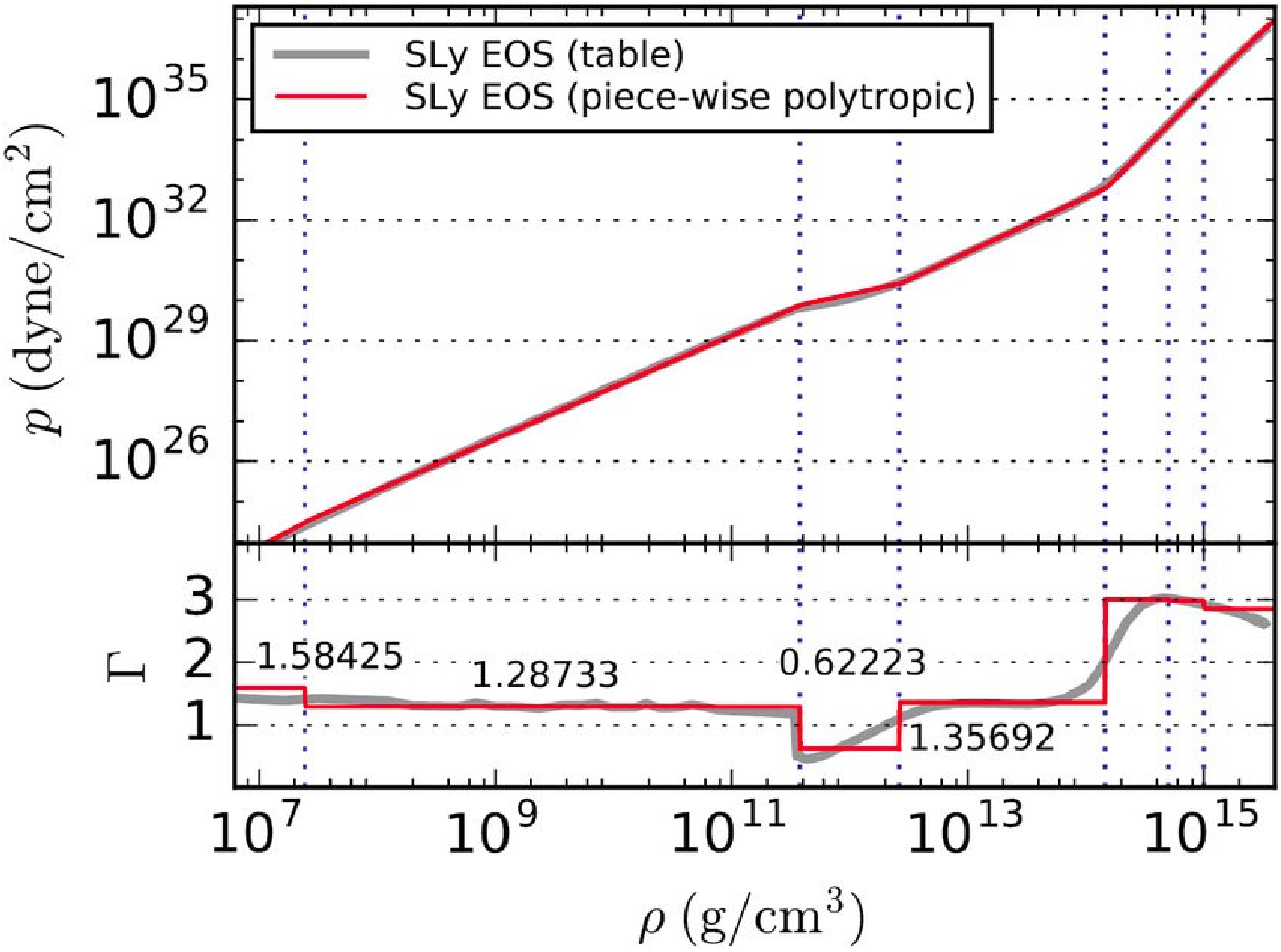


- Each EOS corresponds to a maximum mass
- Stiffer EOS gives larger maximum mass and radius

EOS is highly uncertain

known only at extreme densities

- The EOS of the outer crust is mostly determined using experimentally determined nuclear masses till $\simeq 10^{10}$ **g/cc**.



Feo et al. Class. Quantum Grav. 34, 034001 (2017)

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- We have some constraints on the EOS around the saturation density coming from nuclear physics experiments.

Saturation properties of nuclear matter

The energy/nucleon of nuclear matter can be written as:

$$\begin{aligned} e(\rho,\delta) &\simeq e_{\mathrm{snm}}(\rho) + e_{\mathrm{sym}}(\rho)\delta^2 \\ e_{\mathrm{snm}}(\rho,0) &= \boldsymbol{B} + \frac{1}{2}\boldsymbol{K}_0\chi^2 + \frac{1}{6}J_0\chi^3 + \cdot\cdot \\ e_{\mathrm{sym}}(\rho) &= \boldsymbol{J} + \boldsymbol{L}\chi + \frac{1}{2}K_{\mathrm{sym}}\chi^2 + \frac{1}{6}J_{\mathrm{sym}}\chi^3 + \cdot\cdot \end{aligned}$$

$$\delta = \frac{\rho_n - \rho_p}{\rho} \qquad \chi = \frac{\rho_n - \rho_p}{3\rho_0}, \qquad \rho_0 = \text{saturation density}$$

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\end{aligned}
$$

Latest experimental/empirical bounds at saturation density (ρ_0)

(Oertel et al, Rev. Mod. Phy. 89, 015007 (2017))

$$
\begin{aligned}
\mathrm{Compressibility}:\ 210 &\leq K\,(\mathrm{MeV}) \leq 280 \\
\mathrm{Symmetry\,energy}:\ 28 &\leq J\,(\mathrm{MeV}) \leq 35 \\
\mathrm{Symmetry\,energy\,slope}:\ 30 &\leq L\,(\mathrm{MeV}) \leq 87
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- In the very high-density limit $(\approx 40\rho_0)$ perturbative-QCD (**pQCD**) techniques with quarks and gluon as their degrees of freedom become reliable.

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- This indicates that there is a de-confinement phase transition from hadrons to quarks happening at densities between these two limits.

- The EOS at the intermediate density is very uncertain.
    - Constituents are not known.
    - Interaction between constituents are not fully known.
    - Uncertainties in the many-body description.
- The cores of neutron stars at their heart bears these intermediate densities where phase transition can occur.

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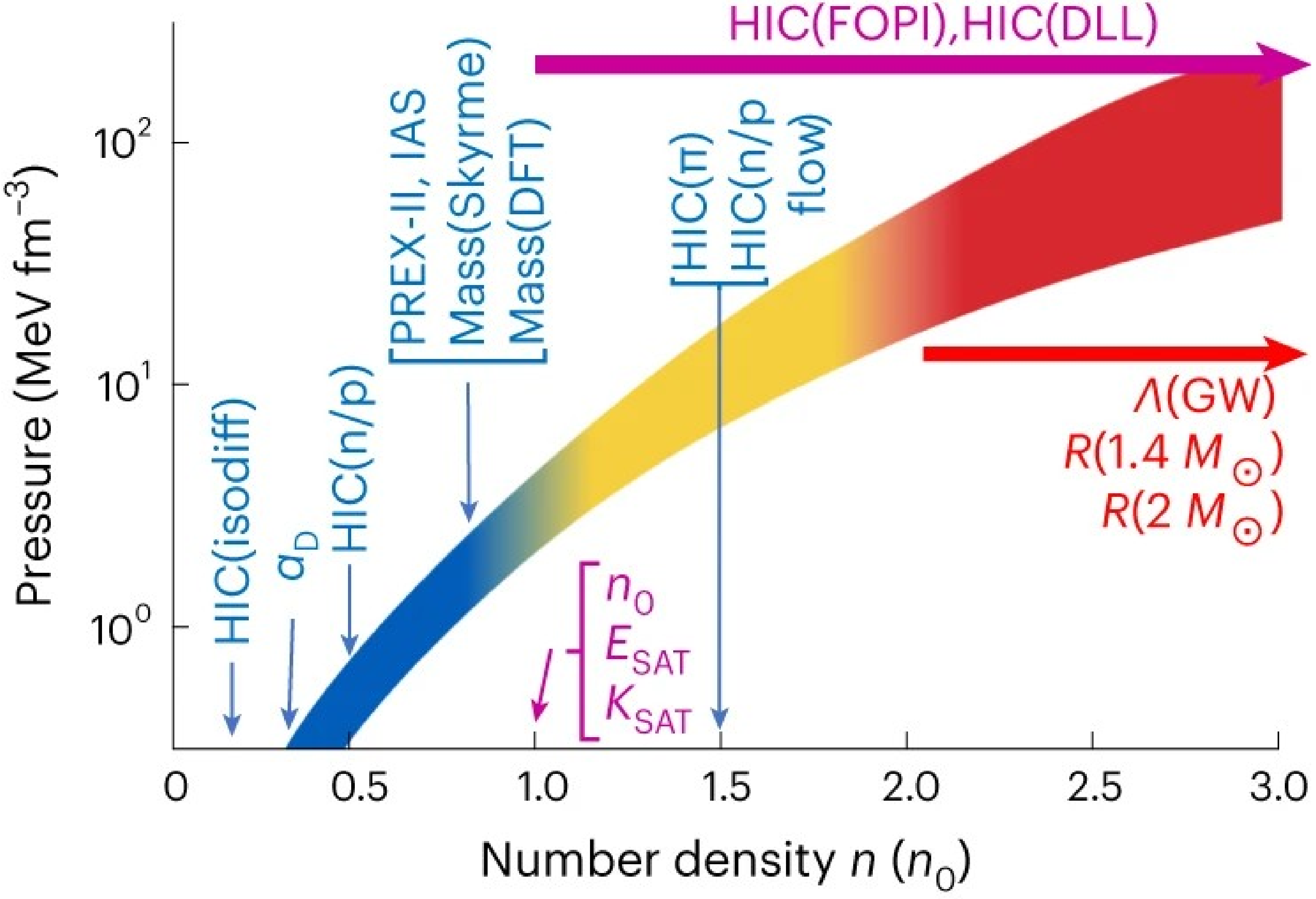
EOS is highly model dependent

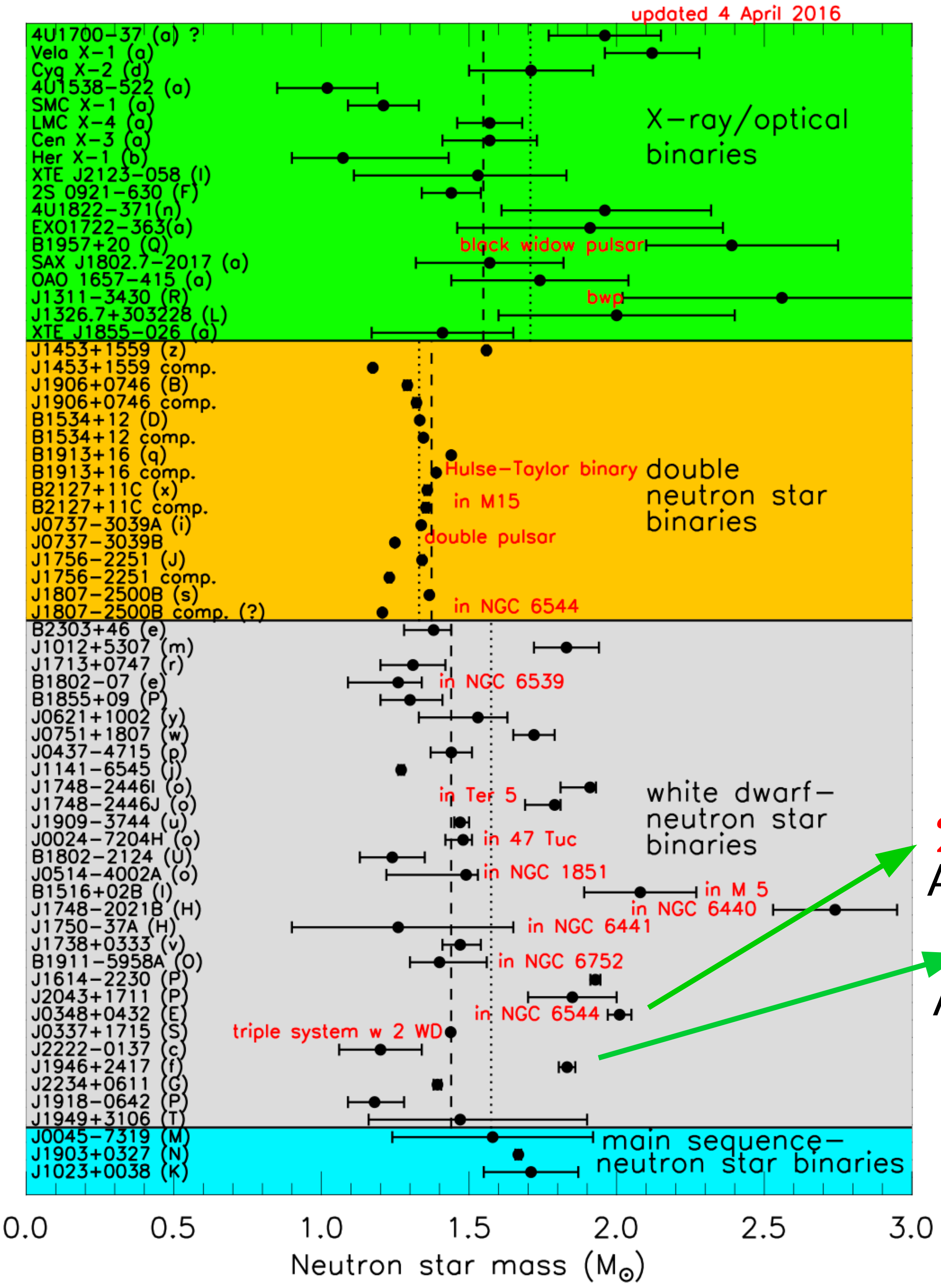
↓

Need to rely on astrophysical observations



Constraints from nuclear experiments and astronomy observations with their corresponding sensitive densities.

Tsang, C.Y. et al., Nat Astron 8, 328(2024)



- First breakthrough

Precise mass
Measurement of
massive NS

Excluded soft EOSs

$2.01 \pm 0.04 M_\odot$

Antoniadis et al *Science* 340 448(2013)

$1.908 \pm 0.016 M_\odot$

Arzoumanian et al *ApJS* 235 37(2018)

$2.08 \pm 0.07 M_\odot$

Fonseca et al *ApJL* 915 L12(2021)

$2.35 \pm 0.17 M_\odot$

Romani et al *ApJL* 934 L17(2022)

Equation of State

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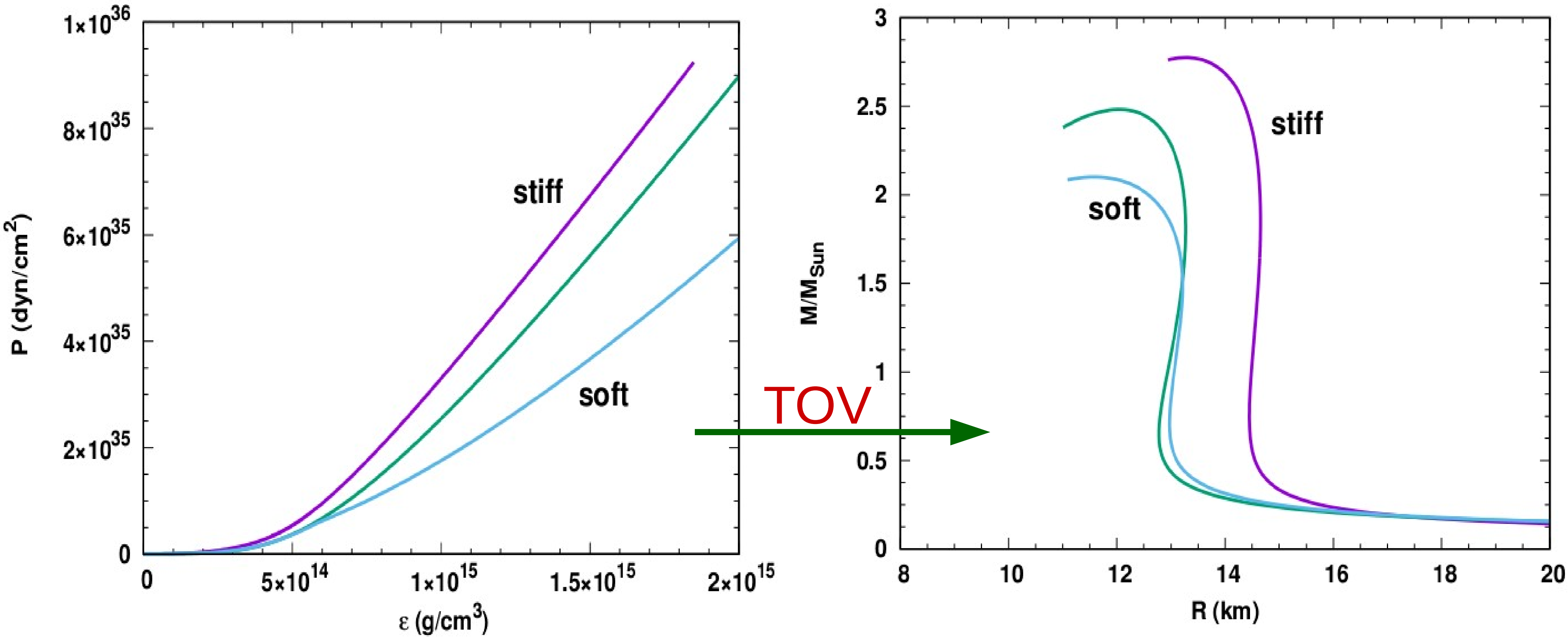

We need precise and simultaneous mass-radius measurements → NICER mission started on 2017.

Tidal deformability

- Another significant constraint came from GW170817 with the measurement of tidal deformability (Λ), an EOS-sensitive quantity:

$\Lambda_{1.4} \leq 800$ LVC, PRL 119, 161101 (2017)

$\Lambda_{1.4} \leq 580$ LVC, PRL 121, 161101 (2018)

$$\Lambda = \lambda/M^5, \quad \lambda = \frac{2}{3} k_2 R^5, \quad k_2 = \text{love number}$$

RMF model

- Interaction between baryons is described via the exchange of mesons
- The most general form of the interaction Lagrangian density:

$$\begin{aligned}\mathcal{L}_{\text{int}} \quad = \quad & \sum_B \bar{\psi}_B \left[g_\sigma \sigma + g_\delta \boldsymbol{\tau} \cdot \boldsymbol{\delta} - \gamma^\mu \left(g_\omega \omega_\mu + \frac{1}{2} g_\rho \boldsymbol{\tau} \cdot \boldsymbol{\rho_\mu} + \frac{e}{2}(1+\tau_3) A_\mu \right) \right] \psi_B \\ & - \frac{\kappa}{3!}(g_\sigma \sigma)^3 - \frac{\lambda}{4!}(g_\sigma \sigma)^4 + \frac{\zeta}{4!}(g_\omega^2 \omega_\mu \omega^\mu)^2 \\ & + g_\sigma g_\omega^2 \sigma \omega_\mu \omega^\mu \left(\alpha_1 + \frac{1}{2} \alpha_1' g_\sigma \sigma \right) + g_\sigma g_\rho^2 \sigma \boldsymbol{\rho_\mu} \cdot \boldsymbol{\rho^\mu} \left(\alpha_2 + \frac{1}{2} \alpha_2' g_\sigma \sigma \right) \\ & + \frac{1}{2} \alpha_3' g_\omega^2 g_\rho^2 \omega_\mu \omega^\mu \boldsymbol{\rho_\mu} \cdot \boldsymbol{\rho^\mu}\end{aligned}$$

σ, ω_μ, $\boldsymbol{\rho}_\mu$ and $\boldsymbol{\delta}$ are meson fields.

- For density dependent (DD) models coupling parameters $g_{i=\sigma,\omega,\rho,\delta}$ are density dependent and don't have nonlinear terms.

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E_{\rm Sym}(\rho) &= J + L\chi + \frac{1}{2}K_{\rm sym}\chi^2 + \mathcal{O}(\chi^3)
\end{aligned}$$

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$$\begin{aligned}
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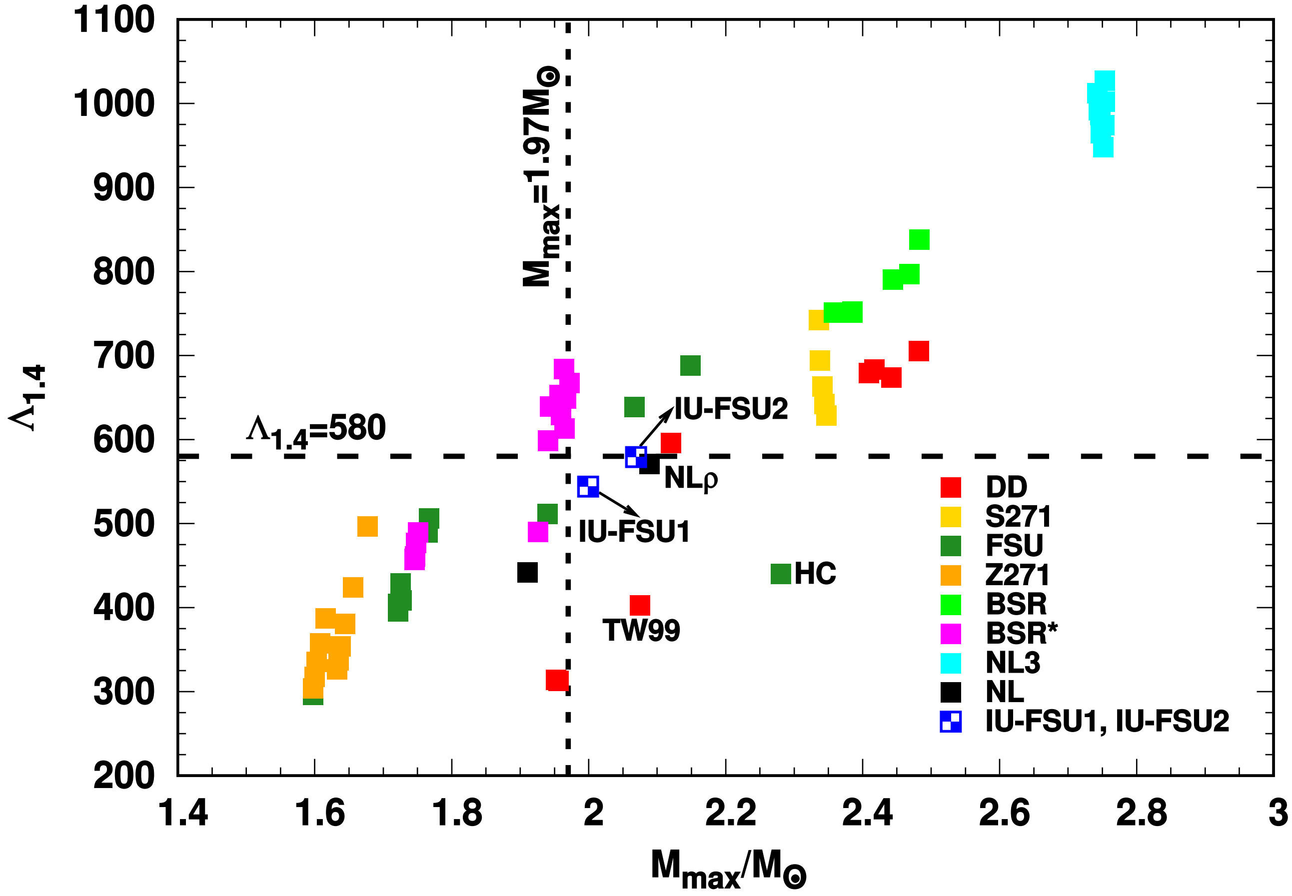
\end{aligned}$$

➡ 67 out of 269 RMF parameter sets satisfy these bounds

R Nandi, P Char and S Pal, PRC 99, 052802(R) (2019)



$M_{\max}$ bound

$$M_{\max} = 2.01 \pm 0.04 M_\odot$$

→ $M_{\max} \geq 1.97 M_\odot$

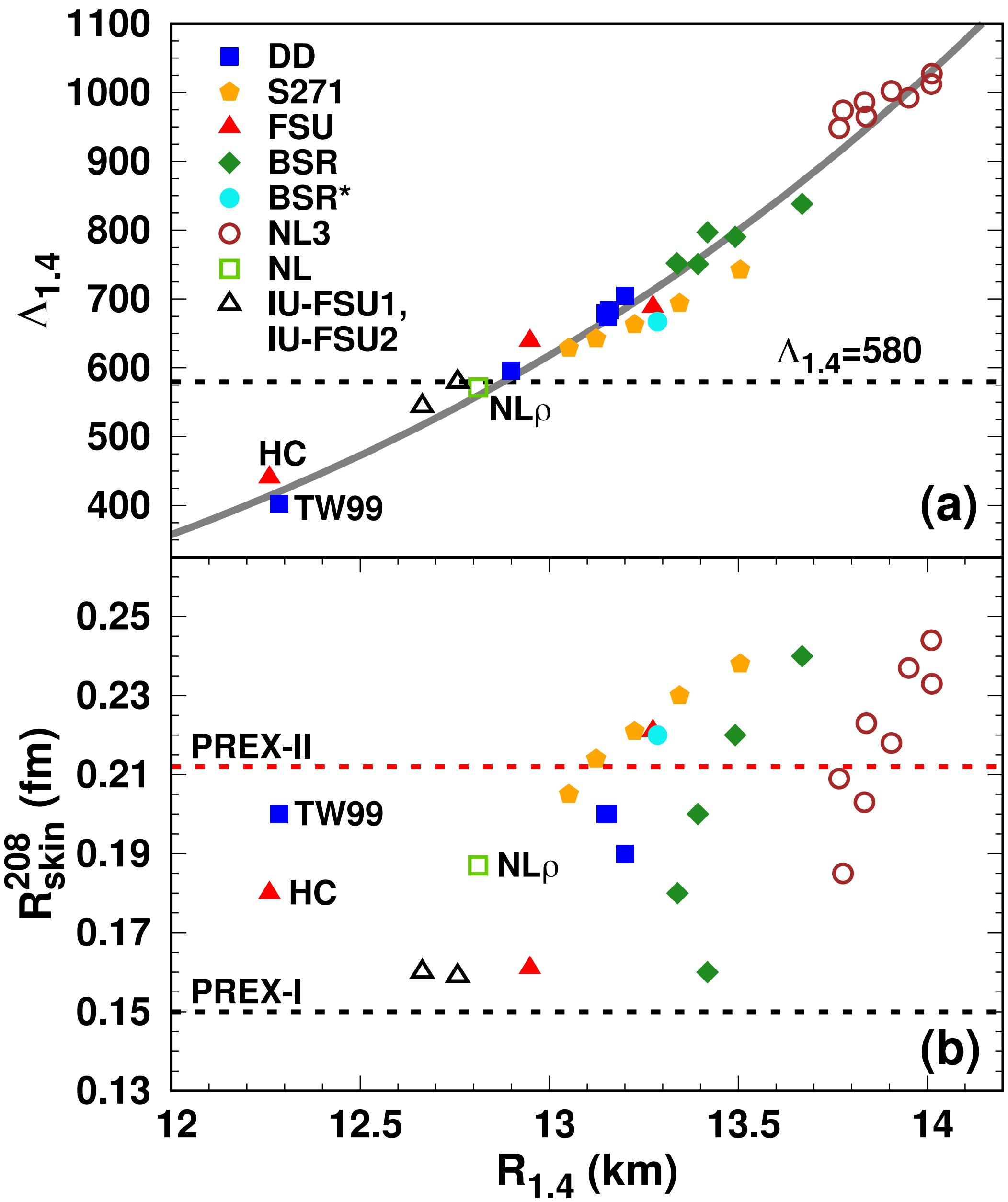
GW170817 bound

$$\Lambda_{1.4} = 190^{+390}_{-120}$$

→ $\Lambda_{1.4} \leq 580$

→ Only 3 EOS (TW99, NLρ and HC) satisfy both the constraints

R Nandi, P Char and S Pal, PRC 99, 052802(R) (2019)



$$
\begin{aligned}
R^{208}_{\rm skin} &\lesssim 0.20\,\mathrm{fm} \\
R_{\rm skin} &= \langle r_n \rangle - \langle r_p \rangle \\
R^{208}_{\rm skin} &= 0.33^{+0.16}_{-0.18}\,\mathrm{fm}
\end{aligned}
$$

PREX-I, PRL108, 112502 (2012)

PREX-II:

$$R^{208}_{\rm skin} = 0.29 \pm 0.07\,\mathrm{fm}$$

PRL126, 172502 (2021).

Quark EOS

- MIT Bag model:

$$\Omega = \sum_i \Omega_i^0 + \frac{3\mu^4}{4\pi^2}(1 - a_4) + B_{\rm eff}\,, \qquad i = u, d, s, e$$

$$P = -\Omega$$

$$\varepsilon = -P + \sum_i \mu_i n_i$$

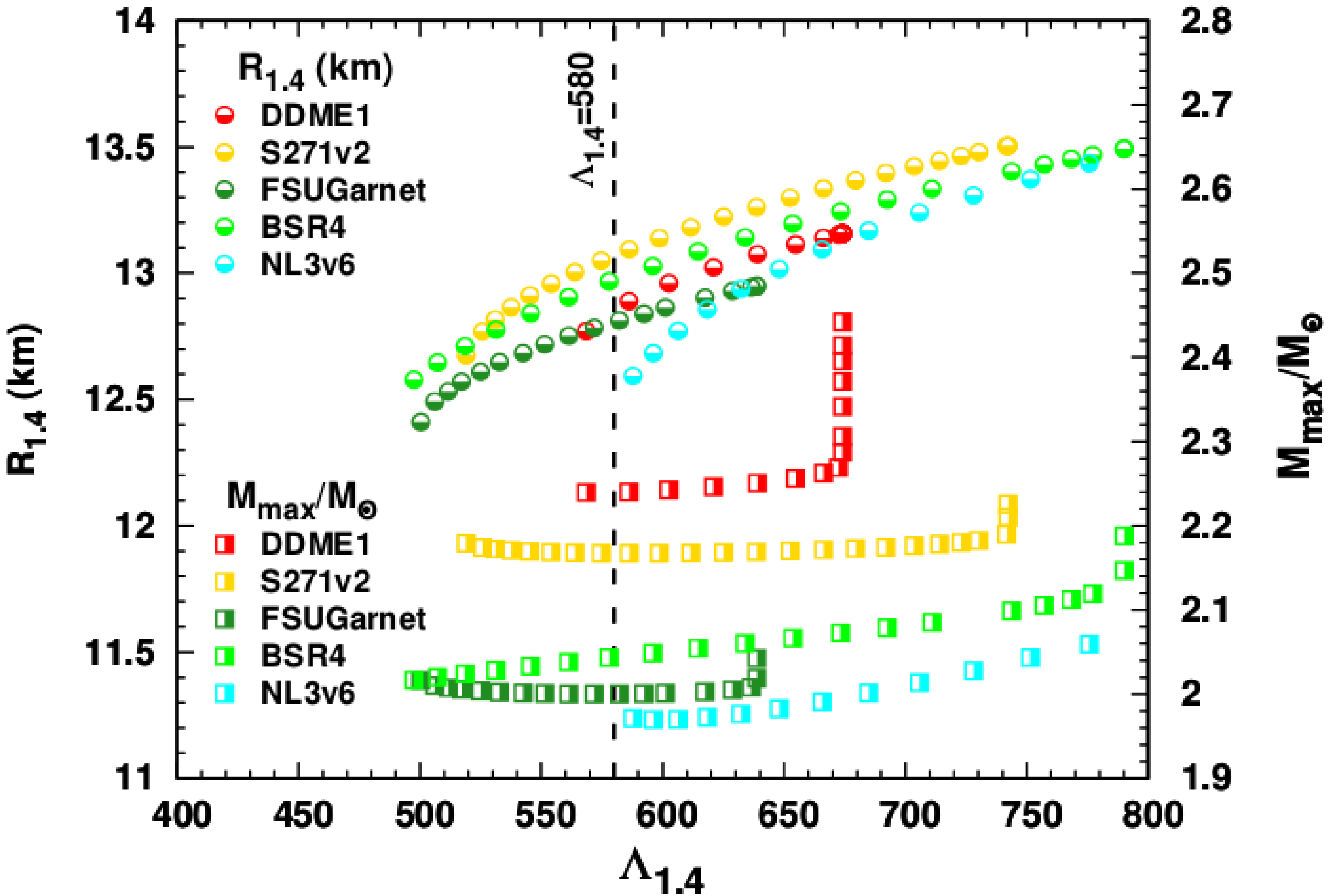
$\Omega_i^0 \rightarrow$ Grand potentials of non-interacting Fermi gas

$\mu \rightarrow$ Baryon chemical potential of quarks

$B_{\rm eff} \rightarrow$ Bag constant

$a_4 \rightarrow$ Interaction parameter

$n_i \rightarrow$ Number density of *i*-th particle



$\longrightarrow B_{\rm eff}^{1/4}$

➡ **Presence of quarks inside NS core is favored within RMF models**

R Nandi, P Char and S Pal, PRC 99, 052802(R) (2019)

- In 2019, NICER provided the First simultaneous measurement of mass-radius for PSR **J0030+0451:**

$$\boldsymbol{M = 1.34^{+0.15}_{-0.16} M_{\odot}, \quad R = 12.71^{+1.14}_{-1.19}\,\mathrm{km}}$$

Riley et al, *ApJL* 887, L21 (2019)

$$\boldsymbol{M = 1.44^{+0.15}_{-0.14} M_{\odot}, \quad R = 13.02^{+1.24}_{-1.06}\,\mathrm{km}}$$

Miller et al, *ApJL* 887, L24 (2019).

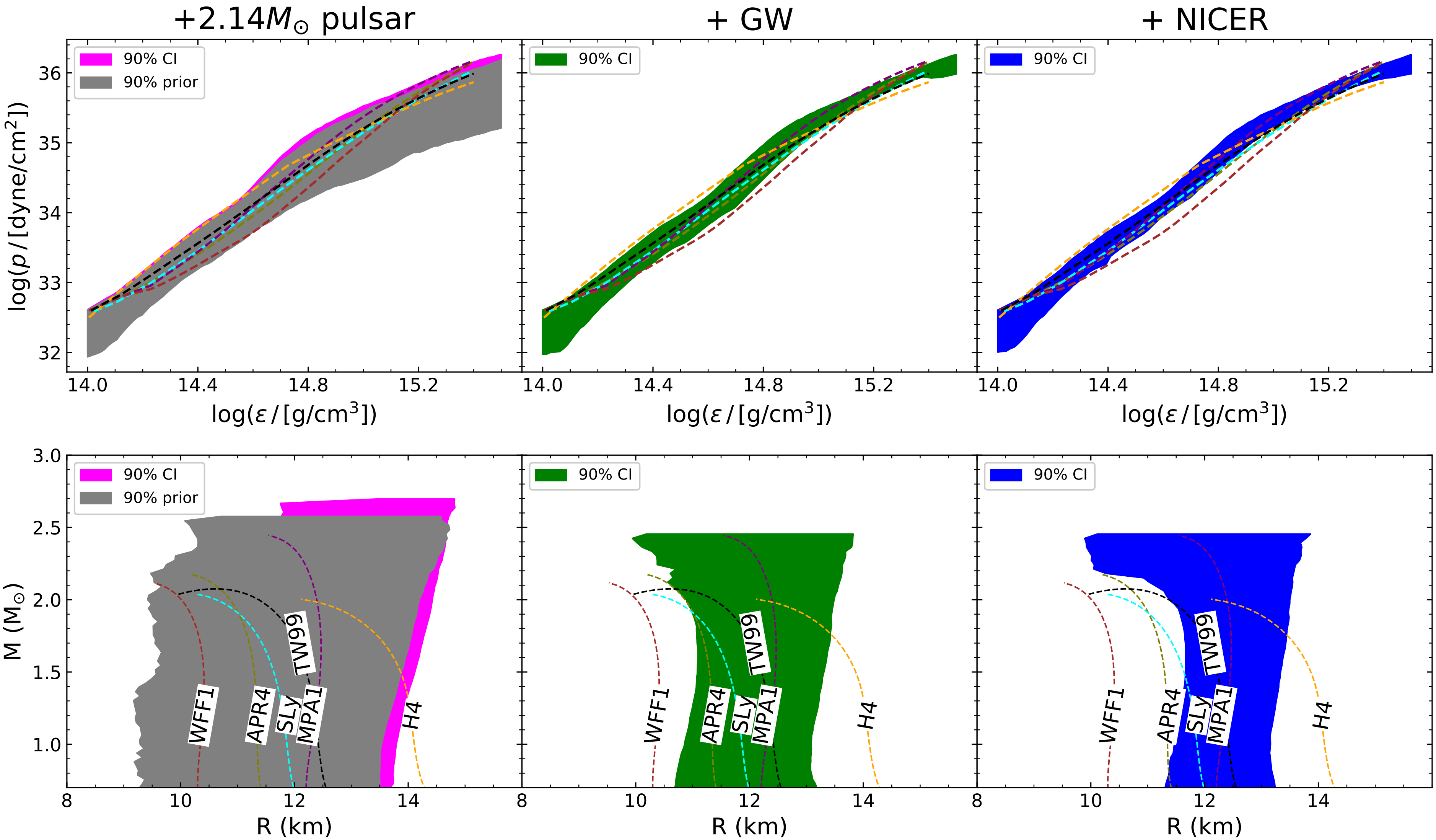
- In 2021, another measurement was reported by analyzing NICER + XMM Newton data of **PSR J0740+6620**:

$$\boldsymbol{M = 2.08 \pm 0.07 M_{\odot}}$$

$$\boldsymbol{R = 13.7^{+2.63}_{-1.68}\,\mathrm{km}}$$

Miller et al, *ApJL* 918, L28 (2021).

Constraint on EOS via Bayesian analysis



Biswas et al., PRD 103, 103015 (2021)

Talk by Tuhin Malik

In 2024, NICER provided two more measurements:

- PSR **J0437-4715** (Choudhury et al, ApJ 971, L20 (2024))

$$\boldsymbol{M = 1.418 \pm 0.037 M_\odot, \quad R = 11.36^{+0.95}_{-0.63}\,\mathrm{km}}$$

- PSR **J1231-1411** (Salmi et al, ApJ 976, 58 (2024))

$$\boldsymbol{M = 1.04^{+0.05}_{-0.03} M_\odot, \quad R = 13.5^{+0.3}_{-0.5}\,\mathrm{km}}$$

- We are now analyzing the effect of these data on the EOS.

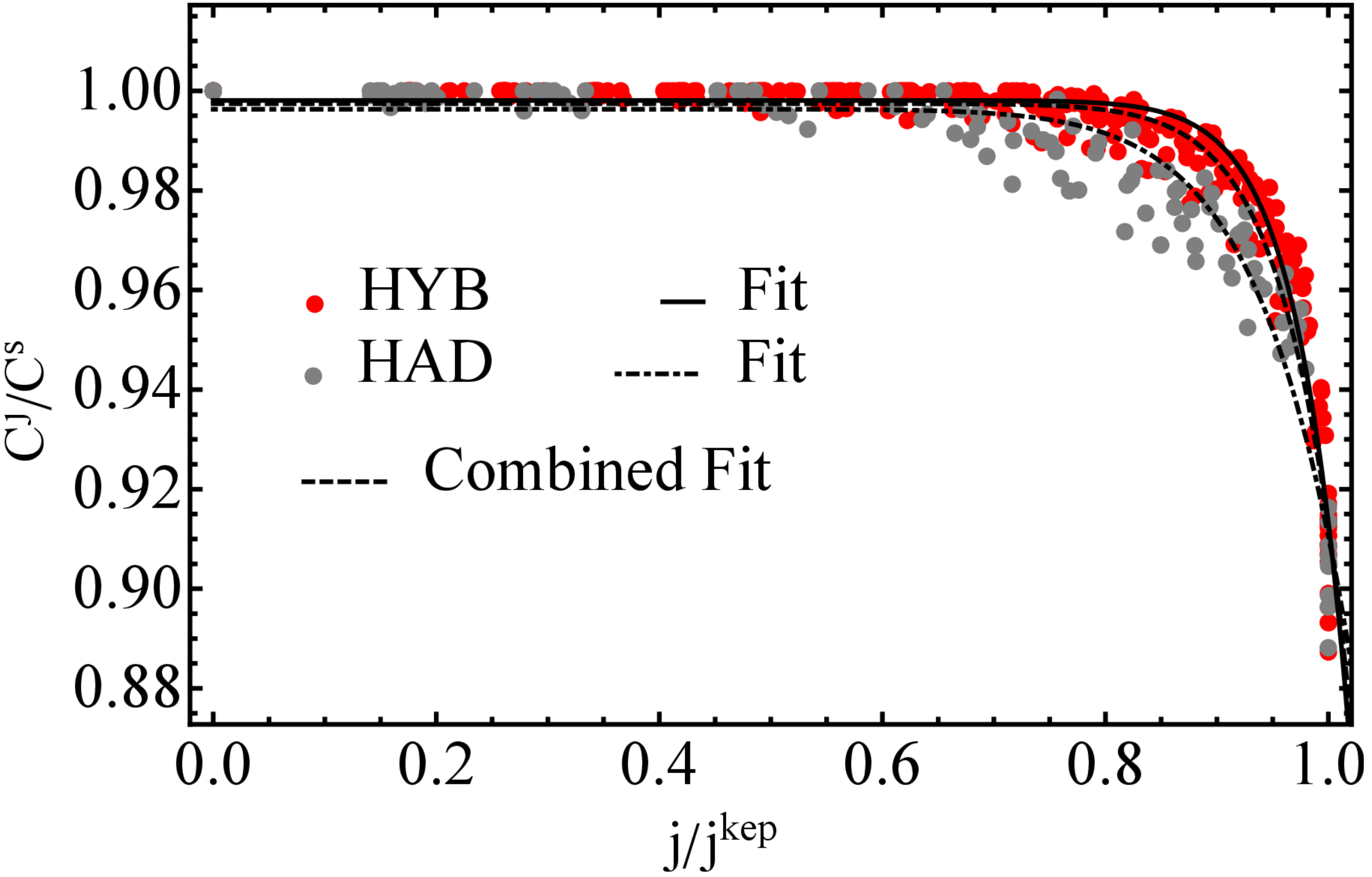
Quark matter

- Still we can’t say whether an NS core can shelter quark matter or not.
- We explored the possibility of distinguishing between neutron stars and neutron stars with a quark core (**Hybrid stars**).

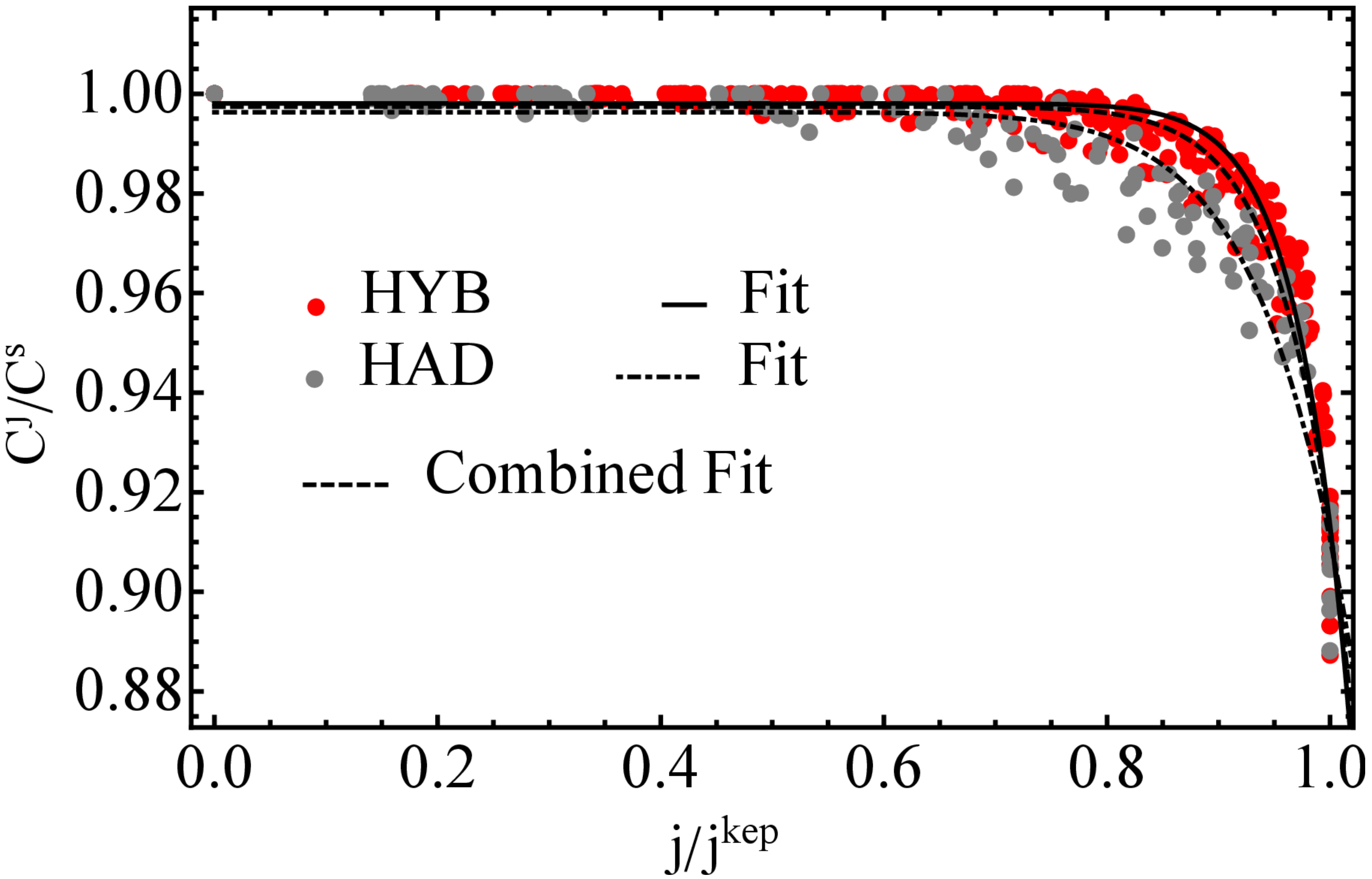
R Mallick, D Kuzur and R Nandi
EpJC 82 512 (2022)

EOS considered

- We considered several Relativistic Mean Field (RMF) EOS to describe the hadronic part.
- For the quark part we adopt the MIT Bag model.
- A hybrid star contains hadronic matter at low densities, pure quark phase at high densities and hadron-quark mixed phase at intermediate densities.
- The transition density and the extent of the mixed phase depend on the hadronic EOS and the parameters of the quark matter EOS.



$C = M/R$ $\qquad$ $j = J/(M^j_{\max})^2$



Hard to distinguish !

R Mallick, D Kuzur and R Nandi
EpJC 82 512 (2022)

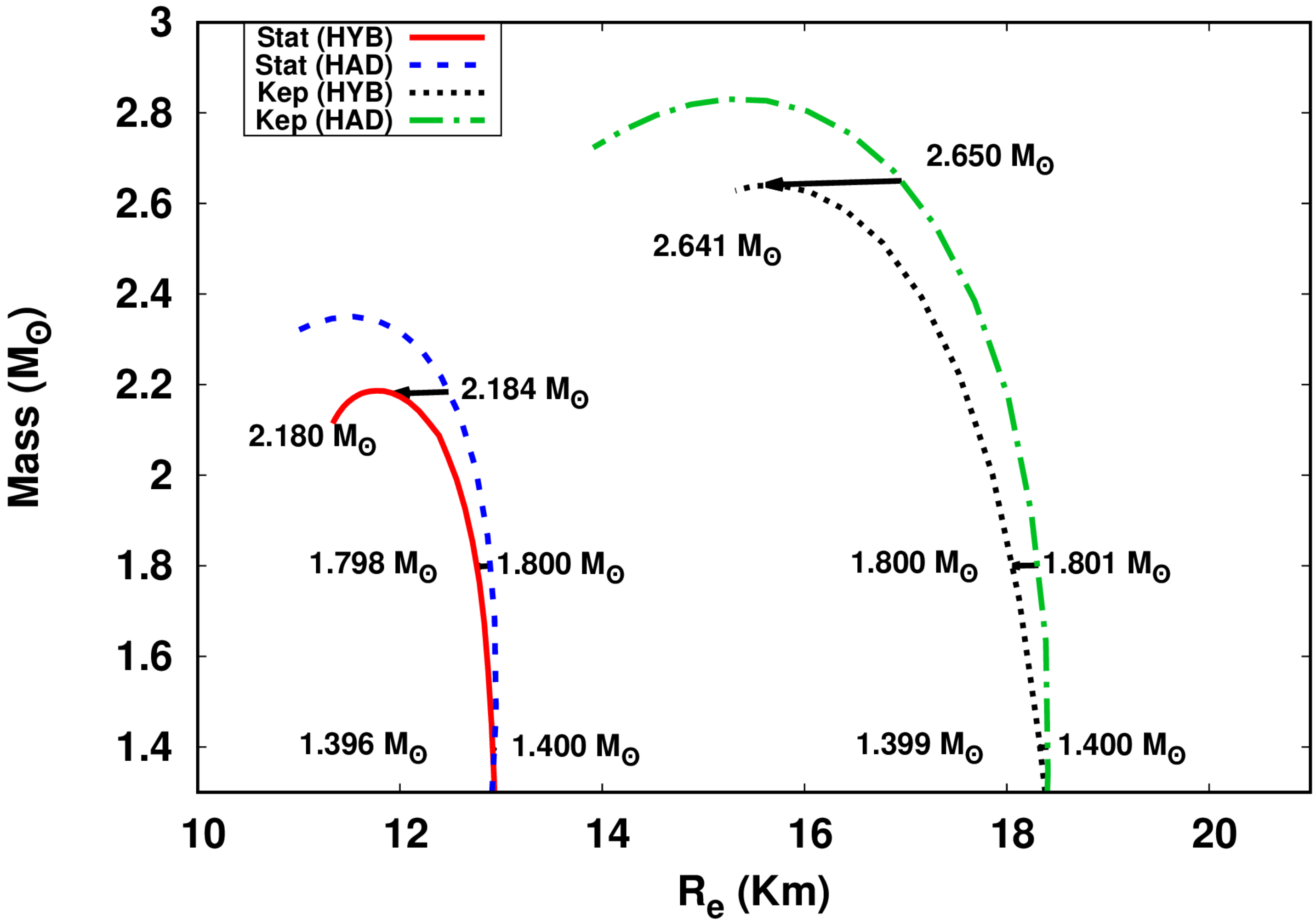
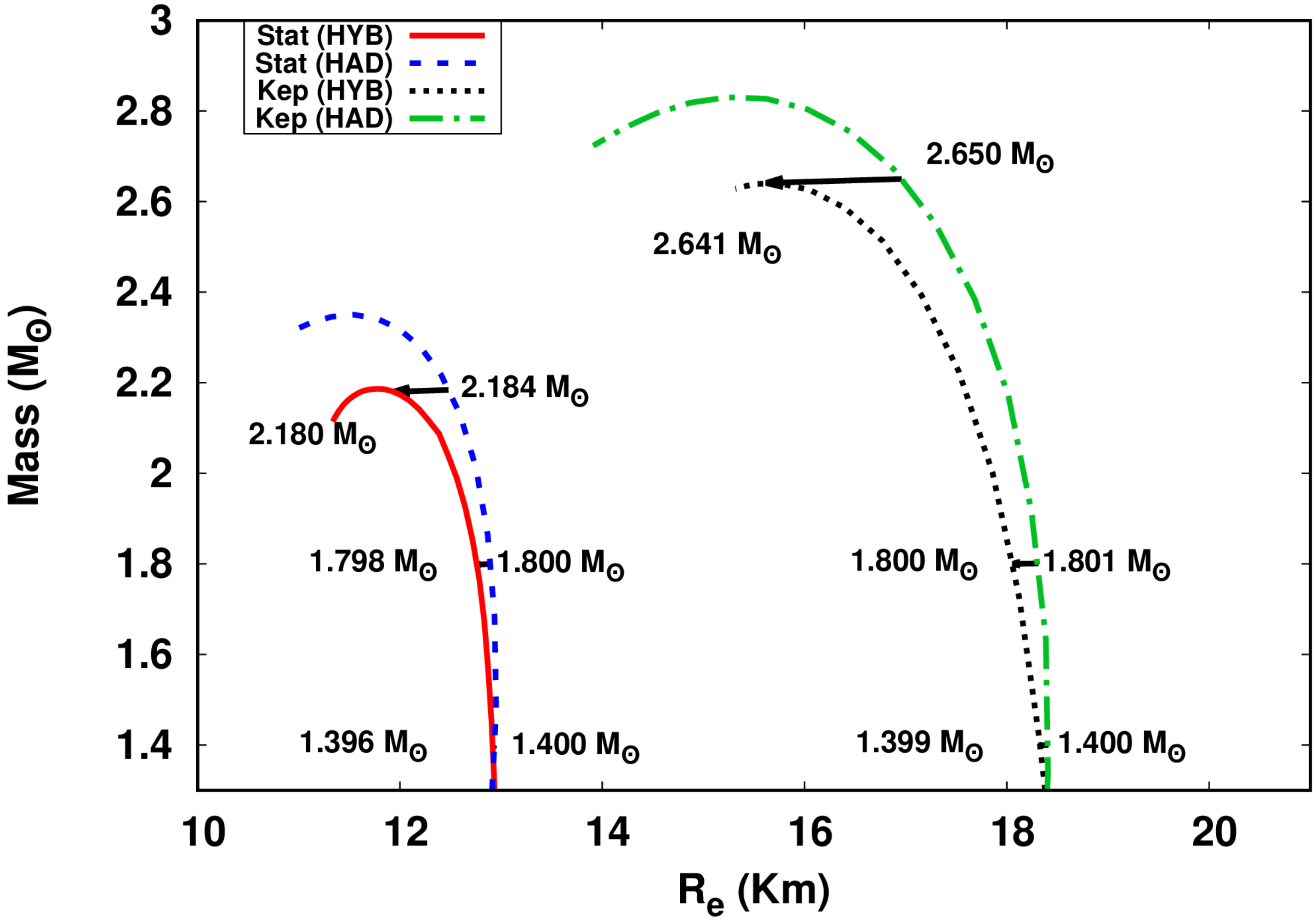
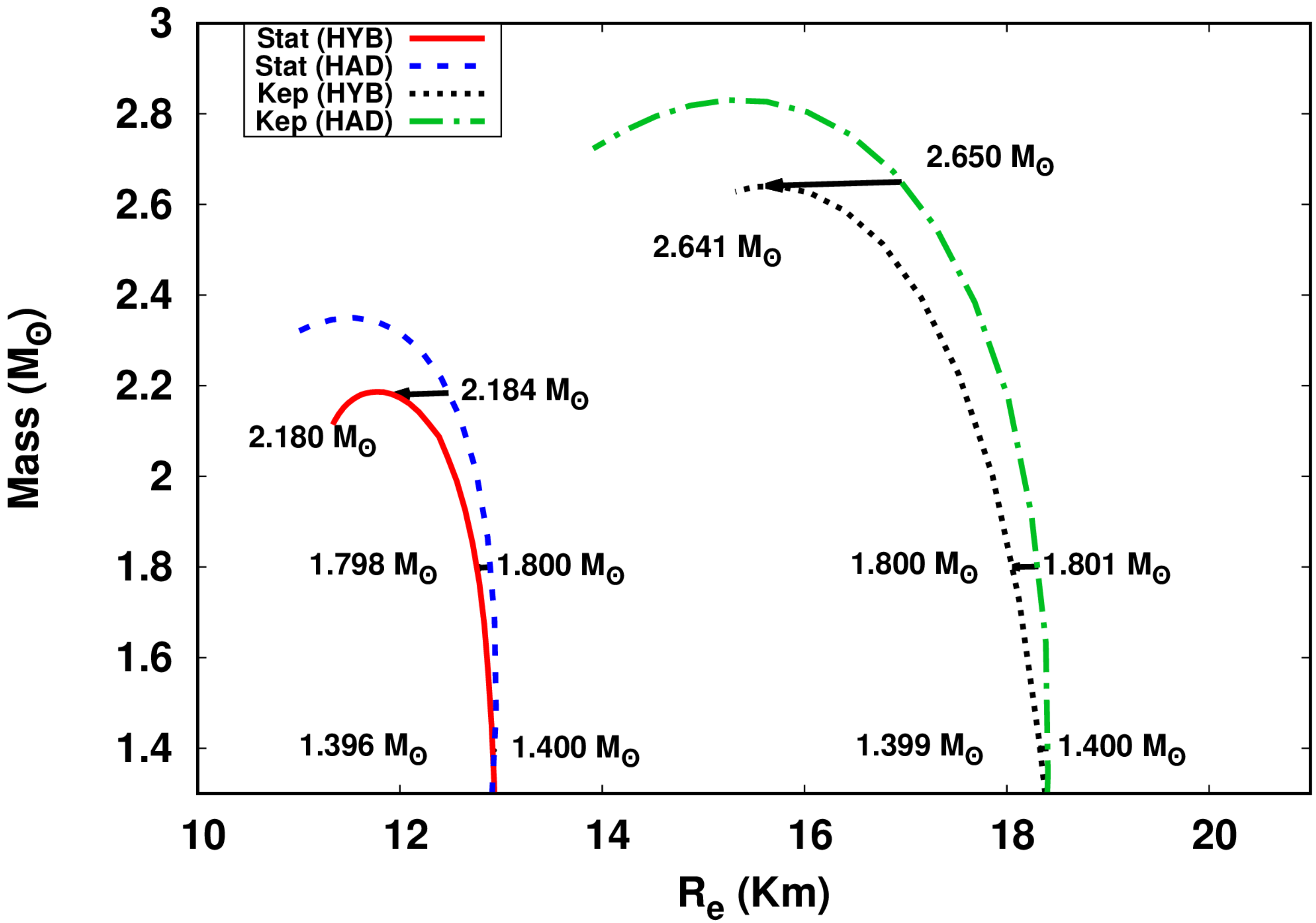


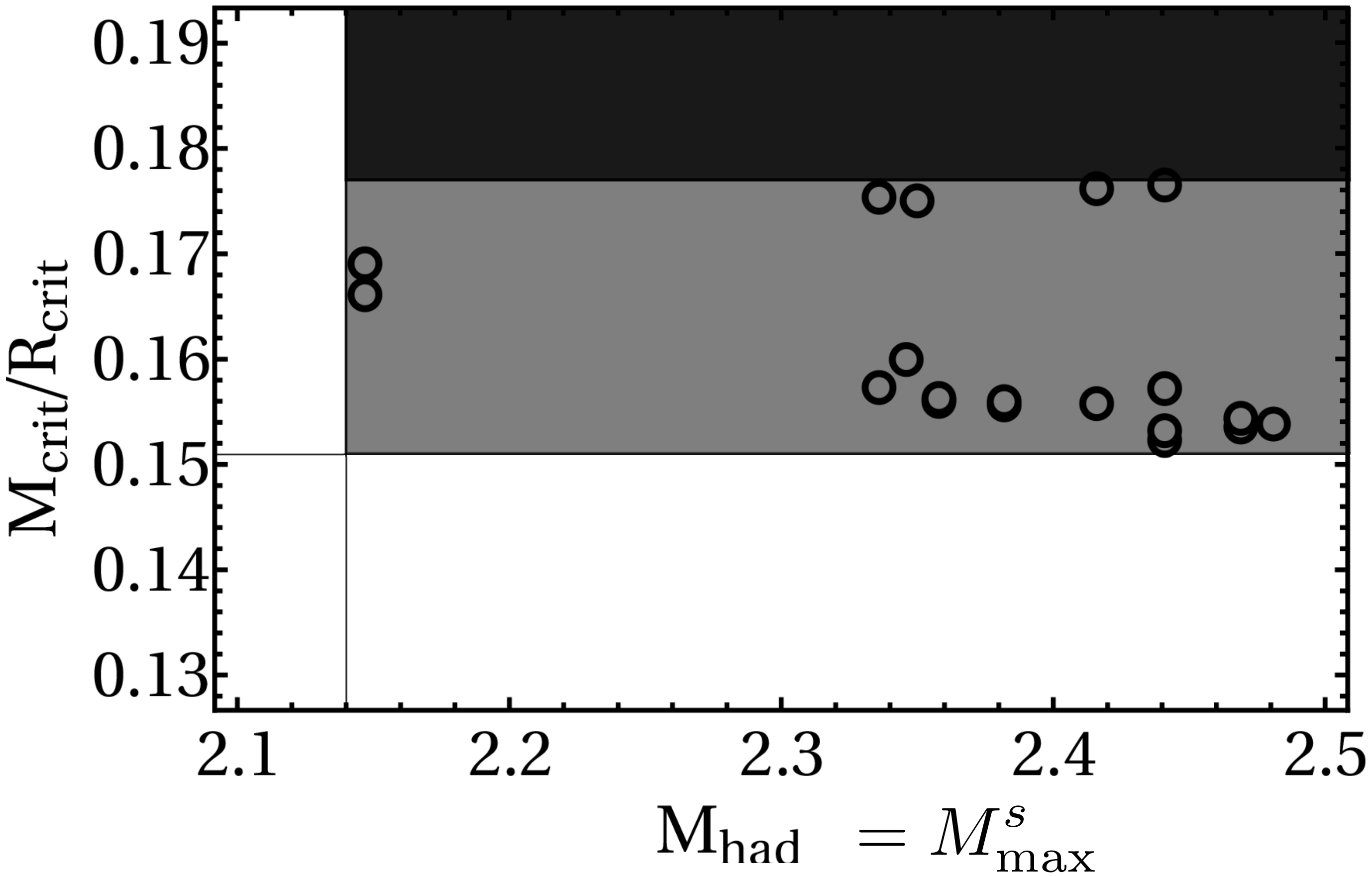
Figure shows how the gravitational mass and radius of a star changes if a hybrid star is formed via phase transition from a NS.



- Although the change in the gravitational mass is relatively small, the radius shrinks considerably.
- Therefore, as phase transition occurs and a quark core is formed inside a star, the star becomes more compact.

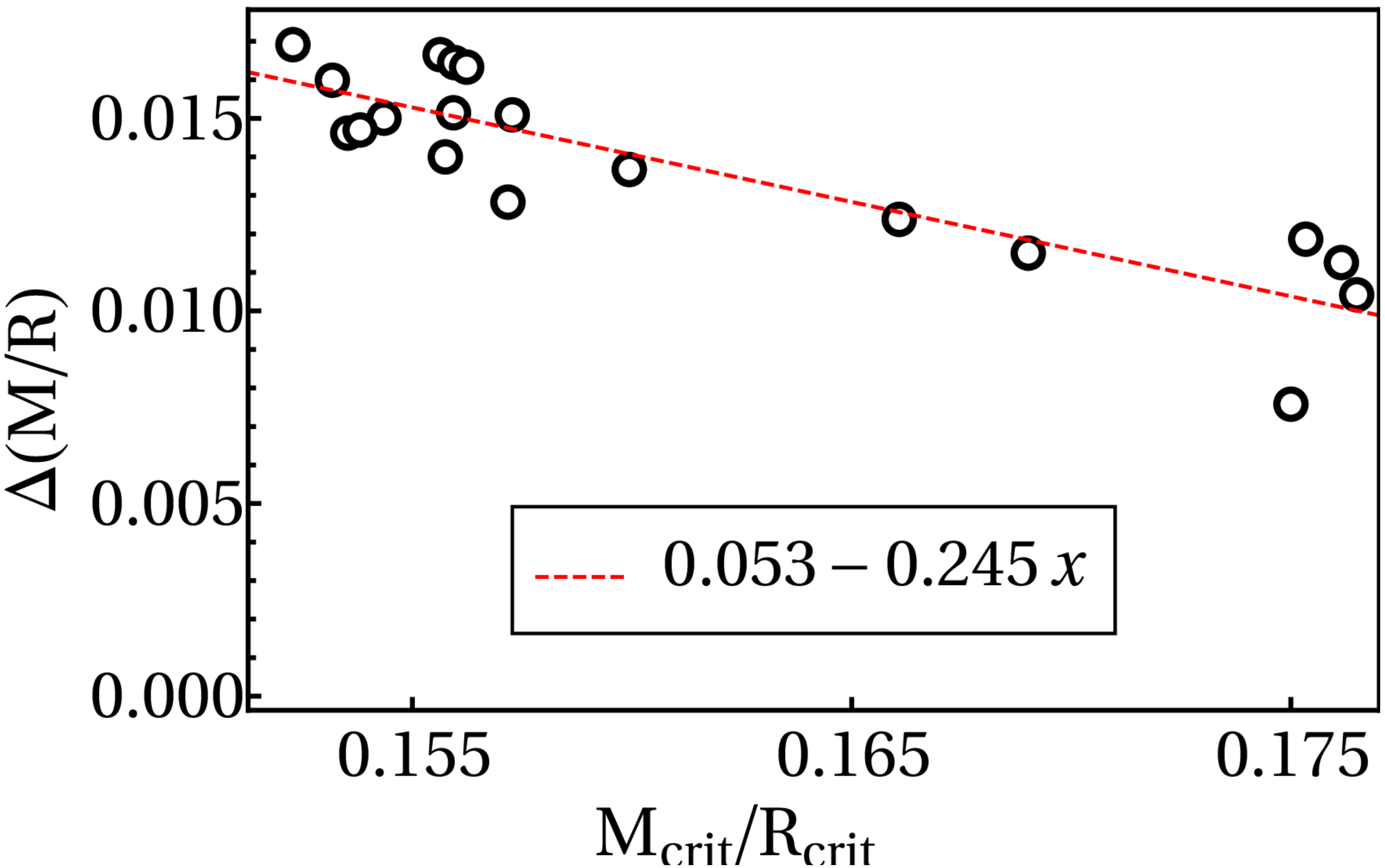


- A massive NS after phase transition can become unstable and probably collapses to a Black Hole.
- For a given EOS one can find the upper bound on mass and radius $M_{\rm crit}$ and $R_{\rm crit}$, beyond which it is not possible to produce a stable hybrid star.



$$\boldsymbol{M_{\mathrm{crit}}/R_{\mathrm{crit}} \lesssim 0.18}$$

If a neutron star undergoing phase transition is more compact it will collapse to a black hole



Change in the compactness of stars due to phase transition.

The change in compactness (and thereby the mass) can be used to estimate the gravitational energy released during the phase transition.

Summary

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- And due to NICER’s simultaneous mass-radius measurements of a few pulsars.
- Existence of quarks inside neutron star core is still debatable.

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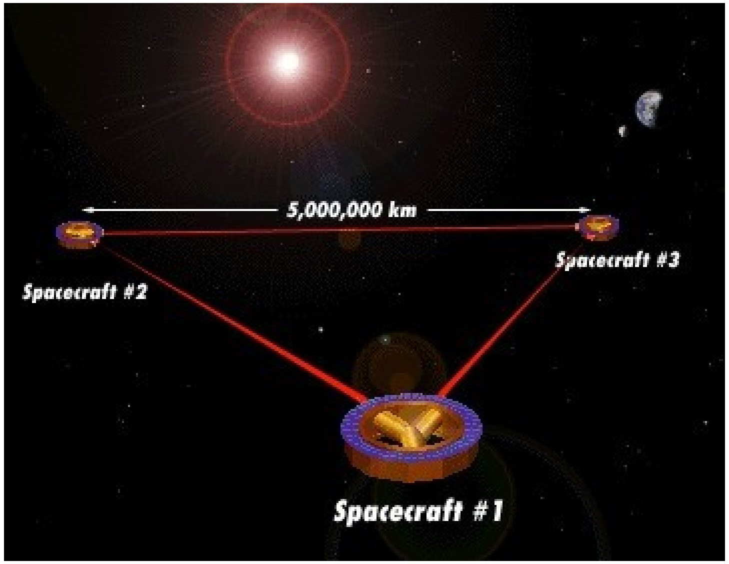
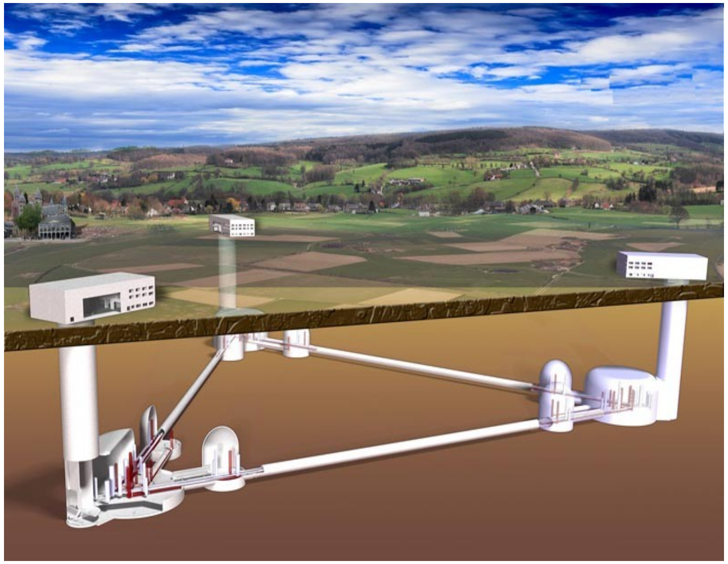
- And due to NICER's simultaneous mass-radius measurements of a few pulsars.
- Existence of quarks inside neutron star core is still debatable.
- Trying to find the possible signatures of phase transition in binary neutron star mergers by performing numerical simulation (**Talk by Ritam Mallick**).

Waiting for...........

- Precise radius measurements by NICER.
- Detection of more GW170817 like events by current/future run of LIGO-Virgo and upcoming detectors:
- Detection of continuous gravitational waves.



LISA



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Thank You