Non-Invertible Peccei-Quinn Symmetry
and
the Massless Quark Solution
to the Strong CP Problem

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Dark World to Swampland 2024: the 9th IBS-IFT Workshop

Opening Remarks

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Naturalness Problems and Global Symmetries

1. Electroweak Hierarch Problem

$$\left(\frac{\text{Gravity}}{\text{weak}}\right) \sim \left(\frac{v}{M_{pl}}\right)^2 \sim \left(\frac{100\ GeV}{10^{19}\ GeV}\right)^2 \sim 10^{-34} \ll 1$$

A source of challenge: **no apparent symmetry** acting on (generic) **scalar Φ**

Exception-1) Shift symmetry: Higgs = PNGB $\Rightarrow$ Composite Higgs / Little Higgs

Exception-2) Chiral symmetry (scalar $\leftrightarrow$ fermion): SUSY $\Rightarrow$ (N)MSSM

In these cases, hierarchy problem becomes **Technical Naturalness Problem.**

Opening Remarks

Naturalness Problems and Global Symmetries

2. Strong CP Problem

$$\tilde{J} = \mathrm{Im}\,\det[y_u^\dagger y_u, y_d^\dagger y_d\,] \propto \sin\delta_{CKM} \sim O(1) \quad \textbf{vs} \quad \bar{\theta} \equiv \arg e^{-i\theta}\det(y_u y_d) \ll 1$$

"Jarlskog invariant"

source of challenge 1: **no clean symmetry structure**

CP (=T), Anomalous $U(1)_{PQ}$, flavor symmetry, …

renormalization of $\bar{\theta}$ from other CPV sources

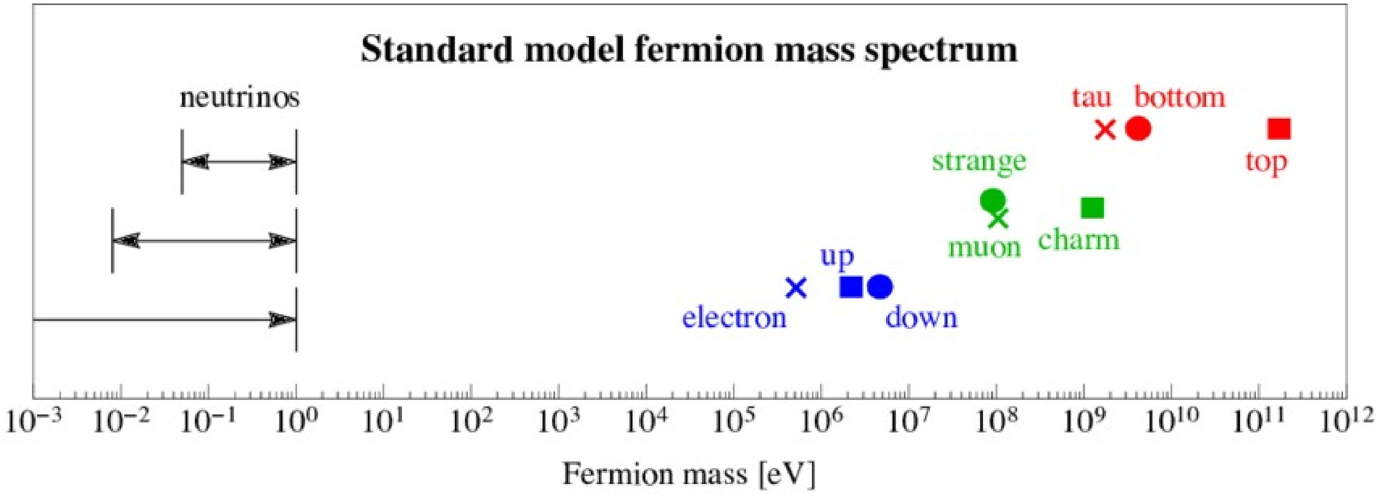
source of challenge 2: the limit $\bar{\theta} \to 0$ does not enhance the symmetry of QFT

Strong CP problem = Dirac Naturalness Problem

Opening Remarks

Naturalness Problems and Global Symmetries

3. Flavor Problem [e.g. m_ν]



https://www.researchgate.net/figure/Mass-spectrum-of-standard-model-fermions-Charged-leptons-up-type-quarks-and-downtype_fig1_361578459

$M_\nu \sim 10^{-2}\, eV$

Requires
Dynamical
Explanation!

Several attractive theories exist.

(1) Seesaw models based on $U(1)_L$
(2) Extradimension, clockwork: localization
(3) …

Opening Remarks

Naturalness Problems and Global Symmetries

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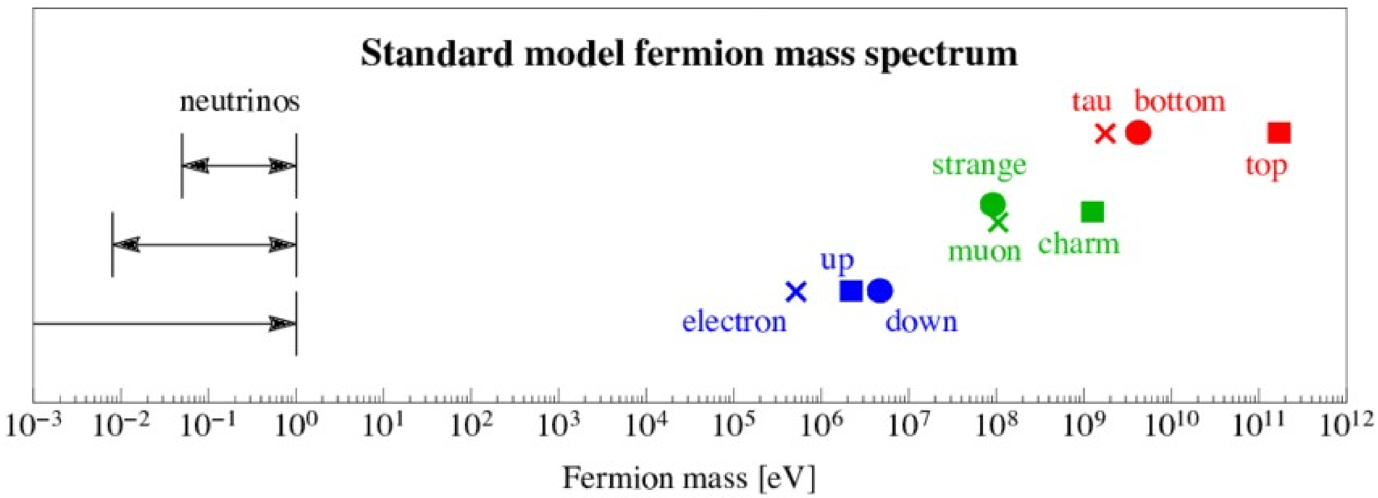

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A source of challenge: ultimate mechanism still to be confirmed.

=> more feasible, testable, and motivating **theoretical ideas** should be laid out.

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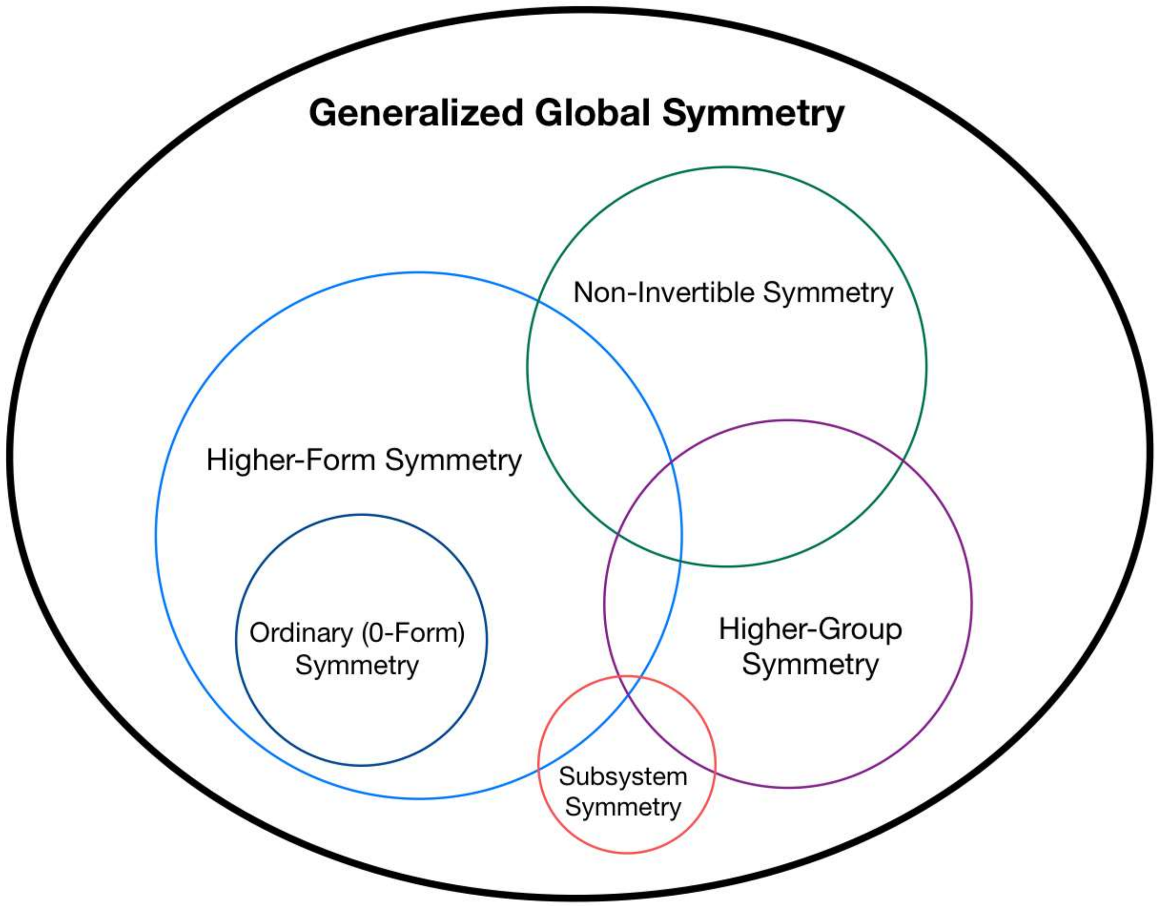
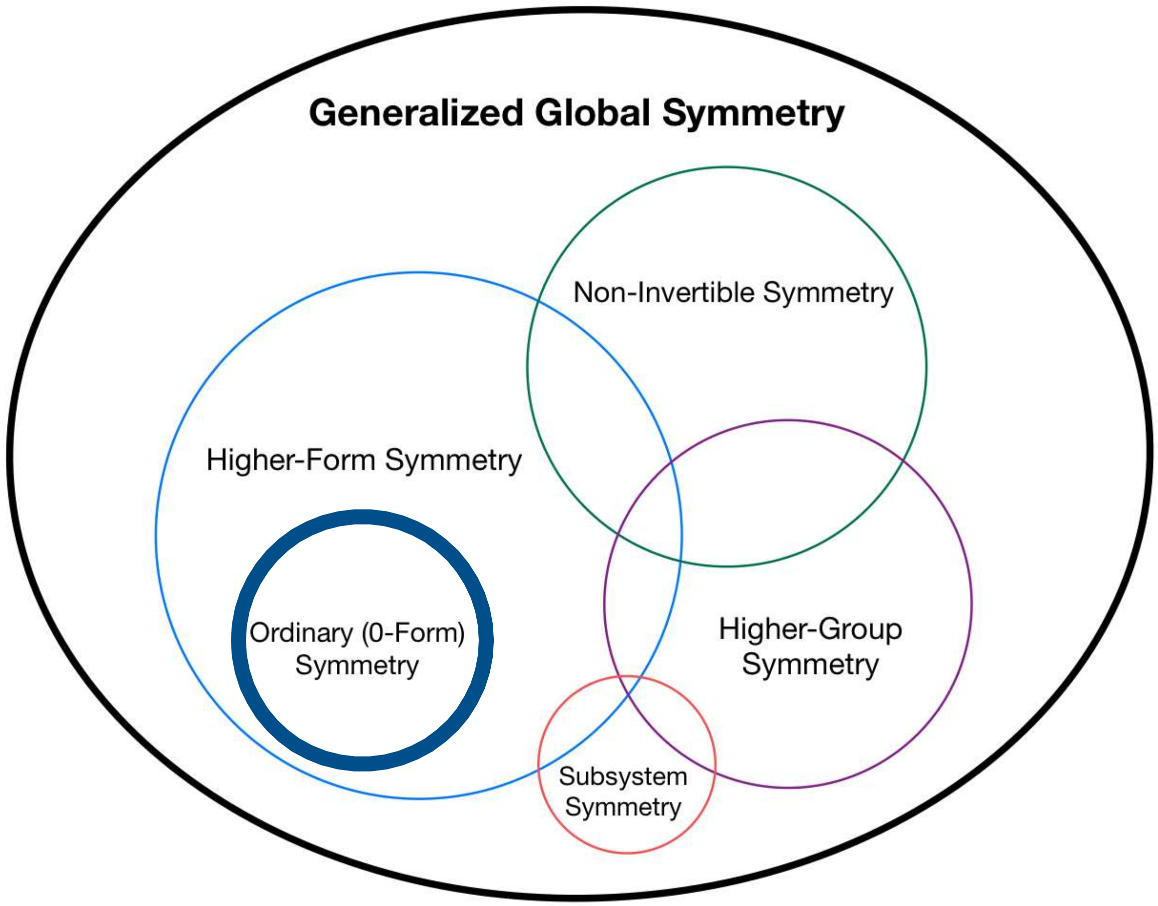
Generalized Global Symmetries



Opening Remarks

Generalized Global Symmetries



Opening Remarks

Generalized Global Symmetries in Physics

Well-motivated and timely to think about new ideas and breakthrough Generalized Global Symmetry can provide.

Opening Remarks

Generalized Global Symmetries in Particle Physics

0. Noninvertible Chiral Symmetry and Exponential Hierarchies '22 (C. Cordova, K. Ohmori)
   Noninvertible Global Symmetries in the Standard Model '22 (Y. Choi, H.T. Lam, S.-H Shao)

1. Neutrino Masses from Generalized Symmetry Breaking '22 (C. Cordova, **SH**, S. Koren, K. Ohmori)

2. Higher Flavor Symmetries in the Standard Model '22 (C. Cordova, S. Koren)

3. Coupling a Cosmic String to a TQFT '23 (T.D. Brennan, **SH**, LT Wang)
   Quantization of Axion-Gauge Couplings and Non-Invertible Higher Symmetries '23 (Y. Choi, M. Forslund, H. T. Lam, S-H. Shao)
   Axion-Gauge Coupling Quantization with a Twist '23 (M. Reece)
   Axion Domain Walls, Small Instantons, and Non-Invertible Symmetry Breaking '23 (C. Cordova, **SH**, L. Wang)
   Axion Couplings in Heterotic String Theory '24 (P. Agrawal, M. Nee, M Reig)

4. Non-invertible Peccei-Quinn Symmetry and the Massless Quark Solution to Strong CP Problem '24 (C. Cordova, **SH**, S. Koren)
   Spontaneously Broken (-1)-Form U(1) Symmetry '24 (D. Aloni, E. Garcia-Valdecasas, M. Reece, M. Suzuki)
   High-Quality Axions from Higher-Form Symmetries in Extra Dimensions '24 (N. Craig, M. Kongsore)

5. Nonperturbative effects in the Standard Model with gauged 1-form symmetry '21 (M. Anber, E. Popptiz)
   Fractional-charge hadrons and leptons to tell the Standard Model group apart '24 (R. Alonso, D. Dimakou, M. West)
   The Standard Model Gauge Group, SMEFT, and Generalized Symmetries '24 (H-L. Li, L-X. Xu)

6. A New Solution to the Callan-Rubakov Effect '23 (T. D. Brennan)
   Monopoles, Scattering, and Generalized Symmetries '23 (M. Beest, P. B. Smith, D. Delmastro, Z. Komargodski, D. Tong)
   Fermion-Monopole Scattering in the Standard Model '23 (M. Beest, P. B. Smith, D. Delmastro, R. Mouland, D. Tong)

Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Strong CP Problem-I: IR to UV

II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

III. Strong CP Problem-II: UV to IR

III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase
III-3. Quality Problem

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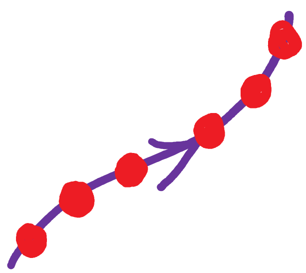
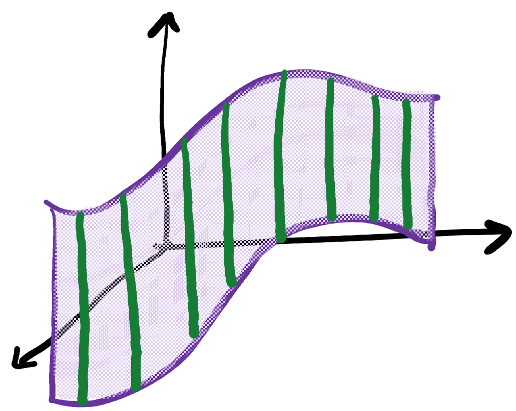
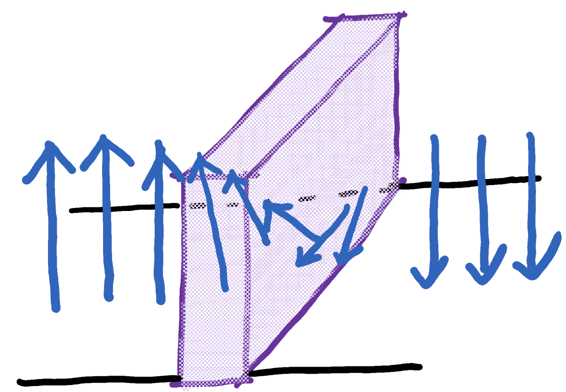
Higher-form symmetries

Various **extended objects** appear in broad class of theories.







Local operator
e.g. particle
0-form symmetry

Line operator
e.g. Wilson line
't Hooft line
1-form symmetry

Surface operator
e.g. Cosmic string
2-form symmetry

Volume operator
e.g. Domain Wall
3-form symmetry

Higher-form symmetries

1. p-form symmetry

0-form $\leftrightarrow$ local op (particle)
0-form $\leftrightarrow$ j_1 (j_μ)
0-form $\leftrightarrow$ A_1 (A_μ)

$$S \supset \int d^4x\, A_\mu j^\mu = \int A_1 \wedge * j_1$$

$$U(\alpha, \Sigma_3) = e^{i\alpha \int * j_1}$$

p-form $\leftrightarrow$ p-dim op
p-form $\leftrightarrow$ j_{p+1}
p-form $\leftrightarrow$ A_{p+1}

$$S \supset \int A_{p+1} \wedge * j_{p+1}$$

$$U\left(\alpha, \Sigma_{d-p-1}\right) = e^{i\alpha \int * j_{p+1}}$$

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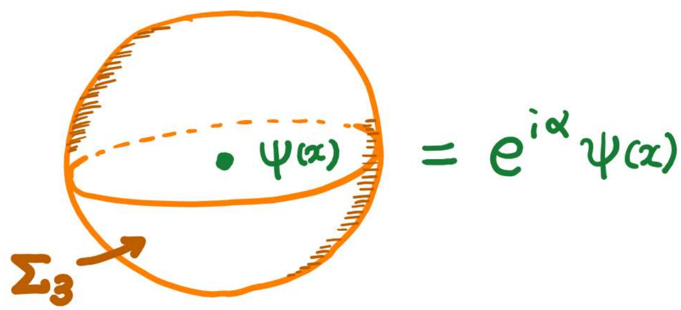
"Symmetry Defect Operator"

$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

$$\langle U(\alpha, \Sigma_3) \psi(x) \rangle \sim e^{i\alpha} \psi(x)$$



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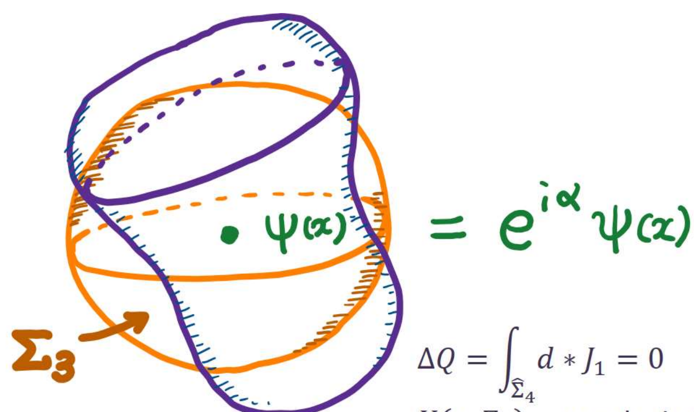

$$\Delta Q = \int_{\hat{\Sigma}_4} d * J_1 = 0$$

$U(\alpha, \Sigma_3) =$ topological

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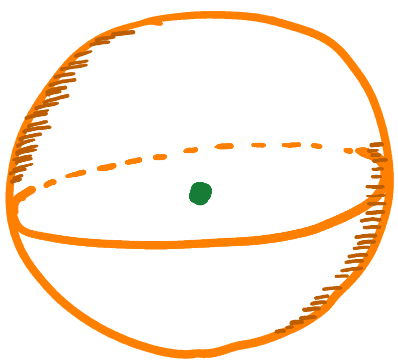
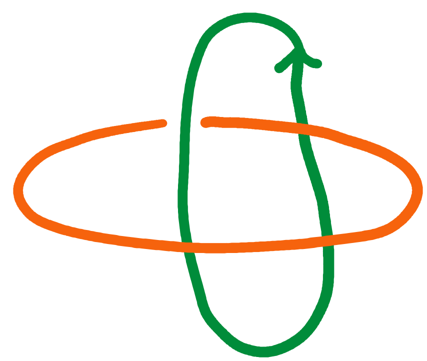
$$U(\alpha, \Sigma_{d-p-1}) = e^{i\alpha \int * j_{p+1}}$$

E.g.) 0- and 1-form symmetry in 3d





Higher-form symmetries

1. p-form symmetry

1-1. $SU(N)$ YM (either pure YM or with only adj matter)

$\exists\ Z_N^{(1)}(e)$: under 0-form center $\Psi \to e^{\frac{2\pi i}{N} * N}\, \Psi$

→ Wilson line with charge $= 0, 1, \cdots, (N-1)$ not screened

$\nexists$ mag 1-form : $\Pi_1\left(SU(N)\right) = \emptyset$

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$\rightarrow$ Wilson line with charge $= 0, 1, \cdots, (N-1)$ not screened

$\nexists$ mag 1-form : $\Pi_1\left(SU(N)\right) = \emptyset$

1-2. $PSU(N) = \frac{SU(N)}{Z_N}$: $Z_N^{(1)}(e)$ is gauged (electric states projected out)

$\nexists$ electric 1-form

$\exists\ Z_N^{(1)}(m) : \Pi_1\left(PSU(N)\right) = Z_N$ or "$N * \frac{1}{N} = 1$"

$\Rightarrow\ \oint G_2 = 2\pi/N\,,\quad \int \mathrm{tr}(G_2 \wedge G_2) \supset \frac{N-1}{2N} \int w_2 \wedge w_2$

Fractional Instanton

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Non-Invertible Symmetry

From Fractional Instanton

e.g. $G = SU(N)$

electric 1-form: Z_N
magnetic 1-form: none

Non-Invertible Symmetry

From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

Non-Invertible Symmetry

From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $\quad S \to S + \frac{2\pi K i}{z} \boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}} + \frac{2\pi K i}{z} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$

$\in Z$ $\qquad \in Z_L$

Non-Invertible Symmetry

From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

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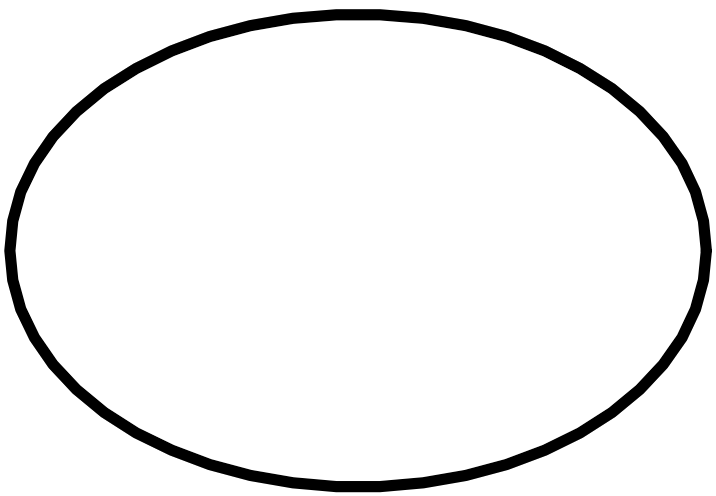
$\in Z$ $\in Z_L$

Global $U(1)_A$



Non-Invertible Symmetry

From Fractional Instanton

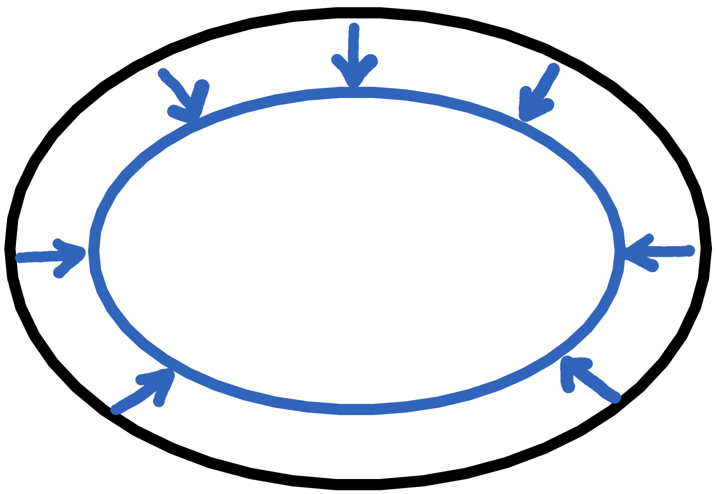
e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $S \to S + \frac{2\pi K i}{z}\boxed{\int_{M_4}\frac{G\wedge G}{8\pi^2}} + \frac{2\pi K i}{z}\boxed{\left(\frac{L-1}{L}\right)\int_{M_4}\frac{w_2\wedge w_2}{2}}$

$\in Z$ $\in Z_L$

Global $U(1)_A$



$\to$ Z Instanton

Non-Invertible Symmetry

From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

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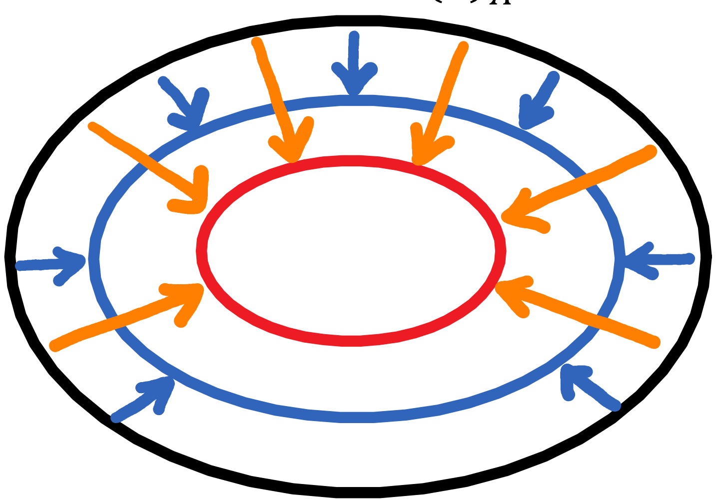
$\in Z$ $\qquad \in Z_L$

Global $U(1)_A$



→ Z **Instanton**

→ Z_L **(fractional) Instanton**

Non-Invertible Symmetry

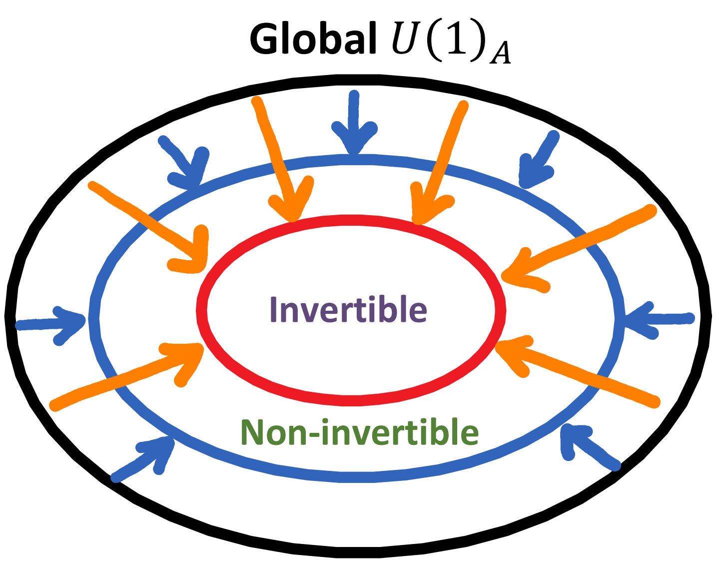
From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$
magnetic 1-form: Z_L

$$U(1)_A \text{ with } \alpha = \frac{2\pi}{z}, \quad S \to S + \frac{2\pi K i}{z} \boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}} + \frac{2\pi K i}{z} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$$

$\in Z$ $\quad$ $\in Z_L$



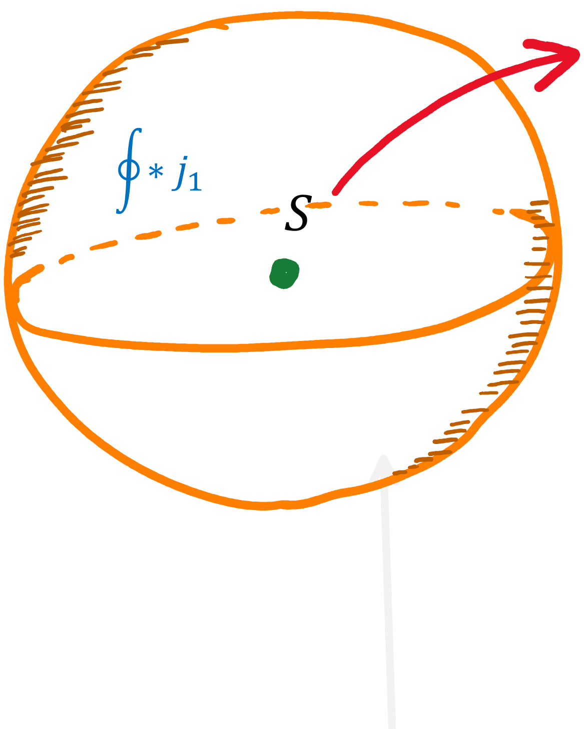
→ Z Instanton

→ Z_L (fractional) Instanton

Non-Invertible Symmetry

From Fractional Instanton

$$U(1)^{(0)}{}_A : \quad \Psi_+ \to e^{i\alpha}\,\Psi_+ \;, \quad \Psi_- \to e^{i\alpha}\,\Psi_- \quad \Rightarrow \quad d * j_A \propto w_2 \wedge w_2$$



$$S \;\to\; S + \frac{2\pi i K \ell}{K}\left(\frac{L-1}{L}\right)\int \frac{w_2 \wedge w_2}{2}$$

$$\underbrace{\exp\left(\frac{2\pi \ell i}{K}\oint * j_1\right)}_{U\left(\frac{2\pi\ell}{K},\,\Sigma_3\right)}$$

Non-Invertible Symmetry

From Fractional Instanton

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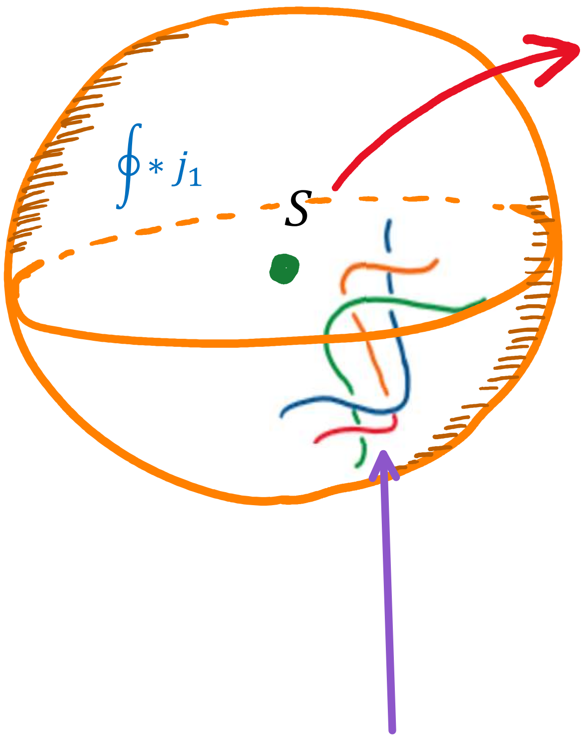

$$S \;\to\; S + \frac{2\pi i K \ell}{K}\left(\frac{L-1}{L}\right)\int \frac{w_2 \wedge w_2}{2} - \frac{2\pi i p}{L}\int \frac{w_2 \wedge w_2}{2} \;\to\; S$$

$$\underbrace{\exp\left(\frac{2\pi \ell i}{K}\oint * j_1\right)}_{U\left(\frac{2\pi\ell}{K},\,\Sigma_3\right)} \times \mathcal{A}^{\mathrm{L,p}}(\mathrm{w}_2)$$

$$S_{3d} = \frac{iL}{4\pi}\int_{\Sigma_3} a_1 \wedge da_2 + \frac{i}{2\pi}\int_{\Sigma_3} a_1 \wedge w_2 \quad (\text{for } p = 1)$$

Non-Invertible Symmetry

From Fractional Instanton

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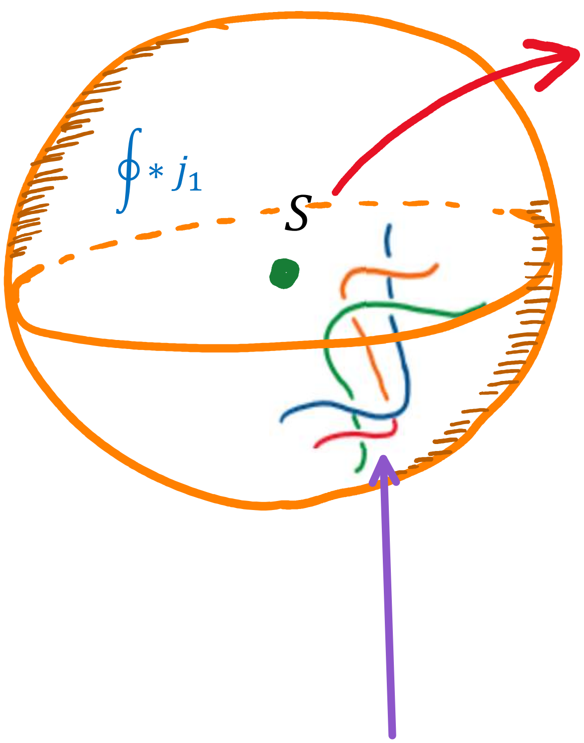

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$$\boldsymbol{\mathcal{D}_\ell(\Sigma_3)} = \underbrace{\exp\left(\frac{2\pi \ell i}{K}\oint * j_1\right)}_{U\left(\frac{2\pi\ell}{K},\,\Sigma_3\right)} \times \mathcal{A}^{\mathrm{L,p}}(\mathrm{w}_2)$$

$$\mathcal{D}_\ell(\Sigma_3) \times \overline{\mathcal{D}}_\ell(\Sigma_3) \sim \sum_S \xi(S)\exp\left(\frac{i}{2\pi L}\int_s w_2\right) \neq 1$$

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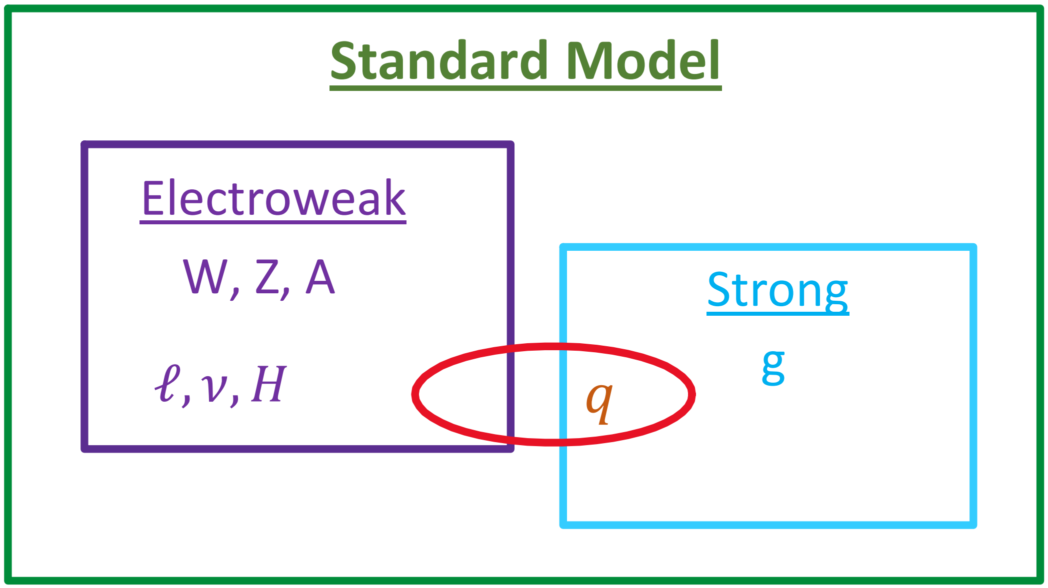
Strong-CP Problem

1. Strong CP Problem



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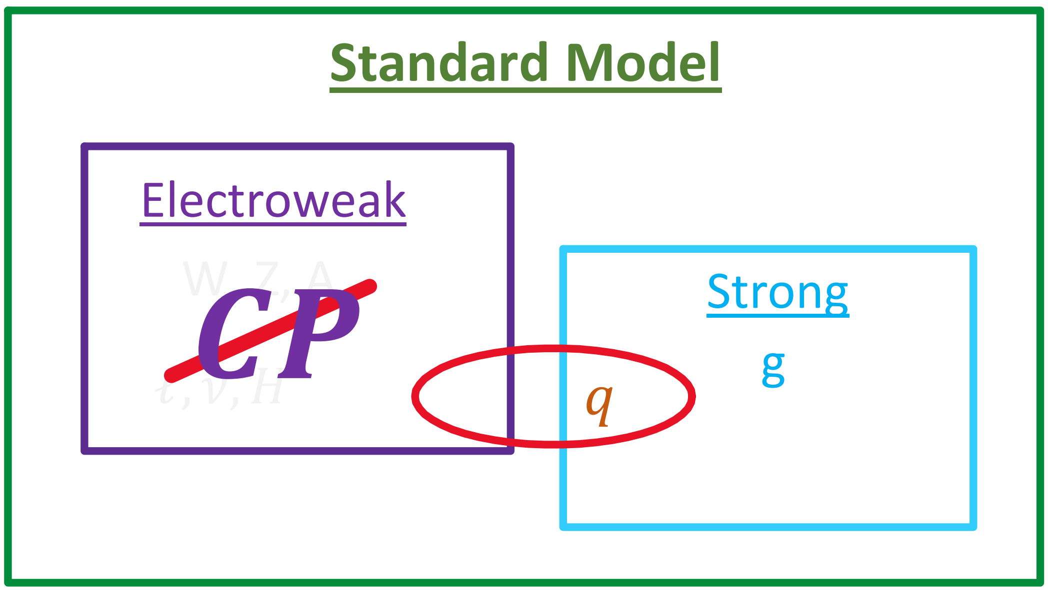
1. Strong CP Problem



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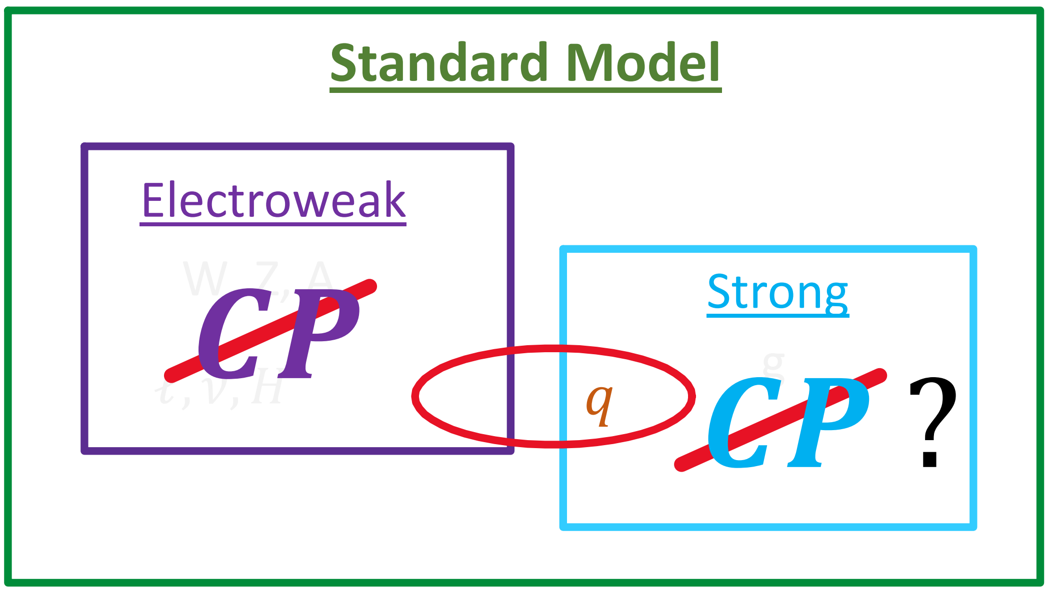


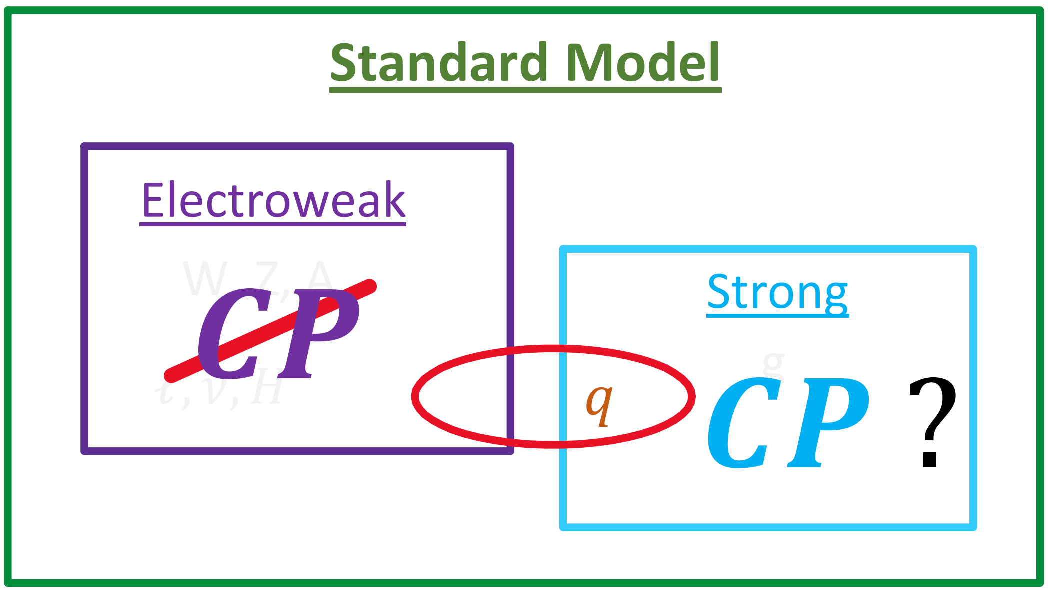
Expectation based on general rules of effective field theory

$$S_{QCD} \supset \frac{i\bar{\boldsymbol{\theta}}}{8\pi^2}\int Tr(G \wedge G)$$

$$\bar{\boldsymbol{\theta}} \sim \boldsymbol{O}(\mathbf{1})$$

Strong-CP Problem

1. Strong CP Problem



Expectation based on general rules of effective field theory

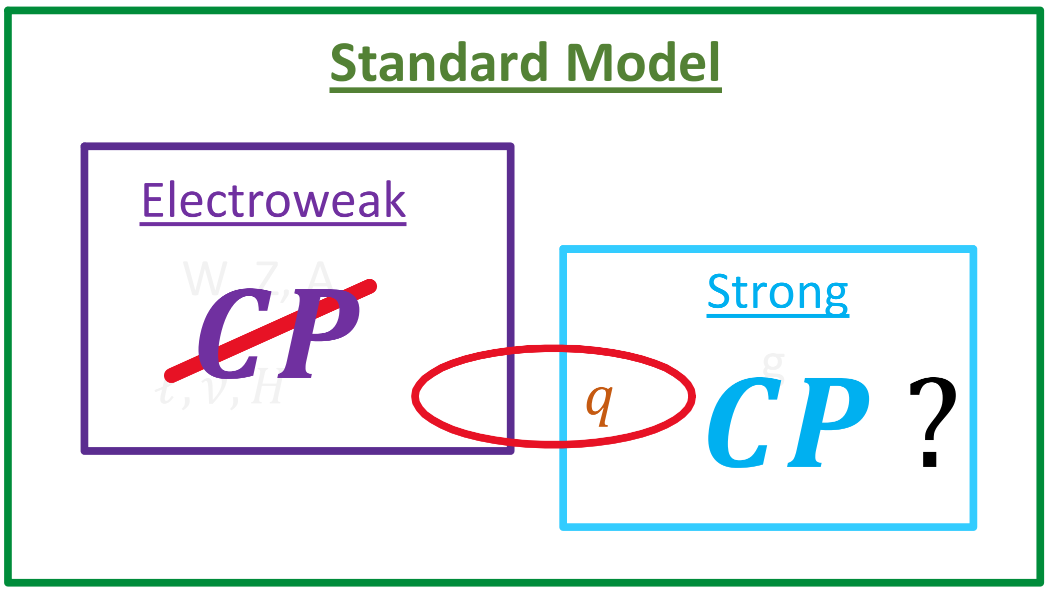
$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$$

Neutron Electric Dipole Moment

$$d_n \sim 3\times 10^{16}\,\bar{\theta} \quad\rightarrow\quad \bar{\theta} < 10^{-10}$$

Strong-CP Problem

1. Strong CP Problem



Expectation based on general rules of effective field theory

$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2} \int Tr(G \wedge G)$$

Neutron Electric Dipole Moment

$$d_n \sim 3 \times 10^{16}\, \bar{\theta} \;\; \rightarrow \;\; \bar{\theta} < 10^{-10}$$

$$\tilde{J} = \mathrm{Im}\,\det[y_u^{\dagger} y_u, y_d^{\dagger} y_d\,] \propto \sin \delta_{CKM} \qquad \text{vs} \qquad \bar{\theta} \equiv \arg e^{-i\theta} \det(y_u y_d)$$

"Jarlskog invariant"

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

Conclusion:

We start with $\mathcal{L} \supset y_u \widetilde{H} Q \bar{u} + y_e H L \bar{e}$ but $\boldsymbol{y_d = 0}$ $(y_d H Q \bar{d})$

So, new symmetries appearing below are approximate symmetries

and $\boldsymbol{y_d}$ is the symmetry breaking spurion (parameter).

Strong-CP Problem

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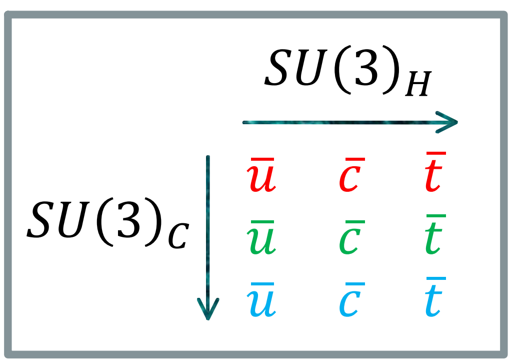
(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

(2) $SU(3)_C \times U(1)_H / Z_3,\ H = B_1 + B_2 - 2B_3$: $\boldsymbol{Z_3^{Q-\bar{u}+\bar{d}}}$ NIS



$B_i \equiv Q_i - \bar{u}_i - \bar{d}_i$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

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(1) $SU(3)_C \times SU(3)_H / \boldsymbol{Z_3}$: $N_c = N_g$ $\boldsymbol{Z_3^{\bar{d}}}$ NIS

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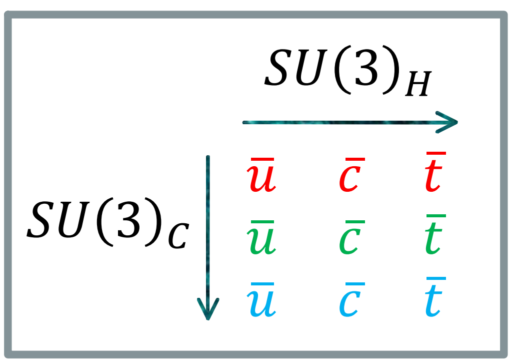

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Strong-CP Problem

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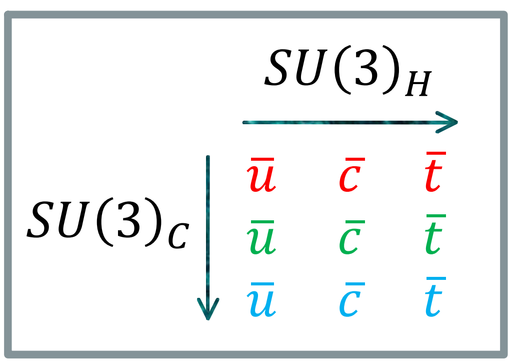

$[Z_3^C \subset SU(3)_C] = [Z_3^H \subset U(1)_H]$: $N_C = N_g$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

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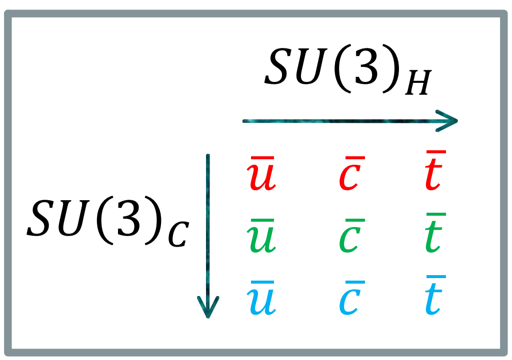
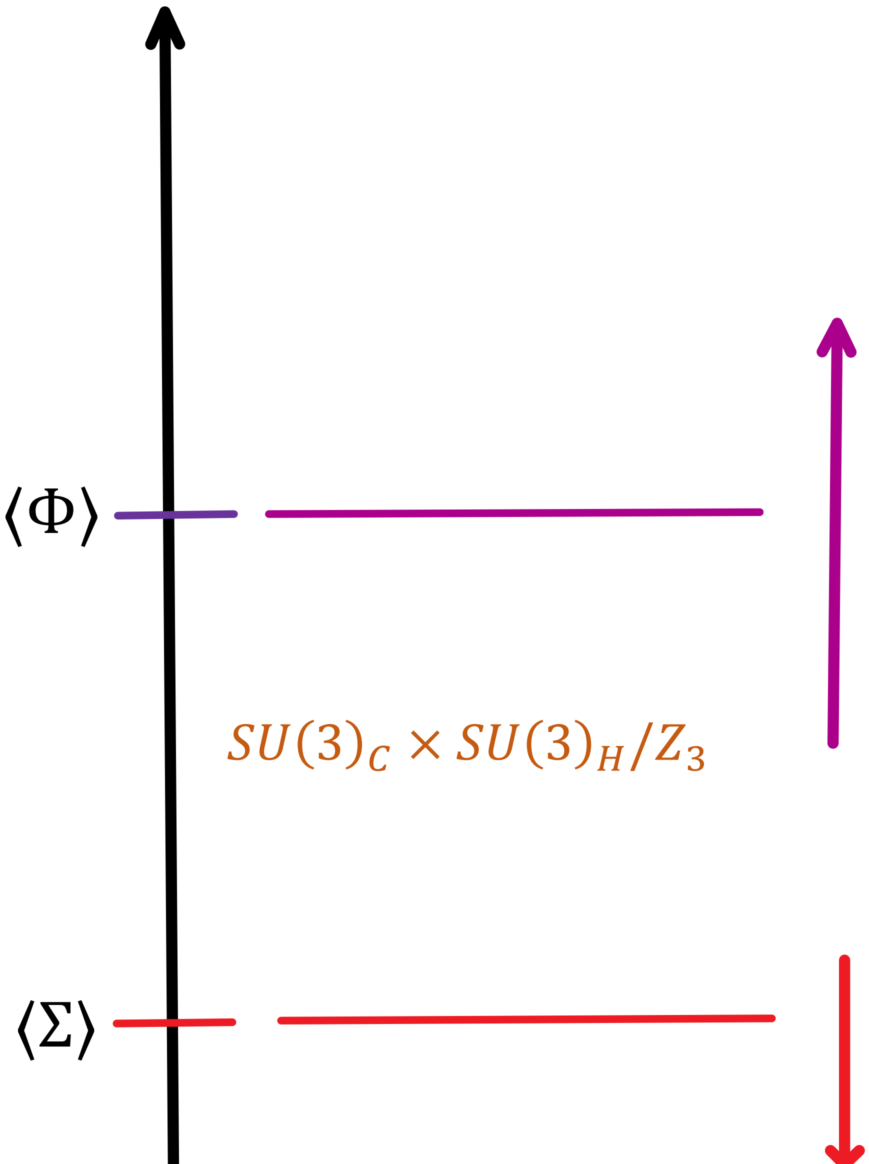


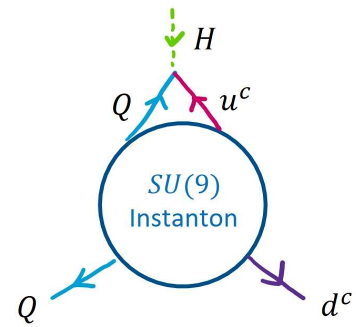
$$B_i \equiv Q_i - \bar{u}_i - \bar{d}_i$$

Solving Strong CP with Non-Invertible Symmetry



- Start with only $y_u \widetilde{H} Q \bar{u}$ $\quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta}$ = unphysical

Solving Strong CP with Non-Invertible Symmetry



$SU(9)$

$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

with $\bar{\theta} = 0$

$\langle \Phi \rangle$

$SU(3)_C \times SU(3)_H / Z_3$

- Start with only $y_u \widetilde{H} Q \bar{u}$ $(y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
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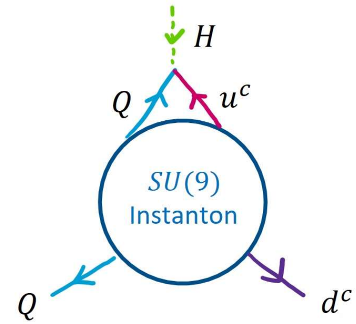
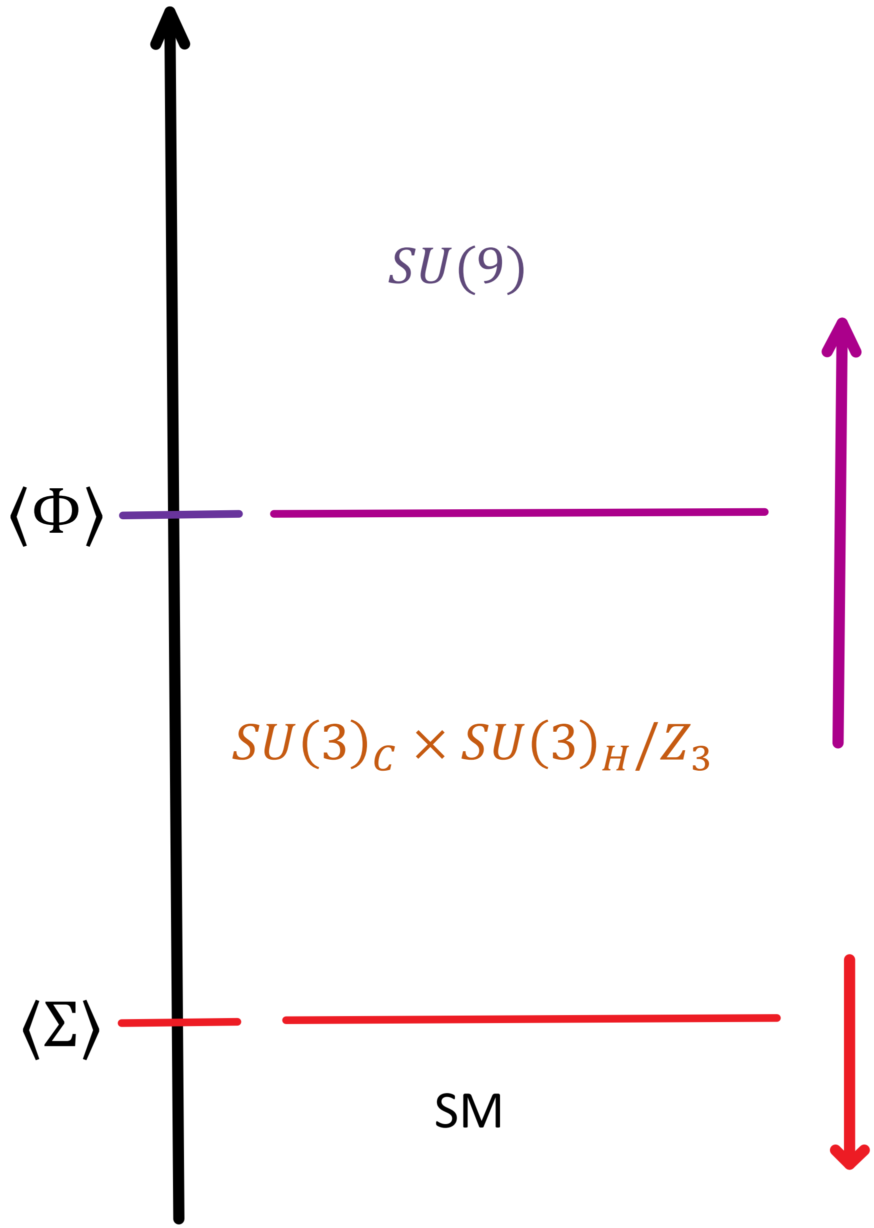
$\langle \Sigma \rangle$

Solving Strong CP with Non-Invertible Symmetry





$\mathcal{L}_{UV} \supset y_u \tilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$

with $\bar{\theta} = 0$

- Start with only $y_u \tilde{H} Q \bar{u}$ $(y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta} =$ unphysical

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

with $\bar{\theta} = 0$ or $\bar{\theta} \ll 1$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

	$SU(3)_C$	$SU(3)_H$	$U(1)_B$	$U(1)_{\bar{d}}$
$\boldsymbol{Q}$	3	3	$+1$	0
$\boldsymbol{\bar{u}}$	$\bar{3}$	$\bar{3}$	-1	0
$\boldsymbol{\bar{d}}$	$\bar{3}$	$\bar{3}$	-1	$+1$

$\mathcal{L} \sim y_u \tilde{H} Q^i \bar{u}_i$ (flavor-diagonal/universal)

$y_u = c_u \mathbb{I}_3$, $\mathrm{c_u} =$ number

Strong-CP Problem

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CFU Fractional Instanton (**CFU**=**C**olor-(non-abelian)**F**lavor-**U**(1))

Quotient by Z_3: (i) $[Z_3 \in SU(3)_C] \equiv [Z_3 \in SU(3)_H]$

(ii) Under "diagonal" Z_3 entire fields are neutral, more magnetic states

(iii) $\exists Z_3$ magnetic 1-form: $\oint w_2(C) = \oint w_2(H) = 0,1,2 \;\; (\in Z_3)$

(iv) CFU instanton: $\mathcal{N}_C = \frac{1}{3}\int w_2(C) \wedge w_2(C)$, $\mathcal{N}_H = \frac{1}{3}\int w_2(H) \wedge w_2(H)$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

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	$SU(3)_C$	$SU(3)_H$	$U(1)_B$	$U(1)_{\bar{d}}$
$\boldsymbol{Q}$	3	3	$+1$	0
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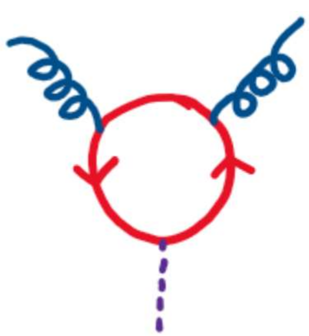

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$$\mathcal{A}_f = \sum_{\Psi_i} q_i I_{\Psi_i} = 3(\mathcal{N}_C + \mathcal{N}_H)(2q_Q + q_{\bar{d}} + q_{\bar{u}})$$

Strong-CP Problem

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$y_u = c_u \mathbb{I}_3$, $\mathrm{c_u}$ = number

	$U(1)_B$	$U(1)_{\bar{d}}$
$[SU(3)_C]^2$	0	N_g
$[SU(2)_L]^2$	$N_c N_g$	0
$[U(1)_Y]^2$	$-18 N_c N_g$	$4 N_c N_g$
$[SU(3)_H]^2$	0	N_c
$[CH]$	0	2

Strong-CP Problem

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Symmetry

(i) Without including $[CH]$ instanton:

$$\frac{U(1)_B}{Z_3} \times U(1)_{\bar{d}} \to Z_3^B \times Z_3^{\bar{d}}$$

non-abelian instantons dominant → No NIS

Strong-CP Problem

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non-abelian instantons dominant → No NIS

(ii) With $[CH]$ instanton:

$$Z_3^B \times Z_3^{\bar{d}} \to Z_3^B \times \emptyset$$

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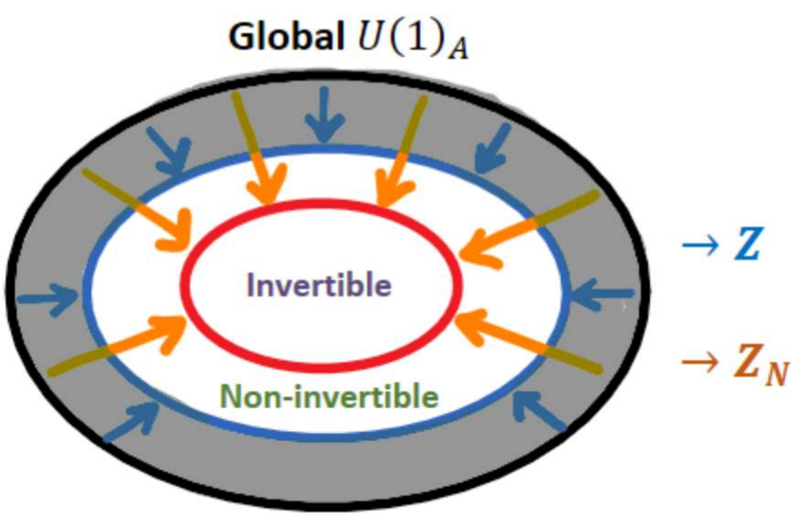

Symmetry

(i) Without including $[CH]$ instanton:

$$\frac{U(1)_B}{Z_3} \times U(1)_{\bar{d}} \rightarrow Z_3^B \times Z_3^{\bar{d}}$$

non-abelian instantons dominant → No NIS

(ii) With $[CH]$ instanton:

$$Z_3^B \times Z_3^{\bar{d}} \rightarrow Z_3^B \times \emptyset \quad \Rightarrow \quad \boldsymbol{Z_3^{\bar{d}}}$$ **NIPQ Symmetry**

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

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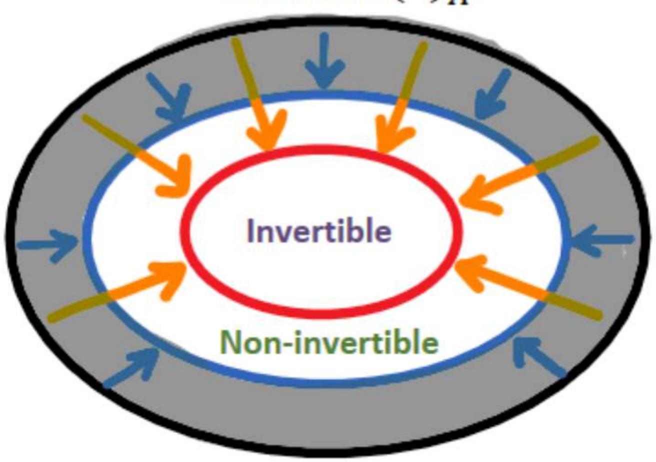

Symmetry

(i) Without including $[CH]$ instanton:

$$\frac{U(1)_B}{Z_3} \times U(1)_{\bar{d}} \to Z_3^B \times Z_3^{\bar{d}}$$

non-abelian instantons dominant → No NIS

(ii) With $[CH]$ instanton: $N_c = N_g$

$$Z_3^B \times Z_3^{\bar{d}} \to Z_3^B \times \emptyset \quad \Rightarrow \quad Z_3^{\bar{d}}$$ **NIPQ Symmetry**

Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Strong CP Problem-I: IR to UV

II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

III. Strong CP Problem-II: UV to IR

III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase
III-3. Quality Problem

Strong-CP Problem

3. Massless Quark Solution to the Strong CP Problem

(1) $SU(3)_C \times SU(3)_H/Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

$\mathcal{L} \sim y_d H Q \bar{d}$ term is forbidden by $Z_3^{\bar{d}}$ non-invertible Peccei-Quinn symmetry

Down quarks (d, s, b) are massless if $Z_3^{\bar{d}}$ is exact.
(y_d = non-invertible $Z_3^{\bar{d}}$ breaking spurion (paramter))

Strong-CP Problem

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(y_d = non-invertible $Z_3^{\bar{d}}$ breaking spurion (paramter))

Massless Quark Solution:

1. $$\mathcal{L}_{SM} \supset y_u \widetilde{H} Q \bar{u} + y_d H Q \bar{d} + \frac{\theta}{32\pi^2} G\tilde{G}$$
$$= m_u e^{i\varphi_u} u\bar{u} + m_d e^{i\varphi_d} d\bar{d} + \frac{\theta}{32\pi^2} G\tilde{G}$$

$$u \to e^{i\alpha} u, \quad \varphi_u \to \varphi_u + \alpha, \quad \theta \to \theta + \alpha$$
$$d \to e^{i\beta} d, \quad \varphi_d \to \varphi_d + \beta, \quad \theta \to \theta + \beta$$

Strong-CP Problem

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$= m_u e^{i\varphi_u} u\bar{u} + m_d e^{i\varphi_d} d\bar{d} + \frac{\theta}{32\pi^2} G\tilde{G}$

$u \to e^{i\alpha} u, \quad \varphi_u \to \varphi_u + \alpha, \quad \theta \to \theta + \alpha$
$d \to e^{i\beta} d, \quad \varphi_d \to \varphi_d + \beta, \quad \theta \to \theta + \beta$

$\boldsymbol{\bar{\theta} \equiv \arg e^{-i\theta} \det(y_u y_d)}$

Neutron electric dipole moment $d_n \sim 3 \times 10^{16}\, \bar{\theta} \;\to\; \bar{\theta} < 10^{-10}$

Strong-CP Problem

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Down quarks (d, s, b) are massless if $Z_3^{\bar{d}}$ is exact.
($y_d =$ non-invertible $Z_3^{\bar{d}}$ breaking spurion (paramter))

Massless Quark Solution:

2. In the presence of massless chiral fermion, e.g. $m_d = 0$, "$\boldsymbol{\bar{\theta}}$ is unphysical"

$$\mathcal{L}_{SM} \supset y_u \widetilde{H} Q \bar{u} + \frac{\theta}{32\pi^2} G\tilde{G} = m_u e^{i\varphi_u} u\bar{u} + \frac{\theta}{32\pi^2} G\tilde{G}$$

Field redefinition: $d \to e^{i\beta} d \quad \Rightarrow \quad \delta S = \frac{i}{8\pi^2}\left(\bar{\theta} + \beta\right) \int \mathrm{tr}(G_2 \wedge G_2)$

Strong-CP Problem

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Massless Quark Solution:

2. In the presence of massless chiral fermion, e.g. $m_d = 0$, "$\boldsymbol{\bar{\theta}}$ is unphysical"

$$\mathcal{L}_{SM} \supset y_u \tilde{H} Q \bar{u} + \frac{\theta}{32\pi^2} G\tilde{G} = m_u e^{i\varphi_u} u\bar{u} + \frac{\theta}{32\pi^2} G\tilde{G}$$

Field redefinition: $d \to e^{i\beta} d \quad \Rightarrow \quad \delta S = \frac{i}{8\pi^2}\left(\bar{\theta} + \beta\right) \int \mathrm{tr}(G_2 \wedge G_2)$

Note: $M \equiv e^{-i\theta} \det(y_u y_d) \in \mathbb{C}$ and CP: $\mathrm{Im}(M) \to -\mathrm{Im}(M)$

M behaves smoothly as $|M| \to 0$

CP-invariance $\leftrightarrow$ $M \in \mathbb{R}_+$

Strong-CP Problem

3. Massless Quark Solution to the Strong CP Problem

(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

$\mathcal{L} \sim y_d H Q \bar{d}$ term is forbidden by $Z_3^{\bar{d}}$ non-invertible Peccei-Quinn symmetry

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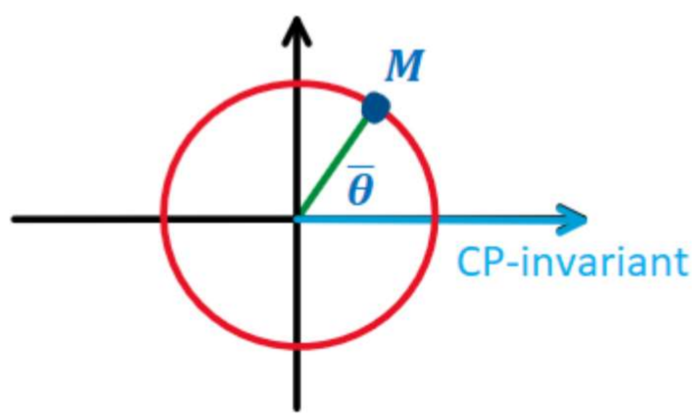

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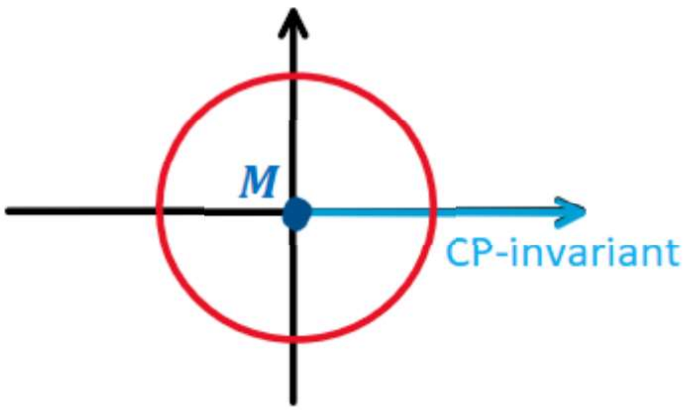

Strong-CP Problem

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Down quarks (d, s, b) are massless if $Z_3^{\bar{d}}$ is exact.
(y_d = non-invertible $Z_3^{\bar{d}}$ breaking spurion (paramter))

Massless Quark Solution:

3. In SM, "massless up quark solution" tried.

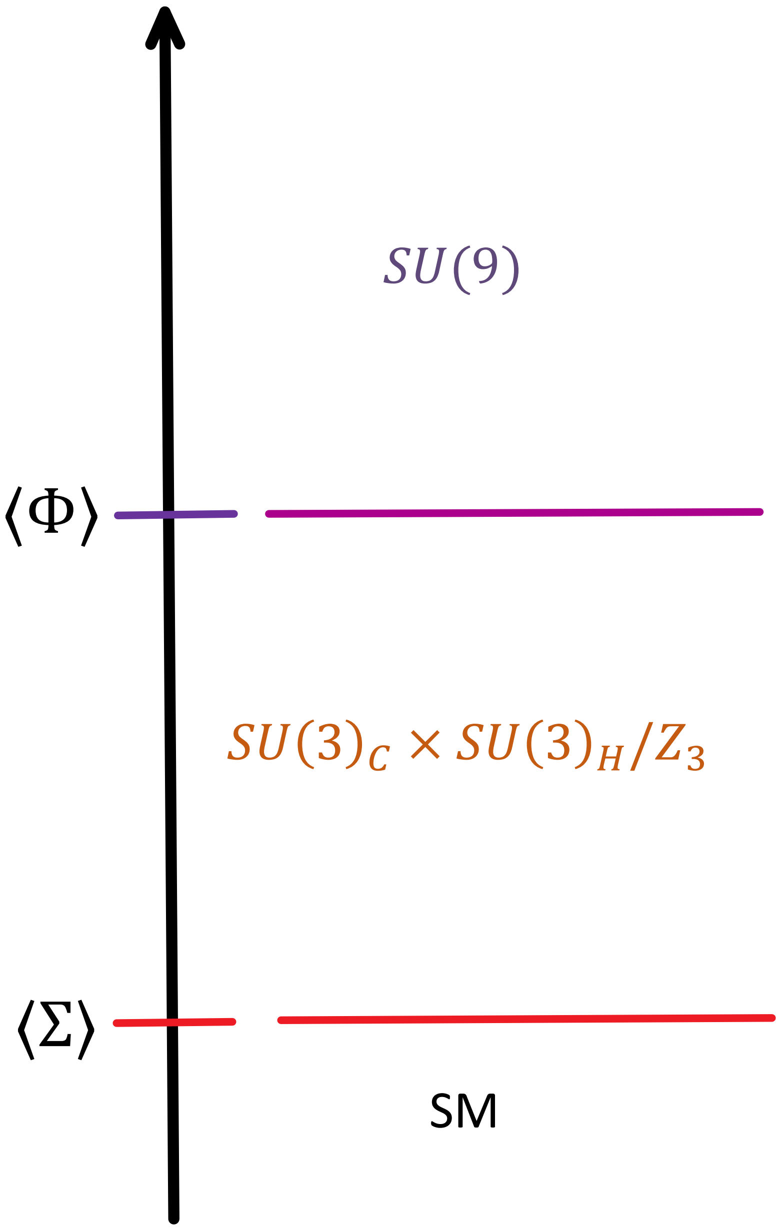
In nature, up quark seems to be massive

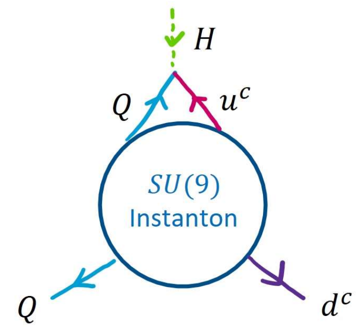
e.g. Chiral-PT + observed hadron mass : $m_u/m_d \sim 0.6$

QCD instanton calculation not under analytic control

Lattice QCD : QCD instanton **not sufficient** in size

Solving Strong CP with Non-Invertible Symmetry





$$\mathcal{L}_{UV} \supset y_u \tilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

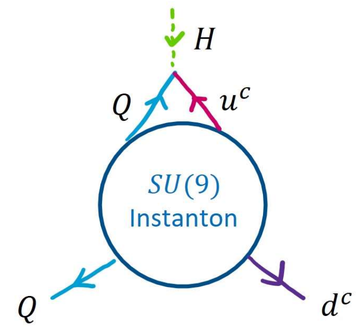
with $\bar{\theta} = 0$

- Start with only $y_u \tilde{H} Q \bar{u}$ $(y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta}$ = unphysical

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

with $\bar{\theta} = 0$ or $\bar{\theta} \ll 1$

Solving Strong CP with Non-Invertible Symmetry



Small (G/H) Instanton

$SU(9)$

$$\mathcal{L}_{UV} \supset y_u \tilde{H} Q \bar{u} + \boldsymbol{y_u^* e^{-\frac{2\pi}{\alpha_9}}} H Q \bar{d} + \cdots$$

with $\bar{\theta} = 0$

$\langle \Phi \rangle$

$SU(3)_C \times SU(3)_H / Z_3$

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$\langle \Sigma \rangle$

SM

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

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Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Strong CP Problem-I: IR to UV

II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

III. Strong CP Problem-II: UV to IR

III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase
III-3. Quality Problem

Color-Flavor Unification

1. $SU(9)$ Unification and the Strong CP phase $\overline{\theta}$

$$SU(3)_C \downarrow \overset{SU(3)_H \;\longrightarrow}{\begin{array}{ccc} \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \end{array}} \;=\; 9\ (\square)\ \text{of}\ SU(9)$$

Color-Flavor Unification

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	$SU(9)$	$U(1)_{Q-\bar{u}}$	$U(1)_{\bar{d}}$
$\boldsymbol{Q} = (\boldsymbol{u}, \boldsymbol{d})^t$	9	$+1$	0
$\bar{\boldsymbol{u}}$	$\bar{9}$	-1	0
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Color-Flavor Unification

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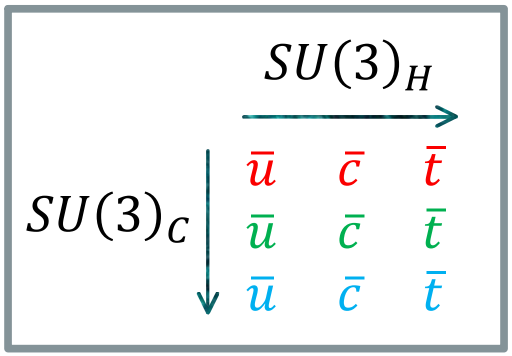

$= 9\ (\square)$ of $SU(9)$

	$SU(9)$	$U(1)_{Q-\bar{u}+\bar{d}}$
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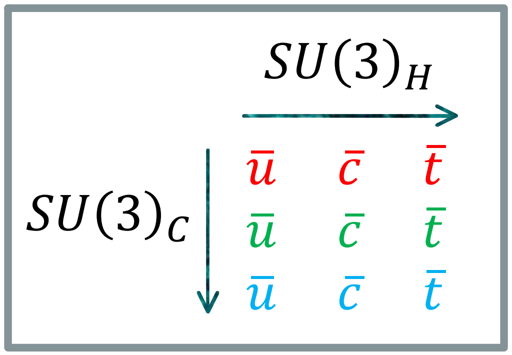

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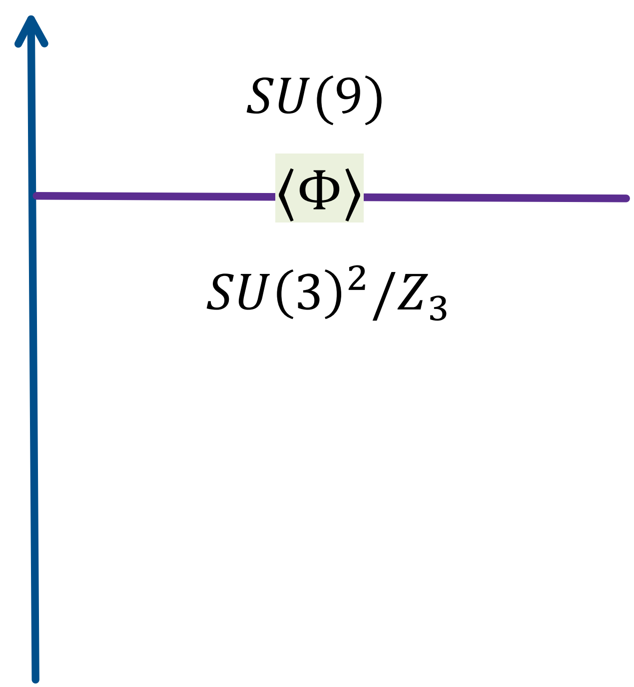

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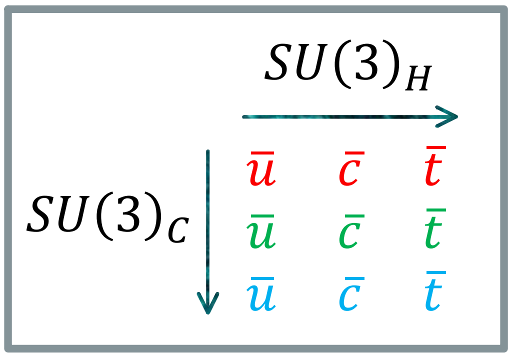

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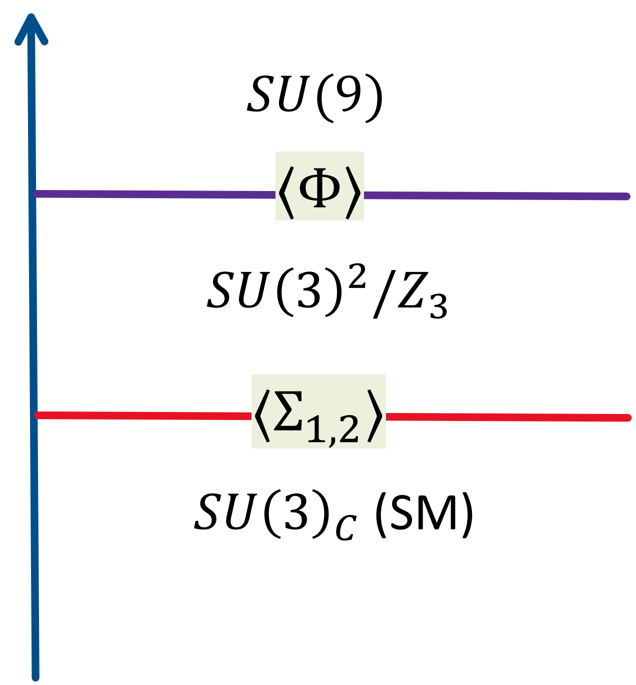

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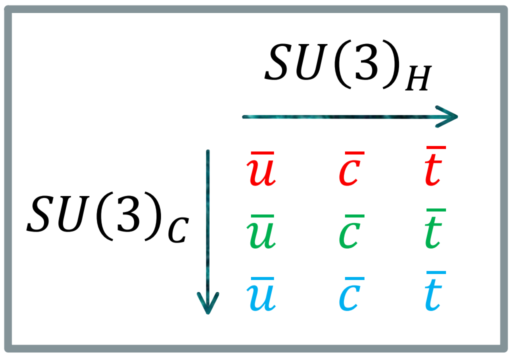

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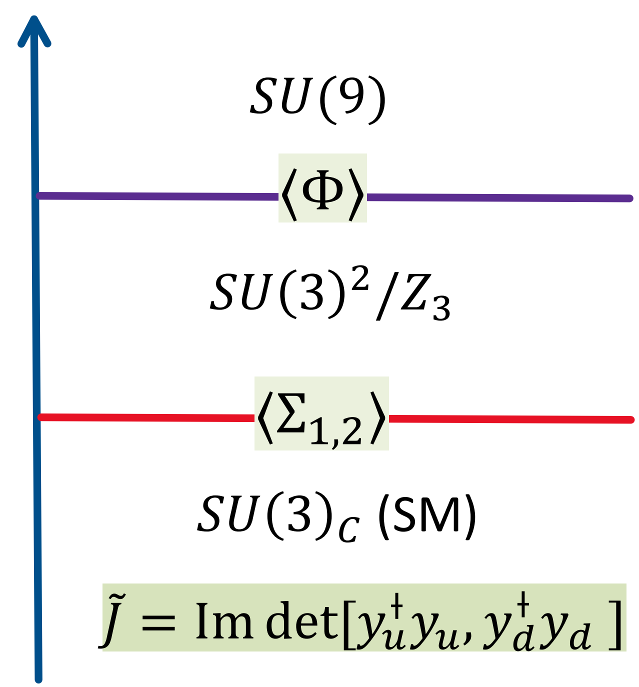

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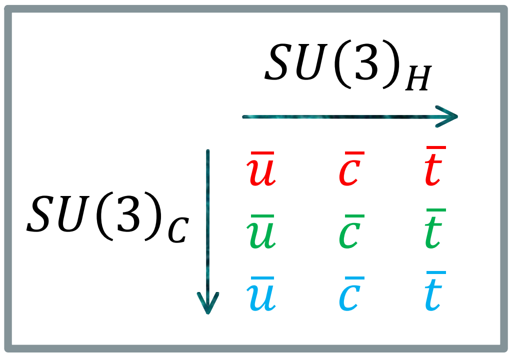

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$$\mathcal{L}_0 = y_t \tilde{H} \boldsymbol{Q} \bar{\boldsymbol{u}} + h.c. + \frac{i\theta_9}{32\pi^2} \int F\tilde{F}$$

$$y_t \sim O(1)$$

y_d perturbatively protected by $U(1)_{\bar{d}}$

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$$y_t \sim O(1)$$

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$$U(1)_{Q-\bar{u}}[SU(9)]^2 = U(1)_{\bar{d}}[SU(9)]^2 = 1$$

$\Rightarrow$ [Anomaly Free] $U(1)_{B=Q-\bar{u}-\bar{d}}$

[Anomalous] $U(1)_{Q-\bar{u}+\bar{d}}$ or $U(1)_{\bar{d}}$

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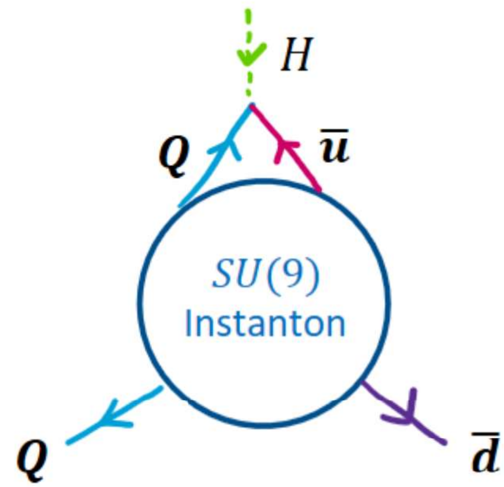

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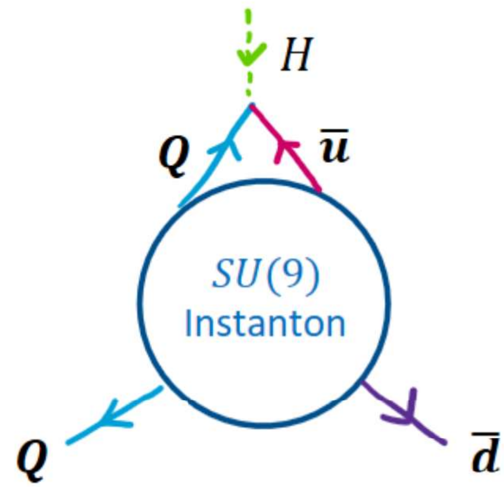

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$$y_d \sim y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda_9)}}$$

$$\bar{\theta} = \arg e^{-i\theta_9} \det(y_u y_d) = -\theta_9 + \arg |y_t|^2 e^{i\theta_9} = 0 \quad \checkmark$$

Color-Flavor Unification

2. $SU(9) \rightarrow SU(3)_C \times SU(3)_H / Z_3$

	$SU(9)$	$U(1)_{Q-\bar{u}+\bar{d}}$
Φ	165 (3S)	0
$\Sigma_{1,2}$	80 (adj)	0
ρ	9	-1
χ	1	0

(i) SSB by $\langle \Phi^{\mathrm{ABC}} \rangle = \Lambda_9 \epsilon^{abc} \epsilon^{ijk}$

(ii) $9(\mathrm{Q}, \bar{u}, \bar{d}, \rho) \rightarrow \ (3,3)$

(iii) Z_3 Quotient: $Q \rightarrow g_C\, Q\, g_H^{\dagger}$

(iv) $165 \rightarrow (10,10) + (\mathbf{8},\mathbf{8}) + (1,1)$

$(\Phi_{\{ai,bj,ck\}} \sim \phi_{\{abc\}} \cdot \tilde{\phi}_{\{ijk\}} + \phi_{[ab]c} \cdot \tilde{\phi}_{[ij]k} + \phi_{[abc]} \cdot \tilde{\phi}_{[ijk]})$

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$$\to y_t \widetilde{H} \boldsymbol{Q} \bar{\boldsymbol{u}} + y_t^* e^{i\theta_9} e^{-\frac{2\pi}{3\alpha_S(\Lambda_9)}} H \boldsymbol{Q} \bar{\boldsymbol{d}} + \frac{i3\theta_9}{32\pi^2} \int \left(G\tilde{G} + K\widetilde{K}\right)$$ (Yukawa$\propto \mathbb{I}_3$, Flavor-diag)

<u>Index of embedding</u>

$G \to H :$ (1 – H – instanton) = (n – G – instanton)

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$(\Phi_{\{ai,bj,ck\}} \sim \phi_{\{abc\}} \cdot \tilde{\phi}_{\{ijk\}} + \phi_{[ab]c} \cdot \tilde{\phi}_{[ij]k} + \phi_{[abc]} \cdot \tilde{\phi}_{[ijk]})$

(v) $80 \to (8,8) + (8,1) + (1,8)$

$$\mathcal{L}_0 = y_t \widetilde{H} \boldsymbol{Q}\boldsymbol{\bar{u}} + y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9}} H \boldsymbol{Q}\boldsymbol{\bar{d}} + \frac{i\theta_9}{32\pi^2} \int F\tilde{F}$$ (Yukawa=single number)

$$\to y_t \widetilde{H} \boldsymbol{Q}\boldsymbol{\bar{u}} + y_t^* e^{i\theta_9} e^{-\frac{2\pi}{3\alpha_s(\Lambda_9)}} H \boldsymbol{Q}\boldsymbol{\bar{d}} + \frac{i3\theta_9}{32\pi^2} \int \left(G\tilde{G} + K\tilde{K}\right)$$ (Yukawa $\propto \mathbb{I}_3$, Flavor-diag)

$$\bar{\theta} = -3\theta_9 + \arg\det|y_t|^2 e^{i\theta_9} = -3\theta_9 + 3\theta_9 = 0 \quad \checkmark$$

From now on, we set $\boldsymbol{\bar{\theta}_9 = 0}$ and take **real yuakwas.**

Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

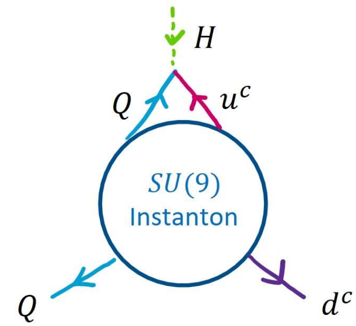
II. Strong CP Problem-I: IR to UV

II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

III. Strong CP Problem-II: UV to IR

III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase
III-3. Quality Problem

Solving Strong CP with Non-Invertible Symmetry



$SU(9)$

$\langle\Phi\rangle$

$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

with $\bar{\theta} = 0$

- Start with only $y_u \widetilde{H} Q \bar{u} \quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta} =$ unphysical

$SU(3)_C \times SU(3)_H / Z_3$

$\langle\Sigma\rangle$

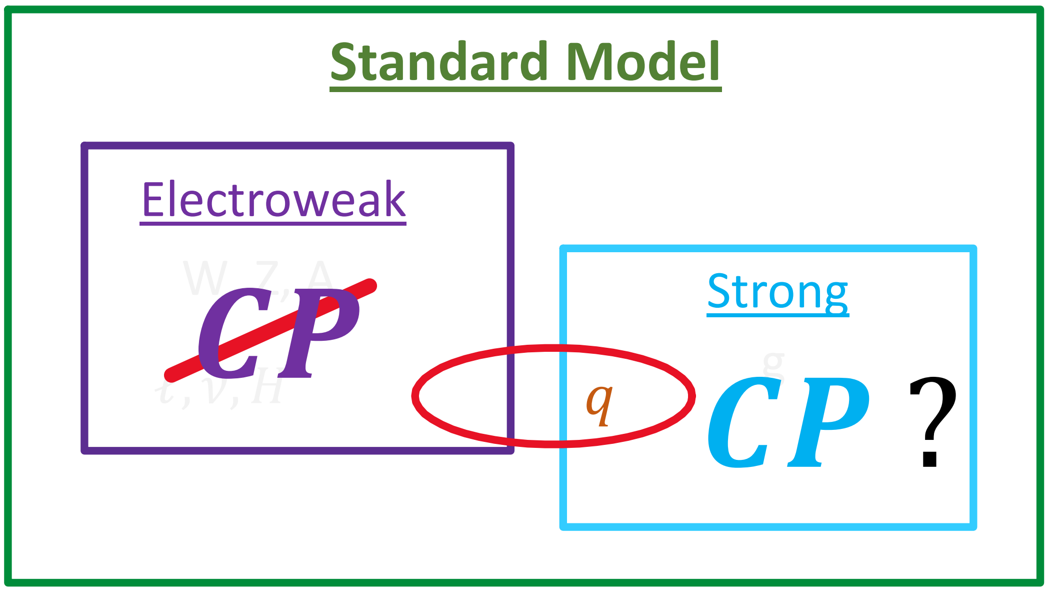
SM

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

with $\bar{\theta} = 0$ or $\bar{\theta} \ll 1$

Strong-CP Problem

1. Strong CP Problem



Expectation based on general rules of effective field theory

$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$$

Neutron Electric Dipole Moment

$$d_n \sim 3 \times 10^{16}\,\bar{\theta} \quad \rightarrow \quad \bar{\theta} < 10^{-10}$$

$$\tilde{J} = \mathrm{Im}\,\det[y_u^\dagger y_u, y_d^\dagger y_d\,] \propto \sin\delta_{CKM} \qquad \text{vs} \qquad \bar{\theta} \equiv \arg e^{-i\theta}\det(y_u y_d)$$

"Jarlskog invariant"

Color-Flavor Unification

3. $SU(3)_C \times SU(3)_H / Z_3 \to SU(3)_C$: Flavor Structre and CKM CPV

	$SU(9)$	$U(1)_{Q-\bar{u}+\bar{d}}$
Φ	165 (3S)	0
$\Sigma_{1,2}$	80 (adj)	0
ρ	9	-1
χ	1	0

(i) Two $SU(3)_H$ adjoint scalar $\Sigma_{1,2}$: $SU(3)_H \to \emptyset$

(ii) Textures of y_u, y_d generated by structure of $\langle \Sigma_{1,2} \rangle$

(iii) Required CKM CPV phase from $V(\Sigma)$

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Combine $\Sigma = \Sigma_1 + i\Sigma_2$ $\quad (U(1)_\Sigma)$

Consider a simple case with Z_4 invariant potential $V(\Sigma)$
(our mechanism works regardless of this simplifying assumption)

$$V(\Sigma) = \eta_1 \mathrm{Tr}(\Sigma^4) + \eta_2 \left(\mathrm{Tr}(\Sigma^2)\right)^2 + h.c. + \xi \mathrm{Tr}\left(\Sigma^\dagger \Sigma\right)^2 + \cdots \text{(terms with real coeffs)}$$

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Field redefinition invariant CPV : $\boldsymbol{\eta_1^\dagger \eta_2}$

$$\Sigma \to e^{-i\varphi_1/4}\Sigma: \quad |\eta_1|e^{i\varphi_1}\mathrm{Tr}(\Sigma^4) + |\eta_2|e^{i\varphi_2}\left(\mathrm{Tr}(\Sigma^2)\right)^2 \to |\eta_1|\mathrm{Tr}(\Sigma^4) + |\eta_2|e^{i(\varphi_2-\varphi_1)}\left(\mathrm{Tr}(\Sigma^2)\right)^2$$

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Complete breaking $\Leftrightarrow$ $[\Sigma_1, \Sigma_2] = \frac{[\Sigma^\dagger, \Sigma]}{2i} \neq 0$

(ii) Textures of y_u, y_d generated by structure of $\langle \Sigma_{1,2} \rangle$

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$\Rightarrow$ **Our mechanism:** generate **Hermitian Yukawas**

(I) all CPV in scalar sector

(II) CPV transferred to SM fermions via bosonic mediation

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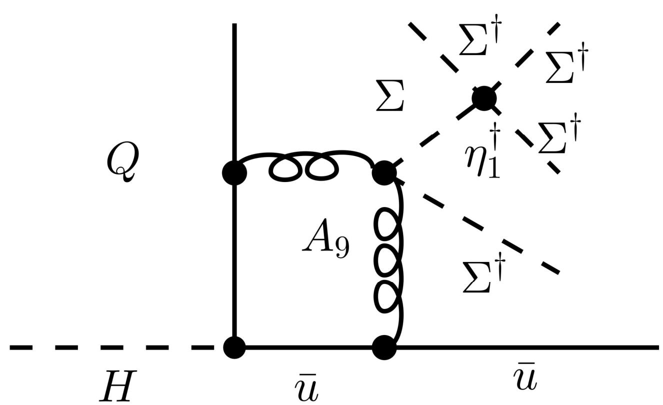

+ diagram with $h.c.\left(\eta \leftrightarrow \eta^\dagger, \Sigma \leftrightarrow \Sigma^\dagger\right)$

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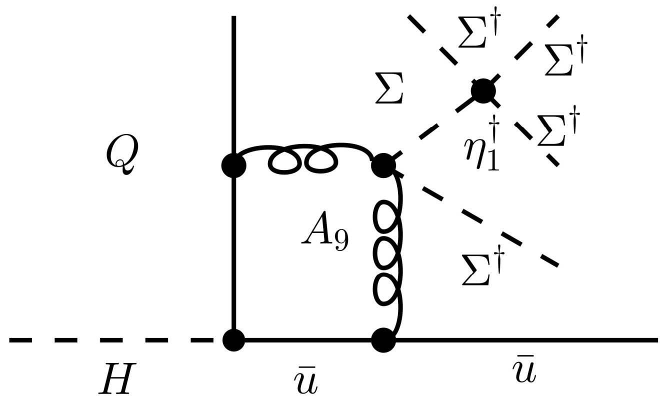

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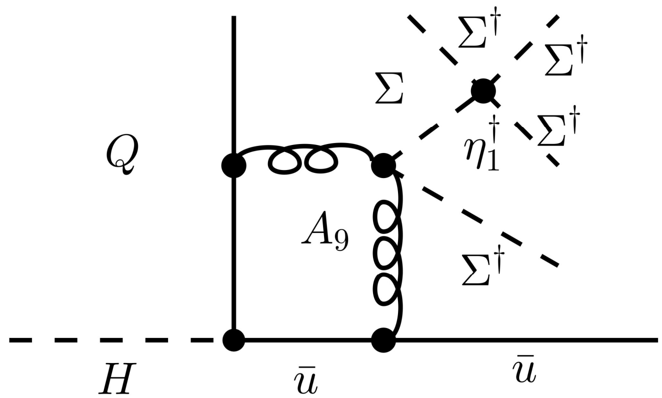

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$$(y_u)^i_j \sim y_t \left(1 + \frac{\alpha_9}{4\pi}\frac{\{\Sigma^\dagger, \Sigma\}}{2\Lambda_9^2} + \left(\frac{\alpha_9}{4\pi} \frac{\eta_1^\dagger (\Sigma^\dagger)^4 + \eta_2^\dagger (\Sigma^\dagger)^2 (\Sigma^\dagger)^2}{\Lambda_9^4} + h.c. \right) \right)^i_j$$

Hermitian **Real** **Complex**

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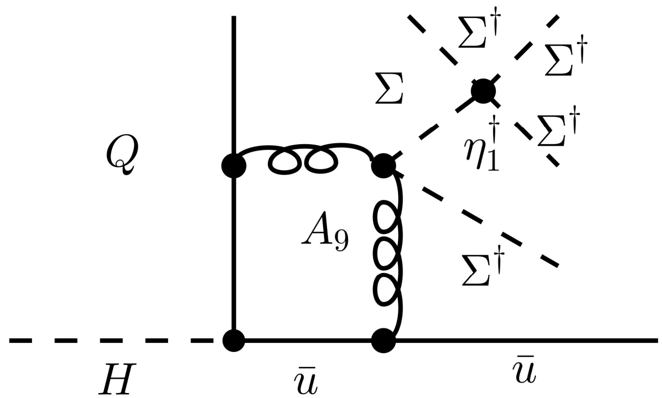

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Hermitian **Real** **Complex**

$y_u, y_d \Rightarrow$ real eigenvalues $\Rightarrow$ $\bar{\theta} = \arg e^{-i\theta} \det(y_u y_d) = \arg\det(y_u y_d) = 0$ ✓

Color-Flavor Unification

3. $SU(3)_C \times SU(3)_H/Z_3 \to SU(3)_C$: Flavor Structre and CKM CPV

(iii) Generate $O(1)$ CKM CPV phase δ_{CKM} (without destabilizing $\bar{\boldsymbol{\theta}} = \mathbf{0}$)

Field-redefinition invariant definition of CKM CPV phase

$\tilde{J} = \mathrm{Im}\,\det\left[y_u^\dagger y_u, y_d^\dagger y_d\right] \propto \sin\delta_{CKM}$ "Jarlskog invariant"

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$$\tilde{J} = \mathrm{Im}\det\underbrace{\left[y_u^\dagger y_u, y_d^\dagger y_d\right]}_{\text{"misalignment" of } y_u \text{ and } y_d} \propto \sin\delta_{CKM}$$

"Jarlskog invariant"

"misalignment" of y_u and y_d

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$$y_u \sim y_t\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta\Sigma^4 + h.c.) + \cdots\right)$$

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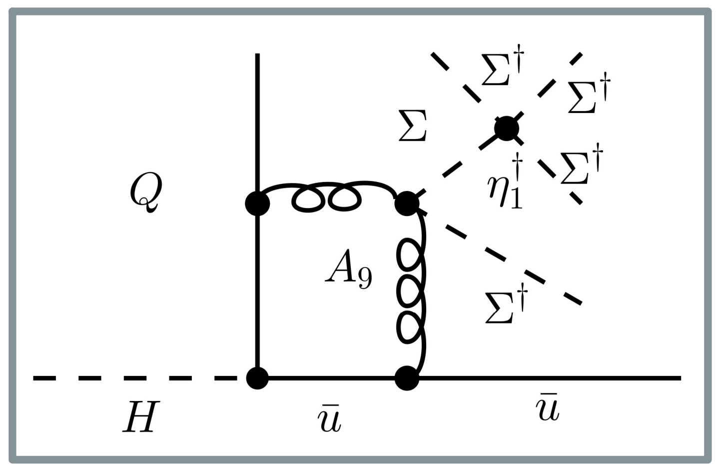

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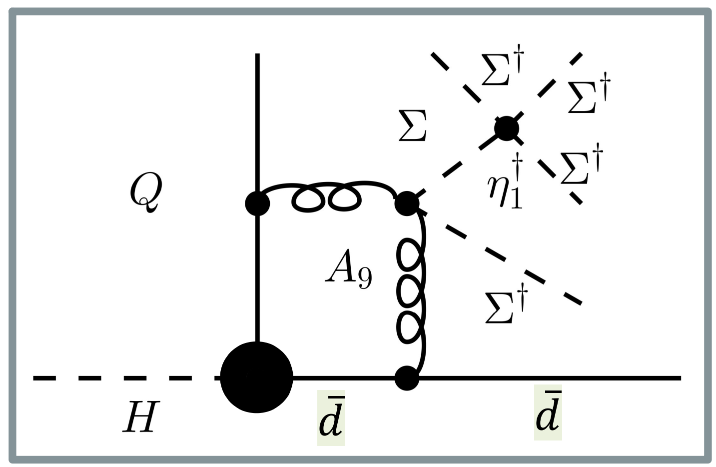

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$\boldsymbol{y_d} \propto \boldsymbol{y_u}$ as a matrix

$\Rightarrow\ \tilde{\boldsymbol{J}} = \mathbf{0}$

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$\boldsymbol{y_d \propto y_u}$ as a matrix

$\Rightarrow \boldsymbol{\tilde{J} = 0}$

We need extra ingredients to **misalign $\boldsymbol{y_d}$ vs $\boldsymbol{y_u}$** : **'down-philic' interactions**

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$$\mathcal{L}_{\rho\chi} \sim \lambda_d \bar{d} \rho \chi + \rho\left(c_1 \Sigma^\dagger \Sigma + c_2 \Sigma \Sigma^\dagger\right)\rho^\dagger$$

$$+ a_1 \rho \Sigma \rho^\dagger + a_2 \rho \Sigma \Sigma \rho^\dagger + h.c.$$

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- Use χ rotation to set $\lambda_d \in \mathbb{R}$
- $c_{1,2} \in \mathbb{R}$
- $a_{1,2} \in \mathbb{C} \;\; \to \;\; a_1^2 a_2^\dagger, \eta_1^\dagger a_2^2$: new CPV source
- $a_{1,2} = 0$ if Z_4^Σ is imposed

  (again, our mechanism works regardless)

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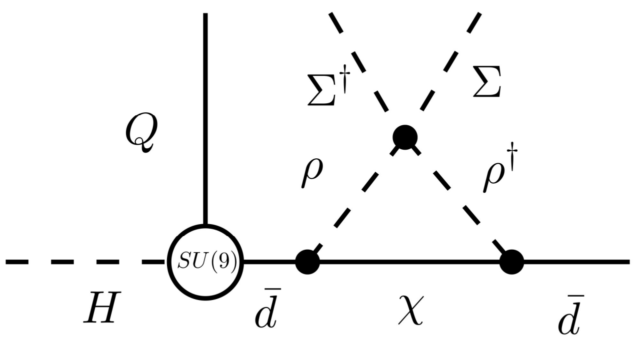

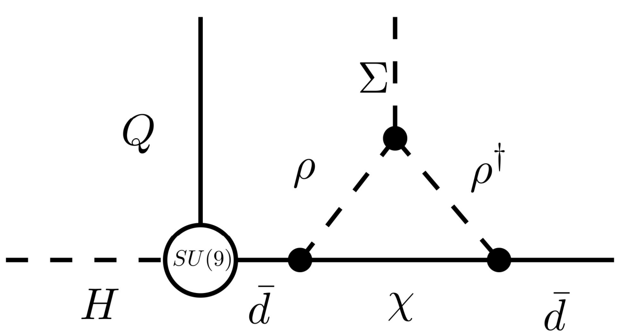

Without "down-philic" interactions

$$y_u \sim y_t\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta\Sigma^4 + h.c.) + \cdots\right)$$

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$$\tilde{J} = 0$$

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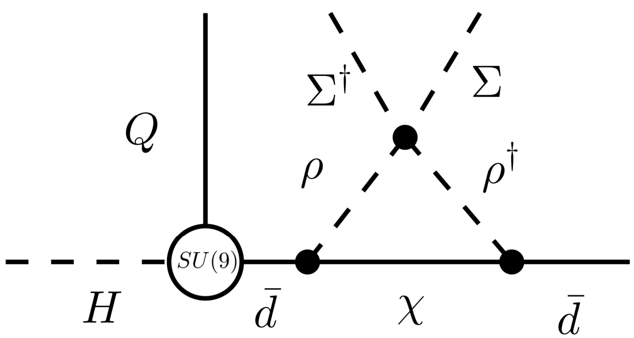

With "down-philic" interactions ($a_{1,2} = 0$)

$$y_u \sim y_t\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta \Sigma^4 + h.c.) + \cdots\right)$$

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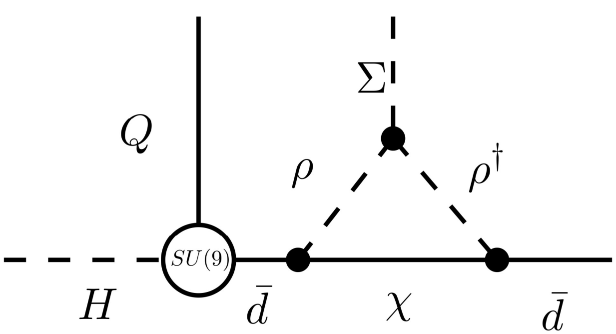

$$\tilde{J} \sim \mathrm{Im}\,\det\left(4r^2\left[\eta\Sigma^4 + \eta^\dagger \Sigma^{\dagger 4}, c\Sigma^\dagger\Sigma\right]\right), \qquad r \sim e^{-\frac{2\pi}{\alpha_9}}$$

$$\propto \mathrm{Im}\,\det\left(\eta\begin{pmatrix}[\Sigma,\Sigma^\dagger]\Sigma^4 + \Sigma[\Sigma,\Sigma^\dagger]\Sigma^3 + \Sigma^2[\Sigma,\Sigma^\dagger]\Sigma^2 \\ +\Sigma^3[\Sigma,\Sigma^\dagger]\Sigma\end{pmatrix} - h.c.\right)$$

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No/Less Quality Problem

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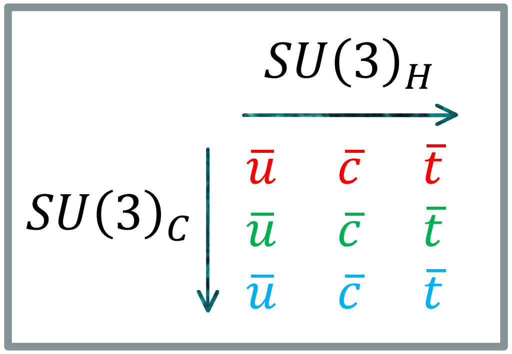


$= 9\ (\square)$ of $SU(9)$

	$SU(9)$	$U(1)_{Q-\bar{u}}$	$U(1)_{\bar{d}}$
$\boldsymbol{Q} = (\boldsymbol{u}, \boldsymbol{d})^t$	9	$+1$	0
$\bar{\boldsymbol{u}}$	$\bar{9}$	-1	0
$\bar{\boldsymbol{d}}$	$\bar{9}$	0	$+1$
H	1	0	0

$$\mathcal{L}_0 = y_t \tilde{H} \boldsymbol{Q} \bar{\boldsymbol{u}} + h.c. + \frac{i\theta_9}{32\pi^2} \int F\tilde{F}$$

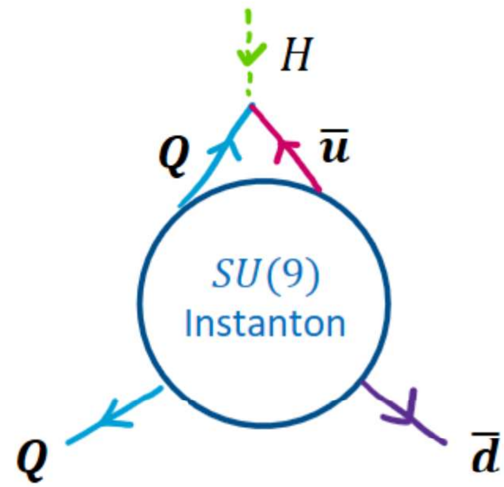
$$U(1)_{Q-\bar{u}}[SU(9)]^2 = U(1)_{\bar{d}}[SU(9)]^2 = 1$$

$\Rightarrow$ [Anomaly Free] $U(1)_{B=Q-\bar{u}-\bar{d}}$

[Anomalous] $U(1)_{Q-\bar{u}+\bar{d}}$ or $U(1)_{\bar{d}}$

No/Less Quality Problem

1. Our solution requires "high-quality" $U(1)_{PQ}$ in the UV



$SU(3)_H$ (→), $SU(3)_C$ (↓)

$\bar{u}$	$\bar{c}$	$\bar{t}$
$\bar{u}$	$\bar{c}$	$\bar{t}$
$\bar{u}$	$\bar{c}$	$\bar{t}$

$= 9\ (\square)$ of $SU(9)$

	$SU(9)$	$U(1)_{Q-\bar{u}}$	$U(1)_{\bar{d}}$
$\boldsymbol{Q} = (\boldsymbol{u}, \boldsymbol{d})^t$	9	+1	0
$\bar{\boldsymbol{u}}$	$\bar{9}$	−1	0
$\bar{\boldsymbol{d}}$	$\bar{9}$	0	+1
H	1	0	0

$$\mathcal{L}_0 = y_t \tilde{H} \boldsymbol{Q}\bar{\boldsymbol{u}} + h.c. + \frac{i\theta_9}{32\pi^2}\int F\tilde{F}$$

$$+ y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9}} H \boldsymbol{Q}\bar{\boldsymbol{d}}$$

$$U(1)_{Q-\bar{u}}[SU(9)]^2 = U(1)_{\bar{d}}[SU(9)]^2 = 1$$

⇒ [Anomaly Free] $U(1)_{B=Q-\bar{u}-\bar{d}}$

[Anomalous] $U(1)_{Q-\bar{u}+\bar{d}}$ or $U(1)_{\bar{d}}$

No/Less Quality Problem

1. Our solution requires "high-quality" $U(1)_{PQ}$ in the UV

2. Estimation of Needed Quality from Quantum Gravity Effects

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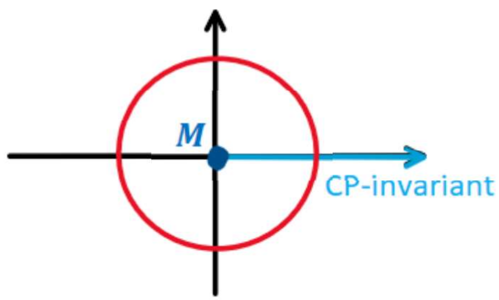

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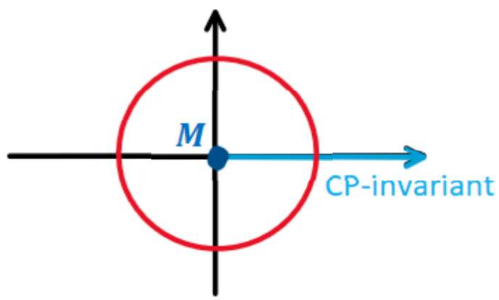

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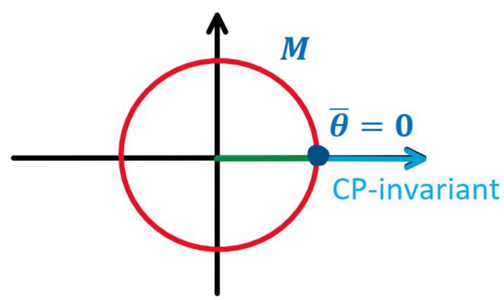

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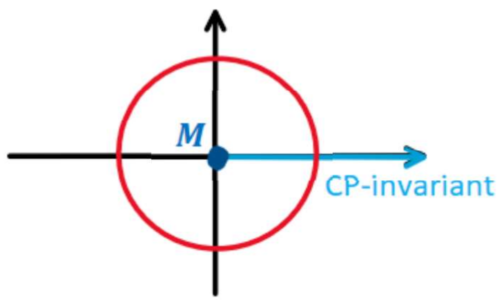

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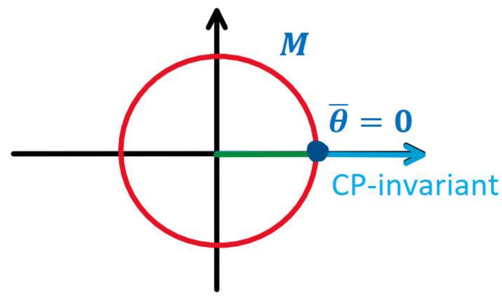

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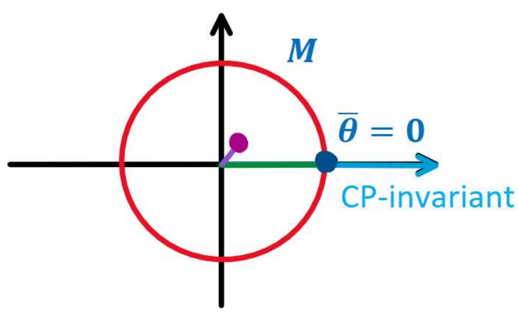

QG breaking of $U(1)_{PQ}$ with O(1) phase

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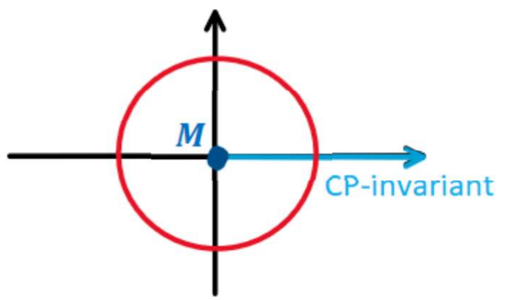

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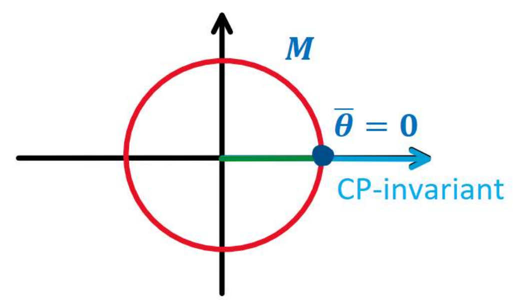

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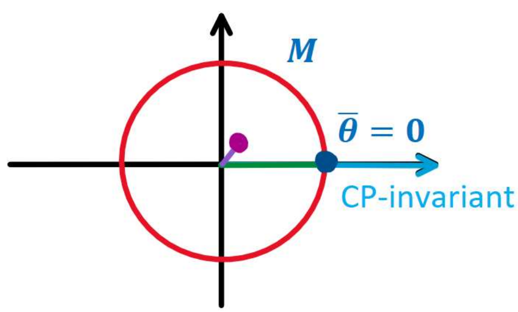

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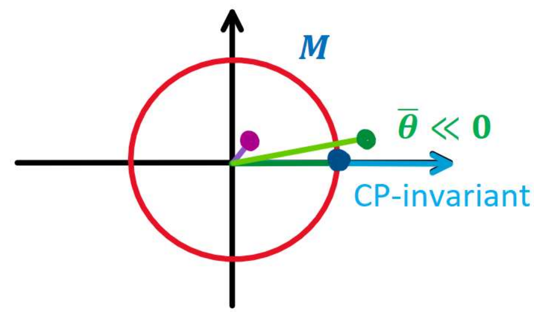

$\bar{\theta} \ll 1$ if

$|\text{QG effect}| \ll |SU(9)\text{inst}|$

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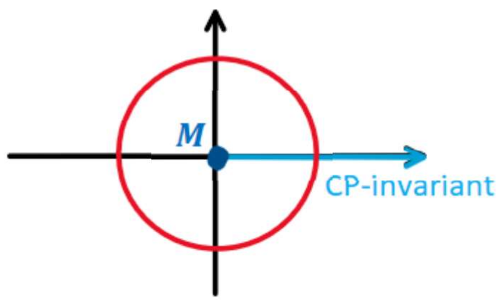

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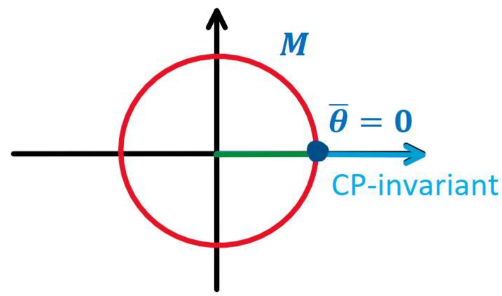

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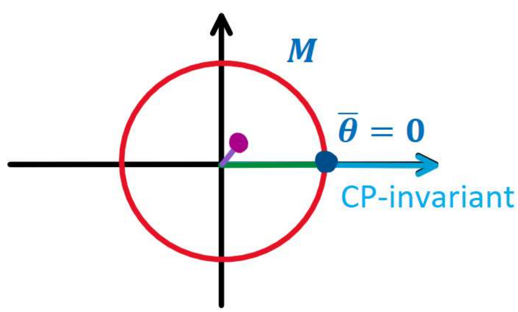
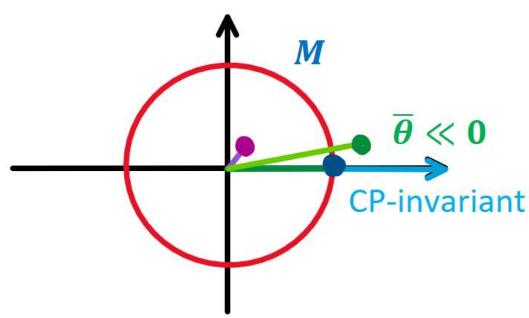


QG breaking of $U(1)_{PQ}$ with $O(1)$ phase



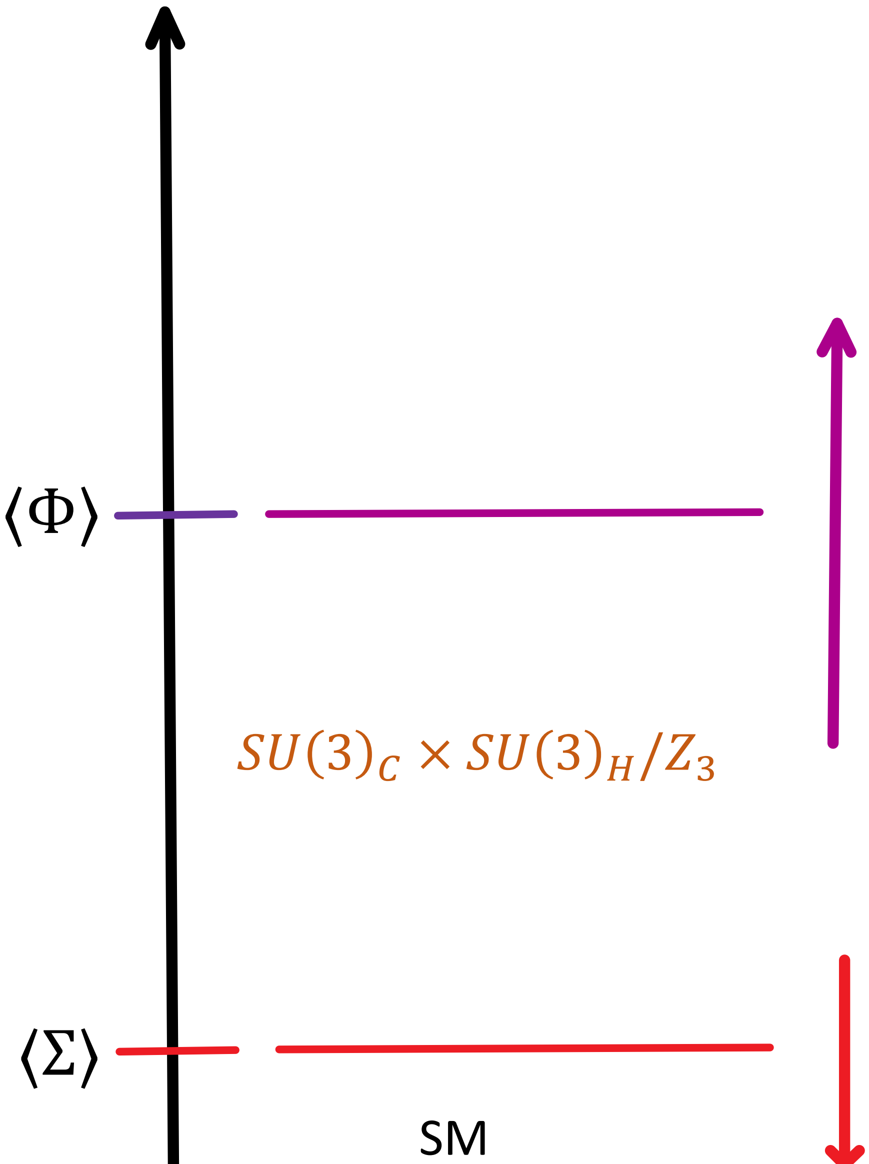
$\bar{\theta} \ll 1$ if

$|\text{QG effect}| \ll |SU(9)\text{inst}|$

$\frac{\lvert\Phi\rvert^2 HQ\bar{d}}{M_{\text{Pl}}^2}$	$\rightarrow$	$\langle\Phi\rangle \sim \Lambda_9 < 10^{13}$ GeV
$\frac{HQ\Sigma\bar{d}}{M_{\text{Pl}}}$	$\rightarrow$	$\langle\Sigma\rangle \sim \Lambda_3 < 10^{8}$ GeV

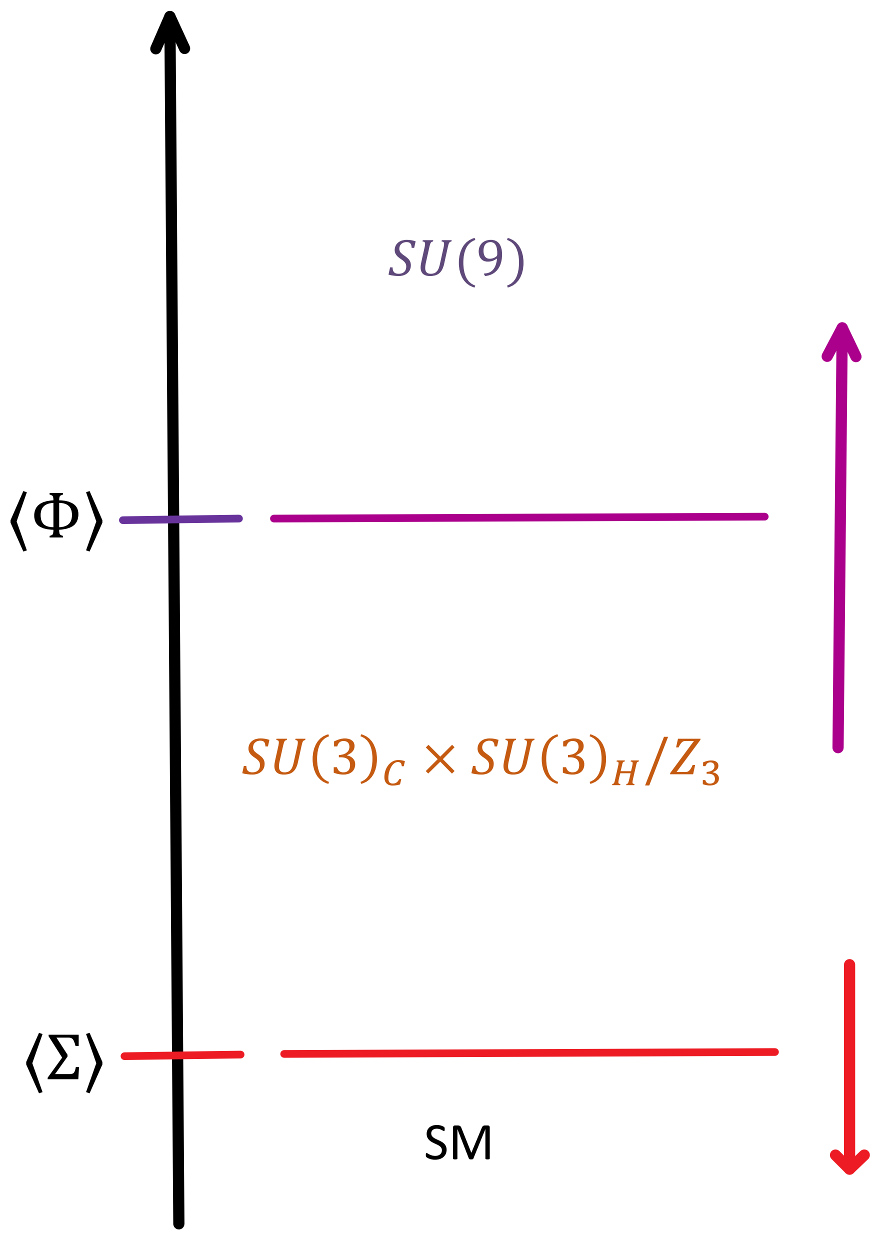
Conclusion

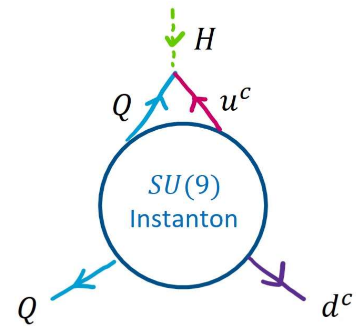
Solving Strong CP with Non-Invertible Symmetry



- Start with only $y_u \widetilde{H} Q \bar{u}$ $\quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons ($\boldsymbol{N_C = N_g}$)
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta}$ = unphysical

Solving Strong CP with Non-Invertible Symmetry





Small (G/H) Instanton

$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

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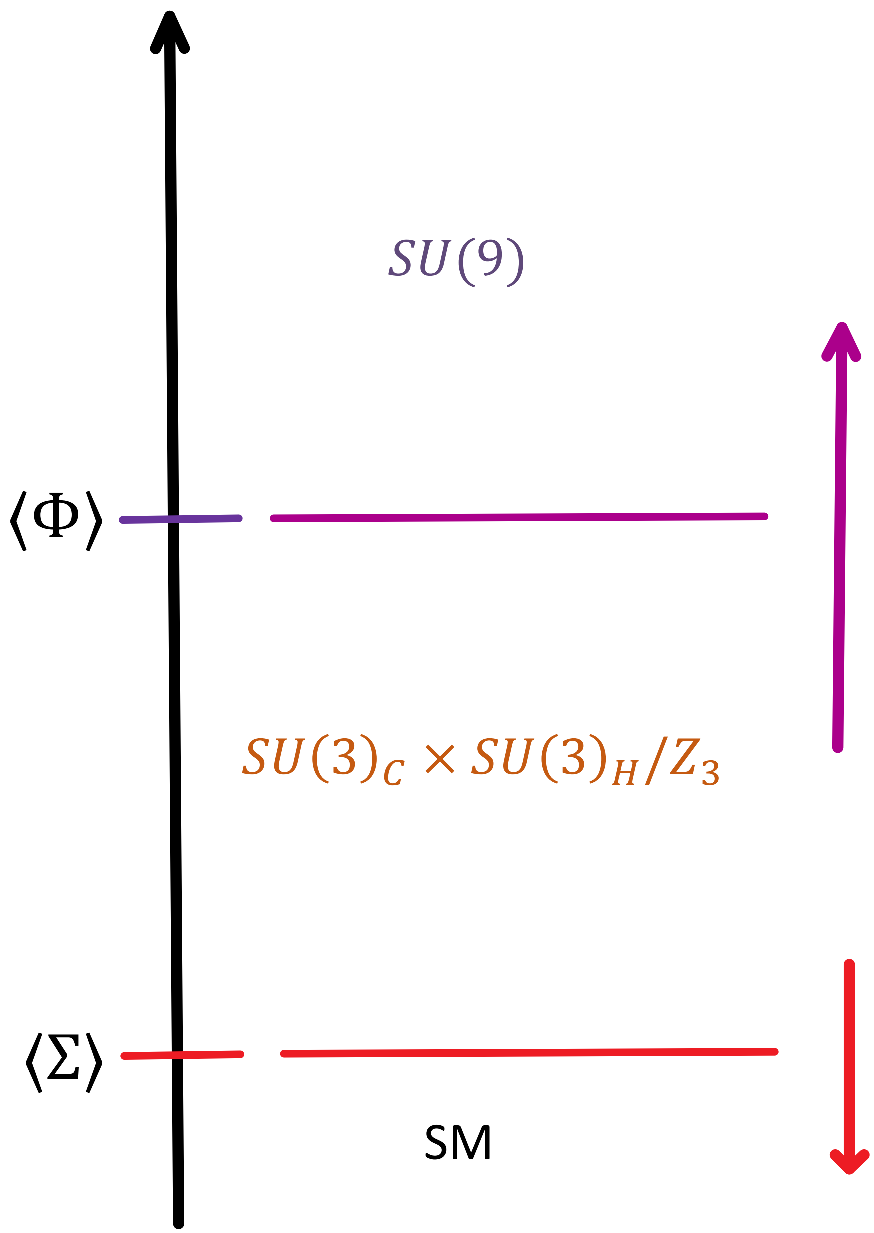

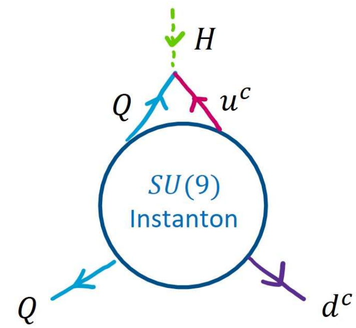

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- $\bar{\theta} =$ unphysical

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)
(Hermitian Yukawa + down-philic interactions)

THANK YOU

FOR

YOUR ATTENTION!

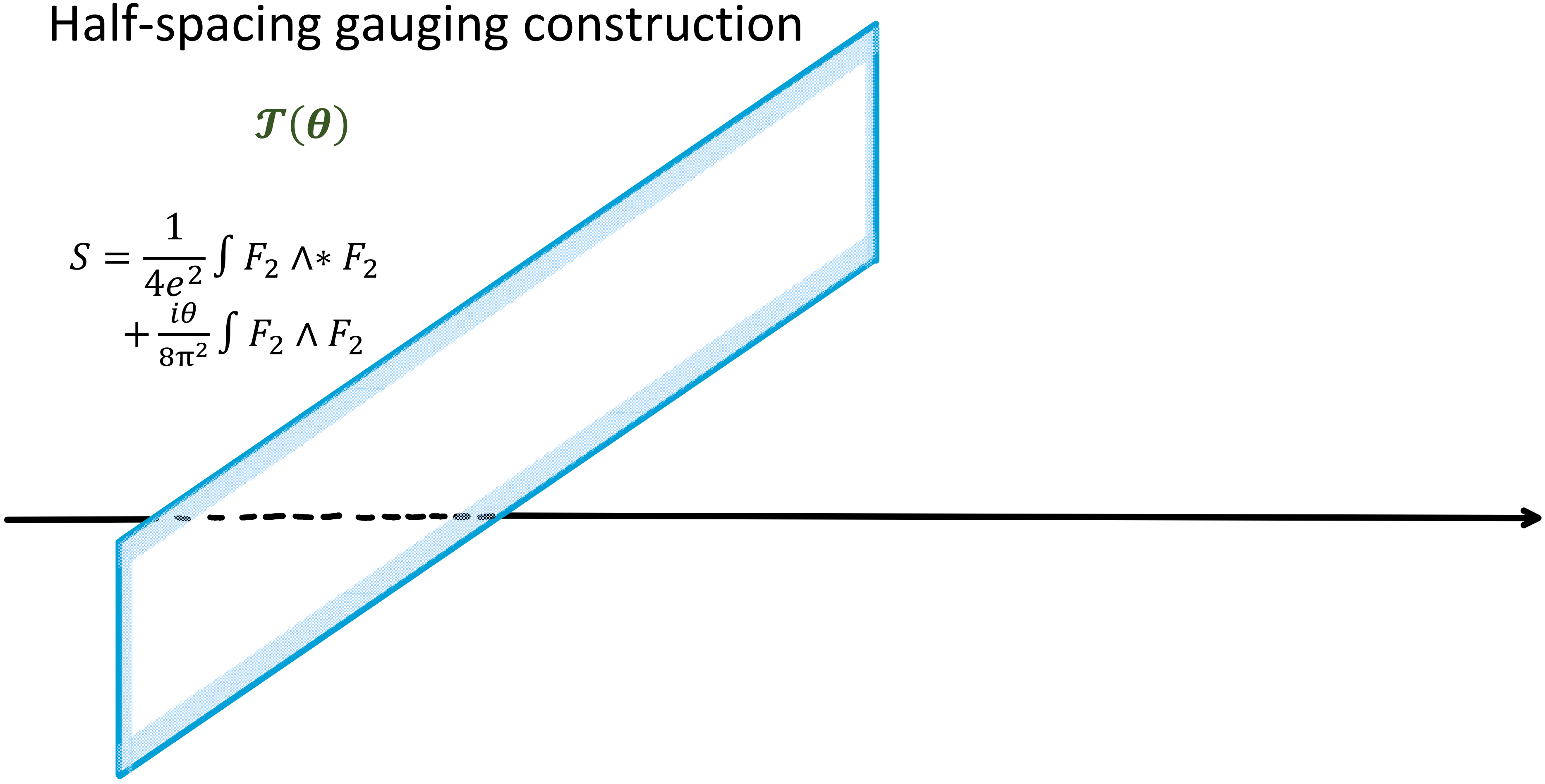
Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction

$\mathcal{T}(\theta)$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$



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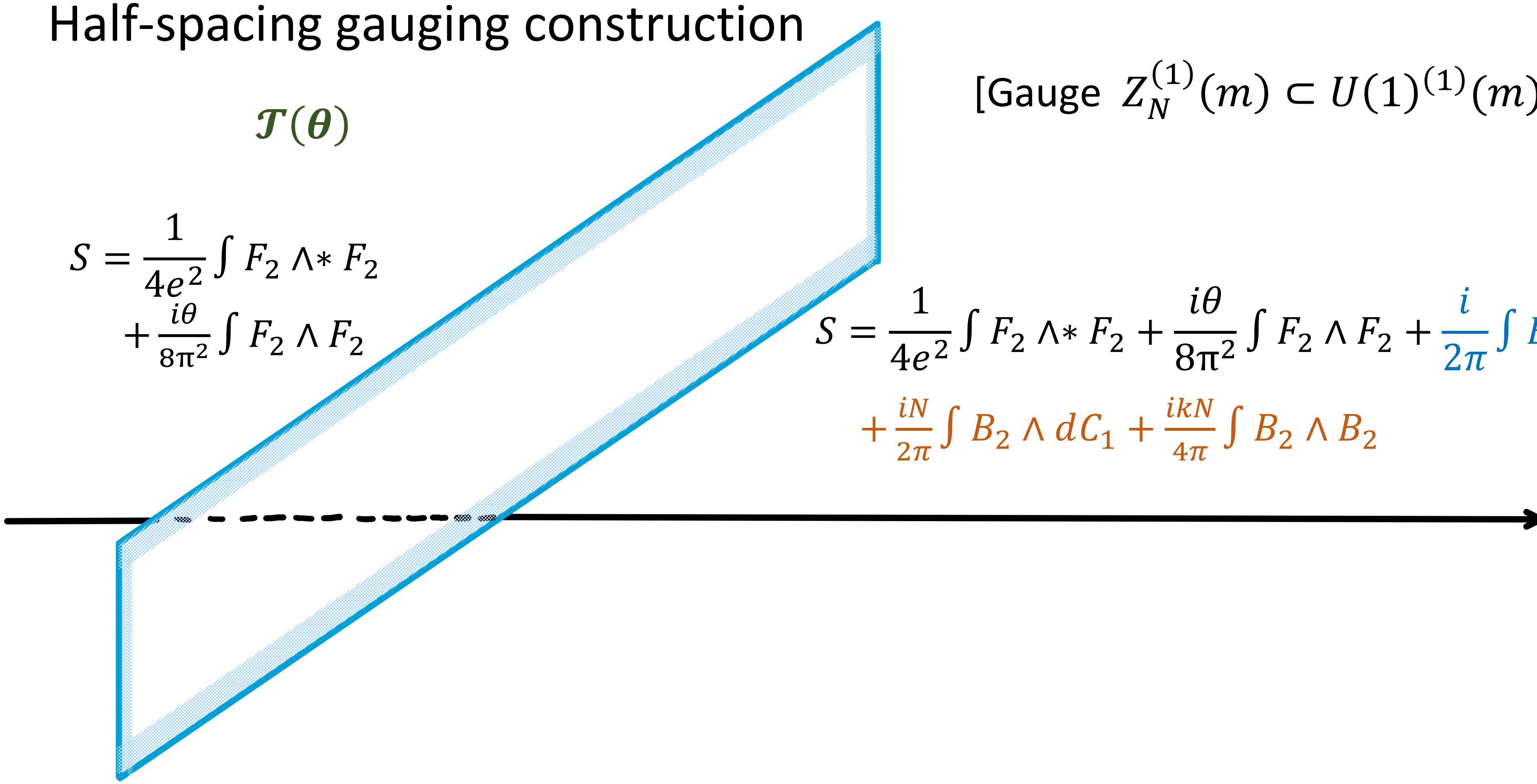

$\mathcal{T}(\boldsymbol{\theta})$

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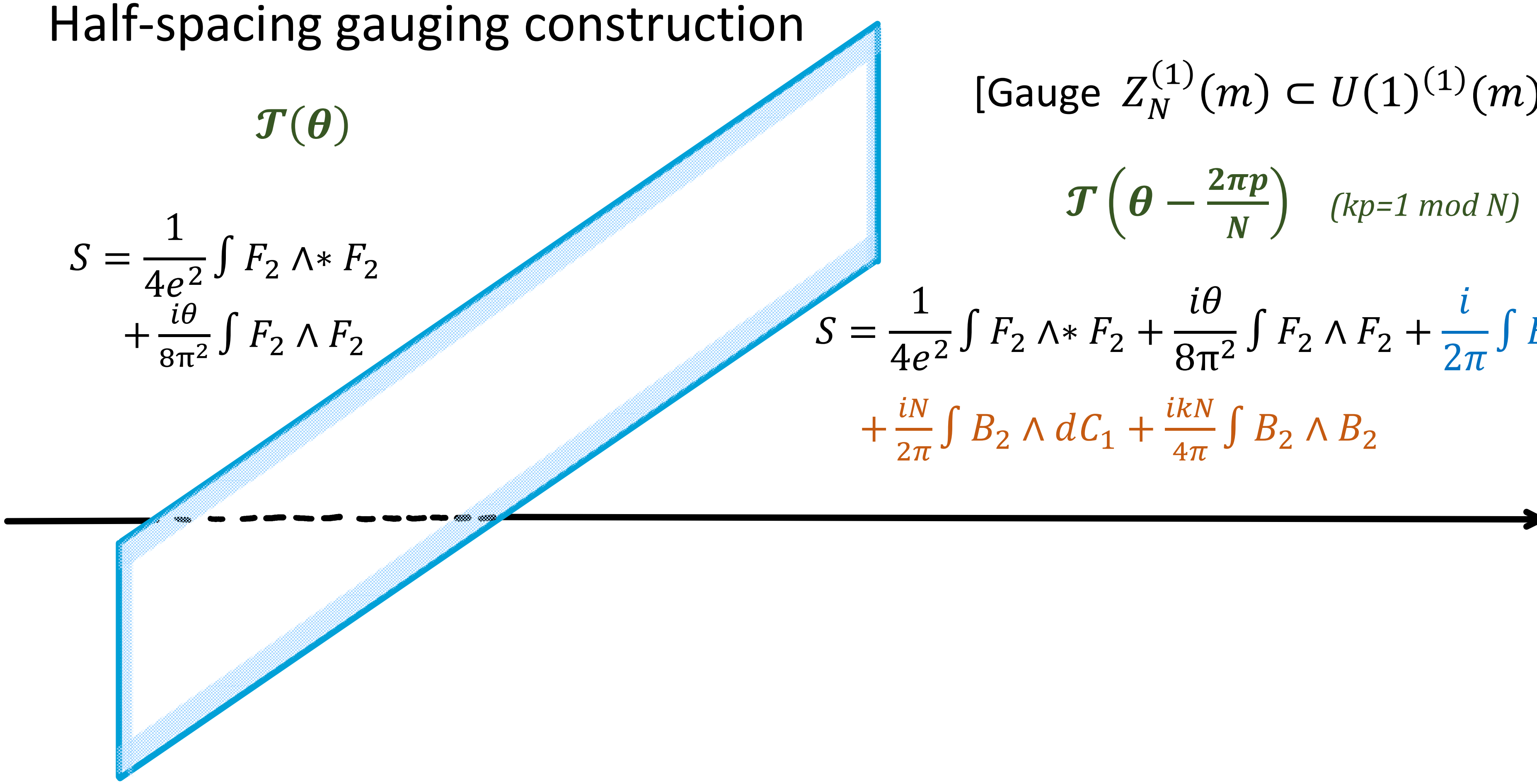

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[Gauge $Z_N^{(1)}(m) \subset U(1)^{(1)}(m)$]

$\mathcal{T}\left(\theta - \frac{2\pi p}{N}\right)$ $(kp=1 \bmod N)$

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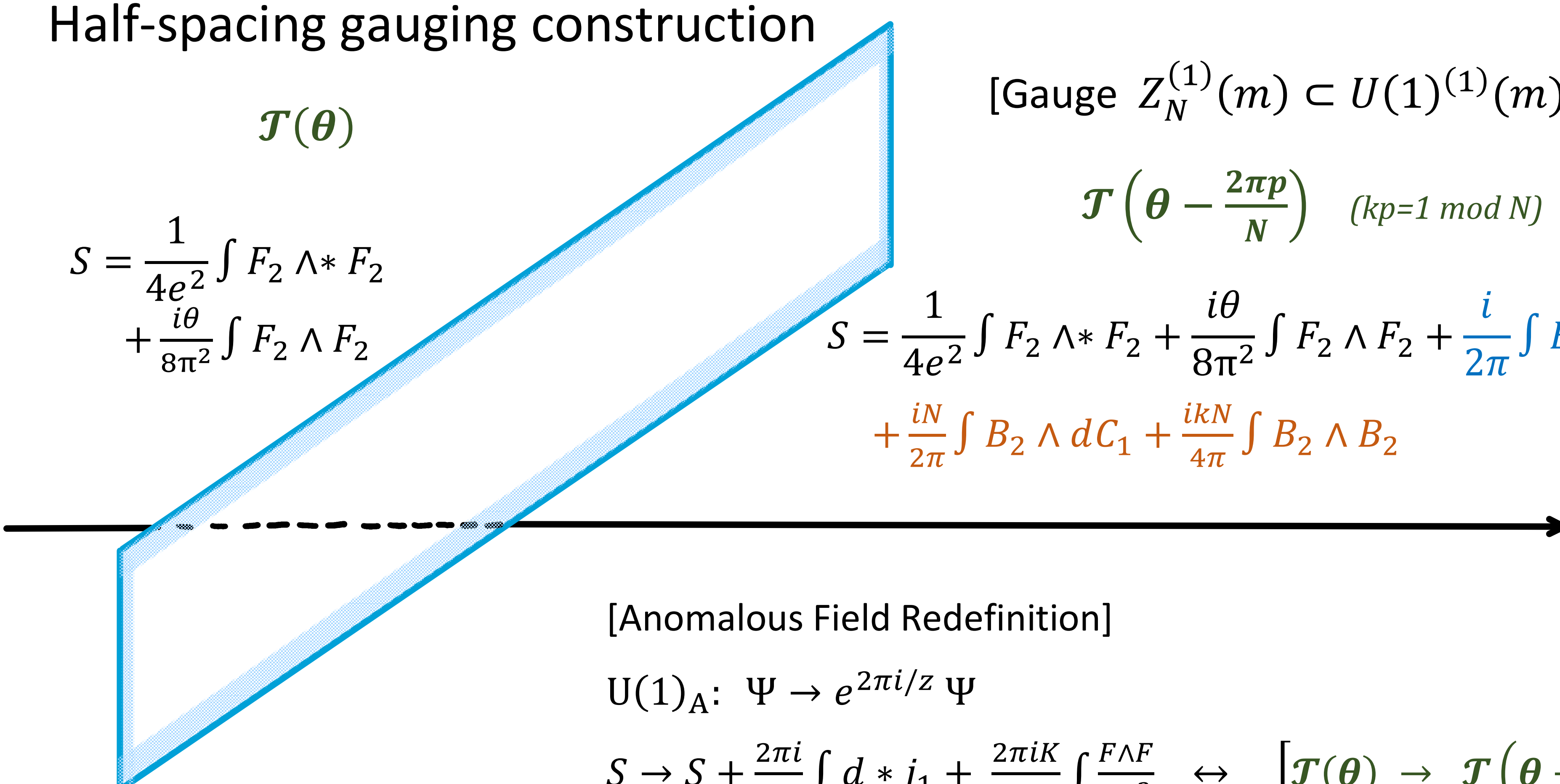

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[Anomalous Field Redefinition]

$\mathrm{U}(1)_\mathrm{A}$: $\Psi \to e^{2\pi i/z}\,\Psi$

$$S \to S + \frac{2\pi i}{z}\int d * j_1 + \frac{2\pi i K}{z}\int \frac{F \wedge F}{8\pi^2} \quad \leftrightarrow \quad \left[\mathcal{T}(\boldsymbol{\theta}) \to \mathcal{T}\left(\boldsymbol{\theta} + \frac{2\pi K}{z}\right)\right]$$

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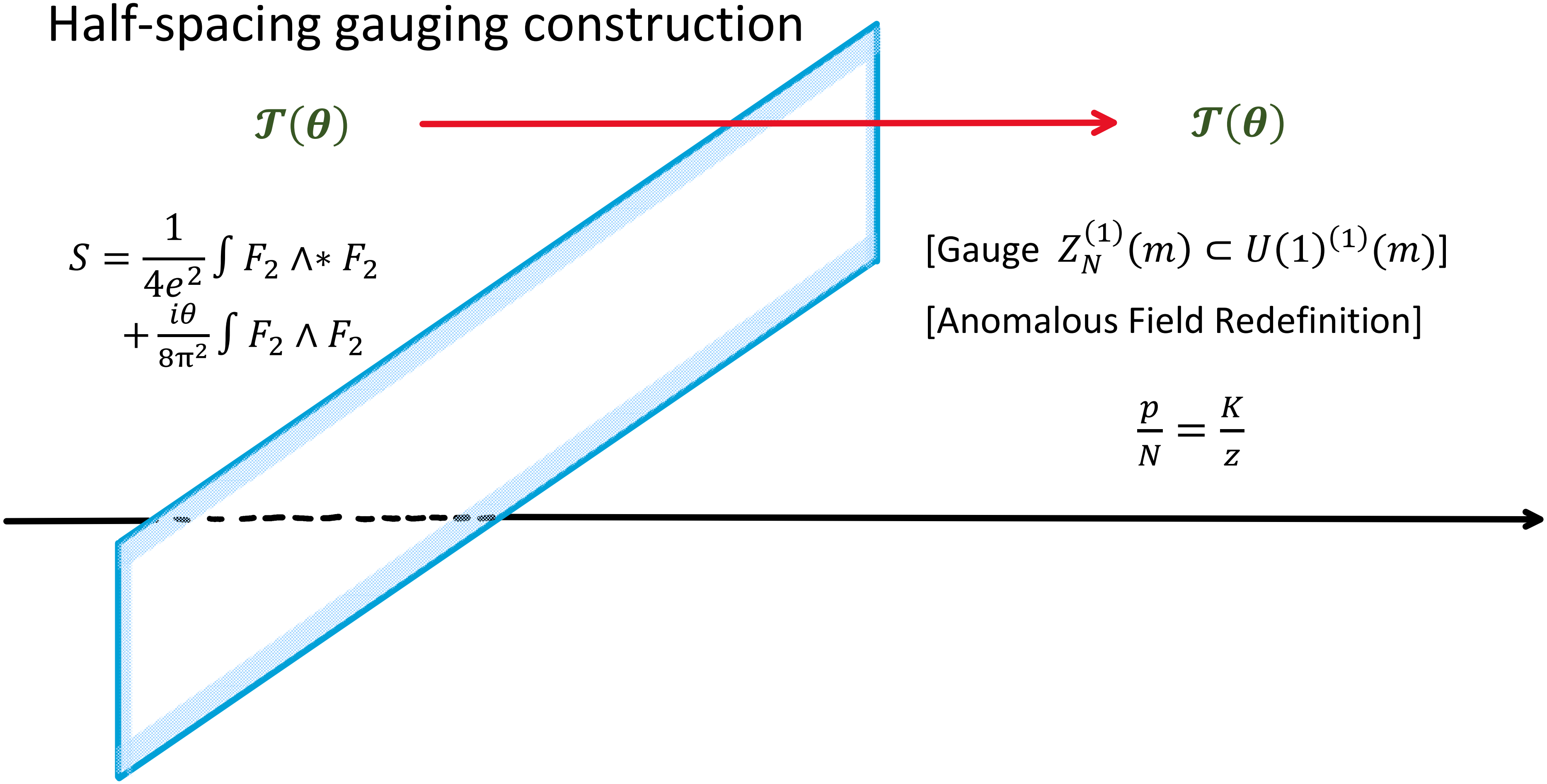
Half-spacing gauging construction

$\mathcal{T}(\boldsymbol{\theta})$ → $\mathcal{T}(\boldsymbol{\theta})$



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[Anomalous Field Redefinition]

$$\frac{p}{N} = \frac{K}{z}$$

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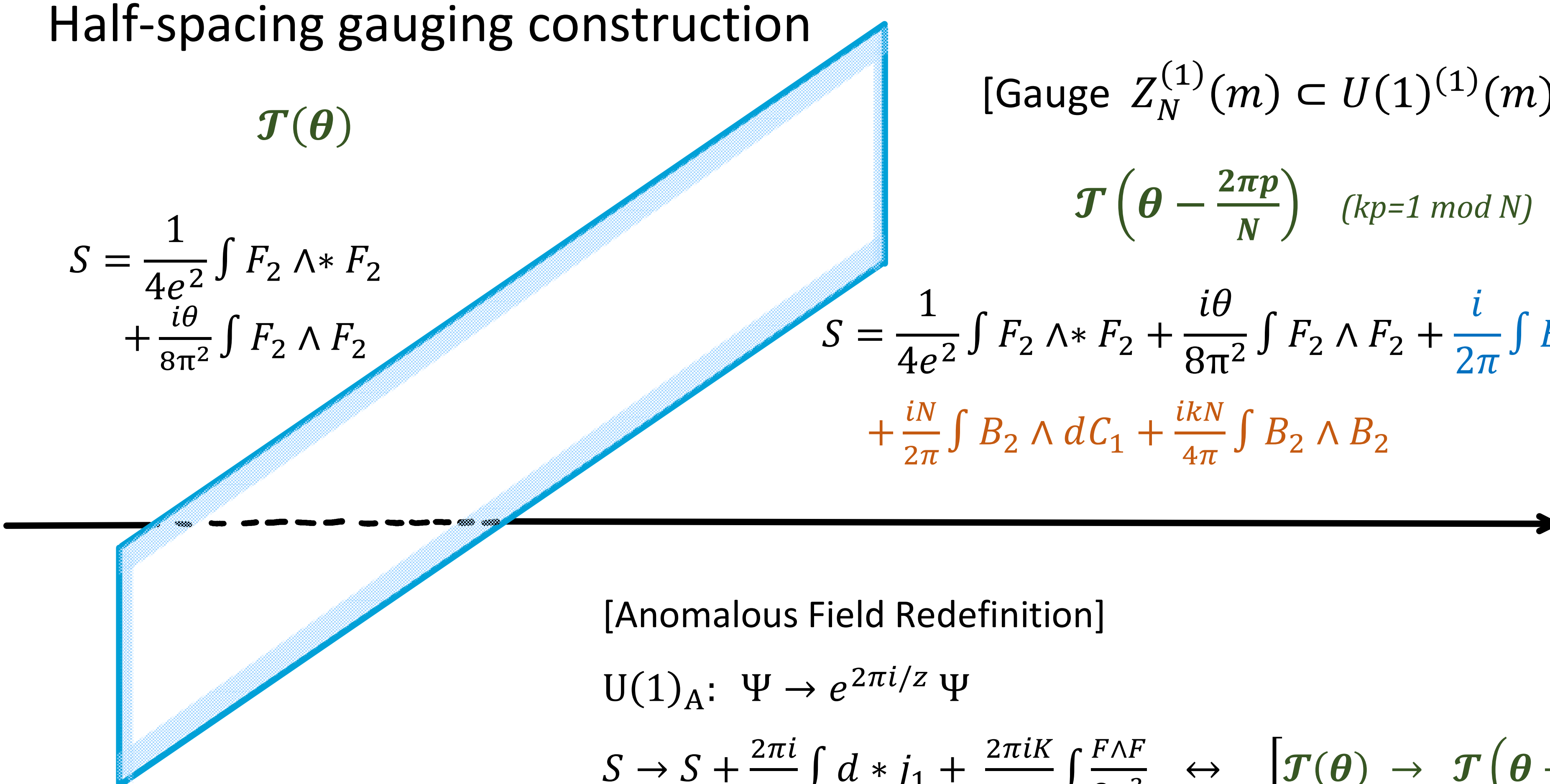
Half-spacing gauging construction



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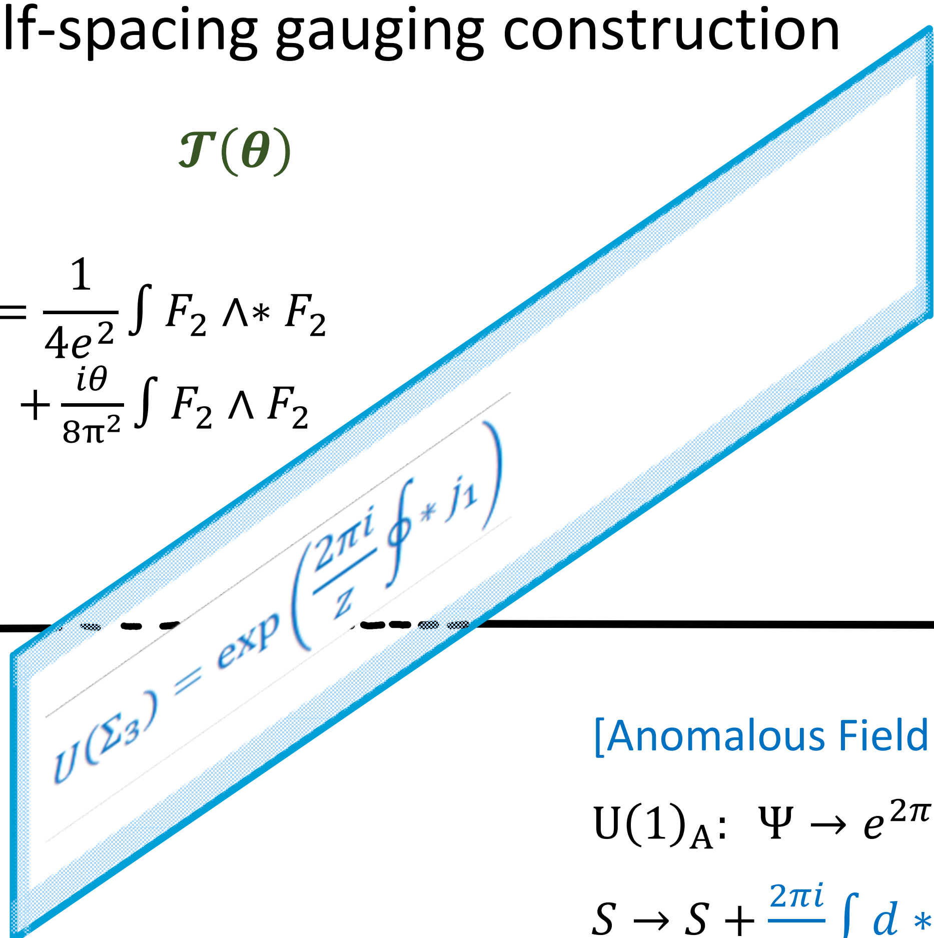

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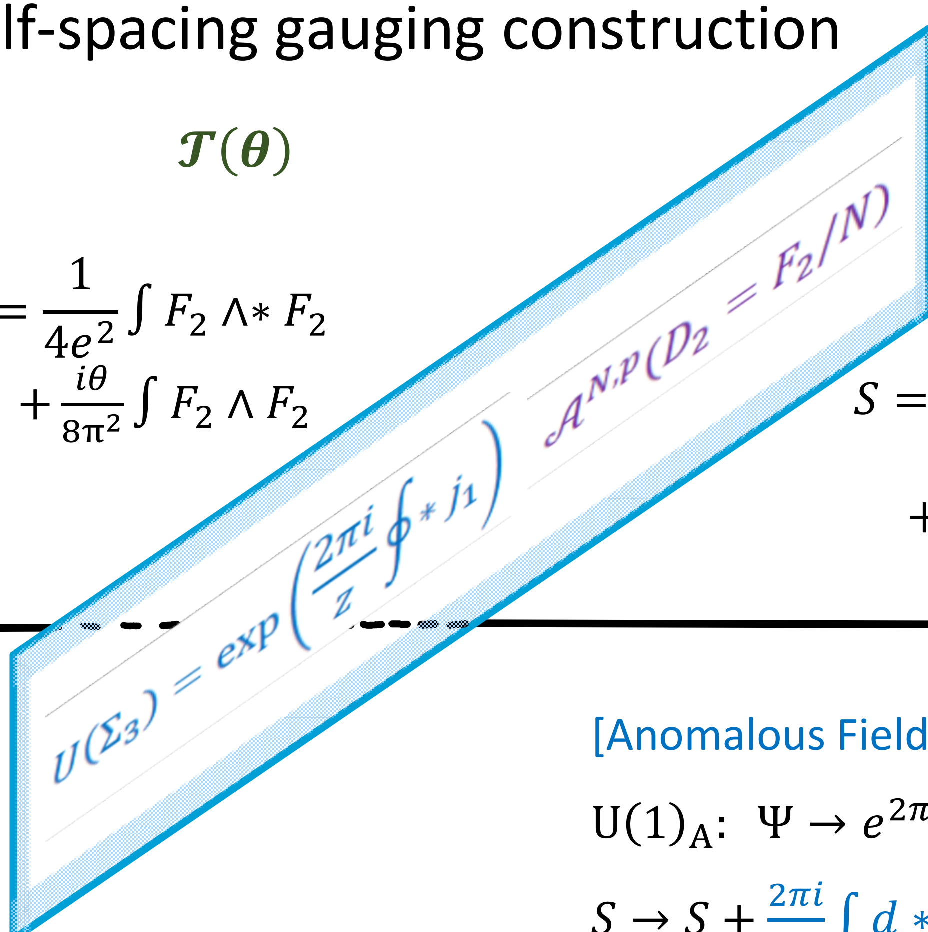

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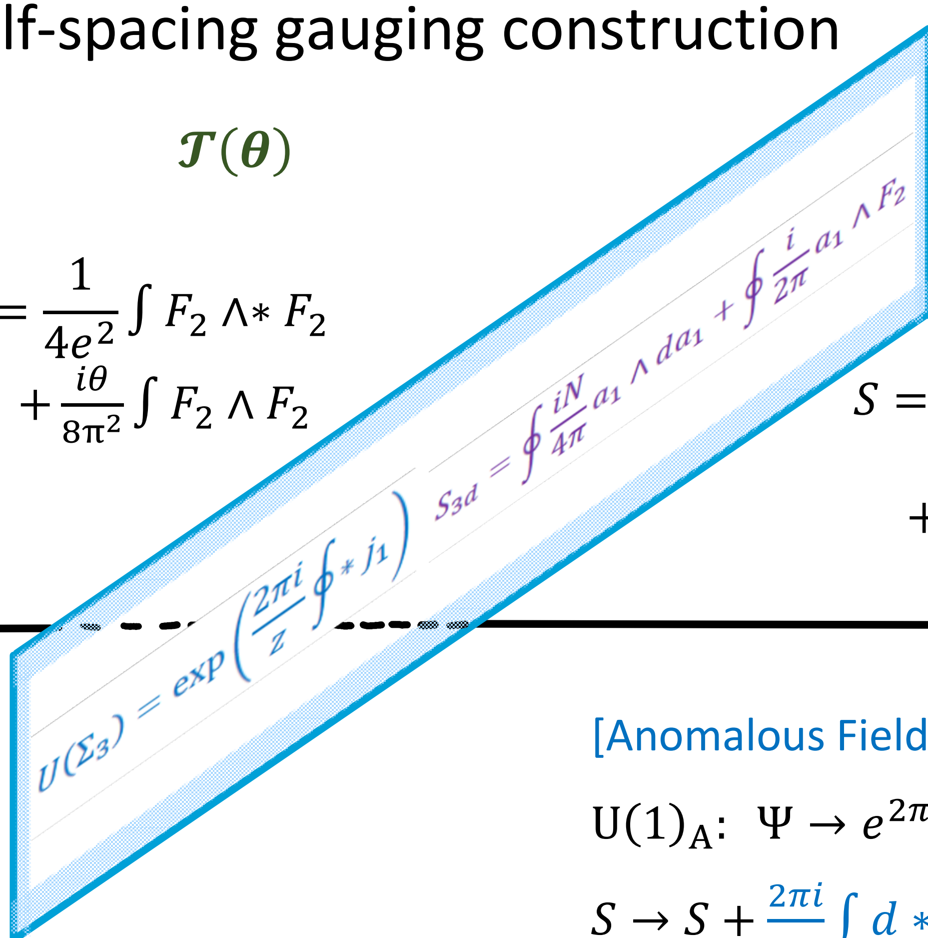

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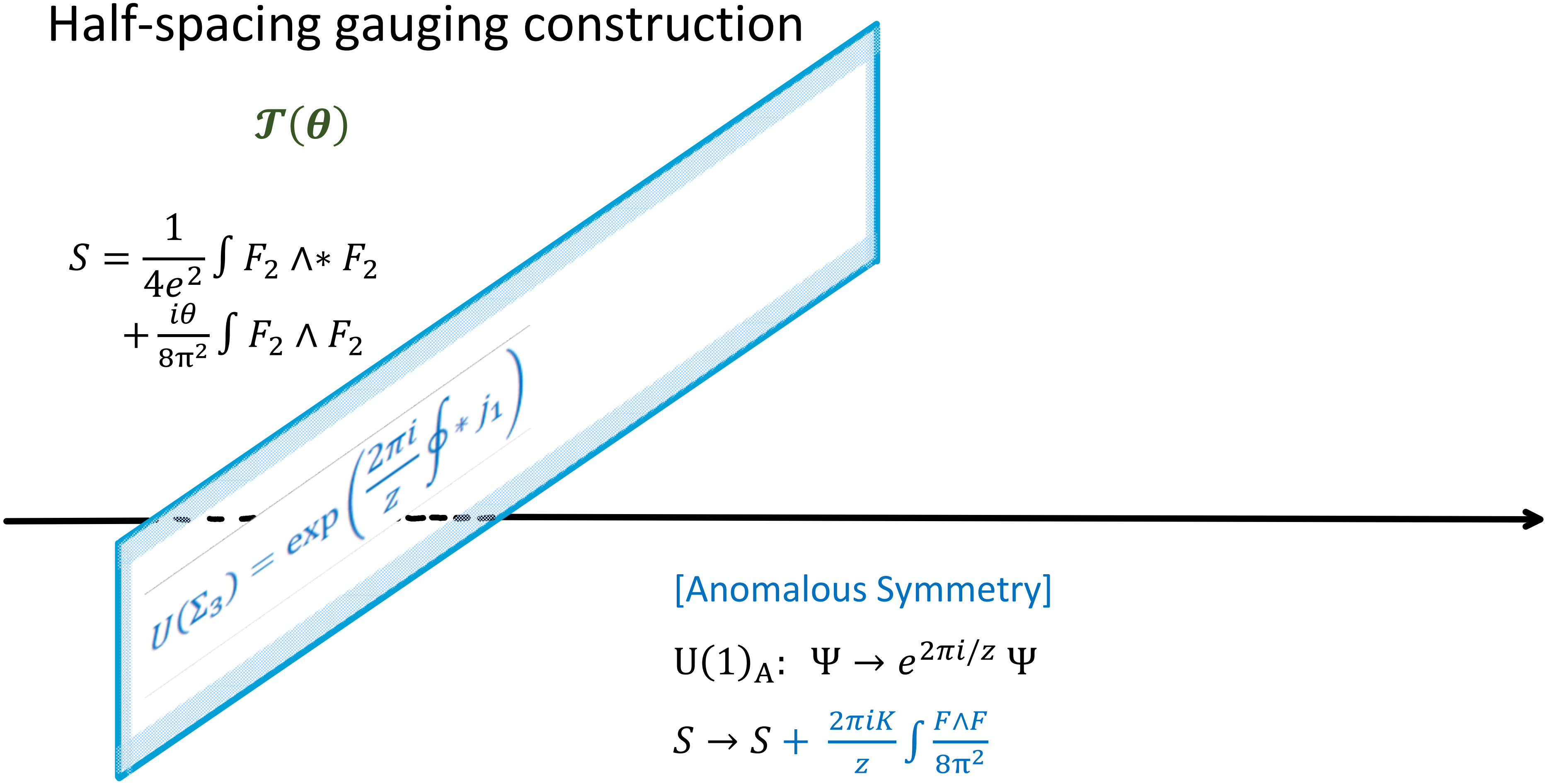

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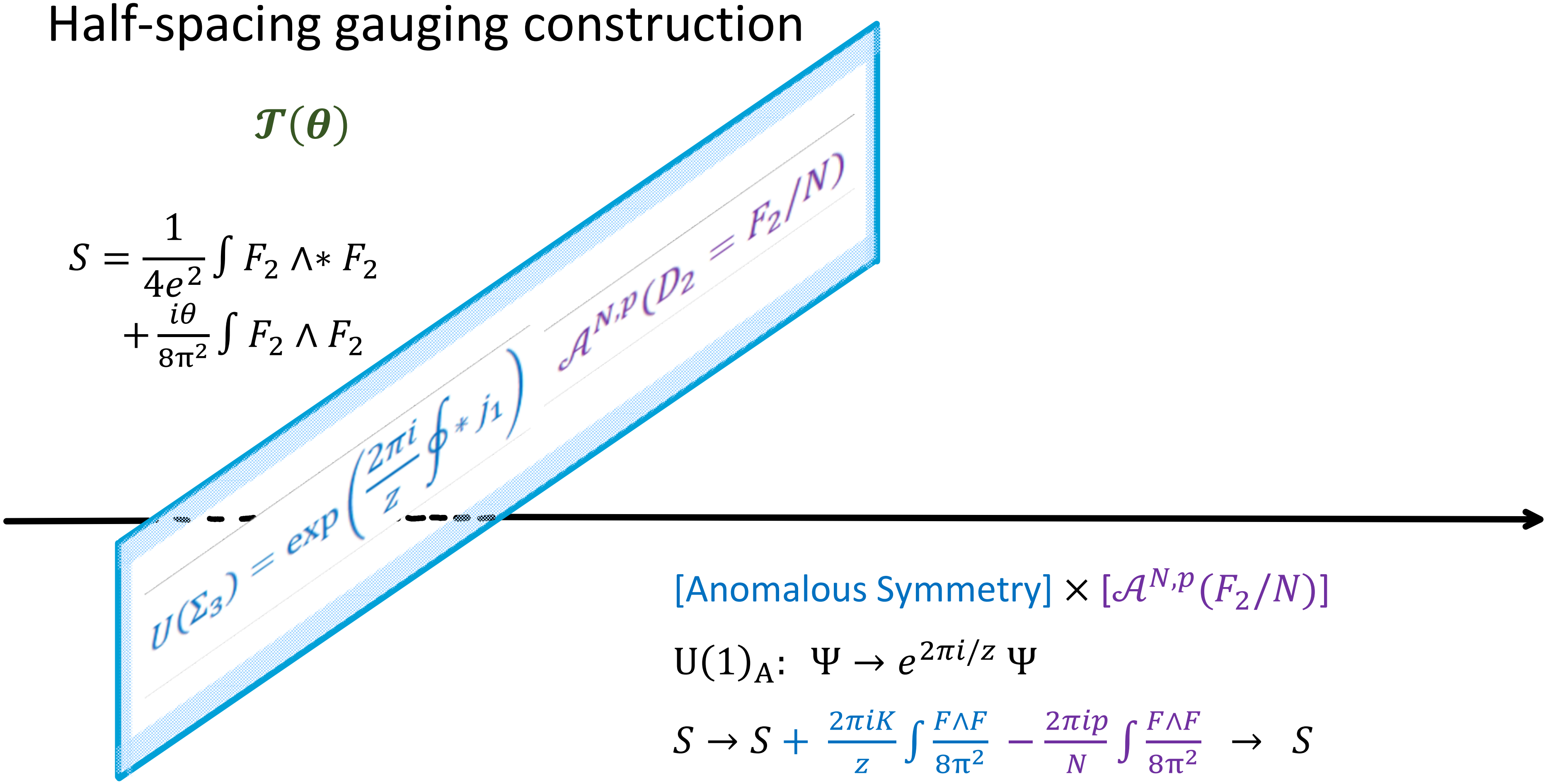

[Anomalous Symmetry] $\times$ [$\mathcal{A}^{N,p}(F_2/N)$]

$\mathrm{U}(1)_\mathrm{A}$: $\Psi \to e^{2\pi i/z}\,\Psi$

$$S \to S + \frac{2\pi i K}{z}\int \frac{F \wedge F}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F\wedge F}{8\pi^2} \;\to\; S$$

Non-Invertible Symmetry

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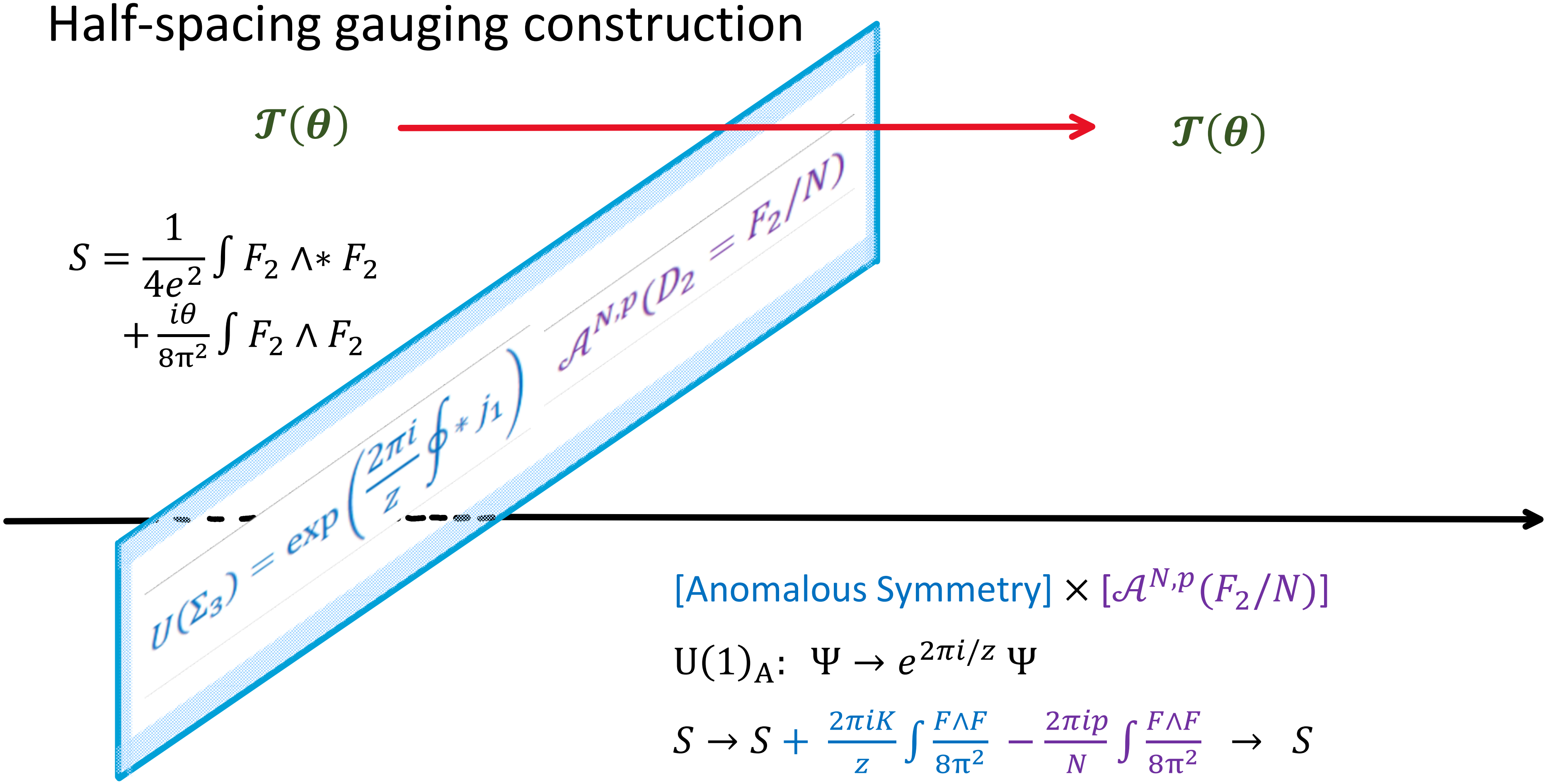

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