Anisotropic Electrical Conductivity in Rotating Hadron Gas

Asian Triangle Heavy-Ion Collision (ATHIC 2025)

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Introduction

- In non-central Heavy Ion Collisions, a large magnetic field as well as angular velocity can be produced.
  F. Becattini, F. Piccinini, and J. Rizzo, Phys. Rev. C 77, 024906 (2008)

$\Longrightarrow$ $\sqrt{s_{NN}} = 200\,\text{GeV}$, $L_0 = 5 \times 10^5\,\hbar$ at an impact parameter $b = 5\,\text{fm}$.
Even 1% angular momentum transfer $\Rightarrow 10^3\,\hbar$ angular momentum of QGP.

- The QGP exhibits a significantly high average vorticity.
- Then this vorticity is effectively transferred to the hadronic matter as the system undergoes hadronization.
- **Question:**
  Is it possible for rotation to alter the transport coefficients of nuclear matter?
- Here, I discuss how rotation affects electrical conductivity in hadronic matter.

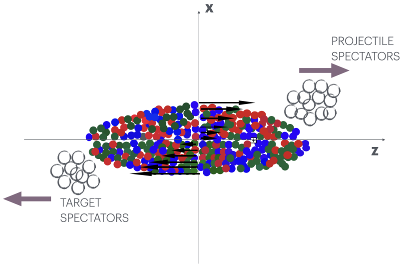


Figure: Schematic diagram of Heavy Ion Collision.

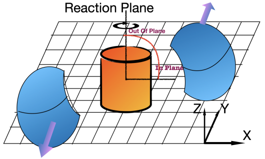


Figure: Geometry of non-central heavy ion Collision.

Motivation

- In 2017, the STAR collaboration has seen the experimental evidence of vorticity (Nature 548 (2017) 62-65).

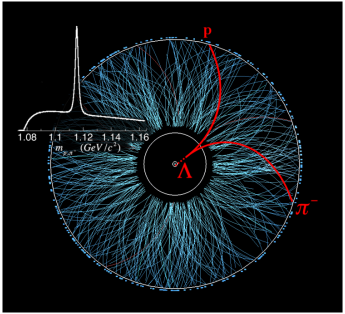


Figure: Source: Nature 548 (2017) 62-65

- In Phys.Rev.C 109 (2024) 3, 034914, the authors have studied the how the rotation affects the electrical conductivity in quark matter in a non-relativistic framework.
- In the present work we have tried to explore the electrical conductivity of hadronic matter in the relativistic framework.

Formalism

The current density representations of hadrons at the microscale and macroscopic levels under an applied electric field ($\vec{\tilde{E}} \equiv \tilde{E}\hat{e}$) are expressed as:

$$J^i = \sum_r J_r^i = \sum_r g_r q_r \int \frac{d^3\vec{p}_r}{(2\pi)^3} \frac{p_r^i}{E_r} \delta f_r = \sum_r \sigma_r^{ij} \tilde{E}_j \equiv \sigma^{ij} \tilde{E}_j, \quad (p_{r0} \equiv E_r) \tag{1}$$

(Here, δf_r represents the deviation of the system from local equilibrium.)
The equation of motion (EOM) of a hadron in a rotating frame can be expressed using connection coefficients ($\Gamma^{\alpha}_{\mu\lambda}$) as:

$$\frac{dp^{\alpha}}{d\tau} + \frac{1}{m} p^{\mu} p^{\lambda} \Gamma^{\alpha}_{\mu\lambda} = F^{\alpha}. \tag{2}$$

To see the similarity with the classical non-relativistic EOM in the rotating frame, the equation can be expressed as:

$$\frac{d\vec{p}}{dt} = \gamma_v m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} + 2\gamma_v m(\vec{v} \times \vec{\Omega}). \tag{3}$$

Key Forces in a Rotating Frame:

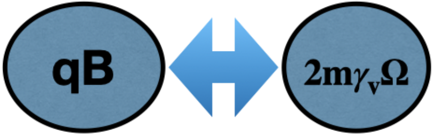
Coriolis force: $-2\gamma_v m(\vec{\Omega} \times \vec{v})$, Centrifugal force: $-\gamma_v m \left[(\vec{r} \times \vec{\Omega}) \times \vec{\Omega} \right]$.

Formalism (Contd.)

In our study, we focus solely on the Coriolis force and its effect on the transport characteristics of the system.

$$\vec{F}_{\text{Lor}} = q(\vec{v} \times \vec{B}), \quad \vec{F}_{\text{Cor}} = 2\gamma_v m(\vec{v} \times \vec{\Omega}).$$

Substituting qB with $2m\gamma_v\Omega$, the two forces become mathematically analogous:



Comparison: Lorentz Force vs. Coriolis Force

- **Lorentz force**: Arises from electromagnetic interactions (q, B).
- **Coriolis force**: Due to rotation of the reference frame (m, Ω).

Formalism (Contd.)

We use the covariant **Boltzmann Transport Equation (BTE)** to study statistical effects:

$$m_r \frac{df_r(x^\mu, \vec{p}_r)}{d\tau} = C[f_r] \implies p_r^\mu \frac{\partial f_r}{\partial x^\mu} - \Gamma^i_{\mu\lambda} p_r^\mu p_r^\lambda \frac{\partial f_r}{\partial p_r^i} + m_r F_r^i \frac{\partial f_r}{\partial p_r^i} = C[f_r]. \tag{4}$$

For systems slightly out of equilibrium, $f_r = f_r^0 + \delta f_r$, where:

$$f_r^0 = \frac{1}{[e^{(p_r^\alpha u_\alpha - \mu)/T} - \xi]}.$$

($\xi = -1$ for baryons, $\xi = +1$ for mesons.)

In the Relaxation Time Approximation (RTA):

$$p_r^\mu \frac{\partial f_r^0}{\partial x^\mu} - \Gamma^\alpha_{\mu\lambda} p_r^\mu p_r^\lambda \frac{\partial}{\partial p_r^\alpha}(f_r^0 + \delta f_r) + q_r F^{\alpha\beta} p_{r\beta} \frac{\partial}{\partial p_r^\alpha}(f_r^0 + \delta f_r) = -\frac{(u^\alpha p_\alpha)\delta f_r}{\tau_c}. \tag{5}$$

After solving:

$$\frac{\partial f^0}{\partial E} \frac{\vec{p}}{p_0} \cdot (q\vec{\tilde{E}}) + 2m\gamma_v(\vec{v} \times \vec{\Omega}) \cdot \frac{\partial \delta f}{\partial \vec{p}} = -\frac{\delta f}{\tau_c}.$$

Formalism (Contd.)

In a rotating HRG, the angular velocity $\vec{\Omega} = \Omega\hat{\omega}$ introduces anisotropy. To solve the BTE, we take the initial guess as:

$$\delta f = -\vec{p}\cdot\vec{X}\left(\frac{\partial f^0}{\partial E}\right),$$

with $\vec{X}$ expressed as:

$$\vec{X} = \alpha\hat{e} + \beta\hat{\omega} + \gamma(\hat{e}\times\hat{\omega}),$$

After solving the values of α, β, and γ, the current density can be expressed as:

$$J_r^i = -g_r q_r^2 \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{p^i p^l}{E^2}\frac{\partial f_r^0}{\partial E}\left(\frac{\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\right)\left[\delta_{jl} + \left(\frac{\tau_c}{\tau_\Omega}\right)^2\omega_j\omega_l + \left(\frac{\tau_c}{\tau_\Omega}\right)\epsilon_{ljk}\omega_k\right]\tilde{E}_j. \quad (6)$$

Comparing the macroscopic expression for Ohm's law, $J_r^i = \sigma_r^{ij}\tilde{E}_j$, the electrical conductivity can be expressed as:

$$\implies \sigma^n = \sum_r \frac{g_r q_r^2}{3T}\int \frac{d^3p}{(2\pi)^3}\frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)^n}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\frac{p^2}{E^2}f_r^0(1+\xi f_r^0).$$

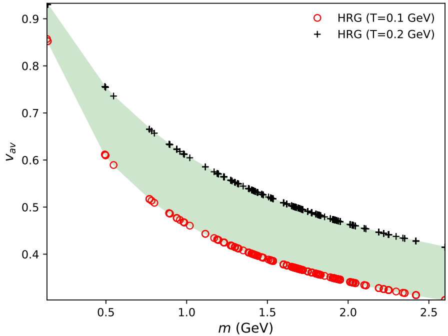
Result and Discussion

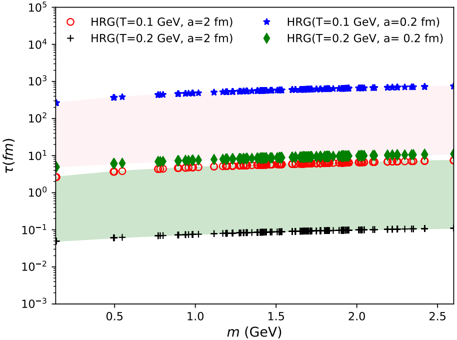
For determining the relaxation time, we adopt the **Hadron Resonace Gas (HRG)** model

$$\tau_c = 1/(n^{id}_{\rm HRG} v^2_{\rm av} \pi a^2) \ , \quad (7)$$

Here, $n^{id}_{\rm HRG}$ represents the system's number density, the parameter **'a'** is taken from the hard sphere scattering model, and the velocity is defined as:

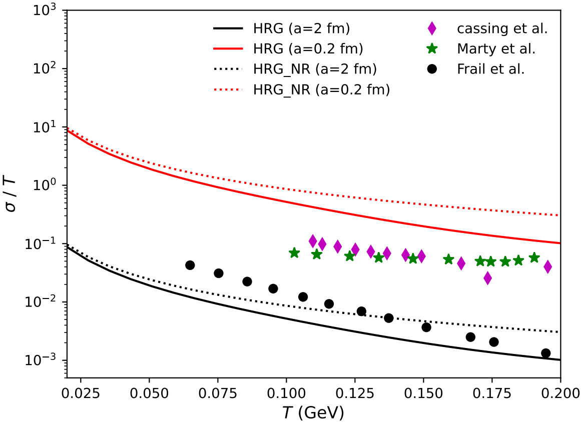
$$v_{\rm av} = \int \frac{d^3p}{(2\pi)^3} \frac{p}{E} f_0 \Big/ \int \frac{d^3p}{(2\pi)^3} f_0 \ . \quad (8)$$





Electrical Conductivity

We have shown the variation of σ/T with temperature (T). The results are compared with data from: Phys. Rev. Lett. 110, 182301 (2013), Phys. Rev. C 88, 045204 (2013), and Phys. Rev. D 73, 045025 (2006), while varying the scattering length **'a'** from 0.2–2 fm.



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- Phys. Rev. C 88, 045204 (2013): **Marty et al.**
- Phys. Rev. D 73, 045025 (2006): **Frail et al.**

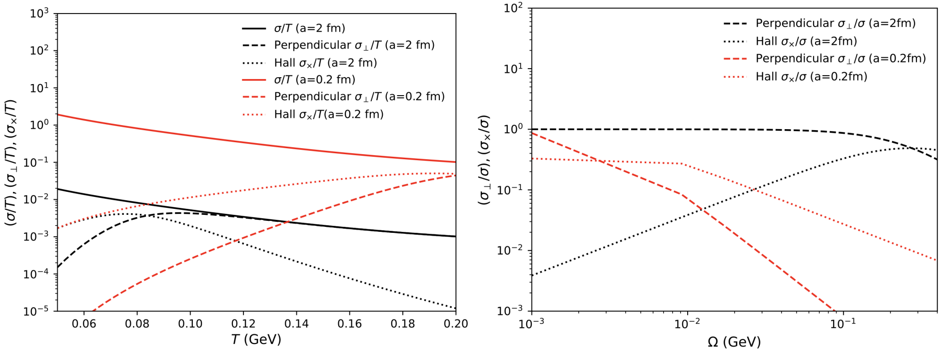
Anisotropic Electrical Conductivity

Due to the presence of Coriolis force, the electrical conductivity components split into three parts: the parallel, perpendicular, and Hall components of electrical conductivity.

$$\sigma^{||}_{\rm HRG} \equiv \sigma_{\rm HRG} = \sum_B \frac{g_B q_B^2}{3T} \int \frac{d^3p}{(2\pi)^3} \tau_c \frac{p^2}{E^2} f^0(1-f^0) + \sum_M \frac{g_M q_M^2}{3T} \int \frac{d^3p}{(2\pi)^3} \tau_c \frac{p^2}{E^2} f^0(1+f^0)$$

$$\sigma^{\perp}_{HRG} = \sum_B \frac{g_B q_B^2}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} \frac{p^2}{E^2} f^0(1-f^0) + \sum_M \frac{g_M q_M^2}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} \frac{p^2}{E^2} f^0(1+f^0)$$

$$\sigma^{\times}_{HRG} = \sum_B \frac{g_B q_B^2}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} \frac{p^2}{E^2} f^0(1-f^0) + \sum_M \frac{g_M q_M^2}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} \frac{p^2}{E^2} f^0(1+f^0)$$



N. Padhan et al.(Phys.Rev.C 110 (2024) 2, 024904)

Summary

- We have calculated the rotational effect in electrical conductivity using the Hadron Resonance gas model. We used BTE in relativistic framework using relaxation time approximation.
- We observed the anisotropic electrical conductivity tensor (σ), analogous to the Lorentz force, with the following components: **Parallel** ($\sigma^{\parallel}$): Unaffected by rotation ,**Perpendicular** ($\sigma^{\perp}$) ,**Hall** ($\sigma^{\times}$)
- We observed at low angular velocity, $\sigma^{\perp} > \sigma^{\times}$, while at high angular velocity, $\sigma^{\times} > \sigma^{\perp}$.
- We have also observed Smaller scattering lengths cause earlier transitions from $\sigma^{\perp}$ dominance to $\sigma^{\times}$ dominance.

References

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Thank You