Axion DM Detection with Superconducting Qubits

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arXiv 2212.03884 [PRL 131 (2023) 211001]

arXiv 2311.10413 [PRL 133 (2023) 021801]

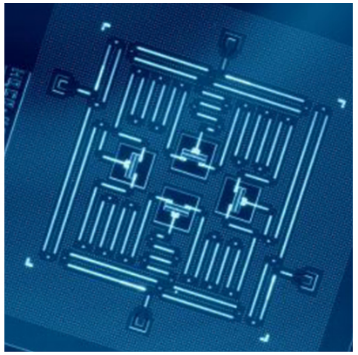
arXiv 2407.19755

Dark World to Swampland: 9th IBS-IFT Workshop, Daejeon, Korea, '24.11.05

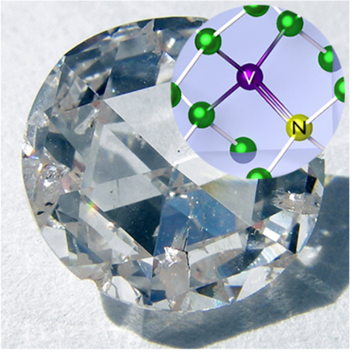
1. Introduction

Quantum technologies are rapidly developing

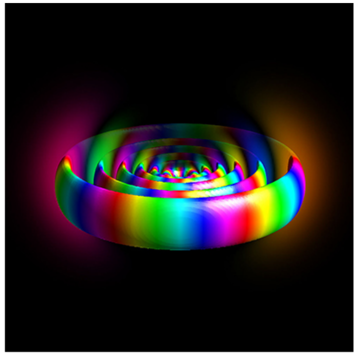
- Quantum computer is (probably) a primary driving force
- Many quantum devices are excellent quantum sensors, sensitive to external fields



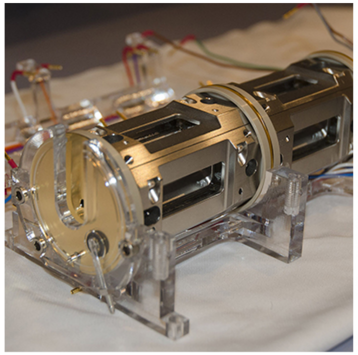
(Transmon) Qubit



NV Center



Rydberg Atom



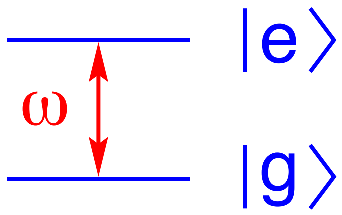
Ion Trap

and more ...

[All the pictures are from Wikipedia]

$\Rightarrow$ They can be (potentially) used to detect BSM physics

What I discuss today: Axion DM search with qubits



Qubit: Two-level quantum system

- Qubit is an essential component for quantum computers
- Various types of qubits have been proposed and realized
- Qubits are excellent quantum sensors for DM detection
  [Dixit et al. ('21); Chen, Fukuda, Inada, TM, Nitta, Sichanugrist ('22, '23, '24); Engelhardt, Bhoonah, Liu ('23); Chigusa, Hazumi, Herbschleb, Mizuochi, Nakayama ('23); Agrawal et al. ('23); Ito, Kitano, Nakano, Takai ('23); Braggio et al. ('24)]

Outline:

1. Introduction

2. Superconducting Qubit

3. DM Detection with Qubits

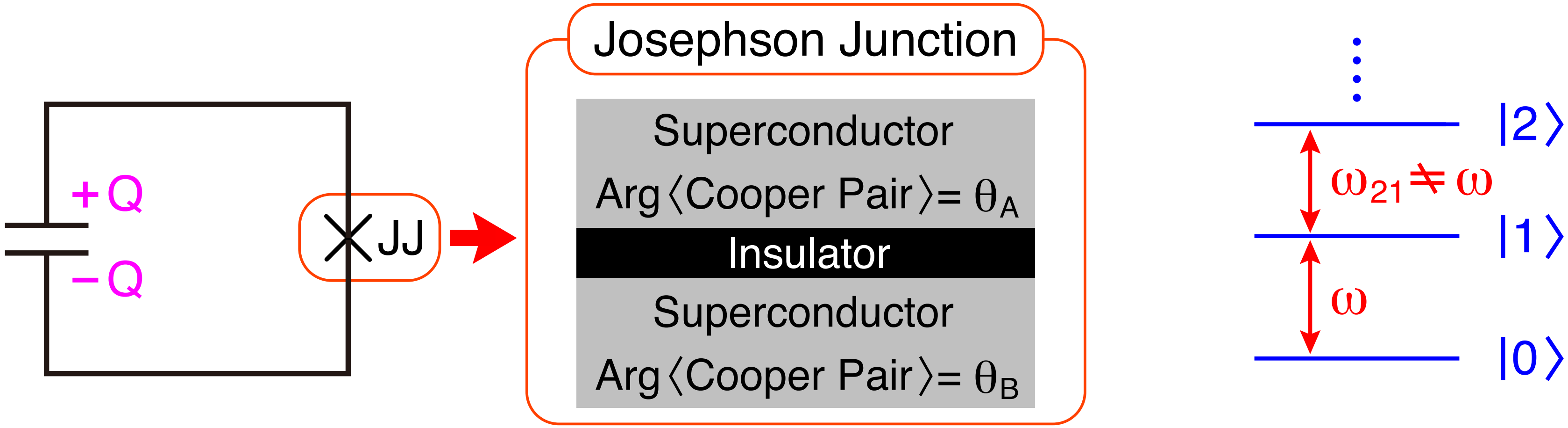
4. Experimental Status

5. Quantum Enhancement / Cavity Effect

6. Summary and Outlook

2. Superconducting Qubit

Superconducting qubit: Capacitor + Josephson junction (JJ)



$\theta = \theta_B - \theta_A$: canonical variable of this system

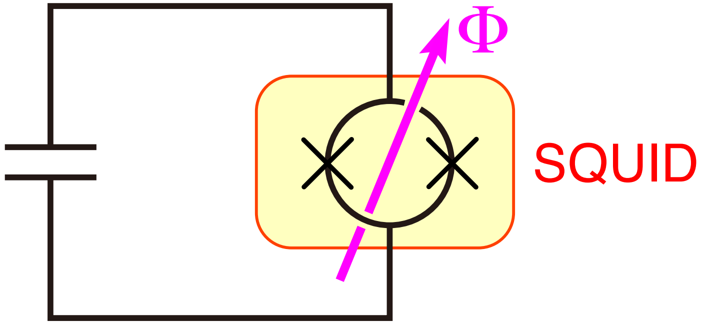
$$\Rightarrow \; H_0 = \frac{1}{2C}Q^2 - J\cos\theta \simeq \frac{1}{2}\frac{C}{(2e)^2}\dot{\theta}^2 - J\cos\theta \;\; \Leftrightarrow Q = CV \simeq C\frac{\dot{\theta}}{2e}$$

Superconducting qubit has discrete energy levels

$\Rightarrow$ $|0\rangle$ and $|1\rangle$ can be used as $|g\rangle$ and $|e\rangle$, respectively

Frequency tunability with SQUID

SQUID: superconducting quantum interference device

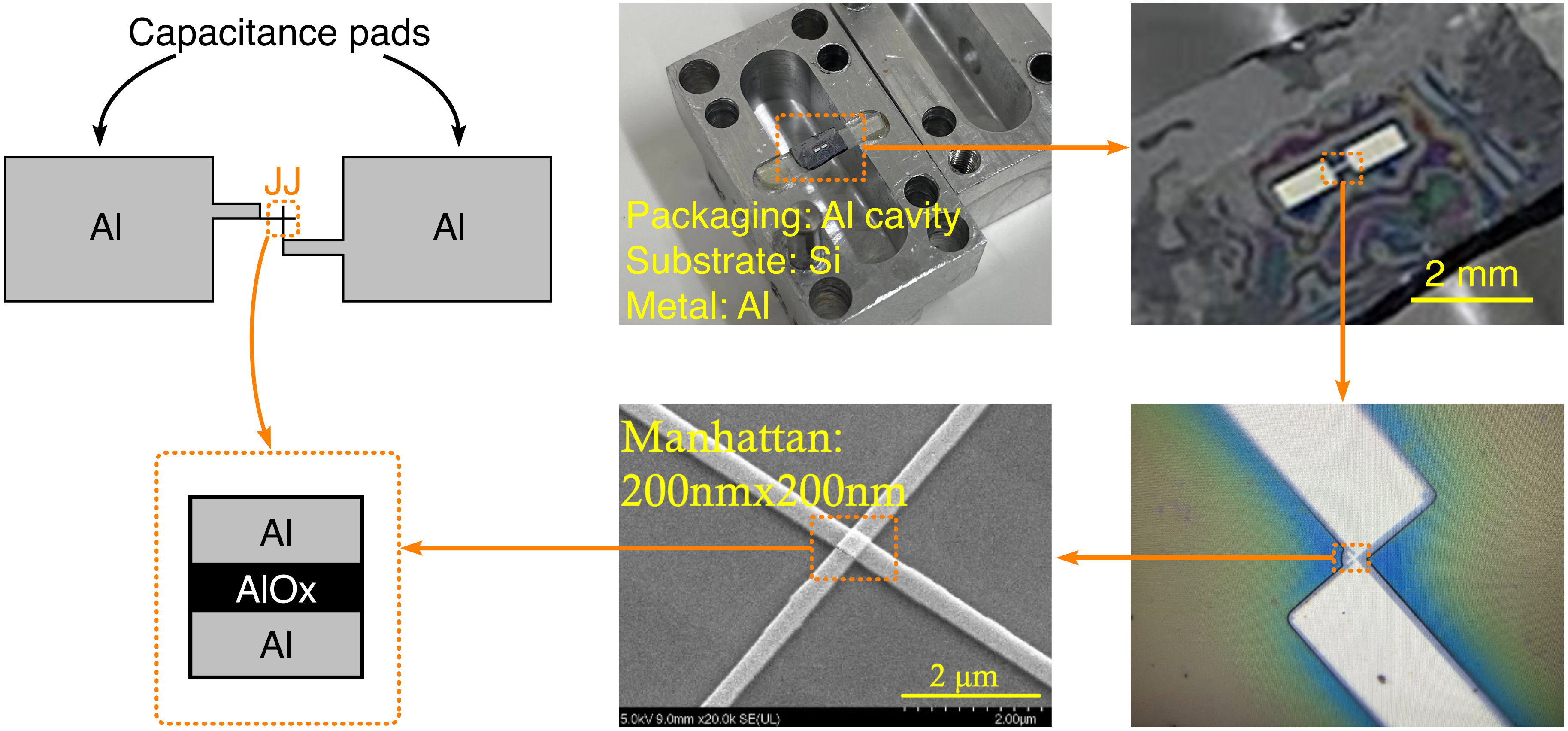


$$\Rightarrow \; H_{\rm SQUID} \simeq -2J\cos(e\Phi)\cos\theta \simeq J\cos(e\Phi)\theta^2 + \cdots$$

$$\Rightarrow \; \omega \simeq \sqrt{\frac{2J}{(2e)^{-2}C}\cos(e\Phi)}$$

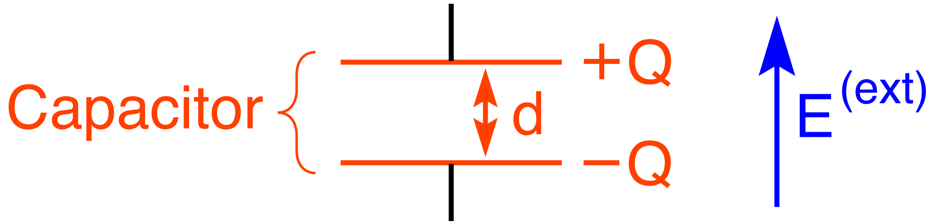
Φ: magnetic flux going through the SQUID loop

Qubit developed by our colleagues (prototype)



- 2D object, fabricated on the surface of a substrate
- Operated with very low temperature $\sim O(10)$ mK

Superconducting qubit couples to external electric field



$\Leftrightarrow \; H_{\rm int} = QdE^{\rm (ext)}$

Charge operator in the transmon limit: $CJ \gg (2e)^2$

$$Q \simeq \frac{C}{2e}\dot{\theta} \simeq \sqrt{\frac{C\omega}{2}}\Big(|g\rangle\langle e| + |e\rangle\langle g| \Big)$$

$|g\rangle \leftrightarrow |e\rangle$ transition occurs if DM field generates electric field

- Axion (with external magnetic field)
- Hidden photon
- $\cdots$

3. DM Detection with Qubits

AC electric field due to oscillating DM field:

$$E^{(\mathrm{DM})} = \bar{E}\cos(m_X t + \alpha) \quad \text{with } m_X = \text{DM mass}$$

Hamiltonian for qubit + DM system

$$H = \omega|e\rangle\langle e| - 2\eta\cos(m_X t + \alpha)\big(\,|e\rangle\langle g| + |g\rangle\langle e|\,\big)$$

$$\eta \simeq \frac{1}{2\sqrt{2}} d\sqrt{C\omega}\bar{E}$$

Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle \quad \Rightarrow \quad |\psi(t)\rangle = U_{\mathrm{DM}}(t)\,|\psi(0)\rangle$$

$$|\psi(t)\rangle \equiv \psi_g(t)|g\rangle + e^{-i\omega t}\,\psi_e(t)|e\rangle$$

Resonance limit $\omega = m_X$ (for $\eta t \ll 1$)

$$\begin{pmatrix} \psi_g(t) \\ \psi_e(t) \end{pmatrix} = U_{\rm DM}(t) \begin{pmatrix} \psi_g(0) \\ \psi_e(0) \end{pmatrix} \simeq \begin{pmatrix} 1 & ie^{-i\alpha}\eta t \\ ie^{i\alpha}\eta t & 1 \end{pmatrix} \begin{pmatrix} \psi_g(0) \\ \psi_e(0) \end{pmatrix}$$

$|g\rangle \to |e\rangle$ transition probability (assuming $|\psi(0)\rangle = |g\rangle$)

$$|\psi_e(t)|^2 \simeq \begin{cases} \eta^2 t^2 & : \omega = m_X \text{ (on-resonance)} \\ \sim \eta^2 (\omega - m_X)^{-2} & : \omega \neq m_X \text{ (off-resonance)} \end{cases}$$

Excitation can be the signal of wave-like DM

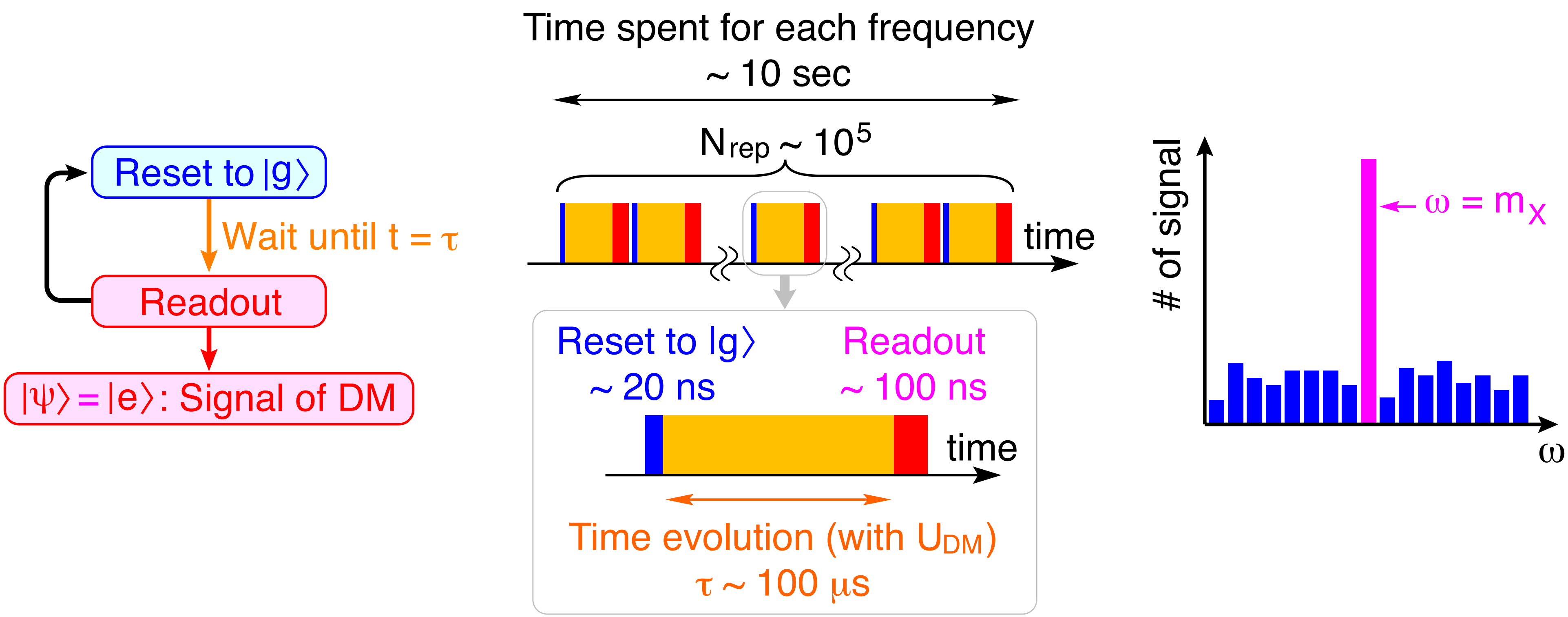
- When $\omega \simeq m_X$, the transition rate is proportional to t^2

  $\Rightarrow$ We should take t as long as the coherence time τ

- DM mass is unknown, so we should scan the frequency

Search strategy (with frequency-tunable SQUID qubits)

- For fixed ω, repeat the measurement cycle (reset, wait, and readout) as many time as possible
- Scan the qubit frequency ω



One of possible targets: hidden photon X_μ

$$\mathcal{L} \ni -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{2}\epsilon F_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu$$

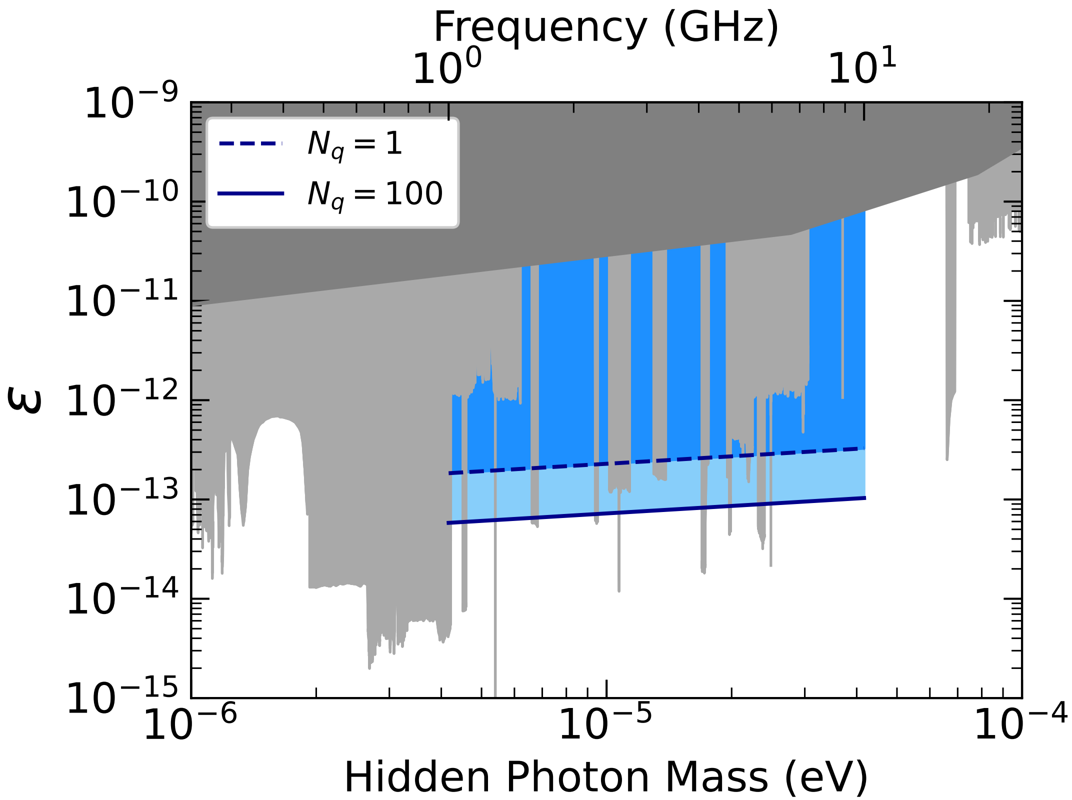
$F_{\mu\nu}$: EM field

Hidden photon DM induces effective electric field

$$\vec{X} \simeq \bar{X}\vec{n}\sin(m_X t + \alpha) \quad \text{with} \quad \rho_{\rm DM} = \frac{1}{2}m_X^2 \bar{X}^2$$

$$\vec{E}^{\rm (DM)} = -\epsilon\dot{\vec{X}} = -\epsilon\, m_X \bar{X}\,\vec{n}\cos(m_X t + \alpha) \Leftrightarrow |\vec{E}^{\rm (DM)}| = \epsilon\sqrt{2\rho_{\rm DM}}$$

Hidden photon DM: 1 year frequency scan ($1 \leq f \leq 10$ GHz)



- $d = 100\ \mu$m
- $C = 0.1$ pF
- $\mathcal{Q} = 10^6$
- Error rate / qubit $= 0.1\ \%$

$\Leftrightarrow$ For $C = 0.1$ pF and $d = 100\ \mu$m:

$$p_{g \to e} \simeq 0.1 \times \left(\frac{\epsilon}{10^{-11}}\right)^2 \left(\frac{f}{1\ \text{GHz}}\right) \left(\frac{\tau}{100\ \mu\text{s}}\right)^2$$

Axion DM detection with qubits

Magnetic field is necessary to convert axion to electric field

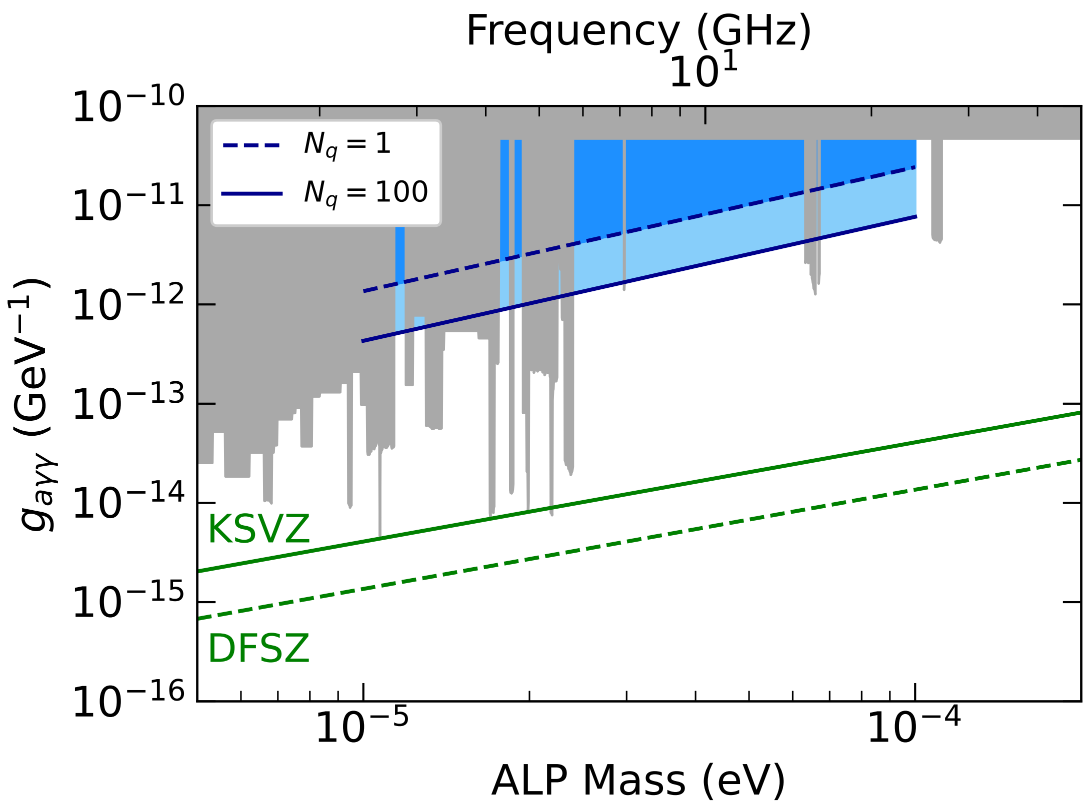
$$\mathcal{L}_{\rm int} = g_{a\gamma\gamma} a \vec{E}\vec{B} \quad \Rightarrow \quad \vec{E} \simeq g_{a\gamma\gamma} a \langle \vec{B}^{(\rm ext)} \rangle$$

Magnetic field onto the superconductor may be a concern

- Transmon qubit is fabricated on the surface of Si substrate (2D object)
- Transmon qubit works with magnetic field of ~ 1 T, if the magnetic field is parallel to the surface
  [Krause et al., 2111.01115]

$\Leftrightarrow$ More detailed study is underway

Axion DM search: 1-year scan



- $B = 5$ T
- $d = 100\ \mu$m
- $C = 0.1$ pF
- $\mathcal{Q} = 10^6$
- Error rate / qubit $= 0.1$ %

$\Leftrightarrow$ For $C = 0.1$ pF and $d = 100\ \mu$m:

$$p_{g\to e} \simeq 0.1 \times \left(\frac{g_{a\gamma\gamma}}{10^{-10}\ \text{GeV}^{-1}}\right)^2 \left(\frac{m_a}{1\ \mu\text{eV}}\right)^{-1} \left(\frac{B}{1\ \text{T}}\right)^2 \left(\frac{\tau}{100\ \mu\text{s}}\right)^2$$

4. Experimental Status

Now, our real search experiment is in progress

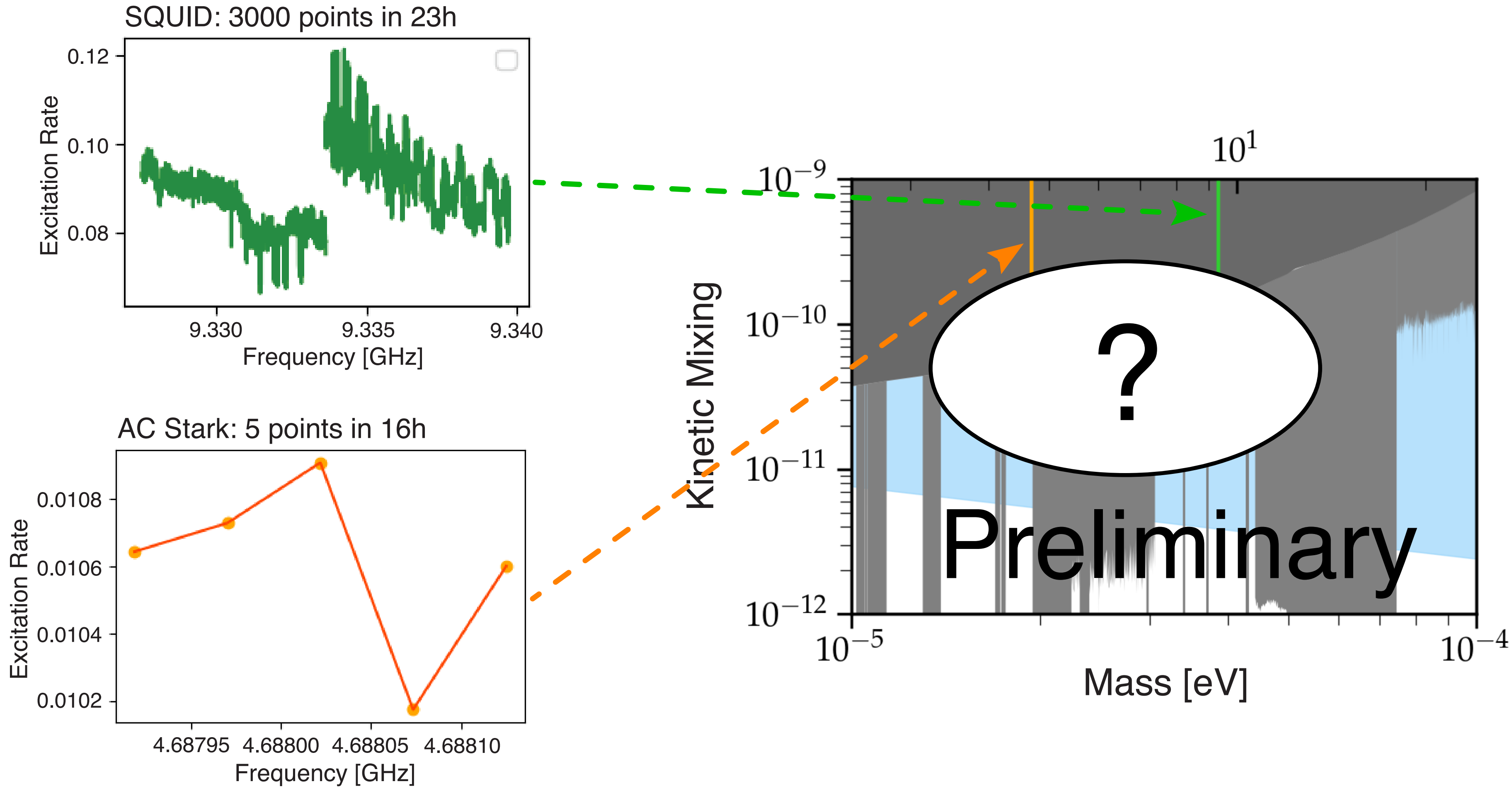
"DarQ" Collaboration: Dark matter hunting with Qubits

Ex.: Watanabe, Chen, Iiyama, Inada, Nakazono, Nitta, Noguchi, Sawada, Shirai, Terashi

Th.: Fukuda, TM, Sichanugrist

- Qubit development is underway
  - ⇒ Currently, $\tau_{\rm q} \sim O(1-10)\ \mu{\rm sec}$
  - ⇒ We hope to realize $\tau_{\rm q} \gtrsim O(100)\ \mu{\rm sec}$
- The first (pilot) run was performed this summer
  - ⇒ We hope to finish the analysis soon

Preliminary results from 2024 summer



⇒ Next run will probably happen within this year

5. Quantum Enhancement / Cavity Effect

Signal rate can be $O(N_\mathrm{q}^2)$ with quantum operations

We may perform quantum operations onto qubits

$\Rightarrow$ "DM detection with quantum computers"

U_DM induces pure phase rotation of its eigenstates

E.g. for $\alpha = 0$: $U_\mathrm{DM} \simeq \begin{pmatrix} \cos\delta & i\sin\delta \\ i\sin\delta & \cos\delta \end{pmatrix}$ **with** $\delta \equiv \eta\tau$

$\Rightarrow$ $U_\mathrm{DM}|\pm\rangle = e^{\pm i\delta}|\pm\rangle$ **with** $|\pm\rangle \equiv \frac{1}{\sqrt{2}}\big(|g\rangle \pm |e\rangle\big)$

$\Rightarrow$ $U_\mathrm{DM}^{\otimes N_\mathrm{q}}|\pm\rangle^{\otimes N_\mathrm{q}} = e^{\pm i N_\mathrm{q}\delta}|\pm\rangle^{\otimes N_\mathrm{q}}$

We can design the following quantum (unitary) operation

$$U_{\mathrm{GHZ}} : \begin{pmatrix} |g\rangle^{\otimes N_{\mathrm{q}}} \\ |e\rangle^{\otimes N_{\mathrm{q}}} \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} |+\rangle^{\otimes N_{\mathrm{q}}} + |-\rangle^{\otimes N_{\mathrm{q}}} \\ |+\rangle^{\otimes N_{\mathrm{q}}} - |-\rangle^{\otimes N_{\mathrm{q}}} \end{pmatrix}$$

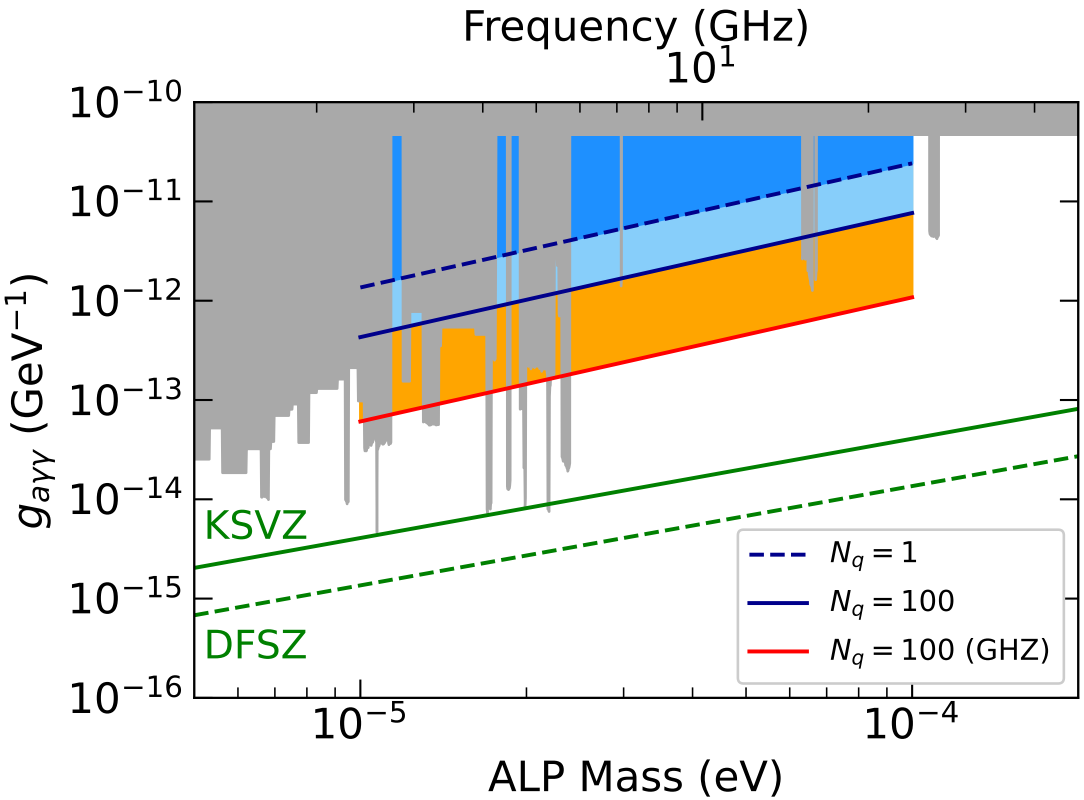
Starting with $|g\rangle^{\otimes N_{\mathrm{q}}}$:

1. Apply U_{GHZ}: $\frac{1}{\sqrt{2}}\left(|+\rangle^{\otimes N_{\mathrm{q}}} + |-\rangle^{\otimes N_{\mathrm{q}}}\right)$
2. Evolution with DM: $\frac{1}{\sqrt{2}}\left(e^{iN_{\mathrm{q}}\delta}|+\rangle^{\otimes N_{\mathrm{q}}} + e^{-iN_{\mathrm{q}}\delta}|-\rangle^{\otimes N_{\mathrm{q}}}\right)$
3. Apply U_{GHZ}^{-1}: $\cos N_{\mathrm{q}}\delta|g\rangle^{\otimes N_{\mathrm{q}}} + i\sin N_{\mathrm{q}}\delta|e\rangle^{\otimes N_{\mathrm{q}}}$

Transition probability:

$$p_{g\to e} = \sin^2 N_{\mathrm{q}}\delta \simeq N_{\mathrm{q}}^2\delta^2 \quad \Rightarrow \quad \frac{S}{\sqrt{B}} \propto N_{\mathrm{q}}^{3/2}\delta^2$$

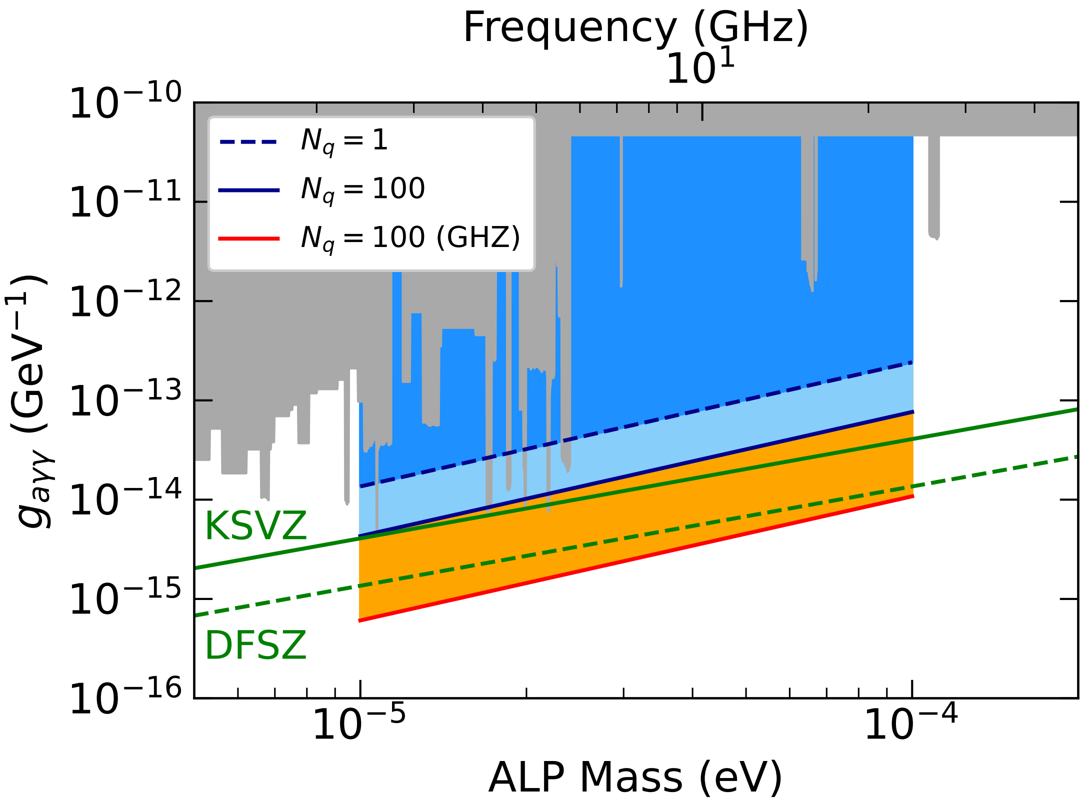
Axion DM search: 1-year scan with the entangled state



- $\kappa = 1$
- $B = 5$ T
- Error rate / qubit $= 0.1$ %

- We need reliable quantum gates
- Frequencies of all the qubits should be equal

Axion DM search: 1-year scan with the entangled state



- $\kappa = 100$
- $B = 5$ T
- Error rate / qubit $= 0.1$ %

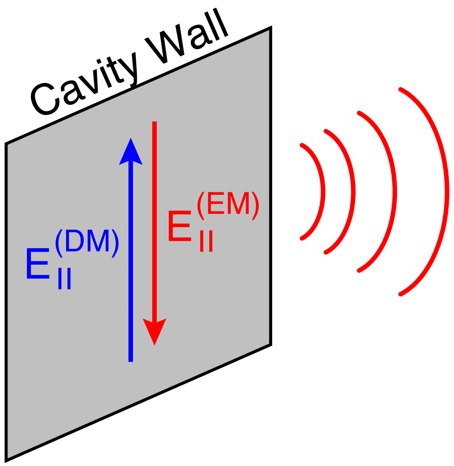
Signal rate can be enhanced with cavity effect

$\Leftrightarrow$ Qubits are usually set in a "cavity"

Effective electric field: $\vec{E}^{(\mathrm{eff})} \equiv \vec{E}^{(\mathrm{DM})} + \vec{E}^{(\mathrm{EM})} \equiv \kappa \vec{E}^{(\mathrm{DM})}$

$$\Box \vec{E}^{(\mathrm{EM})} = 0 \quad \text{and} \quad \vec{\nabla} \vec{E}^{(\mathrm{EM})} = 0$$

$$[\vec{E}_{\parallel}^{(\mathrm{EM})} + \vec{E}_{\parallel}^{(\mathrm{DM})}]_{\mathrm{wall}} = 0 \quad \Leftrightarrow \quad \vec{E}_{\parallel}^{(\mathrm{eff})} = 0 \text{ at the cavity wall}$$

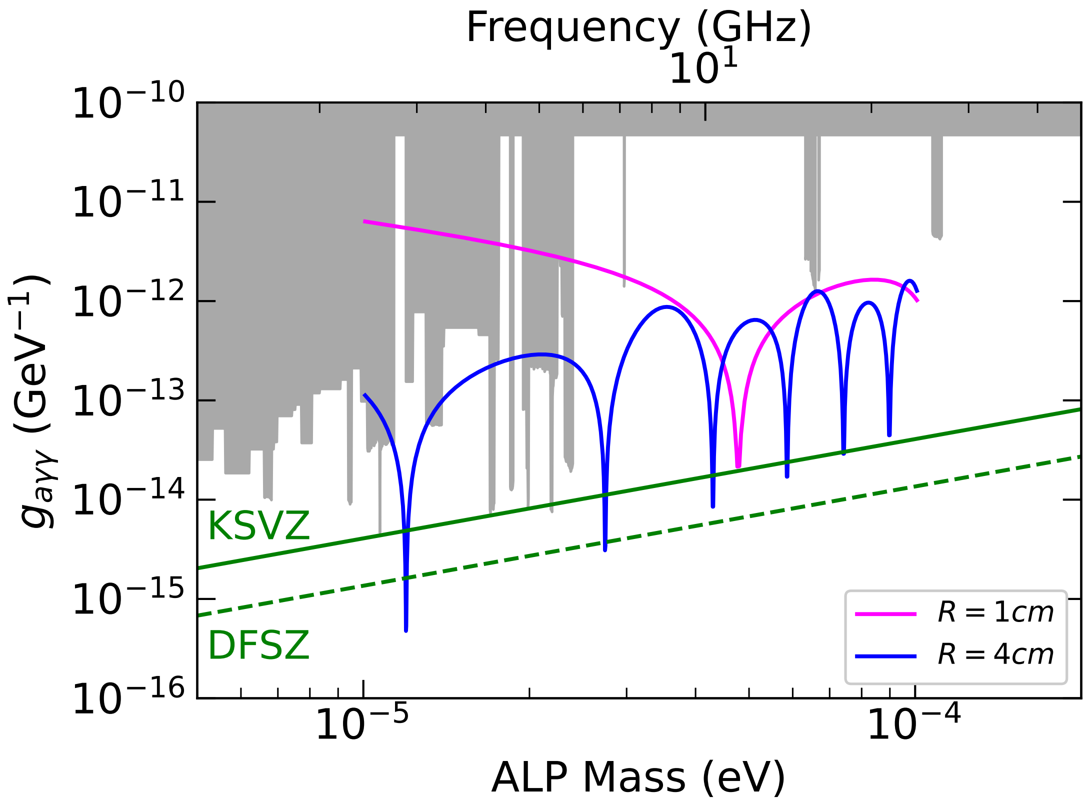


$$\Rightarrow H_{\mathrm{int}} = QdE^{(\mathrm{eff})}$$

$$\Rightarrow p_{g\to e} \propto \kappa^2$$

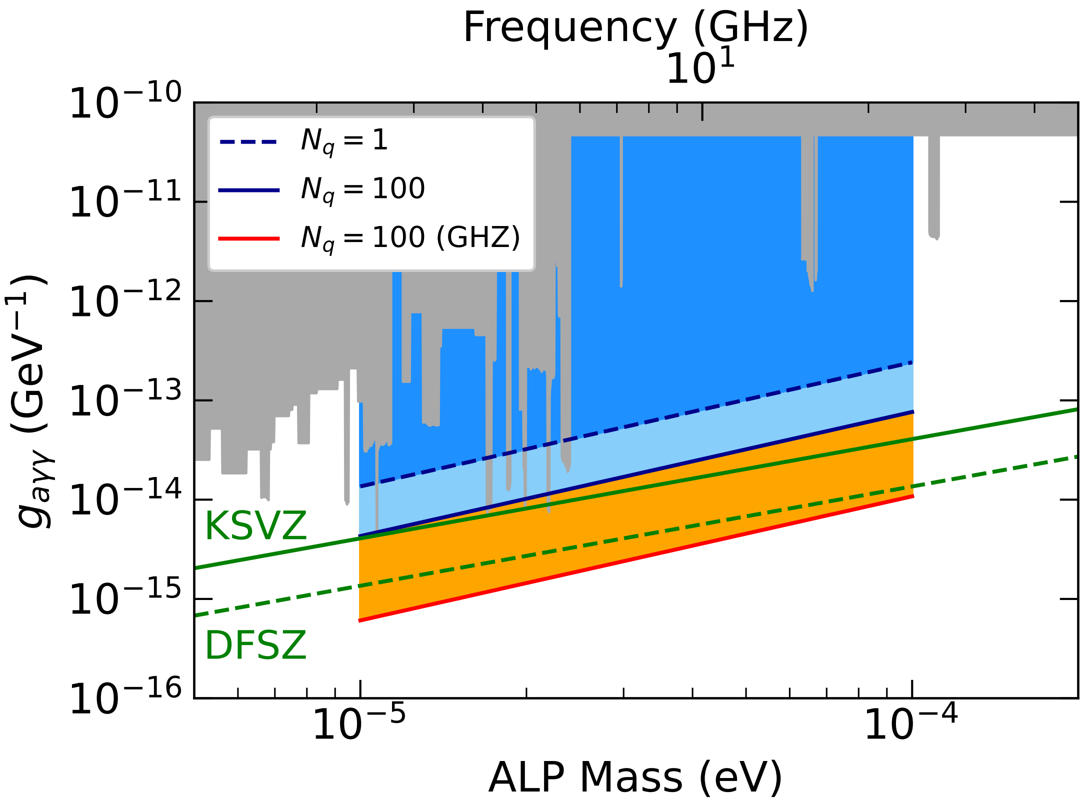
$\Rightarrow$ $\kappa \gg 1$, if m_{DM} is close to one of cavity frequencies

Axion DM search: with fixed cavity geometry (cylinder)



- $B = 5$ T
- $N_{\rm q} = 1$
- Error rate / qubit $= 0.1$ %

Axion DM search: with frequency-tunable cavity



- $B = 5$ T
- $\kappa = 100$

$\Rightarrow$ In order to always realize $|\kappa| \gg 1$, cavity frequency should be tuned during the frequency scan

6. Summary and Outlook

Superconducting qubit is an excellent DM detector

- We may reach parameter region which is unexplored

The real experiment has been started

- We need to fabricate high-quality qubit
- First run was performed this summer
- Effects of the magnetic field is under study (for axion DM detection)
- We hope to announce the first result soon, so stay tuned!

Backup: Hidden Photon DM

Case of hidden photon X_μ

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2}\,\epsilon\, F'_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu$$

$F'_{\mu\nu}$: EM field (in gauge eigenstate)

Vector bosons in the mass eigenstates

$A_\mu \simeq A'_\mu - \epsilon X_\mu$ and X_μ

Interaction with electron

$$\mathcal{L}_{\rm int} = e\bar{\psi}_e \gamma^\mu A'_\mu \psi_e = e\bar{\psi}\gamma^\mu \psi (A_\mu + \epsilon X_\mu)$$

Hidden photon as dark matter

$$\vec{X} \simeq \bar{X}\vec{n}_X \cos m_X t$$

Energy density of hidden photon DM

$$\rho_{\rm DM} = \frac{1}{2}\dot{\vec{X}}^2 + \frac{1}{2}m_X^2\vec{X}^2 \simeq \frac{1}{2}m_X^2\bar{X}^2$$

$$\Leftrightarrow \ \rho_{\rm DM} \sim 0.45\ {\rm GeV/cm^3}$$

Effective electric field induced by the hidden photon

$$\vec{E}^{(X)} = -\epsilon\dot{\vec{X}} = \bar{E}^{(X)}\vec{n}_X \sin m_X t$$

$$\bar{E}^{(X)} = \epsilon m_X \bar{X} = \epsilon\sqrt{2\rho_{\rm DM}}$$

Backup: Transmon Qubit

Hamiltonian

$$H_0 = \frac{1}{2C}Q^2 - J\cos\theta = \frac{1}{2Z}n^2 - J\cos\theta$$

$$Z \equiv (2e)^{-2}C$$

Transmon limit: $CJ \gg (2e)^2 \quad \Rightarrow \quad \langle\theta^2\rangle \ll 1$

[Koch et al. ('07); see also Roth, Ma & Chew (2106.11352)]

$$\Rightarrow \; H_0 = \frac{1}{2Z}n^2 + \frac{1}{2}J\theta^2 + O(\theta^4)$$

$$\hat{a} \equiv \frac{1}{\sqrt{2\omega Z}}(n - i\omega Z\theta), \quad \hat{a}^\dagger \equiv \frac{1}{\sqrt{2\omega Z}}(n + i\omega Z\theta)$$

$$\Rightarrow \; [\hat{a}, \hat{a}^\dagger] = 1$$

In the transmon limit, anharmonicity is small:

$$\Rightarrow\ |e\rangle \simeq \hat{a}^\dagger |g\rangle$$

$$\Rightarrow\ \omega_{21} \simeq \left(1 - \frac{1}{8}\frac{2e}{\sqrt{CJ}}\right)\omega$$

Charge operator in the transmon limit

$$Q = 2en = \sqrt{\frac{C\omega}{2}}\left(\hat{a} + \hat{a}^\dagger\right) \simeq \sqrt{\frac{C\omega}{2}}\left(\, |g\rangle\langle e| + |e\rangle\langle g|\,\right)$$

Interaction Hamiltonian

$$H_{\rm int} = QdE^{\rm (ext)} \simeq \sqrt{\frac{C\omega}{2}} dE^{\rm (ext)}\left(\, |g\rangle\langle e| + |e\rangle\langle g|\,\right)$$

Backup: Schrödinger Equation

Effective Hamiltonian

$$H = \omega|e\rangle\langle e| + 2\eta \sin m_X t \big(|e\rangle\langle g| + |g\rangle\langle e| \big)$$

η: Small parameter

Schrödinger equation:

$$i\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

$$|\Psi(t)\rangle \equiv \psi_g(t)|g\rangle + e^{-i\omega t}\,\psi_e(t)|e\rangle$$

$$\Rightarrow\; i\frac{d}{dt}\begin{pmatrix} \psi_g \\ \psi_e \end{pmatrix} = 2\eta \sin m_X t \begin{pmatrix} 0 & e^{-i\omega t} \\ e^{i\omega t} & 0 \end{pmatrix}\begin{pmatrix} \psi_g \\ \psi_e \end{pmatrix}$$

Solution with $|\Psi(0)\rangle = |g\rangle$ (for $|\omega \pm m_X|^{-1} \ll t \ll \eta^{-1}$)

$$\psi_g(t) \simeq 1 + O(\eta^2)$$

$$\psi_e(t) \simeq \eta \left(\frac{e^{i(\omega - m_X)t} - 1}{i(\omega - m_X)} - \frac{e^{i(\omega + m_X)t} - 1}{i(\omega + m_X)} \right)$$

Resonance limit: $\omega \to m_X$

$\Rightarrow \ \psi_e(t) \to \eta t +$ (non-growing)

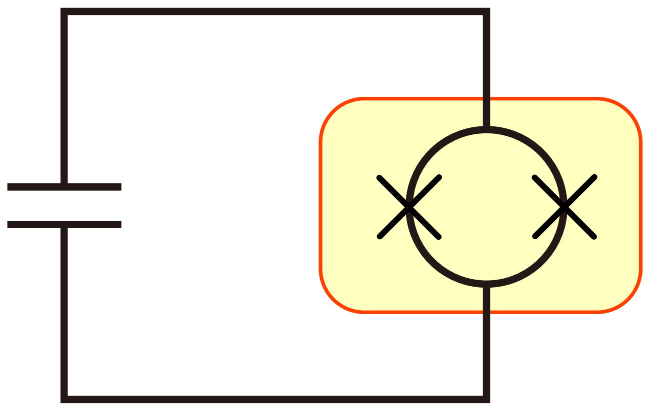
$|g\rangle \to |e\rangle$ transition rate (for $t \ll \eta^{-1}$)

$$P_{ge} = |\psi_e(t)|^2 \simeq \begin{cases} \sim \eta^2 (\omega - m_X)^{-2} & : \omega \neq m_X \\ \eta^2 t^2 & : \omega = m_X \end{cases}$$

Backup: Frequency Scan

Frequency scan

Frequency scan is possible with qubit consisting of SQUID and capacitor

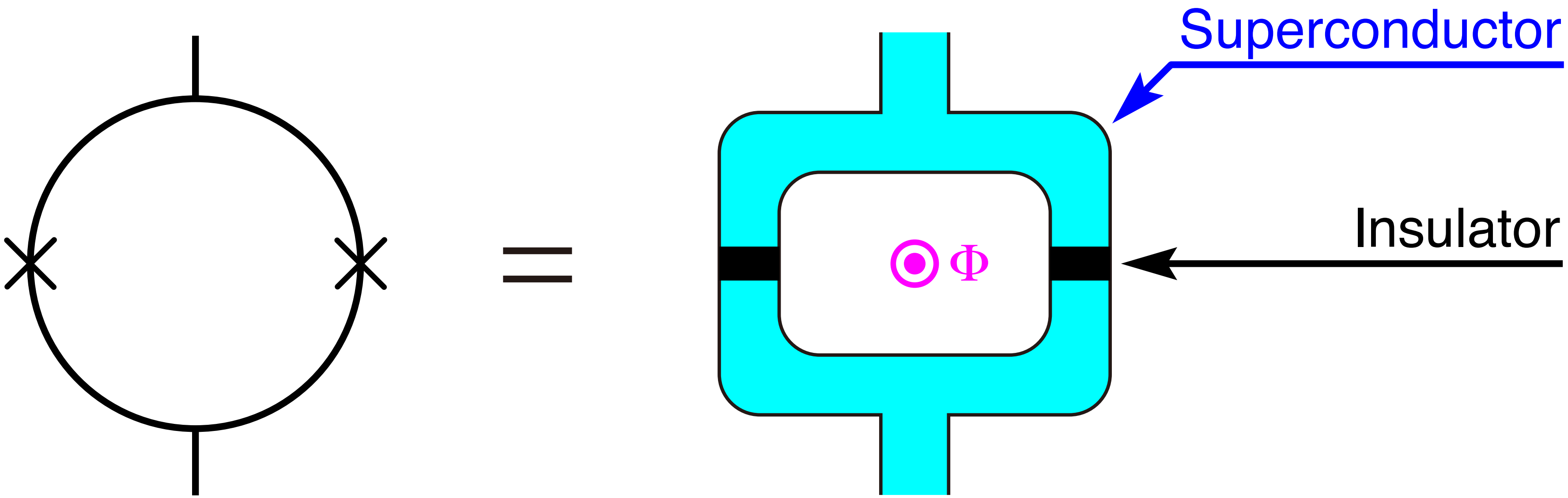


SQUID: superconducting quantum interference device

- Quantum device sensitive to magnetic flux
- With SQUID, the qubit frequency ω can be changed

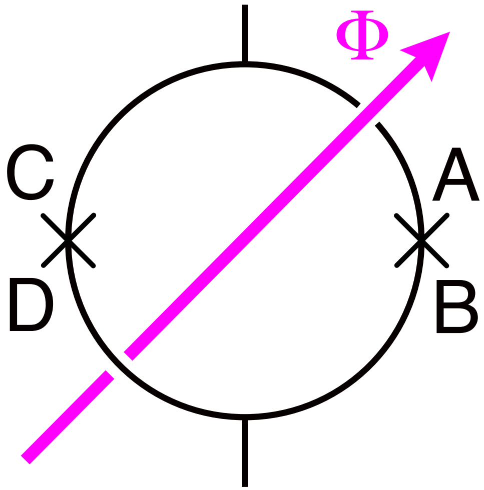
SQUID

- Loop-shaped superconductors separated by insulating layers



- We consider the case with external magnetic flux Φ going through the loop

Phases in the presence of magnetic flux



$$\theta_C - \theta_A = (2e) \int_{A \to C} \vec{A}(\vec{x})\, d\vec{x}$$

$$\theta_B - \theta_D = (2e) \int_{D \to B} \vec{A}(\vec{x})\, d\vec{x}$$

$$\theta_{BA} - \theta_{DC} = (2e) \oint \vec{A}(\vec{x})\, d\vec{x} = (2e)\, \Phi = \frac{2\pi}{\Phi_0} \Phi$$

$$\theta_{YX} = \theta_Y - \theta_X$$

$\Phi_0 = \dfrac{h}{2e}$: magnetic flux quantum

Define: $\theta \equiv (\theta_{BA} + \theta_{DC})/2$

$$H_{\mathrm{SQUID}} \simeq -J\,(\cos\theta_{BA} + \cos\theta_{DC}) = -2J\cos(e\Phi)\cos\theta$$

Based on the previous analysis with $J \to 2J\cos(e\Phi)$

$$\omega \simeq \sqrt{\frac{2J}{Z}\cos(e\Phi)}$$

$$Z = (2e)^{-2}C$$

The excitation energy depends on Φ

$\Rightarrow$ Frequency scan is possible with varying the external magnetic field

Backup: Quantum Circuit

Basic unitary operations (quantum gates)

- Z gate

$$Z = |g\rangle\langle g| - |e\rangle\langle e| \quad \Rightarrow \; |+\rangle \xrightarrow{Z} |-\rangle \;\; \textbf{with} \;\; |\pm\rangle \equiv \frac{1}{\sqrt{2}}\big(|g\rangle \pm |e\rangle\big)$$

- Hadamard gate

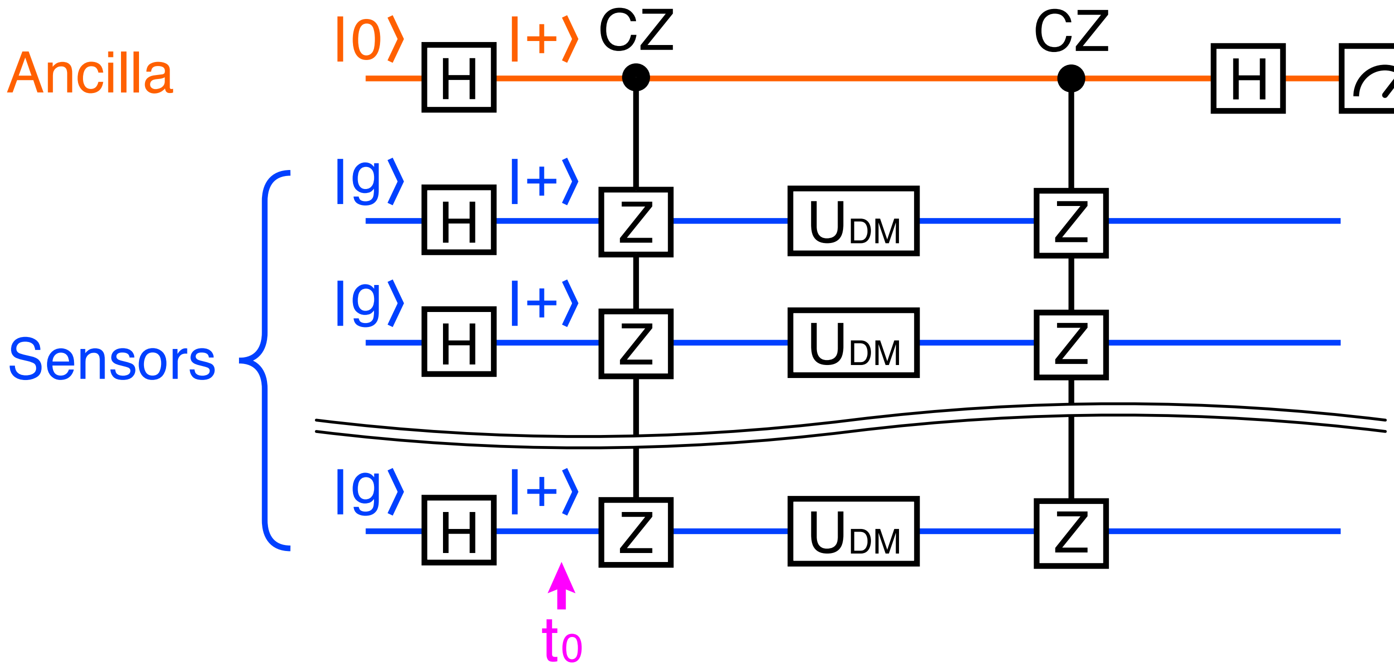
$$H = |+\rangle\langle g| + |-\rangle\langle e| \quad \Rightarrow \; |g\rangle \xrightarrow{H} |+\rangle, \;\; |e\rangle \xrightarrow{H} |-\rangle$$

- Controlled Z gate

$$CZ = |0\rangle\langle 0| \otimes \mathbf{1} + |1\rangle\langle 1| \otimes Z$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |+\rangle \xrightarrow{CZ} \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle$$

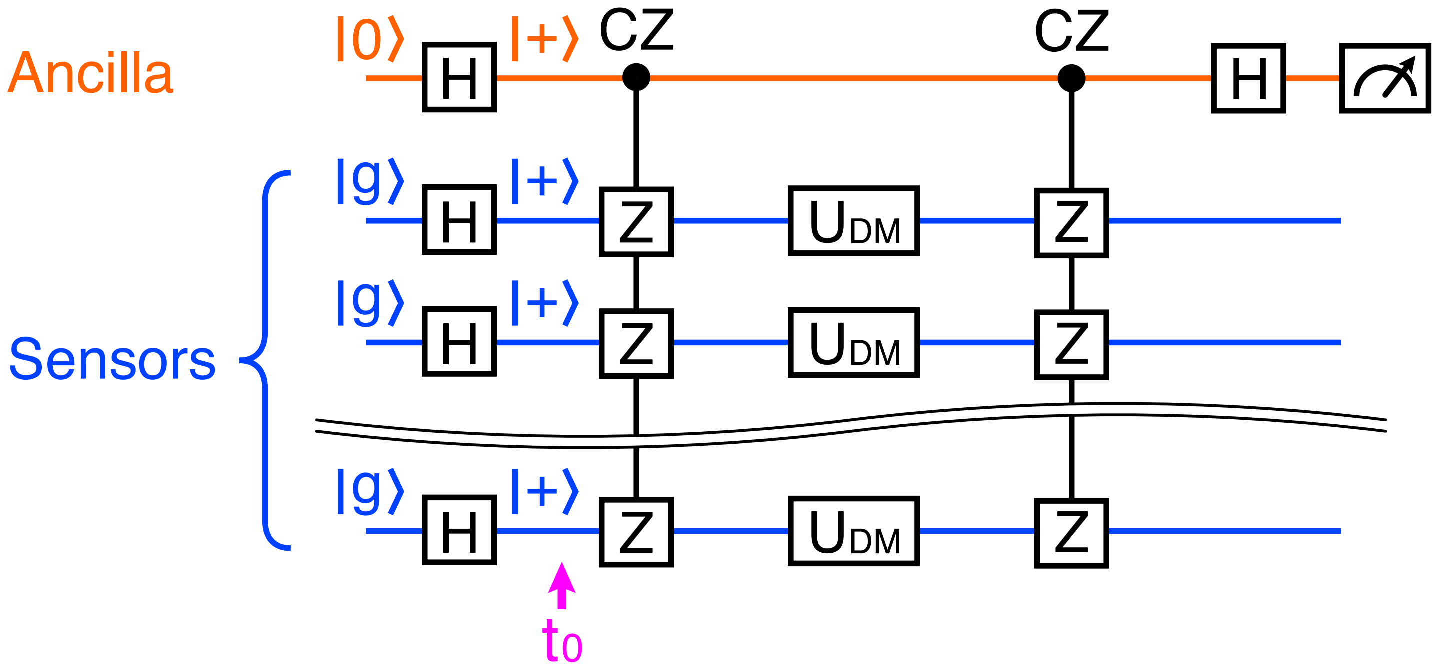
One measurement cycle for the signal enhancement



The above is an example of the quantum circuit

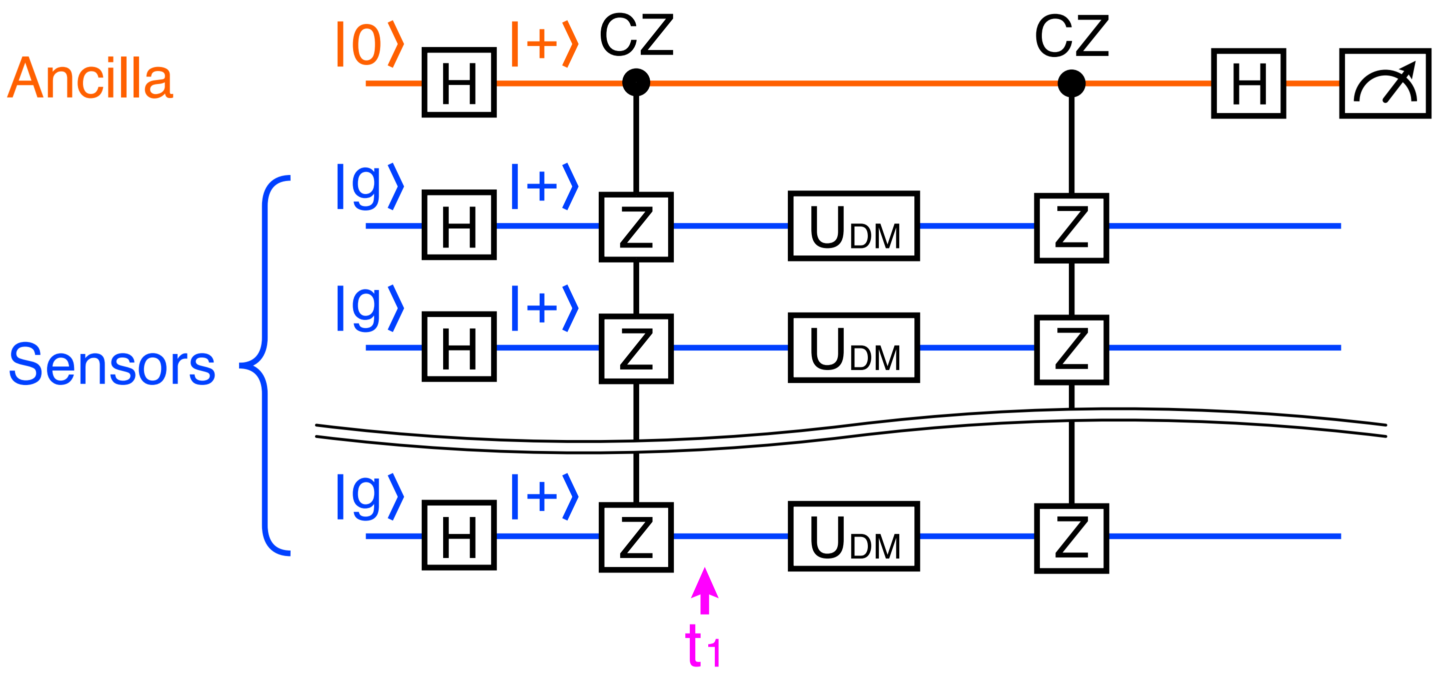
$\Rightarrow$ Let us first see how it works when $\alpha = 0$

One measurement cycle for the signal enhancement



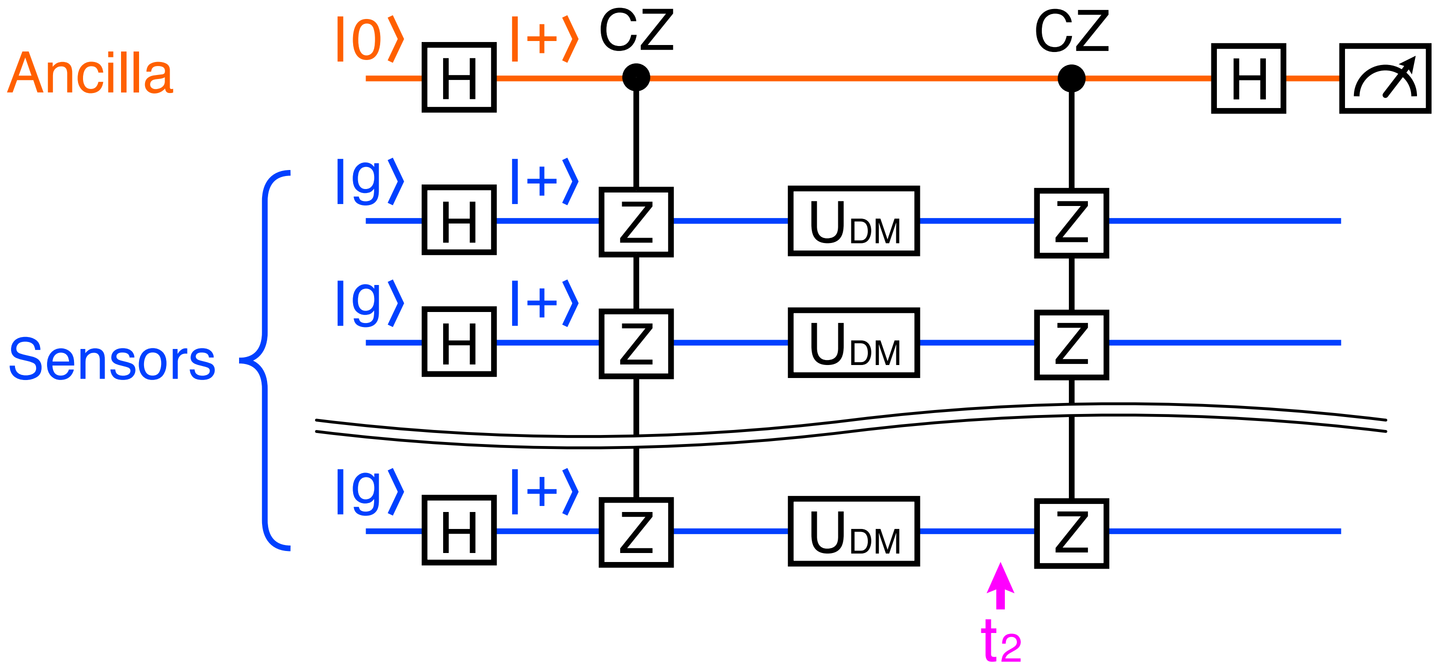
$$|\Psi(t_0)\rangle = |+\rangle \otimes |+\rangle^{\otimes N_\mathrm{q}} = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_\mathrm{q}} + \frac{1}{\sqrt{2}}|1\rangle \otimes |+\rangle^{\otimes N_\mathrm{q}}$$

One measurement cycle for the signal enhancement



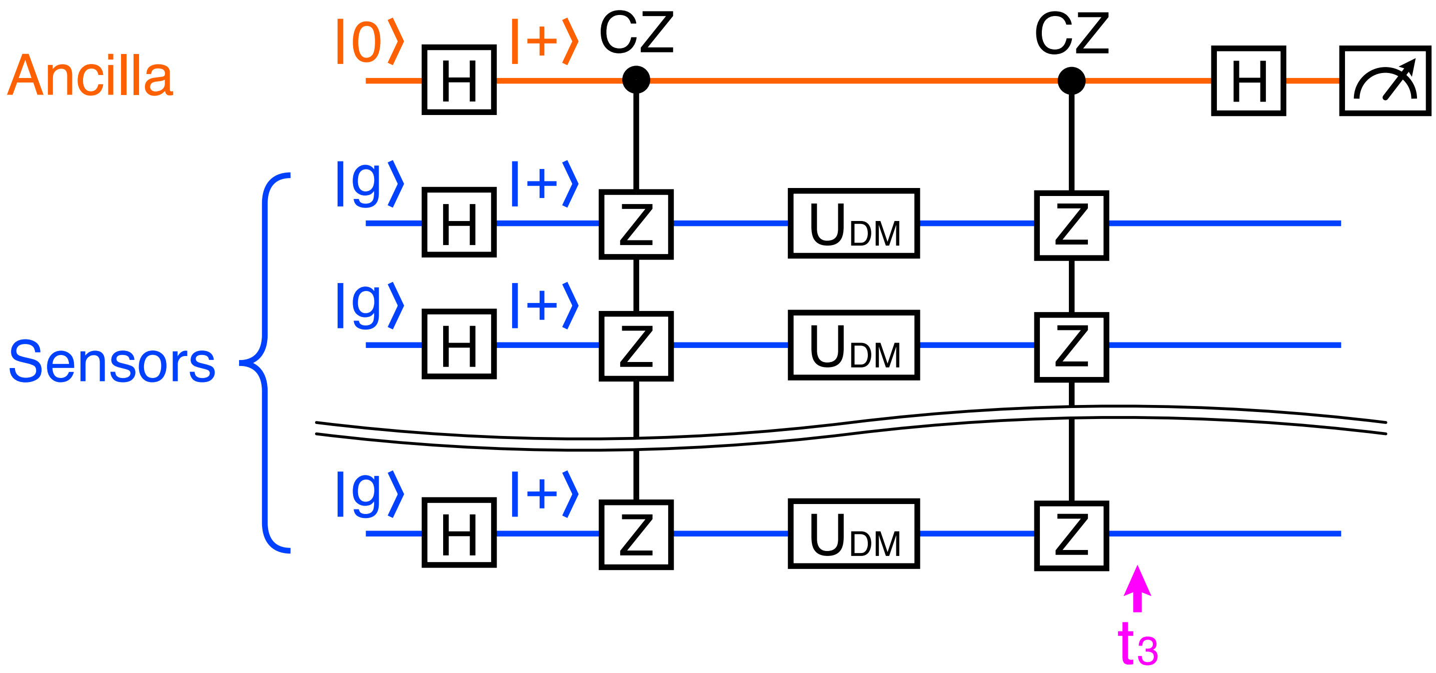
$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_\mathrm{q}} + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle^{\otimes N_\mathrm{q}}$$

One measurement cycle for the signal enhancement



$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}} e^{iN_\mathrm{q}\delta} |0\rangle \otimes |+\rangle^{\otimes N_\mathrm{q}} + \frac{1}{\sqrt{2}} e^{-iN_\mathrm{q}\delta} |1\rangle \otimes |-\rangle^{\otimes N_\mathrm{q}}$$

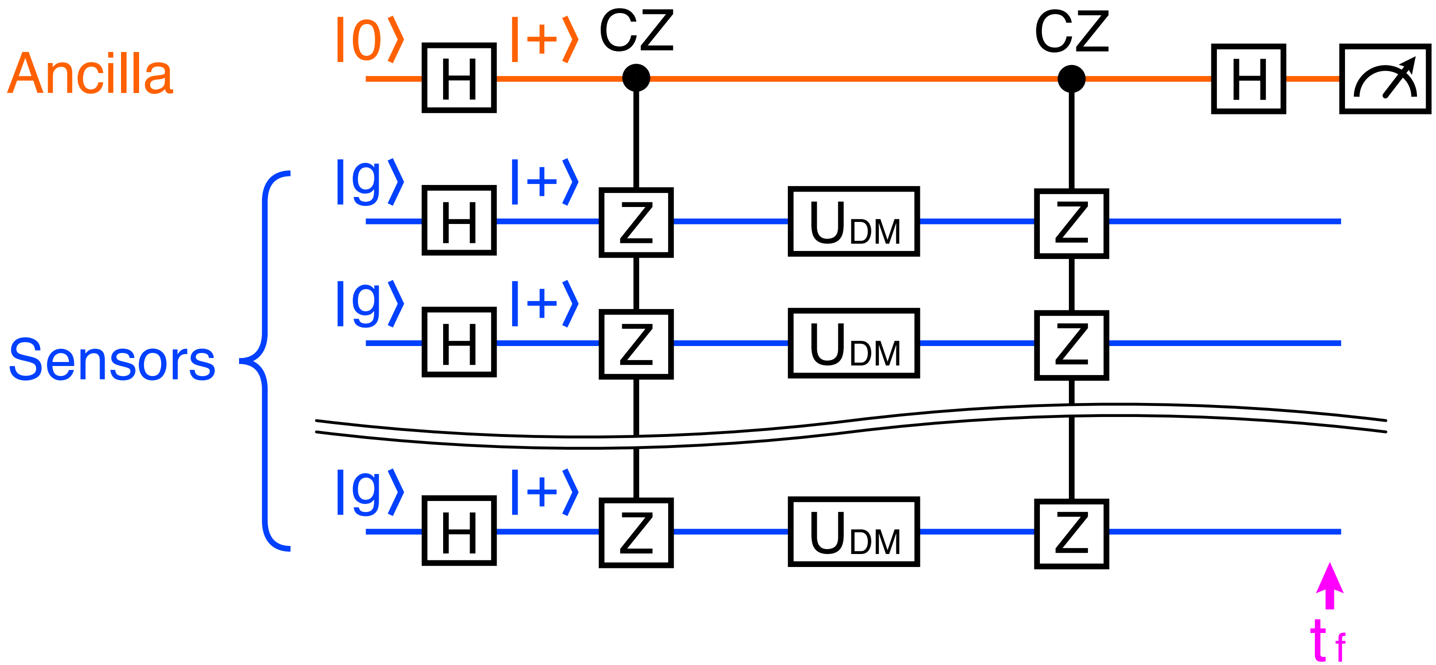
One measurement cycle for the signal enhancement



$$|\Psi(t_3)\rangle = \frac{1}{\sqrt{2}} e^{iN_\mathrm{q}\delta}|0\rangle \otimes |+\rangle^{\otimes N_\mathrm{q}} + \frac{1}{\sqrt{2}} e^{-iN_\mathrm{q}\delta}|1\rangle \otimes |+\rangle^{\otimes N_\mathrm{q}}$$

$$= \left(\cos\, N_\mathrm{q}\delta\,|+\rangle + i\sin\, N_\mathrm{q}\delta\,|-\rangle\right) \otimes |+\rangle^{\otimes N_\mathrm{q}}$$

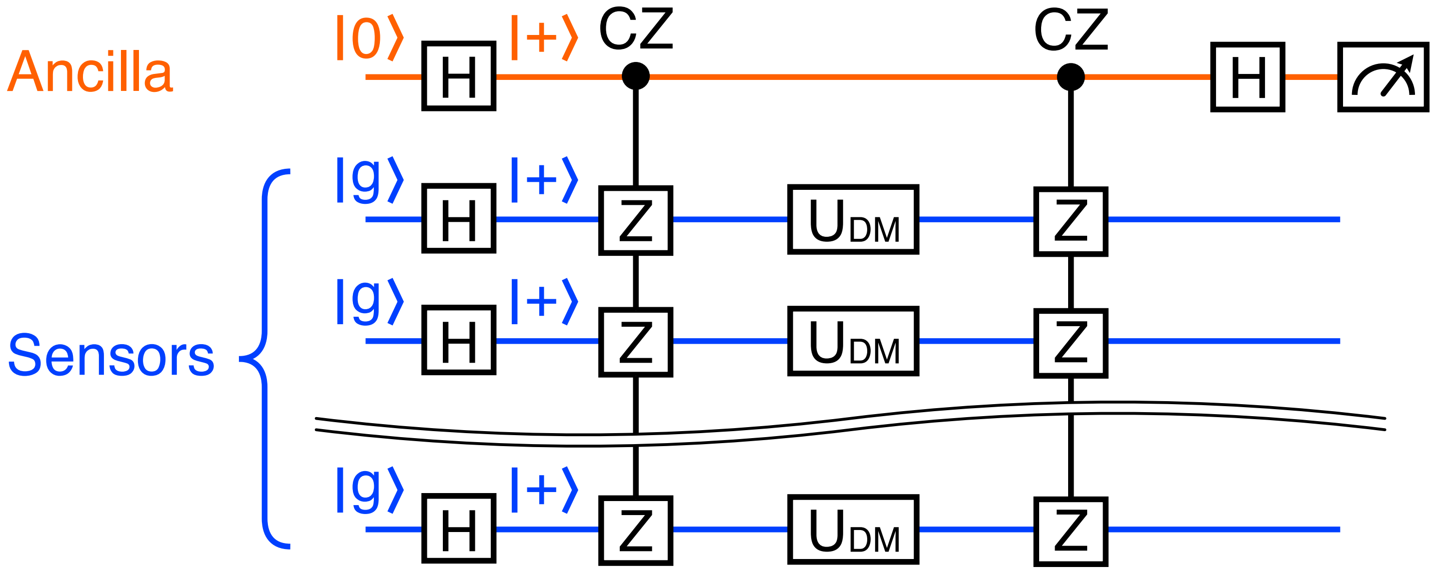
One measurement cycle for the signal enhancement



$$|\Psi(t_{\mathrm{f}})\rangle = \left(\cos\, N_{\mathrm{q}}\delta\,|0\rangle + i\sin\, N_{\mathrm{q}}\delta\,|1\rangle\right) \otimes |+\rangle^{\otimes N_{\mathrm{q}}}$$

$\Rightarrow$ Ancilla qubit can be excited: $P_{0\to 1} \simeq \sin^2 N_{\mathrm{q}}\delta \simeq N_{\mathrm{q}}^2\delta^2$

The phase α is unknown in the actual search, but...



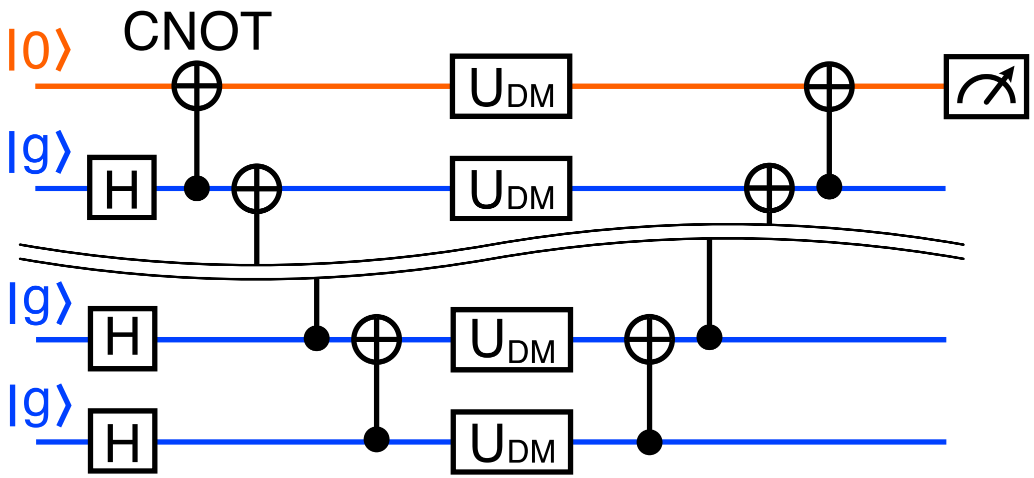
$$P_{0\to1} \simeq N_{\rm q}^2 \delta^2 \cos^2\alpha \to \frac{1}{2} N_{\rm q}^2 \delta^2$$

$\Rightarrow$ Signal rate can be of $O(N_{\rm q}^2)$

$\Rightarrow$ The number of gate operation can be $O(N_{\rm q})$

Circuit only with nearest neighbor interactions

$\Rightarrow$ (# of gates) $\sim O(N_\mathrm{q})$



$$\Rightarrow P_{0\to1} \simeq \frac{1}{2} N_\mathrm{q}^2 \delta^2$$

CNOT (Controlled-NOT) $= |g\rangle\langle g| \otimes \mathbf{1} + |e\rangle\langle e| \otimes X$

$\Rightarrow$ (# of signals) $\sim O(N_\mathrm{q}^2)$

$\Rightarrow$ (# of errors & noises) $\sim O(N_\mathrm{q}) \ll$ (# of signals), for $N_\mathrm{q} \gg 1$