EIC Physics: Recent Theoretical Advances from the BLFQ Collaboration

Xingbo Zhao
With
Chandan Mondal, Siqi Xu, Yang Li, Chenbo Tao,
Yumeng Han, James P. Vary
Institute of Modern Physics, Chinese Academy of Science

10th Asian Triangle Heavy-Ion Conference (ATHIC 2025)
Indian Institute of Science Education and Research Berhampur,
Berhampur, Odisha, India, 14/01/2025

Outline

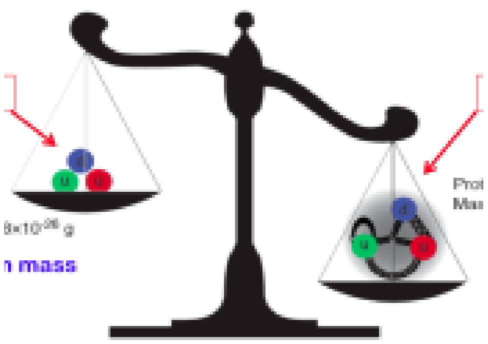
- EIC physics and Basis Light-front Quantization (BLFQ)

- Recent progress in BLFQ
  - Input Hamiltonian
  - Observables
  - High-performance computing

- Conclusion and Outlook

Major Questions in EIC Physics

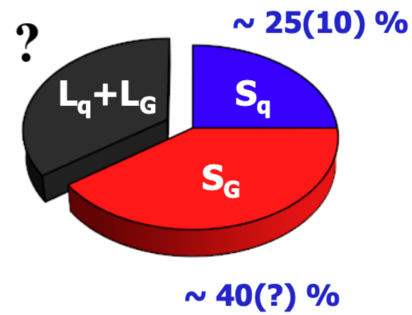
Origin of mass



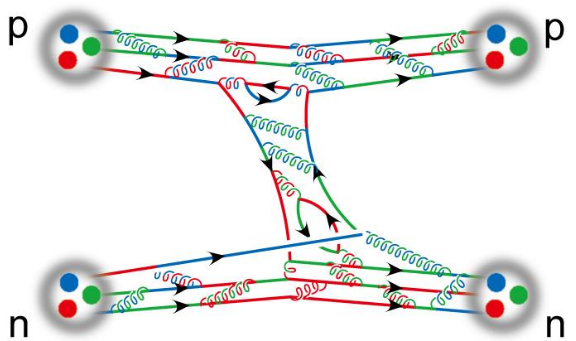
Spin puzzle

Ortibal angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

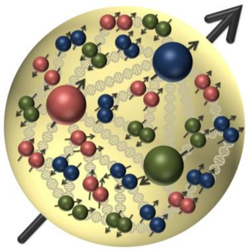


Nuclear force



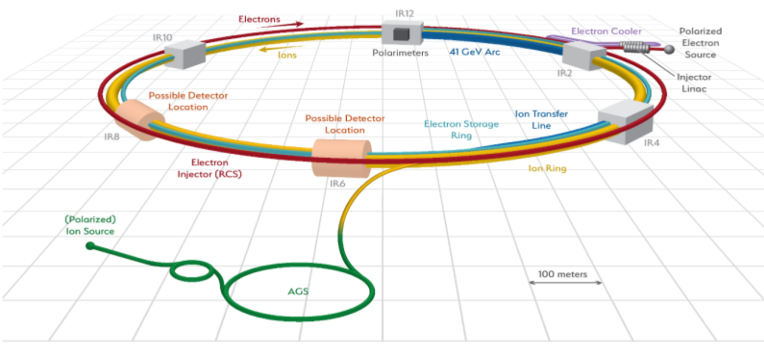
We need to know 3D tomography of nucleon and how it emerges from QCD ?

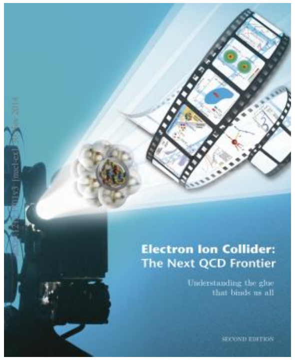
$$\mathcal{L}_{QCD} = \left(\bar{\psi}_q (i\not{D} - m_q)\psi_q \right) - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha} \xrightarrow{?}$$

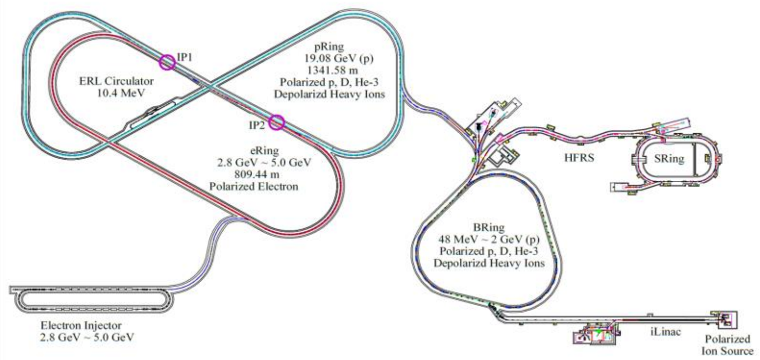


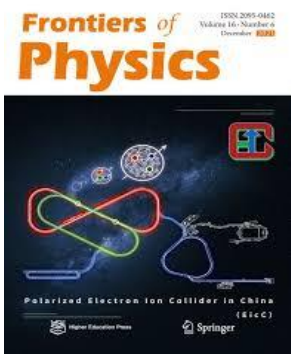
Upcoming Electron-Ion Colliders

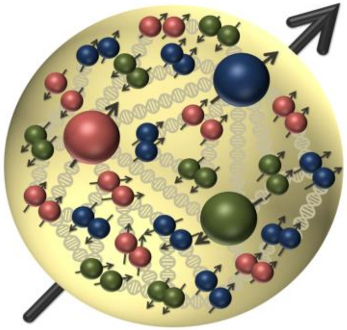
- Electron-Ion colliders with large collision energy and high luminosity











- EIC in the US is under construction by BNL@New York
- EicC in China is planned by IMPCAS@Huizhou

Complimentarity

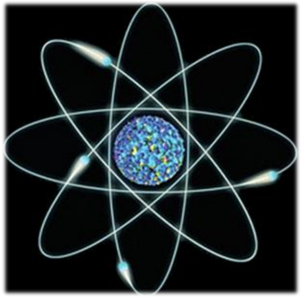
Nonperturbative Approach

- Stationary Schrödinger equation universally describes bound-state structure

$$H|\psi\rangle = E|\psi\rangle$$

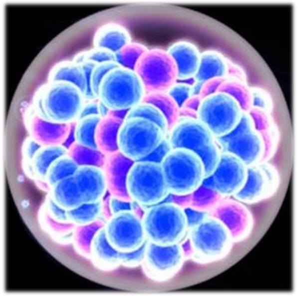
- Eigenstates $|\psi\rangle$ encode full information of the system

Nonrelativistic



atom

Nonrelativistic



nucleus

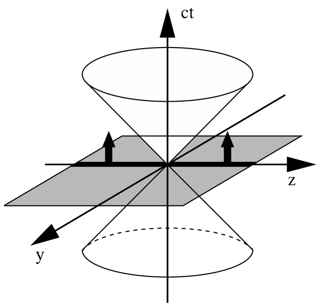
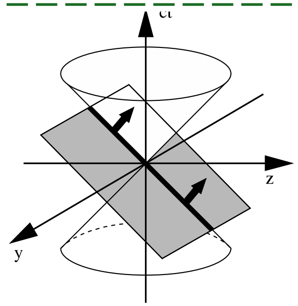
Relativistic

nucleon

- Major challenges from relativity: retardation effects

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$\tau \equiv x^0$	$\tau \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\psi(t)\rangle = H\lvert\psi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\psi(x^+)\rangle = \frac{1}{2}P^-\lvert\psi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Main advantage:

- Frame-independent wave functions
- Minkowski spacetime

$$\Phi^{[\gamma^+]}(x, Q^2) \sim \langle P', \Lambda | \bar{\psi}(x)\gamma^+\psi(0) | P, \Lambda \rangle \Big|_{x^+ = x^\perp = 0}$$

- No square roots in dispersion relation

Basis Light-Front Quantization

- Hamiltonian eigenvalue equation: [Vary, et.al, 2010]

$$P^-|N\rangle = P_N^-|N\rangle$$

- $\boldsymbol{P^-}$: Light-Front Hamiltonian
- $|\boldsymbol{N}\rangle$: Eigenstates
- $\boldsymbol{P_N^-}$: Eigenvalues for eigenstates

- Basis setup:
  - Fock sector expansion: $|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$
  - single particle basis:

$$|qqq\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$$

2-d harmonic oscillator (2DHO): $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$

Discretized longitudinal momentum: $\sum_i k_i^+ = K_{\max}$

Helicity and color: $\mathrm{m}_J = \sum_i (\lambda_i + m_i)$

- Advantages for 2D HO:
  - rotational symmetry in transverse plane
  - center-of-mass motion is factorizable

Dimension of Basis Space

- Expansion in BLFQ basis

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + |qqq\,gg\rangle + |qqq\,ggg\rangle + |qqq\,q\bar{q}\,g\rangle$$

$N_{max} = 7, K_{max} = 16$

	$\|qqq\rangle$	$\|qqqg\rangle$	$\|qqq\,q\bar{q}\rangle$	$\|qqq\,gg\rangle$	$\|qqq\,ggg\rangle$	$\|qqq\,q\bar{q}\,g\rangle$
dimension	35,088	592,960	3,901,500	5,169,360	19,603,584	7,128,576
color config	1	2	3	6	22	8

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,u\bar{u}\rangle + |qqq\,d\bar{d}\rangle + |qqq\,s\bar{s}\rangle$$

Basis Dimension= 12,332,548

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,u\bar{u}\rangle + |qqq\,d\bar{d}\rangle + |qqq\,s\bar{s}\rangle + |qqq\,gg\rangle$$

Basis Dimension= 17,501,908

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + |qqq\,gg\rangle + |qqq\,ggg\rangle$$

Basis Dimension= 37,105,492

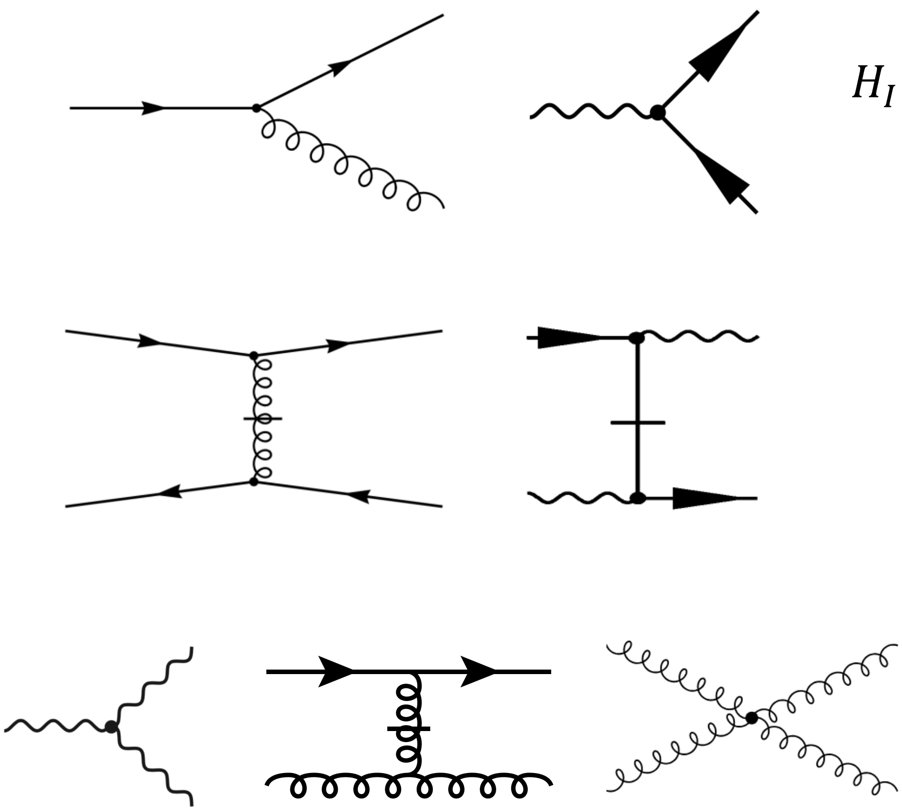
$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + |qqq\,gg\rangle + |qqq\,ggg\rangle + |qqq\,q\bar{q}\,g\rangle$$

Basis Dimension= 58,491,220

QCD Light-front Hamiltonian

- QCD light-front Hamiltonian from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} \;\Longrightarrow\; P^{-}_{QCD} = H_K + H_I \qquad A^+ = 0$$



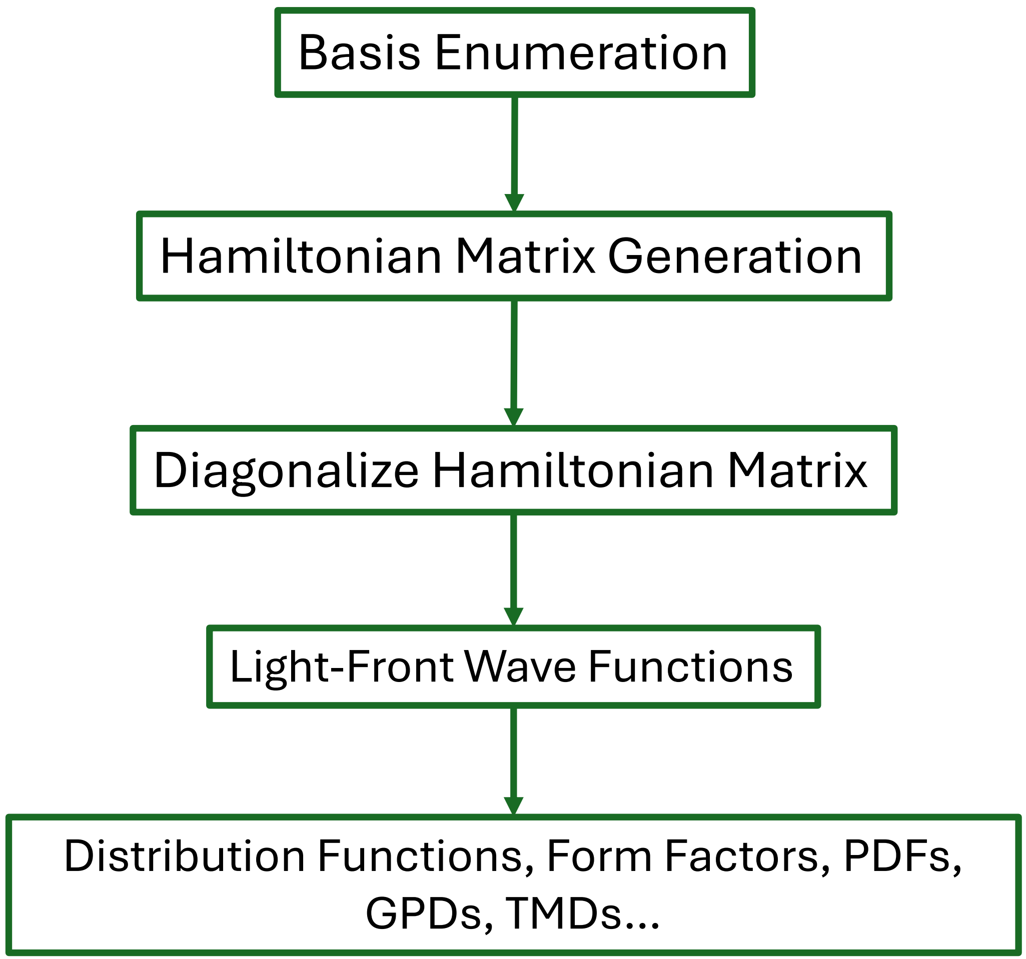
$$H_K = \tfrac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\psi - \tfrac{1}{2}\int \mathrm{d}^3x\, A^i_a (\mathrm{i}\partial^\perp)^2 A^i{}_a$$

$$\begin{aligned}
H_I = &+ g\int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \psi \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{\mathrm{i}\partial^+}\gamma_\nu A^\nu \psi \\
&- \mathrm{i}g^2 \int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^+ T^c \psi \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\mu_a A_{\mu b}\right) \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma^+ T^a \psi \frac{1}{(\mathrm{i}\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \\
&+ \mathrm{i}g \int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\
&- \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade}\mathrm{i}\partial^+ A^\mu_b A_{\mu c} \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\nu_d A_{\nu e}\right) \\
&+ \tfrac{1}{4}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade} A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.
\end{aligned}$$

ψ: quark field operator
A^a_μ: gluon field operator

7 terms in H_I

BLFQ Algorithm Flowchart



Progress toward First Principles

$$|\mathrm{N}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, u\bar{u}\rangle + |qqq\, d\bar{d}\rangle + |qqq\, s\bar{s}\rangle + \cdots$$

- **Wave Functions:**

  [PRD,102,016008] (2019) **[PRD,108 9, 094002] (2023)** **[arXiv:2408.11298] (2024)**

- **GPDs:**

  [PRD,104,094036] (2021) **[PLB,847,138305] (2023)**

  [PRD,105,094018] (2022) **[PRD,110.056027] (2024)**

  [PRD,109,014015] (2024) **[PLB,860,139153] (2025)**

  [PLB,855,138809] (2024)

- **TMDs:**

  [PLB,833,137360] (2022) **[PLB,855 138831] (2024)**

  [PRD,108,036009] (2023)

- **Higher-twist Distribution (GPD,TMD,DPD):**

  [PRD,109,034031] (2024) **[PLB,855 138829] (2024)**

  [arXiv:2410.11574] (2024)

- **Gravitational Form Factors:**

  [PRD,110,056027] (2024)

Progress toward First Principles

$$|\text{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}u\bar{u}\rangle + |q\bar{q}d\bar{d}\rangle + |q\bar{q}s\bar{s}\rangle + \cdots$$

- **Wave Functions:**

  PLB,758,118-124(2016) PLB,825,136890(2022) [xxxx.xxxxx](2025)
  PRD,96,016022(2017)
  PRC,99,035206 (2019)

- **PDFs:**

  PRL,122,172001(2019) 2406.18878 [hep-ph]
  PRD,101,034024(2019)
  PRD,102,014020(2020)

- **GPDs/TMDs:**

  PRD,104,11401(2021) PLB,851,138563(2024)

- **Transition FFs:**

  PRD,104,094034(2021) 2408.06870 [hep-ph]

- **Higher-twist Distribution (GPD,TMD,DPD):**

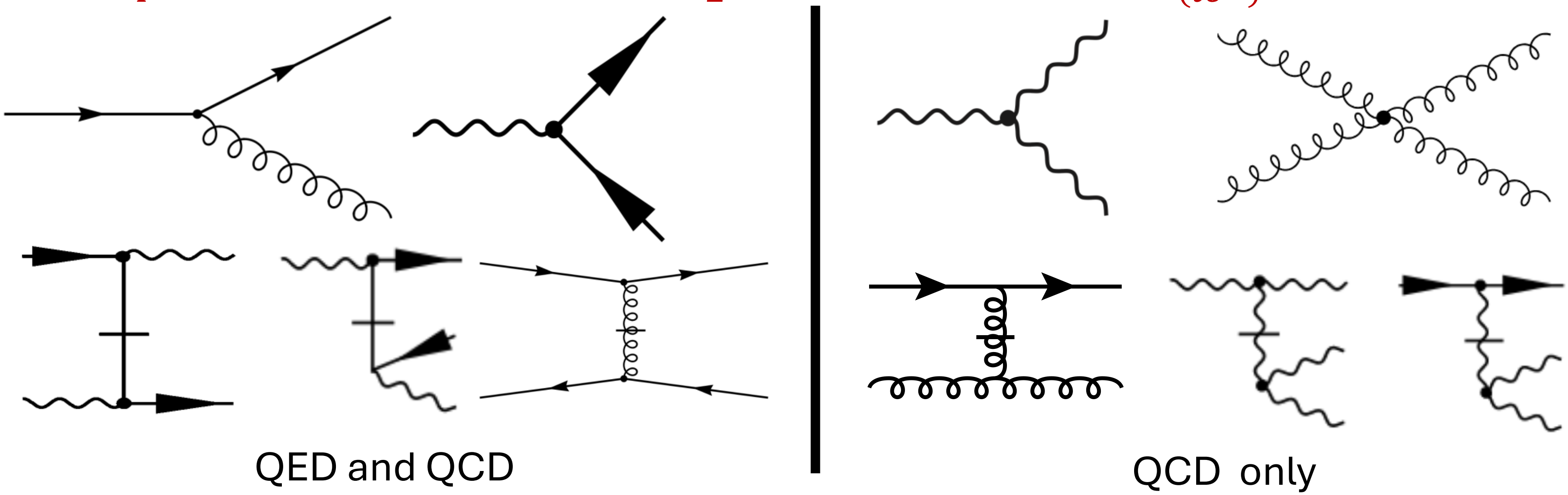
  PLB,839,137808(2023)

Proton with QCD Hamiltonian

$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle$$

$$P^- = H_K + H_I \qquad H_K = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_I = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+ + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+} A_\nu \gamma^\nu \psi$$
$$-g^2 C_F \bar{\psi}\gamma^+\psi \frac{1}{(i\partial^+)^2} i\partial^+ A_\mu^a A_b^\mu + igf^{abc} i\partial^\mu A_a^\nu A_\mu^b A_\nu^c$$
$$+\frac{1}{4}g^2 f^{abc} f^{ade} A_b^\mu A_c^\nu A_{\mu d} A_{\nu e} - \frac{1}{2}g^2 f^{abc} f^{ade} i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2}(i\partial^+ A_d^\nu A_{\nu e})$$



Fock Sector Decomposition

$$|N\rangle \to |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

$|qqq\ q\bar{q}g\rangle \sim$ 8 color singlet state

3 singlet ⊗ octet ⊗ octet

4 octet ⊗ octet ⊗ octet

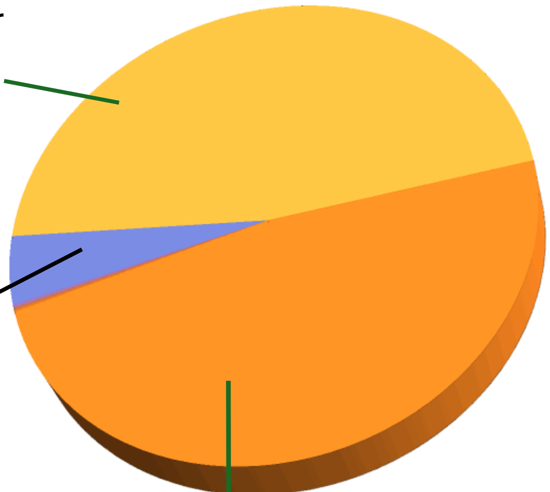
1 decuplet ⊗ octet ⊗ octet

$|qqq\ ggg\rangle \sim$ 22 color singlet state

2 singlet ⊗ singlet

16 octet ⊗ octet

4 decuplet ⊗ octet ⊗ octet ⊗ octet

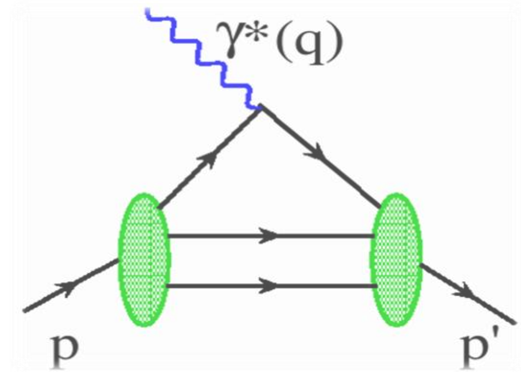


m_u	m_d	m_s	m_f	g	b	b_{inst}
0.5 GeV	0.40 GeV	0.6 GeV	2.2 GeV	2.5	0.6 GeV	3.0 GeV

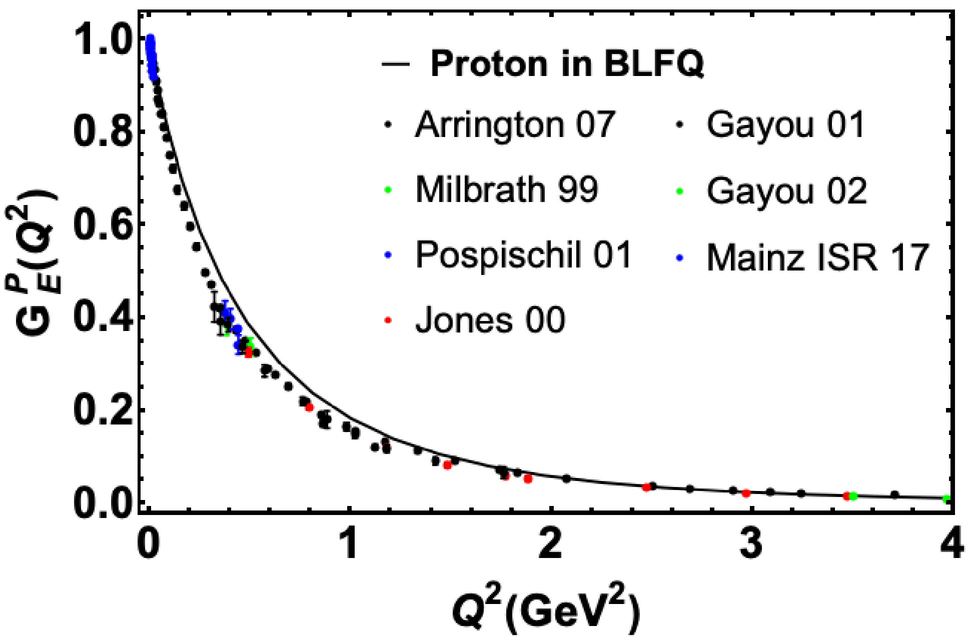
Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 10$

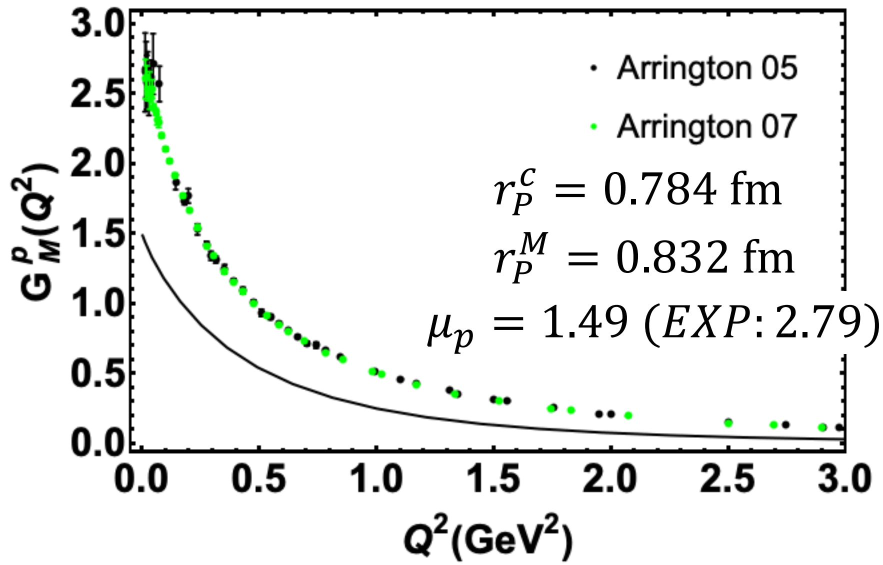
Nucleon Form Factors

$$\langle N(p')|J^{\mu}(0)|N(p)\rangle =$$
$$\bar{u}(p')\left[\gamma^{\mu}\underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N}q_{\nu}\underbrace{F_2(q^2)}\right]u(p)$$



Preliminary





- BLFQ results qualitatively agree with the experimental data for Dirac and Pauli FFs

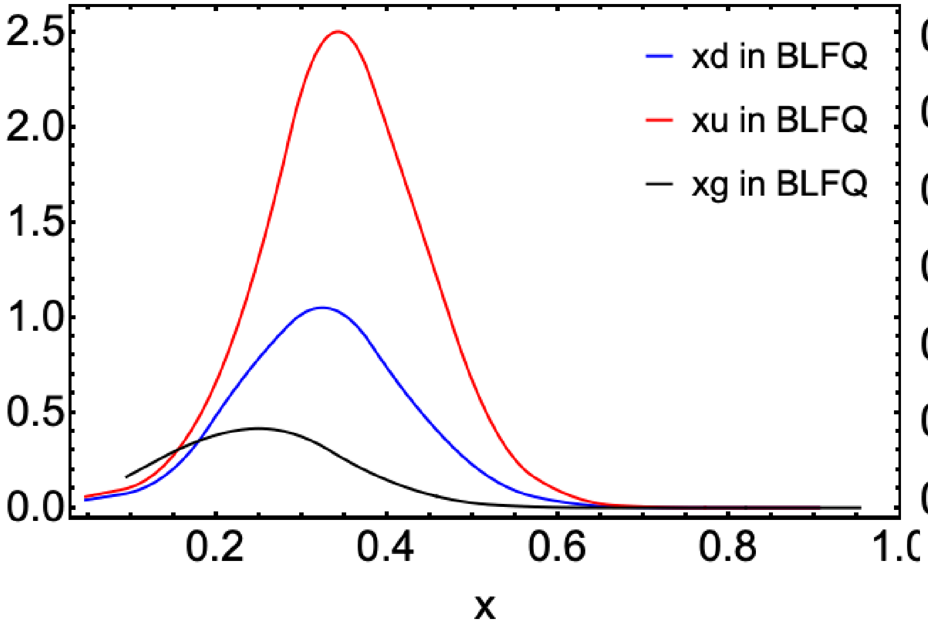
Unpolarized Parton Distribution Functions

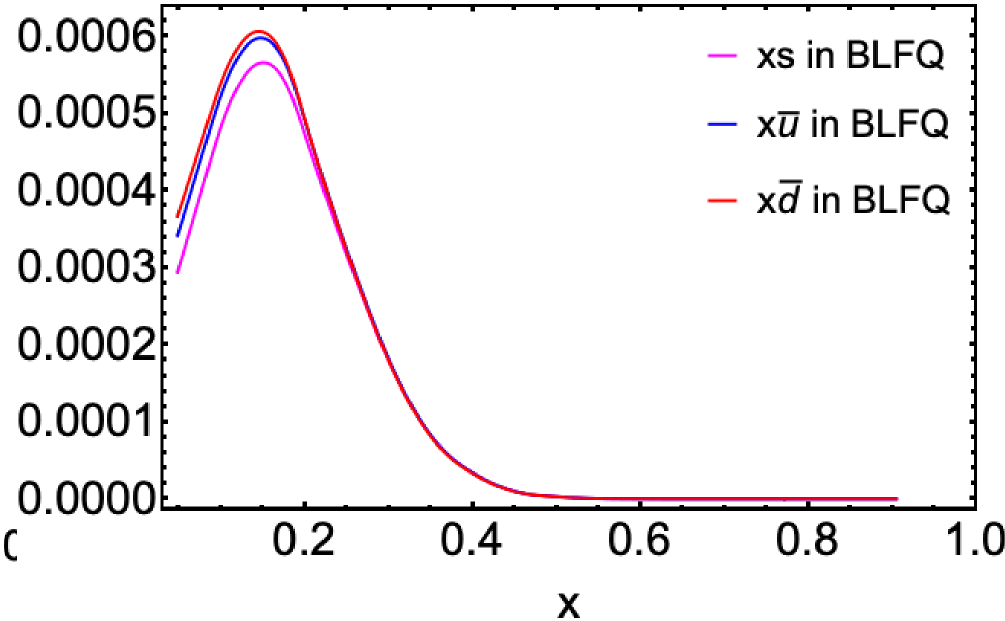
➢ Parton distribution functions with 6-parton Fock sectors

- Qualitative behavior agree with experimental results
- Endpoint behavior improves with $|qqqq\bar{q}g\rangle$ and $|qqqggg\rangle$ Fock sector included

Preliminary

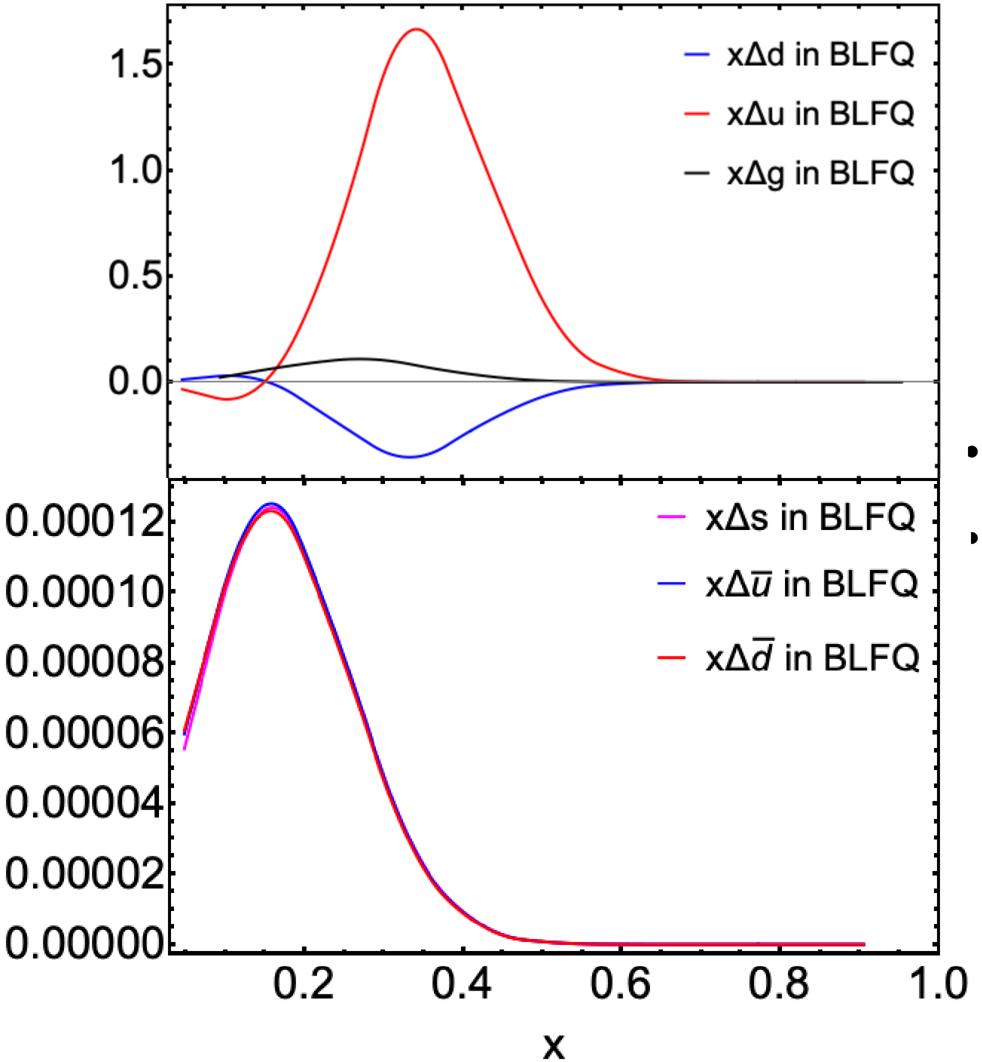
All results are at the initial scale



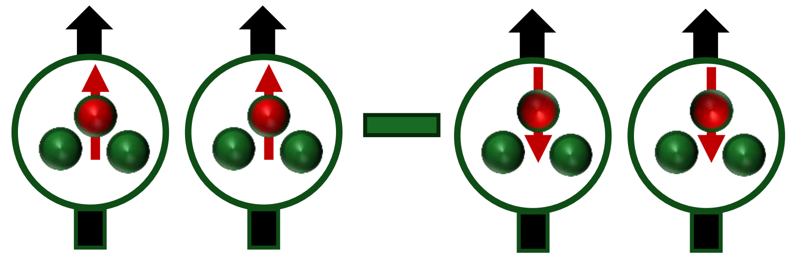


Helicity Parton Distribution Functions

$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$
$+ |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle$



Helicity PDFs:



- Including Higher Fock sectors
- Significantly increasing the helicity contribution of gluon to proton spin

$\Delta\Sigma_u = 0.94 \quad \Delta\Sigma_u = 0.21 \quad \Delta\Sigma = 0.73$

$$\Delta G = 0.12 \text{ (JAM: 0.2)}$$

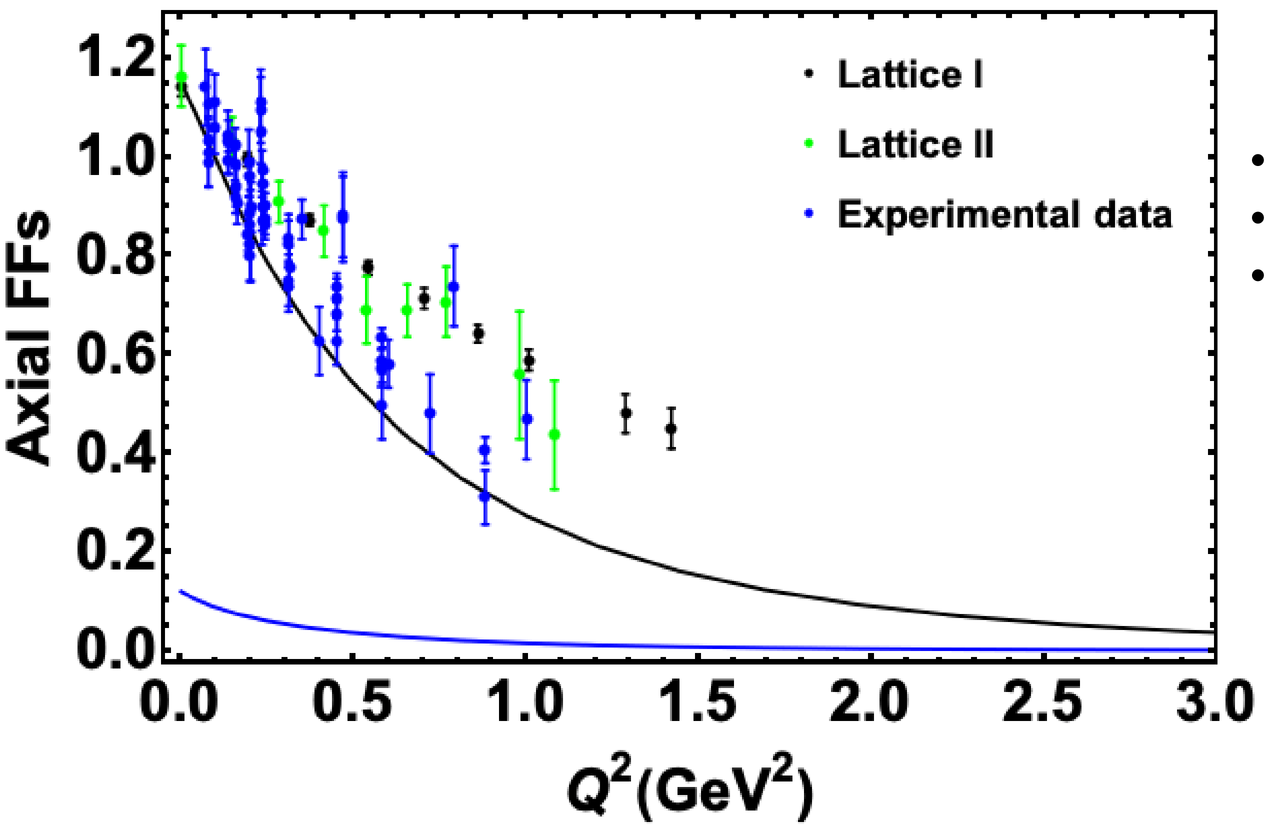
Preliminary

Axial Form Factor of The Proton

➢ Provide information on axial charge distributions

$$\langle N(p')|A^a_\mu|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu G_A(t) + \frac{(p'-p)_\mu}{2m} G_P(t)\right]\gamma_5 \frac{\tau^a}{2} u(p)$$

$$A^a_\mu = \bar{q}\gamma_\mu\gamma_5 T^a q \qquad G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$



- Black line: **valence quark**
- Blue line: **gluon**
- Valence quark contributions agree with the experimental data

Dimension of Basis Space

- Expansion in BLFQ basis

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + |qqq\, gg\rangle + |qqq\, ggg\rangle + |qqq\, q\bar{q}\, g\rangle$$

$N_{max} = 7, K_{max} = 16$

	$\lvert qqq\rangle$	$\lvert qqqg\rangle$	$\lvert qqq\, q\bar{q}\rangle$	$\lvert qqq\, gg\rangle$	$\lvert qqq\, ggg\rangle$	$\lvert qqq\, q\bar{q}\, g\rangle$
dimension	35,088	592,960	3,901,500	5,169,360	19,603,584	7,128,576
color config	1	2	3	6	22	8

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, u\bar{u}\rangle + |qqq\, d\bar{d}\rangle + |qqq\, s\bar{s}\rangle$$

Basis Dimension= 12,332,548

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, u\bar{u}\rangle + |qqq\, d\bar{d}\rangle + |qqq\, s\bar{s}\rangle + |qqq\, gg\rangle$$

Basis Dimension= 17,501,908

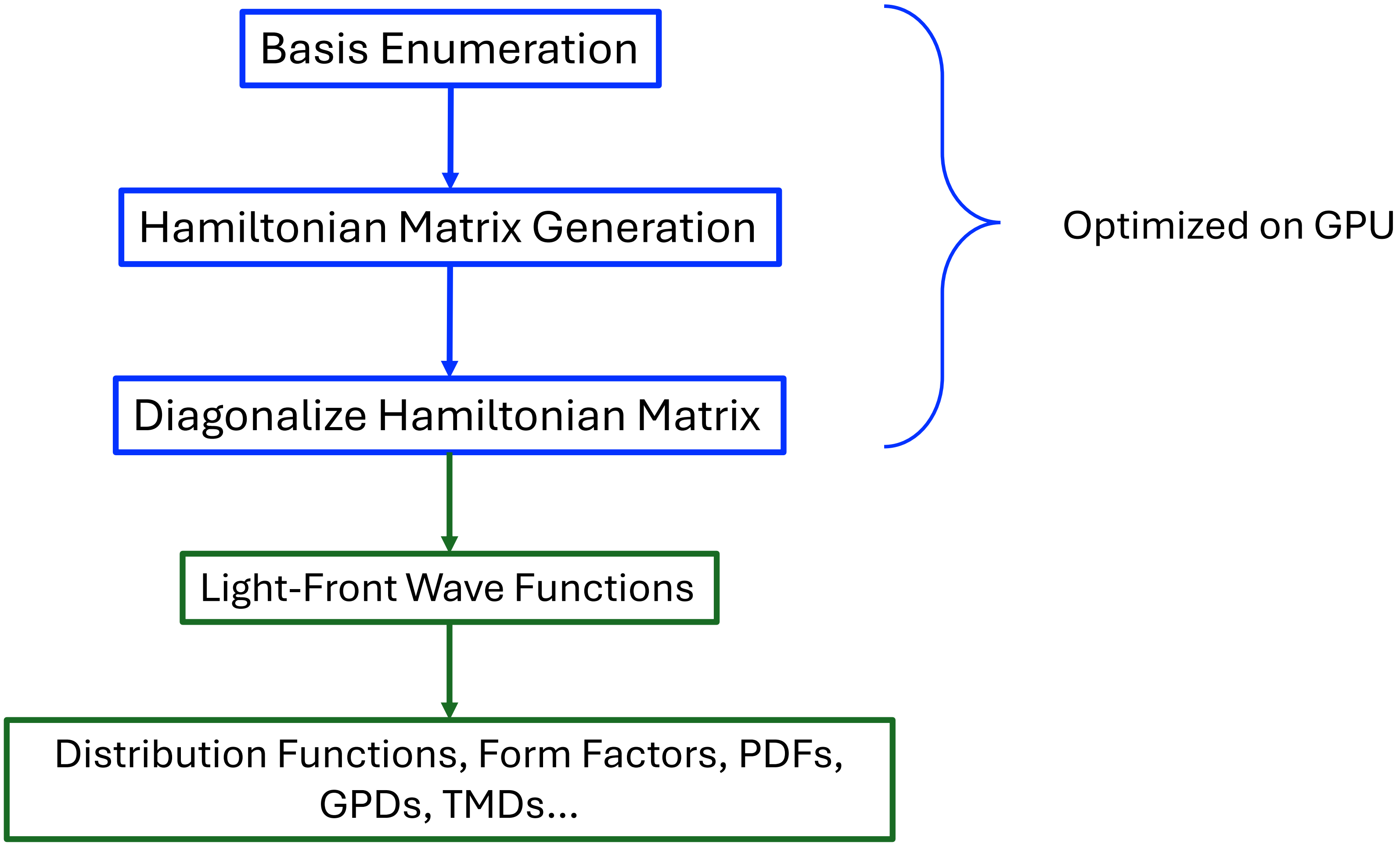
$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + |qqq\, gg\rangle + |qqq\, ggg\rangle$$

Basis Dimension= 37,105,492

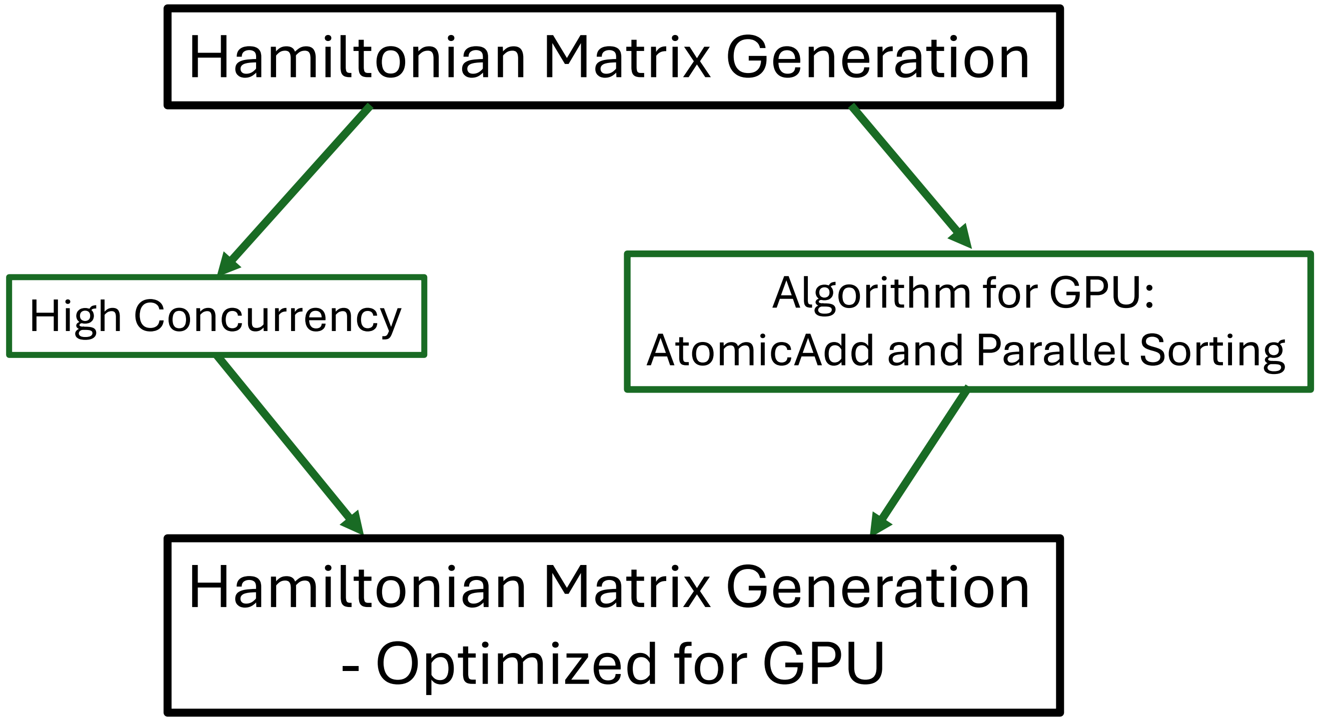
$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + |qqq\, gg\rangle + |qqq\, ggg\rangle + |qqq\, q\bar{q}\, g\rangle$$

Basis Dimension= 58,491,220

BLFQ with Heterogeneous Computing



BLFQ Optimization - Hamiltonian



	n=7 k=8	n=7 k=10	n=9 k=6	n=7 k=14	n=7 k=16
CPU(s)	**245.9**	**1098.699**	**2318.8**	**10805.2**	**30793.2**
GPU(s)	**73.3**	**343.7**	**188.8**	**920.3**	**1887.5**
Ratio	**3.35**	**3.2**	**12.28**	**11.74**	**16.31**

BLFQ Optimization - Diagonalization

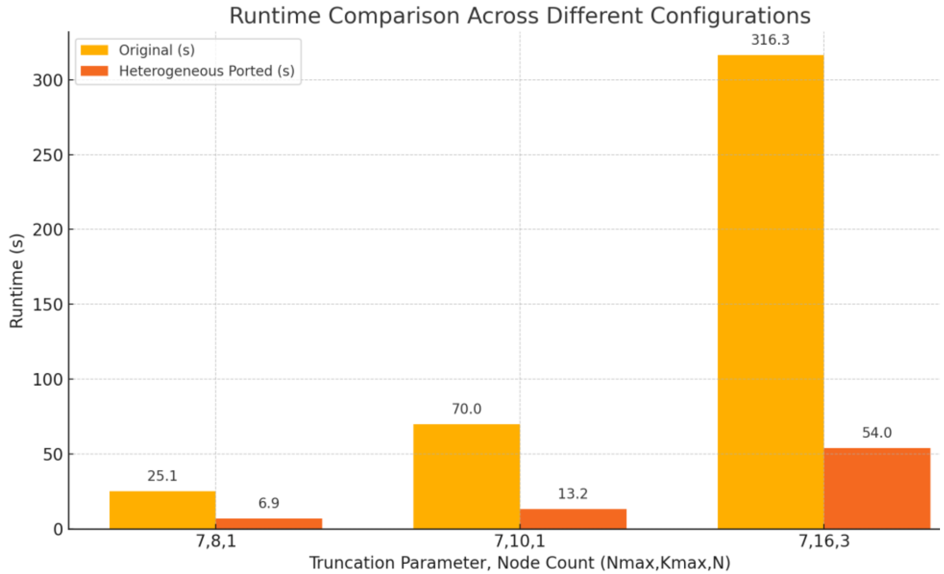
https://github.com/opencollab/arpack-ng

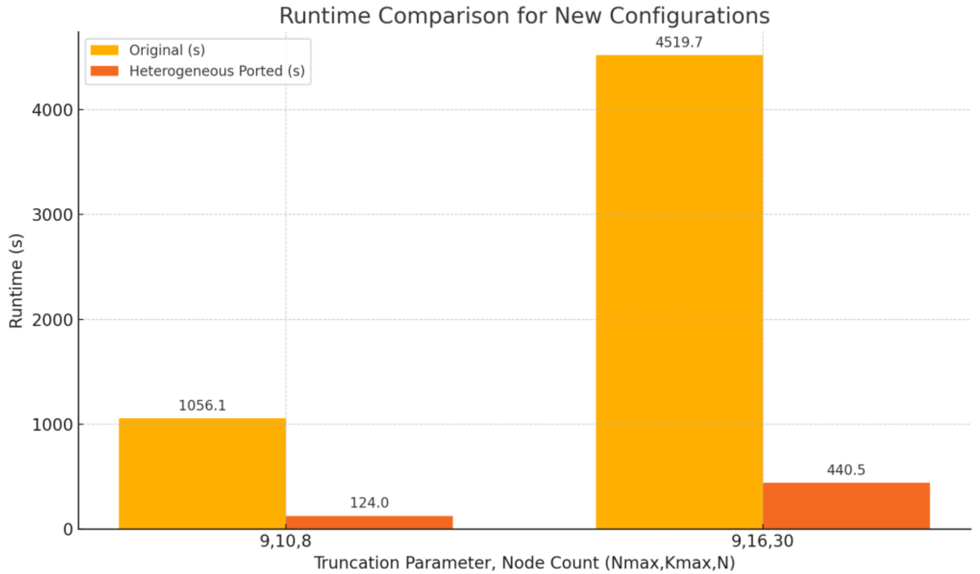
Diagonalization: Arpack

Replace the Basic function
BLAS -> HIPBLAS
LAPACK -> GPU Adaptation

Reprogram the Kernel:
Arpack Kernel, Matrix Multiplication

56 CPU threads vs 4 GPUs





Conclusions

- Basis Light-front Quantization: non-perturbative approach to QFT in Minkowski spacetime
- Systematically extendable toward first-principle calculations
- Light-front wave function available for evaluating nucleon 3D tomography at EICs
- Results improve with increasing Fock space ➡ Fock sector expansion works
- Recent progress:
  - Expanding Fock sectors
  - Incorporating all QCD interactions
  - Higher-twist observables: correlation between partons
  - Developing GPU/CPU hybrid codes

(Expanding Fock sectors and Incorporating all QCD interactions } Toward first-principles)

Outlook

Current status

Full QCD interaction

Fock sector expansion
$|qqq\ q\bar{q}\ g\rangle$ and $|qqq\ ggg\rangle$......

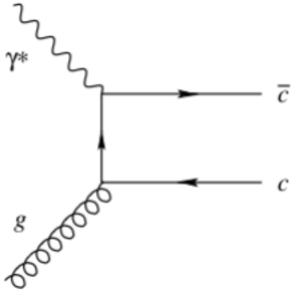
Deutron calculation
$|qqq\ qqq\rangle + |qqq\ qqq\ g\rangle$

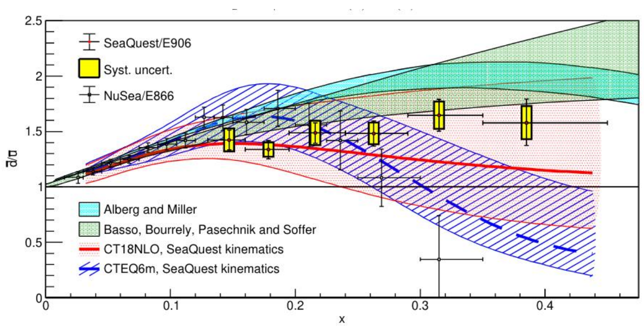
EMC effect

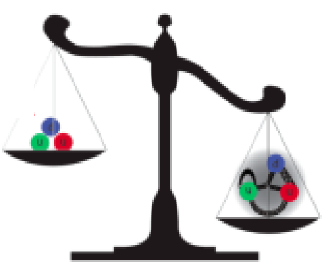
Intrinsic charm

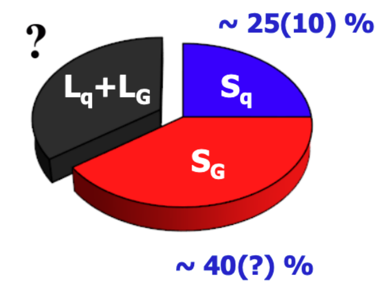
Sea asymmetry

Origin of spin and mass









Thank you!

You are welcome to visit Huizhou!☺