Relativistic Resistive Magnetohydrodynamic Framework to Study Heavy-Ion Collisions

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- **Electric conductivity of QCD matter in high-energy heavy-ion collisions**
  - Charge dependent flow
  - Elliptic flow of photons
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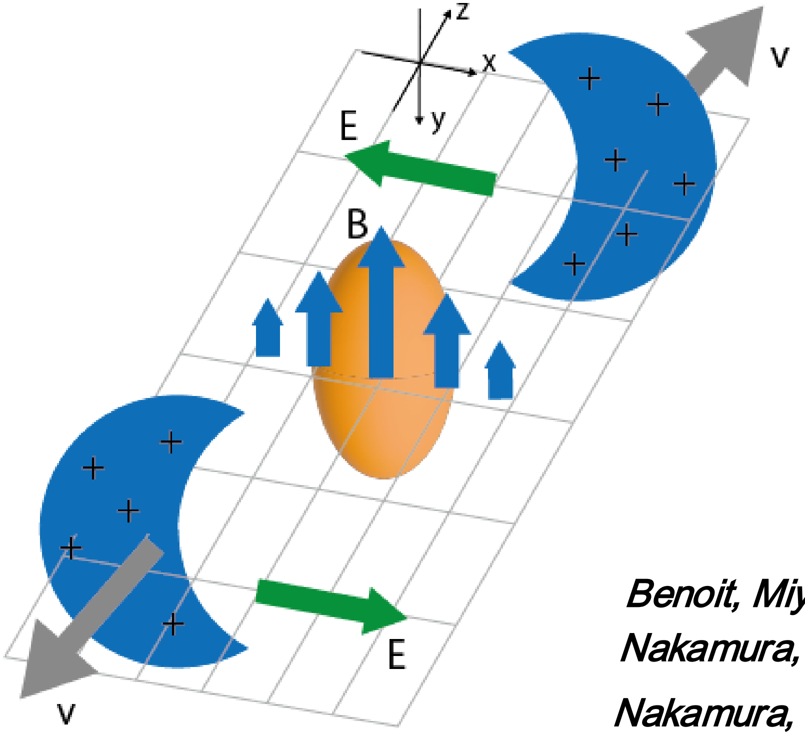
Electromagnetic Field in Heavy Ion Collisions

- **Strong Electromagnetic field ?**
  - Au + Au ($\sqrt{s_{NN}} = 200$ GeV) : 10^{14} T ~10 m_π^2
  - Pb + Pb ($\sqrt{s_{NN}} = 2.76$ TeV) : 10^{15} T



Benoit, Miyoshi, C. N., Sakai and Takahashi, in preparation

Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107, (2023) 014901

Nakamura, Miyoshi, C. N. and Takahashi, Eur.Phys.J.C 83 (2023) 3, 229.

Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107 (2023) 3, 034912

Electromagnetic Field in Heavy-Ion Collisions

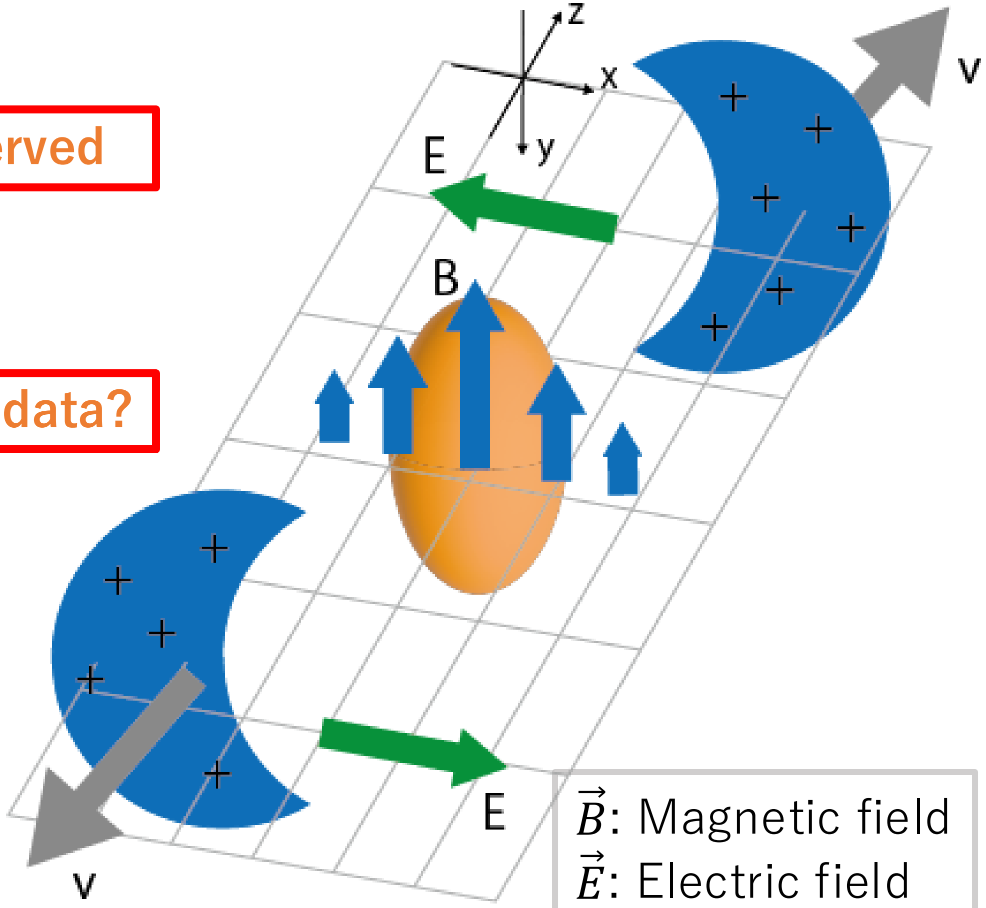
- **Electromagnetic field in heavy-ion collisions**
  - ➢ **Production of strong magnetic field**
    - Au + Au ($\sqrt{s_{NN}} = 200$ GeV) : 10^{14} T ~10 m_π^2
    - Pb + Pb ($\sqrt{s_{NN}} = 2.76$ TeV) : 10^{15} T

Not clearly observed

- **Response to electromagnetic field**
  - Electric conductivity
  - Lattice QCD: $\sigma \sim 0.023$ fm^{-1} @ $T \sim 250$ MeV

Experimental data?

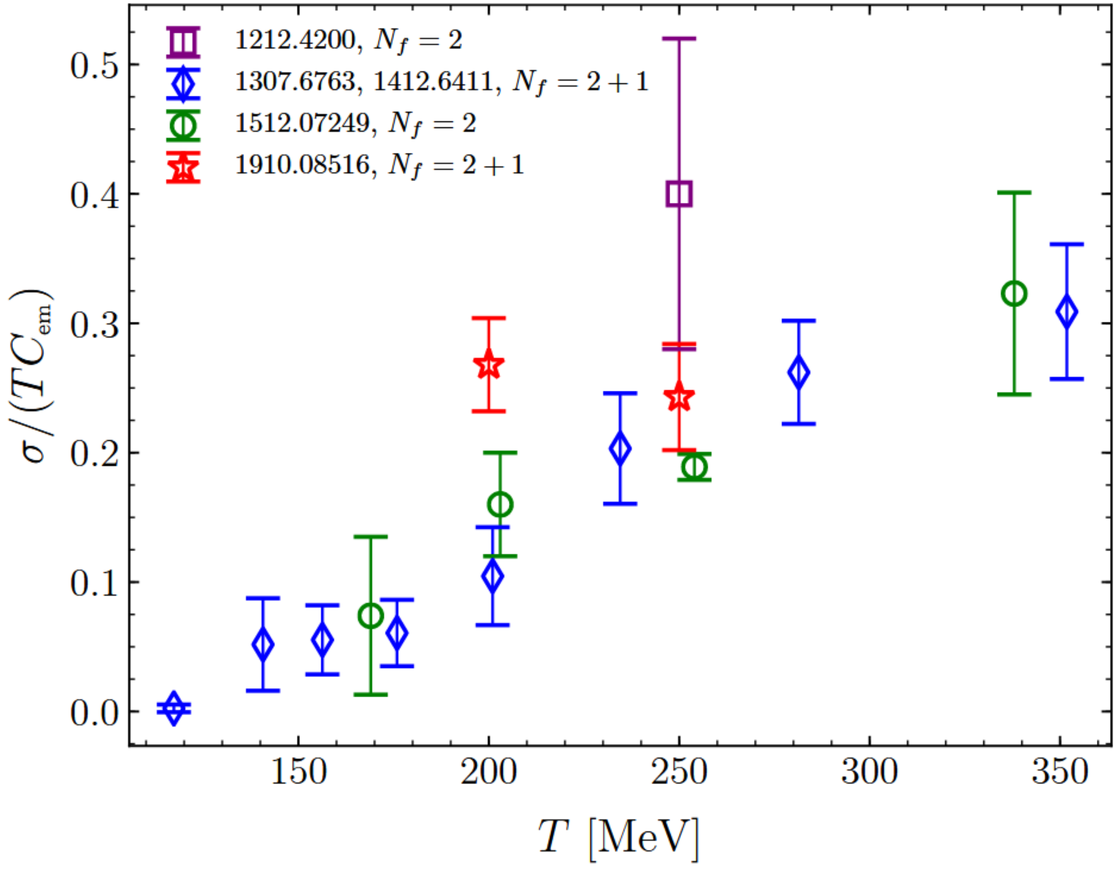
Gert Aarts, et al.
Phys. Rev. Lett., 99:022002, 2007.



$\vec{B}$: Magnetic field
$\vec{E}$: Electric field
v: velocity

Electric Conductivity of QCD Matter

- **Lattice QCD**



Electric Conductivity on the Lattice

$$\sigma = \frac{1}{6}\frac{\partial}{\omega}\left(\int d^4x e^{i\omega t}\langle[j_\mu^{\rm em}(t,x), j_\mu^{\rm em}(0,0)]\rangle\right)\Big|_{\omega=0}$$

Uses linear-response theory (Kubo formula)
Low energy limit of the electromagnetic spectral function

- Does not include external magnetic field effects
- Uses approximately realistic pion mass
- General agreement among results using a variety of methods and parameters

Aarts, Nikolaev, EPJ.A 57, 118 (2021); 2008.12326 [hep-lat]

Electromagnetic Field in Heavy-Ion Collisions

- **Electromagnetic field in heavy ion collisions**
  - ➢ **Production of strong magnetic field**
    - Au + Au ($\sqrt{s_{NN}} = 200$ GeV) : 10^{14} T ~$10\ m_\pi^2$
    - Pb + Pb ($\sqrt{s_{NN}} = 2.76$ TeV) : 10^{15} T

Not clearly observed

- **Response to electromagnetic field**
  - **Electric conductivity from lattice QCD**
  - $\sigma \sim 0.023$ fm^{-1} @ $T \sim 250$ MeV

Gert Aarts, et al.
Phys. Rev. Lett., 99:022002, 2007.

Experimental data?

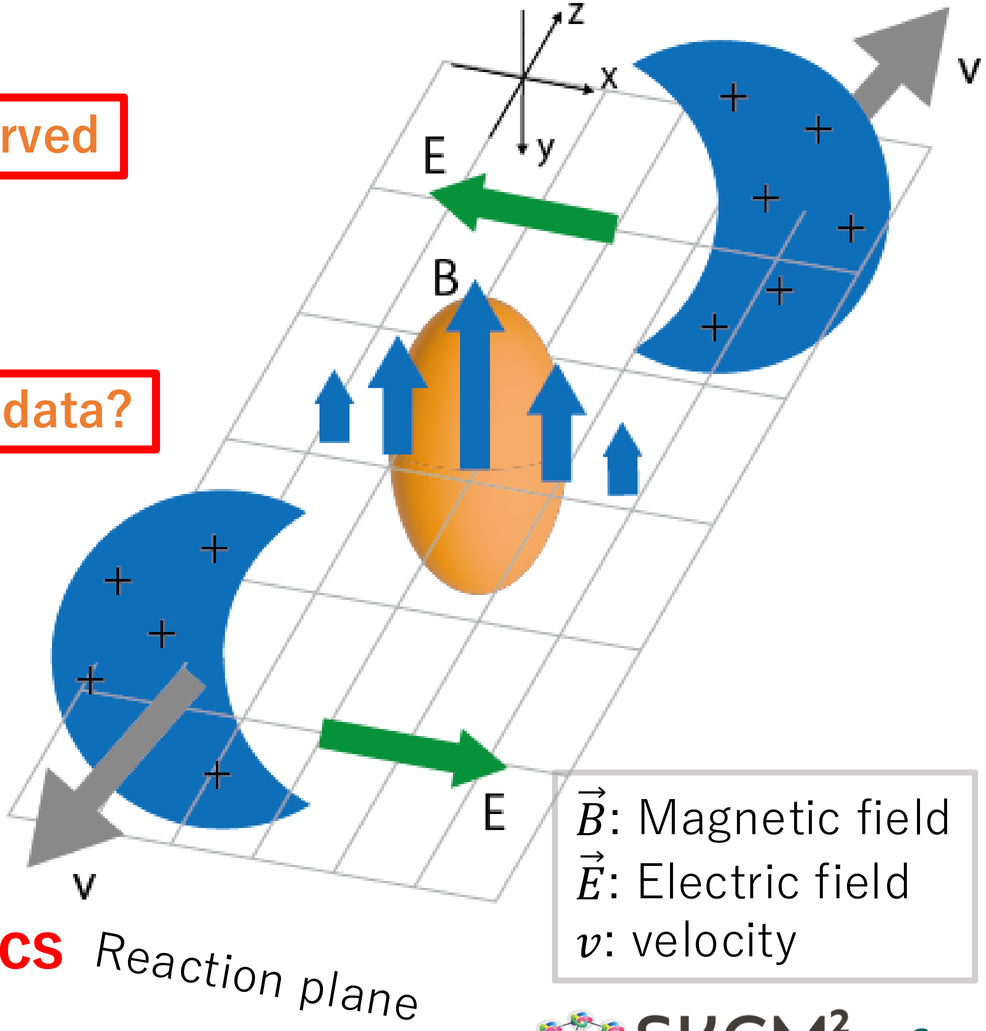
- ➢ **Magnetohydrodynamics** $(\sigma \to \infty)$

Inghirami, et al, Eur. Phys. J. C (2020) 80:293

  - Focus only on magnetic field
  - Quantitative analysis on electric conductivity

Electric conductivity ⟷ **experimental data**

Relativistic Resistive Magnetohydrodynamics



$\vec{B}$: Magnetic field
$\vec{E}$: Electric field
v: velocity

Electromagnetic Fields and Property of QGP

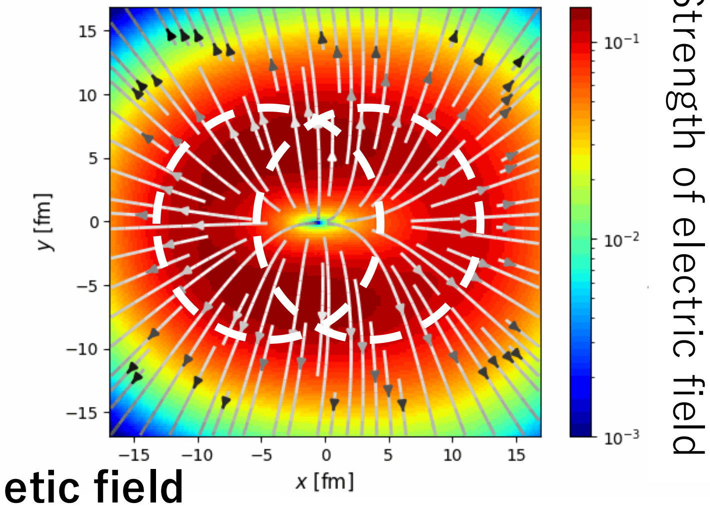
- **Electric Conductivity**
  - **Dissipation of electric field**
    - Ampere's law： $\partial_t \vec{E} - \nabla \times \vec{B} = -\vec{j}$

$\vec{B}$: magnetic field
$\vec{E}$: electric field

Ohm's law makes electric field dissipate

➡ **Dissipated energy to fluid（medium)**

Electric field

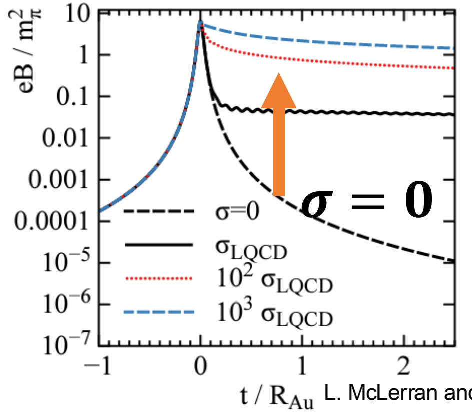


  - **Charge is induced by electric field**
    - Induced charge depends on charge conductivity
  - **Dissipation of magnetic field**

Charge conductivity of QGP

⬅ **dissipation of electromagnetic fields and charge distribution QGP**

Magnetic field



L. McLerran and V. Skokov, Nucl. Phys. A 929 (2014), 184-190

$\sigma \neq 0$

Suppresses of dissipation

Electric field is dissipated.

Understanding of QGP Property

Charge conductivity of QGP from analysis of high-energy heavy-ion collisions

Physical property	Observables	Quantitative analysis
Charge conductivity	?	×
Shear viscosity	Azumithal anisotoropy v_n	○
Bulk viscosity	P_T distributions	○
Diffusion coefficient	Jet energy loss	○

Charge dependent directed flow
Asymmetic collisions → i.e., Hirono, Hongo, and Hirano, PRC 90, 021903 (2014).
Symmetric collisions

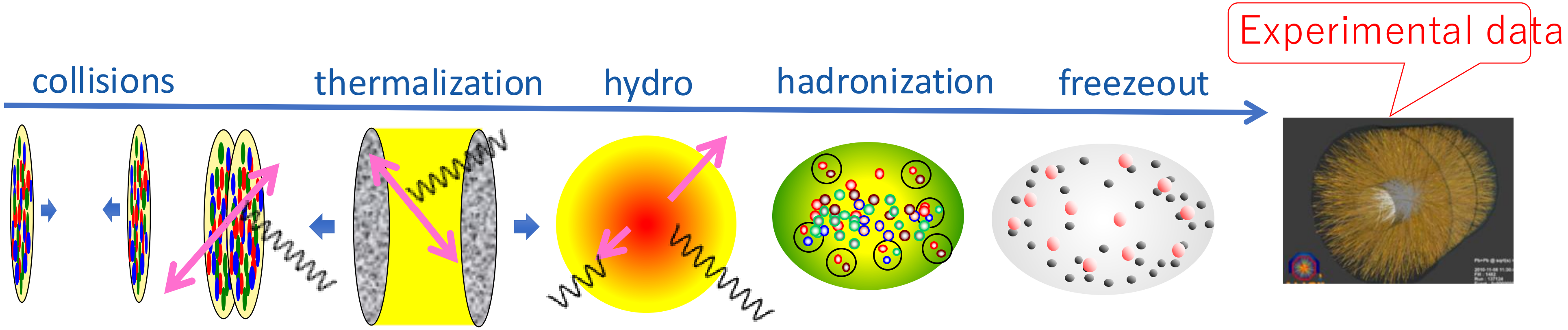
Proposed EM observables
Dileptons → i.e., Akamatsu, Hamagaki, Hatsuda, and Hirano, PRC 85, 054903 (2012).
Photons → i.e., Sun and Yan, PRC 109, 034917 (2024).

Construction of relativistic resistive magnetohydrodynamics

Relativistic Resistive Magnetohydrodynamics

Relativistic Resistive Magneto-Hydrodynamics (RRMHD)

Nakamura, Miyoshi, CN and Takahashi, PRC107, no.1, 014901 (2023)



Initial conditions

Glauber model
+approximate solutions of Maxwell eq.

Hydrodynamics

Hydrodynamic eq. + Maxwell eq. + Ohm's law

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad J^\mu = \sigma e^\mu$$

Relativistic Resistive Magneto-Hydrodynamics (RRMHD)

Nakamura, Miyoshi, CN and Takahashi, PRC107, no.1, 014901 (2023)

- **RRMHD equation**
  - Conservation law + Maxwell eq. + Ohm's law

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

$$J^\mu = J_c^\mu + q u^\mu$$

e: energy density
p: pressure
$p_{em} = (E^2 + B^2)/2$

$\varepsilon = (e+p)\gamma^2 - p + p_{em}$
$m^i = (e+p)\gamma^2 v^i + \epsilon^{ijk} B_j E_k$
$\Pi^{ij} = (e+p)\gamma^2 v^i v^j + (p + p_{em}) g^{ij} - E^i E^j - B^i B^j$

Energy Conservation

$$\partial_t \varepsilon + \nabla \cdot m = 0$$

Momentum conservation

$$\partial_t m^i + \nabla \cdot \Pi^i = 0$$

Faraday's law

$$\partial_t \vec{B} + \nabla \times \vec{E} = 0$$

Ohm's law

$$\vec{J} = q\vec{v} + \underbrace{\sigma\gamma[\vec{E} + \vec{v} \times \vec{B} - (\vec{v} \cdot \vec{E})\vec{v}]}_{=:\vec{J}_c}$$

Ampere's law

$$\partial_t \vec{E} - \nabla \times \vec{B} = -\vec{J}$$

$\rightarrow$ $\partial_t \vec{E} - \nabla \times \vec{B} = q\vec{v}$

$\rightarrow$ $\partial_t \vec{E} = \vec{J}_c$

operator splitting

- Integration with quasi-analytic solutions

$$\vec{E}_\perp = -\vec{v} \times \vec{B} + \left(E_\perp^0 + \vec{v} \times \vec{B}\right) \exp(-\sigma\gamma t)$$

$$\vec{E}_\parallel = E_\parallel^0 \exp(-\sigma t/\gamma)$$

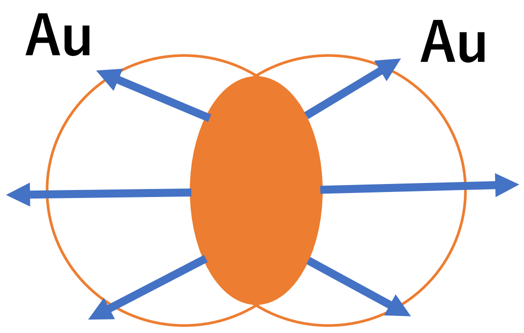
Komissarov, Mon. Not. R. Astron. Soc. 382, 995-1004 (2007)

- **Development in Milne coordinates**
- **Code validation**

Symmetric and Asymmetric Systems

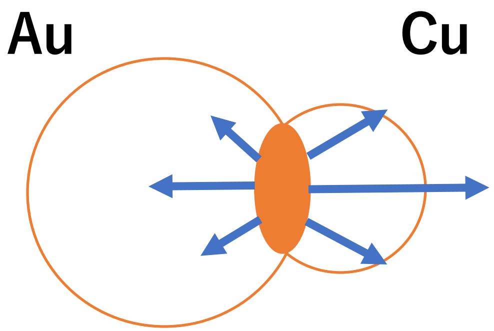
RHIC $\sqrt{s_{NN}} = 200\ \mathrm{GeV}$

■Au-Au collisions



- Symmetric pressure gradient
- Almond-shaped medium

■Cu-Au collisions



- Asymmetric pressure gradient
- Distorted Almond-shaped medium

Hirono, Hongo, Hirano

Initial Condition : QGP Medium

Bozek, et al, Phys. Rev. C 81, 054902(2010)

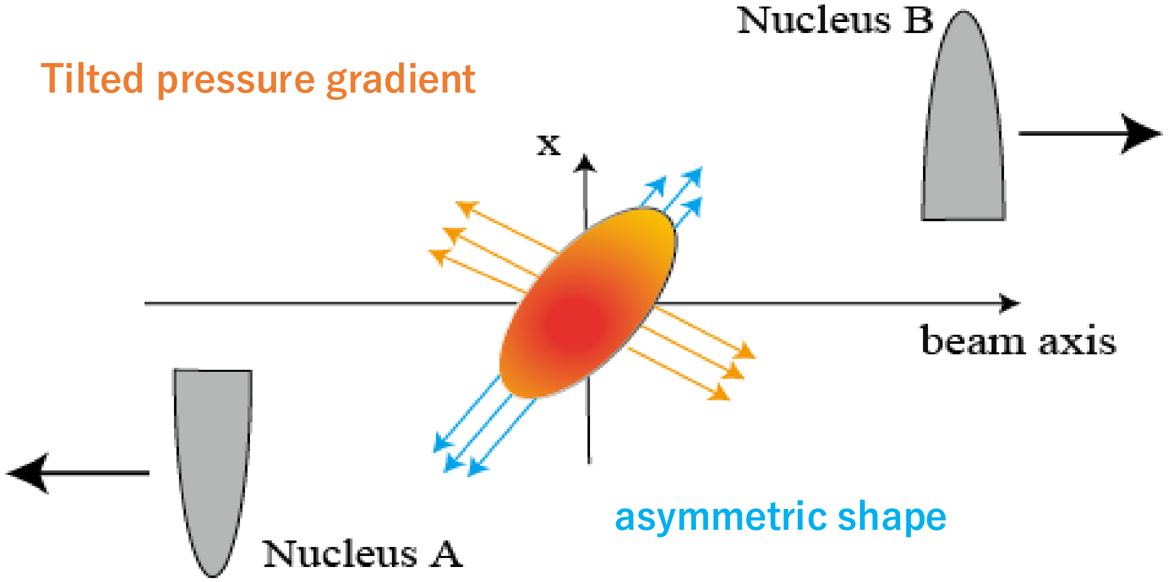
n_p : number of participants
n_c : number of collisions

■ Tilted Glauber model

- Energy density is scaled by n_p and n_c
- Tilted distribution in the longitudinal direction

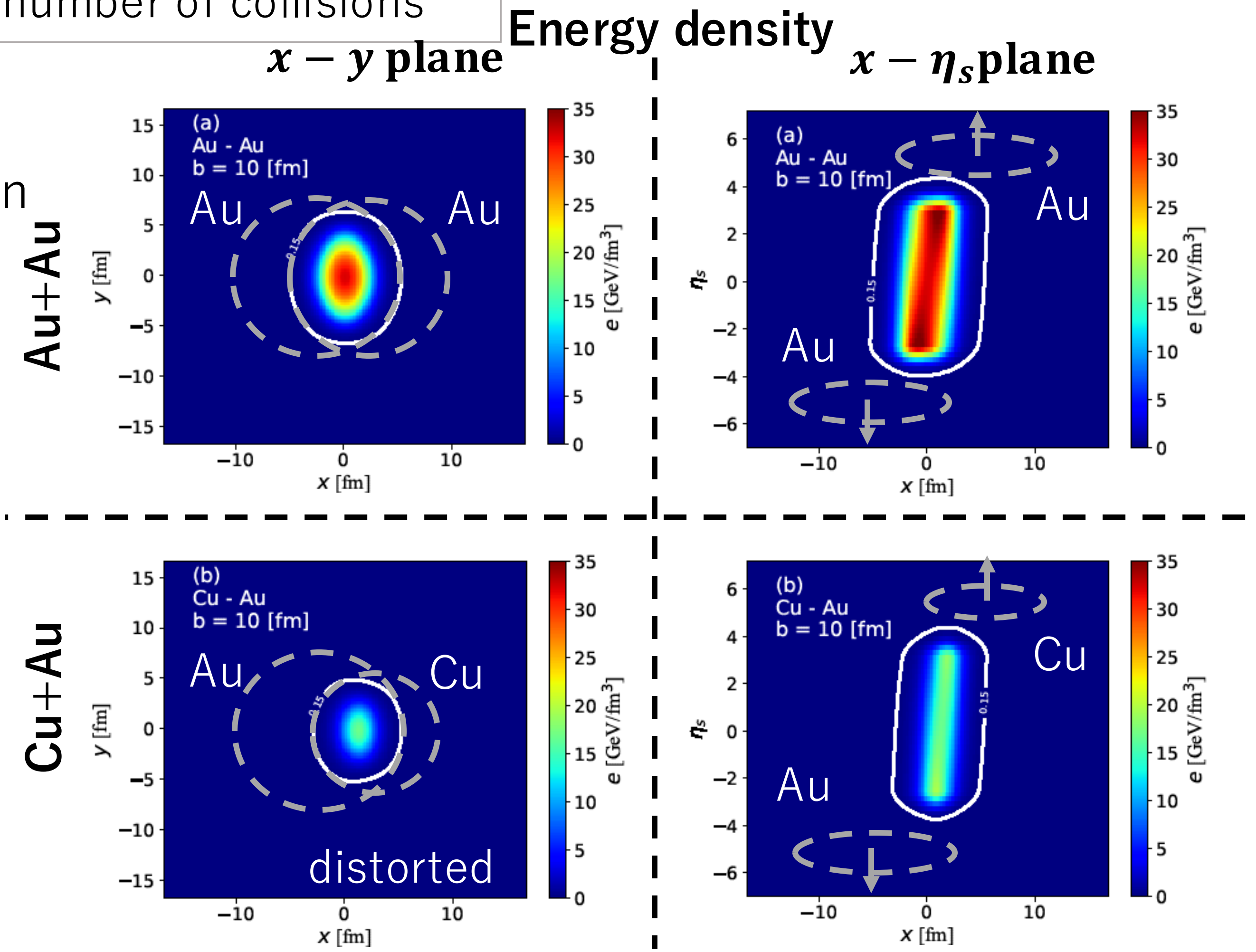
For directed flow v_1

Tilted pressure gradient



asymmetric shape

Freezeout hypersurface



Initial Condition：Electromagnetic Fields

Tuchin, Phys.Rev.C88,024911(2013)

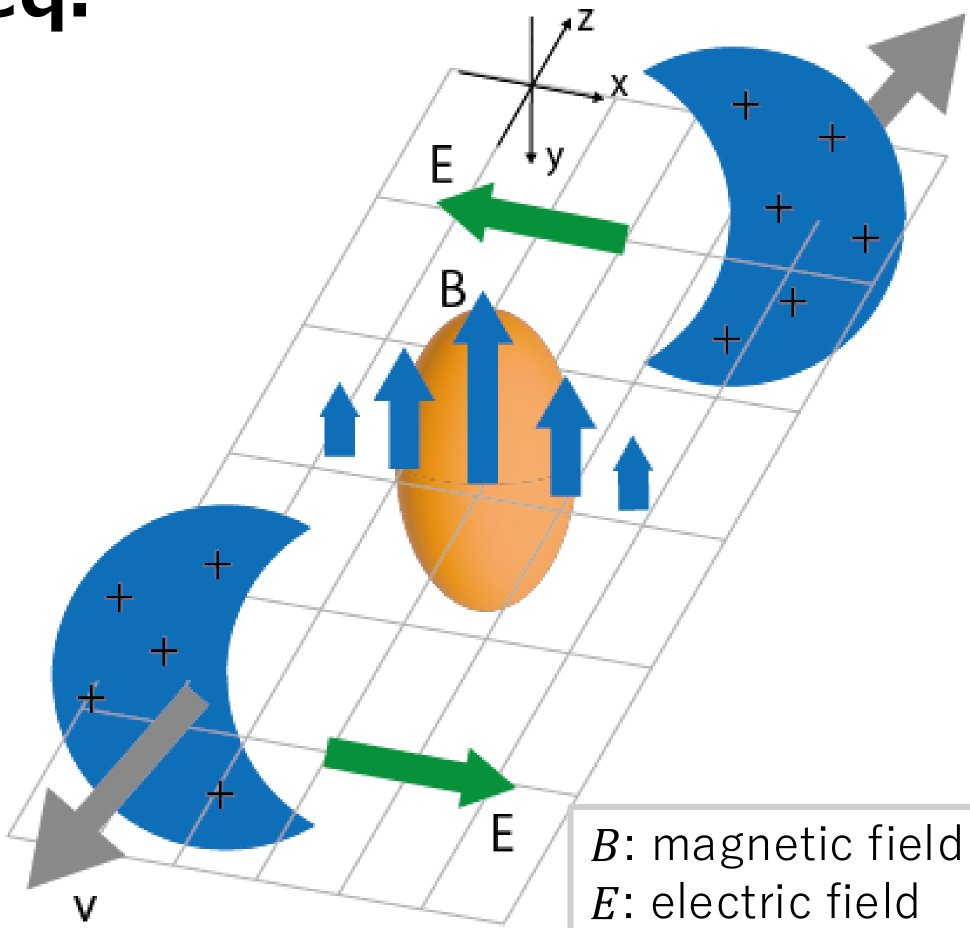
■Asymptotic solution of Maxwell eq.

➢ Electromagnetic field made by point charge moving in the longitudinal axis

- Proton distribution in nucleus : uniform sphere
- Constant charge conductivity (σ =0.023 fm^{-1})

$$\nabla \cdot \boldsymbol{B} = 0\,, \quad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}\,,$$
$$\nabla \cdot \boldsymbol{D} = e\delta(z - vt)\delta(\boldsymbol{b})\,,$$
$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \sigma \boldsymbol{E} + ev\hat{z}\delta(z - vt)\delta(\boldsymbol{b})$$

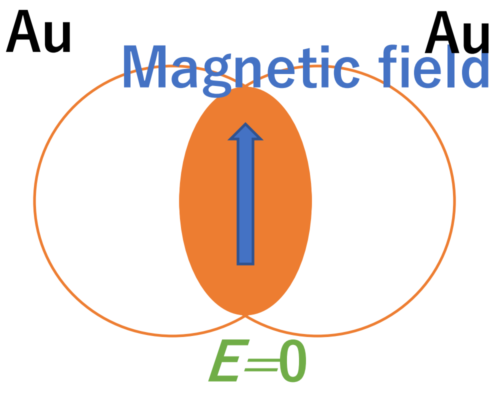
Integration of the asymptotic solutions over the charge distribution inside of nucleus



B: magnetic field
E: electric field
v: velocity of heavy ion

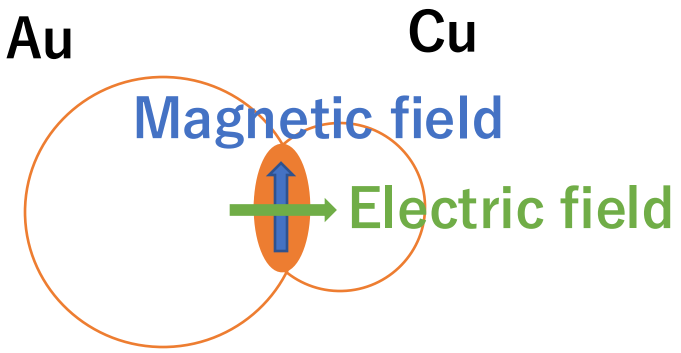
Electromagnetic Field in Symmetric and Asymmetric Systems

■Au-Au collisions



- Magnetic field
  - Strong magnetic field
- Electric field
  - No electric field

■Cu-Au collisions



- Magnetic field
  - Strong magnetic field
- Electric field
  - $E \neq 0$ **due to the asymmetry of the charge distribution**

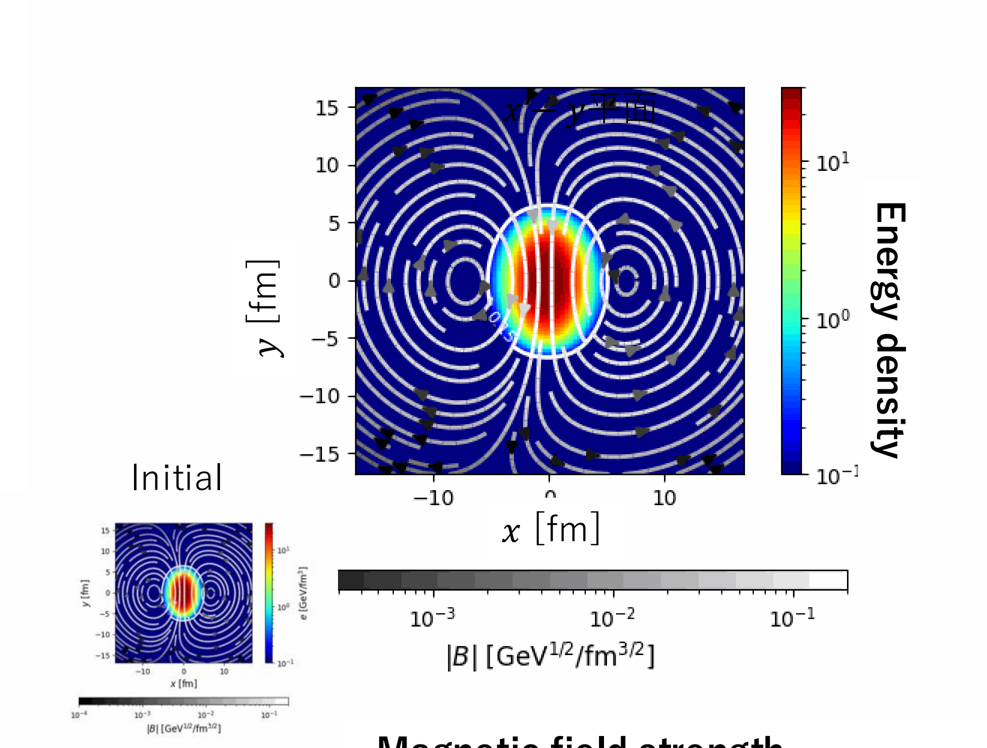
Hirono, Hongo, Hirano

Space-time Evolution

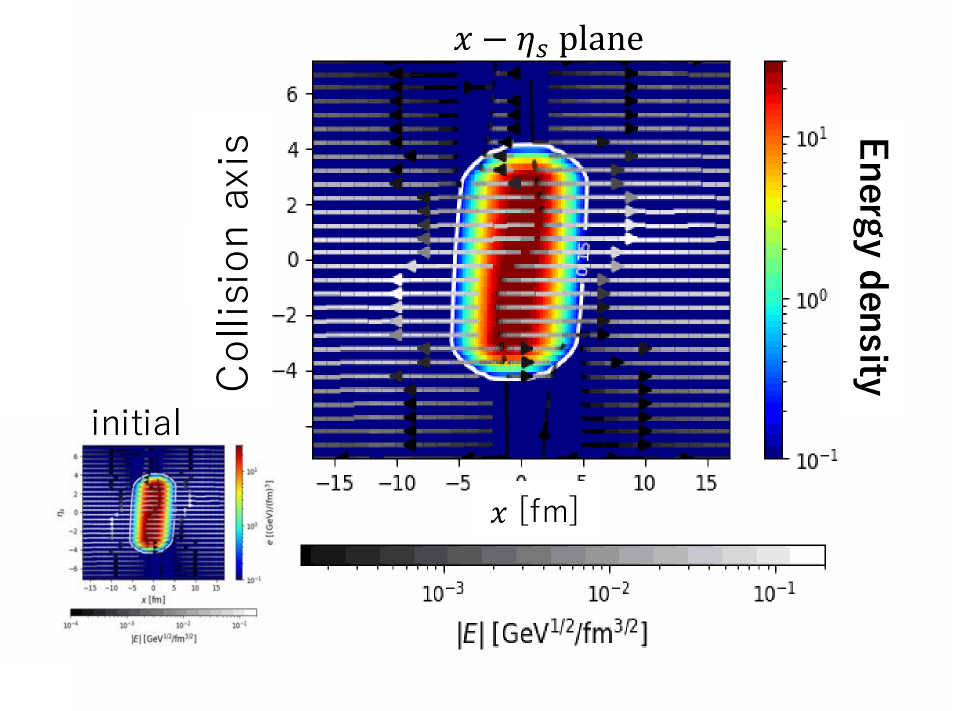
Nakamura, Miyoshi, CN and Takahashi, PRC 107, no.1, 014901 (2023)

Au+Au collision system

First calculation in HIC with RRMHD code



Magnetic field strength



Electric field strength

Analysis of Heavy Ion Collisions

Electrical Conductivity of QCD Matter in HIC

- **Charge Dependent Flow**

  Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107 (2023) 3, 034912

- **Photon**

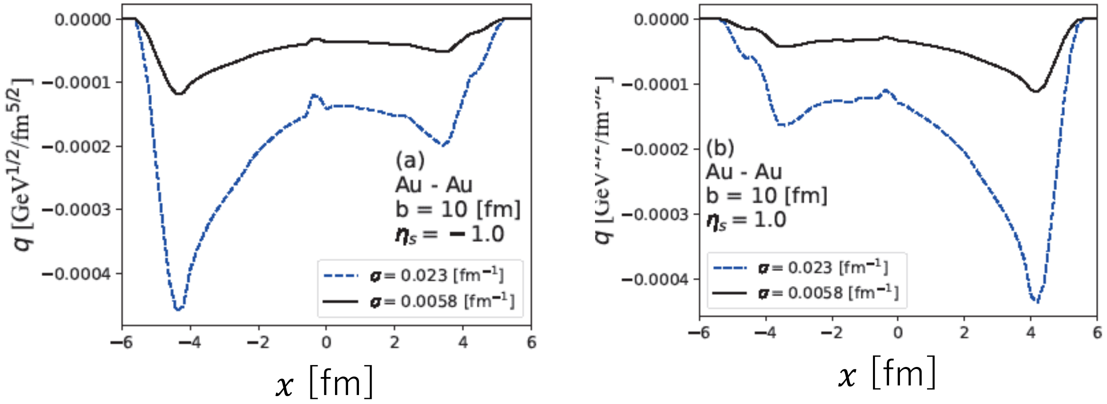
  Benoit ,Miyoshi, CN , Sakai and Takahashi, in preparation

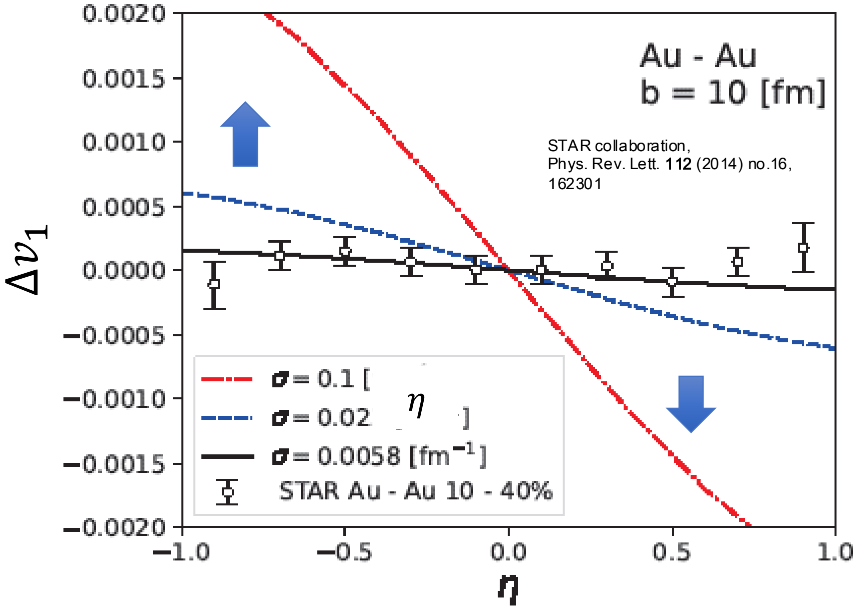
Charge Dependence of Δv_1 : Au + Au

- $\Delta v_1 = v_1^{\pi^+}(\eta) - v_1^{\pi^-}(\eta)$
  - Clear dependence of charge conductivity
    - Proportion to electric conductivity
    - Negative charge induced in the opposite direction of fluid flow
      suppression of v_1 of negative charge
  - Δv_1 with finite σ is consistent with STAR data
    - $\sigma = 0.0058\ \mathrm{fm}^{-1}$
      ex. $\sigma_{LQCD} = 0.023\ \mathrm{fm}^{-1}$
      from lattice QCD
      Gert Aarts, et al.
      Phys. Rev. Lett., 99:022002, 2007.

✓ QGP electrical conductivity from high-precision measurement of Δv_1

Charge distribution on freezeout hypersurface





$\sigma = 0.1$ fm^{-1}
$\sigma = 0.023$ fm^{-1}
$\sigma = 0.0058$ fm^{-1}
○ : STAR data

$$\eta = \frac{1}{2}\ln\frac{|p| + p_z}{|p| - p_z}$$

Charge Dependence of Δv_1 : Cu + Au

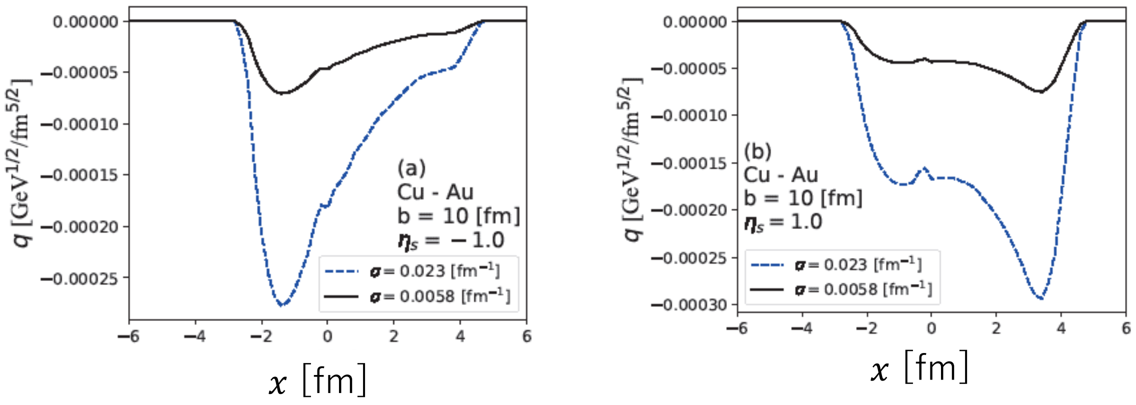
Nakamura, Miyoshi, CN and Takahashi, Phys. Rev. C 107 (2023) 3, 034912

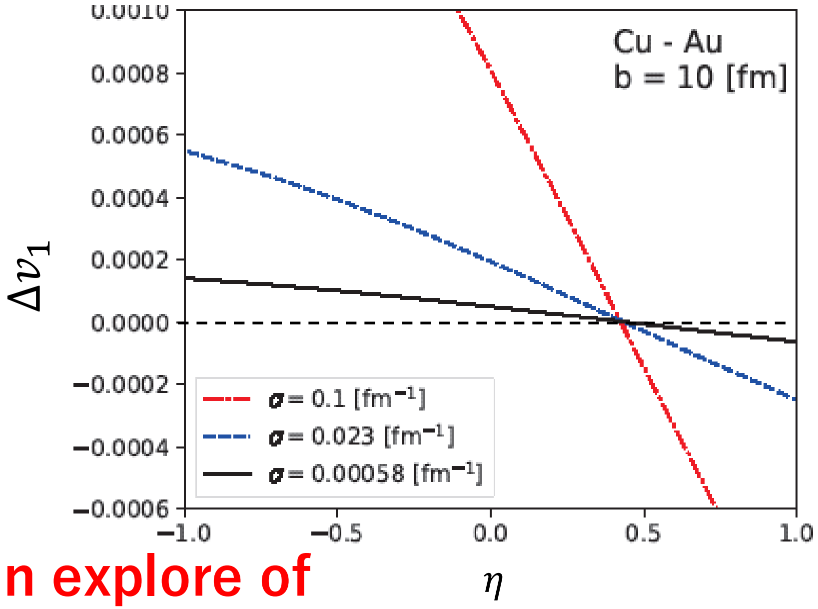
- $\Delta v_1 = v_1^{\pi^+}(\eta) - v_1^{\pi^-}(\eta)$
  - **Electric field created by initial condition**
    - Δv_1 is finite at $\eta = 0$
    - Asymmetry structure to $\eta = 0$
  - **Proportion to electric conductivity**
    - Δv_1 vanishes at $\eta = 0.5$.

✓ **Electrical conductivity <- Δv_1 at $\eta = 0$**

✓ **Initial electrical field from η dependence of Δv_1**

Charge distribution on freezeout hypersurface





$\sigma = 0.1$ fm^{-1}
$\sigma = 0.023$ fm^{-1}
$\sigma = 0.0058$ fm^{-1}

Asymmetric system has advantage in explore of QGP electrical conductivity.

Electrical Conductivity of QCD Matter in HIC

- **Charge Dependent Flow**

  Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107 (2023) 3, 034912

- **Photon**

  Benoit ,Miyoshi, CN , Sakai and Takahashi, in preparation

Electromagnetic Dissipation for QGP Photon

Benoit

- **Electromagnetic fields inside QGP**
  - EM fields penetrating QGP drive charge carriers out-of-equilibrium

$$J^\mu = qu^\mu + \sigma F^{\mu\nu} u_\nu$$

First order dissipation from the EM fields

  - Taking the Boltzmann equation in the relaxation time application

$$k^\mu \partial_\mu f_a + \underline{eQ_a F^{\mu\nu} k_\mu \frac{\partial f_a}{\partial k^\nu}} = -\frac{k^\mu u_\mu}{\tau_R} \delta f^{(n)}_{a,EM}$$

Sun and Yan, PRC 109, 034917 (2024).

Vlasov term for the external EM fields

Order "n" corrections
to the quark distribution function

$$\delta f^{(1)}_{a,EM}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el} k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

Electric conductivity of QGP from
Landau matching with the current

EM fields in the fluid rest frame

$$e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \epsilon^{ijk} v_j B_k)$$

Electromagnetic Dissipation for QGP Photon

Benoit

- **Electromagnetic fields inside QGP**
  - The fluid + EM field contributions from hydrodynamics
  - All of those values can be calculated self-consistently using relativistic resistive magneto-hydrodynamics (RRHMD)

Temperature and four velocity

$$\delta f^{(1)}_{a,EM}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

Electric susceptibility of QGP

$$\chi_{a,el} = -\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3 E_p}(p^\sigma p^\nu \Delta_{\sigma\nu})\frac{-f_{a,eq}(1-f_{a,eq})}{p^\mu u_\mu}$$

Spacetime dependent EM fields in QGP medium

$$e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \epsilon^{ijk} v_j B_k)$$

Photon production from QGP and EM fields

Benoit

- **Rate of QGP photon production should be increased by the EM fields**

$$E_k \frac{d\mathcal{R}}{d^3\vec{k}} = E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\mathrm{QGP}} + E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\mathrm{EM}}$$

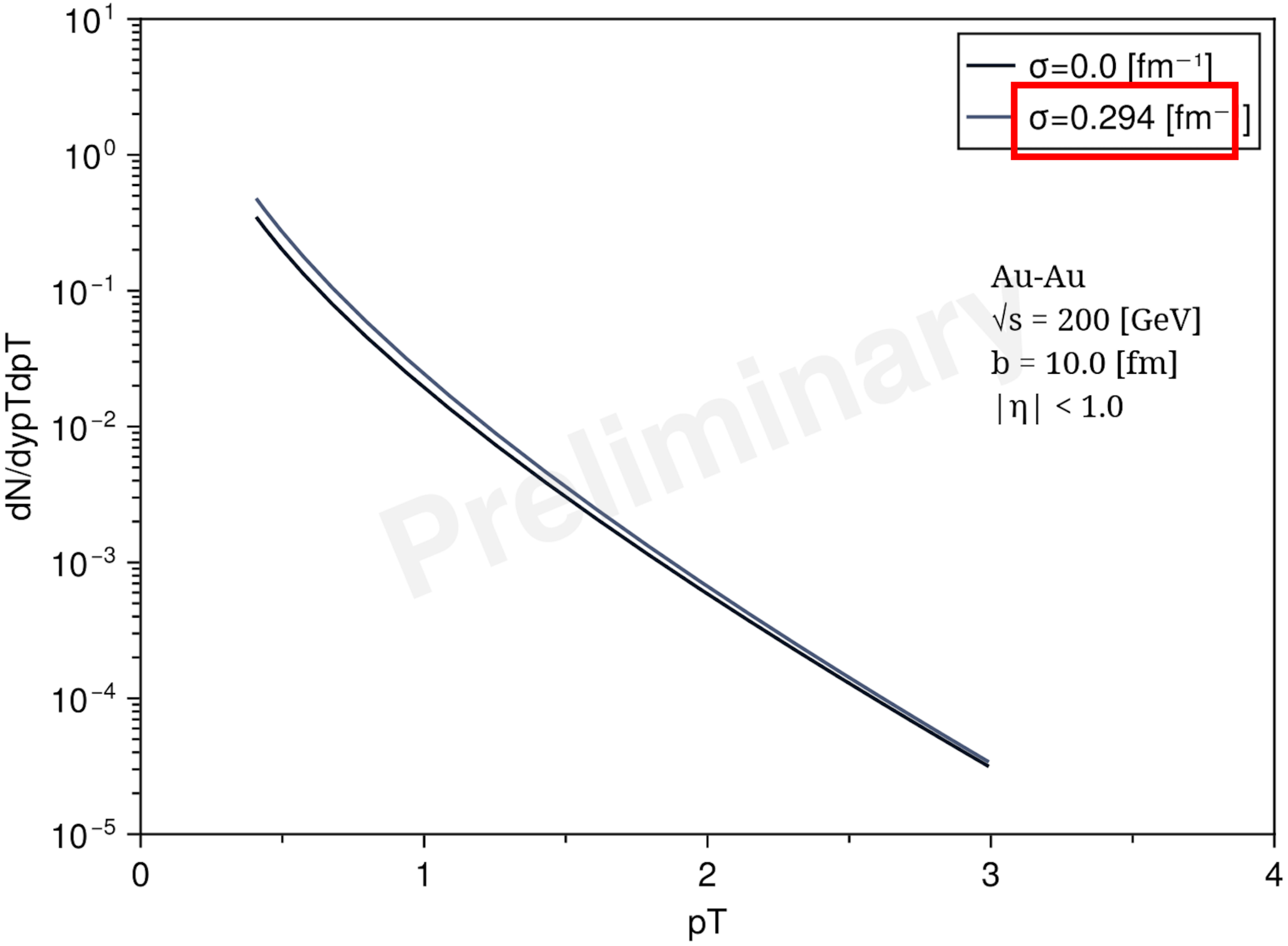
$$E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\mathrm{EM}} \sim C\alpha_s\alpha_{\mathrm{EM}}\mathcal{I}\mathcal{L}_c \sum_a \delta f^{(1)}_{a,\mathrm{EM}}(X,k)$$

We focus on effect of EM dissipation

We neglect viscous dissipation effect

P_T Spectra of Direct Photon

Benoit



$$E_k \frac{d\mathcal{R}}{d^3\vec{k}}^{\text{EM}} \sim C\alpha_s\alpha_{\text{EM}}\mathcal{I}\mathcal{L}_c \sum_a \delta f^{(1)}_{a,\text{EM}}(X,k)$$

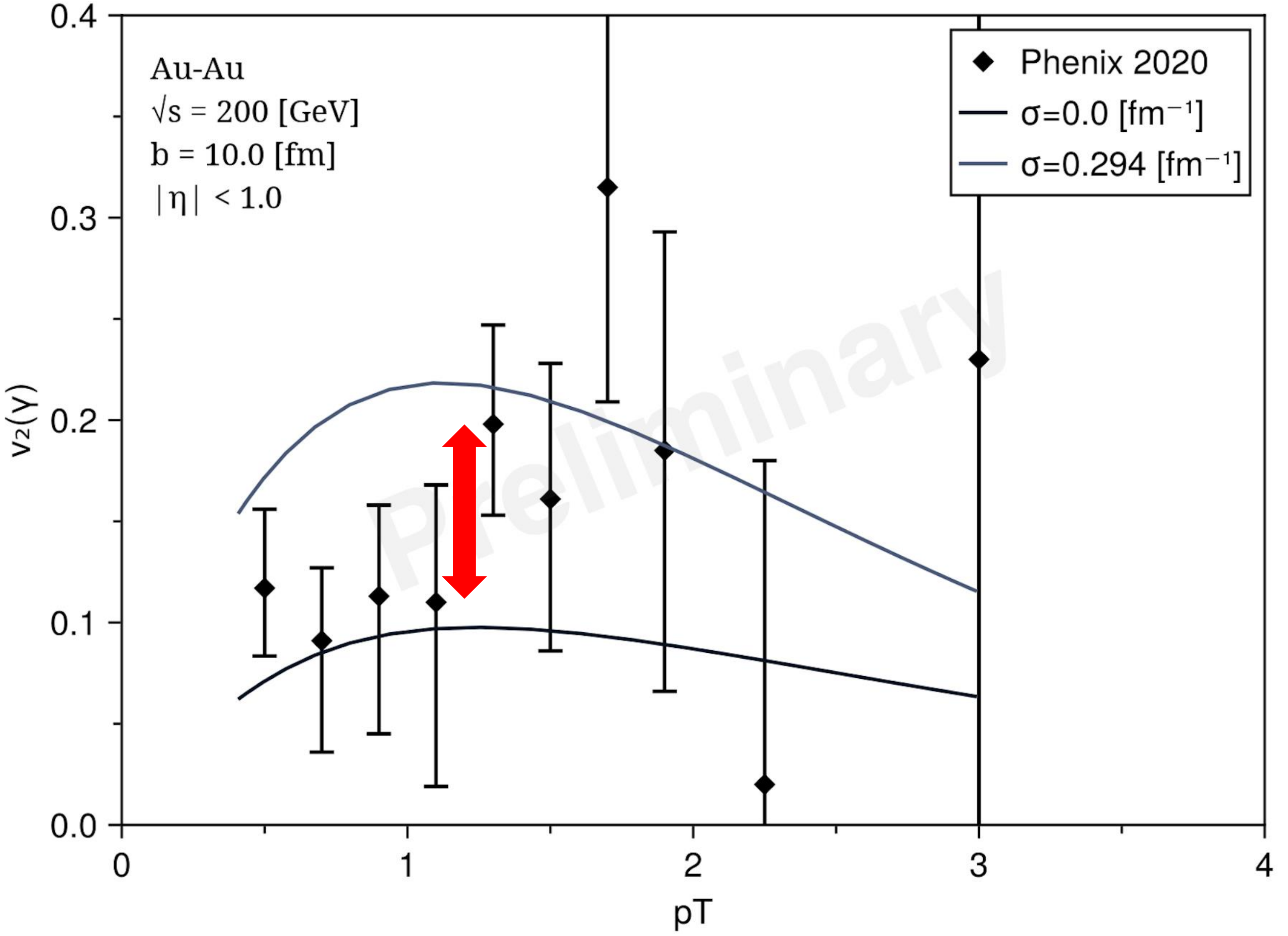
From Lattice QCD

$\sigma = 0.029$ [fm^{-1}]

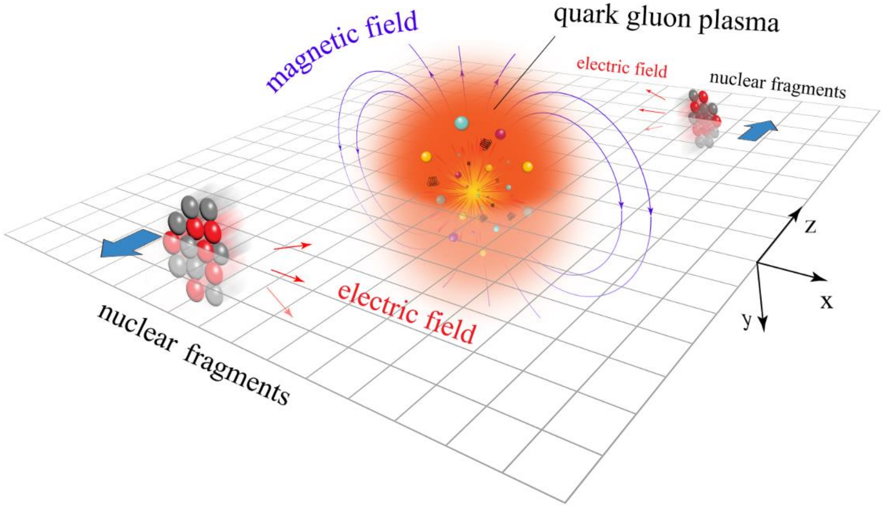
Small contribution to P_T spectra

Elliptic Flow of Direct Photon

Benoit



$$v_2(\gamma) \equiv \frac{v_0 v_2 + v_0^{EM} v_2^{EM}}{v_0 + v_0^{EM}}$$



Large enhancement is observed.

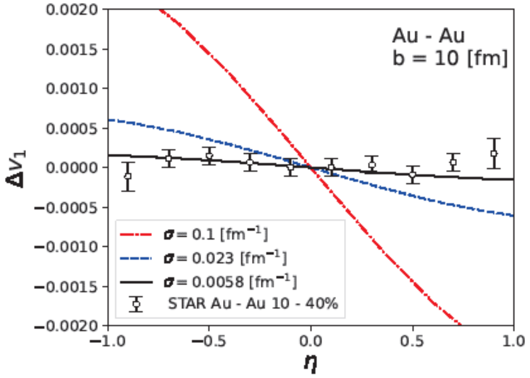
Since largest magnetic field has an elliptic orientation, a larger impact from the EM corrections on elliptic flow appears.

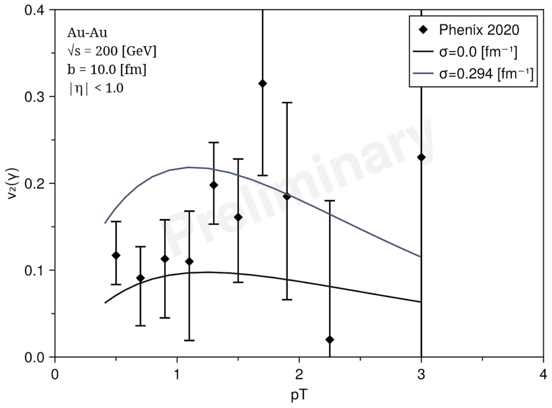
Summary

Electric conductivity of QCD Matter

- **Construction of RRMHD code in Milne coordinates**
- **Application to high-energy heavy-ion collisions**
  - Charge dependent flow
    - Au+Au and Au+Cu systems at RHIC energy
  - Elliptic flow of photons
  - Virtual photon polarization@LHC

Kimura, Benoit, Ishikawa, CN, Shigaki, ArXiv:2411.01406





Future work:

Event-by-event fluctuation
Finite density
Nuclear structure Ru+Ru, Zr+Zr
Vortex
Chiral magnetohydrodynamics
....

Backup

RRMHD Equation in Milne Coordinates

New

- **Milne coordinates**
  - **Expanding systems in the longitudinal direction (τ, x, y, η_s)**
    - Strong expansion in the longitudinal direction is effectively included.
    - Number of grid of fluid is saved.

$$\tau = \sqrt{t^2 - z^2}$$
$$\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}$$

RRMHD Equation

$$\partial_\tau(\tau U) + \partial_i(\tau F^i) = \tau S$$

$$U = \begin{pmatrix} D \\ m_j \\ \varepsilon \\ B^j \\ E^j \\ q \end{pmatrix}, \mathrm{F}^\mathrm{i} = \begin{pmatrix} Dv^i \\ \Pi^{ji} \\ m^i \\ \varepsilon^{jik}E_k \\ \varepsilon^{jik}B_k \\ J^i \end{pmatrix}, S = \begin{pmatrix} 0 \\ \frac{1}{2}T^{ik}\partial_j g_{ik} \\ -\frac{1}{2}T^{ik}\partial_0 g_{ik} \\ 0 \\ J_c^i \\ 0 \end{pmatrix}$$

The first RRMHD code in Milne coordinates

Validation of the Code

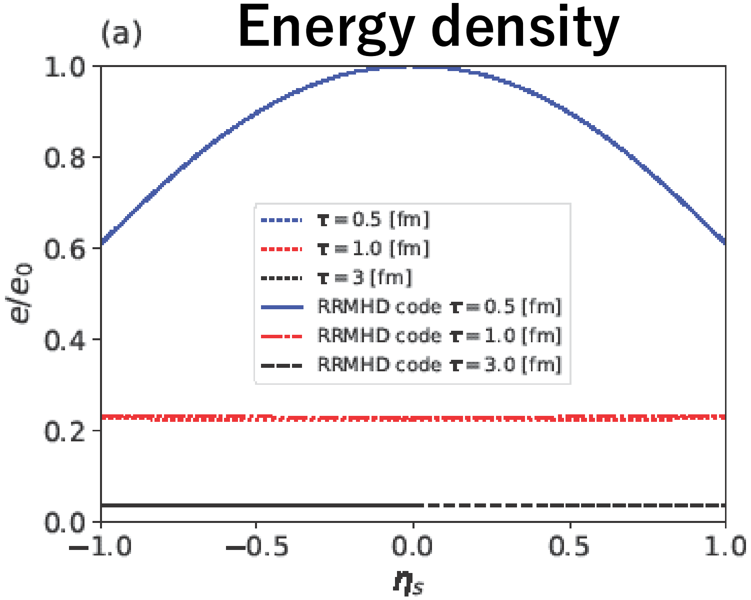
- **RRMHD in the Milne coordinates**

Nakamura, Miyoshi, CN and Takahashi, Eur.Phys.J.C 83 (2023) 3, 229.

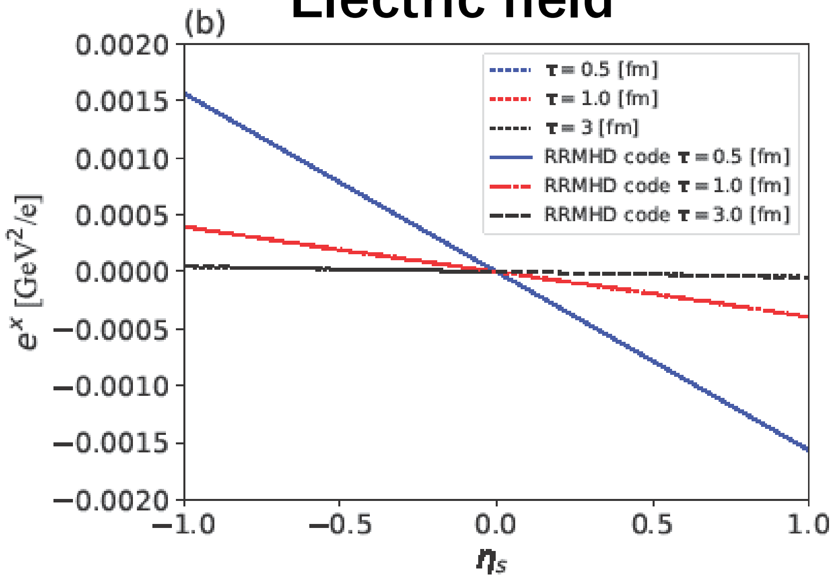
New Test Problem

- **(1+1) dimensional expansion system** $u^\mu = (\cosh Y, 0, 0, \sinh Y)$
  - Comparison between quasi-analytical solution and RRMHD simulation

Energy density



Electric field



Solid line : RRMHD code
Dashed line: quasi-analytical solution

➡ **Application to Heavy Ion Collisions**

Charge Dependent Directed Flow

Benoit, Miyoshi, C. N., Sakai and Takahashi, in preparation