Symmetries of Non-SUSY Orbifold Theories

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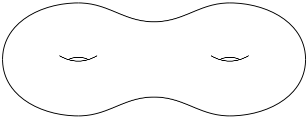
Dark World to Swampland 2024: the 9th IBS-IFT Workshop,

Tuesday November 12th, 2024

Motivation

Focus of Talk: $\mathcal{N} = 0$ String Constructions (IIA/IIB)

Background: $\mathbb{R}^{1,3} \times X$



Global Model

Motivation

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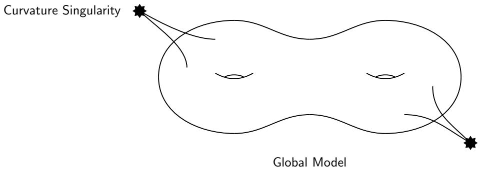

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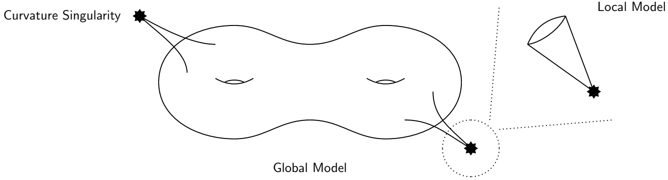

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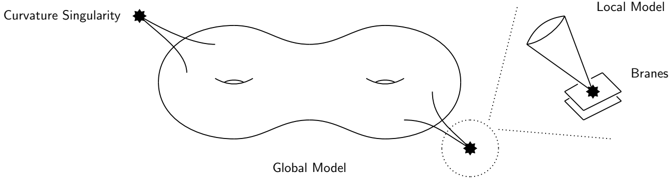
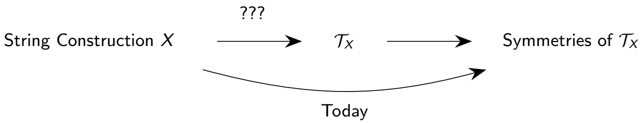


Motivation

Generalized Symmetries → New Tools to study QFTs

[Gaiotto, Kapustin, Seiberg, Willett; 2014] . . . [Choi, Lam, Shao; 2022] . . .

New Tools to study String Theory? String engineered theories: [Del Zotto, Heckman, Park, Rudelius; 2015], [Morrison, Schäfer-Nameki, Willett; 2020], [Lakshya Bhardwaj, MH, Schäfer-Nameki; 2021], . . . , [García Etxebarria; 2022], [Apruzzi, Bah, Bonetti, Schäfer- Nameki; 2022], [Heckman, MH, Torres, Zhang; 2022], [Del Zotto, Heckman, Meynet, Moscrop, Zhang; 2022], . . .



Theme: Focus on Topological Features, study $\mathcal{T}_X$

Related: [McNamara, Vafa; 2019], [Montero, Vafa; 2020], [Dierigl, Heckman, Montero, Torres; 2022], [Debray, Dierigl, Heckman, Montero; 2023], [Basile, Debray, Delgado, Montero; 2023], [Cvetič, Heckman, MH, Torres; 2023], [Kaidi, Ohmori, Tachikawa, Yonekura; 2023], [Heckman, McNamara, Montero, Sharon, Vafa, Valenzuela; 2024], [Kaidi, Tachikawa, Yonekura; 2024], . . .

Setting and Goal

We consider [Chakrabhavi, Braeger, Heckman, Hübner; 2024]

- IIA on $X = \mathbb{R}^6/\Gamma$ with $\Gamma \subset SU(4) \subset \text{Spin}(6)$
  $\rightarrow$ Non-SUSY 4D Theory $\mathcal{T}_X$ (Closed String Tachyons)

  [Adams, Polchinski, Silverstein; 2001], [Morrison, Narayan, Plesser; 2004]

- IIB on X with D3-brane probes
  $\rightarrow$ Non-SUSY 4D Quiver Gauge Theory $\mathcal{T}_{X,\text{D3}}$
  (Double Trace Ops with 1-loop β-function $\neq 0$)

  [Kachru, Silverstein; 1997], [Nekrasov, Lawrence, Vafa; 1998], [Adams, Silverstein; 2001], [Dymarsky, Klebanov, Roiban; 2004], [Pomoni, Rastelli; 2008], ... [Horowitz, Orgera, Polchinski; 2007], [Ooguri, Vafa; 2016], [García Etxebarria, Montero, Sousa, Valenzuela; 2020], ...

and characterize instabilities via symmetries [Heckman, Hübner; 2024]

Steps

Analysis separates into steps (for IIA)

- Step 1: Fixed time t

  Fermionic Quivers & Chen-Ruan Orbifold Cohomology

- Step 2: Short time evolution $t + \delta t$

  Deformations & Cobordisms

- Step 3: Large time evolution $t + \Delta t$

  Interfaces & Deformations

With equivalent steps in IIB setup (time $\leftrightarrow$ scale). However in IIB there's an additional parameter:

- Step 4: Large / Small 't Hooft Coupling λ

  Intermediate-Value-Type Considerations for $0 \ll \lambda \ll \infty$

Plan of the Talk

1. Recall: Calabi-Yau Orbifold $\mathbb{C}^3/\mathbb{Z}_N$ in IIA
2. Contrast: Non-Susy Orbifold $\mathbb{R}^6/\mathbb{Z}_N$ in IIA
3. T-duality Related Setup: D3-brane Probes of $\mathbb{R}^6/\mathbb{Z}_N$ in IIB
4. Deform to Non-Perturbative Regime: $0 \ll \lambda \ll \infty$

Recall: Calabi-Yau Orbifold $\mathbb{C}^3/\mathbb{Z}_N$ in IIA

Consider the Crepant Resolution $\widetilde{X} \to X = \mathbb{C}^3/\mathbb{Z}_N$

Preserves 4D $\mathcal{N} = 2$ SUSY & introduces 2-/4-cycles

D2-/D4-branes $\to$ electric and magnetic particles $\to \mathcal{T}_X$ AD-type Theory

many many references ... for review see [Heckman, Rudelius; 2024]

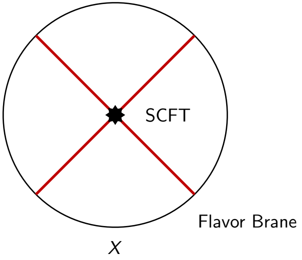
Geometry: Singularity Types

1) Codimension-6:

SCFT degrees of freedom localize

2) Codimension-4:

"Flavor 5-Branes"

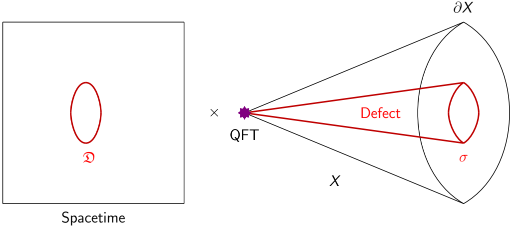


Recall: Calabi-Yau Orbifold $\mathbb{C}^3/\mathbb{Z}_N$ in IIA

Defect group from p-branes: [Del Zotto, Heckman, Park, Rudelius; 2015], [Morrison, Schäfer-Nameki, Willett; 2020], [Cvetič, Heckman, Hübner, Torres; 2022], ...

$$\mathbb{D} = \bigoplus_n \mathbb{D}^{(n)} \qquad \text{with} \qquad \mathbb{D}^{(n)} \cong \bigoplus_{p-k=n} \frac{H_{k+1}(X, \partial X)}{H_{k+1}(X)}$$

E.g., $\mathbb{D}^{(1)}$ are Lines á la Wilson and 't Hooft from D2-/D4-branes wrapped on non-compact cycles. In pictures:



Contrast: Non-Susy Orbifold $\mathbb{R}^6/\mathbb{Z}_N$ in IIA

Closed String Tachyon $\rightarrow$ No 4D CFT

Potential on scalar moduli space $\rightarrow$ rolling solution

Rolling is geometrized

[Adams, Polchinski, Silverstein; 2001], [Morrison, Narayan, Plesser; 2004]

$$\begin{array}{ccccccccc} & \nearrow & & \nearrow & & \nearrow & & & \\ \mathbb{R}^6/\Gamma & \rightarrow & \mathbb{R}^6/\Gamma' & \rightarrow & \mathbb{R}^6/\Gamma'' & \rightarrow & \dots & \rightarrow & \text{Susy Config.} = \begin{cases} \text{Flat Space} \\ \text{Susy Singularity} \end{cases} \\ & \searrow & & \searrow & & \searrow & & & \end{array}$$

Geometry: Singularity Types

1) Codimension-6: Tip of the Cone

2) Codimension-4: “5-Branes” of the form $\mathbb{R}^4/\mathbb{Z}_N$

3) Codimension-2: “7-Branes” of the form $\mathbb{R}^2/\mathbb{Z}_N$ (NEW)

Contrast: Non-Susy Orbifold $\mathbb{R}^6/\mathbb{Z}_N$ in IIA

Defect group from p-branes:

$$\mathbb{D} = \bigoplus_n \mathbb{D}^{(n)} \qquad \text{with} \qquad \mathbb{D}^{(n)} \cong \bigoplus_{p-k=n} H_k^{\mathrm{CR}}(S^5/\mathbb{Z}_N)$$

Sequence of Defect Groups as the Geometry decays

$$\begin{array}{ccccccccc} & \nearrow & & \nearrow & & \nearrow & & & \\ \mathbb{D} & \to & \mathbb{D}' & \to & \mathbb{D}'' & \to & \dots & \to & \text{Susy Defect Groups} \\ & \searrow & & \searrow & & \searrow & & & \end{array}$$

Contrast: Non-Susy Orbifold $\mathbb{R}^6/\mathbb{Z}_N$ in IIA

Defect group from p-branes: [Chen, Ruan; 2000]

$$\mathbb{D} = \bigoplus_n \mathbb{D}^{(n)} \qquad \text{with} \qquad \mathbb{D}^{(n)} \cong \bigoplus_{p-k=n} H_k^{\mathrm{CR}}(S^5/\mathbb{Z}_N)$$

Alternative Derivation / Check:

charge lattice via wrapped fractional branes, entire basis of fractional branes already access by D0-brane probe, D0-brane probe Quiver QM, Fermionic Adjacency Matrix → Dirac Pairing → Defect Group

susy analysis: [Closset, Del Zotto, 2022], [García Etxebarria, Del Zotto; 2022]

Sequence of Defect Groups as the Geometry decays

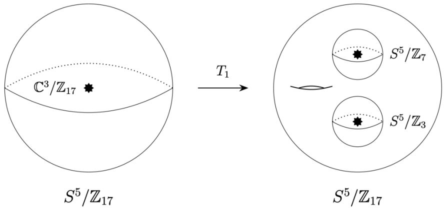
$$\begin{array}{ccccccccc} & \nearrow & & \nearrow & & \nearrow & & & \\ \mathbb{D} & \rightarrow & \mathbb{D}' & \rightarrow & \mathbb{D}'' & \rightarrow & \dots & \rightarrow & \text{Susy Defect Groups} \\ & \searrow & & \searrow & & \searrow & & & \end{array}$$

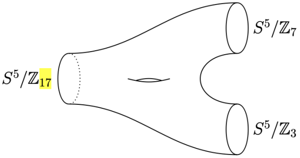
Example: $\mathbb{R}^6/\mathbb{Z}_{17}$ in IIA

Consider IIA on the purely geometric background $X = \mathbb{R}^6/\mathbb{Z}_{17}^{(4,6,10,14)}$

$\rightarrow$ Isolated Singularity with link $S^5/\mathbb{Z}_{17}$

Tachyon driven resolution: $\mathbb{R}^6/\mathbb{Z}_{17}$ $\nearrow \mathbb{C}^2/\mathbb{Z}_3$, $\searrow \mathbb{C}^2/\mathbb{Z}_7$ $\qquad$ $\mathbb{D}^{(1)} \cong \mathbb{Z}_{17}^2$ $\nearrow \mathbb{Z}_3^2$, $\searrow \mathbb{Z}_7^2$



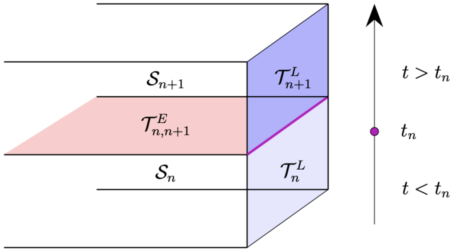


Beyond Defect Groups

Tachyonic instabilities result in a pulsing off of Bordisms

GeoEng of Symmetry Theories: Bordisms → SymTFT interfaces

[Cvetič, Donagi, Heckman, Hübner, Torres; 2024], [Baume, Heckman, Hübner, Torres, Turner, Yu; 2023]



Comment: Instantaneous Pulse Approximation

D3-brane Probes of $\mathbb{R}^6/\mathbb{Z}_N$ in IIB

D0-brane probe of $\mathbb{R}^6/\mathbb{Z}_N$ in IIA $\leftrightarrow$ D3-brane probe of $\mathbb{R}^6/\mathbb{Z}_N$ in IIB

Assumption: Suitable limit to decouple the tachyonic closed string modes, leaving a 4D gauge theory defined by just the open string sector.

Infer (at small 't Hooft coupling) results on the D3-brane Quiver Gauge Theory.

- Radius of $X \sim$ Energy $\Rightarrow$ Bordisms along RG flow
- Defects map across T-duality

D3-brane Probes of $\mathbb{R}^6/\mathbb{Z}_N$ in IIB

Idealized Setup:

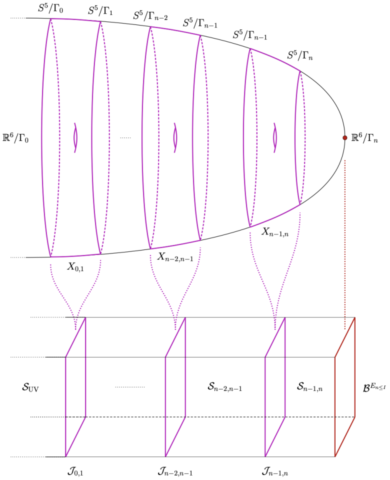
$$\mathbb{R}^6/\Gamma_0 \to \mathbb{R}^6/\Gamma_1 \to \cdots \to \mathbb{R}^6/\Gamma_n$$

with boardisms $X_{i,i+1}$ with

$$\partial X_{i,i+1} = S^5/\Gamma_i - S^5/\Gamma_{i+1}$$

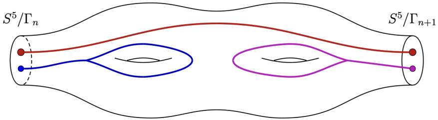
which map to interface theories

$$\mathcal{J}_{i,i+1}\,.$$



D3-brane Probes of $\mathbb{R}^6/\mathbb{Z}_N$ in IIB

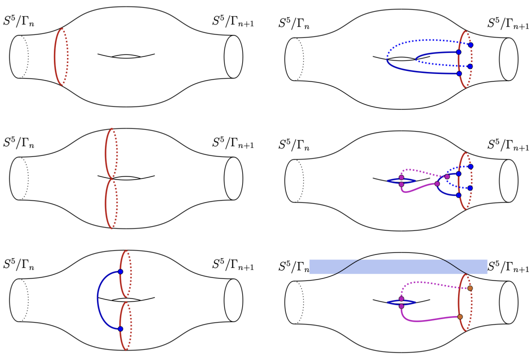
Interfaces describe degrees of freedom which decouple. Defects are removed, added, or remain:



The interface theory is IIB reduced on the manifold with boundary given by the bordism! [Cvetič, Donagi, Heckman, Hübner, Torres; 2024], [Baume, Heckman, Hübner, Torres, Turner, Yu; 2023]

D3-brane Probes of $\mathbb{R}^6/\mathbb{Z}_N$ in IIB

Symmetry operators are deformed between boundaries $S^5/\Gamma_n \to S^5/\Gamma_{n+1}$:



Summary (Up to Now)

We considered:

- Non-susy Orbifold $\mathbb{R}^6/\Gamma$ in IIA
- D3-brane Probes of Non-susy Orbifold $\mathbb{R}^6/\Gamma$ in IIA (at small 't Hooft coupling)

As a function of time / scale, the effective geometry changed:

$$\mathbb{R}^6/\Gamma \underset{\searrow}{\overset{\nearrow}{\rightarrow}} \mathbb{R}^6/\Gamma' \underset{\searrow}{\overset{\nearrow}{\rightarrow}} \mathbb{R}^6/\Gamma'' \underset{\searrow}{\overset{\nearrow}{\rightarrow}} \; \dots \; \rightarrow \text{Susy Config.} = \begin{cases} \text{Flat Space} \\ \text{Susy Singularity} \end{cases}$$

Bordisms mapped to Euclidean / Lorentzian symmetry theory interfaces.
Geometric analysis matched by Quiver analysis.

We made step function approximations.

Summary (Up to Now)

We considered:

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Bordisms mapped to symmetry theory interfaces. Geometric analysis matched by Quiver analysis.

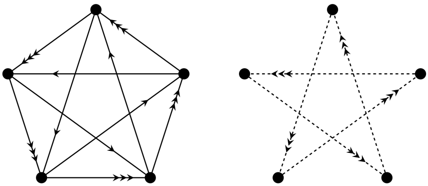
We made step function approximations.

Revisiting D3-brane Probes

Weak coupling $\lambda \ll 1$

[Kachru, Silverstein; 1997], [Nekrasov, Lawrence, Vafa; 1998], [Adams, Silverstein; 2001], [Dymarsky, Klebanov, Roiban; 2004], [Pomoni, Rastelli; 2008], ...

$\mathcal{N} = 0$ Quiver Gauge Theory

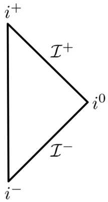


closed string tachyon, $\beta_{\text{single-trace}} = 0$, $\beta_{\text{double-trace}} \neq 0$

probed geometry is time-dependent

Revisiting D3-brane Probes

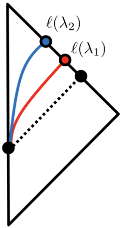
Characterize time-dependence via Penrose-Diagram:



with coordinates T, R. T is time, and R is radius of internal space X.

Revisiting D3-brane Probes

Watch the instability evolve:



where tachyonic mass m and 't Hooft coupling are related by

$$\alpha' m^2 = \kappa_\Gamma \lambda - \xi_\Gamma$$

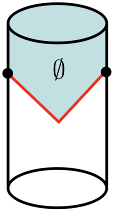
with $O(1)$ constants $\kappa_\Gamma, \xi_\Gamma$. Note $\lambda \ll 1$, so increasing λ (while keeping it small), decreases m, the instability pulse slows down: $\lambda_1 < \lambda_2$ in Fig.

Revisiting D3-brane Probes

Strong coupling $\lambda \gg 1$

[Horowitz, Orgera, Polchinski; 2007], [Ooguri, Vafa; 2016], [García Etxebarria, Montero, Sousa, Valenzuela; 2020]

Attempt near horizon limit, find bubble of nothing

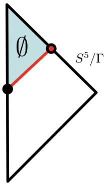


Instability nucleated at some finite time.

Revisiting D3-brane Probes

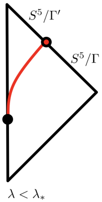
Strong coupling $\lambda \gg 1$

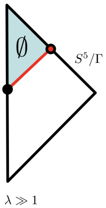
Away from near horizon limit, still bubble of nothing



Revisiting D3-brane Probes

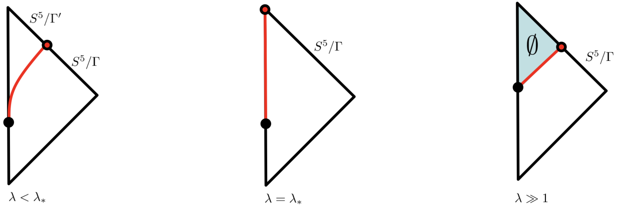
"Intermediate-Value-Theorem": $\lambda \ll 1 \rightarrow \lambda \gg 1$





Revisiting D3-brane Probes

"Intermediate-Value-Theorem": $\lambda \ll 1 \to \lambda \gg 1$



There exists λ_* at which one stability transitions into the other. This is compatible with potentials flattening out, symmetries read off from boundary being preserved, the occurrence of a non-susy D3-brane CFT.

Summary

We considered 2 non-susy setups

- IIA on $\mathbb{R}^6/\Gamma_{\text{non-susy}} \times \mathbb{R}^{1,3}$
- IIB on $\mathbb{R}^6/\Gamma_{\text{non-susy}} \times \mathbb{R}^{1,3}$ with D3-brane probes

and characterized the topological symmetry structures as instabilities evolved.

In the IIB setup, we argued that the instabilities at small and large 't Hooft coupling are of different type. "Intermediate-Value-Theorem"-type arguments then resulted in a scenario where at finite 't Hooft coupling, but away from any perturbative regime, the different instabilities cancel against another.

Compare with "Holomorphy".

Outlook

All of our analysis was in very specific local models.

Goal 1: Widen Construction [Chakrabhavi, Braeger, Heckman, Hübner; WIP]

Goal 2: Return to a global model.

Dream: Construct finely tuned stable non-susy QG vacua from D3-branes + Singularities.

Goal 3: Explore Intermediate-Value-Type arguments between perturbative limits.

Thanks!

Thank you for your time!