ELECTRON-ION COLLIDER (EIC)- THEORY PERSPECTIVE: SPIN PHYSICS

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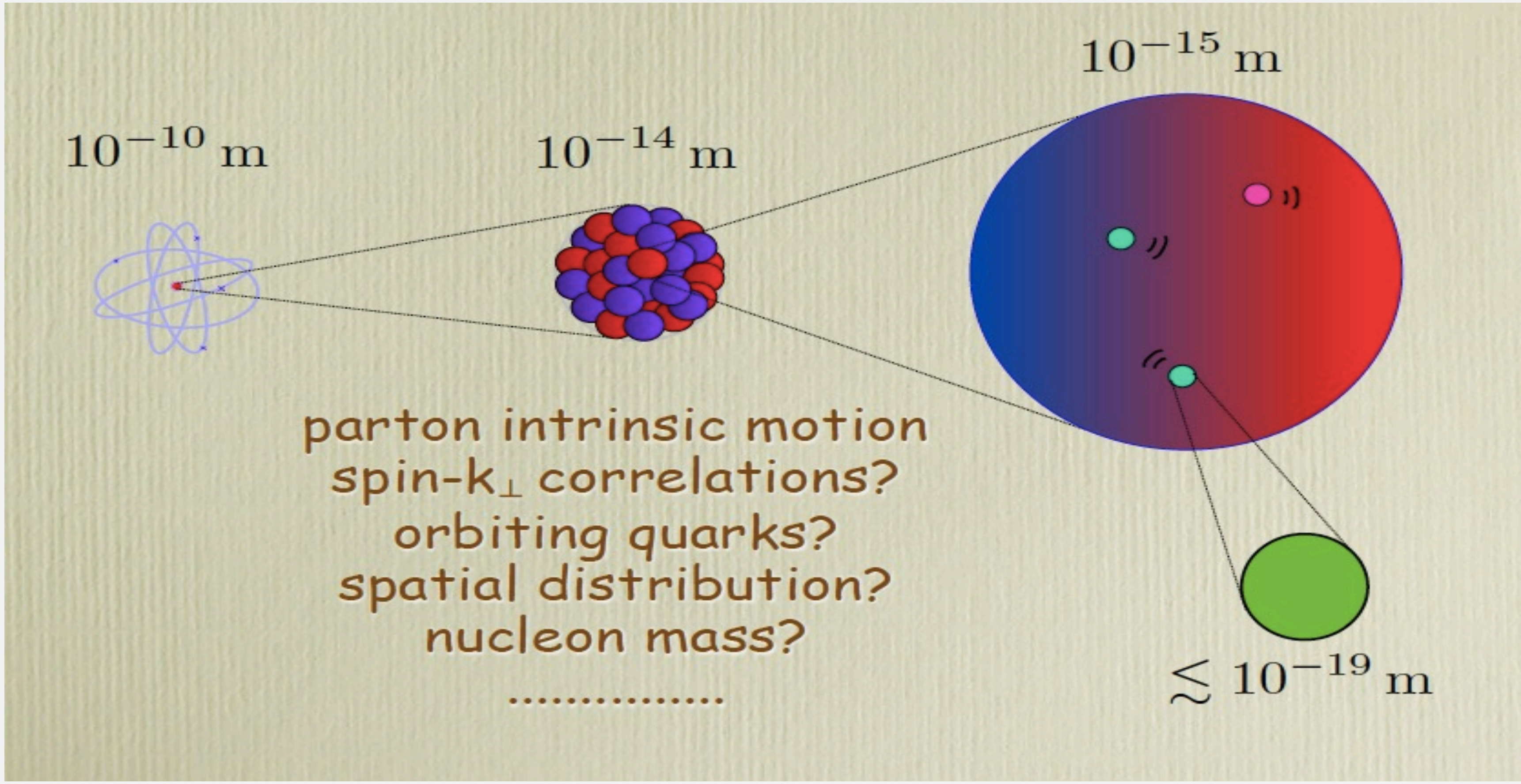
STRUCTURE OF THE NUCLEONS IN TERMS OF QUARKS AND GLUONS



Pic : M. Anselmino

How to study the internal structure of a watermelon ?

Method 1 : smashing two watermelons against one another

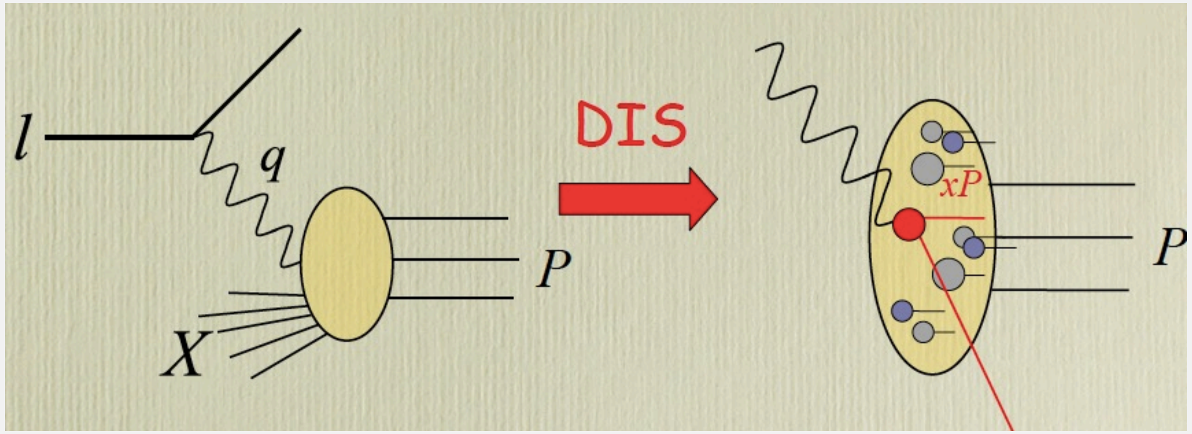
A-A/p-p collision at the relativistic heavy ion collider at BNL (RHIC)

Method 2 : slicing the watermelon by a knife

Deep inelastic e-p collision

Pic and example : Abhay Deshpande

NUCLEON STRUCTURE : PROBED THROUGH ELECTRON-PROTON DEEP INELASTIC SCATTERING



$$Q^2 = -q^2 \rightarrow \infty$$

$$x = \frac{Q^2}{2P \cdot q} \quad \text{fixed}$$

Virtual photon 'sees ' the partons (quarks) inside the proton

Proton is Lorentz contracted, like a pancake in transverse plane

Target is a collection of partons moving with fraction x of proton momentum, and collinearly with the proton

In the deep inelastic limit, the electron passes target at almost zero time, sees partons frozen in transverse plane.

Electron can interact with the partons only if the impact parameter is less than 1/Q. Electron-parton scattering happens at a much shorter time scale than the hadronization scale of proton remnants

FACTORIZED FORM OF THE CROSS SECTION

$$\frac{d\sigma}{dx\, dQ^2} = \sum_f \left(\frac{d\hat{\sigma}}{dQ^2}\right)_f e_f^2\, \phi_f(x)$$

Probability density of finding a parton of momentum fraction x inside the proton

Parton model : Partons are non-interacting

Differential scattering cross section

Elastic electron-parton scattering

Incoherent sum over all partons

Factorization of the hard part, that is interaction of electron with the parton, and the soft part, that is the parton distributions in the cross section

In parton model, parton distributions show scaling : they are functions of x only

Hard part can be calculated perturbatively but the parton distributions are non-perturbative. They are also not dependent on the process

Bjorken & Paschos, Phys. Rev D185, 1975, (1969).

PARTON MODEL TO QCD

$$\frac{d\sigma}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4} A(y)\ F_2(x)$$

$$F_2(x) = \sum_f e_f^2\, x\, \phi_f(x)$$

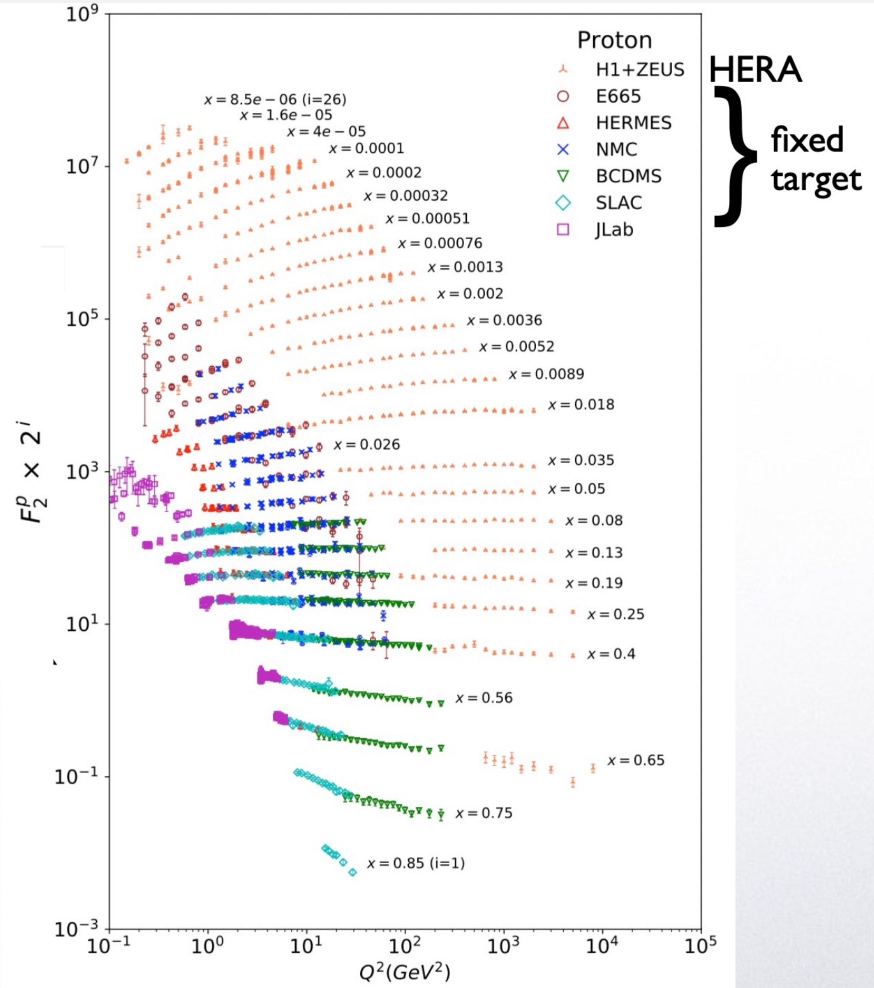
y = electron inelasticity $\sim \dfrac{\nu}{E}$
$A(y) = 1 + (1-y)^2$

F_2 varies also with Q^2 : scale evolution

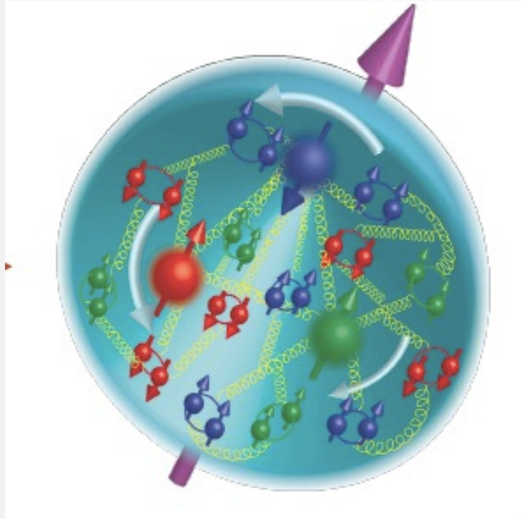
Scale evolution can be calculated using evolution equations
partons, or quarks are not free : they interact through gluons !

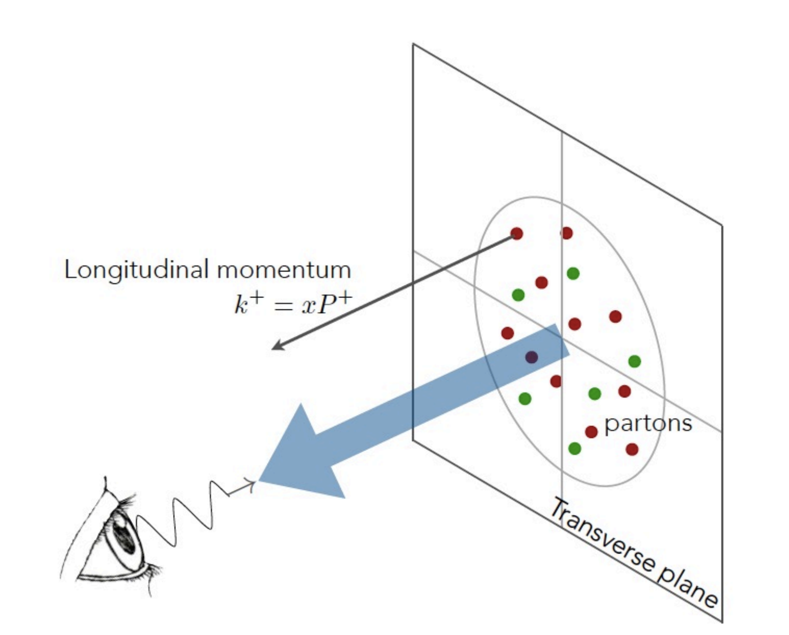
Interaction of quarks and gluons are called strong interaction or QCD

$$\frac{d\sigma}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[A(y)\, F_2(x,Q^2) - y^2\, F_L(x,Q^2)\right]$$



COLLINEAR PDFS : NUCLEON STRUCTURE IN 1-D





Motion of quarks in the transverse plane ignored

Non-perturbative : Is extracted by fitting experimental data

Scale evolution of pdfs can be calculated using **Dokshitzer–Gribov–Lipatov–Altarelli–Parisi** (DGLAP) evolution equations

Independent of process : once extracted can be used to predict cross section of another process as the scale evolution is known

One can also perform a polarized scattering experiment : probes polarized structure functions

NUCLEON SPIN PUZZLE

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

Proton spin

Quark spin

Quark OAM

Gluon spin

Gluon OAM

EMC (European Muon Collaboration) at CERN in 1989 measured spin asymmetry in polarized muon-proton scattering experiment, and found that the contribution coming from the intrinsic spin of quarks is very small

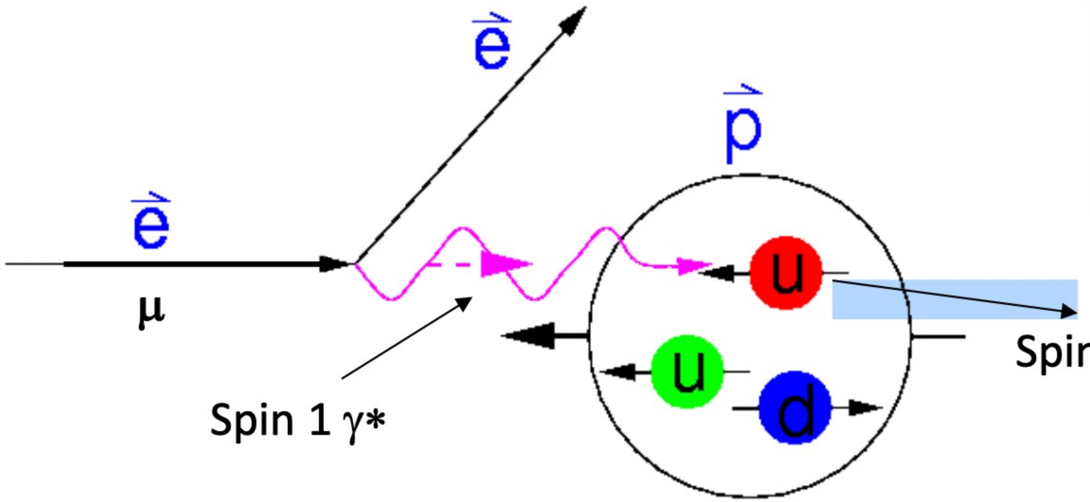
$\Delta\Sigma /2 = (0.12)$ +/- (0.17) (EMC, 1989)

Significant contribution comes from gluons as well as the orbital angular momentum of quarks and gluons

How to measure the orbital angular momentum ? Observables ? Can one separate the gluon part intro intrinsic and orbital in a gauge invariant way ?

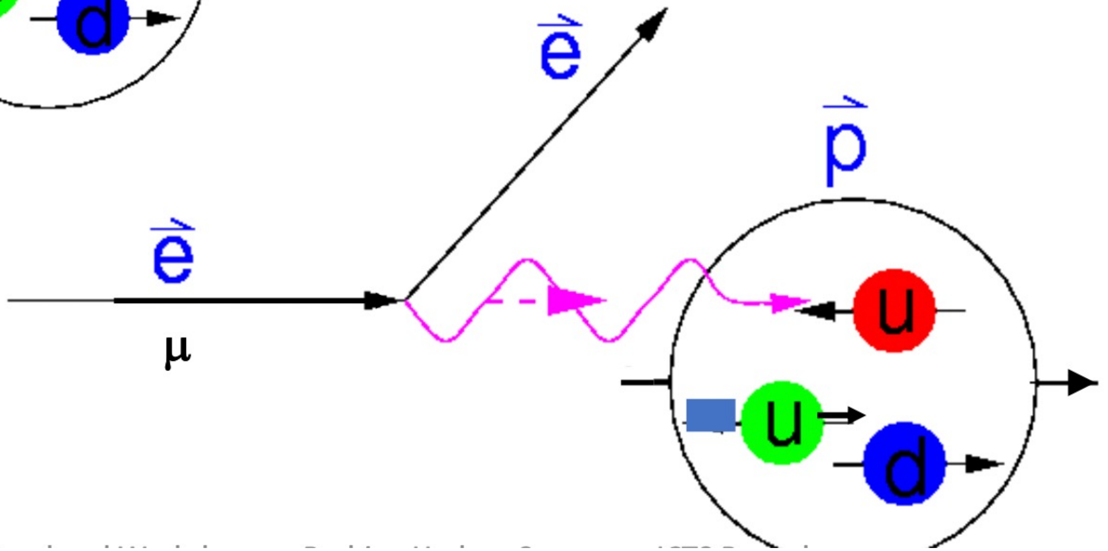
How to measure quark contribution to the spin ?

Polarized deep inelastic scattering experiment : electron and proton longitudinally polarized



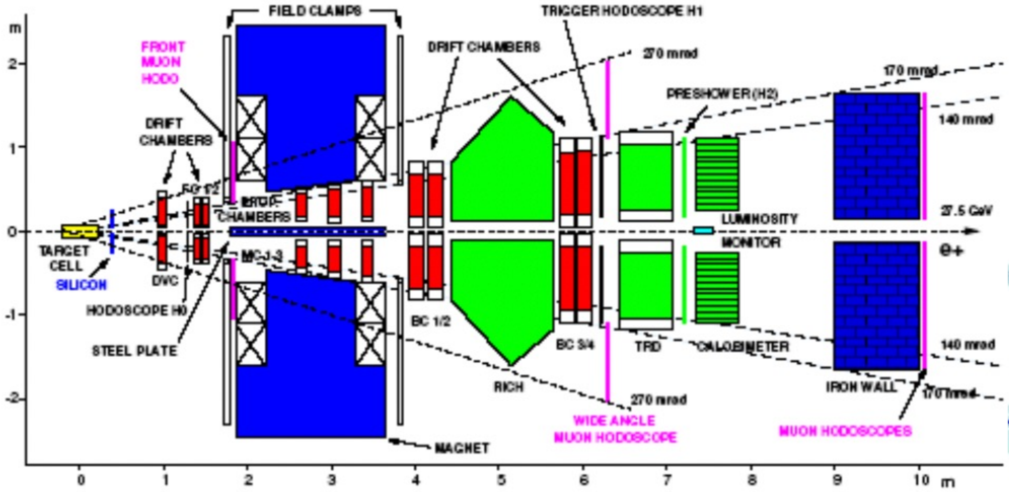
$$A_{\gamma * p} \propto \frac{N^{\uparrow\downarrow} - N^{\uparrow\uparrow}}{N^{\uparrow\downarrow} + N^{\uparrow\uparrow}}$$

One can measure double longitudinal spin asymmetry : sensitive to the intrinsic spin carried by quarks

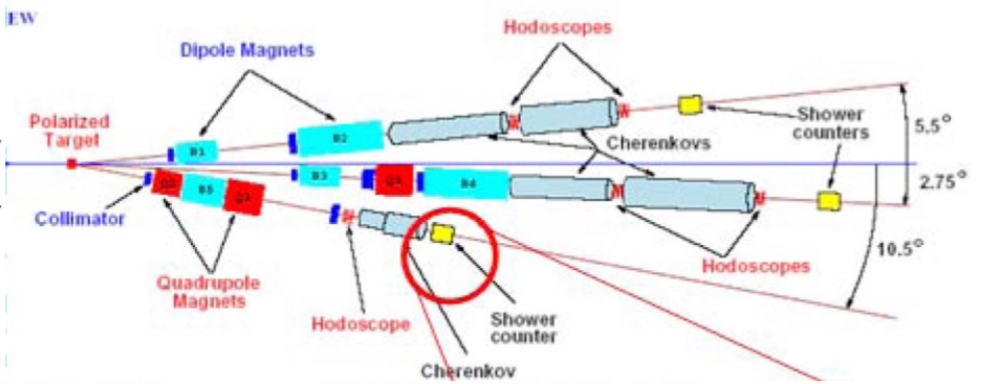


Pic : Abhay Deshpande

Experiments

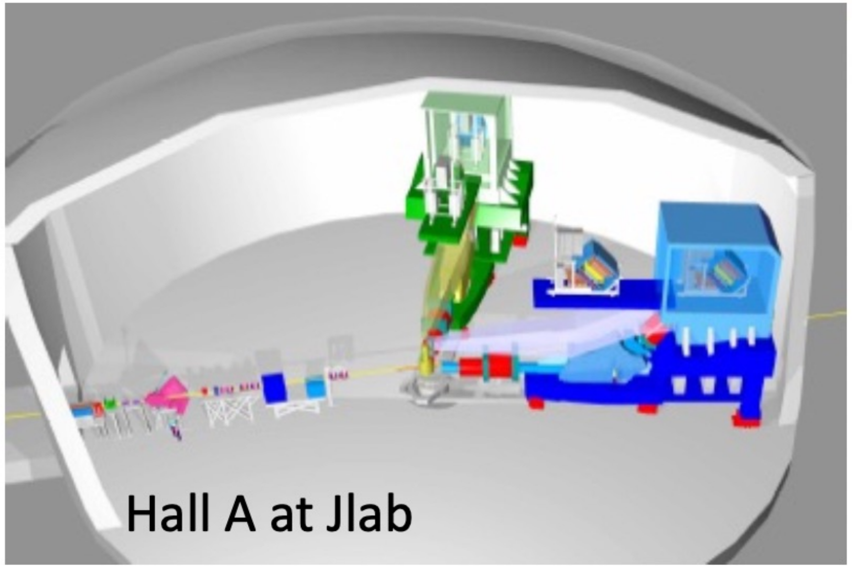


HERMES at DESY

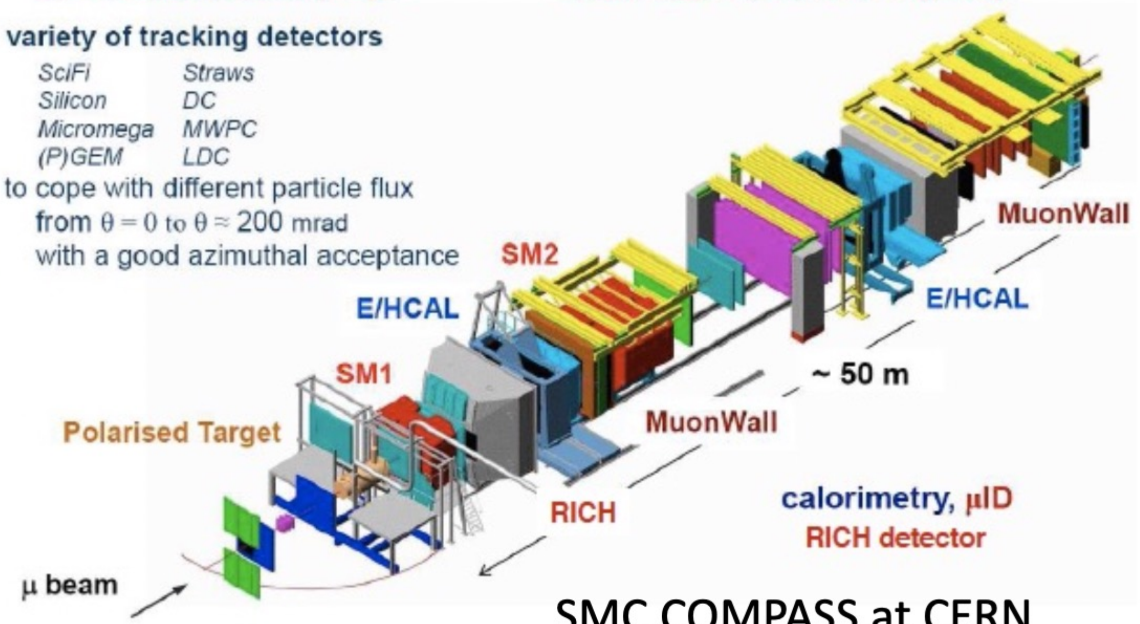


- high energy beams
- large angular acceptance
- broad kinematical range

two stages spectrometer
Large Angle Spectrometer (SM1)
Small Angle Spectrometer (SM2)



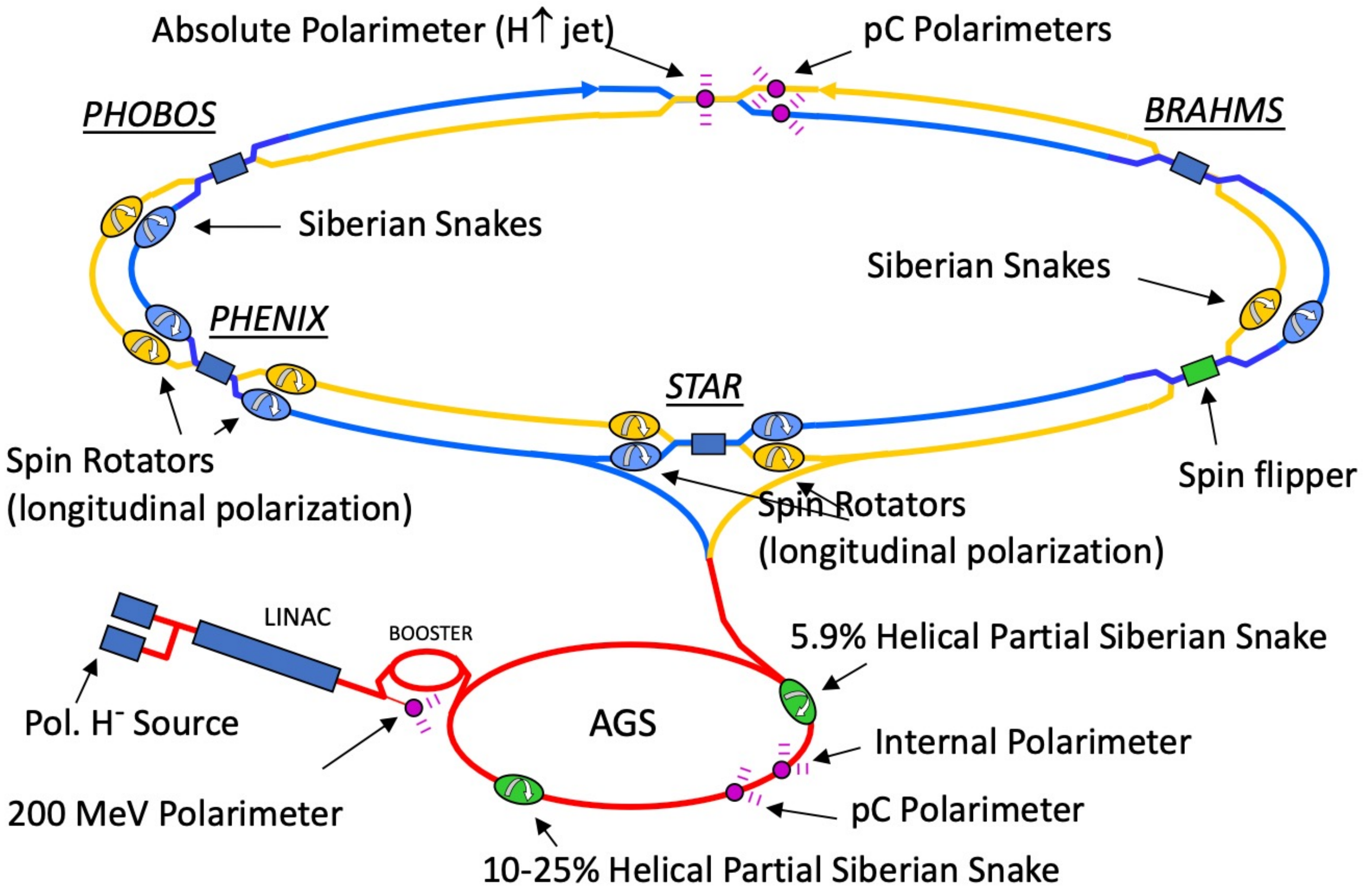
Hall A at Jlab



SMC,COMPASS at CERN

Slide : Abhay Deshpande

RHIC as a Polarized Proton Collider



Slide : Abhay Deshpande

Without Siberian snakes: $\nu_{sp} = G\gamma = 1.79\ E/m \rightarrow$ ~1000 depolarizing resonances
With Siberian snakes (local 180° spin rotators): $\nu_{sp} = ½ \rightarrow$ no first order resonances
Two partial Siberian snakes (11° and 27° spin rotators) in AGS

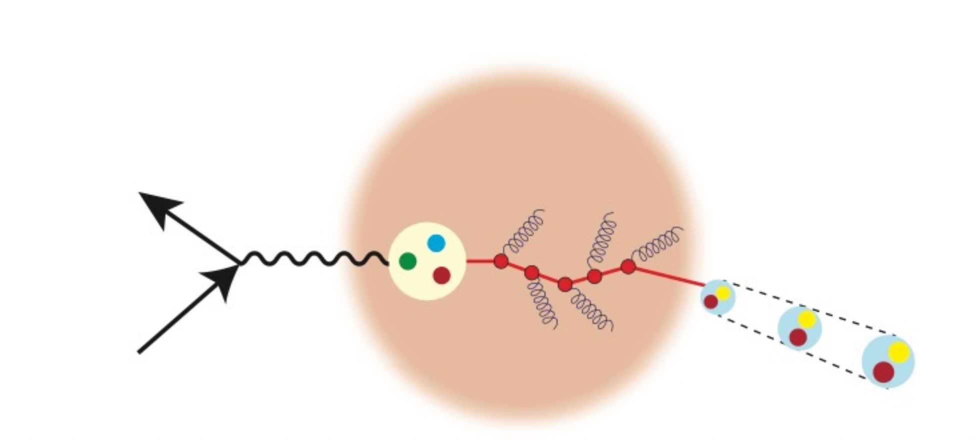
UPCOMING ELECTRON-ION COLLIDER (EIC)

The EIC to be built at Brookhaven National Lab, USA will collide highly energetic electron beam with proton/heavy ion to take 'snapshots' at high accuracy --tomography of the nucleon

How the quarks and gluons are distributed in space inside the nucleon

How do quarks and gluons bind together and for the nucleon ? What is the Origin of the mass of the nucleon ?

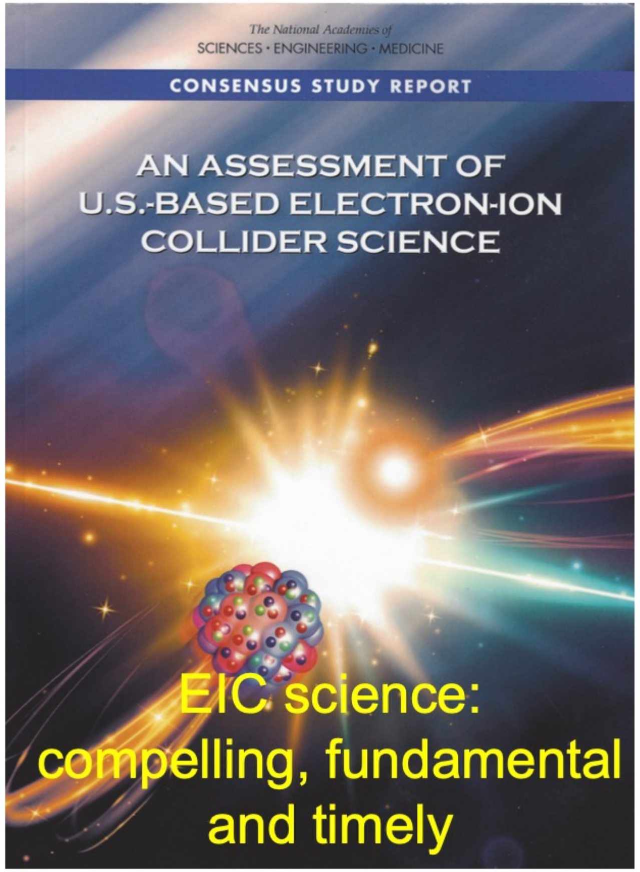
How the spin (1/2) of the proton is made from the spin and orbital angular momentum of the quarks and gluons



Will explore the correlations between spin/OAM and intrinsic transverse momentum

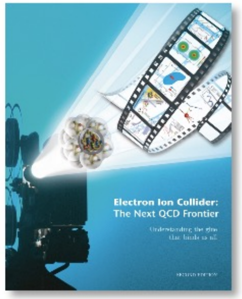
How does a dense nuclear environment affect quarks and gluons and their interactions ?

National Academy's Assessment

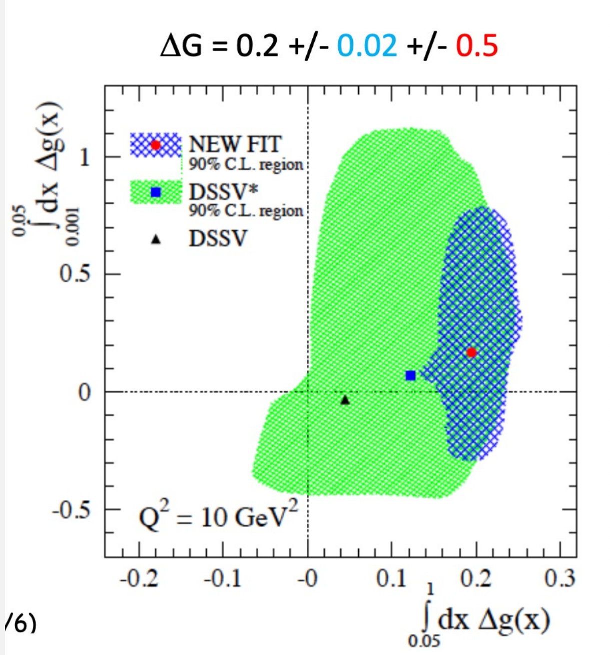


Machine Design Parameters:

- High luminosity: up to 10^{33}-10^{34} $cm^{-2}sec^{-1}$
  - a factor ~100-1000 times HERA
- Broad range in center-of-mass energy: ~20-100 GeV upgradable to 140 GeV
- Polarized beams e-, p, and light ion beams with flexible spin patterns/orientation
- Broad range in hadron species: protons…. Uranium
- Up to two detectors well-integrated detector(s) into the machine lattice



NUCLEON SPIN PUZZLE



D. deFlorian et al., arXiv:1404.4293

RHIC data shows significant contribution from gluon spin.

Several lattice calculations of quark and gluon angular momentum contributions

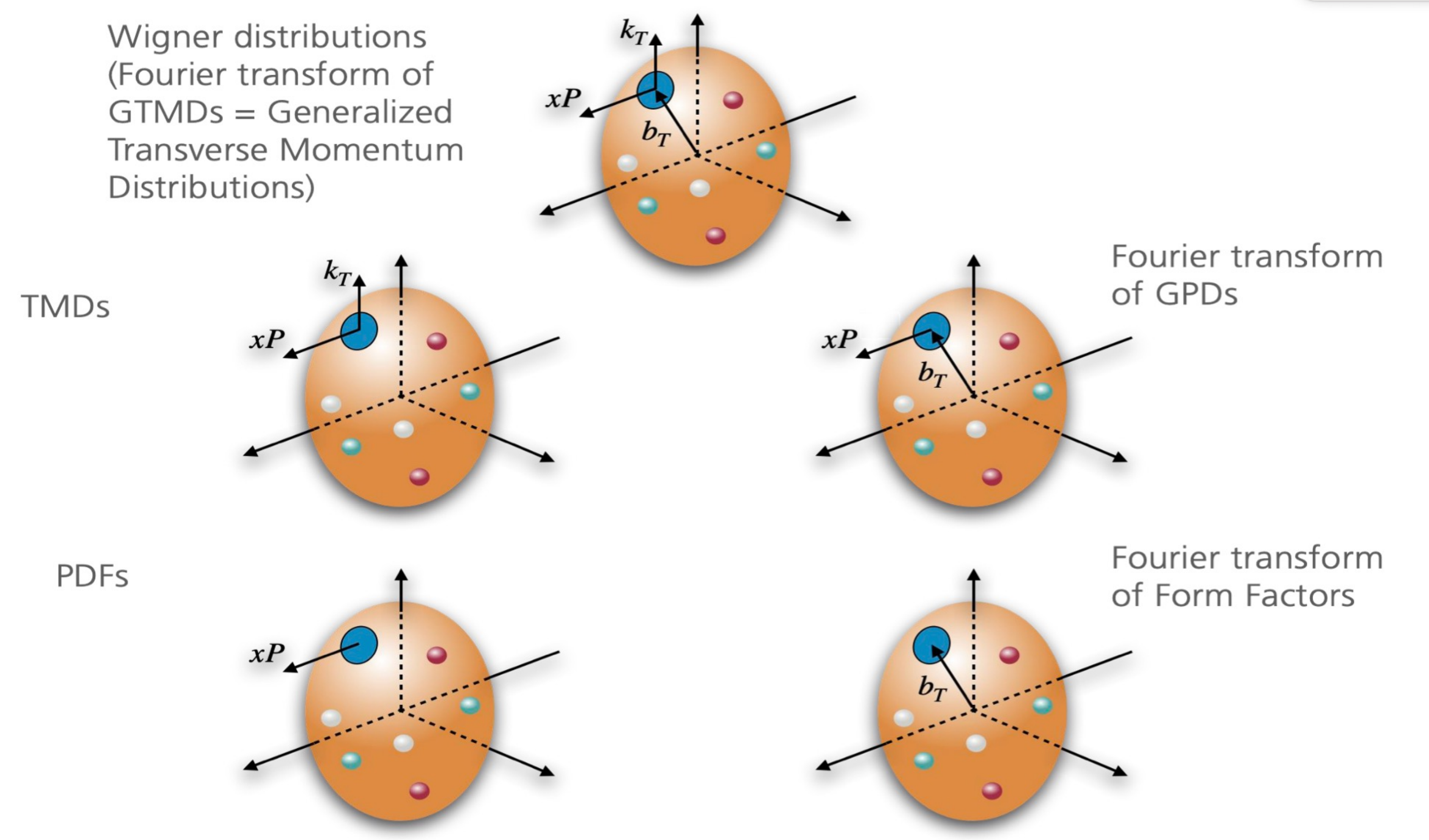
Total quark angular momentum contribution about 54-57 %, total gluon angular momentum about 38-46 %, quark OAM about 13-18 %

How to measure OAM of quarks and gluon experimentally ? Intrinsic transverse momentum ?

Quark and gluon OAM are related to most general Wigner distributions-several theoretical proposals to access them at the EIC, for example longitudinal double spin asymmetry in exclusive dijet production at EIC

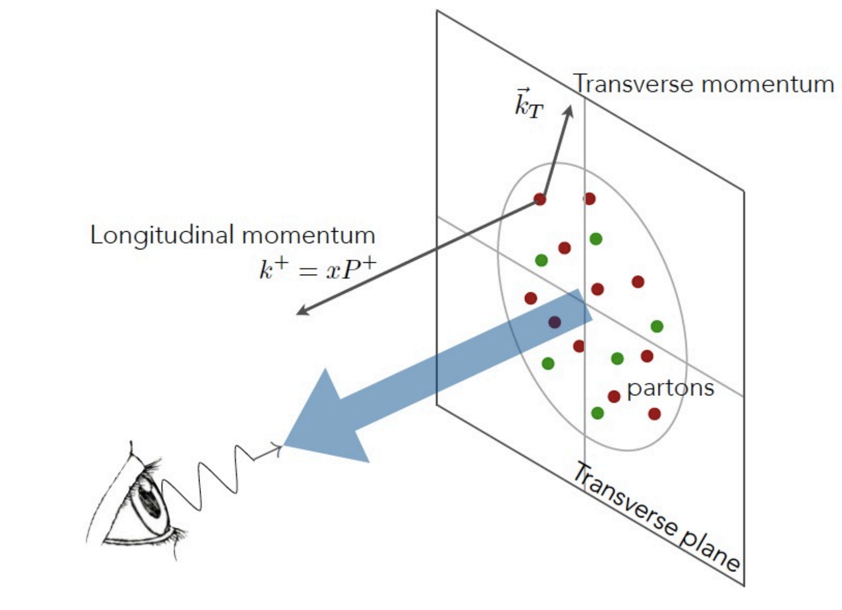
S. Bhattacharya, R. Boussarie, Y. Hatta ; 2404.04209 [hep-ph]

HADRON STRUCTURE IN THREE DIMENSIONS



Lorce, Pasquini, Vanderhaeghen, JHEP 1105 (11)

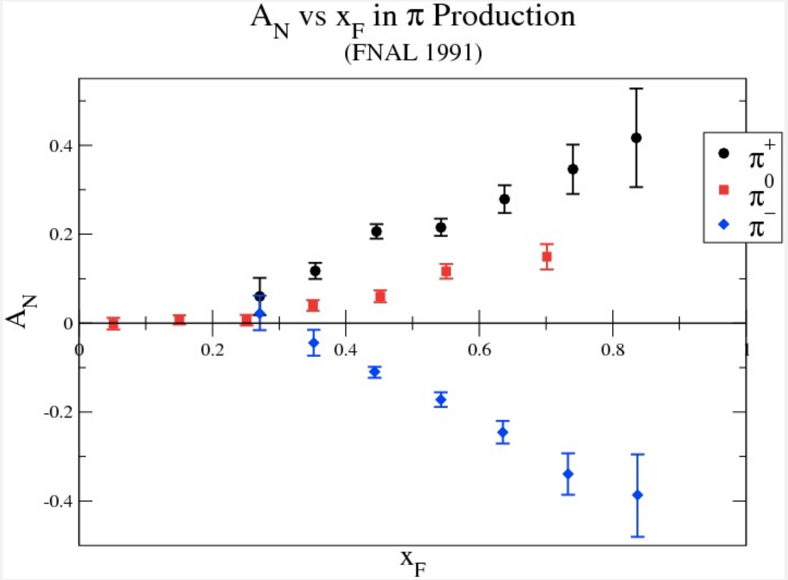
TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTIONS (TMDS)



Large (30-40%) Single transverse spin asymmetries were seen at FermiLab and RHIC experiments

Such large asymmetries cannot be explained in terms of collinear leading twist pdfs : need TMDs, or twist three pdfs

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

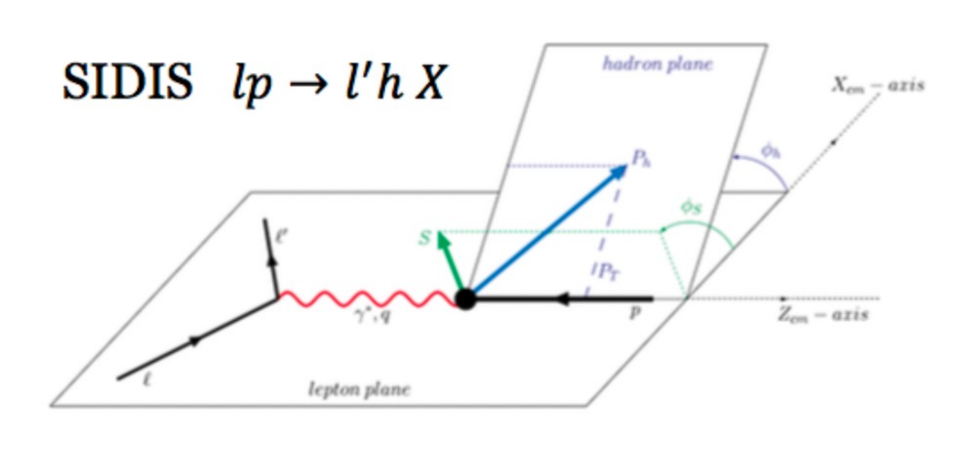


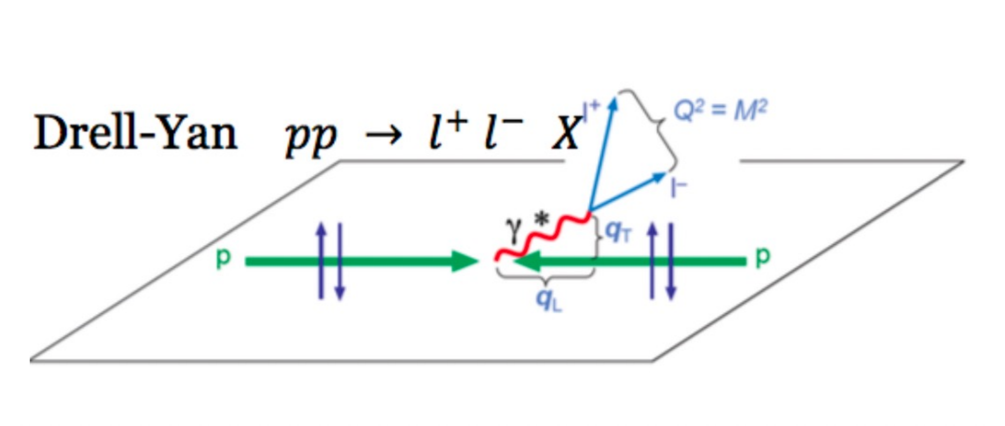
TMDs : functions of x and intrinsic transverse momentum : Gives a 3 D picture of the nucleon in momentum space

Correlations of spin, OAM and k_T : in terms of TMDs

TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

TMDs play a role in processes where two scales are present $Q^2 >> q_T^2$





For SIDIS and DY, TMD factorization is proven to all orders in α_s and leading twist

Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

For some processes, attempts have been made to prove TMD factorization at one loop and beyond leading twist

TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

$$\underbrace{}_{}\mathrm{d}\sigma^{\ell p\to\ell hX} = \sum_q \overset{\text{TMD-PDFs}}{f_q(x,\boldsymbol{k}_\perp;Q^2)} \otimes \overset{\text{hard scattering}}{\mathrm{d}\hat{\sigma}^{\ell q\to\ell q}(y,\boldsymbol{k}_\perp;Q^2)} \otimes \overset{\text{TMD-FFs}}{D_q^h(z,\boldsymbol{p}_\perp;Q^2)}$$

Fragmentation function for final hadron

TMDs play an important role in single spin and azimuthal asymmetries

Process dependent due to the gauge link or Wilson line in the operator

Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P,S \left| \overline{\psi}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,\psi(\xi) \right| P,S \right\rangle$$

$$\mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\mathrm{exp}\left(-ig\int_{\mathcal{C}[0,\xi]} \mathrm{d}s_\mu A^\mu(s)\right)$$

Φ : quark correlator, parametrized in terms of TMDs

Gauge link : resummation of initial and/or final state gluon exchanges : process dependent

QUARK TMDS

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T}, h_{1T}^{\perp}$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

There are eight quark TMDs at leading twist

Only three of them survive after transverse momentum integration

Two TMDs, Sivers function and Boer-Mulders function are odd under time reversal

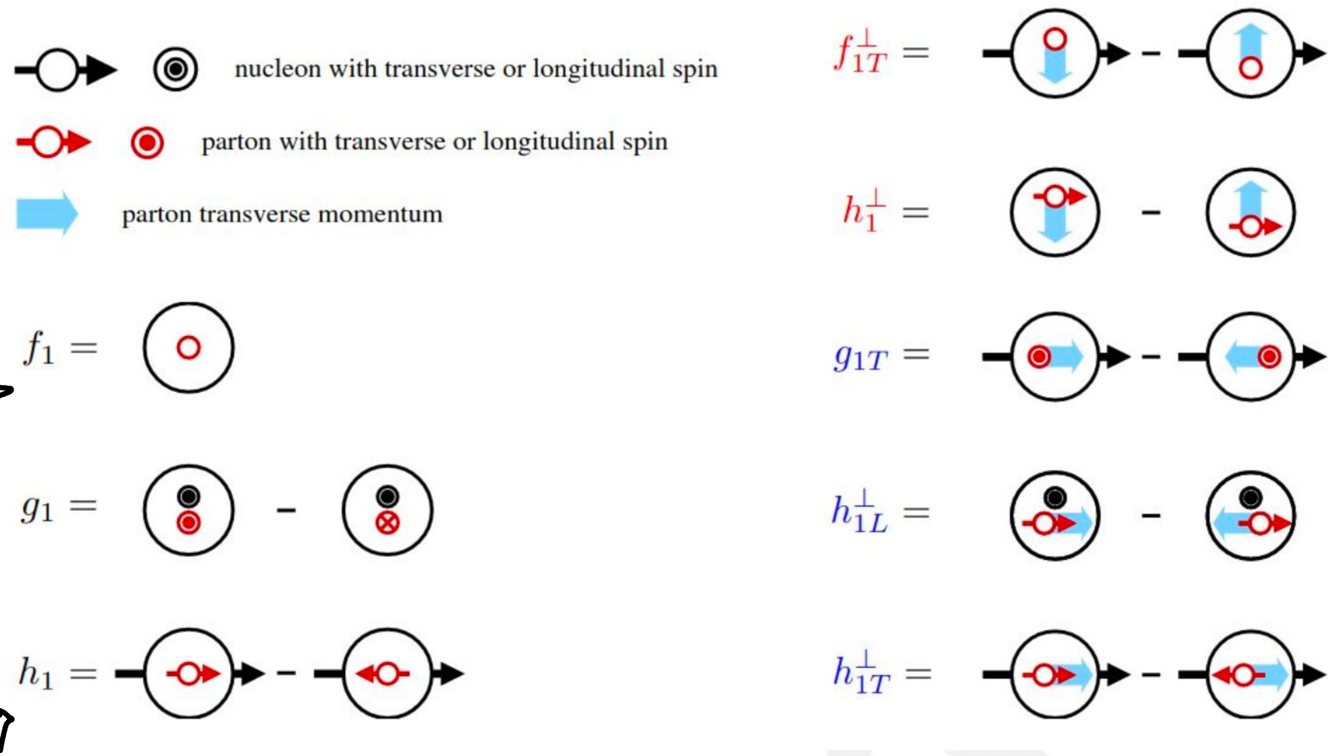
TMDs contribute in different azimuthal angle asymmetries

Extraction of unpolarized TMD as well as the Sivers function Upto N^3LL

Pavia 2017, JHEP 06 (2017)
Scimemi, Vladimirov, JHEP 06 (2020)
MAP Collaboration, JHEP (2022)

Bury, Prokudin, Vladimirov, PRL 126 (2021)
Echevarria, Kang, Terry, JHEP 01 (2021)
Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

QUARK TMDS FOR THE NUCLEON



Unpolarized

Helicity

Transversity

Sivers

Boer Mulders

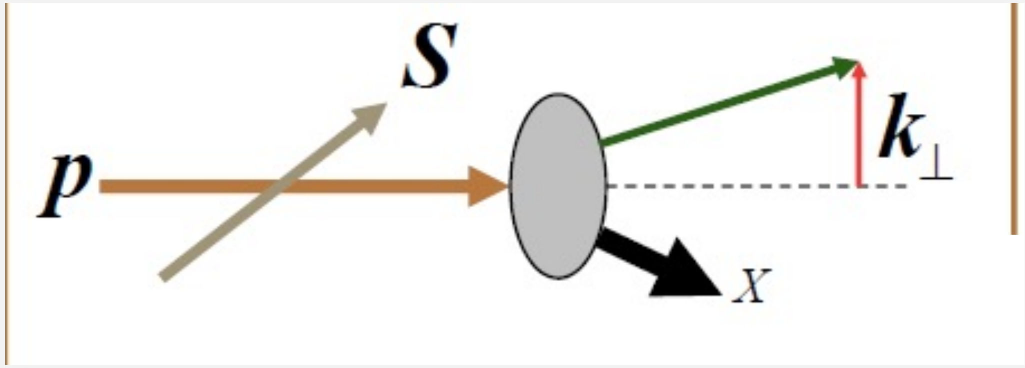
Worm-gear

Pretzelocity

Figure 3.5: Probabilistic interpretation of twist-2 transverse-momentum-dependent distribution functions. To avoid ambiguities, it is necessary to indicate the directions of quark's transverse momemtum, target spin and quark spin, and specify that the proton is moving out of the page, or alternatively the photon is moving into the page.

Lecture notes, A. Bacchetta

SIVERS FUNCTION



$$f_{q/p,S}(x,k_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

Sivers function TMD (D. Sivers; PRD (1990)) is related to the probability to find an unpolarized quark inside a transversely polarized nucleon

It includes the correlation between the quark intrinsic transverse momentum and the transverse spin of the proton

In some models it is related to the orbital angular momentum of the quarks

Meissner, Metz, Goeke, PRD 76 (2007), 034002

T-odd function ; depends on gauge link, or Wilson line

CROSS SECTION FOR SIDIS

Diff cross section for SIDIS with transversely polarized proton can be written as

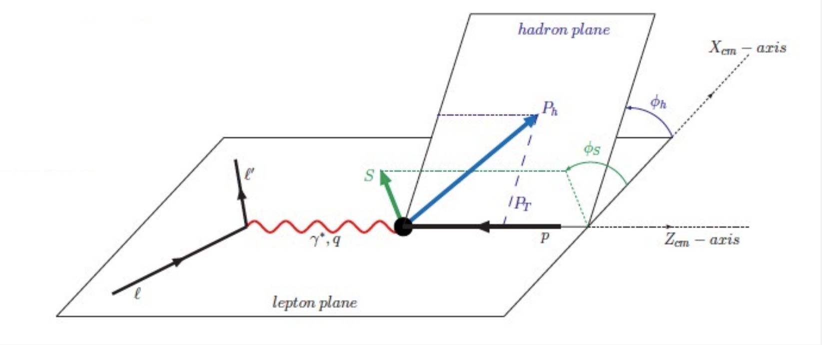
$$\frac{d\sigma^{\ell+p(S_T)\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = \frac{2\,\alpha^2}{Q^4} \times$$

$$\Bigg\{\frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+\Bigg[\frac{1+(1-y)^2}{2}\,\sin(\phi_h-\phi_S)\, F_{UT}^{\sin(\phi_h-\phi_S)} + (1-y)\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+\ (1-y)\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\ (2-y)\sqrt{1-y}\,\Big(\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Big)\Bigg]\Bigg\}$$



$$F_{UT}^{\sin(\phi-\phi_S)} \sim \sum e_a^2\, f_{1T}^{\perp a} \otimes D_1^a$$

F functions contain different TMDs : each come with a different azimuthal modulation

Sivers Function

GLUON TMDS

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g,\ h_{1T}^{\perp g}$

$h_1^{\perp\, g}$: Linearly polarized gluon distribution in unpolarized hadron; T even

$f_{1T}^{\perp g}$ Gluon Sivers function in Transversely polarized proton

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

$$h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2}\, h_{1T}^{\perp\, g}$$

Vanish under p_T integration

In contrast to quark TMDs, very little is known about gluon TMDs

$$\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P,S|\,\mathrm{Tr}_c\big[\,F^{+\nu}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}^{\mathcal{C}'}_{[\xi,0]}\,\big]\,|P,S\rangle$$

Gluon TMDs need two gauge links for gauge invariance

Mulders, Rodrigues, PRD 63 (2001)
Buffing, Mukherjee, Mulders, PRD 88 (2013)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

PROCESS DEPENDENCE OF TMDS

Gauge link is also present in collinear pdfs : but it is possible to choose a gauge (light-front gauge) where the gauge link becomes unity.

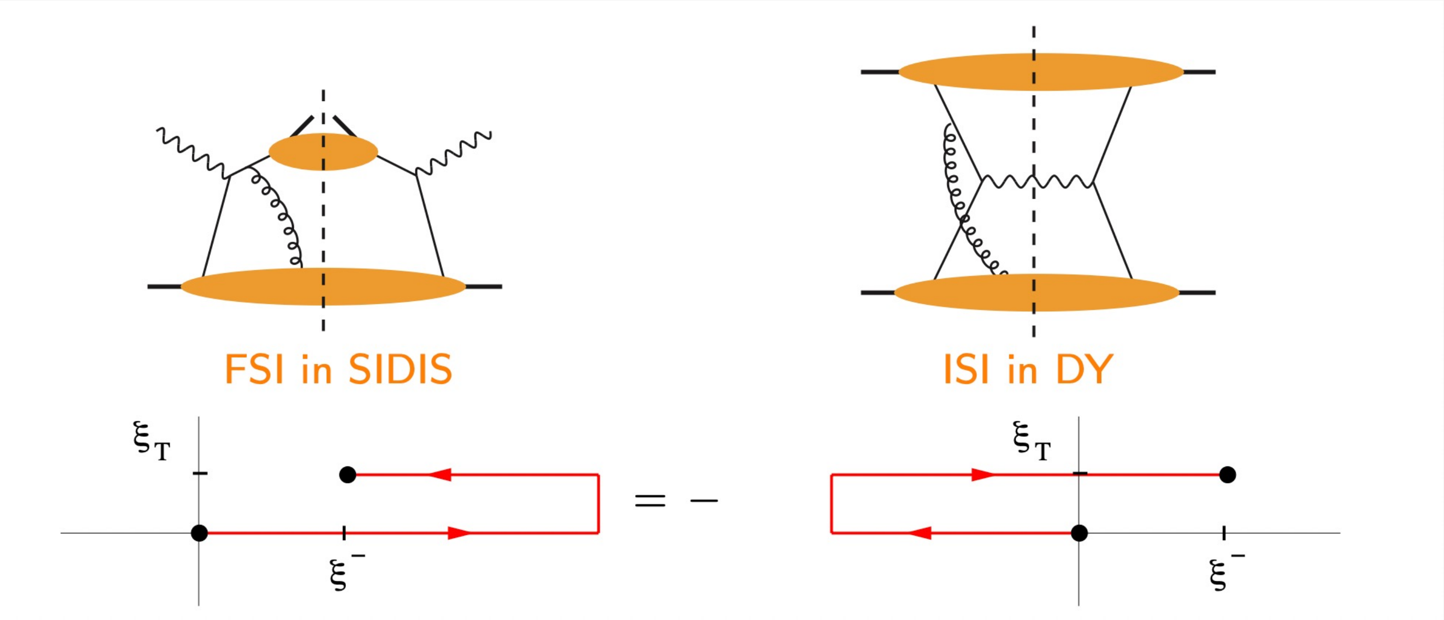
This is because the collinear pdf operator is bilocal only in the longitudinal direction but TMD operator is bilocal both in longitudinal and transverse direction

$\bar{\psi}(y^-)\Gamma\psi(0)$ $\quad$ $\bar{\psi}(y^-, y^\perp)\Gamma\psi(0)$ $\quad$ $y^- = y^0 - y^3$

For TMDs, even if one chooses the light cone gauge the effect of the gauge link remains and in fact plays a very important role for the T-odd TMDs like Sivers function.
Such TMDs would be zero if the gauge link is not taken into account

J. C. Collins, Phys. Lett. B 536 (2002) 43

SIVERS FUNCTION : PROCESS DEPENDENCE



Gauge link : depends on specific process . Example : SIDIS (final state interaction, future pointing gauge link) and Drell Yan (initial state interaction, past pointing gauge link)

Sivers function in Drell-Yan process is same in magnitude but opposite in sign compared to the Sivers function probed in semi-inclusive DIS

Collins, PLB (2002); Boer, Mulders, Pijlman, Nucl. Phy. B (2003)

$$f_{1T}^{\perp\,[DY]}(x, \boldsymbol{k}_\perp^2) = -f_{1T}^{\perp\,[SIDIS]}(x, \boldsymbol{k}_\perp^2)$$

Data from RHIC in favour of the sign change; EIC will play an important role to establish this.

LINEARLY POLARIZED GLUON DISTRIBUTIONS

Linearly polarized gluon distributions were first introduced in

Mulders and Rodrigues, PRD 63, 094021 (2001)

Operator structure of unintegrated gluon distributions can be different in different processes. In the literature, at small x, Weizsacker-Williams (WW) gluon distribution contains both past or both future pointing gauge links and dipole distributions contain one past and one future pointing gauge link. These are also called f and d type distributions, contribute in different processes

Extensive literature on unintegrated gluon distributions.

Linearly polarized gluon TMD : Measures an interference between an amplitude when the active gluon is polarized along x (or y) direction and a complex conjugate amplitude with the gluon polarized in y (or x) direction in an unpolarized hadron

Affects unpolarized cross section as well as generates a $\cos 2\phi$ asymmetry

GLUON SIVERS FUNCTION (GSF)

Distribution of quarks and gluons in a transversely polarized proton is not left-right symmetric with respect to the plane formed by the momentum and spin directions – this generates an asymmetry called Sivers effect

D. Sivers, PRD 41, 83 (1990)

Highly sensitive to the color flow of the process and on initial/final state interactions (T-odd)

In some models, the Sivers function TMD is related to the orbital angular momentum of the quarks

Very little is known about GSF apart from a positivity bound

Depending on the gauge link in the operator structure there can be two different gluon Sivers function, f-type and d-type

Bomhof and Mulders, JHEP 02, 029 (2007),
Buffing, AM, Mulders, PRD 88, 054027 (2013)

Burkardt's sum rule, which states that the total transverse momentum of all quarks and gluon in a transversely polarized proton is zero, still leaves some room for GSF (30 %), moreover d type GSF is not constrained by it.

M. Burkardt, Phys. Rev. D 69, 091501 (2004)

Back-to-back J/ψ-photon/jet/pion production processes in eP collision are effective ways to probe the gluon TMDs : expect TMD factorization. Only f-type gluon TMDs contribute

GLUON TMDS IN J/ψ PRODUCTION PROCESSES AT THE EIC

Semi-inclusive J/ψ production in eP collision is a good cannel to probe gluon TMDs

AM and Rajesh EPJC (2017)

For low transverse momentum region, TMD factorization is expected to hold and for large transverse momentum collinear factorization is applicable. In the intermediate region, results from these two formalisms should match

TMD factorized description of the process needs smearing effects to be taken into account in the form of TMD shape functions. The perturbative tail of the shape function can be obtained through a matching procedure.

M. G. Echevarria, JHEP (2019), Boer et al, JHEP (2023)

Also gluon TMDs can be probed in back-to-back production of J/ψ and photon/jet/pion, TMD factorization is expected to be valid. The small scale is provided by the transverse momentum of the pair. By varying the invariant mass of the pair scale evolution of the TMDs can be studied

So far the smearing effects and the shape functions are not calculated by matching procedure

PRODUCTION OF J/Ψ IN NRQCD

In NRQCD the heavy quark pair is produced in the hard process either in color octet or in color singlet configuration

Then they hadronize to form a color singlet quarkonium state of given quantum numbers through soft gluon emission

Hard process is calculated perturbatively and soft process is given in terms of long distance matrix elements (LDMEs) that are determined from data

The LDMEs are categorized by performing an expansion in terms of the relative velocity of the heavy quark v in the limit $v \ll 1$

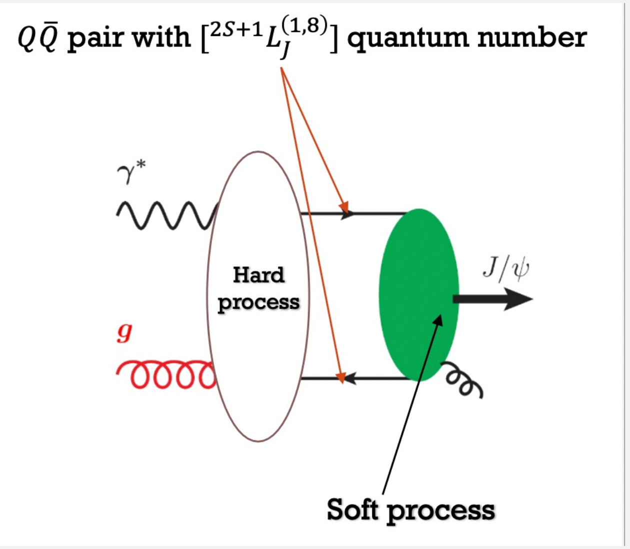
The theoretical predictions are arranged as double expansions in terms of v as well as α_s.

C. E. Carlson and R. Suaya, Phys. Rev. D 14, 3115 (1976).
E. L. Berger and D. L. Jones, Phys. Rev. D 23, 1521 (1981).
R. Baier and R. Ruckl, Phys. Lett. B 102B, 364 (1981).
R. Baier and R. Ruckl, Nucl. Phys. B201, 1 (1982).
E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995).
P. L. Cho and A. K. Leibovich, Phys. Rev. D 53, 150 (1996).

G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 51, 1125 (1995); 55, 5853(E) (1997).

PRODUCTION OF J/Ψ IN NRQCD

J/ψ is a bound state of charm quark and anti-quark ($Q\bar{Q}$)



Long distance matrix elements (LDMEs) : Describes hadronization of of $Q\bar{Q}[n]$ states into final quarkonium state

NRQCD factorization

$$d\sigma^{ab\to J/\psi} = \sum_n d\hat{\sigma}[ab \to c\bar{c}(n)]\langle 0 \mid \mathcal{O}_n^{J/\psi} \mid 0\rangle$$

Perturbative short distance coefficient

G.T. Bodwin et al, PRD51 (1995), Lepage 95

Subprocess cross section for formation of heavy quark pair in particular color, angular momentum and spin state "n" : ${}^{2S+1}L_J$, calculated by perturbative QCD

BACK-TO BACK PRODUCTION OF J/ψ AND JET

$$e^-(l) + p(\mathrm{P}) \to e^-(l') + J/\psi(\mathrm{P}_\psi) + \mathrm{jet}(\mathrm{P}_j) + X,$$

$$Q^2 = -q^2, \qquad s = (P + l)^2, \qquad W^2 = (P + q)^2,$$

$$x_B = \frac{Q^2}{2P \cdot q}, \qquad y = \frac{P \cdot q}{P \cdot l}, \qquad z = \frac{P \cdot \mathrm{P}_\psi}{P \cdot q}.$$

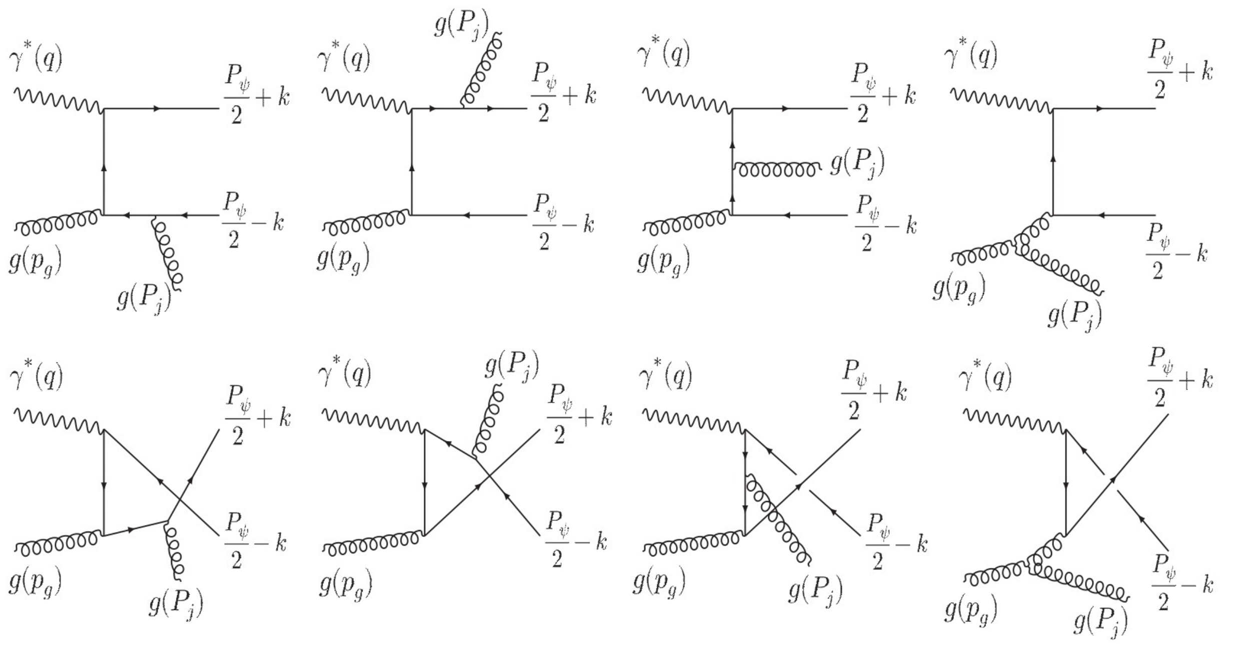


FIG. 1. Feynman diagrams for the partonic process $\gamma^*(q) + g(p_g) \to J/\psi(\mathrm{P}_\psi) + g(\mathrm{P}_j)$.

Use TMD factorization in the kinematics where the outgoing J/ψ and (gluon) jet are almost back-to back

Use NRQCD to calculate the J/ψ production

Also compare with the color singlet (CS) model result

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

CALCULATION OF AMPLITUDE USING NRQCD

The amplitude can be written as

D. Boer and C. Pisano (2012)

$$M(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}](P_\psi) + g)$$
$$= \sum_{L_z S_z} \int \frac{d^3k}{(2\pi)^3} \Psi_{LL_z}(k) \langle LL_z ; SS_z | JJ_z \rangle \mathrm{Tr}[\mathcal{O}(q,p,P_\psi,k) \mathcal{P}_{SS_z}(P_\psi,k)]$$

Amplitude for production of $Q\bar{Q}$ pair : $\quad \mathcal{O}(q,p,P_\psi,k) = \sum_{m=1}^{8} C_m \mathcal{O}_m(q,p,P_\psi,k)$

The spin projection operator, $\mathcal{P}_{SS_z}(P_\psi,k)$, projects the spin triplet and spin singlet states of $Q\bar{Q}$ pair

$$\mathcal{P}_{SS_z}(P_\psi,k) = \sum_{s_1 s_2} \left\langle \frac{1}{2} s_1 ; \frac{1}{2} s_2 \middle| SS_z \right\rangle v\left(\frac{P_\psi}{2} - k, s_1\right) \bar{u}\left(\frac{P_\psi}{2} + k, s_2\right)$$
$$= \frac{1}{4M_\psi^{3/2}} (-P_\psi + 2k + M_\psi) \Pi_{SS_z} (P_\psi + 2k + M_\psi) + O(k^2)$$

$\Pi_{SS_z} = \gamma^5$ for spin singlet ($S = 0$)

$\Pi_{SS_z} = \epsilon_{S_z}^{\mu}(P_\psi)\gamma_\mu$ for spin triplet ($S = 1$)

BACK-TO-BACK PRODUCTION OF J/ψ AND JET

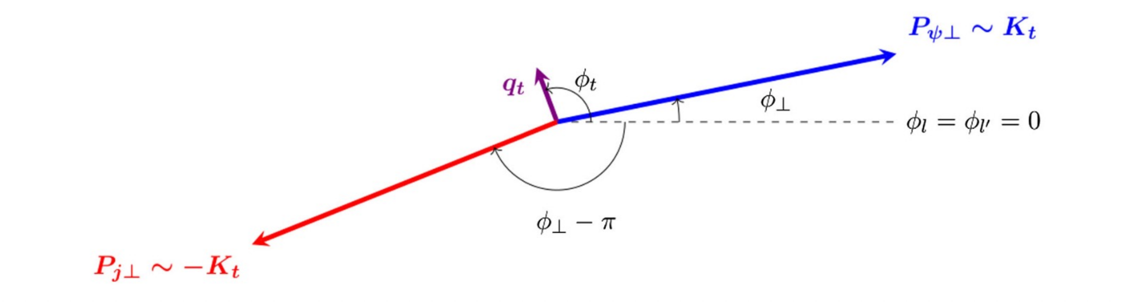
$$d\sigma = \frac{1}{2s}\frac{d^3 l'}{(2\pi)^3 2E_{l'}}\frac{d^3 P_\psi}{2E_\psi (2\pi)^3}\frac{d^3 P_j}{2E_j (2\pi)^3} \times \int dx d^2\mathbf{p}_T (2\pi)^4 \delta^4(q+p_g-P_j-P_\psi) \times \frac{1}{Q^4} L^{\mu\mu'}(l,q)\Phi_g^{\nu\nu'}(x,\mathbf{p}_T^2)\mathcal{M}_{\mu\nu}^{g\gamma^*\to J/\psi g}\mathcal{M}_{\mu'\nu'}^{*g\gamma^*\to J/\psi g}.$$

$$\mathcal{M}(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]g) = \sum_{L_z S_z}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\Psi_{LL_z}(\mathbf{k})\langle LL_z; SS_z|JJ_z\rangle \times \mathrm{Tr}[O(q,p_g,P_\psi,k)\mathcal{P}_{SS_z}(P_\psi,k)],$$

Contribution comes from the color singlet state $({}^3S_1^{(1)})$ And color octet states $({}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_{J(0,1,2)}^{(8)})$

In NRQCD, k, the relative momentum of the charm quark is small.
We have Taylor expanded the amplitude about k=0. The first term gives the S wave contribution and second term the p wave contribution

Formation of the bound state J/ψ from the heavy quark pair is encoded in the non-perturbative long distance matrix elements (LDMEs). These are obtained by fitting data



Upper bound of the asymmetries : U. D'Alesio, F. Murgia, C. Pisano, and P. Taels

ASYMMETRY

$$\mathbf{q}_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}, \qquad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}.$$

$$|\mathbf{q}_t| \ll |\mathbf{K}_t|.$$

$$\frac{d\sigma}{dz dy dx_B d^2\mathbf{q}_t d^2\mathbf{K}_t} = \frac{1}{(2\pi)^4}\frac{1}{16 s z(1-z)Q^4}\Big\{(\mathbb{A}_0 + \mathbb{A}_1\cos\phi_\perp + \mathbb{A}_2\cos 2\phi_\perp) f_1^g(x,\mathbf{q}_t^2)$$
$$+ \frac{\mathbf{q}_t^2}{M_P^2} h_1^{\perp g}(x,\mathbf{q}_t^2)(\mathbb{B}_0\cos 2\phi_t + \mathbb{B}_1\cos(2\phi_t - \phi_\perp) + \mathbb{B}_2\cos 2(\phi_t - \phi_\perp)$$
$$+ \mathbb{B}_3\cos(2\phi_t - 3\phi_\perp) + \mathbb{B}_4\cos(2\phi_t - 4\phi_\perp))\Big\}.$$

$$\langle\cos 2\phi_t\rangle \equiv A^{\cos 2\phi_t} = \frac{\int q_t dq_t \frac{\mathbf{q}_t^2}{M_p^2}\mathbb{B}_0 h_1^{\perp g}(x,\mathbf{q}_t^2)}{\int q_t dq_t \mathbb{A}_0 f_1^g(x,\mathbf{q}_t^2)}.$$

Gaussian parametrization of TMDs :

$$f_1^g(x,\mathbf{q}_t^2) = f_1^g(x,\mu)\frac{1}{\pi\langle\mathbf{q}_t^2\rangle}e^{-\mathbf{q}_t^2/\langle\mathbf{q}_t^2\rangle},$$

$$h_1^{\perp g}(x,\mathbf{q}_t^2) = \frac{M_p^2 f_1^g(x,\mu)}{\pi\langle\mathbf{q}_t^2\rangle^2}\frac{2(1-r)}{r}e^{1-\frac{\mathbf{q}_t^2}{r\langle\mathbf{q}_t^2\rangle}},$$

Boer and Pisano, PRD, 2012

$$\langle\mathbf{q}_t^2\rangle = 0.25\ \mathrm{GeV}^2. \qquad r=1/3$$

Spectator model :

$$F^g(x,\mathbf{q}_t^2) = \int_M^\infty dM_X \rho_X(M_X)\hat{F}^g(x,\mathbf{q}_t^2;M_X).$$

Spectral function

$$\rho_X(M_X) = \mu^{2a}\left[\frac{A}{B+\mu^{2b}} + \frac{C}{\pi\sigma}e^{-\frac{(M_X-D)^2}{\sigma^2}}\right],$$

M_X : mass of spectator : continuous

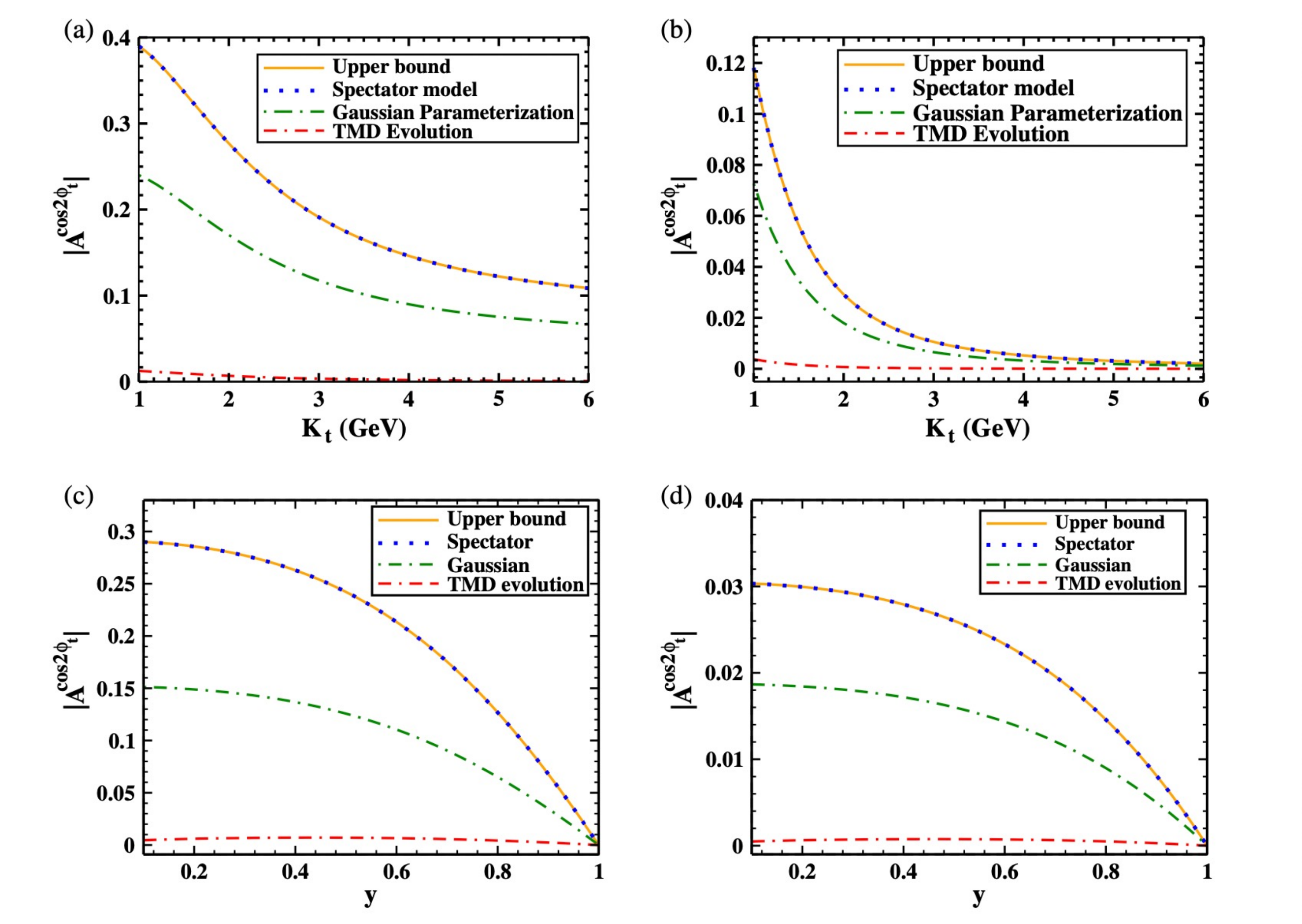
$$L_X^2(\Lambda_X^2) = xM_X^2 + (1-x)\Lambda_X^2 - x(1-x)M^2.$$

$$\hat{f}_1^g(x,\mathbf{q}_t^2;M_X) = -\frac{1}{2}g^{ij}[\Phi^{ij}(x,\mathbf{q}_t,S) + \Phi^{ij}(x,\mathbf{q}_t,-S)]$$
$$= [(2Mxg_1 - x(M+M_X)g_2)^2[(M_X - M(1-x))^2 + \mathbf{q}_t^2]$$
$$+ 2\mathbf{q}_t^2(\mathbf{q}_t^2 + xM_X^2)g_2^2 + 2\mathbf{q}_t^2M^2(1-x)(4g_1^2 - xg_2^2)][(2\pi)^3 4xM^2(L_X^2(0) + \mathbf{q}_t^2)^2]^{-1},$$

$$\hat{h}_1^{\perp g}(x,\mathbf{q}_t^2;M_X) = \frac{M^2}{\varepsilon_t^{ij}\delta^{jm}(p_t^j p_t^m + g^{jm}\mathbf{q}_t^2)}\varepsilon_t^{ln}\delta^{nr}[\Phi^{nr}(x,\mathbf{q}_t,S) + \Phi^{nr}(x,\mathbf{q}_t,-S)]$$
$$= [4M^2(1-x)g_1^2 + (L_X^2(0) + \mathbf{q}_t^2)g_2^2] \times [(2\pi)^3 x(L_X^2(0) + \mathbf{q}_t^2)^2]^{-1}.$$

A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taels, Eur. Phys. J. C 80, 733 (2020).

UPPER BOUND OF THE ASYMMETRY COMPARED WITH DIFFERENT RESULTS



NRQCD

CS

$y = 0.3$ In upper panels $\sqrt{s} = 140\ GeV$

$K_t = 0.2\ GeV$ In lower panels

Result in spectator model in the kinematics considered overlaps with the upper bound saturating the positivity bound

Result is Gaussian parametrization lower than in spectator model

Asymmetries in CS smaller than in NRQCD

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

SUMMARY AND OUTLOOK

The upcoming EIC at BNL will pay a very important role in understanding the nucleon in three dimensions in terms of quarks and gluons and their interaction

The sign change of Sivers function-once confirmed by experiment- will validate fundamental theory aspects like TMD factorization

EIC will have the potential to measure the still unknown orbital angular momentum of quarks and gluons-this is crucial to understand the spin sum rule of the nucleon.

EIC will be able to access in particular the less known gluon TMDs through single spin and azimuthal asymmetries

As an example J/ψ production processes are promising channels to probe the gluon TMDs, for example the linearly polarized gluon distribution and gluon Sivers function : extensive theoretical work in recent years

EIC will also explore the Winger distributions through GTMDs- most generalized quark-gluon description of the nucleon

Exciting years ahead !