QCD-Gravity double-copy in the Regge regime via shockwave collisions

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In collaboration with Raju Venugopalan

[2311.03463, 2312.03507, 2406.10483] + WIP

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Introduction

$$\begin{array}{c}\text{perturbative QCD}\\ \Big\downarrow{\scriptstyle \text{BCJ}}\\ \text{perturbative gravity}\end{array}$$

There has been tremendous progress in understanding the relationship between perturbative QCD amplitudes and gravity amplitudes. [Bern, Carrasco, Johannson]

The BCJ double copy states that Yang-Mills amplitudes can be mapped onto their gravity counterparts by applying a simple set of well-defined color-to-kinematics replacement rules

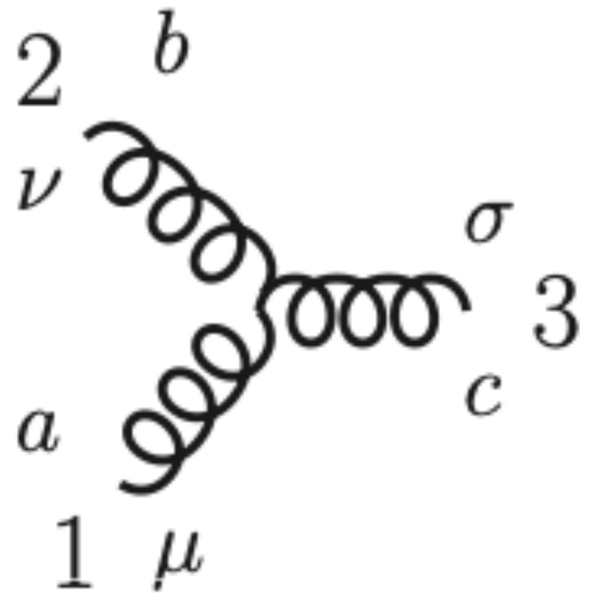
Introduction

perturbative QCD

$$\Big\downarrow \text{BCJ}$$

perturbative gravity

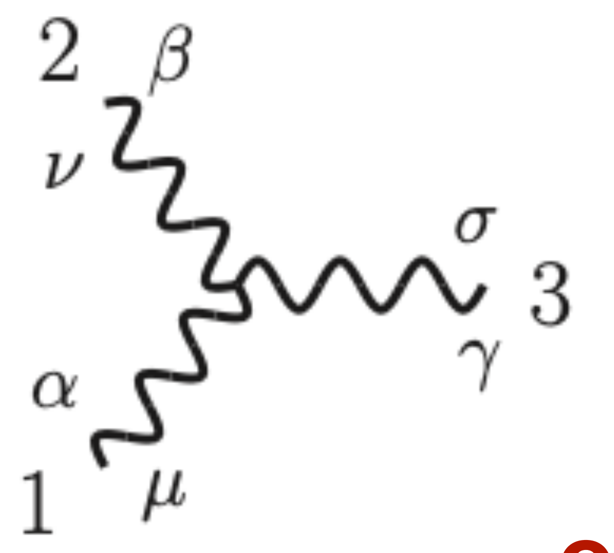
Example: bare 3-point interaction vertex



$$V^{abc}_{3\mu\nu\sigma}\left(p_1, p_2, p_3\right) = g f^{abc}\left[\left(p_1 - p_2\right)_\sigma \eta_{\mu\nu} + \text{ cyclic }\right]$$

$g \to \kappa$ | replace f^{abc} as:

$$\frac{\kappa^2}{8\pi} = G = \frac{1}{M_p^2} = \ell_p^2$$



$$V_{3\mu\alpha,\nu\beta,\sigma\gamma}\left(p_1, p_2, p_3\right) = -i\kappa\left[\left(p_1 - p_2\right)_\gamma \eta_{\alpha\beta} + \text{ cyclic }\right]\left[\left(p_1 - p_2\right)_\sigma \eta_{\mu\nu} + \text{ cyclic }\right]$$

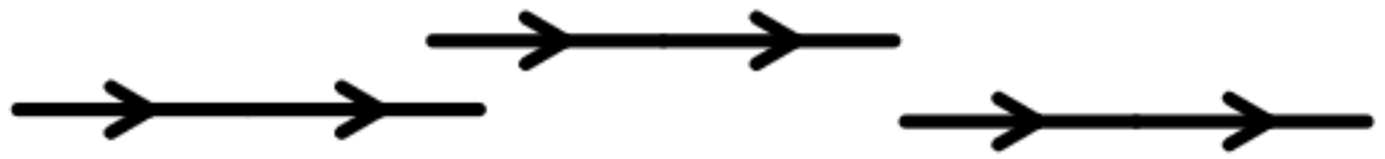
on-shell methods for perturbative gravity calculations [Bohr, Donoghue, Vanhove [1309.0804]]

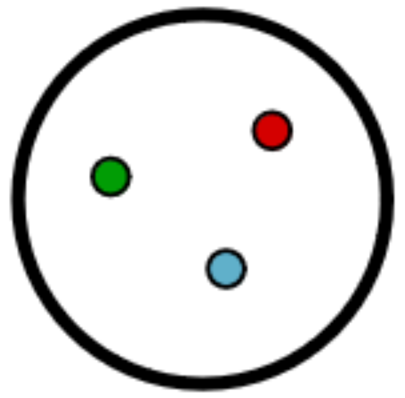
Introduction

$$\begin{array}{ccc} \text{perturbative QCD} & \xrightarrow{\text{high energies}} & \begin{array}{l}\text{QCD at high occupancy}\\ \text{(high parton densities)}\end{array} \\ \downarrow{\scriptstyle \text{BCJ}} & & \\ \text{perturbative gravity} & & \end{array}$$

Introduction

$$\begin{array}{ccc} \text{perturbative QCD} & \xrightarrow{\text{high energies}} & \text{QCD at high occupancy (high parton densities)} \\ \Big\downarrow {\scriptstyle \text{BCJ}} & & \\ \text{perturbative gravity} & & \end{array}$$



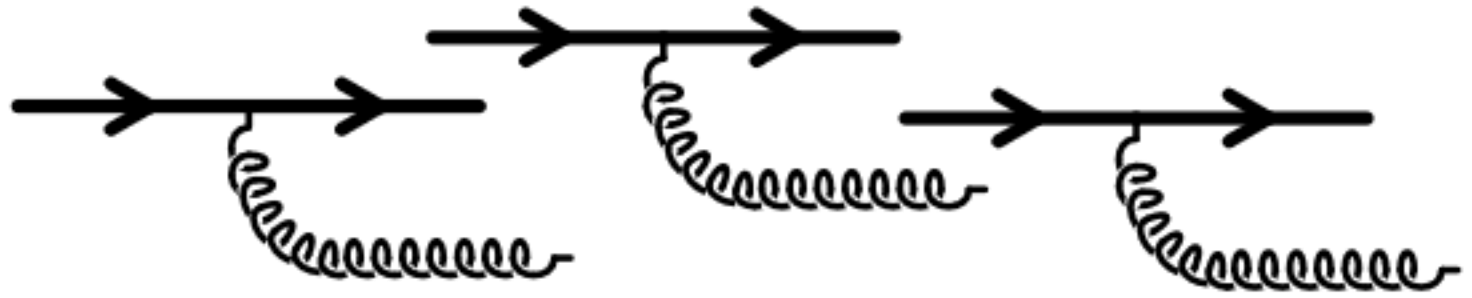


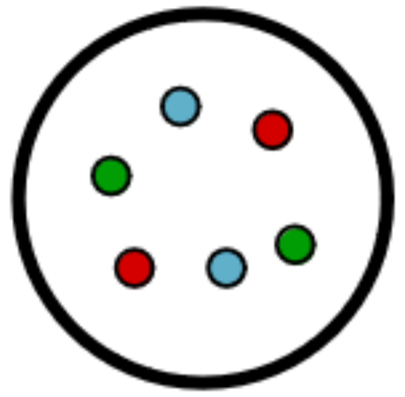
Introduction

perturbative QCD $\xrightarrow{\text{high energies}}$ QCD at high occupancy (high parton densities)

$\downarrow$ BCJ

perturbative gravity



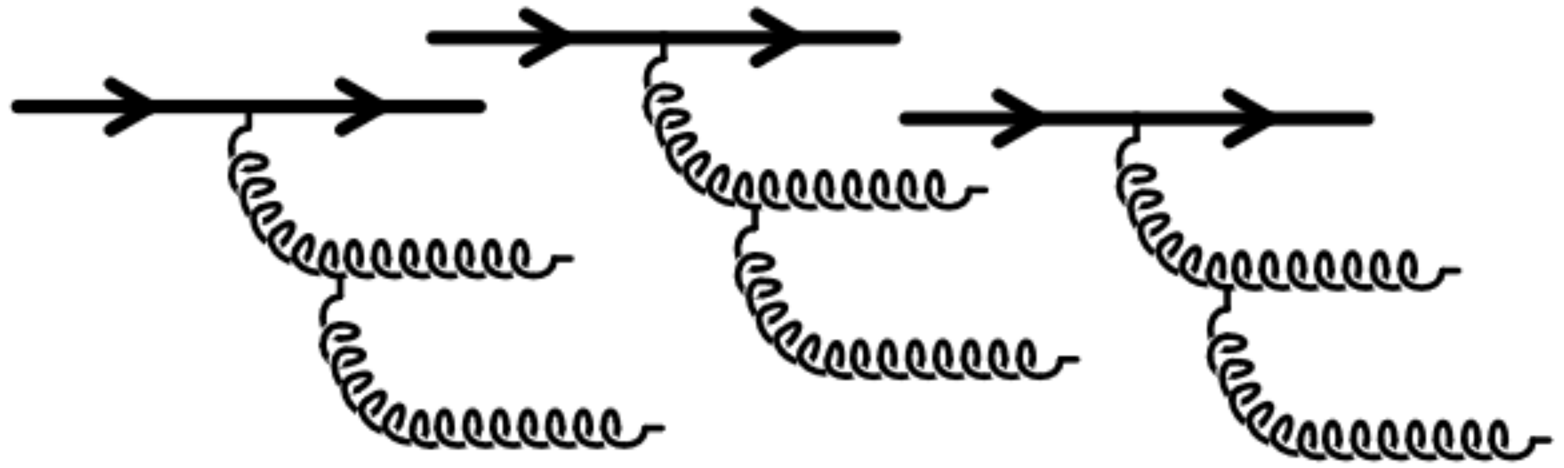


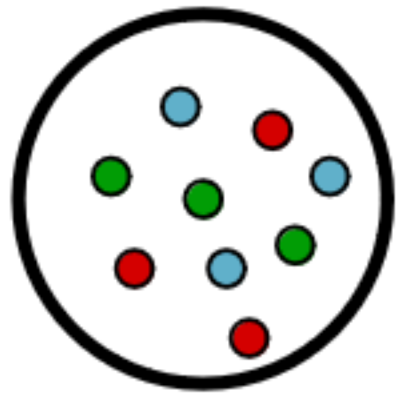
Introduction

perturbative QCD $\xrightarrow{\text{high energies}}$ QCD at high occupancy (high parton densities)

BCJ ↓

perturbative gravity

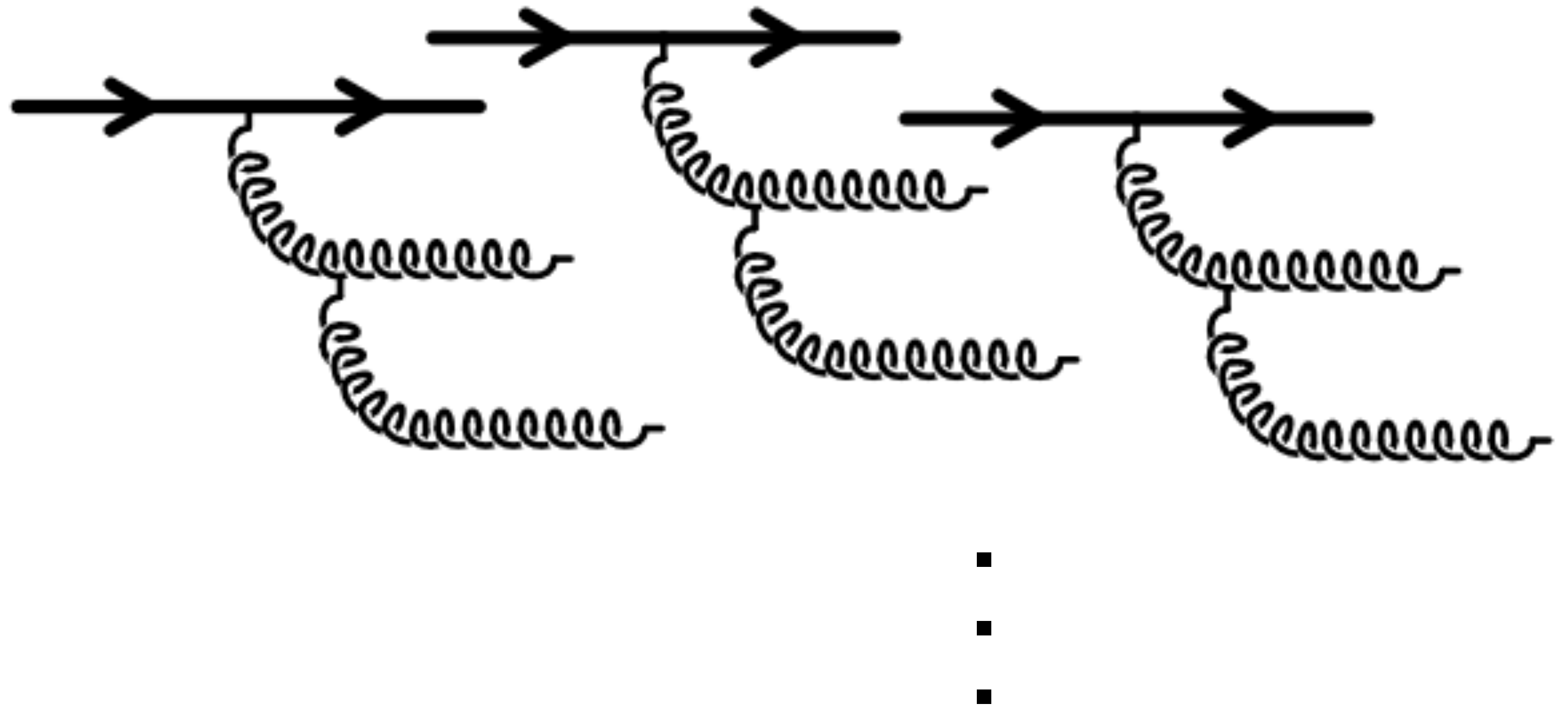




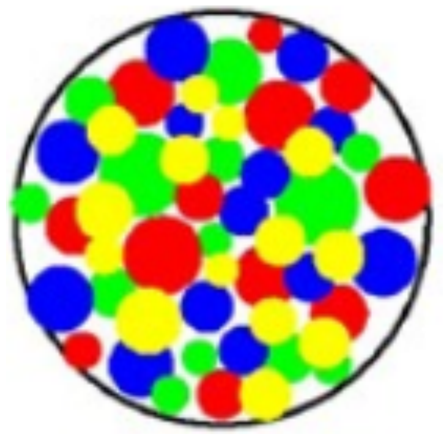
Introduction

perturbative QCD $\xrightarrow{\text{high energies}}$ QCD at high occupancy (high parton densities)

perturbative QCD $\xrightarrow{\text{BCJ}}$ perturbative gravity



Parton cascade → high parton densities



High occupancy system of N particles

$$\alpha_s N \sim 1$$

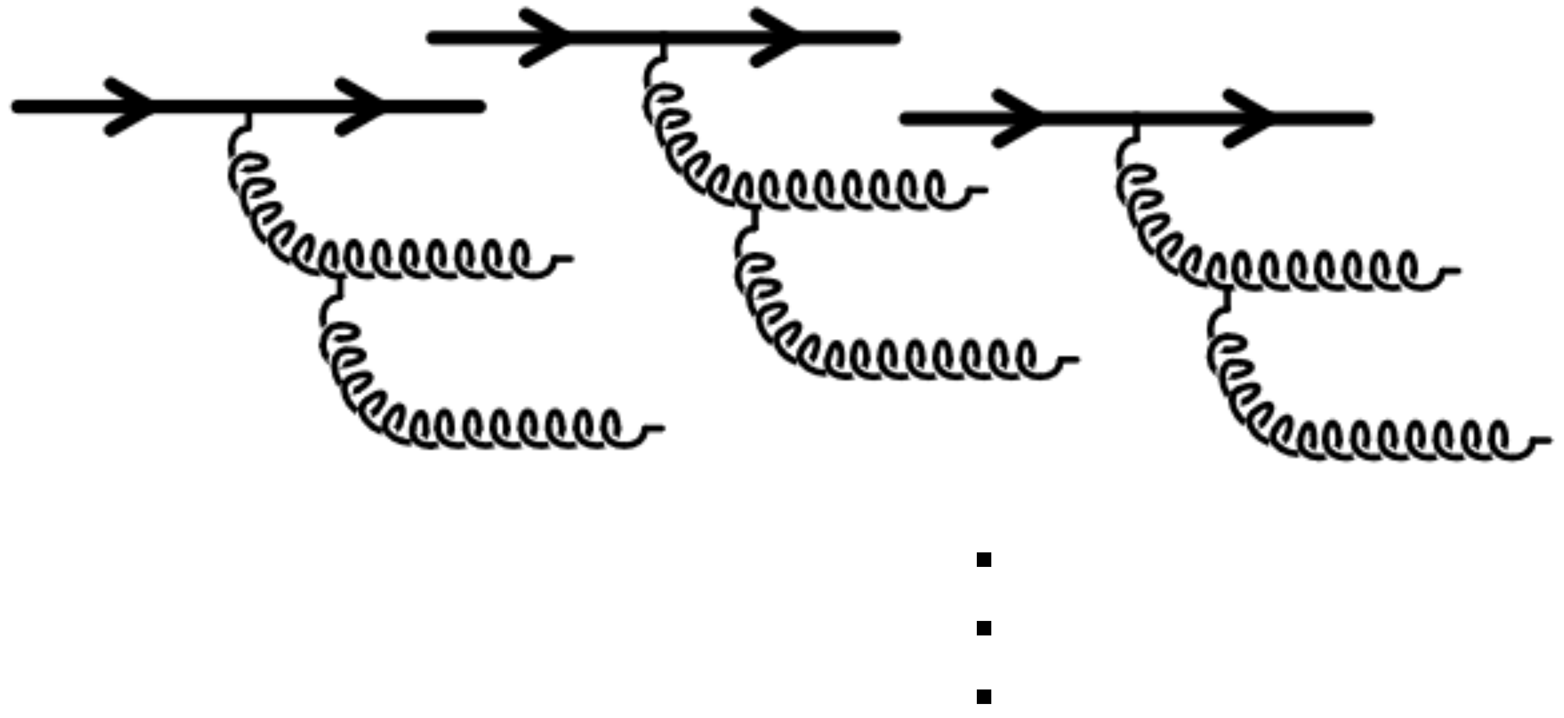
Saturation scale

$$Q_s \sim A^{1/3}\Lambda_{QCD}$$

Introduction

perturbative QCD $\xrightarrow{\text{high energies}}$ QCD at high occupancy (high parton densities)

perturbative QCD $\xrightarrow{\text{BCJ}}$ perturbative gravity



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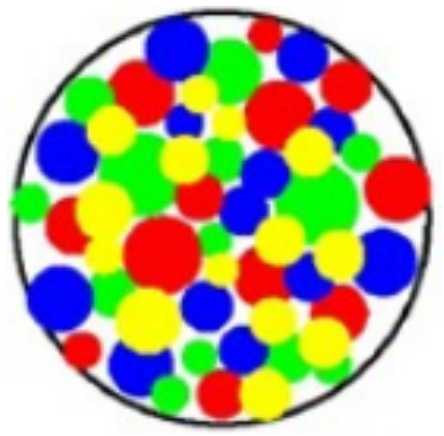


Dense close-packed classical configuration $\dfrac{1}{Q_S}$

Color Glass Condensate

[McLerran, Venugopalan]

Introduction

$$\begin{tikzcd}
\text{perturbative QCD} \arrow[r, "\text{high energies}"] \arrow[d, "\text{BCJ}"'] & \begin{array}{l}\text{QCD at high occupancy}\\ \text{(high parton densities)}\end{array} \arrow[d, "{[?]}"] \\
\text{perturbative gravity} \arrow[r, "\text{high energies}"'] & \text{Gravity at high occupancy}
\end{tikzcd}$$

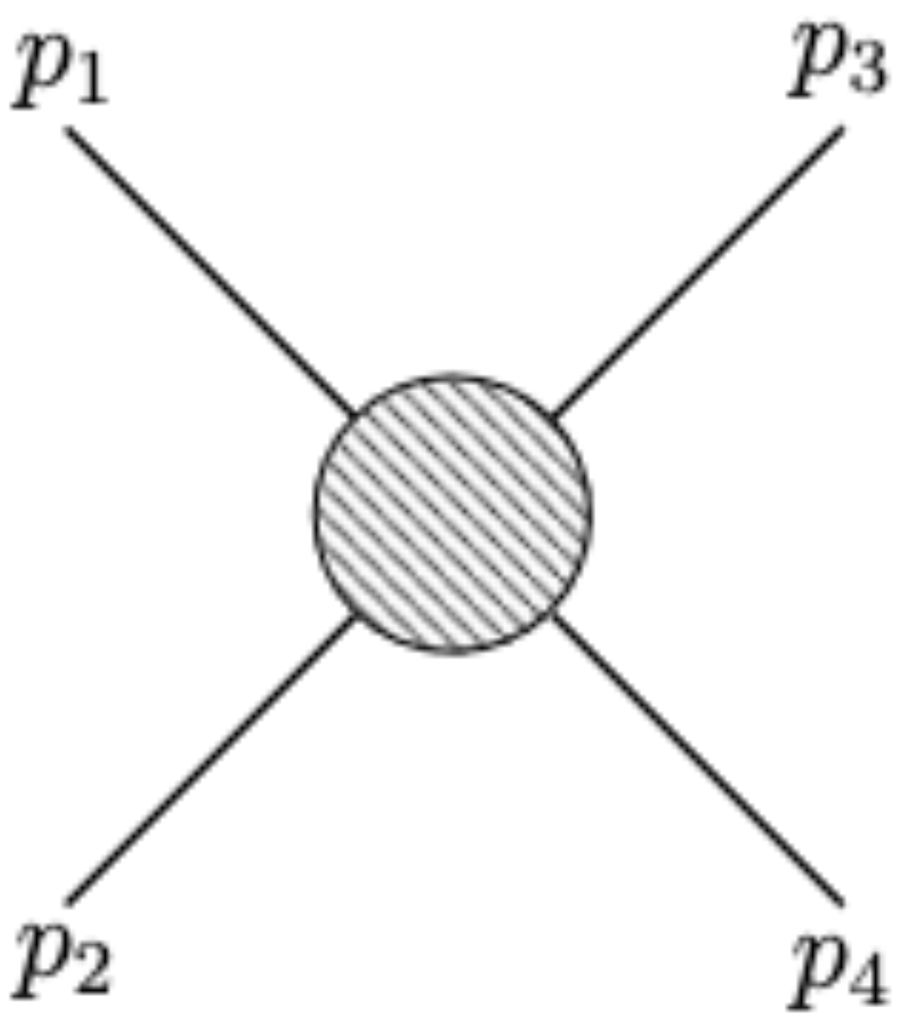
Does a double-copy exist in the high occupancy / strong field regime?

If so, what is its precise structure (does it follow the usual / similar rules of CK duality)?

Given the progress in the QCD side such an identification might have observational implications for gravitational radiation produced in high-energy scattering in gravity

High energy kinematics

Regge kinematics



$2 \to 2$ amplitude

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2$$

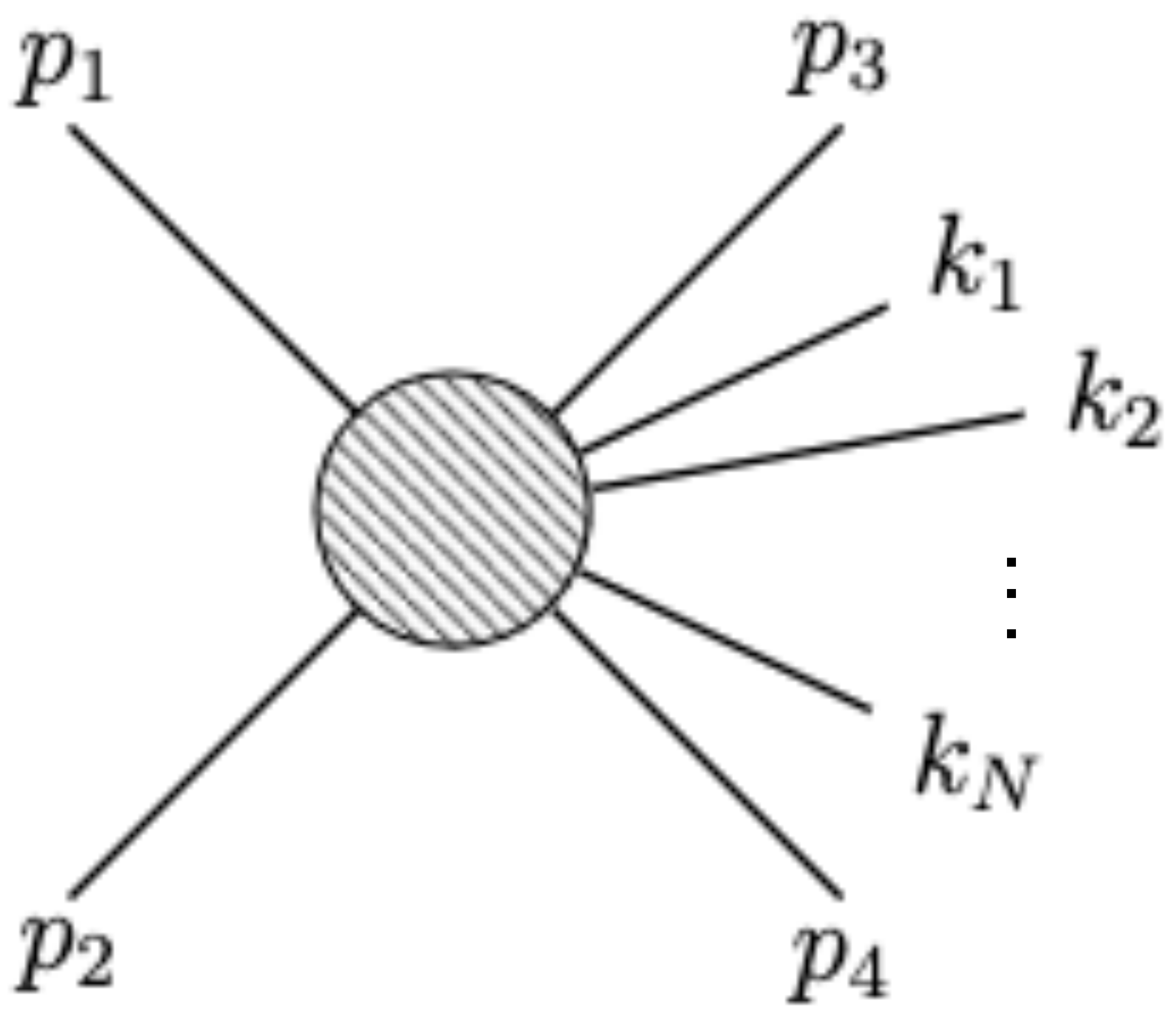
$$s \gg |t| \gg \Lambda^2_{\mathrm{QCD}}$$

equivalently strong ordering in rapidity

$$y_3 \gg y_4 \qquad \boldsymbol{p}_3 \sim \boldsymbol{p}_4$$

High energy kinematics

Multi-Regge kinematics



$2 \to 2 + N$ amplitude

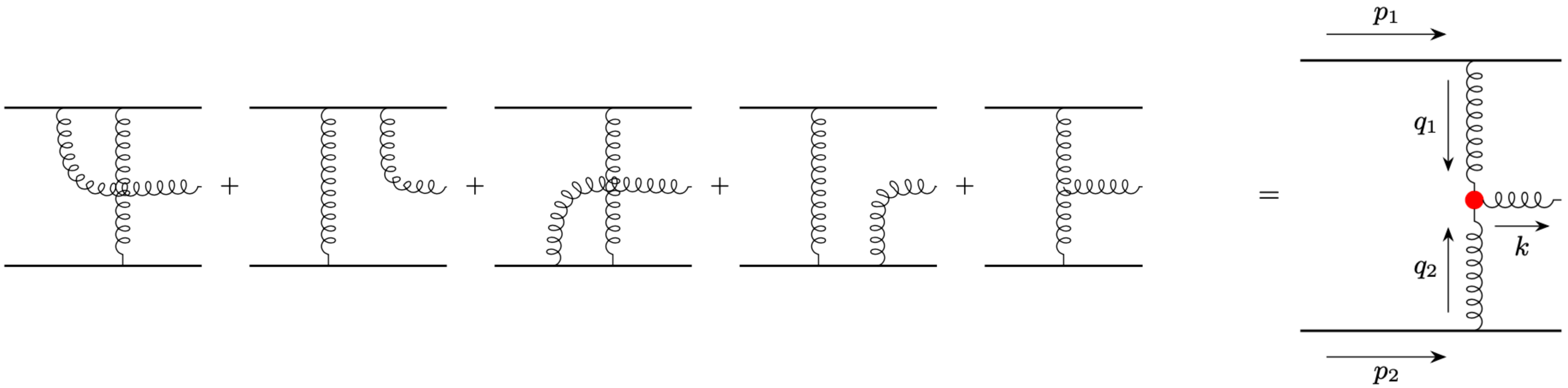
$$p = \left(p^+ = |\boldsymbol{p}|\,e^{+y}, p^- = |\boldsymbol{p}|\,e^{-y}, \boldsymbol{p}\right)$$

strong ordering in the final state light cone momentum

$$y_0 \gg y_1 \gg y_2 \gg \cdots \gg y_N \gg y_{N+1} \quad \text{with} \quad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

gives dominant contribution to the inelastic
$2 \to 2 + N$ multi-particle production

QCD in the Regge limit



$C_\mu(q_1, q_2)$

$$C_\mu\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

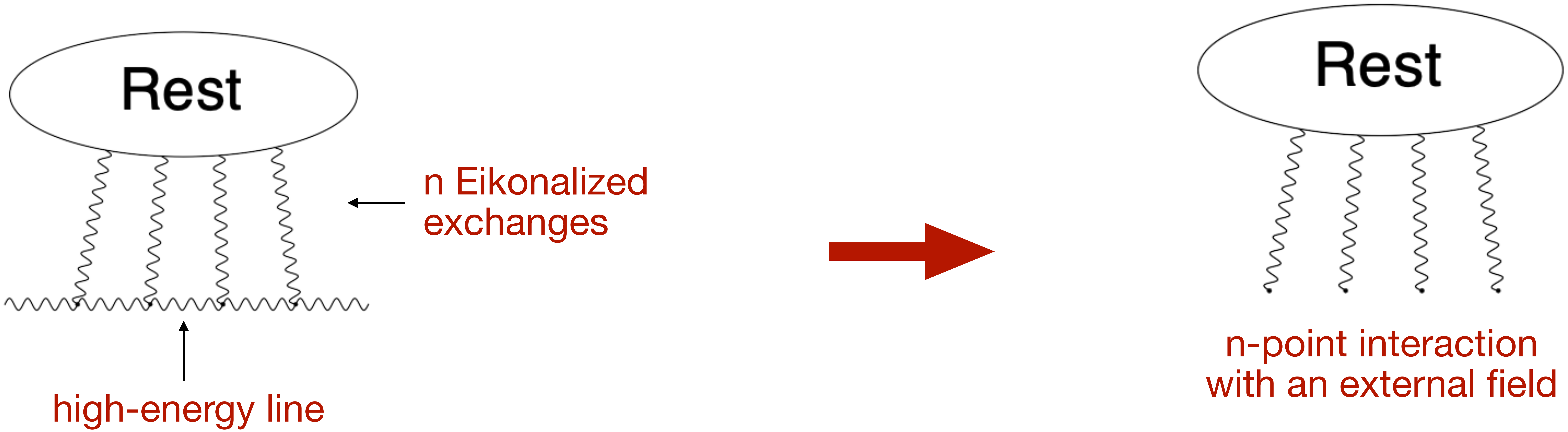
[Lipatov]

Gauge invariant $k^\mu C_\mu\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right) = 0$

Connection to shockwaves

Replace a fast moving particle in interaction with the rest of an arbitrary Feynman diagram by an external source

Eikonal scattering [Akhoury, Saotome]



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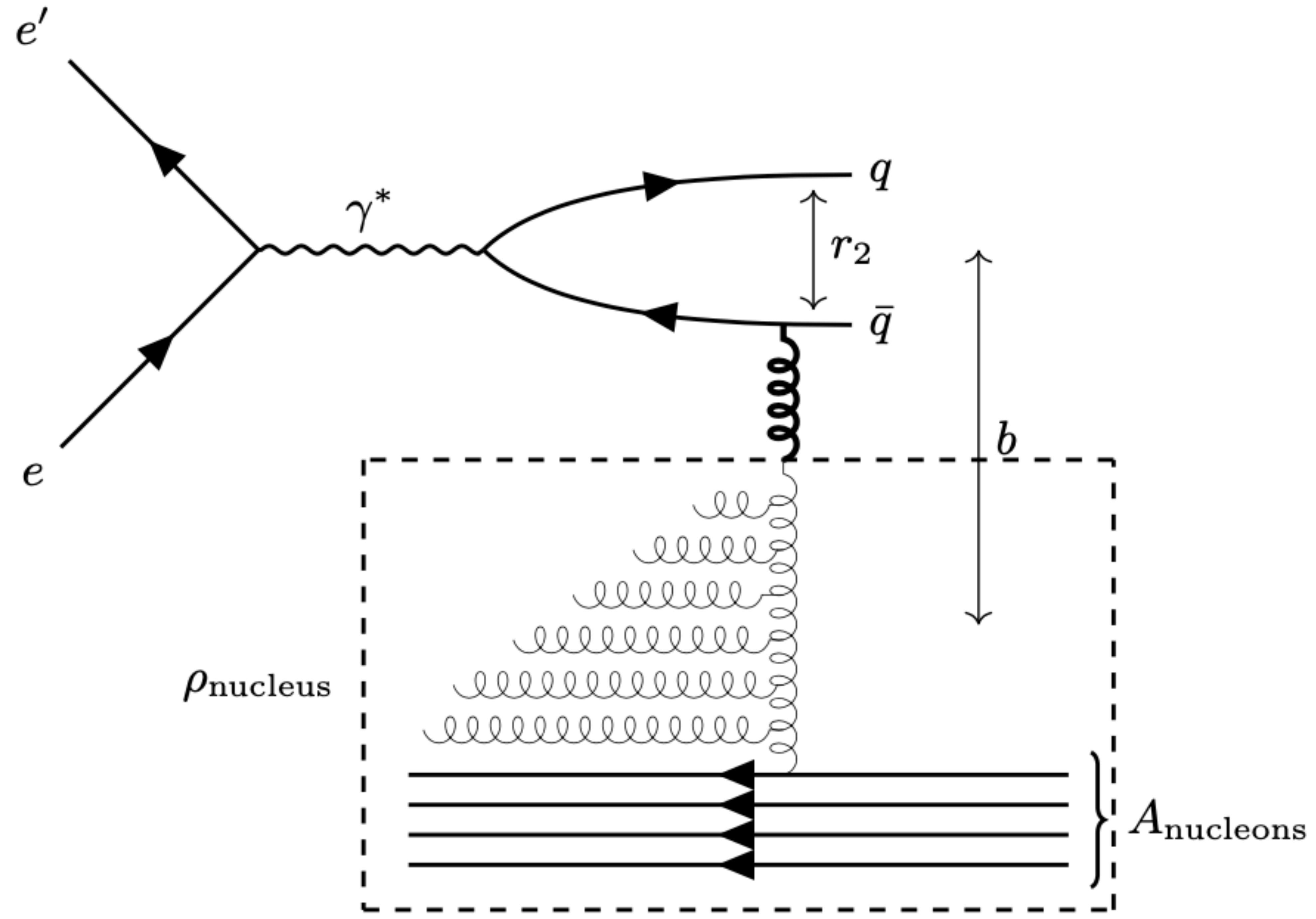

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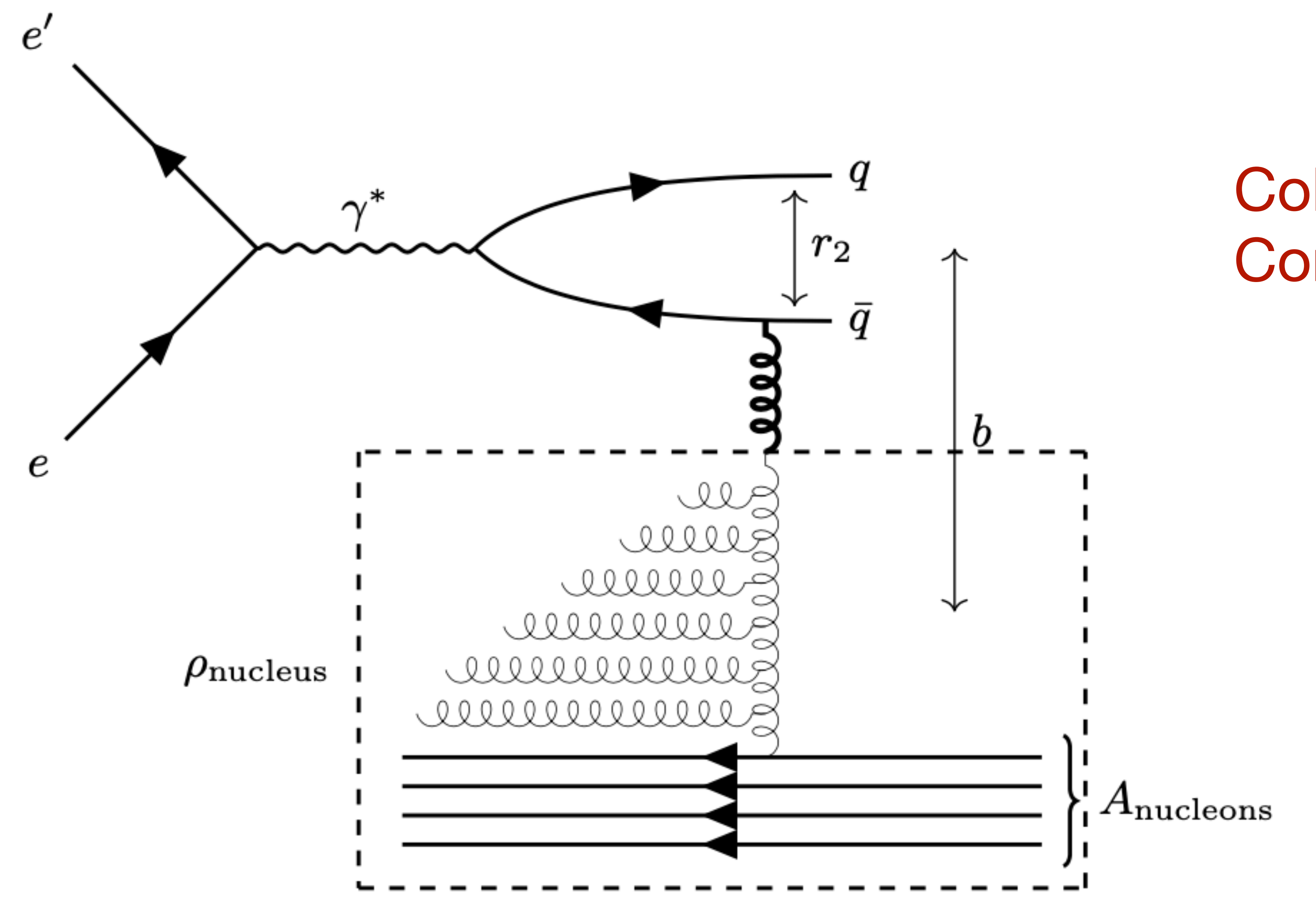
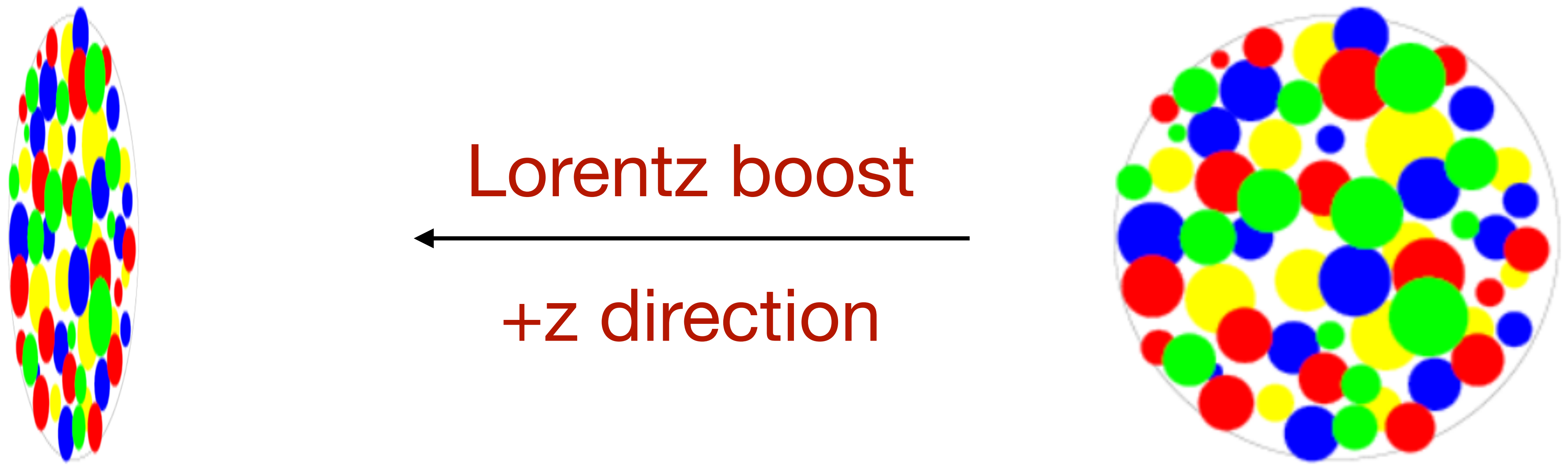


Dense close-packed classical configuration $\dfrac{1}{Q_S}$

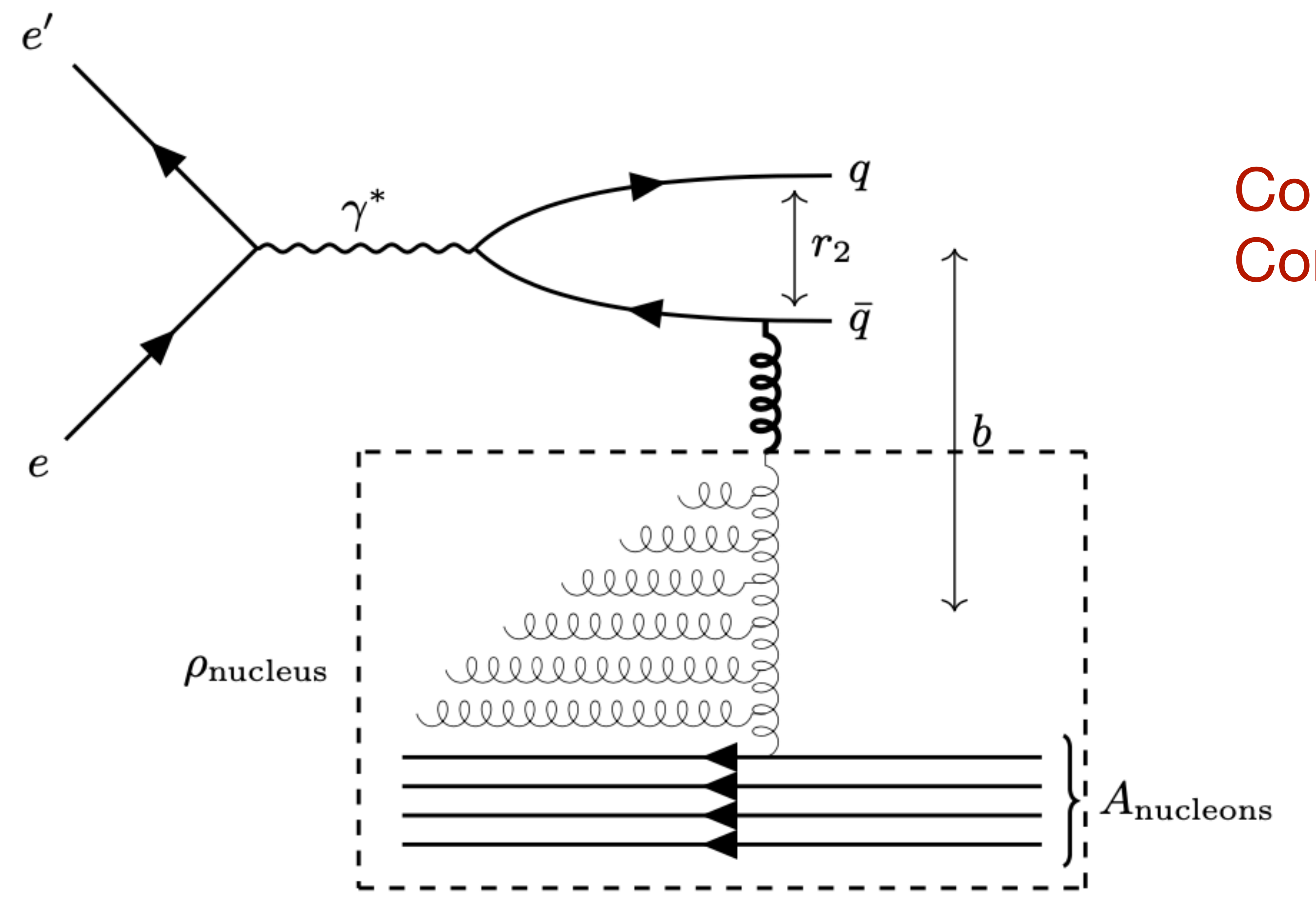
Color Glass Condensate



The sheet with transverse profile $\rho(\boldsymbol{x})$ travels in the x^+direction while sitting at $x^- = 0$

Connection to shockwaves

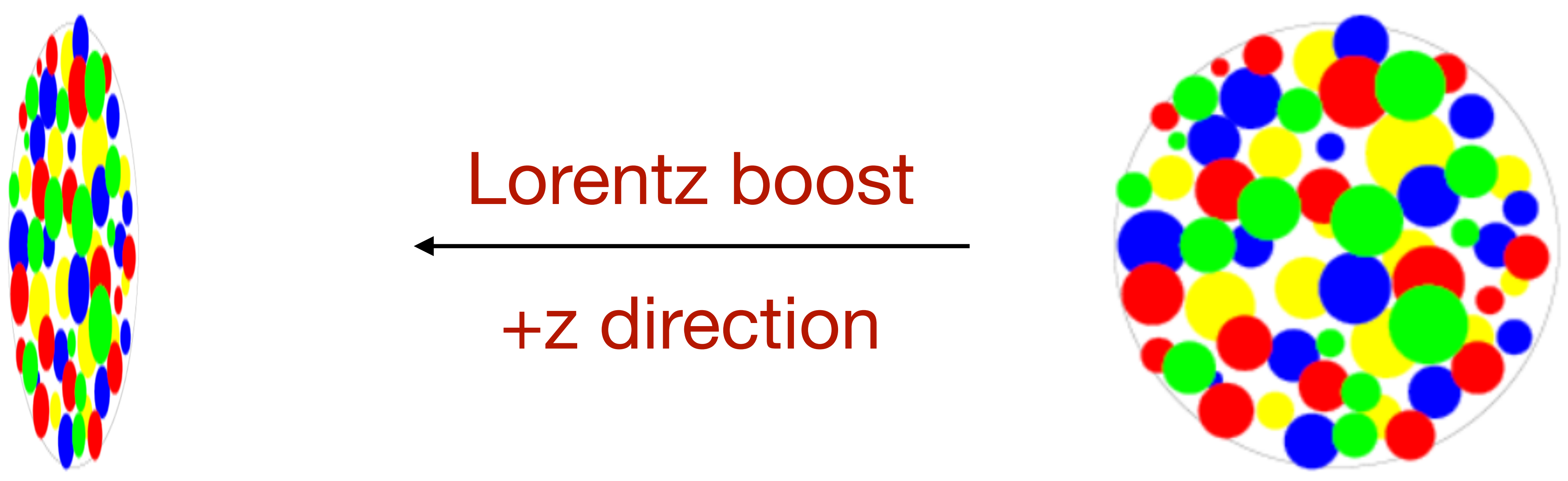
Replace a fast moving particle in interaction with the rest of an arbitrary Feynman diagram by an external source



Model this blob with a gluon shockwave

Dense close-packed classical configuration $\frac{1}{Q_S}$

Color Glass Condensate



The sheet with transverse profile $\rho(\boldsymbol{x})$ travels in the x^+direction while sitting at $x^- = 0$

$$J_-^a = g\delta(x^-)\,\rho(\boldsymbol{x})T^a$$

Gluon shockwave

Gluon shockwave is an exact solution to YM equations with covariantly conserved source

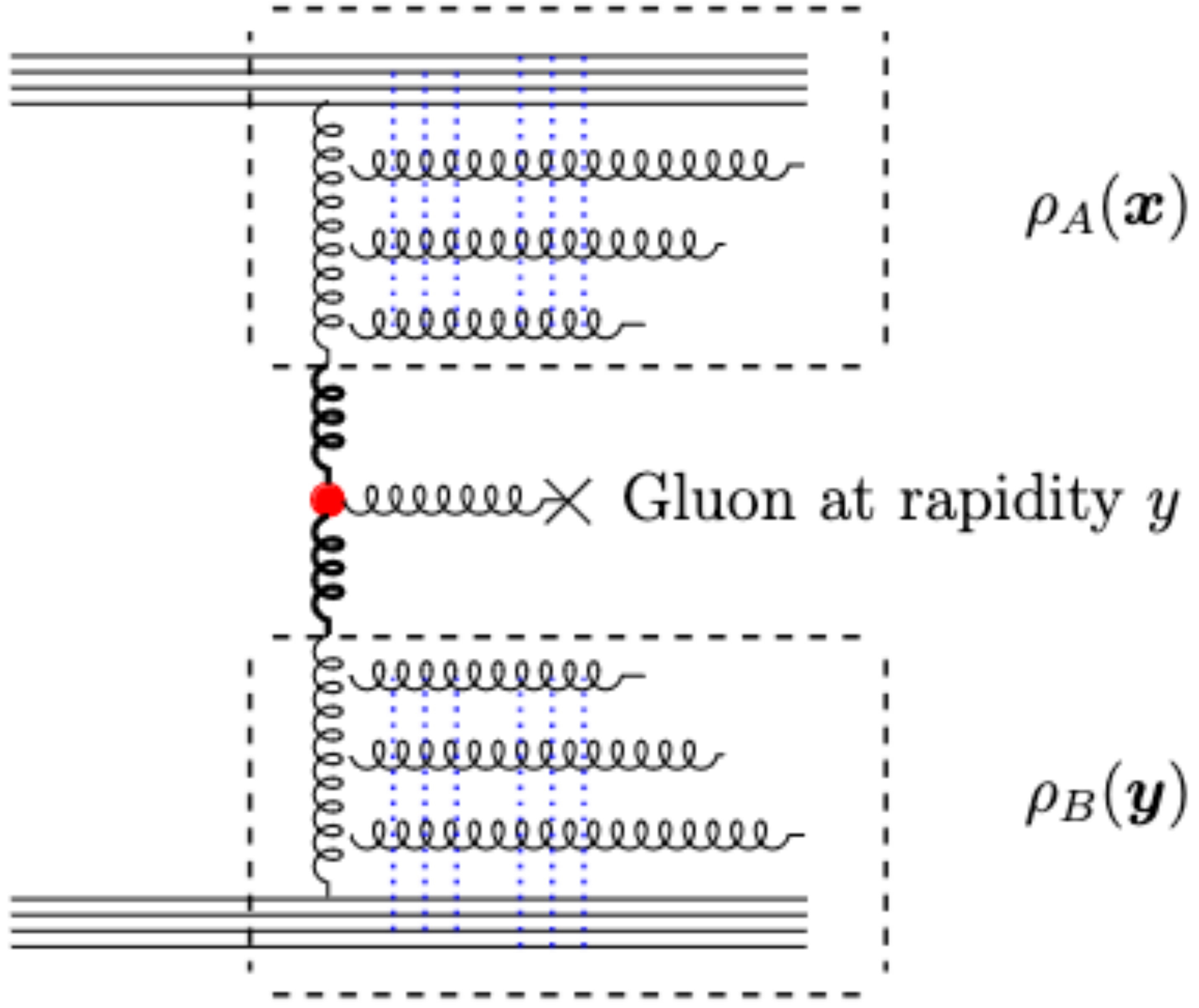
$$J_-^a = g\delta\left(x^-\right)\rho(\boldsymbol{x})T^a$$

shockwave solution takes the form

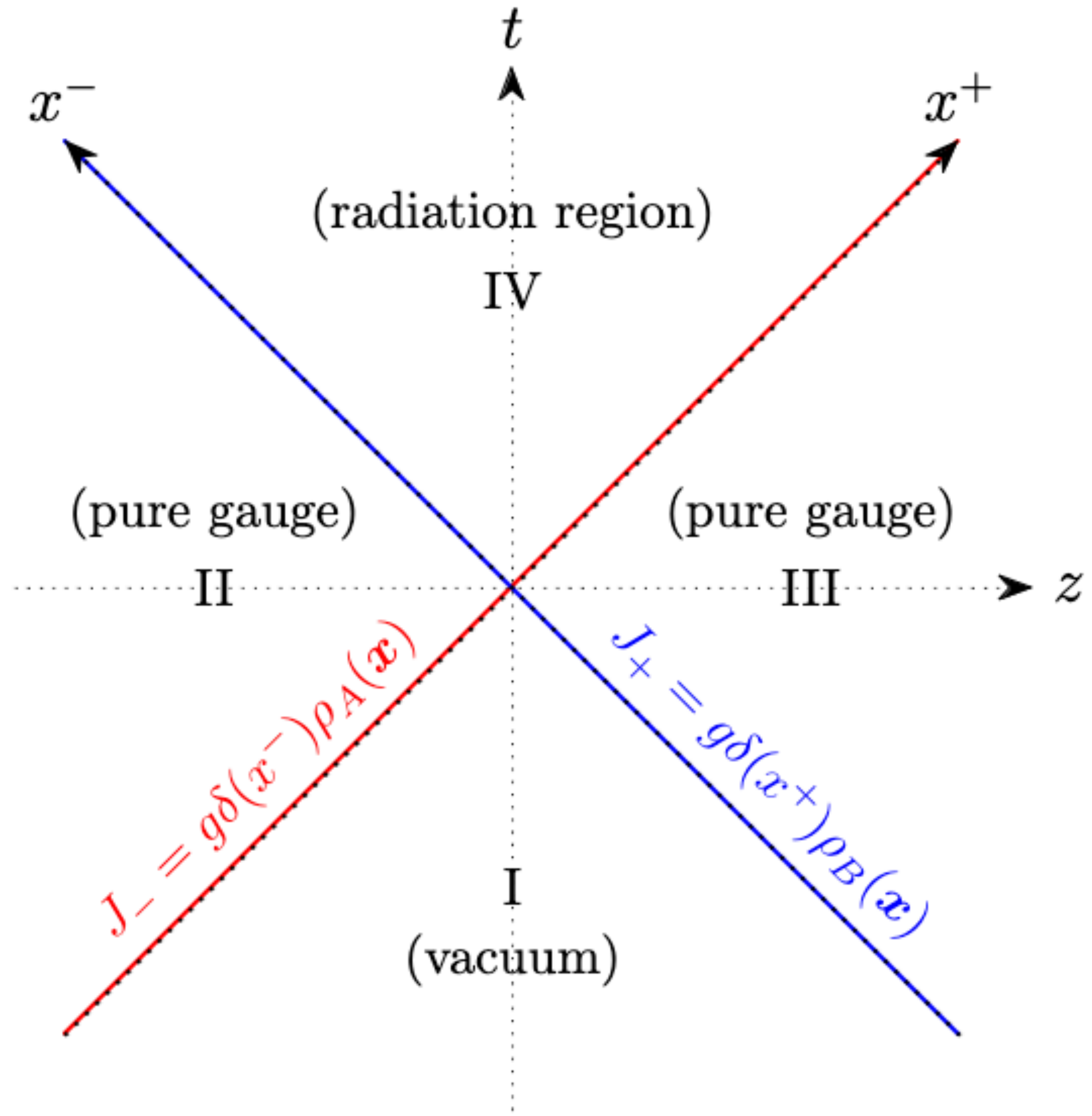
$$A_i = \frac{i}{g}\Theta\left(x^-\right)U\partial_i U^\dagger \quad \text{where} \quad U = \exp\left(ig^2\frac{\rho(\boldsymbol{x})}{\Box_\perp}\right)$$

Field strength vanishes on either sides ($x^- > 0,\ x^- < 0$) but the vacuum is not the same

Gluon shockwave scattering



collision of two nuclei at sufficiently high energies can be approximated by gluon shockwave collisions



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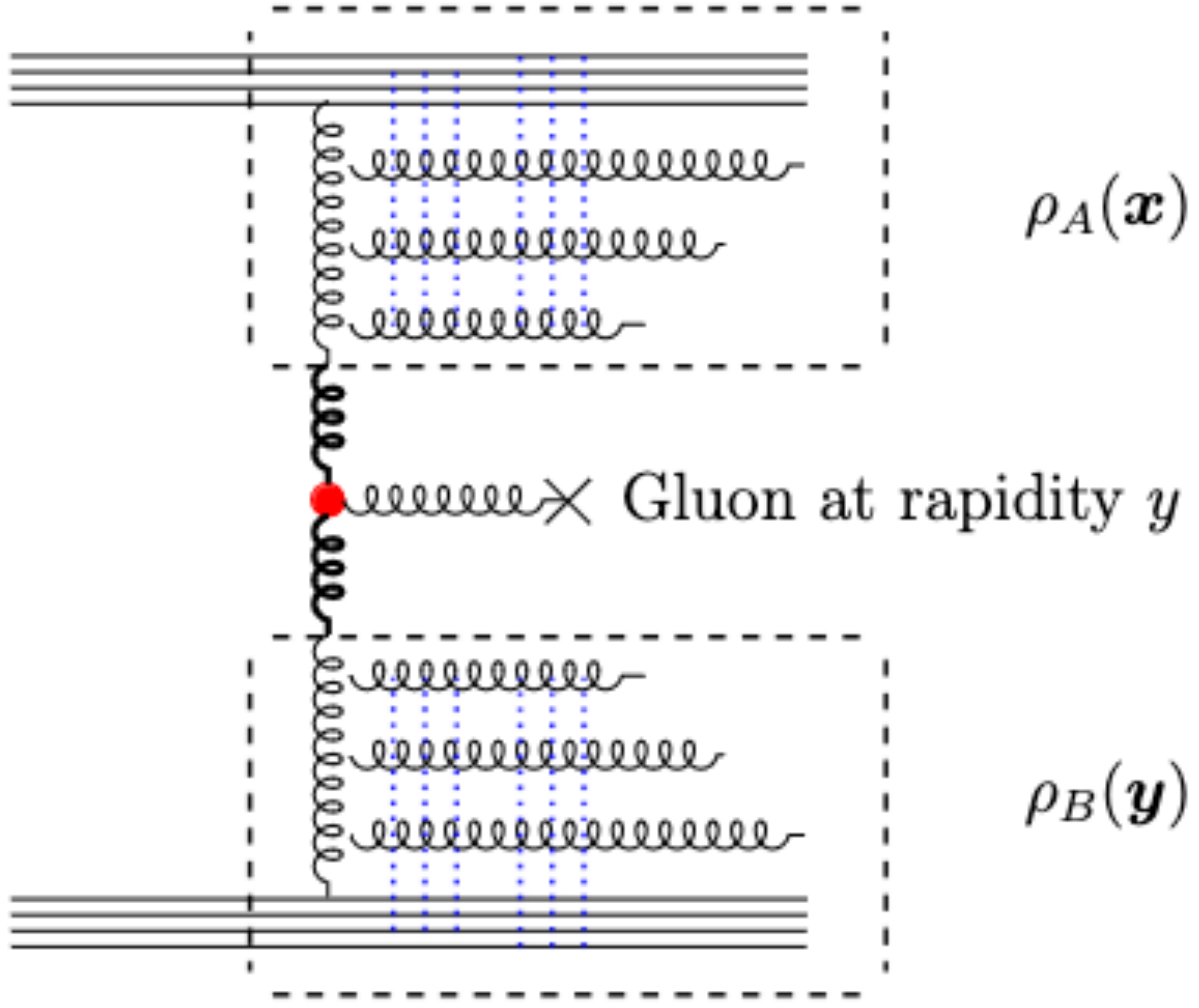

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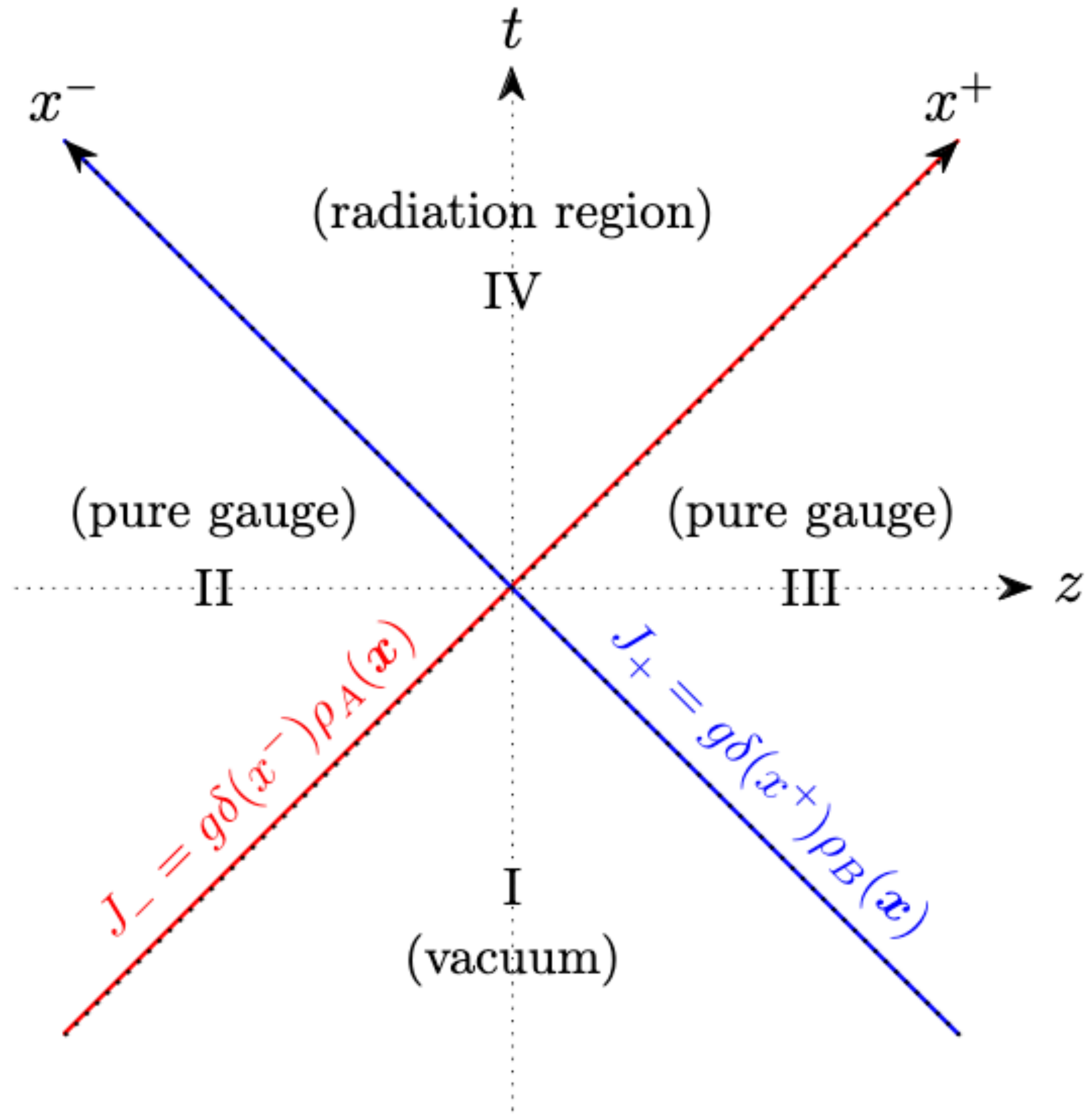
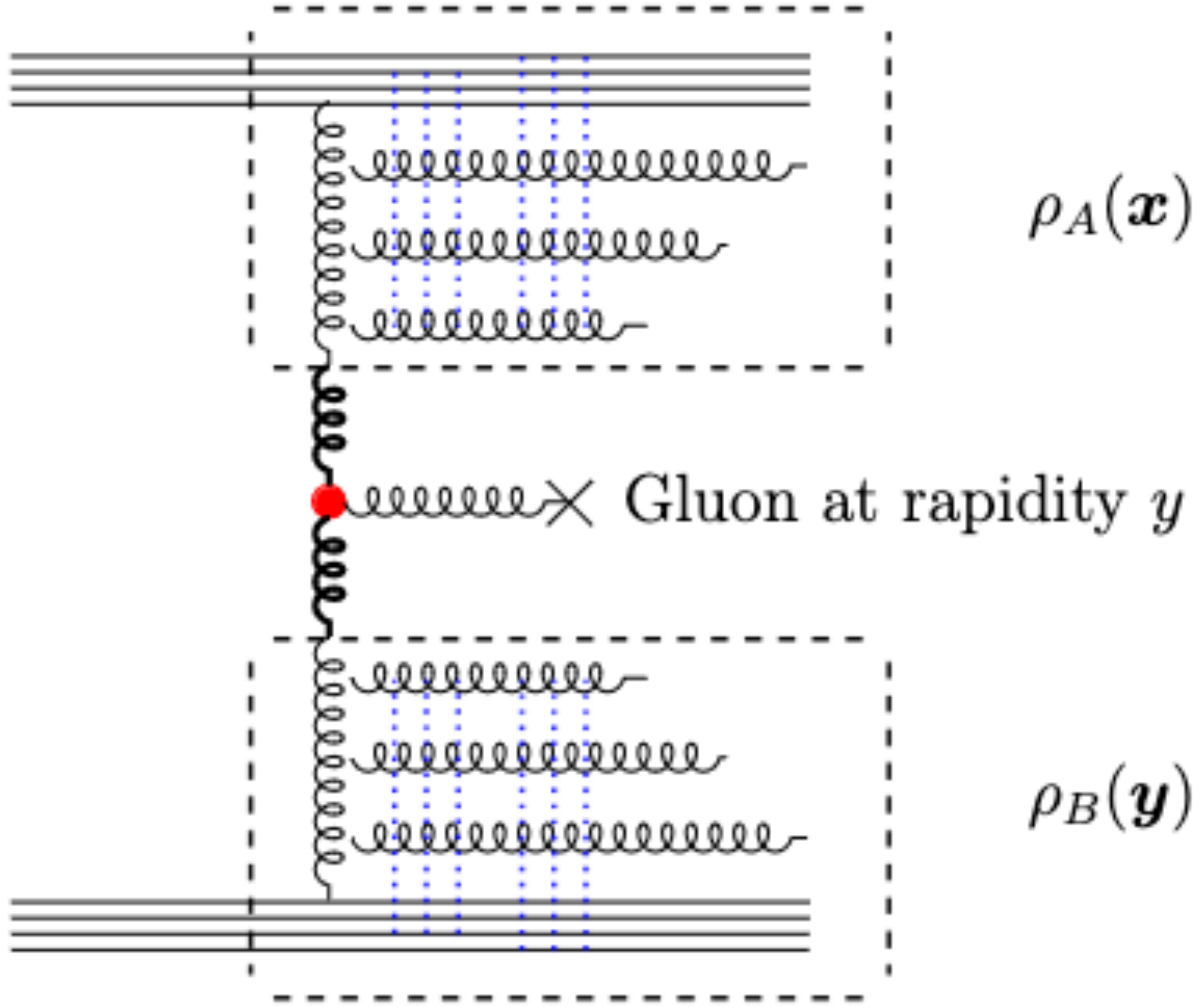
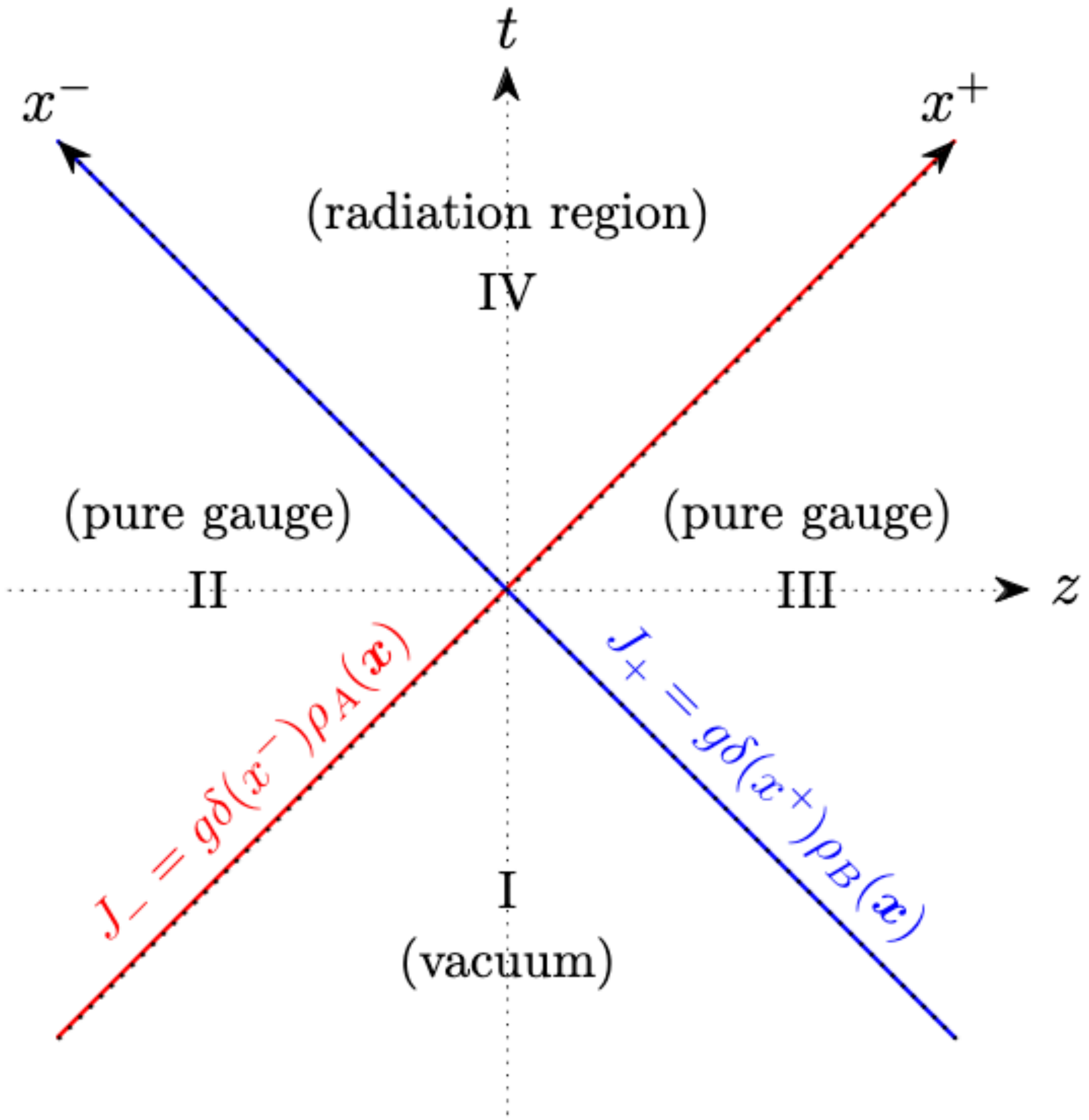


Radiation in gluon shockwave collision?

Gluon shockwave scattering



collision of two nuclei at sufficiently high energies can be approximated by gluon shockwave collisions

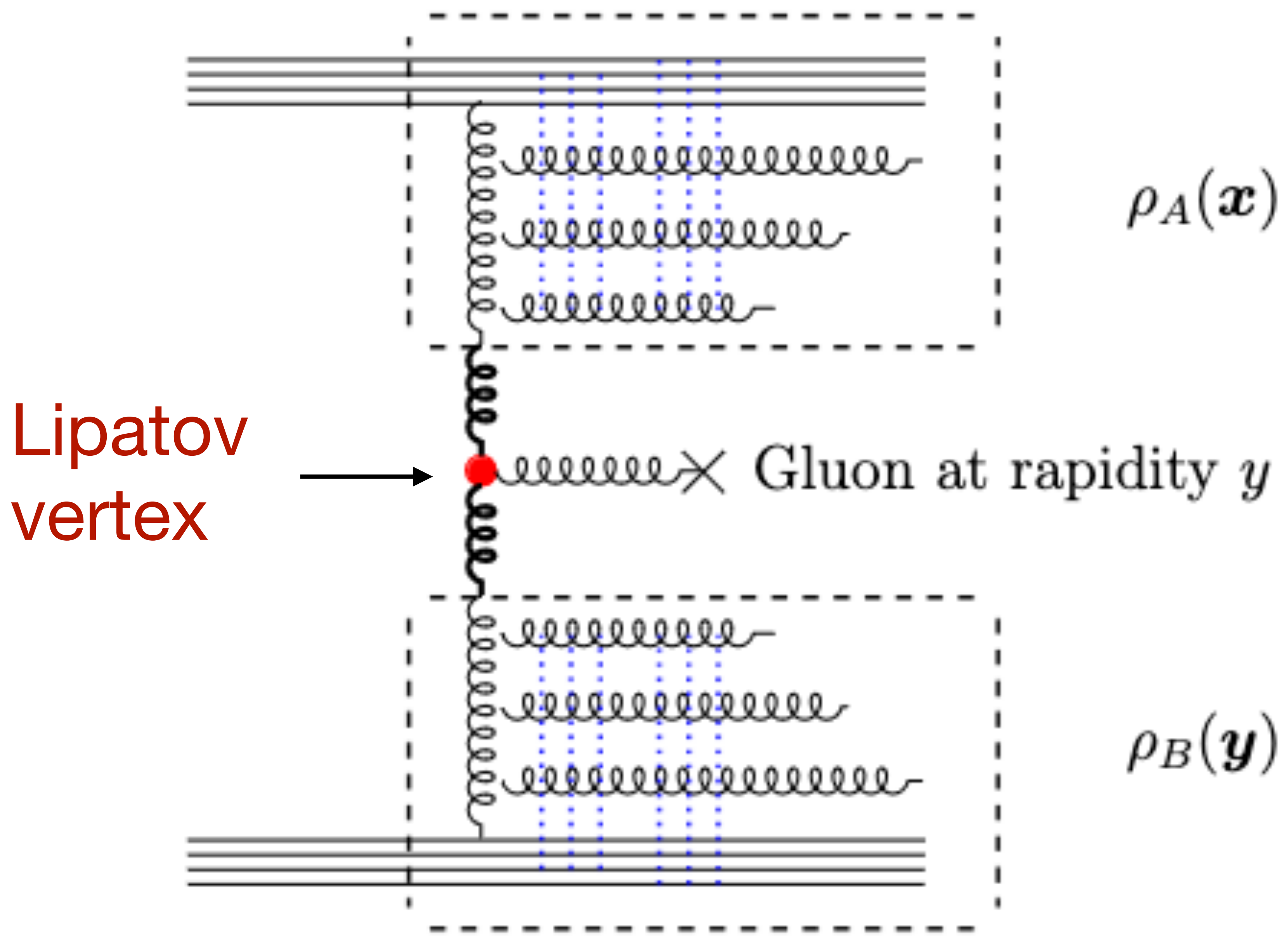


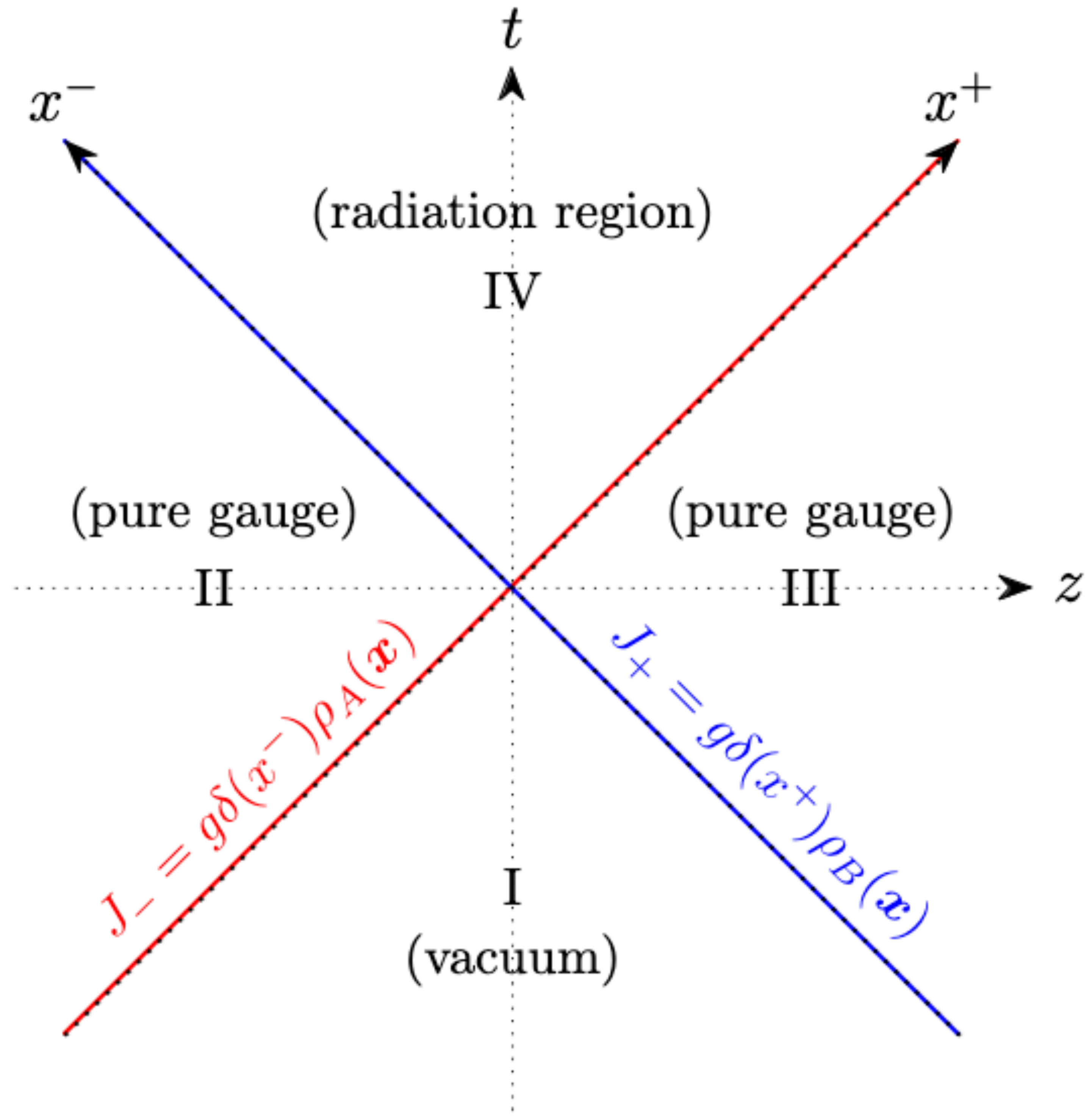
Radiation in gluon shockwave collision?

Can be computed from classical YM equation

[Kovner, Mclerran, Weigert]
[Krasnitz, Venugopalan]

Gluon shockwave scattering





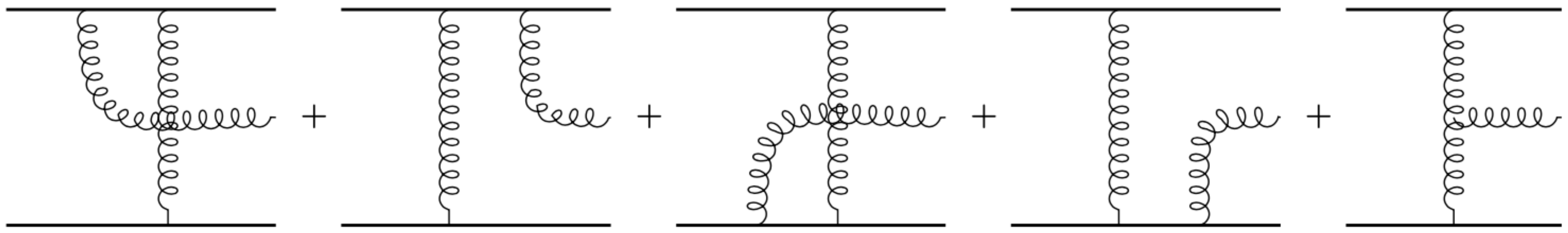
Solving classical equations of motion $\Longrightarrow$ perturbative solution for the radiation field

Lipatov vertex

$$a_i^a(k) = -\frac{ig^3}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_A(\boldsymbol{q}_1)}{\boldsymbol{q}_1^2} \frac{\rho_B(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} f^{abc} T_b T_c$$

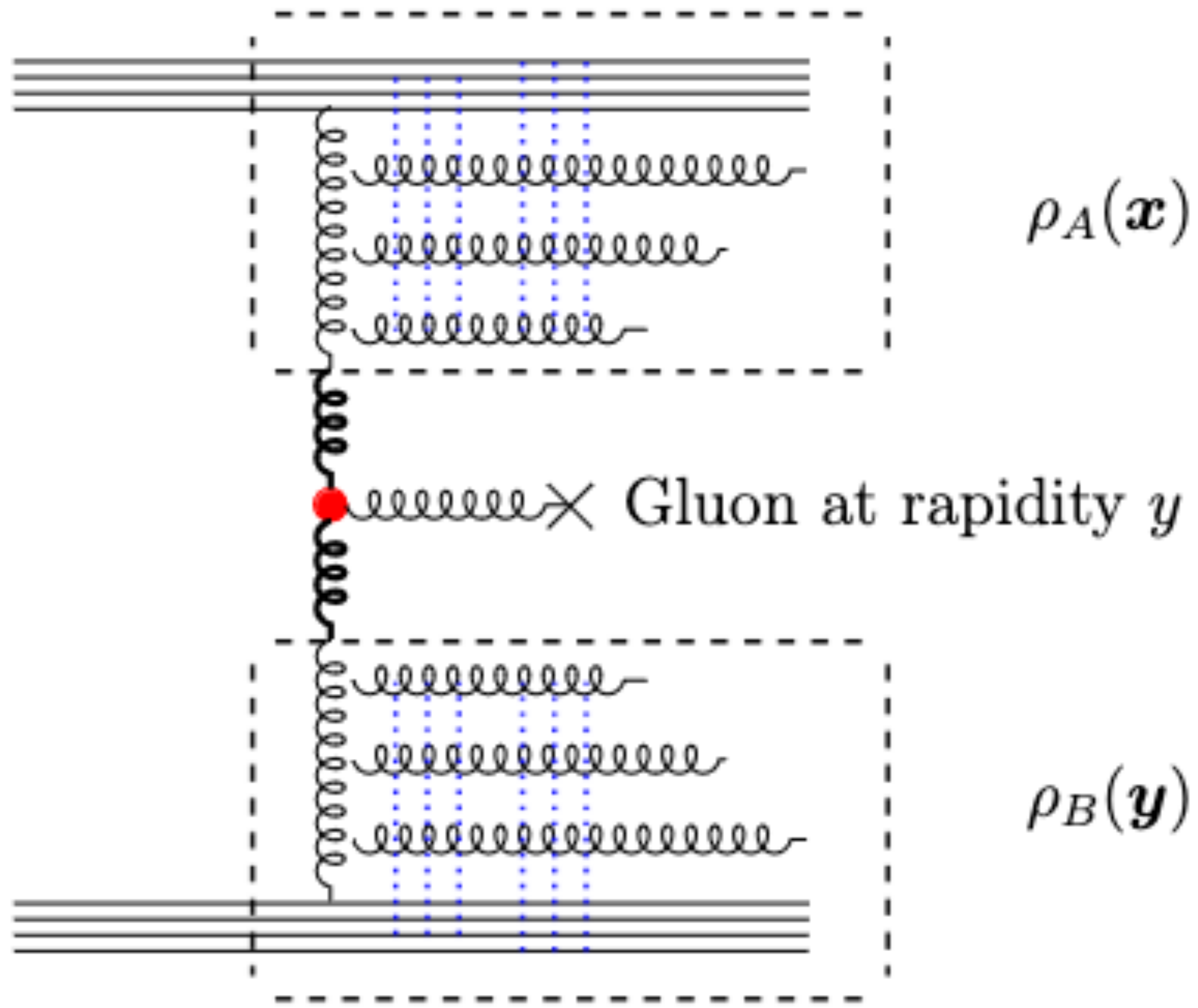
Gluon shockwave scattering

Upshot:



pQCD

equivalent in the dilute regime

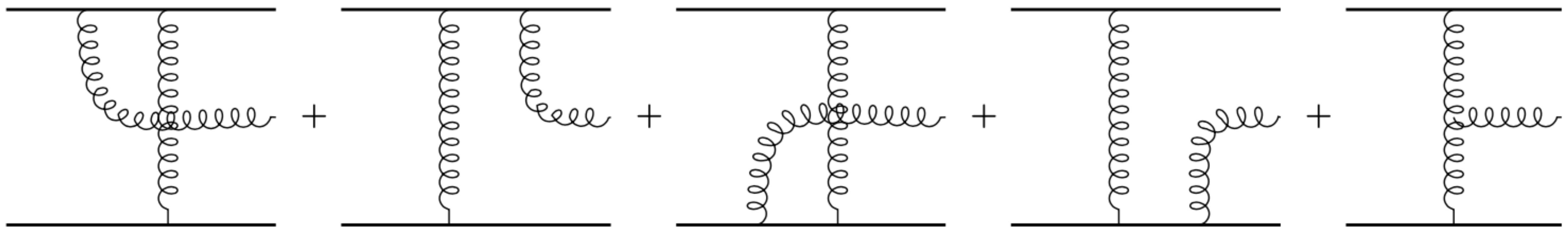


powerful framework for the high occupancy regime

Shockwave formalism

Gluon shockwave scattering

Upshot:



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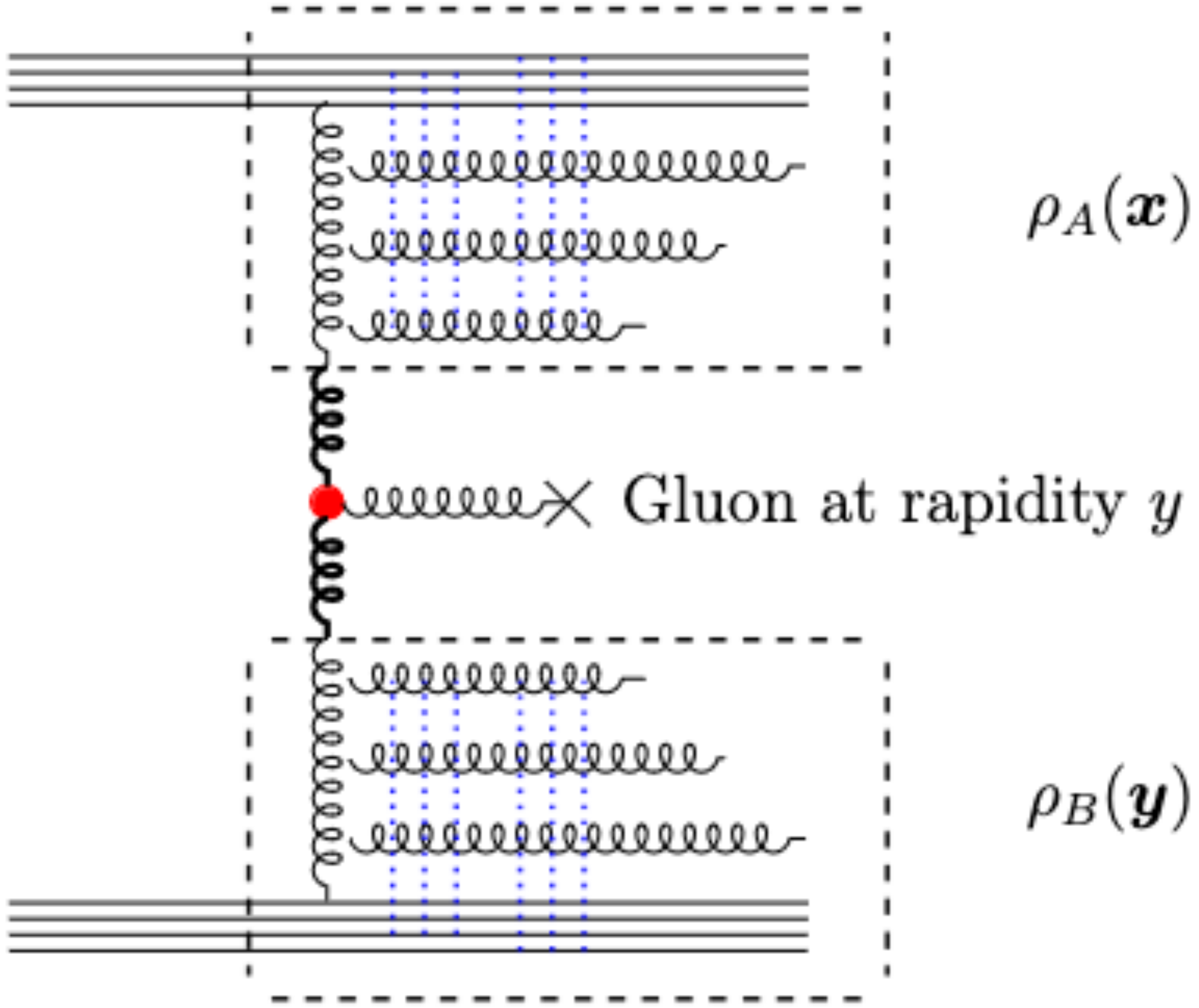


Efficient access to the dense regime: Balitsky-Kovchegov evolution equation that lead to unitarization the BFKL Pomeron via saturation!

Shockwave formalism

Gravity in the high energy regime

Scales and dimensionful constants in the problem:

$$\frac{\kappa^2}{8\pi} = G = \frac{1}{M_p^2} = \ell_p^2 \qquad R_S \equiv G\sqrt{s} = \frac{\sqrt{s}}{M_p^2} = \frac{2M}{M_p^2} \qquad b$$

Newton's constant   Schwarzschild radius   Impact parameter

Trans-Planckian scattering regime:

$$Gs = \frac{4M^2}{M_p^2} \gg 1 \qquad \ell_p \ll R_s \ll b$$

We organize an expansion in dimensionless ratios:

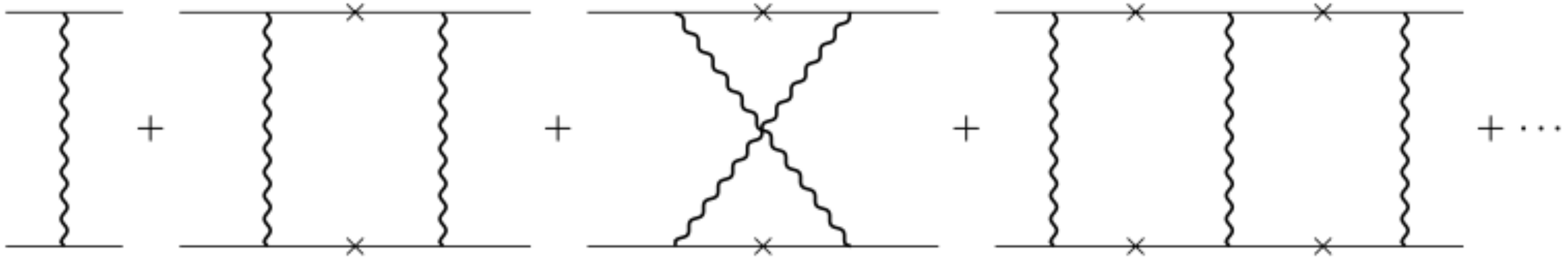
$$\left(\frac{R_s}{b}\right)^m \left(\frac{\ell_p}{b}\right)^n$$

Classical corrections   quantum gravity corrections

$$n = 0,\ m = 0,2,4,\ldots$$

Gravity in the high energy regime

$2 \to 2$ Regge amplitude at large impact parameter $(b \gg R_s)$ eikonalizes

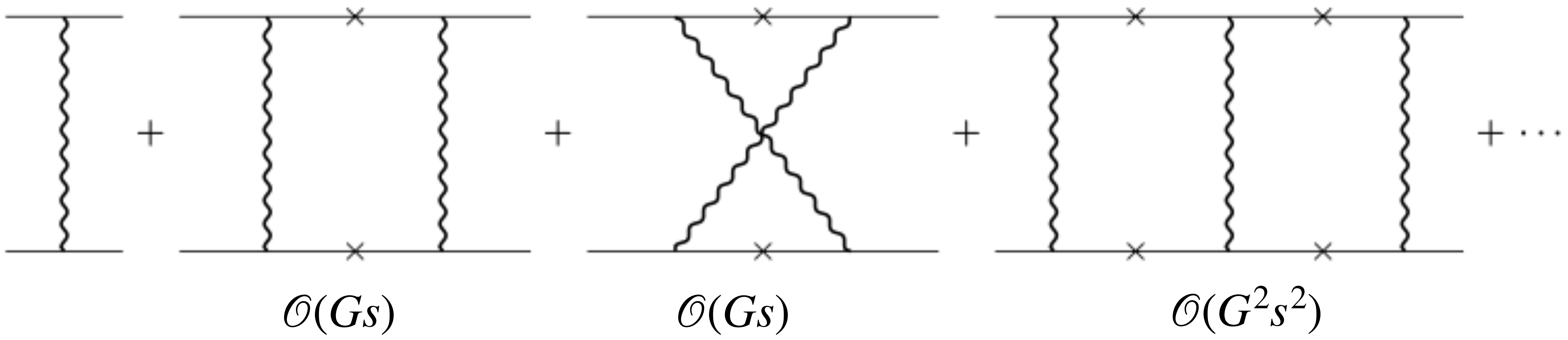


$\mathcal{O}(Gs)$ $\mathcal{O}(Gs)$ $\mathcal{O}(G^2 s^2)$

[Muzinich, Soldate]
[Kabat, Ortiz]
[Amati, Ciafaloni, Veneziano]

Gravity in the high energy regime

$2 \to 2$ Regge amplitude at large impact parameter $(b \gg R_s)$ eikonalizes



[Muzinich, Soldate]
[Kabat, Ortiz]
[Amati, Ciafaloni, Veneziano]

$$i\mathcal{M}_{\text{Eik}} = 2s \int d^2\boldsymbol{b}\, e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right)$$

eikonal amplitude

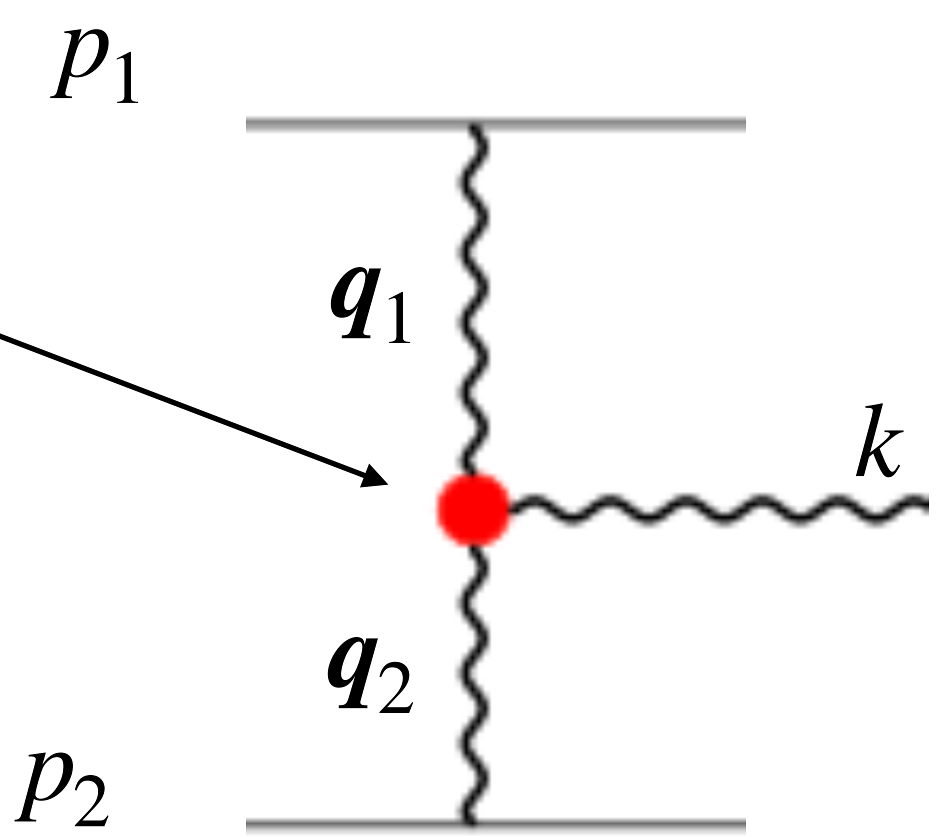
$$\chi(\boldsymbol{b}, s) = \frac{\kappa^2 s}{2} \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2} e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$$

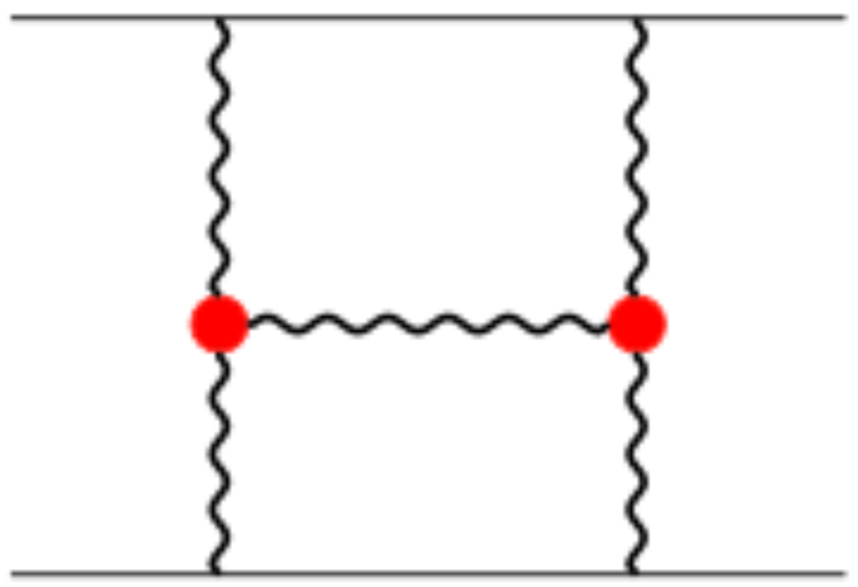
eikonal phase

Gravity in the high energy regime

At $O(R_s^2/b^2)$ we have contributions from H diagrams

Gravitational Lipatov vertex
$\Gamma_{\mu\nu}$





leading correction to the eikonal scattering series

[Amati, Ciafaloni, Veneziano]

$$\Gamma_{\mu\nu}\left(\boldsymbol{q}_1,\boldsymbol{q}_2\right)=\frac{1}{2}C_\mu\left(\boldsymbol{q}_1,\boldsymbol{q}_2\right)C_\nu\left(\boldsymbol{q}_1,\boldsymbol{q}_2\right)-\frac{1}{2}N_\mu\left(\boldsymbol{q}_1,\boldsymbol{q}_2\right)N_\nu\left(\boldsymbol{q}_1,\boldsymbol{q}_2\right)$$ [Lipatov]

two copies of
QCD Lipatov vertex

soft photon factors

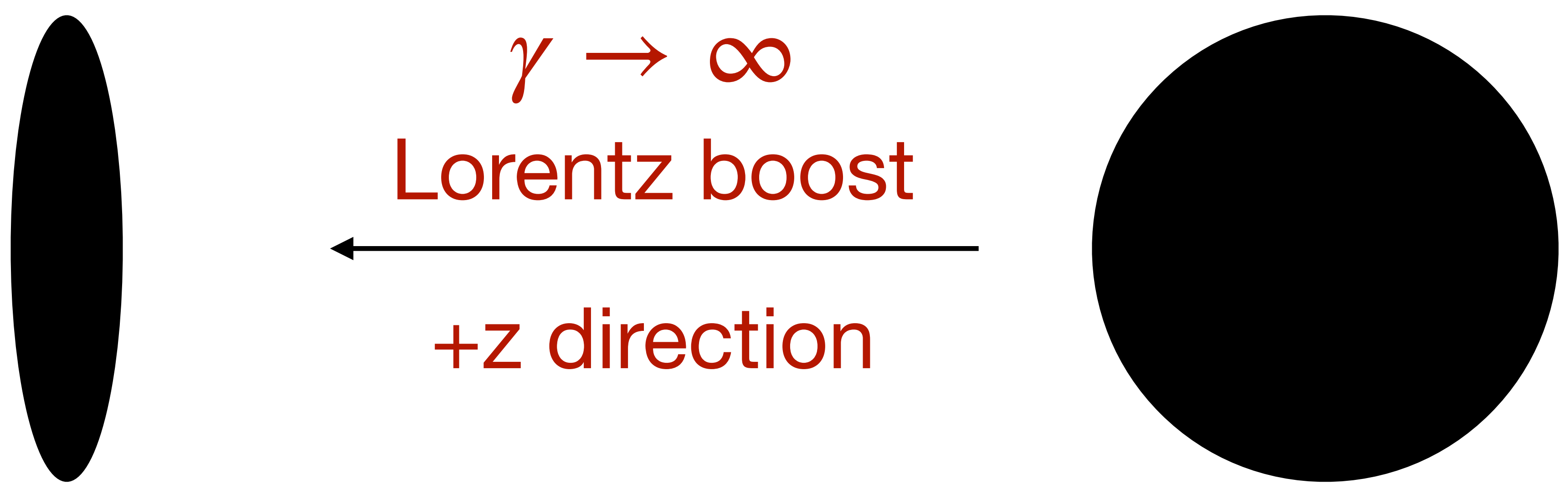
$$N_\mu\left(\boldsymbol{q}_1,\boldsymbol{q}_2\right)=\sqrt{\boldsymbol{q}_1^2\boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1\cdot k}-\frac{p_{2\mu}}{p_2\cdot k}\right)$$

Required for cancellation of simultaneous overlapping poles

$$C_\mu\left(\boldsymbol{q}_1,\boldsymbol{q}_2\right)\simeq-\boldsymbol{q}_{1\mu}+\boldsymbol{q}_{2\mu}+p_{1\mu}\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_{2\mu}\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)$$

Gravitational shockwave

Analogous connection to gravitational shockwaves obtained by boosting a Schwarzschild black hole [Aichelburg, Sexl]



The sheet with transverse profile $\rho(\boldsymbol{x})$ travels in the x^+direction while sitting at $x^- = 0$

$$T_{--} = \mu\delta\left(x^-\right)\rho(\boldsymbol{x})$$

$$\mu = \lim_{\gamma\to\infty} m_{BH}\gamma$$

Gravitational shockwave

Gravitational shockwave is an exact solution to Einstein equations with covariantly conserved source

$$T_{--} = \mu\delta\,(x^-)\,\rho(\boldsymbol{x})$$

shockwave takes the form [Aichelburg, Sexl]

$$ds^2 = 2dy^+dy^- - g_{ij}dy^i dy^j$$

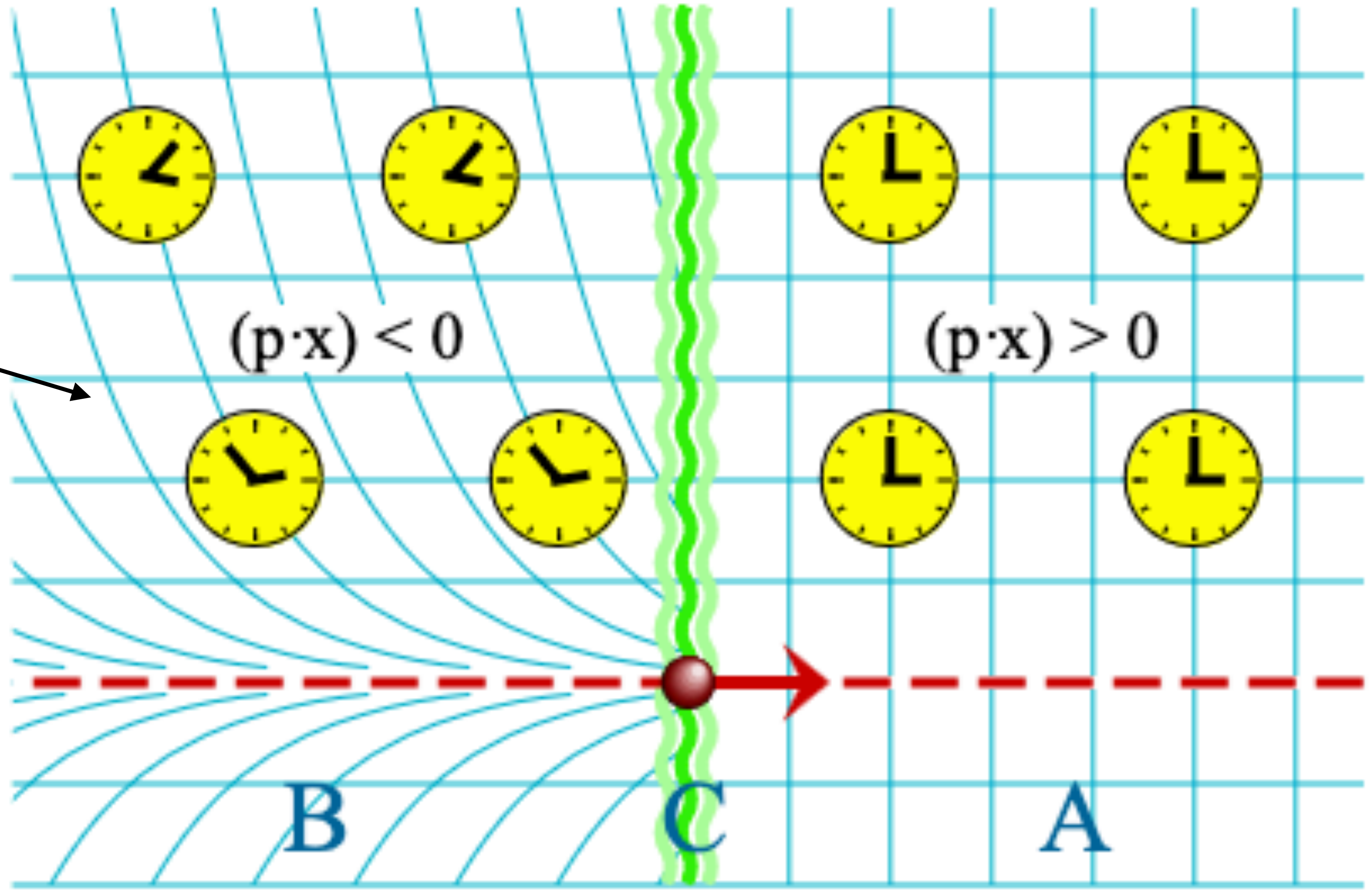
$$g_{ij} = \delta_{ij} - y^-\Theta\left(y^-\right)\left[2\kappa^2\mu\frac{\partial_i\partial_j}{\Box_\perp}\rho(\boldsymbol{y}) - \kappa^4\mu^2 y^-\left(\frac{\partial_i\partial_k}{\Box_\perp}\rho(\boldsymbol{y})\right)\left(\frac{\partial_j\partial_k}{\Box_\perp}\rho(\boldsymbol{y})\right)\right]$$

Curvature vanishes on either sides ($x^- > 0,\ x^- < 0$) but the vacuum is not the same

Gravitational shockwave

the metric is flat

$R_{ijkl} = 0$



4. Snapshot of the gravitational shock wave caused by a highly relativistic particle. If we have a rectangular grid and synchronized clocks before the particle passed by (region A), then, behind the particle (region B), the grid will be deformed, and the clocks desynchronized. The shift is proportional to the logarithm of the transverse distance.

['t Hooft, Dray]

A color-kinematic double copy relation for shockwaves

There exists (in an appropriate gauge) a double copy relation between gluon and gravitational shockwaves

[Akhoury, Saotome]

$$\frac{1}{g}A^a_\mu = iq_\mu T^a \frac{\delta(x^-)}{2q_0}\int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\frac{1}{\mathbf{k}_\perp^2} \qquad T^a \to iq_\mu \qquad \frac{1}{\kappa}g_{\mu\nu} = -\, q_\mu q_\nu \frac{\delta(x^-)}{2q_0}\int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\frac{1}{\mathbf{k}_\perp^2}$$

Does such / similar relationship also exists for radiation produced in shockwave collisions?

Gravitational shockwave scattering

Solving classical equations of motion $\Longrightarrow$ perturbative solution for the radiation field

Gravitational Lipatov vertex

$$\tilde{h}_{ij}^{(2)}(k) = \frac{\kappa^3 s}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \Gamma_{ij}\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right) \frac{1}{\boldsymbol{q}_1^2} \frac{1}{\boldsymbol{q}_2^2}$$

$$\Gamma_{\mu\nu}\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right) = \frac{1}{2} C_\mu\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right) C_\nu\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right) - \frac{1}{2} N_\mu\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right) N_\nu\left(\boldsymbol{q}_1, \boldsymbol{q}_2\right)$$

[HR, Venugopalan]

Gravitational shockwave scattering

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[HR, Venugopalan]

Compare this with classical radiation field in gluon shockwave collision

$$a^a_i(k) = -\frac{ig^3}{k^2 + i\epsilon k^-}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}C_i(\boldsymbol{q}_1,\boldsymbol{q}_2)\frac{1}{\boldsymbol{q}_1^2}\frac{1}{\boldsymbol{q}_2^2}f^{abc}T_bT_c$$

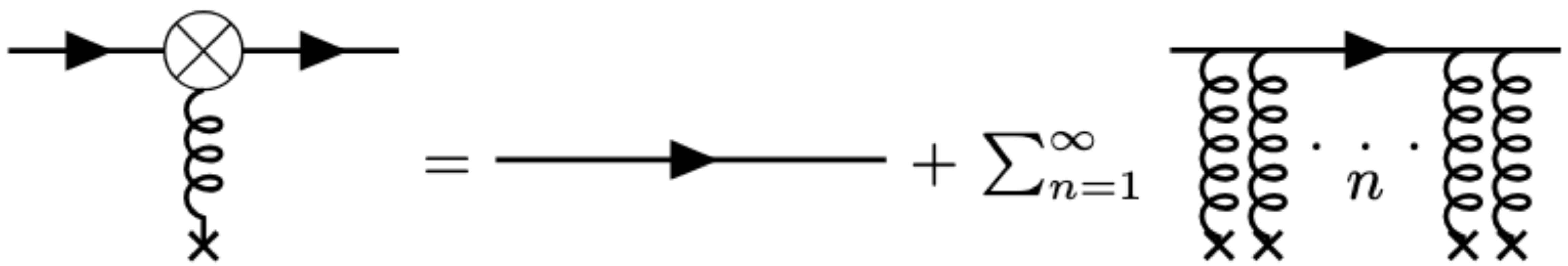
$$-if^{abc}T_bT_cC_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2) \xleftrightarrow[\text{CK relation}]{\text{There also exists a}} s\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2)$$

[HR, Venugopalan]

Propagators in shockwave background

Propagators of various quantum fields in background of gluon and graviton shockwaves also satisfy double-copy relations

$$\tilde{G}_R\left(p,p'\right) = \tilde{G}^0_R(p)(2\pi)^4\delta^{(4)}\left(p-p'\right) + \tilde{G}^0_R(p)\mathscr{T}\left(p,p'\right)\tilde{G}^0_R\left(p'\right)$$



The dependence on the shockwave is captured by the $\mathscr{T}(p,p')$ function given in terms of light-like Wilson line that satisfy a double-copy relation between gauge theory and gravity [Melville, Naculich, Schnitzer, White], [HR, Venugopalan]

$$U(\boldsymbol{x}) = P\exp\left(ig\int dz^- A_-(z^-,\boldsymbol{x})\cdot T\right)$$

QCD

$$U(\boldsymbol{x}) = P\exp\left(\frac{1}{2}\int dz^- g_{--}(z^-,\boldsymbol{x})\cdot\partial_+\right)$$

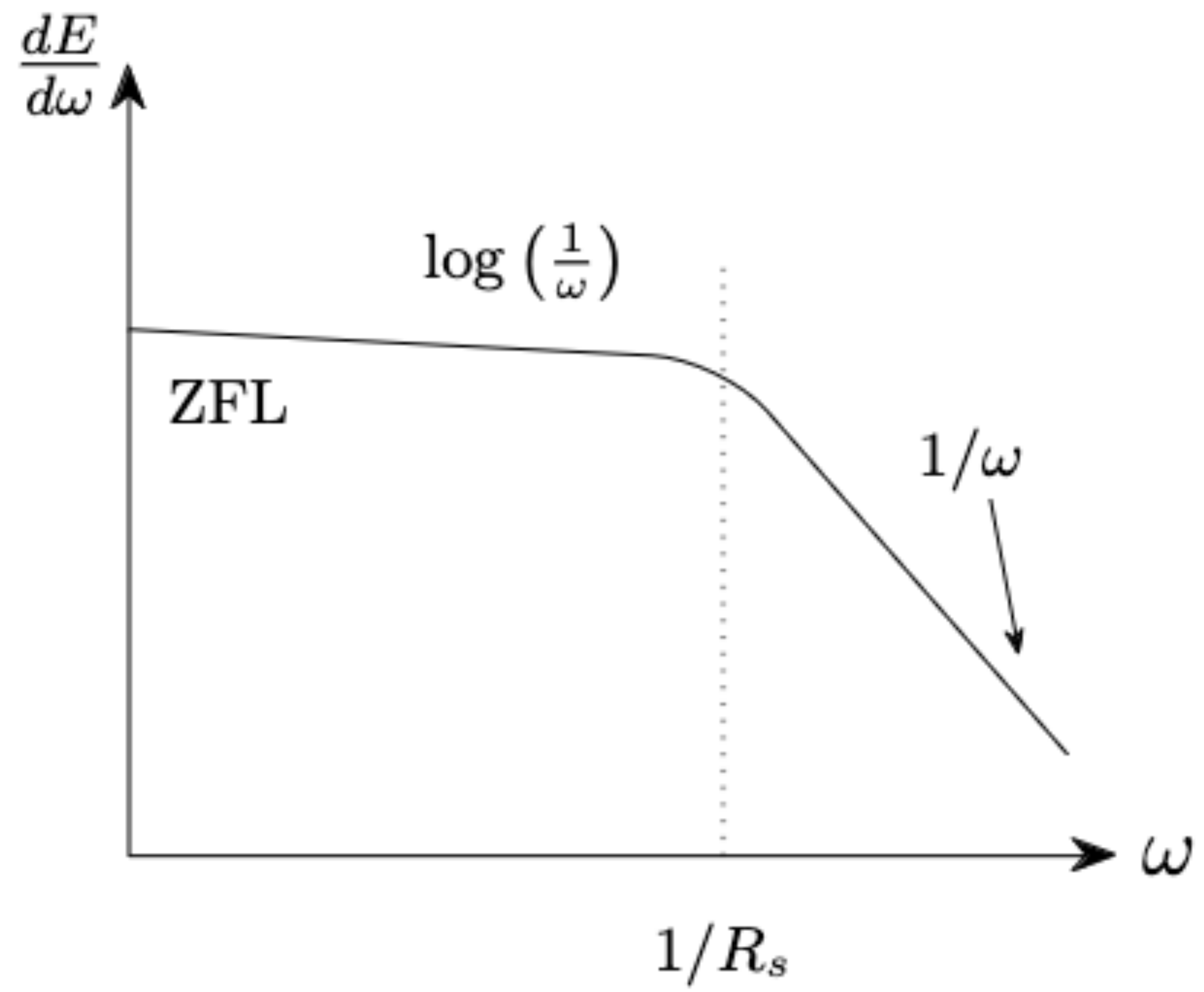
Gravity

Conclusion

These results can be used to compute the spectrum for gravitational bremsstrahlung

[Gruzinov, Veneziano]
[Ciafaloni, Colferai, Veneziano]
[Ciafaloni, Colferai, Coradeschi, Veneziano]

$$\frac{dE^{\mathrm{GW}}}{d\omega d\Omega} = \frac{1}{2\pi^2}\omega^2 \sum_\lambda \left| \mathscr{M}^{(\lambda)} \right|^2 \qquad \mathscr{M}^{(\lambda)} = k^2 \tilde{h}^{(2)}_{ij}(k)\varepsilon^{(\lambda)}_{ij}$$



gravitational radiation
frequency spectrum

Conclusion

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[Gruzinov, Veneziano]
[Ciafaloni, Colferai, Veneziano]
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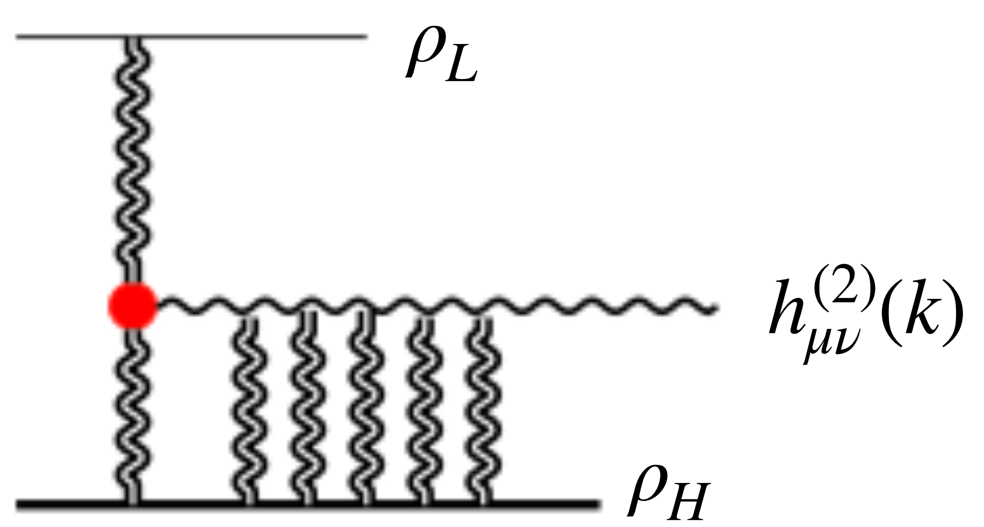
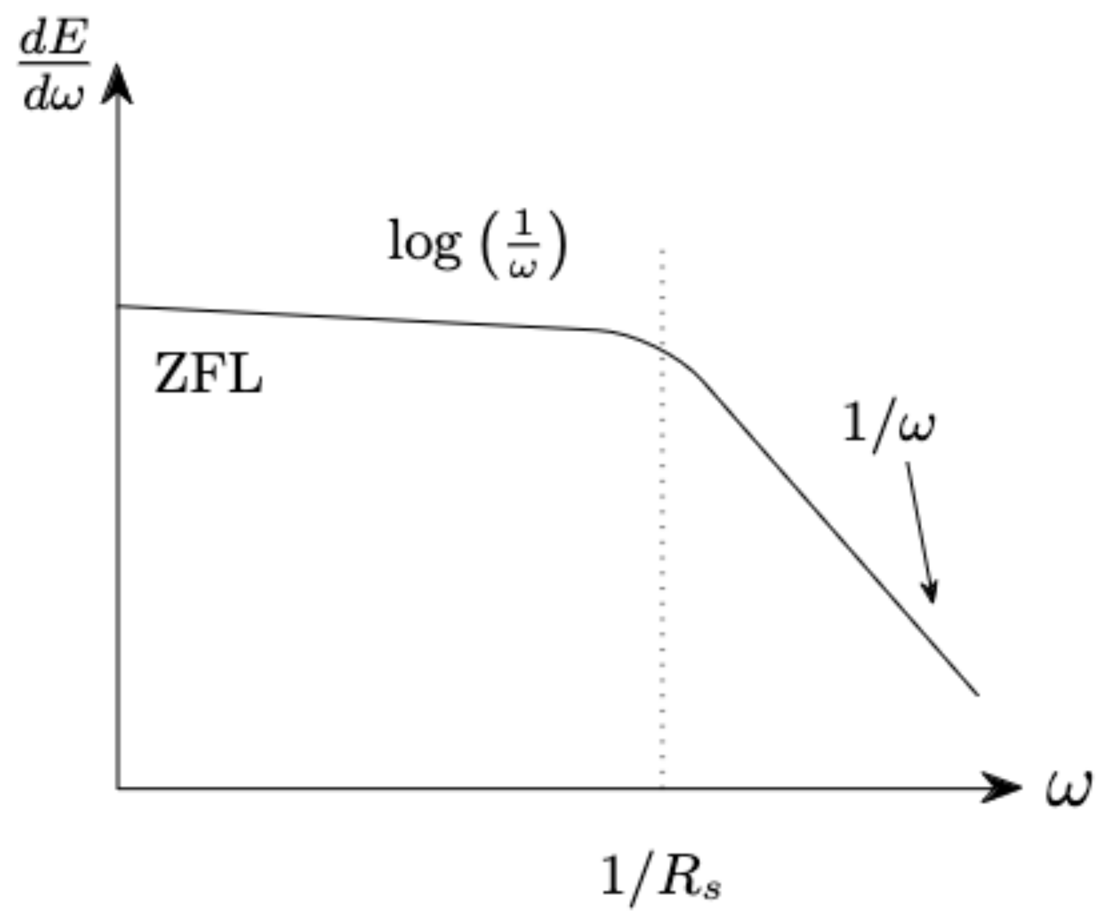
Because of the dilute-dilute approximation, this result does not account for multiple re-scattering effects of the emitted radiation off the dense source





gravitational radiation frequency spectrum

This calls for a dilute-dense analysis. Double copy ?

Conclusion

Need for non-trivial evolution of sources in the transverse direction for dilute dense analysis (Raychaudhuri equation)

More generally: CK duality for amplitudes in shockwave spacetime

$2 \to N$ differential cross-section and radiation spectrum using the shockwave description and the BFKL framework [HR, Stasto, Venugopalan] work in progress

Question 1: A sharp prediction for the high-frequency radiation spectrum possibly accessible at next generation GWOs

Question 2: In QCD, the shockwave formalism immensely facilitated the derivation of the Balitsky-Kovchegov RG (that unitarizes the BFKL pomeron) whose fixed point is the CGC. Does an analogous RG hold in gravity and can black-hole formation be seen as a fixed point of such an RG?

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Thank you for your attention